Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

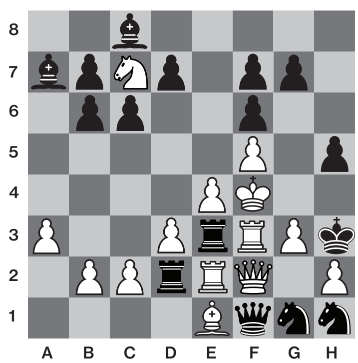
Oh my gosh, we did it again! For the second time, we have referred to our very-much-alive contributor as "the late Dick Hess". The first time is easy to explain: We, mostly I, screwed up.

The second time, in the 2018 Nov/Dec issue, is a real mystery to me as the words in the Nov/Dec restatement of J/A 2, were copied from the original presentation in Jul/Aug, where no "the late" appellation appeared. Some downstream gremlin must have cached the appellation from our first goof years ago, and the recent reference to Hess must have hit in that cache. Whatever the cause, we are embarrassed and once again apologize for the error.

Game-related problems (e.g., chess, bridge, winks, etc) are in critical supply.

Problems

M/A 1. Another "whose turn is it" chess problem from Timothy Chow. Can the diagrammed position be reached by a sequence of legal moves from the standard starting position if chess? If so, can you determine whose turn it is to move?



M/A 2. Nob Yashigahara wants you to place the digits 1-9 once each into the 9 boxes to yield a valid equation.

$$\frac{\square}{\square\,\square} + \frac{\square}{\square\,\square} + \frac{\square}{\square\,\square} = 1$$

M/A 3. In a number of two person games, e.g., tennis or table tennis, a game can reach a so called "deuce" state in which the score is tied and the game proceeds as follows. Player's alternate serving and each serve leads to a point for one of the players. When one player is up by 2 points, they win the game.

Assume player A, when serving, wins the point with probability P_A and player B, when serving, wins the point with probability P_B.

What is the probability that player A wins the game assuming A serves first at deuce? Is it better to serve first at deuce or second?

Speed department

Ermanno Signorelli spoke to a woman who said that she was 28 2 days ago and will be 31 next year. How?

Solutions

Larry Kells wants you to fine a Bridge hand in which N-S can make some contract against best defense, even though neither of them has a card higher than a 9.

Jarek Langer apparently is well familiar with bad hands. He writes.

South can make 1 spade on the layout below. If the lead is a minor suit, ruff in hand, and then crossruff three hearts and three more minor suit cards (2 of the minor that wasn't led and 1 of the minor that was), ending in hand. If the lead is ace of hearts, let it win (do not ruff) and then play as above. East cannot overruff and pull trump before South has 7 ruffing tricks.

North:
♠ 9 8 7
♥
♦ 6 5 4 3 2
♣ 6 5 4 3 2

West:
♠
♥ A
♦ A K Q J 10 9
♣ A K Q J 10 9

East:
♠ A K Q J 10
♥ K Q J 10
♦ 8 7
♣ 8 7

South:
♠ 6 5 4 3 2
♥ 9 8 7 6 5 4 3 2
♦
♣

N/D 2. Dave Blackston tells us that number theorists call a positive integer p-smooth if all its prime factors are at most p. The very large integer $N = 6^{100000}$ is clearly 3 smooth. Blackston asks for the smallest 3-smooth number that is larger than N.

I received a number of wonderful solutions to this problem, space concerns prevent me from including more of them. I wasn't going to select any that used floating point arithmetic until I encountered one, by Tony Bielecki, that actually analyzed the rounding error. This solution is on the Puzzle Corner website.

Also on the web site is Richard Bumby's solution, which includes a "continued fraction"-like analysis.

John Chandler explains easily why N/D 2 is just a (feasible) search problem (assuming your searching apparatus includes unbounded integers). Chandler writes.

3-smooth integers are all of the form $2_A \times 3_B$, where A and B are non-negative integers. The given number N is just the instance where $A = B = 100{,}000$. Obviously, if we start with a number that is greater than or equal to N and then decrease A or B by 1, we must increase the other exponent by at most 2 to get a new number that is again greater than N. It is therefore necessary only to examine all 200000 pairs of A and B that give numbers minimally larger than N to see which is the smallest. The answer is $A = 225743$, $B = 20665$, yielding the value $e^{179175.9469265}$ (compared with $N = e^{179175.9469228}$).

Jorgen Harmse puts all the cards on the table and includes his program. He writes: The solution must be $2_k \times 3_l$, where k, l are nonnegative integers and $(k\text{-}100{,}000) / (100{,}000\text{-}l)$ is close to ln(3)/ln(2). I don't have MatLab (which does rational approximations), but Python has unlimited integers, and a bruteforce search takes a minute. I found 2ˆ225,743 3ˆ20,665, which exceeds 6ˆ100,000 by about 4 parts in a million.

```
def find_3smooth(k0=100000, l0=100000):
''' Find the next 3-smooth number after 2**k0 * 3**l0. If that
number is 2**k * 3**l then return k,l.
'''
def pow(k,l):
return 2**k * 3**l
N0 = pow(k0,l0)
k1,l1 = k0+1,l0
N1 = pow(k1,l1)
k,l = k0,l0
while k>0:
k -= 1
N = pow(k,l)
if N<N0:
l += 1
elif N<N1:
N1,k1,l1 = N,k,l
k,l = k0,l0
while l>0:
k += 1
l -= 1
N = pow(k,l)
if N<N0:
k += 1
elif N<N1:
N1,k1,l1 = N,k,l
assert 3*N1<4*N0, "This code doesn't work for small inputs."
return k1,l1
>>> 125743*np.log(2)-79335*np.log(3)
3.664710675366223e-06
```

N/D 3. Our last regular problem this issue is one John Astolfi calls an "old time safari puzzle".

The year was 1888 and famed explorer Sir Rigglesworth was stymied. He wished to cross on foot a totally barren desert that would take a person 6 days to cross. But a person could only carry only 4 days rations of food and water. Fortunately, two of his bearers, Al and Zack, put their heads together and came up with a plan to get Sir Rigglesworth successfully across. How did they do it?

There are ambiguities in this problem. First, are we permitted to leave Al and Zach without supplies in the middle of the desert? I decided that we are not. Are we permitted to have the bearers start at the destination? Again, I vote no. Must all three get to the destination or may the bearers end at the start.

This time I decided to permit both interpretations.

Jessica Winter-Stolzman lets the bearers return to base and gets Rigglesworh accross in 6 days as follows

Al, Zack, and Rigglesworth start out together, each carrying 4 days' rations. After 1 day, Al turns around, carrying only the 1 ration she needs to get home, and gives her extra 2 days' worth to Zack and Rigglesworth. After the 2nd day of travel, Zack also retreats with the necessary 2 days' worth of rations, and gives his extra 1 to Rigglesworth. Now Rigglesworth has 4 days' worth of rations and 4 days of desert left to cross: success.

Rik Anderson gets all three across, but needs 8 days.

Days 1-4: Al and Zack make two 2-day round trips to Camp 1, caching 2 dayâs rations each on each trip; Rigglesworth joins them on one of the trips, so the cache has 10 daysâ rations total.

Days 5 & 6: All 3 go from Base to Camp 2, each picking up one cached ration at Camp 1, leaving 7 there and arriving at Camp 2 with 3 days' rations each, 9 daysâ rations total.

Day 7: Rigglesworth sets out to cross the desert with 4 dayâs rations, arriving across on Day 10; Al and Zack return to Camp 1 with 1 daysâ rations each, leaving 3 days' rations cached at Camp 2.

Day 8: Al and Zack return to Camp 2 with 5 dayâs rations, which together with the 3 previously cached there give the two of them enough to cross the remaining distance on Days 9-12, arriving two days after Rigglesworth.

By comparison, David Goldfarb shows that, without the bearers, Sir Rigglesworth can cross the desert in 12 days as follows.

Day 1: Load up on the full four days' rations. Walk out into the desert.

Day 2: Cache two days' rations, use the one day's worth left to return.

Day 3: Load up on four days' rations again. Go to the previous cache.

Day 4: Take one day's rations from the cache, and go one day further into the desert.

Day 5: Cache two days' rations, then return to the first cache.

Day 6: Return to camp.

Days 7-8: Load up full again, go to the second cache.

Days 8-12: Sir Rigglesworth consumed two days' rations getting to the cache. He refills there and is back up to four days' worth, with four days' worth of desert left to cross.

Better late than never

2018 J/A 1. Timothy Chow notes that many mate-in-two chess problems have a large number of variations and refers interested readers to https://goo.gl/GGtxqZ.

The key is unique (1.Qh5) but there are a lot of variations. After a "random" move by Black, any of the 13 moves by the bishop on d5 delivers checkmate, so we get well over a hundred variations this way, not to mention all the other variations where Black defends intelligently.

Other responders

Responses have also been received from R. Anderson, A. Andersson, M. Astolfi, F. Bachner, M. Barr, J.Bergmann, S. Berkenblit, E. Berlin, T. Bielecki, M. Bolotin, M. Branicky, M. Brill, R. Bumby, A. Cann, S. Carpenter, M. Chackerian, G. Chan, J. Chandler, T. Chase, T. Chow, G. Coram, R. Cross, B. Currier, R. DeJong, S. de Reil, D. Emmes, K. Feigl, G. Fischer, M. Galatin, D. Garbin, G. Garmaise, R. Giovanniello, D. Goldfarb, S. Gourley, D. Grunberg, A. Guthmiller, J. Hardis, J. Harmse, A. Hirshberg, J. Jamison, U. Kausch, B. Kulp, J. Langer, V. Levy, T. Mattick, J. McGrew, J. Mermelstein, B. Meyer, T. Mita, E. Moass, R. Morgen, G. Muldowney, S. Nelson, E. Nelson-Melby, S. Olgiati, A. Ornstein, R. Orr, S. Peters, B. Rhodes, K. Rosato, J. Russell, E. Signorelli, S. Silberberg, S. Silverstein, S. Ulens, E. Smolisky, R. Virgile, J. Ward, D. Worley, K. Zeger, M. Zuker, J. Larsen, H. Joseph, and S. Sperry/.

Solution to speed problem

She was born on 31 Dec and was speaking on Jan 1.