Readers of this column will probably be interested in the Museum of Mathematics (momath.org), which is slated to open next year. Even its phone number is "puzzling" (see the speed problem below).

Since this is the first issue of a new academic year, let me once again review the ground rules. In each issue I present three regular problems, the first of which is normally related to bridge (or chess or some other game), and one "speed" problem. Readers are invited to submit solutions to the regular problems, and two columns (i.e., four months) later, one submitted solution is printed for each; I also list other readers who responded. For example, the current issue contains solutions to the regular problems posed in May/June.

The solutions to the problems in this issue will appear in the January/February column, which I will need to submit in mid-October. Please try to send your solutions early to ensure that they arrive before my deadline. Late solutions, as well as comments on published solutions, are acknowledged in subsequent issues in the "Other Responders" section. Major corrections or additions to published solutions are sometimes printed in the "Better Late Than Never" section, as are solutions to previously unsolved problems.

For speed problems the procedure is quite different. Often whimsical, these problems should not be taken too seriously. If the proposer submits a solution with the problem, that solution appears at the end of the same column in which the problem is published. For example, the solution to this issue's speed problem is given below. Only rarely are comments on speed problems published.

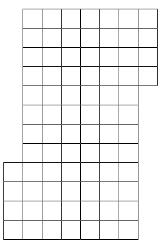
There is also an annual problem, published in the first issue of each year, and sometimes I go back into history to republish problems that have remained unsolved.

PROBLEMS

S/O 1. I know that the shortest chess game ending in checkmate is the two-move "fool's mate" (1. f3 e5; 2. g4 Qh4) but have never seen the following related question, from Sorab Vatcha: What is the shortest chess game ending in stalemate?

S/O 2. Jerry Grossman wonders if there exists an infinite number of sets such that the intersection of every two distinct sets in the collection is nonempty, but the intersection of every three sets in the collection is empty.

S/O 3. There were no solutions to M/J 2, which surprised me until I noticed that the diagram was wrong! It needs to be one box wider. The problem is reopened here with the corrected figure.



Solomon Golomb's October 1987 installment of "Golomb's Gambits" asks us to dissect the figure at right into four congruent pieces. The pieces do *not* have to be similar to the original.

SPEED DEPARTMENT

Glen Whitney reports that inserting two mathematical symbols into the Museum of Mathematics' phone number, 212-542-0566, produces a valid equation. He adds that one of the symbols must be an equals sign.

SOLUTIONS

M/J 1. All responders agree that the correct contract is 6 spades. Roy Schweiker notes that if West has the AK of hearts and leads them, you can't make a grand slam, 6 no-trump, or 6 hearts, which are the only contracts that yield at least as many points as 6 spades. John Chandler describes the play at 6 spades as follows.

South has three cards that aren't guaranteed winners, one in each populated suit, but South will play after East on two rounds, and each time it will convert a loser into a winner.

The opening lead is the first such occasion. If West leads a diamond or club, South takes the trick with the lowest possible card, ending up with the three highest cards in that suit. South then draws the three as-yet-unseen trumps, cashes the three tricks in the first suit, and leads the remaining spades from the top down. The last spade (the 9) falls to East's 10, but then East has only one suit left and must lead from it, setting up South to take all four remaining tricks.

If West leads a spade, South again takes the trick with the lowest possible card and ends up with the four highest spades, which are immediately cashed. South then chooses either minor suit and plays the four cards in turn, from the top down, surrendering the last to East's jack, whereupon the play ends as described above.

If West leads a heart, the situation is slightly more complicated. If East ruffs, South proceeds just the same as if West had led a spade. If East discards a diamond or club, South ruffs with the 9 of spades, draws four rounds of spades, and then plays all four cards of the suit East discarded. On the fourth such trick, South's 10 is the high card, but East is out and therefore has the option of discarding or ruffing. A ruff leads to the same ending play as above. A discard leaves South still on lead with three winners in the remaining minor suit, surrendering only the 13th trick to East's remaining spade.

M/J 2. A corrected version of this problem is now S/O3.

M/J 3. I received a number of excellent solutions to this problem. In addition to printing the offering from Tim Barrows (which itself contains, as a bonus extra, a paradox and its resolution), I include a chart from Joseph Falcone that shows how close the ship would get to the North Pole for all heading angles.

Barrows writes: "A ship starts out at the equator and always maintains a heading of true northeast. Assuming that Earth is a perfect sphere, the position P of the ship can be described using spherical

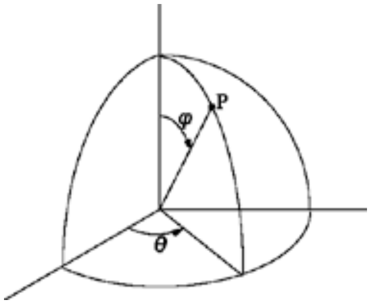


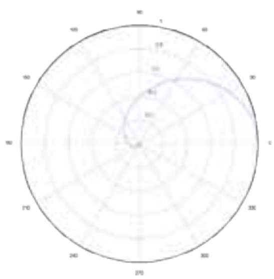
Figure 1

coördinates as shown in Figure 1. Here ϕ is the co-latitude. To simplify the problem, assume the ship starts out at the prime meridian, so that θ = east longitude. Let du = distance traveled east when θ changes by an amount $d\theta$, and dv = distance traveled north when ϕ changes by an amount $d\phi$. Then $du = R_e \sin\phi \, d\theta$, and $dv = R_e \, d\phi$, where R_e is the radius of Earth. Since the ship always maintains a northeast heading, we have $du = dv$. Equating the foregoing expressions, we get $d\phi/\sin\phi = -d\theta$

"Integrating both sides gives $\theta = -\int \frac{d\phi}{\sin\phi} = \log(\tan\frac{\phi}{2})$.

It is not necessary to include a constant of integration, because the formula as given gives $\theta = 0$ when $\phi = \pi/2$, corresponding to the initial condition that the ship starts out at the equator.

"The nature of this equation is shown in Figure 2. The radial lines in this figure correspond to lines of longitude. The dashed circles represent circles of constant distance from Earth's axis—they are not the same as equally spaced circles of latitude. An isometric view is shown in Figure 3. This figure has circles representing each 10 degrees of latitude, plus a great circle representing the prime meridian.



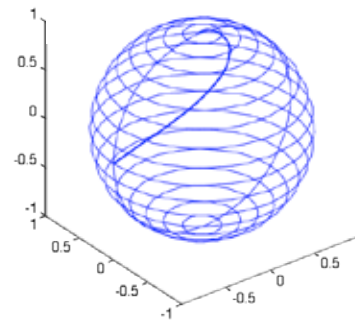


Figure 2, left, view looking down at the North Pole, and Figure 3, right, isometric view of the trajectory.

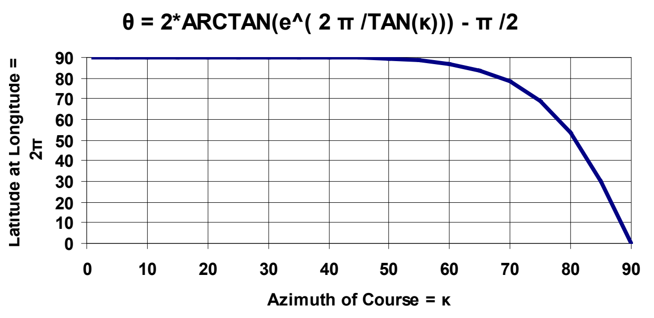
"We wish to know the latitude when the longitude has changed by a full circle—i.e., when $\theta = 2\pi$ and we're back at the prime meridian. Inserting this value for θ into the above expression and inverting, we obtain $\phi = 2\tan^{-1}(e^{-2\pi}) = .00373488$ radians = 0°12' 50"

"This is the co-latitude. The answer to the puzzle is the corresponding latitude, 89°57'10".

"The problem raises an interesting paradox. What happens if the ship continues farther? Does it ever reach the North Pole? The analysis above tells us that $\theta = \infty$ when $\phi = 0$, leading to the notion that it can never get there. However, if the ship can maintain a constant forward velocity in spite of a high turn rate, the northern component of the velocity—i.e., the velocity in the direction of the pole—will also be constant, at the same rate as when it started out at the equator.

"Thus, there is a definite instant at which the ship reaches the pole. But what if we start at that instant and move backward? What direction will the ship take as it backs away from the pole? Clearly the only possible direction is south, but this is a contradiction of the condition that the forward direction was northeast. The resolution of this dilemma is that the turn rate at the instant the ship reaches the pole is infinite. Thus, the heading is indeterminate. If you ever find yourself on such a ship, be sure to move to the exact center of gravity. Any other point you will experience infinite centrifugal force."

Falcone was not content with considering just a pure northeast direction and computed the ending latitude for all headings. The result is shown in the graph below. He notes that the azimuth of course must be about 70° (or angling only 20° above of the equator) before the latitude where the first circuit occurs is under 80°.



OTHER RESPONDERS

Responses have also been received from R. Ackerberg, F. Albisu, A. Anderson, J. Boyton, P. Cassady, M. Chartier, J. Feil, R. Giovanniello, R. Hess, H. Hodara, A. LaVergne, B. Layton, P. Lemieux, R. Llewellyn, T. Mita, E. Nelson-Melby, M. Perkins, L. Satori, I. Shalom, E. Signorelli, and A. Ucko.

PROPOSER'S SOLUTION TO SPEED PROBLEM

212 − 542 = −05 × 66 ■

Send problems, solutions, and comments to Allan Gottlieb, New York University, 715 Broadway, Room 712, New York, NY 10003, or to gottlieb@nyu.edu. For other solutions and back issues, visit the Puzzle Corner website at cs.nyu.edu/~gottlieb/tr.