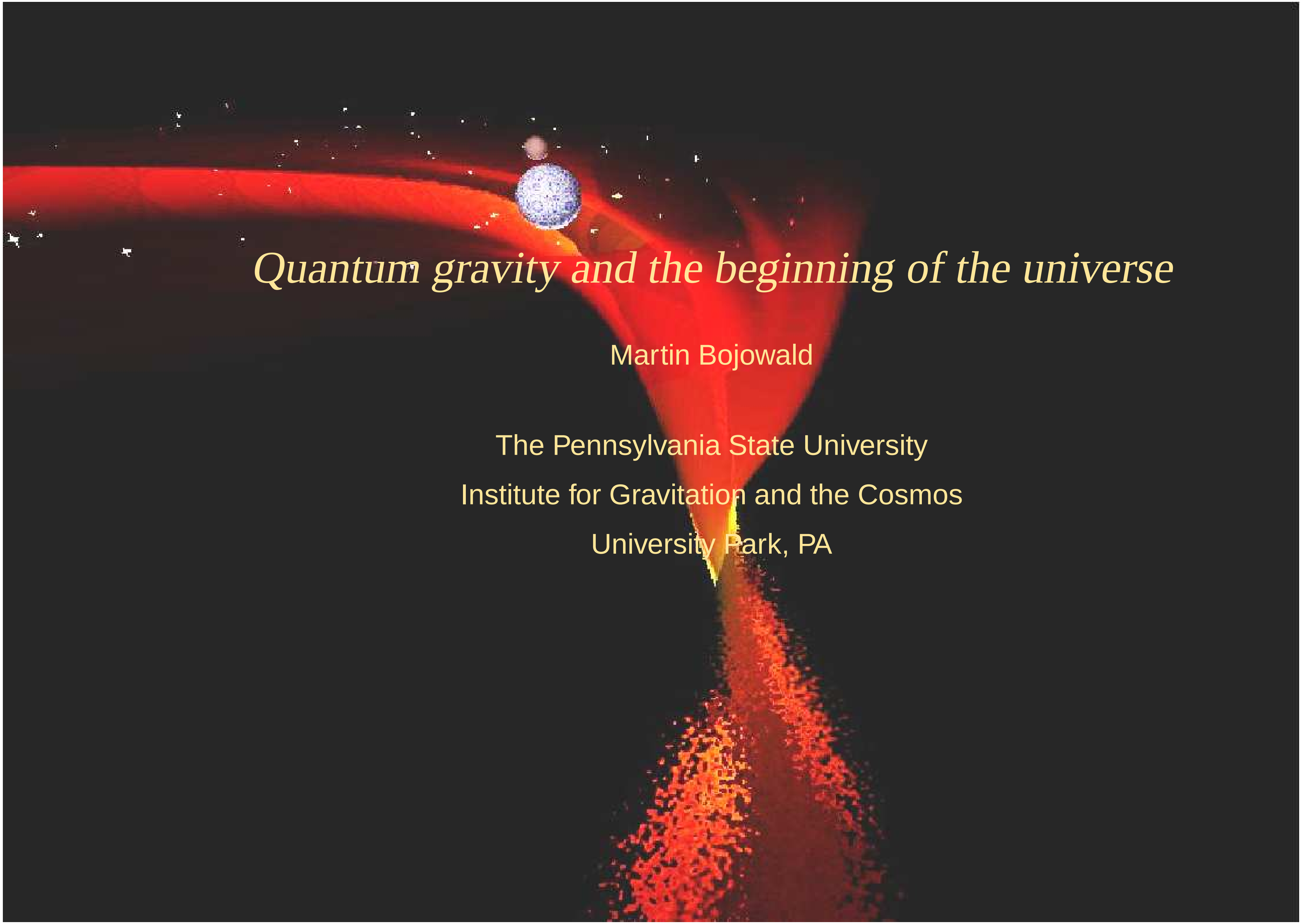

Quantum gravity and the beginning of the universe

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Problem/arrow of time

Quantum space-time: Non-classial degrees of freedom crucial for concepts of space and time?

Several indications: Squeezing of quantum matter (or gravitational wave) state, growth of quantum correlations.

[Gasperini, Giovannini; Kruczenski, Oxman, Zaldariagga; Koks, Matacz, Hu; Kiefer, Polarski, Starobinsky]

Here: Explore relation to quantum gravity state. Emergent concept of time in quantum description of universe models.

[Based on MB, R. Tavakol: PRD 78 (2008) 023515.]

Friedmann equation

Free, massless scalar as matter:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{4\pi G}{3}\frac{p_\phi^2}{a^6} + \Lambda$$

with momentum p_ϕ of ϕ. (Here, $k = 0$, $\Lambda < 0$ to be specific.)

Canonical formulation: $V = a^3/4\pi G$, $P = \dot{a}/a$, $\{V, P\} = 1$

$$p_\phi = \pm\sqrt{24\pi G}\, V\sqrt{P^2 - \Lambda} =: H(V, P)$$

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Take $H(V, P)$ as Hamiltonian for evolution in ϕ, not proper time.

Quadratic $H \propto |VP|$ for $k = 0 = \Lambda$: harmonic cosmology.
Exactly solvable quantum system; *no quantum back-reaction.*

Perturbation theory for $\Lambda \neq 0$ or with matter potential.

Quantum variables

Parameterize (density) state by expectation values $V = \langle \hat{V} \rangle$, $P = \langle \hat{P} \rangle$ and moments

$$G^{\overbrace{V \cdots V}^{a} \overbrace{P \cdots P}^{b}} = \langle (\hat{V} - \langle \hat{V} \rangle)^a (\hat{P} - \langle \hat{P} \rangle)^b \rangle_{\mathrm{Weyl}}$$

Form infinite dimensional phase space.

Subject to constraints, such as uncertainty relation

$$G^{VV} G^{PP} - (G^{VP})^2 \geq \frac{\hbar^2}{4}$$

Especially interesting: covariance $G^{VP} = \frac{1}{2} \langle \hat{V} \hat{P} + \hat{P} \hat{V} \rangle - VP$ as a measure for squeezing.

Quantum evolution

Don't take detour of solving for state first, then computing expectation values. Instead: Evolution of quantum variables.

$$\frac{\mathrm{d}O}{\mathrm{d}\phi} = \frac{\langle[\hat{O},\hat{H}]\rangle}{i\hbar}$$

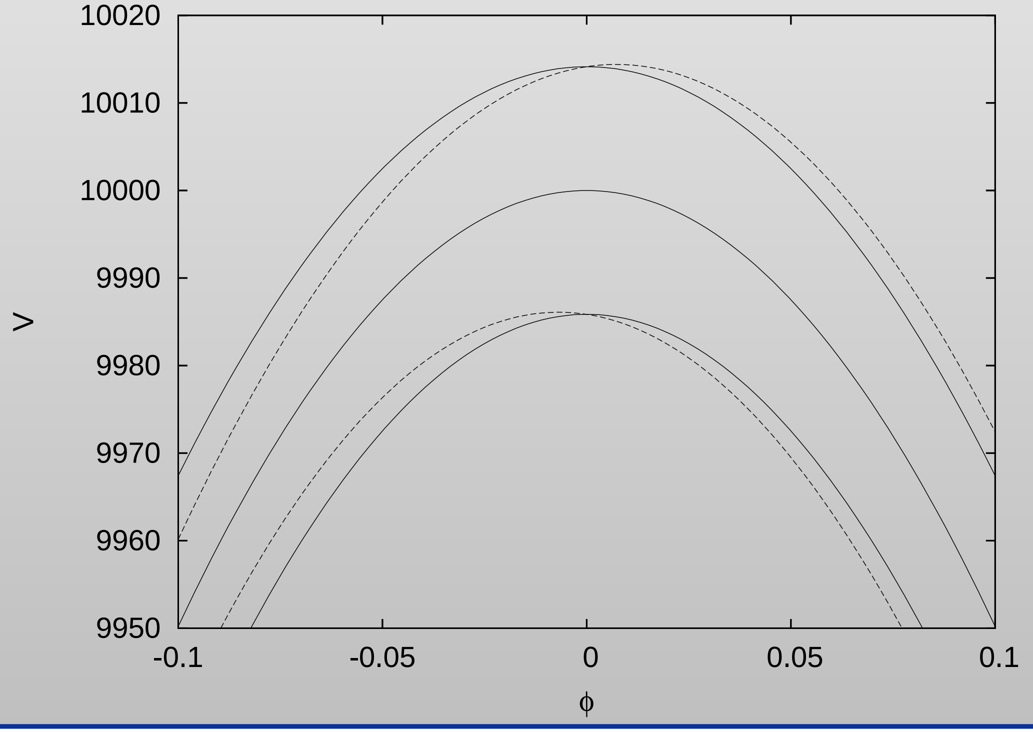
expanded as

$$\begin{aligned}
\frac{\mathrm{d}V}{\mathrm{d}\phi} &= \frac{3}{2}\frac{VP}{\sqrt{P^2+|\Lambda|}} - \frac{9}{4}|\Lambda|\frac{VP}{(P^2+|\Lambda|)^{5/2}}G^{PP} \\
&\quad + \frac{3}{2}|\Lambda|\frac{G^{VP}}{(P^2+|\Lambda|)^{3/2}} + \cdots \\
\frac{\mathrm{d}P}{\mathrm{d}\phi} &= -\frac{3}{2}\sqrt{P^2+|\Lambda|} - \frac{3}{4}|\Lambda|\frac{G^{PP}}{(P^2+|\Lambda|)^{3/2}} + \cdots
\end{aligned}$$

Monotonicity

$$\frac{\mathrm{d}G^{VP}}{\mathrm{d}\phi} = \frac{3}{2}|\Lambda|\frac{V}{(P^2+|\Lambda|)^{3/2}}G^{PP}$$

with positive $G^{PP} = (\Delta P)^2$. Positivity of fluctuations, uncertainty implies fixed tendency for correlations.



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So far: free ϕ, monotonic. With potential $W(\phi)$:

$$\begin{aligned}\frac{\mathrm{d}G^{VP}}{\mathrm{d}\phi} &= \frac{3}{2}\frac{V(|\Lambda|-8\pi GW(\phi)/3)}{(P^2+|\Lambda|-8\pi GW(\phi)/3)^{3/2}}G^{PP} \\ &\quad +\frac{\pi GW(\phi)(P^2+|\Lambda|-2\pi GW(\phi))}{(P^2+|\Lambda|-8\pi GW(\phi)/3)^{3/2}}G^{VV}\end{aligned}$$

G^{VP} still monotonic at least for small P.

Before the big bang

Loop quantum cosmology: higher order terms to Friedmann equation.

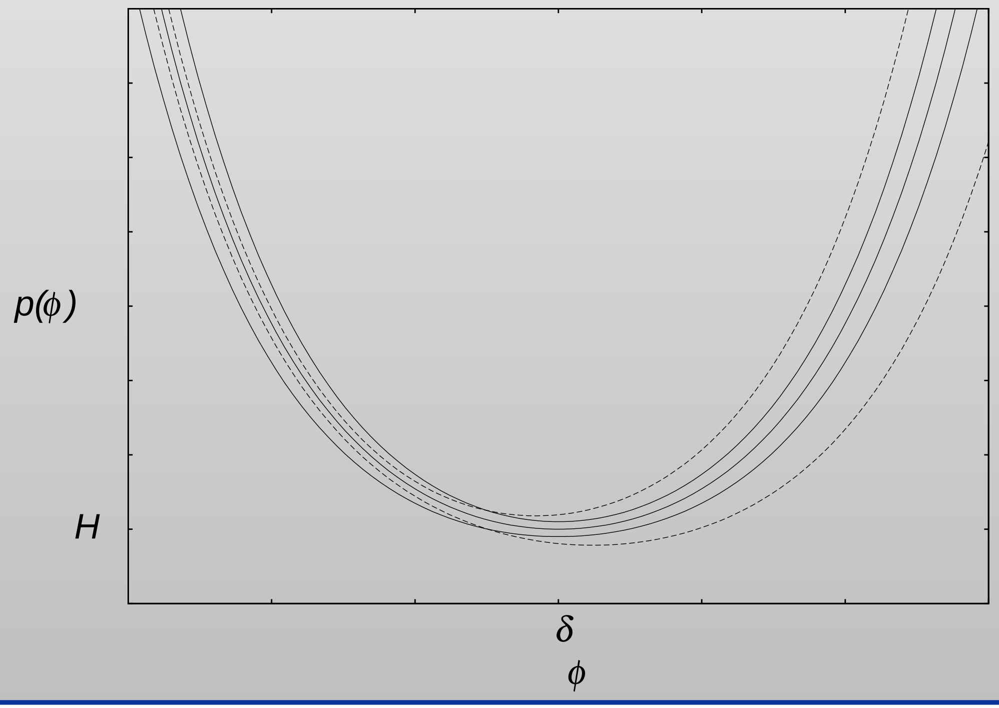
$$\frac{\sin^2(\mu P)}{\mu^2} = \frac{1}{12\pi G}\frac{p_\phi^2}{V^2}$$

Hamiltonian for ϕ-evolution not quadratic in V and P even for $k = 0 = \Lambda$: non-perturbative at strong curvature $P \gg 1/\mu$.

But "resummable": solvable in variables V, $J = V\exp(iP)$.

$\mathrm{Re}J = \langle\hat{V}\cos(\hat{P})\rangle$
$\approx -\langle\hat{V}(\hat{P} - \pi/2)\rangle$

Suggests constant correlations.



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Behavior of covariance

Started with proper time or internal time ϕ, but covariance serves as single-valued time variable more generally: even through turning points of V and ϕ.

Same behavior in expanding or contracting universe phase. No strong change through big bang (?).

Would require quantum gravity to formulate evolution.