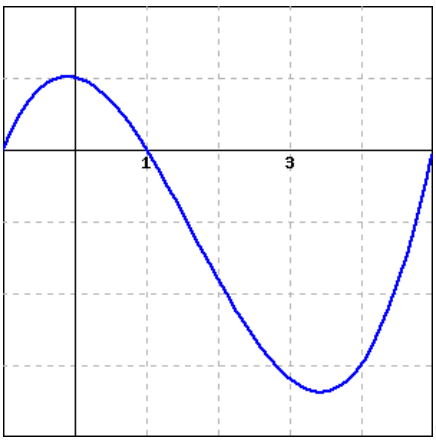
Final Exam draft Name_____

Show all work to support answers for full credit, regardless if multiple choice or not. All problems 3 points, except where noted.

1. Find the ABSOLUTE maximum value attained by the function $f(x) = x^3 - 3x^2 - 9x$ on the closed interval [-2, 10].

2. Graph the function $f(x) = \frac{2x-6}{x-1}$ below. Compute, draw, and label all appropriate asymptotes and intercepts in your graph.

3. 

Given the graph of f(x), which is TRUE about f(x) on the entire interval 3< x <5?

a) $f'(x) > 0$ and $f''(x) > 0$
b) $f'(x) < 0$ and $f(x) < 0$
c) $f''(x) > 0$
d) $f''(x) < 0$
e) None of the above

4. Use implicit differentiation to find an expression for $\frac{d^2y}{dx^2}$, given the equation below. Your answer should be in terms of the variables x or y only (or both).

$$2x^2 - 2y^2 = 8$$

5. Find the interval(s) on which the function $f(x) = 2x^3 + 6x^2 + 7$ is a DECREASING function.

a) (-2, 0) b) $(-\infty, -2) \cup (0, \infty)$ c) none d) $(-\infty, 0)$ e) $(2, \infty)$

6. For the function $f(x) = 4x^3 - 12x + 15$, use the 2nd derivative sign chart to find and to verify any possible points of inflection (x, y) in the graph of the function.

a) (1, 7) only b) (0, 15) only c) (1, 7) and (-1, 23) d) (-1, 23) e) (3, 12)

$$f(x) = 1x^3 + (3) \cdot x^2 + 6x - 5.$$

7. Find the interval on which the given function is CONCAVE DOWN:

$a.\ (-\infty, -1)$ b. $(-1, \infty)$ c. $only\ at\ x = -1$ d. $(-\infty, \infty)$ e. $(-\infty, -1) U (1, \infty)$

8. Find the interval(s) on which a function $f(x)$ is DECREASING, given that the derivative is $f'(x) = (x + 3)^2(2x - 6)$. NOTE: this is the DERIVATIVE , $f'(x)$!

a) (-3, 0) b) $(-\infty$, -3) U (3, $\infty)$ c) (-3,3) d) (3, ∞) e) none

9. Graph the function $f(x) = x^3 - 6x^2 + 8$ below. Plot and label any critical points and inflection points in your graph. [4 POINTS]

10. $\frac{d}{dx}\ln(4x^2) =$

a) $\frac{1}{4x^2}$ b) 2x c) $\frac{2}{x}$ d) $\frac{1}{32x^3}$ e) 8x

11. $\int x^2 e^{5x^3}\, dx =$

a) $\frac{1}{15}e^{5x^3} + C$ b) $\frac{1}{5}e^{5x^3} + C$ c) $-\frac{1}{15}e^{5x^3} + C$ d) $\frac{1}{15}e^{5x^3} \cdot x^2 + C$ e) $-\frac{1}{5}xe^{5x^3} + C$

12. Use a Reimann Sum to directly compute the APPROXIMATE area between the graph of $y = x^2$ and the x-axis between $x = 0$ and $x = 2$. For your approximation, use 4 rectangles of equal width and use the right endpoint of each rectangle base to determine the height of the rectangle. Draw a picture.
a) 64/3 b) 15/4 c) 30/4 d) 30 e) 64

13. Which of the following is the correct Sigma notation for the Riemann Sum used to compute the area between $f(x) = 2x + 3$ and the x-axis between x=1 and x= 5 with 16 rectangles, using right endpoints?

a) $\sum_{i=1}^{16}\left[2\left(1+\frac{i}{16}\right)+3\right]\left(\frac{16}{1}\right)$

b) $\sum_{i=1}^{16}\left[2\left(1+\frac{i}{4}\right)+3\right]\left(\frac{1}{4}\right)$

c) $\sum_{i=1}^{15}\left[2\left(\frac{i}{4}\right)+3\right]\left(\frac{1}{4}\right)$

d) $\sum_{i=1}^{15}\left[2\left(1+\frac{i}{4}\right)+3\right](4)$

e) $\sum_{i=1}^{16}\left[2\left(\frac{i}{4}\right)+3\right]\left(\frac{1}{4}\right)$

15. Given: the graph of $f(x) = \frac{lnx}{x}$ has a relative maximum for x>0. Find it. You must complete a 1st derivative sign chart to justify your answer.

a) Rel min at x=1 b) rel max at x=e c) rel min at x=e d) saddle at x=e e)NONE.

16. How long will it take, in years, to DOUBLE a bank account balance if the annual interest rate is 6%, assuming continuous compounding?

a) 0.06 ln 2000 b) $\frac{\ln 2}{0.06}$ c) $\frac{0.06}{\ln 2}$ d) $\ln\frac{1}{2} \cdot 0.06$ e) $\frac{0.06}{12}\ln 2$

17. Let $R(x) = -10x^2 + 160\sqrt{x} - 100$, in thousands of dollars. Use differentials to approximate the change in the Revenue when production level x is increased from 4 to 4.1.

a) -2000 b) - 4000 c) -5000 d) -3000 e) -8000

18. Find the value of the derivative of the inverse function, $(f^{-1})'(x)$, at $x = -3$, given that $f(x) = 2x^3 + 5x + 4$. Note that f(-1)= -3.

a) -55 b) $-\frac{1}{65}$ c) $-\frac{1}{49}$ d) $\frac{1}{59}$ e) $\frac{1}{11}$

19. Evaluate the indefinite integrals:

$$\int (e^x + 4\sqrt{x})\, dx$$

20. $\int \frac{2x-\sqrt{x}}{x^2} dx$

21. Evaluate $ln\sqrt{\frac{1}{e^3}}$

a) 2/3 b) $e^{\frac{-3}{2}}$ c) -3/2 d) $e^{\frac{2}{3}}$ e) none of these

22. If a marginal cost function is $C'(x) = 20x - 3$, find the total cost function $C(x)$,given that the fixed overhead costs are known to be \$300. $C(x) =$

a) 20 b) $10x^2 - 3x + 280$ c) $10x^2 - 3x + 300$ d) $20x + 297$ e) 300

23. On what intervals is the function $f(x) = xe^x$ decreasing? You must show a 1st derive. sign chart to justify.

a) $(-\infty, -1)$ b) $(-\infty, \frac{1}{e})$ c)$(-1, \infty)$ d) $(-\infty, \infty)$ e) $(-e, -1)$

24. The ONE critical point for the function $f(x) = x\ lnx$, x>0 occurs at the point x=1/e. What type of critical point is it? You MUST complete either a 1st derivative sign chart or use the 2nd derivative test to at appropriate test points to justify your answer.

25. A sample of radioactive material decays from 8 g. to 4 g. in 100 days (i.e.it'shalf life). Assuming that the decay rate is proportional to the amount present at each instant, what is the decay constant k?

a) (ln 2)/100 b) (- ln2)/100 c)– ln(100) d)-200 e) -100ln 2

26. Evaluate: $\int x(x^2+4)^5 dx$

27. Evaluate: $\int_1^e \frac{(\ln x)^3}{x}\, dx$

a) $\frac{1}{2^2} = \frac{1}{4}$ b) 2 c) $\frac{2}{e}$ d) $\frac{1}{2e}$ e) 2e

28. What is the area in between the functions $f(x) = \frac{1}{4}x^2 - 1$ and $f(x) = x - 1$?

a) $\frac{32}{3}$ b) $\frac{16}{3}$ c) $\frac{8}{3}$ d) $\frac{2}{3}$ 3) 6

29. Find the linearization for the given function at the indicated point: $f(x) = \sqrt{x^2 + 5}$, at x=2.

a) $y - 3 = \frac{2}{3}(x + 2)$ b) $y - 3 = \frac{4}{3}(x - 2)$ c) $y - 3 = \frac{2}{3}(x - 2)$ d) $y - 3 = \frac{2}{3}(x + 2)$ e) $y - 3 = \frac{1}{3}(x - 2)$

30. Find dy/dx (answer may be in terms of both x and y),given that $xe^y + x = xy$.

a) $\frac{y-1}{xe^y - x}$ b) $\frac{y-1-e^y}{xe^y}$ c) $\frac{y-1-e^y}{xe^y - x}$ d) $\frac{y-e^y}{xe^y}$ e) $\frac{-e^y}{xe^y - x}$

31. Find the relative maximum value for the function $f(x) = x^3 - 15x$.

a) -√15 b) √15 c) -10√5 d) 10√5 e) 12