The City College Department of Mathematics Fall 2009

MATH 20500 Final Exam

1) Turn-off cell phones and put them and all notes out of sight.
2) NO CALCULATORS, NO scrap paper (use sheets provided)
3) Leave all numbers in exact form (Simplify answers when reasonable, but leave them in terms of π, $\sqrt{\ }$, e, $\ln$, and fractions).
4) Points will be deducted if a solution is given without written proof of your work

SHOW ALL WORK

PART 1: Answer ALL questions in this part. (50 points)

1) Find $\frac{dy}{dx}$ and simplify where reasonable (15 points):

[5 pts] **1-a)** $y = 2(x^3 - 1)(3x^2 + 1)^4$ [5 pts] **1-b)** $y = \frac{e^{4x}}{4 + x}$ [5 pts] **1-c)** $y = (\ln x)^2 - \ln(x^2)$

2) Simplify the following (6 points):

[3 pts] **a)** $\ln x - \ln x^2 + \ln x^4$ [3 pts] **b)** $e^{(\ln x^2 - 3\ln y)}$

3) Find the integral and simplify where reasonable (20 points):

[5 pts] **3-a)** $\int \frac{3}{x \ln(x^2)}\, dx$ [5 pts] **3-b)** $\int \frac{3e^x}{3 - 2e^x}\, dx$

[5 pts] **3-c)** $\int (3 - x)(x^2 - 6x)^5\, dx$ [5 pts] **3-d)** $\int_3^5 x\sqrt{x^2 - 9}\, dx$

4) Let $P(t)$ be the population of a colony of bacteria. At 10AM there are 50 bacteria and at 3PM there are 350. Assume exponential growth. (9 points)

[3 pts] **a)** Find $P(t)$ and simplify.
[3 pts] **b)** What is the size of the population at 4PM?.
[3 pts] **c)** When will the population reach 1500?

PART 2: Answer 5 complete of the 7 questions (1 question worth 10 points in each page). If you answer more than 5, cross-out work not to be graded.

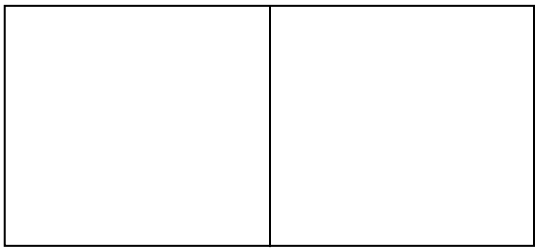
5) For the function $f(x) = \frac{2}{4 - 5x}$:

a) Using the definition of derivative (limits) to compute $f'(2)$.
b) Use the result of part (**a**) to find an equation of the line tangent to the curve $y = f(x)$ at the point for which $x = 2$.

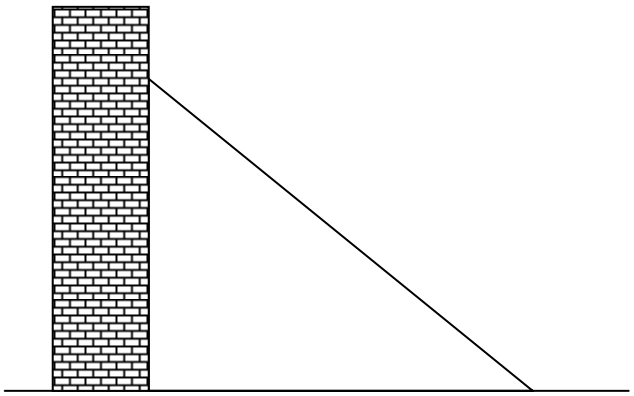
6) Graph the curve $y = \frac{4}{3}x^3 - 2x^2 + x$. Find the *y*-intercept, points where tangent is horizontal, where the graph is increasing and decreasing, where concave up and down, and inflection points. **Label** the preceding points on your graph.

7) A storage shed is to be built in the shape of a box with a square base. It is to have a volume of 300 cubic feet. The concrete for the base costs \$8 per square foot, the material for the roof costs \$4 per square foot, and the material for the sides costs \$5 per square foot. Find the dimensions of the most economical shed.

8) A rectangular corral of 162 square meters is to be fenced off and then divided by a fence into two sections, as shown in figure below. Find the dimensions of the corral so that the amount of fencing is minimized.



9) In the figure, a 20-foot ladder is leaning against a wall. Suppose that the foot of the ladder is being pulled along the ground at the rate of 6 feet per second. How fast is the top of the ladder sliding down the wall at the time when the foot of the ladder is 12 feet from the wall?



10) A toy rocket fired straight up into the air has height $s(t) = 160t - 16t^2$ feet after *t* seconds.

a) What is the rocket's initial velocity?

b) What is the acceleration when $t = 3$?

c) What will be its velocity when it hits the ground?

11) A bank pays 5% interest compounded continuously. Suppose you make an initial deposit of \$4000 in the account.

a) Write a differential equation together with initial condition whose solution gives the amount in your account at any future time.

b) At what rate (in \$/year) is your account increasing when the principal reaches \$6000.

c) How long will it take until the principal in the account reaches \$5000?