Bayesian Modelling:  
Joint C-SHRP / Agency Applications

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In July 1994, the Canadian Strategic Highway Research Program (C-SHRP) initiated a project to encourage the use of Bayesian statistical methods and software by the provincial agencies. Modelling applications were pursued jointly between C-SHRP and eight agencies. In July 1995, detailed reports on the individual applications were received and this technical brief summarizes the results. Readers interested in more information are referred to four papers included in the proceedings of the C-SHRP session, Pavement Performance Modelling Using a Bayesian Statistical Approach, at the 1995 Annual Conference of the Transportation Association of Canada (TAC). The eight original agency reports are available for loan from the TAC library.

BAYESIAN STATISTICAL ANALYSIS

An important objective of the Canadian Long-Term Pavement Performance (C-LTPP) study is to evaluate Canadian practice in the rehabilitation of flexible pavements and to develop methodologies and strategies for pavement management. During the initial phase (1989-1994) of the planned fifteen year C-LTPP life, test sites were constructed and the systematic monitoring of those sites for in-service pavement performance began. Also during the initial phase, C-SHRP identified the need for early and ongoing analysis of the C-LTPP performance data. To facilitate that effort, C-SHRP developed computer software for Bayesian statistical analysis. Developed for LTPP analysis, the software is, nonetheless, broadly applicable to other modelling problems.

Bayesian regression analysis is a methodology for the development of models or predictive relationships between variables. As an example, a predictive equation may take the following linear form.

\[ y = b_0 + b_1x_1 + b_2x_2 + \ldots \]

In this equation, \( y \) represents the dependent value to be predicted from \( x_i \), the independent variables which contribute to \( y \). The problem is to establish quantitative estimates for the coefficients \( b_j \).

The classical statistical approach to the problem is to gather data and estimate the coefficients by regressing \( y \) on \( x \) to best fit the data. However, there are a number of drawbacks with the

The Strategic Highway Research Program (SHRP) was established by the United States Congress in 1987 as a five-year, $150 million research program to improve the performance and durability of highways and to make them safer for motorists and highway workers. As a follow-on program to SHRP, Congress established in the Intermodal Surface Transportation Efficiency Act of 1991 programs to implement SHRP products and to continue SHRP’s long-term pavement performance (LTPP) program. The Canadian Strategic Highway Research Program (C-SHRP) is directed at extracting the benefit of the US work for Canada.
classical approach. First, the collection of data can be expensive. In particular, collecting data through the observation of in-service pavements can prove difficult. Pavements change so slowly that nothing substantive may be learned for many years. Furthermore, the estimates of \( b \) based on small sample sizes may not be meaningful. Finally the classical approach to model development does not incorporate judgement in any way. In practice, results are sometimes modified to reflect the judgement or experience of the modeller.

The Bayesian statistical approach to model development, represented in Figure 1, is to systematically combine prior knowledge and experience with data to improve the predictive relationship. The Bayes approach calculates a meaningful and credible answer without relying solely on a small sample size database. In doing so, the Bayes technique allows decisions to be made in the short term while improvements to the data, judgement, and the model continue to be made. The Bayes solution achieves a balance between two solutions based on data or judgement alone. That balance is not arbitrary. The calculation is mathematically rigorous, based upon a theorem first developed in the 1700s by Thomas Bayes. Since the 1950s, the Bayes concept has been refined for regression analysis by mathematicians including Raiffa, Schlaiffer, Press, Pratt and Zellner.

The differences between classical regression and Bayesian regression are most apparent during the data collection stage of model development. In assembling information for Bayesian regression, data collected in the traditional manner is supplemented with prior knowledge. The so-called prior may be drawn from expert judgement, “old” data sets, or knowledge that is generally accepted in the field. Expert judgement can be encoded by polling experts and asking them to estimate the value of the dependent variable for a combination of contributory variables. Once collected, the experts’ “observations” are used exactly like traditional data.

When the data and the prior for a model have been collected, the next step is to conduct the analysis to estimate the regression coefficients and other useful statistics. BSTAT and XLBAYES software were developed under contract for C-SHRP to assist this process. The software provides a platform for the organization of data and prior information, allows the user to easily formulate the performance model, performs the Bayesian calculations, and organizes the results in numerical and graphical form. BSTAT and XLBAYES run as an add-ins to Microsoft Excel and require, as a minimum, a 386 PC with 4 Mb of RAM, 5 Mb of hard drive disk space, MS DOS version 3.1, Windows version 3.1, and Excel version 4.0.

**JOINT C-SHRP / AGENCY MODELLING APPLICATIONS**

In Canada, C-SHRP has promoted the use of Bayesian techniques and the BSTAT and XLBAYES software through joint modelling applications with the provincial highway agencies. Two important purposes of the joint applications were to encourage early agency use of the techniques and software and to provide the agencies with an analytical tool to solve problems within their own province. C-SHRP provided overall project management and contracted Bayesian training and
technical assistance for the agencies. Each agency provided its own problem and analysts to the project. The applications are described below.

**Alberta**

In its application, Alberta Transportation and Utilities developed a model to predict the riding comfort index (RCI) of asphalt overlays (first rehabilitation cycle) on hot-mix asphalt concrete pavement on granular base courses in central Alberta. The field data used in the analysis consisted of 311 records from the Alberta pavement management system. The judgement of five agency experts were combined to form the input prior. The result is a model which expresses the riding comfort index as a function of the RCI performance of the original pavement (RCI perf), a soil factor, overlay thickness, age, initial RCI, and the cumulative traffic load. The standard error of the estimate for the resultant model is reasonable, at 0.498 RCI.

\[
RCI = 3.41 - 2.69 \text{ (RCI perf)} - 0.11 \text{ (soil factor)} + 0.0027 \text{ (thickness)} - 0.12 \text{ (age)} + 0.63 \text{ (init RCI)} + 0.01 \text{ (total loading)}
\]

**Manitoba**

Manitoba Highways and Transportation developed two models to predict the immediate and long term maximum Benkelman Beam rebound (BBR\(_{\text{max}}\)), or deflection, of a flexible pavement immediately after an asphaltic concrete overlay. The models were intended to verify and optimize pavement rehabilitation design procedures. The limited field data that were available for the project consisted of two to three years of observations from the C-SHRP and SHRP databases. Three experts within the agency contributed their judgement to the analysis. Contributory variables included initial BBR values as well as overlay thickness, traffic, and precipitation. Both models have reasonable standard errors, at 0.205 mm for the first model and 0.210 mm for the second model.

\[
\begin{align*}
\text{BBR}_{\text{max}(i)} &= 2.711 + 0.89 \ln (\text{BBR}_0) - 0.45 \ln \text{ (thickness)} \\
\text{BBR}_{\text{max}(t)} &= 0.281 + 0.93 \ln (\text{BBR}_{\text{max}(i)}) - 0.04 \ln \left(\text{kESAL}\right) + 0.19 \ln \text{ (precip)} + 0.15 \ln \text{ (age)}
\end{align*}
\]

**Ontario**

The Ontario Ministry of Transportation developed a model to predict the distress index (DI) of steel slag dense friction course pavements in southern Ontario. The Bayesian model combines information derived from field observations of 79 existing projects with information elicited from five experts. The model expresses the distress index as a function of age, asphalt content of the mix, and traffic volume. The standard error of the estimate for the model is 19.4 DI units, which was deemed reasonable.

\[
\begin{align*}
\text{DI} &= 94.8 + 6.29 \text{ (age)} - 15.4 \text{ (AC)} - 2.57 \log \text{ (traffic)}
\end{align*}
\]

**Saskatchewan**

Saskatchewan Highways and Transportation undertook a study to evaluate subgrade performance and developed a model to estimate road surface deflections for low volume, unpaved roads. Field data were collected from two gravel haul roads and ten experts were polled for the prior data. The resultant model expresses road surface deflections in terms of crust thickness, crust strength, subgrade material strength and total loading. This first generation model has a large degree of variance due to the limited data. The standard error of the estimate for the model is 5.9 mm and the model should be enhanced with additional field data as it becomes available.

\[
\text{Deflection} = 7.23 - 0.02 \text{ (crust thickness)} - 0.02 \text{ (crust strength)} - 0.09 \text{ (subgrade material strength)} + 0.01 \text{ (total loading)}
\]

**Quebec**

In Quebec, the Ministère des Transports constructed two models of frost action on paved roads. One model predicts distortion (IRI) while another predicts longitudinal cracking. Data for the models were extracted from the C-LTPP database. Data from the US-LTPP database, screened by environment, were also used in the study. Six experts, internal and external to the agency, were polled for their contribution to the Bayesian analysis. The standard error of the estimate for the model is 0.6 on the transformed dependent variable. This value was considered reasonable.

\[
R_t = R_0 + K_r \cdot A^\beta
\]

where

\[
\begin{align*}
K_r &= e^{1.8-0.0013T+0.0003P+0.0009FI+0.0032FS} \\
R_t &= \text{roughness at time } t
\end{align*}
\]
$R_0 =$ initial roughness, value of 1.1 assumed

$A =$ age of pavement

$T =$ total pavement thickness

$P =$ yearly precipitation, equivalent of rain

$FI =$ freezing index

$FS =$ frost susceptibility

**New Brunswick**

The New Brunswick Department of Transportation modelled plastic deformation rutting in thin and thick asphalt concrete overlays used in AC pavement rehabilitation. Field data specific to each model were collected from 14 sites for thin overlays and ten sites for thick overlays. Input from nine experts was included. The resultant models express the rut depth in terms of the percent air voids, percent retained on the 4.75 mm sieve, age of the overlay, and percent crushed particles. The model for thin overlays includes the annual traffic while the model for thick overlays includes the cumulative traffic. The standard error of the estimate is reasonable at 2.4 mm.

For thin overlays:

$$ Rut = 0.12652 - 1.79063 \text{ (voids)} - 0.11282 \text{ (retained)} + 0.88664 \text{ (age)} - 0.16013 \text{ (crushed)} + 4.91527 \log (\text{ESAL/yr}) $$

For thick overlays:

$$ Rut = 3.31691 - 1.3381 \text{ (voids)} - 0.10573 \text{ (retained)} + 0.45418 \text{ (age)} - 0.11993 \text{ (crushed)} + 3.30298 \log (\text{ESAL}) $$

**Newfoundland**

The Newfoundland Department of Works, Services and Transportation developed a model to predict the compressive strength of high performance silica fume concrete to assist the scheduling of construction activities. Approximately 50 records of data were used in conjunction with judgements from three experts. The model expresses the compressive strength as a function of the water-cement ratio (WC), the air content, and the fine to total aggregate ratio (fineagg). The standard error of the estimate is 5.08 MPa, which is reasonable.

$$ \text{Compress} = 146.411 - 138.838 \text{ (WC)} - 0.2906 \text{ (air)} - 48.125 \text{ (fineagg)} $$

**Agency Comments**

An objective of the joint applications was to introduce the Bayes methodology and BSTAT and XLBAYES software to the agencies. This objective was well satisfied through the training and technical assistance provided to the agencies. All participating agencies reported that their introduction to Bayesian techniques was a positive experience and that they found the software to be a good analytical tool which proved helpful in solving problems within their province.

Some agencies found that a major benefit of Bayesian regression is that it can provide early modelling results. First generation models can be developed after collecting data for a relatively short period of time. When more data are available, updating a model will improve its accuracy. Incorporating expert judgement in a modelling exercise early in the design of an experiment can save time and money. The traditional method of collecting data for several years before evaluation is too costly and may mean the wrong data have been collected.

The use of the Bayesian modelling approach provides additional insights into the modelling process. In soliciting expert judgement, practical issues must be addressed and the analyst must use relevant, easy to understand variables. The contribution and significance of information provided through field observations and by experts is quantified. Even when a database alone is sufficient for the development of prediction models, the inclusion of expert judgement using the Bayesian methodology can provide an independent review and endorsement of the models by the experts.
SUMMARY

Early in the C-LTPP project, C-SHRP identified the need for ongoing analysis of the pavement performance data. Bayesian methods offer significant advantages to predictive modelling, addressing small sample size problems, the need for early results and the support of experts. Using Bayesian techniques and the C-LTPP database C-SHRP has had considerable success in the development of first generation models of pavement performance. Provincial highway agencies have also been successful in applying Bayesian methods and the BSTAT and XLBAYES software to problems in their own domain.

The successful application of Bayesian analysis to pavement performance prediction is a significant break through in the field. Recognition of C-SHRP’s success in this area has encouraged other agencies, notably the US Federal Highway Administration, to assess Bayesian analysis techniques for their own operations.
For full details of this research see the following reports available on loan from the Transportation Association of Canada:

**Bayesian Methodologies for Evaluating Rutting in Nova Scotia’s Special “B” Asphalt Concrete Overlays**

**Deterioration of Asphalitic Concrete Surfaces Containing Steel Slag**

**Joint C-SHRP/Alberta Bayesian Application**
M. Kurlanda, Alberta Transportation & Utilities; L. Kajner, VEMAX Management Inc. 1995

**Joint C-SHRP/Manitoba Bayesian Application, Benkelman Beam Rebound - AC Overlay Model**
L. Kavanagh, Manitoba Highways & Transportation, Materials & Research Branch. 1995

**Joint C-SHRP/NBDOT Bayesian Application**
New Brunswick Department of Transportation, Research Section, Engineering Services Division. 1995

**Predicting the Compressive Strength of High-Performance Silica Fume Concrete by Bayesian Methods**
J. English, Department of Works, Services and Transportation, Government of Newfoundland and Labrador. 1995

**Saskatchewan Highways & Transportation’s Subgrade Shear Failures**
A. Widger, R. Schmidt, Saskatchewan Highways and Transportation. 1995

**The Development of Models to Predict Pavement Performance in Frost Conditions**
G. Doré, Québec Ministry of Transportation, Pavement Branch. 1995

See also the Proceedings of the 1995 Annual Conference of the Transportation Association of Canada, Victoria, British Columbia, October 1995.

**Deterioration of Asphalitic Concrete Surfaces Containing Steel Slag**, A. Bradbury, J.J. Hajek, I. Afrani.


**Bayesian Methodologies for Evaluating Rutting in Nova Scotia’s Special “B” Asphalt Concrete Overlays**

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