This being the first issue of a calendar year, we again offer a "yearly problem" in which you are to express small integers in terms of the digits of the new year (2, 0, 1, and 5) and the arithmetic operators. The problem is formally stated in the "Problems" section, and the solution to the 2014 yearly problem is in the "Solutions" section.

Problems

Y2015 How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 5 exactly once each; the operators +, −, × (multiplication), and / (division); and exponentiation? We desire solutions containing the minimum number of operators; among solutions having a given number of operators, those using the digits in the order 2, 0, 1, 5 are preferred. Parentheses may be used; they do not count as operators. A leading minus sign, however, does count as an operator.

J/F 1. Larry Kells offers another "minimum point" problem, but this time against strong opponents. Consider deals in which South has a 4-3-3-3 distribution, North has no high-card points, and South can make 7 no-trump against best defense. What is the minimum number of high-card points South needs to have?

J/F 2. Aaron Ucko noticed that 2 × 65 = 26 × 5 and wonders if there are other triples of distinct nonzero digits *a*, *b*, *c* such that *ab* × *c* = *a* × *bc*. More challenging would be to find quadruples *a*, *b*, *c*, *d* such that at least two of *a* × *bcd*, *ab* × *cd*, and *abc* × *d* are equal.

J/F 3. I just returned from an important dermatology conference in Las Vegas at which my beautiful wife, Alice, gave two talks and was amused to find the following problem from Donald Aucamp at the front of the queue.

A gambler sits in on a table stakes game in which a fair coin is flipped. If the outcome is a head, he makes a 100 percent profit on the amount he bets, whereas he loses 60 percent on a tail. [*Vegas games aren't like this –Ed.*] He decides to use a constant bet fraction *c* in which he bets *cS* when his current stake is *S*. What value of *c* should he choose to maximize his chances in the long run of beating any other constant strategy? What happens in the long run if instead he sets *c* = 1?

Speed Department

Dan Diamond wonders what size cube has the same number of square inches in its surface as it has cubic inches in its volume. What about spheres? Surprise!

Solutions

Y2014 How many integers from 1 to 100 can you form using the digits 2, 0, 1, and 4 exactly once each; the operators +, −, ×, /; and exponentiation? The following solution is from Michael Piazza.

$1 = 1^{204}$	19 = 40 − 21	$42 = 40 + 2^1$
2 = 2 + 0 × 14	$20 = 20 \times 1^4$	43 = 40 + 1 + 2
3 = 12/4 + 0	$21 = 20 + 1^4$	48 = 12 × 4 + 0
4 = 20/(1 + 4)	$22 = 21 + 4^0$	52 = 12 + 40
$5 = 20^1/4$	23 = 20 − 1 + 4	60 = 20 × (4 − 1)
6 = 20 − 14	24 = 20 × 1 + 4	61 = 21 + 40
7 = (10 + 4)/2	25 = 20 + 1 + 4	$64 = 2^{10-4}$
8 = 10 + 2 − 4	$26 = 10 + 4^2$	70 = 140/2
9 = 10/2 + 4	28 = 40 − 12	76 = (20 − 1) × 4
10 = 2 × (0 + 1 + 4)	30 = 120/4	78 = (40 − 1) × 2
$11 = 12 - 4^0$	32 = 42 − 10	79 = 40 × 2 − 1
12 = 10 + 4 − 2	34 = 20 + 14	80 = 20 × 1 × 4
$13 = 12 + 4^0$	$36 = (10 - 4)^2$	81 = 20 × 4 + 1
14 = 24 − 10	37 = 40 − 1 − 2	82 = (40 + 1) × 2
15 = 20 − 1 − 4	$38 = 40 - 2^1$	84 = (20 + 1) × 4
16 = 20 × 1 − 4	39 = 40 + 1 − 2	$96 = 10^2 - 4$
17 = 20 + 1 − 4	40 = 40/(2 − 1)	98 = 102 − 4
18 = 10 + 2 × 4	$41 = 40 + 1^2$	100 = 20 × (1 + 4)

S/O 1. Larry Kells apparently plays bridge against some fairly poor opponents. He asks, what is the weakest combined holding that North-South can have and still be able to make seven spades against worst play by the defense?

Frank Model found this problem rather easy and writes:

"The weakest N-S holding to make seven spades is five points. (If the opponents have the spade KQJ, one must take a trick.) Here's one of the layouts that work. (Partner's hand is irrelevant.)

A J × × × × × × × × ×
2 3
[void]
[void]

K — Q
A K Q J — × × × ×
A K Q J — × × × ×
A K Q J — × × × ×

"West coöperatively leads her spade honor, and then both opponents pitch their hearts on the run of the spades.

"Zero points will do nicely in 7 no-trump. South's hand is again irrelevant.

10 9 8 7 6
10 9 8 7
[void]
10 9 8 7

× — A K Q J
× × — A K Q J 10
× × — A K Q J
A K Q J × × × × — [void]

"West leads a low club, then ducks the clubs while East discards all his spades. On the run of the spades, East discards all his hearts, making North's hearts good."

S/O 2. Scott Silverstein is a fan of a Tommy Tutone song involving a woman named Jenny whose phone number is 867-5309. Silverstein noticed that his own seven-digit number includes exactly four of Jenny's digits (in the same position as in her number). What is the probability that a random seven-digit number has this property?

Reading the solutions, I realize that there are two ways to interpret the problem: we could be asking about seven-digit numbers or legitimate phone numbers. (I guess even harder would be to ask about phone numbers in use at that time.)

Michael Branicky used the first interpretation and writes that there are 7 choose 4 = $7!/(4!3!) = 35$ ways to pick the four places in which the digits will match the phone number exactly. In each of these cases for each of the remaining three digits, there are nine choices that will *not* match. Also, there are a total of 10^7 seven-digit numbers. Thus the probability that a random seven-digit number has the property is

$35 \times 9^3 / 10^7 = 0.0025515$.

Michael Thompson used the second interpretation and began by asking how many valid (North American) phone numbers there are.

"The first digit can be neither 0 nor 1 because these indicate a long-distance phone call. The second and third digits cannot both be 1 because the three-digit sequences n-1-1 are reserved (411, 911, etc.). Also, numbers of the form 555-xxxx are reserved for use in entertainment and examples. So the total number of valid phone numbers is $8 \times 10^6 - 8 \times 10^4 - 1 \times 10^4 = 7{,}910{,}000$." See his full solution on the Puzzle Corner website.

S/O 3. Ermanno Signorelli heard on a PBS broadcast that it would take 17 copies of the planet Mercury to encircle Earth at its equator. Assuming both planets are spheres and the 17 Mercurys fit exactly, how many Mercurys can be placed under the equator inside a hollow sphere the size of Earth?

First, we realize that 17 Mercurys wouldn't fit as described. That number came from PBS. A number of readers sent fine geometric-trigonometric solutions, but I got a special kick from Jay Mackro's submission, which included a careful construction performed with a caliper, compass, scissors, cardboard, and pennies.

Joseph Falcone found that you need to fit at least 10 externally tangent bodies to have two internal tangent bodies (the minimal number). Further experiments suggested that the number of external bodies is always about seven more than the number of internal ones. He performed an analysis showing that when both values are large, their difference is about 2π.

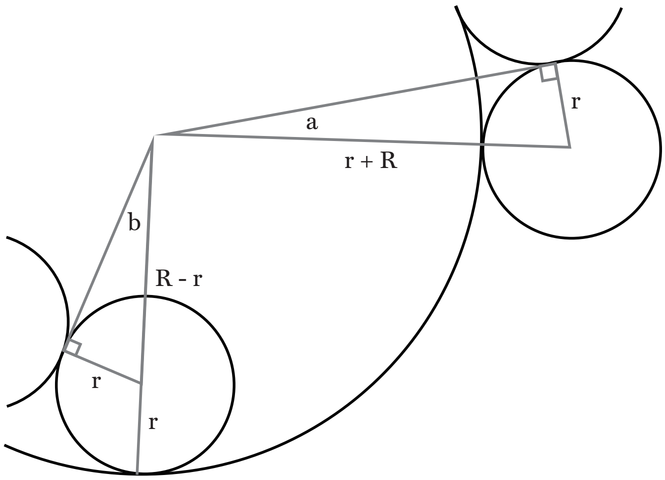
The following analysis comes from Zhe Lu.

Angle a is half the angle subtended by one of the outside Mercurys. Therefore, $a = \pi/17$. Calling the radius of Mercury r and the radius of Earth R, we see that $\sin(\pi/17) = r/(R + r)$ or $\csc(\pi/17) = 1 + R/r$.

Angle b is half the angle subtended by one of the inner Mercurys, so $\sin b = r/(R - r)$ or $\csc b = R/r - 1$.

We easily obtain from the above $\csc b = \csc(\pi/17) - 2$ or $b = \csc^{-1}(\csc(\pi/17) - 2) \approx 0.29$.

Since each inside Mercury occupies an angle of $2b = 0.59$, we can fit $2\pi/(2b) \approx 10.6$ Mercurys inside the equator. Assuming we're not using fractional Mercurys, we can fit 10 with some space left over.



Other Responders

Responses have also been received from L. Azevedo, R. Bird, W. Blake, W. Blank, M. Branicky, M. Brill, R. Bronowitz, W. Burke, P. Cassady, M. Chartier, H. Cortina, J. Eggers, J. Fiel, G. Fisher, J-P. Garric, P. Groot, J. Grossman, J. Harmon, J. Harmse, K. Haruta, Y. Hinuma, A. Hirshberg, J. Ingersoll, M. Kay, J. Korba, P. Kramer, W. Lemnios, D. Lino, Lisa M., N. Markovitz, F. Moezinia, R. Morgen, A. Ornstein, T. Ostrand, G. Perry, J. Prussing, J-P. Radley, K. Rosato, P. Schottler, I. Shalom, E. Sheldon, E. Signorelli, S. Silberberg, T. Sim, J. Simon, J. Steele, A. Stern, D. Stork, R. Virgile, and B. and C. Weggel.

Proposer's Solution to Speed Problem

Edge size = diameter = six inches.

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.