Chapter 4: Systems of Linear Equations

4.1 and 4.2 Solving Systems of Linear Equations

System of two linear equations in two variables means having two linear equations, graphically corresponds to having two lines.

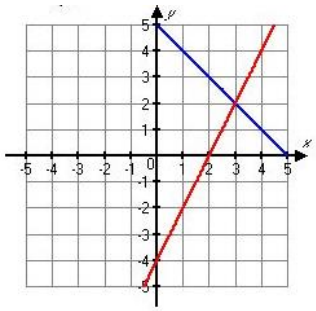
To solve a system, means to find coordinates of intersection point(s) if any. Can be done graphically or algebraically.

Intersecting Lines

One solution

Ex. **x + y = 5**

y = 2x – 4



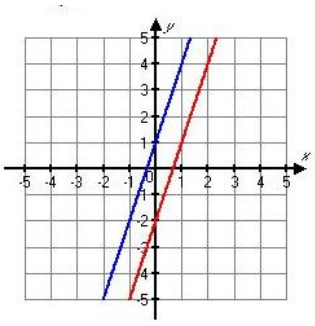
Solution: { (3,2) }

Parallel Lines

No solution

Ex. **y = 3x + 1**

y = 3x – 2



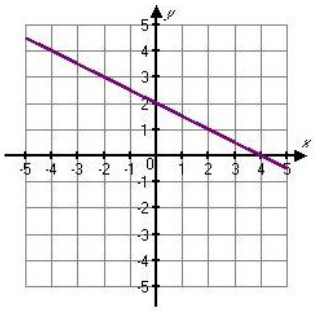
Solution: { }

Coincident Lines

Infinitely many solutions

Ex. **x + 2y = 4**

2x + 4y = 8



Solution: { (x, y) | x + 2y = 4 }

Graphing Method:

Step 1: Graph each equation using any method learned in class.

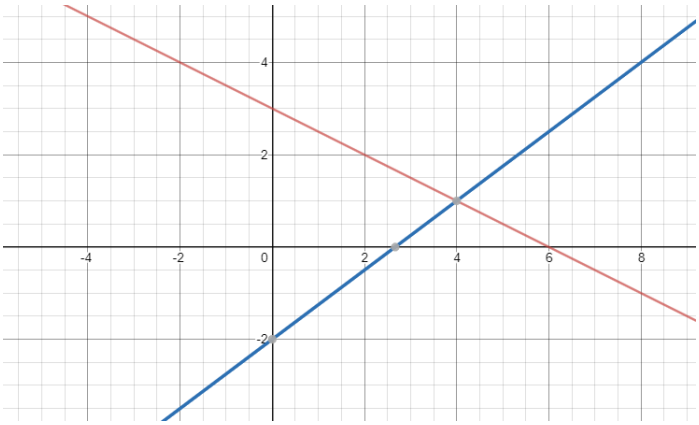
Step 2: Identify whether lines intersect (one solution; the point of intersection), are parallel (no solution) or coincide (infinitely many solutions).

Step 3: Write solution accordingly.

Example 4.1.1: Solve systems of equations graphically.

a) $y = -\frac{1}{2}x + 3$

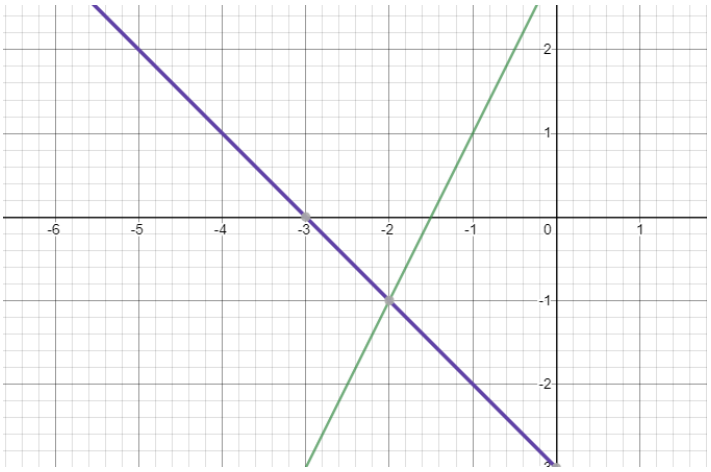
$y = \frac{3}{4}x - 2$



Solution: {(4,1)}

b) $6x - 3y = -9$

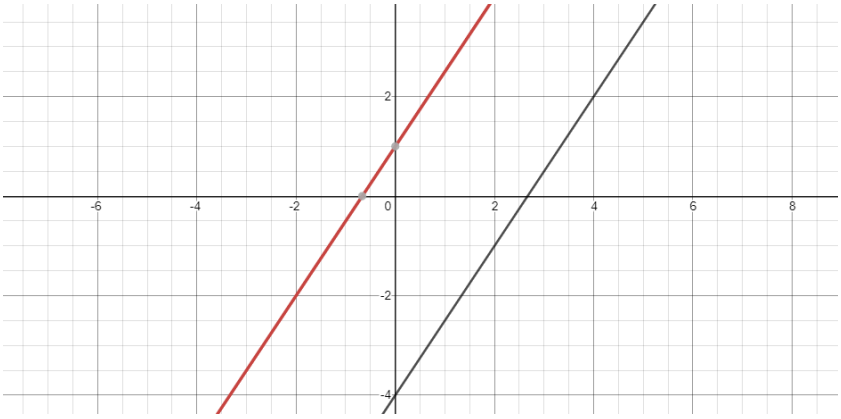
$2x + 2y = -6$



Solution: {(– 2, – 1)}

c) $y = \frac{3}{2}x - 4$

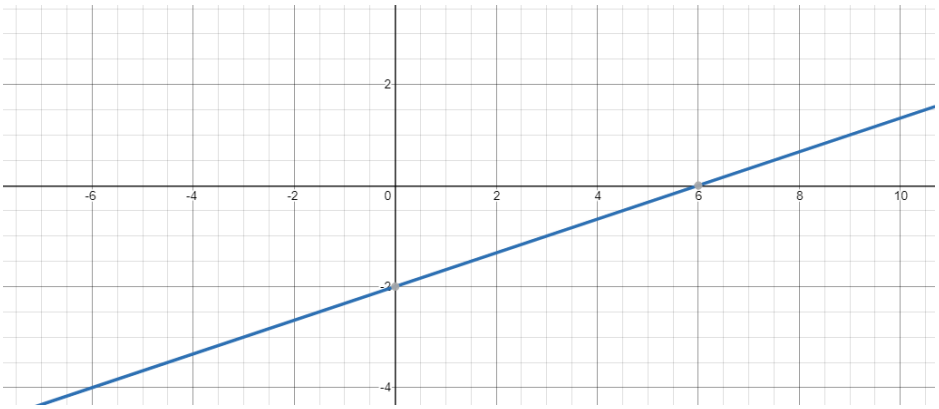
$y = \frac{3}{2}x + 1$



No solution: { }

d) $2x - 6y = 12$

$3x - 9y = 18$



Infinitely many solutions. $\{(x, y) \mid 2x - 6y = 12\}$

Algebraic Methods:

Solving a System of Linear Equations Using Substitution Method:

Ex.

$x + y = 5$

$y - 2x = -4$

Step 1: Solve either of the equations for one of the variables.

$x = 5 - y$

Step 2: Substitute what you got from step 1 into the other equation for the same variable you had solved in the previous step.

$y - 2(5 - y) = -4$

Step 3: Solve the resulting equation for the given variable.

$y - 10 + 2y = -4$

$3y = 6$

$y = 2$

Step 4: Substitute the value you obtained in step 3 into the equation from step 1 to find the value of the other variable.

$x = 5 - 2 = 3$

Step 5: Check your solution. Then write solution as an ordered pair (x, y).

$3 + 2 = 5$

$2 - 2(3) = -4$

Solution: { (3, 2) }

Solving a System of Linear Equations Using Elimination by Addition Method:

Ex.

$x + y = 5$

$y - 2x = -4$

Step 1: Put both equations into standard form Ax + By = C.

$x + y = 5$

$-2x + y = -4$

Step 2: Determine which variable you want to eliminate. Look for the variables having same coefficients but opposite signs, if needed, multiply one or both equations by an appropriate integer that makes coefficients the same with opposite signs.

To eliminate x, multiply both sides of the top equation by 2.

$2(x + y) = (5)2$

$-2x + y = -4$

Step 3: Add two equations; the variable will be eliminated. Then solve for the other variable.

$$\begin{array}{r} 2x + 2y = 10 \\ + \\ \underline{-2x + y = -4} \\ \\ 3y = 6 \\ y = 2 \end{array}$$

Step 4: Substitute the value you obtained in step 3 into the either of the two original equation to find the value of the other variable.

$x + y = 5$

$x + 2 = 5$

$x = 3$

Step 5: Check your solution. Then write solution as an ordered pair (x, y).

$3 + 2 = 5$

$2 - 2(3) = -4$

Solution: {(3, 2)}

Note: *When while using either of the algebraic methods both variables are gone, and you get:*

Number = Number

ex. 0 = 0, which is TRUE, then a system has ***infinitely many solutions*** *(coincident line).*

Number = Different number

ex. 0 = 5, which is FALSE, then a system has ***no solution*** *(parallel lines).*

Example 4.1.2: Solve systems of equations algebraically.

a) $2x - 3y = 7$
$y = 3x - 7$

Solution: $\{(2, -1)\}$

b) $3x + 2y = 1$
$x - 5y = 6$

Solution: $\{(1, -1)\}$

c) $y + 4 = 3x$
$2y - 6x = -8$

$0 = 0$ True Infinitely many solutions.

Solution: $\{(x, y) \mid -3x + y = -4\}$

d) $6x - 3y = -9$
$-2x + y = 5$

$0 = 6$ False No solution

Solution: $\{\ \}$

Example 4.1.3: A personal trainer has been offered two options for his salary as a trainer at the gym. Option A would pay him $ 20,500 plus $20 for each training session. Option B would pay him $ 17,000 plus $ 25 for each training session. After how many sessions will the salary options be equal?

Let x represent the number of training sessions.

With Option A the salary is: 20,500 + 20x

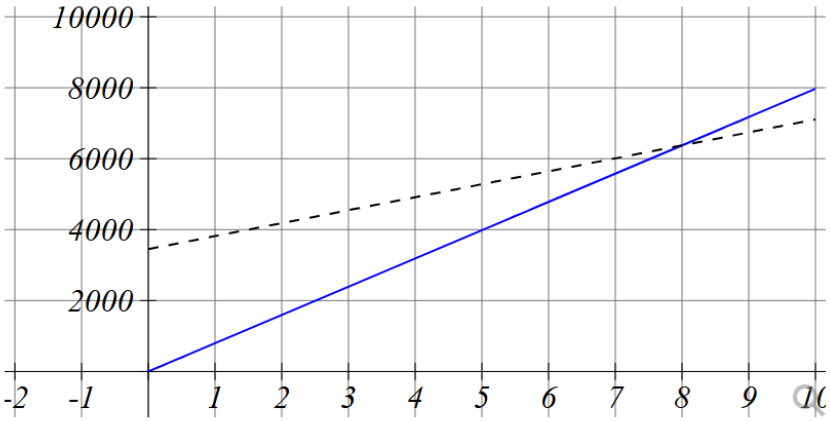
With Option B the salary is: 17,000 + 25x

20,500 + 20x = 17,000 + 25x

x = 700

The salary will be equal after 700 sessions.

Example 4.1.7: In the graph provided, *R(x)* (solid) shows the revenue a bicycle company makes when bikes are sold, and *C(x)* (dashed) is the cost the company incurs for each bike sold. See below.



How many bikes must the company sell to break even?

Company must sell 8 bikes.

Example 4.1.4: A small ice cream company estimates its revenue (the money earned from selling ice cream) to be R = 6x dollars, where x = the number of ice creams sold. The ice cream company estimates its costs by C = 5.5x + 250 dollars, where x = the number of ice creams sold.

a) What is the selling price of each ice cream?

   $ 6

b) Write equation that represents the profit from selling x ice creams.

   P = R – C
   P = 6x – (5.5x + 250) = 0.5x – 250

c) In order to break even, at least how many ice creams must the company sell?

   $P \geq 0$ or $R \geq C$
   $0.5x - 250 \geq 0$
   $x \geq 500$

   Company needs to sell at least 500 ice creams.

d) What will be the profit if 250 ice creams are sold?

   P = 0.5(250) – 250 = – $ 125

   Note: *Negative value represents the loss.*

e) What will be the profit if 550 ice creams are sold?

   P = 0.5(550) – 250 = $ 25

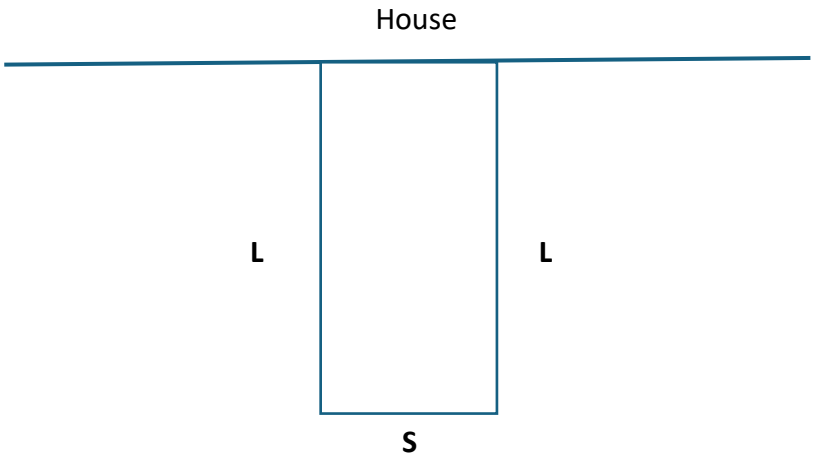
Example 4.1.5: You want to put a fence around the dog run in your back yard. One side of the dog run will be adjacent to the house. There are two long sides and one shorter side parallel to the house. You have 264 feet of fencing to enclose the dog run. The length of the long side is 3 feet less than four times the length of the short side.

a) Write an equation for L, the length of the long side, in terms of S, the length of the short side.

L = 4S – 3

b) Find dimensions in feet of the dog run.

Draw a diagram.



You need perimeter (add up all sides) of the rectangle without one short side which is adjacent to the house.

Fencing = 2L + S = 264

264 = 2(4S – 3) + S

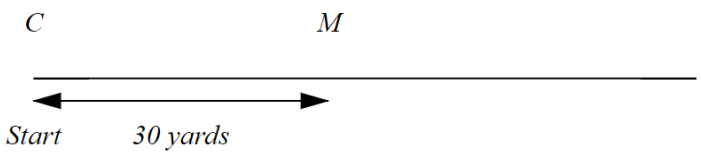
264 = 8s – 6 + s

S = 30

L = 4(30) – 3 = 117

Dimensions of the dog run are 117 by 30 feet.

Example 4.1.6: A mouse and a cat start 30 yards apart. At t = 0, the cat begins chasing the mouse, running towards it at 4 yards/second, while mouse begins running away from the cat at 3 yards/second. See below.



a) Write an equation (in terms of *t*) to find out when the cat catches the mouse.

Distance = Speed • Time

Cat will catch the mouse, when the distance from the starting point for cat and for mouse is the same.

$D_{cat} = 4t + 0$
$D_{mouse} = 3t + 30$

$4t = 3t + 30$

b) Determine when cat catches up with mouse.

$t = 30$

Cat will catch the mouse after 30 seconds.