Through a Glass, Darkly

To illuminate the murky 10-dimensional universe of string theory, Tevian Dray and Corinne Manogue are turning to a bizarre eight-dimensional set of numbers.

by ANIL ANANTHASWAMY

FQXi Awardees: Tevian Dray & Corinne Manogue — January 16, 2009

Ask Tevian Dray and Corinne Manogue what it is like to be a married couple working towards a unified theory of fundamental particles and they'll tell you to ask their children. "They have been known to complain about the dinner-table conversation," says Dray.

Manogue puts it in context: "Tevian is very much the mathematician and I'm very much the physicist. I have a tendency to see the physics that we are striving for, but through a glass, darkly," she says. "I have some sense of where we want to go, but it is cloudy, and kind of befuddled. The first thing that happens is I say, 'we want to do this.' His reaction is, 'I have no idea what you are saying.' And so we go through a very tumultuous period, where he is trying to get me to articulate clearly enough what I mean so that he can do [the mathematics]. It's typically a loud and frustrating time. At the dinner table, most often."

Together Dray and Manogue are trying to tackle a profound question in physics: Why is our universe described so well by the standard model of particle physics? The standard model works in four dimensions—three of space and one of time—and has been extremely successful at explaining how elementary particles interact with each other. And yet there are vagaries that the standard model can't make sense of, such as why these particles have the masses that they do, or why they group together in families of three with similar properties, but different masses. Dray and Manogue, who are both at Oregon State University in Corvallis, are convinced that the answer lies in the mathematics of higher dimensions—no less than 10 dimensions, in fact.


BALANCING FAMILY AND PHYSICS
Tevian Dray (far left) and Corinne Manogue (far right)

> It's typically loud and frustrating. At the dinner table, most often.
>
> *- Corinne Manogue on working with husband Tevian Dray*

If the idea that the universe contains 10 dimensions sounds familiar, it's because it's often bandied about by string theorists. In the mid-1980s, superstring theory was going through a revolution. Physicists had developed equations to describe fundamental particles as strings vibrating in 10 dimensions. But these equations were extremely difficult to solve. At the time, Manogue was working with David Fairlie of Durham University in the UK, and together they realized that a bizarre system of numbers, called the *octonions*, could come to the rescue.

Strange Brood

Octonions are a strange brood, forming an eight-dimensional number system (see "The Crazy Old Uncle of Algebra," page 2). By contrast, the lovable real numbers that we're all comfortable with live in one dimension—that is, they can be written out along a one-dimensional number line; while the complex numbers that some of the more mathematically-inclined dabble with, make up a two-dimensional number system.

In the standard model, particles can be split into *fermions*, which make up matter, and *bosons*, which are associated with the fundamental forces of nature. Fairlie discovered that octonions are

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handy for writing out the equations for how bosons move. Working with Fairlie and with Anthony Sudbury of the University of York, UK, Manogue later discovered that the very same octonions could also be used to describe the behavior of fermions.

Using octonions, Manogue and Dray can now describe the electrons and their cousins, the *muons* and *tau* particles, and also the neutrinos in 10 dimensions. It's a fantastic achievement, but Dray emphasizes that there's still a long way to go. "What we cannot do in our language at all is have them do anything other than sit there," says Dray. "If I stand up in front of a physics audience and say, 'here's my electron, and by the way, I don't yet even know how it interacts with electric fields,' I'll get laughed at."

That's exactly the objection that David Fairlie raises about the work. "There is no answer to questions of particle interactions," he says. He points out that the peculiar mathematics of octonions introduces new problems. In particular, octonions are *non-associative* (see sidebar). However, all known physical processes are associative, so using octonions to characterise particle interactions will be tricky, Fairlie says.

Despite this stumbling block, Manogue and Dray continue to plug away. They have used their octonions to encode the momentum and spin properties of these particles, explain why neutrinos are "left-handed" (that is, why the neutrinos' spins are always oriented in one particular sense relative to the direction in which they move and never in the opposite sense), and even provide clues to why the particles cluster into families of three. These properties seem to be inherent in the language of octonions.

"Dray and Manogue are among the few really good physicists who think hard about the octonions and what they might mean for physics," says John Baez, a mathematical physicist at the University of California, Riverside. "As far as I'm concerned, these questions remain mysteries. But Dray and Manogue have found some tantalizing clues."

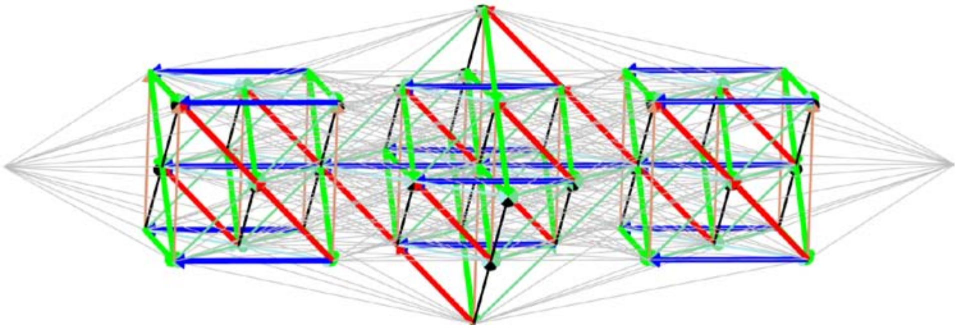
The next step—using a $51,393 grant from FQXi—is to try to use octonions to identify quarks and also to figure out how particles get their charge. "At that stage we might be able to make some experimentally verifiable predictions, like there is no Higgs," says Manogue.

Their ultimate goal is to show that the standard model is just a natural consequence of describing the fundamental particles in 10 dimensions. It if works, octonions could also help solve one of the biggest puzzles facing string theorists: How their hypothetical six extra dimensions of space are folded up so that we only experience four-dimensions in our universe.

Collapsing Dimensions

Currently, string theorists have an infinity of possibilities for how this folding might happen. But Manogue and Dray have discovered that choosing one particular octonion to focus on from their arsenal, while neglecting the rest, "collapses" the 10 dimensions down to four dimensions, in a simple way. Interestingly, it doesn't matter which octonion you choose, you always end up with a working four-dimensional universe. "If this octonionic stuff is right, it tells you uniquely what to do with the extra dimensions and how to handle them," says Manogue.

Octonions may also be hinting at another deep truth about the structure of the universe. In 10 dimensions, octonions can be used to describe a particle's momentum, but not its position. But after the description is collapsed down from 10 to four dimensions, particles can be described in both ways. This may suggest that spacetime isn't a fundamental property of the universe, but only emerges in its four-dimensional description. "That would be incredibly profound, I think," says Manogue.



SPOT THE QUARK

The geometric structure F_4, pictured here, could one day help us visualize how the eight-dimensional octonions describe quarks in our 4-D world.

The Crazy Old Uncle of Algebra

Octonions may be the key to understanding the universe, but what are they? There are four number systems in mathematics, each spanning a different number of dimensions, in which division is possible:

- real numbers (1 dimension);
- complex numbers (2 dimensions);
- quaternions (4 dimensions);
- octonions (8 dimensions).

John Baez at the University of California, Riverside, has written that, "The real numbers are the dependable breadwinner of the family, the complete ordered field we all rely on. The complex numbers are a slightly flashier but still respectable younger brother: not ordered, but algebraically complete."

Quaternions are weirder still because they are *noncommutative*. In mathematics, commutativity is the ability to change the order of terms in your mathematical operation without changing the end result. So, for example, using real numbers, multiplication is commutative because 1 x 2 is the same as 2 x 1. Baez describes quaternions as "the eccentric cousin who is shunned at important family gatherings."

Octonions are worse still because they do not have the property of *associativity*, which means that you get a different result if you change the order in which you carry out your mathematical operations. Multiplication *is* associative for the real numbers because, for example, multiplying 1 and 2 together first and then by 3, is the same as multiplying by 1 the product of 2 and 3 (that is, (1 x 2) x 3 = 1 x (2 x 3)). That isn't true for octonions. "[T]he octonions are the crazy old uncle nobody lets out of the attic," says Baez (www.math.ucr.edu/home/baez/octonions/node1.html).

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