A Variational Problem Involving a Polyconvex Integrand

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Let $f \in C^1(\mathbb{R}^d)$ and $h \in C^2(0, \infty)$ be strictly convex functions. When
\[ \lim_{t \to 0^+} h(t) = \lim_{t \to \infty} h(t)/t = \infty, \]
the current state of the art in the calculus of variations dramatically fails to provide the Euler–Lagrange equations for the minimizers of

\[ (u, \beta) \to I_\varphi(u, \beta) = \int_\Omega f(\nabla \varphi u) + h(\beta) \, dx \]

over $\mathcal{A}$, the set of pairs $(u, \beta)$ such that $u \in W^{1,p}(\Omega, \Omega^*)$, $\beta > 0$ and $u_\beta = \chi_{\Omega^\beta}$.

Here, $\nabla \varphi u$ is the $f$–projection of $\nabla u$. When $\varphi$ is a finite dimensional space, we prove uniqueness of a minimizer of $I_\varphi$ over $\mathcal{A}$, and identify its Euler–Lagrange equation. An approximation argument allows us to make an inference about the case when $\varphi$ is an infinite dimensional space.

Biography

Professor Gangbo was born in Benin, Africa and attended graduate school at Swiss Federale Institute of Technology where he earned his Ph.D. in 1992. He has held postdoctoral positions at Carnegie Mellon University and the Mathematical Sciences Research Institute in Berkeley. He joined the mathematics department of Georgia Tech as an assistant professor in 1995. In 2001, he was promoted to full professor. His research interests include the calculus of variations, nonlinear analysis, partial differential equations, kinetic theory, functional analysis, and fluid mechanics.

He is also the Founder of *EcoAfrica*, an association of scientists involved in several projects in support of African countries. EcoAfrica was founded in 1990 in Switzerland and currently has six members. In the past years, EcoAfrica received funds mostly from Swiss institutions such as DDA, a Swiss governement agency. Among other things, EcoAfrica has organized several workshops in applied mathematics in Senegal and intends to do the same for other countries in Africa.