WARM-UP

- Find the LCM between the two expressions

1. $7x^3 + 21x^2$ and $x^2 + 3x - 10$

   LCM=$7x^2(x+3)(x-2)(x+5)$

2. $x^2 - 8x$ and $x^4 - 4x^3 - 32x^2$

   LCM=$x^2(x+4)(x-8)$

3. $x^3 + 4x^2 + 4x$ and $x^3 - 4x$

   LCM=$x(x+2)^2(x-2)$

8.6 SOLVING RATIONAL EQUATIONS

Solve with cross multiplication

- Use when the expressions are set up as a proportion

$$\frac{a}{b} = \frac{c}{d} \qquad ad = bc$$

Examples:

1. $\dfrac{7}{x-2} = \dfrac{11}{2x-10}$

$$7(2x-10) = (x-2)11$$
$$14x - 70 = 11x - 22$$
$$3x = 48$$
$$\boxed{x = 16}$$

2. $\dfrac{-x}{x-6} = \dfrac{2}{x-1}$

$$-x(x-1) = (x-6)2$$
$$-x^2 + x = 2x - 12$$
$$0 = x^2 + x - 12$$
$$(x+4)(x-3) = 0$$
$$\boxed{x = 3,\ x = -4}$$

Practice:

1. $\dfrac{3}{5x} = \dfrac{2}{x-7}$ $\boxed{x = -3}$

2. $\dfrac{-4}{x+3} = \dfrac{5}{x-3}$ $\boxed{x = \dfrac{-1}{3}}$

3. $\dfrac{1}{2x+5} = \dfrac{x}{11x+8}$ $\boxed{x = -1,\ x = 4}$

4. $\dfrac{x}{x-2} = \dfrac{-3}{x-4}$ $\boxed{x = -2,\ x = 3}$

Solve with Least Common Denominator (LCD)

- Use when the expressions are not set up as a proportion

$$\frac{a}{b} = \frac{c}{d} + \frac{e}{f}$$

$$bdf\left(\frac{a}{b}\right) = bdf\left(\frac{c}{d} + \frac{e}{f}\right)$$

$$dfa = bfc + bde$$

$$LCD = bdf$$

Examples:

3. $\frac{15}{x} = \frac{7}{x} - \frac{4}{5}$

4. $\frac{5}{5-x} = 2 + \frac{x^2}{5-x}$

Solve with Least Common Denominator (LCD)

Examples:

3. $\frac{15}{x} = \frac{7}{x} - \frac{4}{5}$

$LCD = 5x$

$$5x\left(\frac{15}{x}\right) = 5x\left(\frac{7}{x} - \frac{4}{5}\right)$$

$$5(15) = 5(7) - x(4)$$

$$75 = 35 - 4x$$

$$40 = -4x$$

$$\boxed{x = -10}$$

4. $\frac{5}{5-x} = -2 + \frac{x^2}{5-x}$

$LCD = (5-x)$

$$(5-x)\left(\frac{5}{5-x}\right) = (5-x)\left(-2 + \frac{x^2}{5-x}\right)$$

$$5 = -2(5-x) + x^2$$

$$5 = -10 + 2x + x^2$$

$$0 = x^2 + 2x - 15$$

$$0 = (x-3)(x+5)$$

$$\boxed{x = 3,\ x = -5}$$

Practice:

5. $\frac{2a-3}{6} = \frac{2a}{3} + \frac{1}{2}$ $\quad \boxed{a = -3}$

6. $\frac{x}{x+1} + \frac{5}{x-1} = 1$ $\quad \boxed{x = \frac{-3}{2}}$

7. $1 = \frac{2}{n^2} - \frac{1}{n}$ $\quad \boxed{n = -2,\ n = 1}$

8. $\frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}$ $\quad \boxed{x = -3,\ x = -8}$

Warm-Up

1. $\frac{6}{x} - \frac{9}{x-1} = \frac{1}{4}$ $\boxed{x = -3,\ x = -8}$

2. $\frac{7}{x-1} - 5 = \frac{6}{x^2-1}$ $\boxed{x = \frac{-3}{5},\ x = 2}$

Extraneous Solutions:

- These happen when one or more of the solutions will not work in the original equation.
- Example:

$$LCD = (x-6)(x-2)$$

5. $$\frac{4}{x^2 - 8x + 12} = \frac{x}{x-2} + \frac{1}{x-6}$$

$$(x-6)(x-2)\left(\frac{4}{(x-6)(x-2)}\right) = (x-6)(x-2)\left(\frac{x}{x-2} + \frac{1}{x-6}\right)$$

$$4 = x(x-6) + 1(x-2)$$
$$4 = x^2 - 6x + x - 2$$
$$0 = x^2 - 5x - 6$$
$$0 = (x-6)(x+1)$$
$$x = 6,\ x = -1$$

If x=-1 $\frac{4}{21} = \frac{1}{3} + \frac{-1}{7}$ True so $x = -1$

If x=6 $\frac{4}{0(4)} = \frac{3}{2} + \frac{1}{0}$ Not true so $x \neq 6$

Practice:

9. $\dfrac{5x}{x-2} = 7 + \dfrac{10}{x-2}$ $\boxed{\text{No solution}}$

10. $\dfrac{x-6}{2x^2+2x-4} + \dfrac{x}{2x-2} = \dfrac{1}{2}$ $\boxed{x = 2}$

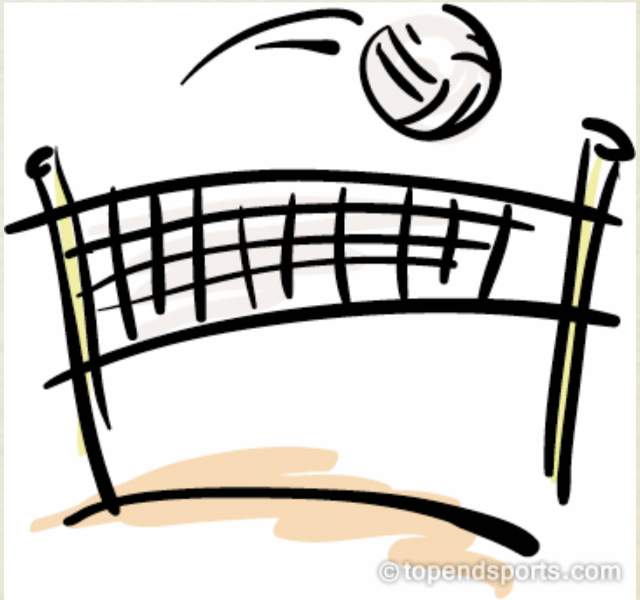
11. $x - \dfrac{2}{x-3} = \dfrac{x-1}{3-x}$ $\boxed{x = -1}$

- So far in your volleyball match, you have put into play 37 of the 44 serves you have attempted. How many consecutive serves do you need in order to raise your service percentage to 90%?

$$\text{Goal service percentage} = \frac{\text{Serves put into play}}{\text{Serves attempted}}$$

$$\frac{90}{100} = \frac{37 + x}{44 + x}$$

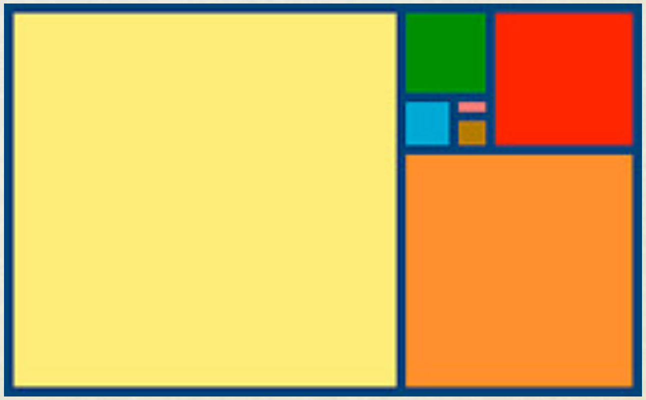
26 serves



- Golden Rectangles are rectangles for which the ratio of the width w to the length l is equal to the ratio of l to l+w. The ratio of the length to the width of these rectangles is called the golden ratio. Find the value of the golden ratio using a rectangle with a width of 1 unit.

$$\text{Ratio of } w \text{ to } l = \text{Ratio of } l \text{ to } l + w \qquad \frac{w}{l} = \frac{l}{l+w}$$

$$\boxed{l = \frac{1+\sqrt{5}}{2}}$$



Solving Rate Problems

Example 7:
Bailey's garden hose can fill her pool in 8 hours. Her neighbor has a hose that can fill the pool in 12 hours. How long will it take the pool to fill if she is using both hoses?

rate of Bailey's hose + rate of her neighbors = rate of both

rate of Bailey's hose $= \frac{1}{8}$

rate of her neighbors $= \frac{1}{12}$

rate of both $= \frac{1}{t}$

Substitute the values $\frac{1}{8} + \frac{1}{12} = \frac{1}{t}$

Solve for t $24\left(\frac{1}{8} + \frac{1}{12}\right) = 24\left(\frac{1}{t}\right)$

$t = \frac{24}{5} = 4.8$ hours