Tangents of σ-Finite Curves and Scaled Oscillation

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In this talk we will discuss two questions. The first question involves necessary regularity properties for continuous functions on $[0,1]^d$ for which oscillation can be controlled locally, but not necessarily at every scale or at every point. It is known that if the function is defined on [0,1] and the set on which this "lower scaled oscillation" of the function is not finite is at most countable, then the function is always differentiable on a set of positive measure. We will show that we can extend the same result to functions defined on $[0,1]^d$ and discuss conditions on the exceptional set that preserve the existence of the set of differentiability.

The second question is whether continuous simple curves with sigma-finite length have tangents at positively many points. The results on sigma-finite curves that we will discuss were initiated by the observation that the graph of a continuous function on [0,1] with the properties of the functions from the first question has sigma-finite 1-Hausdorff measure. We will discuss our conclusion that every sigma-finite curve has a tangent, in the pointwise sense, on a set of positive measure. Surprisingly, we show that this conjecture is false for $d > 1$. This is joint work with M. Csörnyei.

Biography

Bobby Wilson was born and raised in Phoenix, Arizona. He received his B.S. in mathematics from Morehouse College in 2010. He is receiving his Ph.D. in mathematics at the University of Chicago in June. His research interests include classical harmonic analysis, dispersive PDEs, and geometric measure theory.