THE MINIMAL GEOMETRICAL ONTOLOGY FOR A FUNDAMENTAL THEORY

Julian Barbour

"He that attempts natural philosophy without geometry is lost." Galileo 1632

"I am inclined to believe ... that four-dimensional symmetry is not a fundamental property of the physical world."

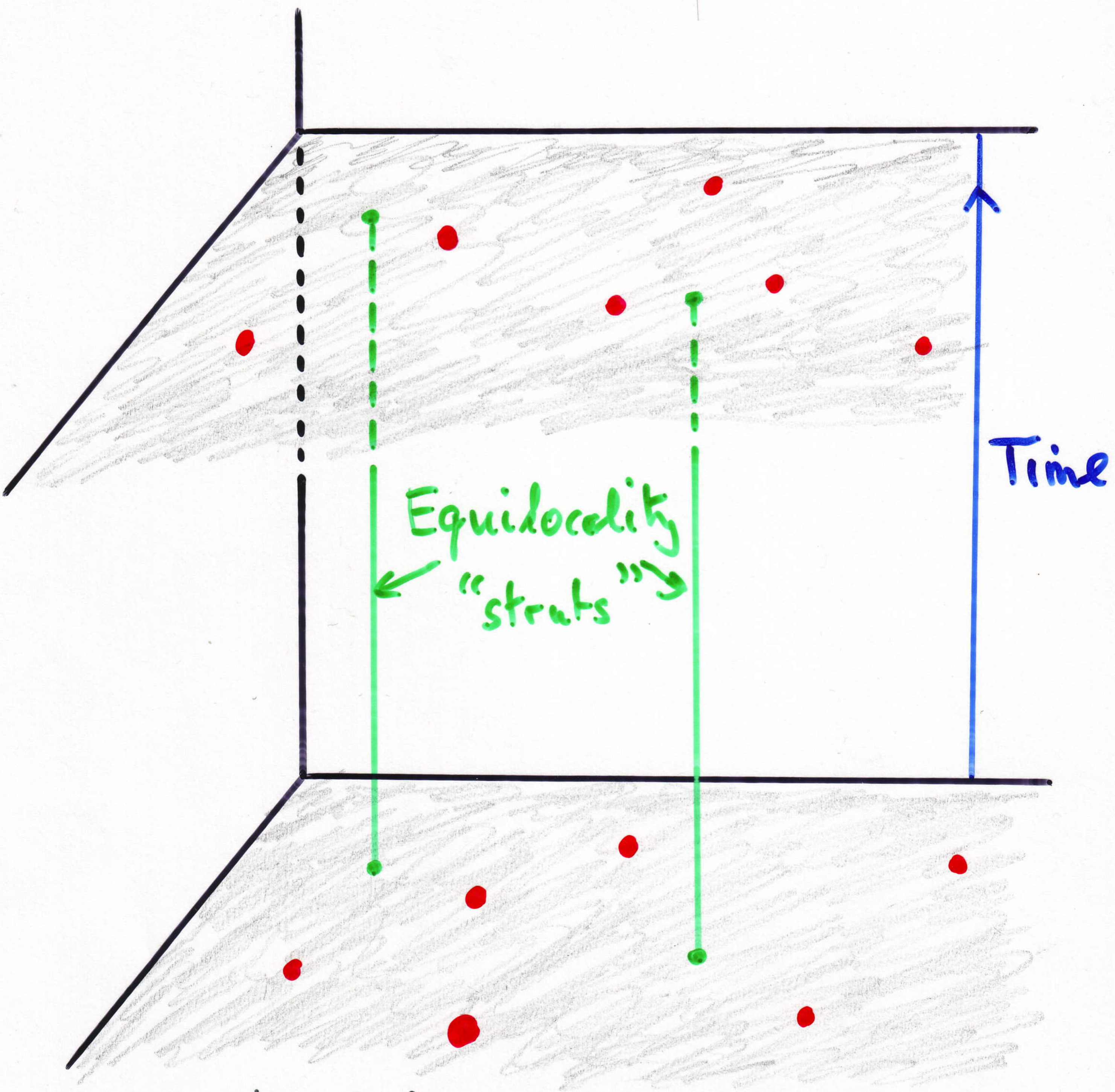
Dirac 1958

What is motion?

OVERVIEW

1. Elimination of redundant Newtonian kinematics by making displacement, time, and size relative. Global groups.
2. Machian interpretation of Einsteinian kinematics in GR using local symmetry groups.
3. Replacement of foliation invariance in GR by 3-dimensional conformal invariance
4. Conclusions/Comments. The argument for continuous conformal 3-geometry as minimal geometrical ontology.
5. The arrow of time – if time permits.

NEWTONIAN KINEMATICS



Newtonian action depends on:

1. Overall rotation in absolute space.
2. Absolute time difference.
3. Absolute scale.

ABSOLUTE & RELATIVE CONFIGURATION SPACES

$$\frac{\mathcal{A}}{\text{SymmetryGroup}} := \mathcal{R}$$

Global G

$$\frac{\text{Newtonian } Q}{\text{Translations \& Rotations}} := \text{Euclidean RCS}$$

$$\frac{\text{Euclidean RCS}}{\text{Dilatations}} := \text{Shape Space}$$

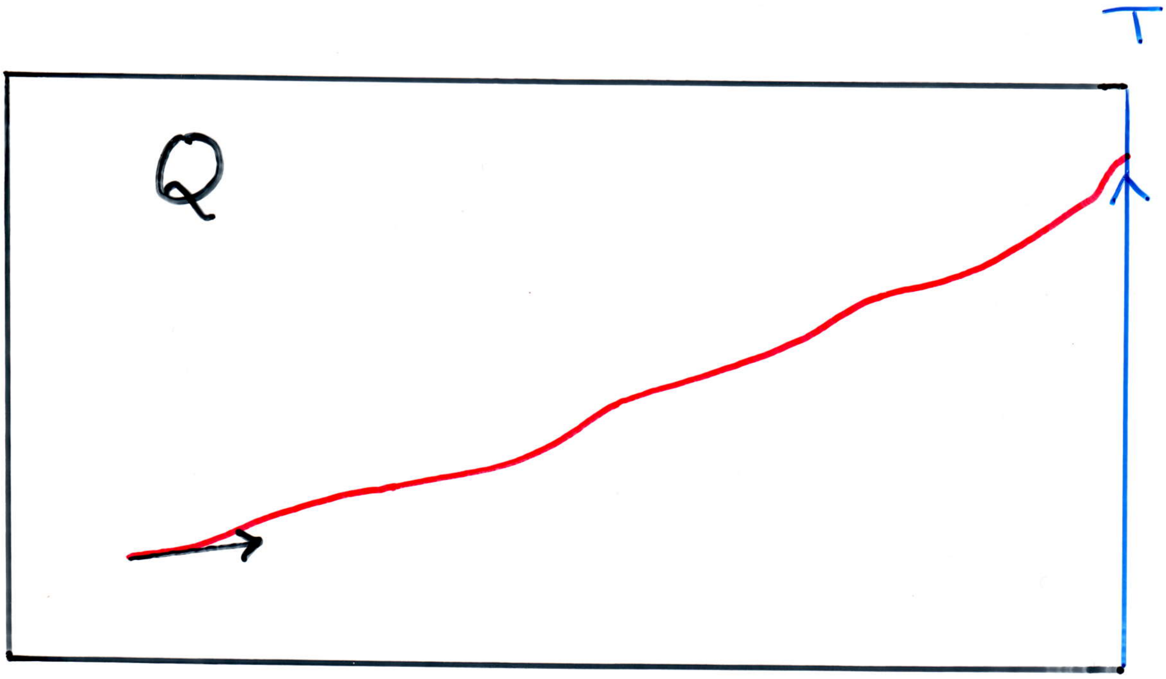
Local Group

$$\frac{\text{Riem}}{\text{3-Diffeomorphisms}} := \text{Superspace}$$

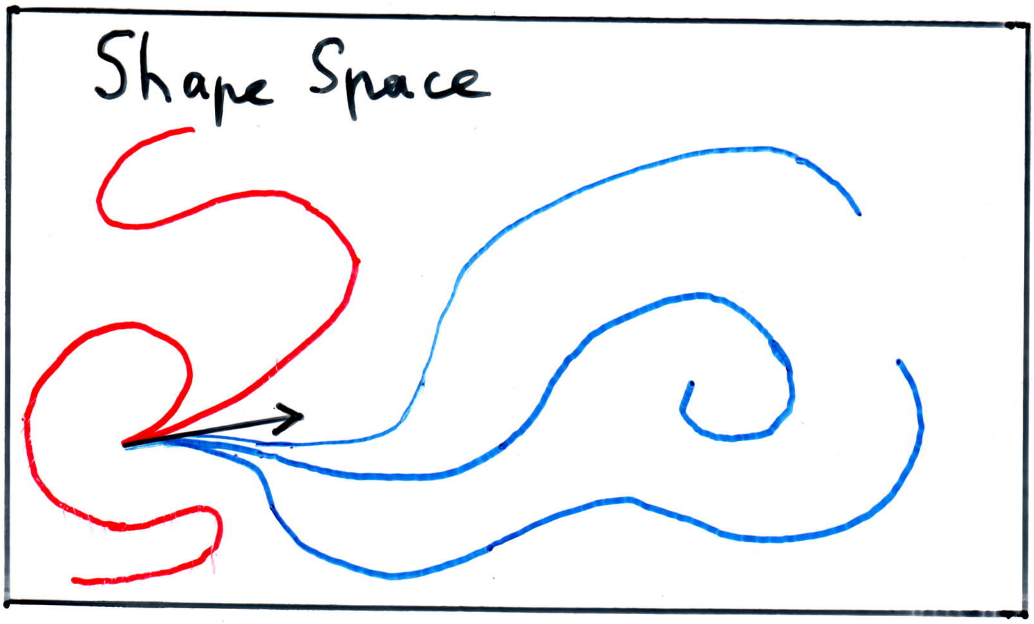
$$\frac{\text{Superspace}}{\text{VPCT}} := \text{CS} + \text{Volume}$$

$$\frac{\text{Superspace}}{\text{CT}} := \text{Conformal Superspace}$$

NEWTONIAN INITIAL-VALUE PROBLEM



Initial point and initial direction in QT determine a unique evolution.

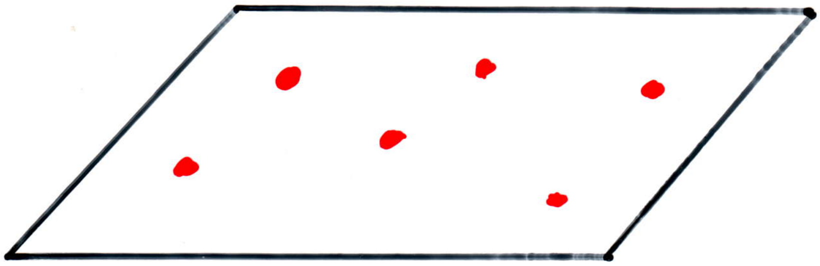


A 5-parameter family of Newtonian solutions all pass through the same initial point and direction in Shape Space.

MACH'S PRINCIPLE

An initial point and direction
in $\mathcal{R}$
must determine a unique evolution

TIMELESS BEST MATCHING

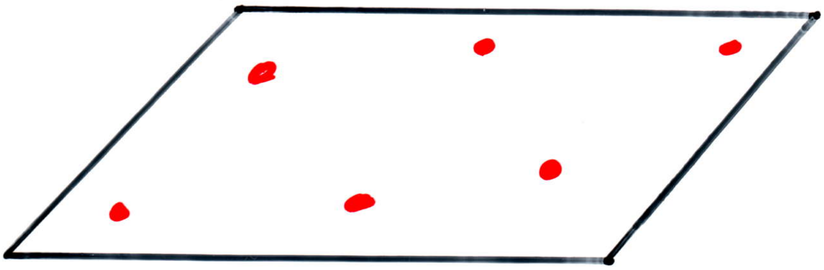


$x_a^i + \delta x_a^i$

x_a^i

$i = 1, 2, 3$

$a = 1, \ldots, N$



Minimize $\sqrt{WT} = \sqrt{W(x) \sum_{i,a} \frac{m_a}{2} \delta x_a^{i\,2}}$

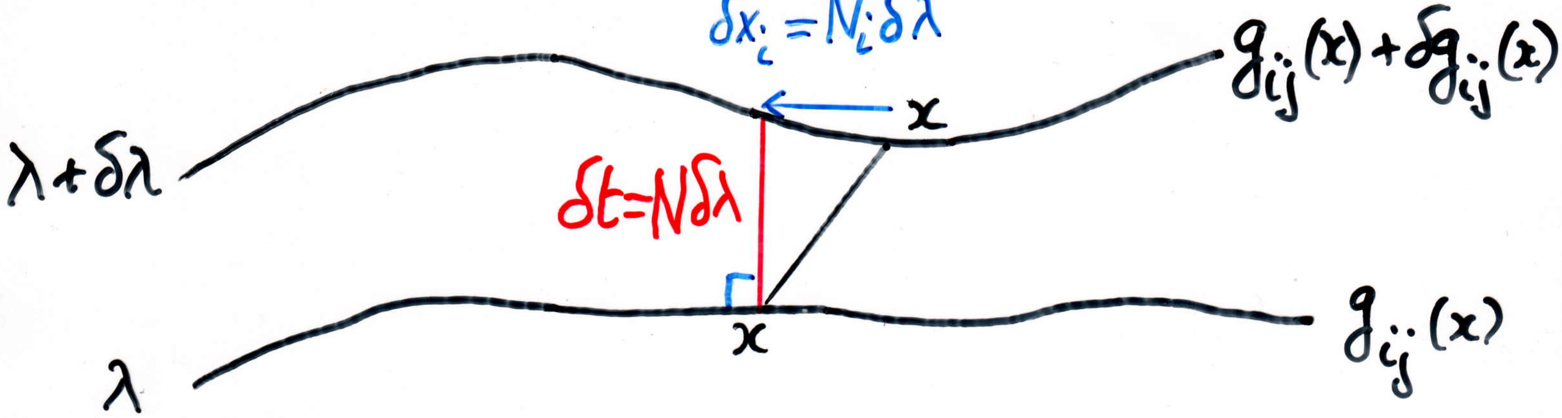
wrt translations, rotations, & dilatations.

Yields geodesics on Shape Space corresponding to Newtonian solutions with $P = 0$, $L = 0$, $D = \sum x_a \cdot p_a = 0$

Emergent Newtonian time $\delta t = \sqrt{\dfrac{\sum \frac{m_a}{2} \delta x_a^{i\,2}}{W}}$

$W = E - V$ for Euclidean RCS

EINSTEINIAN KINEMATICS



g_{ij}:
- 3 coordinate degrees of freedom
- 2 conformal " " " } Evolving
- 1 volume degree " " } 3-Geometry

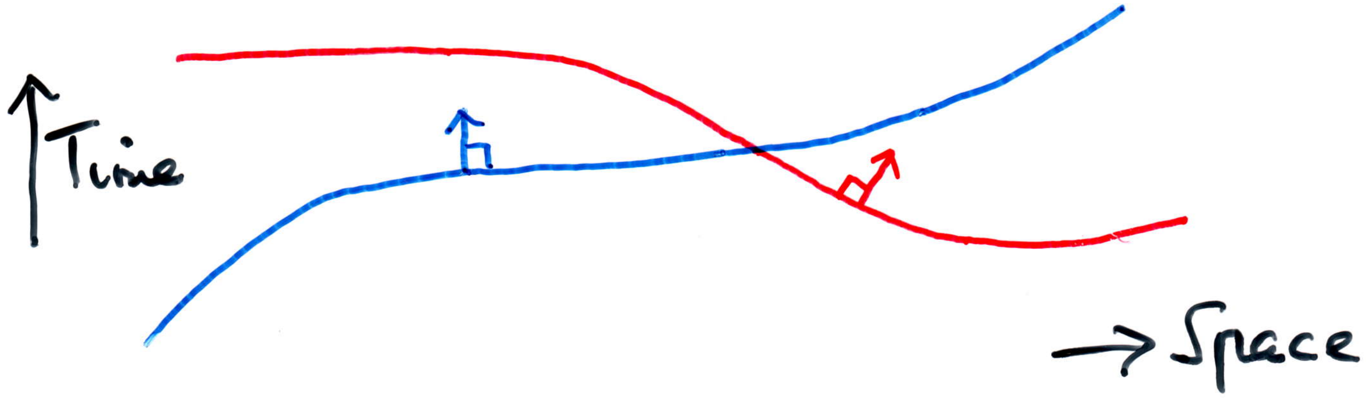
Lapse N and Shift N_i encode Equilocality & Local Proper Time

1. How much of spacetime is real?

2. What did Dirac mean?

The fact that <u>Space evolves</u> breaks the symmetry between space and time.

FOLIATION INVARIANCE



g_{ij}, p^{ij} canonical coordinates

$$\mathcal{H} = \int \sqrt{g}\,(NH + N_i H^i)\, d^3x$$

$$gH = -p^{ij}p_{ij} + \frac{1}{2}p^2 + gR = 0$$

$$\sqrt{g}\, H^i = p^{ij}{}_{;j} = 0$$

What are the two **true** degrees of freedom?

Dirac added gauge condition $p = p^{ij} g_{ij} = 0$

York's CMC condition: $\dfrac{2p}{3\sqrt{g}}$ = spatial constant

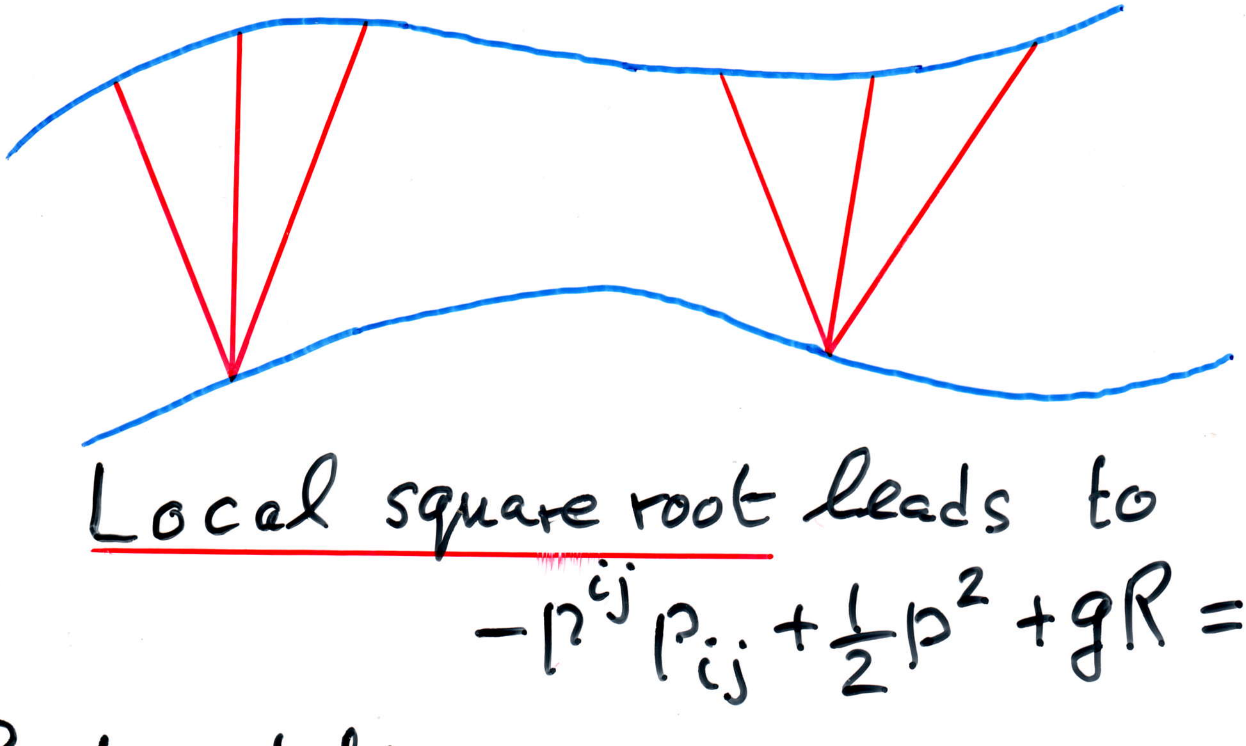
Particle Action $A = 2\int d\lambda \sqrt{(E-V)T}$

$$T = \sum_{ij,\, a\alpha} \frac{m_a}{2}\left(\frac{dx^i_a}{d\lambda} - \dot{\varepsilon}^\alpha \omega^{\alpha i}_{\ \ j} x^j_a\right)\left(\frac{dx^i_a}{d\lambda} - \dot{\varepsilon}^\alpha \omega^{\alpha i}_{\ \ j} x^j_a\right)$$

BAIERLEIN-SHARP-WHEELER ACTION

$$A_{BSW} = \int d\lambda \int \sqrt{g}\sqrt{R}\sqrt{T}\, d^3x$$

$$T = \left(g^{ac}g^{bd} - g^{ab}g^{cd}\right)\left(\frac{\partial g_{ab}}{\partial\lambda} - \dot{\xi}_{(a;b)}\right)\left(\frac{\partial g_{cd}}{\partial\lambda} - \dot{\xi}_{(c;d)}\right)$$



Local square root leads to

$$-p^{ij}p_{ij} + \frac{1}{2}p^2 + gR = 0$$

Best matching leads to $p^{ij}{}_{;j} = 0$

Consistency leads to A_{BSW}

CONFORMAL BEST MATCHING

$$g_{ij} \rightarrow \hat{\phi}^4 g_{ij}, \quad \hat{\phi} = \frac{\phi \int d^3x \sqrt{g}}{\int d^3x \sqrt{g}\, \phi^6}$$

$$A_{BSW} \rightarrow \int d\lambda \int d^3x \sqrt{g}\, \hat{\phi}^4 \sqrt{R - \frac{8\nabla^2\hat{\phi}}{\hat{\phi}}} \sqrt{\hat{T}}$$

$$\hat{T} = \hat{\phi}^{-8} \left(g^{ac} g^{bd} - g^{ab} g^{cd} \right) \frac{\partial(\hat{\phi}^4 g_{ab})}{\partial\lambda} \frac{\partial(\hat{\phi}^4 g_{cd})}{\partial\lambda}$$

Action contains $\frac{\partial\phi}{\partial\lambda}$ and ϕ

Conformal best matching leads to

$\frac{2p}{\sqrt{g}}$ = spatial constant : York's CMC cdn.

$NR - \nabla^2 N + \frac{Np^2}{4} = D$: Lapse-fixing cdn

Local inertial frames, local proper time and local proper distance are all dynamically emergent.

CONCLUSIONS / COMMENTS

Discreteness

1. Continuous symmetry groups acting on continuous spaces in GR & gauge theory.
2. Difficulty of inverse problem.
3. Is discreteness dictated by QM?

Conformal Geometry

4. Tantalizingly close to complete understanding of GR as implementing Machian relativity of Time, Motion, and Size.
5. Hořava gravity.

Arrow of Time

6. Is arrow of time a consequence of the built-in asymmetry of the Universe's configuration space?