Puzzle corner

Send problems, solutions, and comments to Allan Gottlieb at New York University, 60 Fifth Ave., Room 316, New York, NY, 10011, or gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.

This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 9) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2018 yearly problem is in the "Solutions" section.

I note that bridge, chess, and other "game problems" are in short supply. Meanwhile, Tom Volet suggests that anyone interested "retrograde chess analysis" like the first problem in M/J 2018 should browse www.chessproblem.net/viewtopic.php?f=10&t=429.

Problems

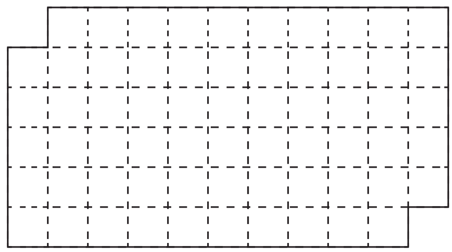
Y2019. How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 9 exactly once each,along with the operators +, −, × (multiplication), / (division), and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 9 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Robert Virgile and some unnamed groundhog sent us the following bedtime story.

Each night, the groundhog thought about aces as he fell asleep. But his dreams always disappointed. His recurring nightmare hand always began with the two of clubs and the two of diamonds. Despite the fact that he never held any aces, and despite the fact that his partner never held any worthwhile cards at all, he did always make 3 no-trump ... night after night after night. He tried to change the outcome but found that he was powerless to do so. As long as he followed suit, he could play his cards in any order; he could try to lose tricks; he could even enlist the opponents to help him. But he still always took exactly nine tricks in 3 no-trump.

Sketch out the hands.

J/F 2. We have another Golomb Gambit. This time you are to divide the figure at the top of the next column into four congruent pieces. There are two solutions.



Speed department

Sorab Vatcha asks if any letter is not in the name of any US state, the District of Columbia, or Puerto Rico.

Solutions

Y2018. Daniel Turek found 55 of the 100 possible values. Thirteen of the solutions, which are printed in bold type, use the digits in order

1 = 821^0	**20 = 20 × 1^8**	64 = 8 × (10 - 2)
2 = 20 - 18	**21 = 20 + 1^8**	65 = 8^2 + 1 + 0
3 = 2 + 81^0	22 = 21 + 8^0	68 = 80 - 12
4 = 8 - 2 + 10	**24 = (2 + 0 + 1) × 8**	72 = 82 - 10
5 = 8 - 2 - 1 + 0	26 = 8 × 2 + 10	74 = 8^2 + 10
6 = 8 × 2 - 10	**27 = 20 - 1 + 8**	77 = 80 - 1 - 2
7 = 8 - 21^0	**28 = 20 × 1 + 8**	78 = 8 × 10 - 2
8 = 8 + 21 × 0	**29 = 20 + 1 + 8**	79 = 80 + 1 - 2
9 = 8 + 21^0	**36 = (2 + 0) × 18**	80 = (2 - 1) × 80
10 = (2 + 0) × 1 + 8	**38 = 20 + 18**	81 = 81 + 2 × 0
11 = 20 - 1 - 8	39 = 80/2 - 1	82 = 81 + 2^0
12 = 20 × 1 - 8	40 = 8/2 × 10	83 = 81 + 2 + 0
13 = 20 + 1 - 8	41 = 1 + 80/2	90 = 180/2
14 = 8/2 + 10	49 = $(8 - 1)^2$ + 0	92 = 82 + 10
15 = 120/8	54 = 108/2	94 = 102 - 8
16 = 8 - 2 + 10	59 = 80 - 21	96 = (10 + 2) × 8
17 = 18 - 2^0	60 = (8 - 2) × 10	100 = (8 + 2) × 10
18 = 28 - 10	61 = 81 - 20	
19 = 20 - 1^8	63 = 8^2 -1 + 0	

S/O 1. The following is from Steve Kanter.

"Here is a solution to the bridge problem from Larry Kells, with one defender holding KJ975 of spades and a side KJ, and the other defender holding the remaining two KJs. South makes seven spades against best defense but cannot make any other grand slam.

	♠ 4, 3, 2 ♥ 5, 4, 3, 2 ♦ A , Q, 10, 9 ♣ A, Q	
♠ ♥ 10, 9, 8, 7, 6 ♦ K, J ♣ K, J, 10, 9, 8, 7		♠ K, J, 9, 7, 5 ♥ K, J ♦ 8, 7, 6, 5 ♣ 6, 5
	♠ A, Q, 10, 8, 6 ♥ A, Q ♦ 4, 3, 2 ♣ 4, 3, 2	

"No matter West's lead, the two AQs give North four entries for three spade finesses and a heart finesse against East, with successful finesses against West's top clubs and diamonds taking down West's diamond KJ along the way. South now takes his last heart, gets back to dummy with the diamond 10, and cashes the diamond 9. On the 12th trick, the low heart lead from North works as a ruffing finesse, and South therefore overtakes the last two cards, trumps, held by East.

"No other grand slam is possible. If played in hearts or clubs, West has natural trump winners to defeat the contracts. In no-trump, declarer will eventually have to lead his fourth and then losing fifth spade for down one. Diamonds will play the same as no-trump since an attempted heart ruff on a third heart would be taken by one of East's higher trump spots."

S/O 2. Barry Kulp begins his solution with a minimality argument and then achieves this minimum. He writes:

"Since the rowboat can only hold two people, and one must row back, then one round trip (two crossings) can deliver one person net across the river until there are two left, in which case one final crossing delivers both of them. With N children plus the father, $N - 1$ pairs of crossings deliver all but two people (either father plus one child, or two children, one of whom must be 1 to make the final crossing). Therefore it requires $2 \times (N - 1) + 1$, or $2N - 1$, total crossings to get the entire family across the river.

"To fully answer the question, we must determine how to combine people to avoid leaving i and $i + 1$ alone together. With $N \geq 5$, 1 can take each of the other odd ones (in any order) and return; then father can take each of the even ones (in almost any order, as long as 2 is not the last one left alone with 1); and finally he and 1 can join the others.

"This requires slight modification for $N \leq 4$. If $N = 4$, then 1 would leave 3, and father could not then leave either 2 or 4 alone with 3 (and taking 4 would leave 1 and 2 alone together). So after 1 delivers all other odd ones (in any order), then father delivers 1 and then all even ones (in any order). For large N, multiple odd ones are left alone with no conflict on the far side, while the remaining odd ones are joined with multiple even ones on the near side. As even ones are delivered, they will be with adjacent siblings, but not alone. For $N = 4$, 1 takes 3, leaving father with 2 and 4. Father takes 1, leaving 2 and 4 on the near side and then leaving 1 and 3 on the far side. Father can then take 2 and 4 in either order. For $N = 3$, 1 takes 3, leaving father with 2. Father takes 1, leaving 2 alone on the near side and then leaving 1 and 3 on the far side. Father then takes 2. For $N = 2$, father takes 1 and then 2. This follows the general method but could actually be reversed. For $N = 1$, father and 1 cross."

S/O 3. I now believe that Robert Bird's football problem is harder than I thought initially (an 18.01 midterm exam question). Timothy Maloney gives solutions for realistic NFL configurations and also shows that his solution for an asymptotically long football field seems to converge to the known right answer. Maloney's solution is on the Puzzle Corner website.

The following solution is from Marc Strauss.

By setting up the parametric equations $y = -0.5 \times g \times t^2 + v \times \sin\theta \times t$ and $x = v \times \cos\theta \times t$ according to ballistic motion (where q is the angle from vertical at which the ball is kicked), we can immediately solve the latter for the time when the football strikes the crossbar as $t = x/(v \times \cos\theta)$.

Then substitute t into the first equation to find an expression for v in terms of θ: $2 \times v^2/(g \times x^2) = 1/(\cos\theta \times (x \times \sin\theta - y \times \cos\theta))$.

To minimize the initial velocity, take the derivative and set it to zero, which (after a couple of chain rule derivatives and trig identities) provides $\theta = \arctan(-x/y)/2$, being sure to take the (non-standard domain) solution where θ is positive. For example, if $x = y$, then $\theta = 67.5°$ from horizontal, which seems fairly intuitively correct. Interestingly, the ideal angle doesn't depend on gravity! Solving for v, however, provides a somewhat less concise result that does:

$$\sqrt{v = g \times x^2/(2\cos\theta \times (x\sin\theta - y\cos\theta))}$$

Other responders

Responses have also been received from M. Branicky, P. Cassady, G. Coss, M. Cramer, R. DeJong, F. DeSimone, A. Hirschberg, T. Kelly, J. Langer, J. Larsen, W. Lemnios, S. Liu, D. Loeb, V. Luchangco, J. Mackro, M. Marinan, J. McNaughton, D. Mellinger, T. Mita, R. Morgen, G. Muldowney, A. Ornstein, J. Prussing, B. Rhodes, L. Schaider, E. Signorelli, J. Sinnett, C. Swift, and D. Worley.

Solution to speed problem

Q is not present (but your editor was born in Queens NY :-)).