

Chapter 9: Quadratic Functions and Equations

9.1 Solve Quadratic Equations by Square Root Property

Square Root Property:

If $a^2 = n$ then $x = \sqrt{n}$ and $x = -\sqrt{n}$

Example 9.1.1: Solve for x .

a) $x^2 = 9$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm \sqrt{9}$$

$$x = \pm 3$$

$$x = 3 \quad \text{and} \quad x = -3$$

b) $x^2 = -5$

$$\sqrt{x^2} = \sqrt{-5}$$

No real solution.

Note: The solution is $x = \pm i\sqrt{5}$

c) $5y^2 - 34 = 26$

$$5y^2 = 60$$

$$y^2 = 12$$

$$\sqrt{y^2} = \sqrt{12}$$

$$y = \pm \sqrt{12}$$

$$y = \pm 2\sqrt{3}$$

$$d) (2x + 4)^2 = 36$$

$$\sqrt{(2x + 4)^2} = \sqrt{36}$$

$$2x + 4 = \pm \sqrt{36}$$

$$2x + 4 = \pm 6$$

$$2x = -4 \pm 6$$

$$x = -2 \pm 3$$

$$x = 1 \quad x = -5$$

9.2 Solve Quadratic Equations by Completing the Square

Perfect Square Trinomials:

Perfect square trinomials are formed by squaring binomials.

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Quadratic equation: $ax^2 + bx + c = 0$ In a perfect square trinomial $c = \left(\frac{b}{2}\right)^2$

Square root property: $x^2 = n$ implies that $x = \sqrt{n}$ and $x = -\sqrt{n}$

Example 9.2.1: Find c – term to make this a perfect square trinomial.

a) $x^2 + 6x + \underline{\hspace{2cm}}$

$$c = \left(\frac{6}{2}\right)^2 = 9$$

b) $y^2 - 5y + \underline{\hspace{2cm}}$

$$c = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$

c) $p^2 + \frac{1}{2}p + \underline{\hspace{2cm}}$

$$c = \left(\frac{\frac{1}{2}}{2}\right)^2 = \frac{1}{16}$$

Solving Quadratics by Completing the Square:

Goal is to make any trinomial a perfect square trinomial on one side of the equation, then use square root property.

- Put equation in $ax^2 + bx + c$ form.

$$\text{Ex. } x^2 - 6x + 7 = 0$$

- If $a \neq 1$, divide both sides by a .

- Move the c -term to the other side.

$$\text{Ex. } x^2 - 6x = -7$$

- Determine the value of $c = \left(\frac{b}{2}\right)^2$ and add it to both sides of the equation.

$$\text{Ex. } x^2 - 6x + \left(\frac{-6}{2}\right)^2 = -7 + \left(\frac{-6}{2}\right)^2$$

- Once you have a perfect square trinomial rewrite it in a factored form as a squared binomial.

$$\text{Ex. } x^2 - 6x + 9 = 2$$

$$(x - 3)^2 = 2$$

- Take square root of each side and continue solving for the given variable.

$$\text{Ex. } \sqrt{(x - 3)^2} = \sqrt{2}$$

$$x - 3 = \pm \sqrt{2}$$

$$x = 3 \pm \sqrt{2}$$

Example 9.2.2: Solve, by completing the square.

a) $2x^2 + 5x + 10 = 0$

After dividing each side by 2

$$x^2 + \frac{5}{2}x + 5 = 0$$

$$x^2 + \frac{5}{2}x + \left(\frac{5}{2}\right)^2 = -5 + \left(\frac{5}{2}\right)^2$$

$$x^2 + \frac{5}{2}x + \frac{25}{16} = -5 + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = -\frac{55}{16}$$

$$\sqrt{\left(x + \frac{5}{4}\right)^2} = \sqrt{-\frac{55}{16}}$$

$$x = -\frac{5}{4} \pm \frac{\sqrt{-55}}{4}$$

No real solution.

Note: Solution is $x = \frac{-5 \pm i\sqrt{55}}{4}$

b) $-3z^2 - 8z + 1 = 0$

After dividing each side by -3

$$z^2 + \frac{8}{3}z - \frac{1}{3} = 0$$

$$z^2 + \frac{8}{3}z + \left(\frac{8}{3}\right)^2 = \frac{1}{3} + \left(\frac{8}{3}\right)^2$$

$$x^2 + \frac{8}{3}x + \frac{16}{9} = \frac{1}{3} + \frac{16}{9}$$

$$\left(x + \frac{4}{3}\right)^2 = \frac{19}{9}$$

$$x = -\frac{4}{3} \pm \frac{\sqrt{19}}{3}$$

$$x = \frac{-4 \pm \sqrt{19}}{3}$$

9.3 Solve Quadratic Equations by Quadratic Formula

Quadratic formula is derived by solving quadratic equation $ax^2 + bx + c = 0$ by completing the square method.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Quadratics by Quadratic Formula:

- Set an equation in the standard form $ax^2 + bx + c = 0$
- Identify values of a, b, and c
- Substitute into the formula and evaluate using order of operations.

Example 9.3.1: Solve using formula.

a) $x^2 + 3x + 2 = 0$

$$a = 1 \quad b = 3 \quad c = 2$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)} = \frac{-3 \pm \sqrt{9 - 8}}{2} = \frac{-3 \pm 1}{2}$$

$$x_1 = \frac{-3 + 1}{2} = -1 \quad x_2 = \frac{-3 - 1}{2} = -2$$

Solution: $\{-2, -1\}$

b) $4x^2 - 12x + 9 = 0$

$$x = \frac{-(-12) \pm \sqrt{144 - 4(4)(9)}}{2(4)} = \frac{12 \pm \sqrt{0}}{8} = \frac{12 \pm 0}{8} = \frac{3}{2}$$

Solution: $\{\frac{3}{2}\}$

Using Discriminant to determine the number and types of solutions to quadratics equations:

For $ax^2 + bx + c = 0$ the expression $b^2 - 4ac$ is called the *discriminant*.

- If $b^2 - 4ac > 0$, then there will be **two real solutions**.

Graphically it implies two x – intercepts.

If $b^2 - 4ac$ is a perfect square; the solutions will be rational numbers.

If $b^2 - 4ac$ is not a perfect square; the solutions will be irrational numbers.

- If $b^2 - 4ac < 0$, then there will be **two imaginary solutions**.

Graphically it implies no x – intercepts.

- If $b^2 - 4ac = 0$, then there will be **one real solution**.

Graphically it implies one x – intercept.

Example 9.3.2: Use discriminant to determine the number and type of solutions to each equation.

a) $2x^2 - 5x + 9 = 0$

$$b^2 - 4ac = (-5)^2 - 4(2)(9) = 25 - 72 = -47$$

Since the discriminant is negative there will be two imaginary solutions.

b) $3x^2 = -x + 2$

$$3x^2 + x - 2 = 0$$

$$b^2 - 4ac = 1^2 - 4(3)(-2) = 1 + 24 = 25$$

Since the discriminant is positive and a perfect square there will be two real and rational solutions.

c) $x^2 + 8x = -16$

$$x^2 + 8x + 16 = 0$$

$$b^2 - 4ac = 8^2 - 4(1)(16) = 64 - 64 = 0$$

Since the discriminant is zero there will be one real solution.

Example 9.3.3: For each equation, find:

- The discriminant and use it to determine if there are any x – intercept(s).
- The x – intercept(s), if they exist.
- The y – intercept.

a) $f(x) = x^2 - 3x + 1$

$$(-3)^2 - 4(1)(1) = 9 - 4 = 5$$

There will be two x – intercepts.

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$X - \text{intercepts are: } \left(\frac{3 + \sqrt{5}}{2}, 0\right) \text{ and } \left(\frac{3 - \sqrt{5}}{2}, 0\right)$$

$$f(x) = 0^2 - 3(0) + 1 = 1$$

Y – intercept: (0, 1)

b) $f(x) = x^2 + x + 2$

$$1^2 - 4(1)(2) = 1 - 8 = -7$$

There will be NO x – intercepts.

$$f(0) = 2$$

Y – intercept: (0, 2)

Example 9.3.4: A blogger earns $R(x) = 6x + 0.031x^2$ in revenue, where x is the number of subscribers, and has a cost of $C(x) = 2350 + 5x$.

Let $P(x)$ be the profit when there are x subscribers.

- a) Write a function giving the blogger’s profit when there are x subscribers.

$$P(x) = R(x) - C(x) = 6x + 0.031x^2 - (2350 + 5x) = 0.031x^2 + x - 2350$$

$$P(x) = 0.031x^2 + x - 2350$$

- b) Use that function to determine the least number of subscribers the blogger needs to break even.

$$0.031x^2 + x - 2350 \geq 0$$

$$0.031x^2 + x - 2350 = 0$$

$$x = \frac{-1 \pm \sqrt{1 - 4(0.031)(-2350)}}{2(0.031)} = \frac{-1 \pm \sqrt{1 - 4(0.031)(-2350)}}{2(0.031)} = \frac{-1 \pm \sqrt{292.4}}{0.062}$$

Since x represents the number of subscribers, we will be only looking at positive solution.

$$x = \frac{-1 + \sqrt{292.4}}{0.062} \approx 259.67$$

Blogger needs at least 260 subscribers.