I. Basic Integration Formulas

SELECTED SOLUTIONS with STEPS

A) $\int k dx = \quad kx + C$

B) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

C) $\int kf(x)dx = \ k\int f(x)dx =$

D) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

Evaluate each indefinite integral.

1. $\int x^8 dx$

SOLNS: $\frac{x^9}{9} + C$

2. $\int \sqrt[5]{x} dx$

Rewrite $\int x^{\frac{1}{5}} dx = \frac{5x^{\frac{6}{5}}}{6} + C$

3. $\int \frac{1}{5x^2} dx$

$= -\frac{1}{5x} + C.$

Find the indefinite integral of the given functions:

4. $f(x) = x^3 - 3x^2$

$\int (x^3 - 3x^2)dx$

By "reverse" power rule

$= \frac{x^4}{4} - \frac{3x^3}{3} + C$

5. $f(x) = \frac{2x^3 - 1}{x^2}$

Rewrite:

$\int (2x - x^{-2})dx$

$= \frac{2x^2}{2} + \frac{x^{-1}}{1} + C$

$= x^2 + \frac{1}{x} + C.$

6. $f(x) = \frac{2}{3x^2}$

Rewrite in power form:

$\int \frac{2}{3} x^{-2} dx = -\frac{2}{3} x^{-1} + C$

$= -\frac{2}{3x} + C.$

7. $\int \frac{1}{(5x)^2}\,dx$

8. $5\int \frac{1}{x\cdot\sqrt[3]{x}}dx$

9. $\int \left(8x^2 - 3x + 4\right)dx$

SOLNS:

direct,“reverse” power!

$$\int \frac{1}{25x^2}\,dx = \int \frac{x^{-2}}{25}\,dx$$

$$= -\frac{1}{25x}.$$

$$= 5\int \frac{1}{x\cdot x^{\frac{1}{3}}}dx = 5\int \frac{1}{x^{\frac{4}{3}}}dx = 5\int x^{-\frac{4}{3}}dx$$

$$= \frac{5}{-1/3}x^{-\frac{1}{3}} + C = -15x^{-\frac{1}{3}} + C$$

10. $\int (x+2)^2\,dx = \int (x^2 + 4x + 4)dx = \frac{x^3}{3} + 2x^2 + 4x + C.$

5. Find the equation for *y*, given the derivative and the indicated point on the curve.

$$\frac{dy}{dx} = 12x^2 - 24x + 1; (1,-2)$$

Integrating, we get

$y = 4x^3 - 12x^2 + x + C.$ Now using the point that is GIVEN, $(1,-2)$, find the CONTSTANT C:

$-2 = 4\cdot 1^3 - 12\cdot 1^2 + 1 + C.$

$-2 = 4 - 12 + 1 + C = -7 + C$, Thus C= 5.

Now that we have found C=5 , we have the PARTICULAR solution to the differential equation:

$y = 4x^3 - 12x^2 + x + 5.$

5. Find the equation for *y*, given the derivative and the indicated point on the curve.

6. Solve the differential equation.

$$f''(x) = 20x^3 - 10,\ f(1) = 1,\ f'(1) = -5$$

7. A ball is thrown vertically upward from ground level with an initial velocity of 20 $ft/\sec$.

How high will the ball go? SOLN for finding the position function:

SOLN: The differential equation for motion in the atmosphere is:

$s''(x) = -32\frac{ft}{\sec^2}$. The initial conditions can be written as: $s(0) = 0, s'(0) = 20\frac{ft}{\sec}$

Now integrate the differential equation, and use the given condition to find C.

$$s''(x) = -32\frac{ft}{\sec^2}$$
$$\Rightarrow s'(x) = -32x + C.$$

Since initial velocity is $s'(0) = 20\frac{ft}{\sec}$, we get

$s'(0) = -32(0) + C = 20.$ So,

$s'(0) = 0 + C = 20 \Rightarrow C = 20.$

Thus, the first antiderivative gives the particular solution: $\underline{s'(x) = -32x + 20.}$ (velocity function)

Now integrate this new function, and use the initial given condition to find the original function $\underline{s(x).}$

$$s'(x) = -32x + 20.$$
$$\Rightarrow s(x) = -16x^2 + 20x + C.$$

The initial condition $s(0) = 0$. This means that C is zero.

Thus, the particular position function is: $\underline{s(x) = -16x^2 + 20x}$. **(position function)**

You can now find the maximum height by finding the maximum of this position function (a parabola!).

8. A ball is dropped from an initial height of 80 ft. How long will it take to reach the ground?

Intuition behind the concept of an antiderivative of a function f(x)?

Let $f(x)$ be a function representing the $rate\ of\ change$ of the given quantity with respect to x.

EXAMPLE:

Draw a graph of $f(t)\ vs.t$ for the following examples:

Ex. If $f(t) = 30\ mph,\ what\ does\ area\ under\ f(t)$

$between\ t = 0\ to\ t = 3\ hours\ represent?$ **The total distance traveled in that time period.**

Draw a graph of the particular antiderivative $F(t)$ of the function $f(t)$ with initial value $F(0) = 0$. What is the value of $F(1)$? **30** $F(2)$? **60** $F(3)$? **90**

$\boldsymbol{F(t) = 30t + C = 30t}$. (since C=0, from initial point).

Ex.2 If an empty tank is filled at the rate $f(t) = 2\ gal/min$, what is the total number of gallons in the tank after 5 minutes? What is the *area under the rate function $f(t)$ from t=0 to t=5*?

Compute the particular antiderivative for $f(t)$ with initial condition $F(0) = 0$.

$\boldsymbol{F(t) = 2t}$ (since C =0, from initial condition)

What is $F(5)$? $\boldsymbol{F(5) = 2(5) = 10}$.

Total water accumulated from t=0 to t=5 is : $\boldsymbol{F(5) - F(0) = 10 - 0 = 10\ gal}$.

Hence, the *antiderivative function $F(t)$ computes* **the total accumulated amount of the quantity whose rate of change is given by the function f(t).**

What is the relationship between $F(t)$ and the area under the rate function up to time t ?

$\boldsymbol{F(t)}$ is also equivalent to the AREA under the graph of $\boldsymbol{f(t)}$ over the time interval up to time $\boldsymbol{t}$.

Ex.3 If an empty tank is filling at the rate given by $f(t) = 2t$ gal/min (the flow rate into the tank is continuously increasing at each t!), what is the total number of gallons in the tank after 5 minutes?

In this case, $F(t) = \int 2t = t^2 + C = t^2$. (since C=0)

Answ: $\boldsymbol{\int_0^5 2tdt = t^2\ ]_0^5 = 5^2 - 0^2 = 25\ gal}$.