Antiderivatives______________________________ Name_____________________

I. Basic Integration Formulas

A) $\int k dx = \quad kx + C$

B) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

C) $\int kf(x)dx = \ k\int f(x)dx =$

D) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

E) $\int e^x dx = e^x + C$; $\int e^{bx} dx = \frac{1}{b} e^x + C$

F) $\int \frac{1}{x} dx = \ln|x| + C$

Evaluate each indefinite integral.

1. $\int x^8 dx$

2. $\int \sqrt[5]{x} dx$

3. $\int \frac{1}{5x^2} dx$

Find the indefinite integral of each function below:

4. $f(x) = x^3 - 3x^2$

5. $f(x) = \frac{2x^3 - 1}{x^2}$

6. $f(x) = \frac{2}{3x^2}$

7. $\int \frac{1}{(5x)^2}\,dx$

8. $5\int \frac{1}{x\sqrt[3]{x}}dx$

9. $\int (8x^2 - 3x + 4)dx$

10. $\int (x+2)^2\,dx$

11. $\int \frac{4}{2x+3}\,dx$

12. $\int \frac{4}{(2x+3)^2}\,dx$

13. $\int 6e^{2x}dx$

14. $\int \frac{6}{x^2}e^{1/x}dx$

5. Find the particular solution *y*, given the derivative and the indicated point on the curve.

$$\frac{dy}{dx} = 12x^2 - 24x + 1;\ (1,-2)$$

5. Find the particular solution *y*, given the derivative and the indicated point on the curve.

$$\frac{dy}{dx} = \int \frac{12x^2 - 4\sqrt{x}}{x};\ (4,50)$$

6. Solve the differential equation.

   $f''(x) = 20x^3 - 10,\ f(1) = 1,\ f'(1) = -5$

The differential equation of motion for objects in the atmosphere is $s''(t) = -32\ {}^{ft}/_{sec^2}$.

7. A ball is thrown vertically upward from ground level with an initial velocity of 20 $ft/\sec$.

   How high will the ball go?

8. A ball is dropped from an initial height of 80 ft. How long will it take to reach the ground?

Intuition behind the concept of antiderivative (or indefinite integral) and area under a function f(x):

Let $f(x)$ be a function representing the $rate\ of\ change$ of the given quantity with respect to x.

EXAMPLE:

Draw a graph of $f(t)\ vs.t$ for the following examples:

Ex. If $f(t) = 30\ mph,\ what\ does\ area\ under\ f(t)$

$between\ t = 0\ to\ t = 3\ hours\ represent?$____________

Draw a graph of the particular antiderivative $F(t)$ of the function $f(t)$ with initial value $F(0) = 0$. What is the value of $F(1)$? $F(2)$? $F(3)$?

Ex.2 If an empty tank is filled at the rate $f(t) = 2\ gal/min$, what is the total number of gallons in the tank after 5 minutes? What is the *area under the rate function $f(t)$ from t=0 to t=5*?

Compute the particular antiderivative for $f(t)$ with initial condition $F(0) = 0$.

What is $F(5)$?

Hence, the *antiderivative function $F(t)$ computes* ____

____.

What is the relationship between $F(t)$ and the area under the rate function up to time t ?
____.

Ex.3 If an empty tank is filling at the rate given by $f(t) = 2t$ gal/min (the flow rate into the tank is continuously increasing at each t!), what is the total number of gallons in the tank after 5 minutes?