Chapter 7: Rational Expressions and Functions

7.1 Rational Expressions and Functions

Rational expression P/Q, where P and Q are algebraic expressions with no common factor other than 1.

Rational expressions are **undefined** at value(s) that:

- Make denominator equal to zero.
- Make radicand negative under even n-th root.

Example 7.1.1: Determine the value(s) of the variable for which the rational expression is undefined.

a) $\frac{x + 1}{x - 1}$ b) $\frac{\sqrt{x+2}}{3}$ c) $\frac{5z}{z^2 - 6z + 8}$ d) $\frac{a^2 + 2a - 3}{15}$

a) Undefined at $x = 1$
b) Undefined at $x < -2$
c) Undefined at $z = 2, z = 4$
d) Defined for all values

Evaluating and Simplifying Rational Expressions:

Example 7.1.2: Evaluate.

a) $\frac{5x - 7}{2x^2 + 3x - 1}$ at $x = 2$

$\frac{3}{13}$

b) $\frac{3p^2 - 1}{p^3 + p^2 + p + 4}$ at $p = -1$

$\frac{2}{3}$

Example 7.1.3: Simplify by reducing to lowest terms (reduce all common factors).

a) $\frac{12ab^3}{4a^2b} = \frac{3b^2}{a}$

b) $\frac{(x-4)(x+5)}{(x-3)(x+5)} = \frac{(x-4)}{(x-3)}$

c) $\frac{t^2-9}{t-3} = \frac{(t+3)(t-3)}{(t-3)} = t+3$

d) $\frac{2z^2+11z+12}{3z^2+11z-4} = \frac{(2z+3)(z+4)}{(3z-1)(z+4)} = \frac{2z+3}{3z-1}$

Operations with Rational Expressions:

Rational expressions are added/subtracted and multiplied/divided the same way as arithmetic fractions.

Example 7.1.4: Perform an indicated operation and simplify.

a) $\frac{x^2-7x+12}{x}\frac{x^2}{x-3} = \frac{(x-4)(x-3)}{x}\frac{x^2}{x-3} = x(x-4) = x^2-4x$

b) $\frac{(a-b)}{3b} \div \frac{(a-b)^2}{b^3} = \frac{(a-b)}{3b}\frac{b^3}{(a-b)^2} = \frac{b^2}{3(a-b)}$

c) $\frac{4}{x+1} + \frac{5x}{x+1} = \frac{5x+2}{x+1}$

d) $\frac{5}{4x} - \frac{2}{3x} = \frac{3}{3}\frac{5}{4x} - \frac{4}{4}\frac{2}{3x} = \frac{15-8}{12x} = \frac{7}{12x}$

e) $\frac{3}{t+2} - \frac{2}{t-2} + \frac{t-1}{t^2-4} = \frac{3(t-2)-2(t+2)+t+1}{(t+2)(t-2)} = \frac{3t-6-2t-4+t-1}{t^2-4} = \frac{2t-11}{t^2-4}$

7.4 and 7.5 Rational Equations

Solving Rational Equations:

Can be solved by using inverse operations to isolate x.

OR

Using LCD method by finding LCD for all present fractions and multiplying each term by LCD to clear all denominators then continue solving the resulting equation (linear, quadratic, etc.).

Ex.

$\frac{5}{x} - \frac{1}{3} = \frac{1}{x}$ LCD = 3x

$\frac{3x}{1}\frac{5}{x} - \frac{3x}{1}\frac{1}{3} = \frac{3x}{1}\frac{1}{x}$

15 – x = 3

x = 12

Example 7.4.1: Solve each of the following equations.

a) $\frac{3}{x-1} + \frac{2x}{x+1} = 2$

LCD = (x + 1)(x – 1)

After multiplying each term by LCD and clearing all fractions we get:

$3(x + 1) + 2x(x - 1) = 2(x^2 - 1)$
$3x + 3 + 2x^2 - 2x = 2x^2 - 2$
$x = -5$

{– 5}

b) $2 - \frac{4}{x} = \frac{x+1}{5}$

LCD = 5x

After multiplying each term by LCD and clearing all fractions we get:

$10x - 20 = x^2 + x$
$x^2 - 9x + 20 = 0$
$(x - 4)(x - 5) = 0$
$x = 4 \qquad x = 5$

{4, 5}

c) $-\frac{2a}{a+1} = 1 + \frac{2}{a+1}$

LCD = a + 1

After multiplying each term by LCD and clearing all fractions we get:

$-2a = a + 1 + 2$

$-3a = 3$

$a = -1$

{ }

Note: *When substituting a = – 1, it makes denominator equal to zero, which is undefined, therefore, there is no solution to this equation.*

d) $\frac{2}{t^2 - 1} - \frac{1}{2} = \frac{1}{t - 1}$

LCD = $2(t + 1)(t - 1)$

After multiplying each term by LCD and clearing all fractions we get:

$2(2) - (t + 1)(t - 1) = 2(t + 1)$
$4 - (t^2 - 1) = 2t + 2$
$t^2 + 2t - 3 = 0$
$(t + 3)(t - 1) = 0$
$t = -3 \quad t = 1$

$\{-3\}$

Example 7.4.2: Shanice can build a brick wall in 5 hours, while her apprentice can do the job in 6 hours.

a) How much of the job does Shanice complete in 1 hour?

$\frac{1}{5}$

b) How much of the job does her apprentice complete in 1 hour?

$\frac{1}{6}$

c) Suppose Shanice and her apprentice work together for t hours. How much of the job will they complete in that time? Write an expression.

$\frac{1}{5}t + \frac{1}{6}t$

d) How long does it take for them to build a wall together? Express your answer as a fraction if necessary.

$30\left(\frac{1}{5}t + \frac{1}{6}t\right) = 1(30)$
$6t + 5t = 30$
$t = \frac{30}{11}$

Example 7.4.3: Suppose there are three pipes filling a tank. The first pipe can fill the tank in 3 hours, the second in 7 hours, and the third in 8 hours. Suppose the pipes run for 3(7)(8) = 168 hours.

a) How many tanks can the first pipe fill?

$\frac{168}{3} = 56$

b) How many tanks can the second pipe fill?

$\frac{168}{7} = 24$

c) How many tanks can the third pipe fill?

$\frac{168}{8} = 21$

d) How many tanks can all three pipes fill in 168 hours?

56 + 24 + 21 = 101

e) How long will it take for all three pipes to fill 1 tank?

$$\frac{168}{56 + 24 + 21} = \frac{168}{101}$$

Example 7.4.4: A cabinet maker can build a kitchen island in 20 hours. If the cabinet maker's apprentice helps, they can construct the kitchen island in 17 hours.

Let *h* represent the number of hours it takes the apprentice to complete the job alone. How long would it take apprentice to complete a kitchen island alone?

$17(\frac{1}{20} + \frac{1}{h}) = 1$

$\frac{17}{20} + \frac{17}{h} = 1$

$20h\ (\frac{17}{20} + \frac{17}{h}) = 1\ (20h)$

$17h + 340 = 20h$

$340 = 3h$

$h = \frac{340}{3}$

It would take apprentice $\frac{340}{3}$ hours to complete a kitchen island alone.