Chapter 2: Solving Linear Equations

2.1 Use General Strategies to Solve Linear Equations

Equation is a statement where two expressions containing are equal to each other, with equal sign "=" placed in between.

An Equation is like a balance scale. Everything must be equal on both sides. Whatever you do to one side of the equation, you must do exactly the same to the other side of the equation, to keep it at balance.

The solution to the equation is a number(s) that make the statement true; left side = right side.

Linear equation is an equation where the highest exponent of a given variable is one.

Ex. $x + 5 = 7$ $2x = 10$ $x - 8 = 5$ $\frac{x}{7} = 49$

To solve a linear equation, we need to isolate given variable. To do so, we need to use inverse operations (addition/subtraction and multiplication/division).

Example 2.1.1: Solve.

$x + 5 = 7$ $2x = 10$ $x - 8 = 5$ $\frac{x}{7} = 49$

$x = 2$ $x = 5$ $x = 13$ $x = 343$

Example 2.1.2: Solve each equation.

a) $20v = -160$

$v = -8$

b) $16x = 320$

$x = 20$

c) $-16 + n = -13$

$n = 3$

d) $p - 8 = -21$

$p = -13$

e) $-15 = x - 16$

$x = 1$

f) $-8k = 120$

$k = -15$

g) $-15 = \frac{x}{9}$

$x = -135$

Note: *When fraction = fraction, we may cross-multiply to solve an equation.*

Example 2.1.3: Solve and check.

a) $5x + 7 = 7$

$5x = 0$
$x = 0$

b) $4 - 2x = 10$

$-2x = 6$
$x = -3$

c) $4x - 20 = -8$

$4x = 12$
$x = 3$

Note: *It is recommended to first start from adding/subtracting then multiply/divide.*

Steps to Solve Linear Equations:

1. Distribute through any parentheses.

2. Combine like terms on each side of the equation.

3. Isolate the variable by bringing it on one side.

4. Solve the remaining 2-step equation.

5. Check your answer by plugging it back to the original equation.

Example 2.1.4: Solve and check.

a) $-21x + 12 = -6 - 3x$

$-18x + 12 = -6$
$-18x = -18$
$x = 1$

b) $-7(x - 2) = -4 - 6(x - 1)$

$-7x + 14 = -4 - 6x + 6$
$-x + 14 = 2$
$-x = -12$
$x = 12$

c) $-6(8k + 4) = -8(6k + 3) - 2$

$-48k - 24 = -48k - 24 - 2$
$-24 = -26$
$0 = -2$
False statement, thus NO solution.

d) $-(n + 8) + n = -8n + 2(4n - 4)$

$-n - 8 + n = -8n + 8n - 8$
$-8 = -8$
$0 = 0$
True statement, thus infinitely many solutions.

Note: *When while solving an equation, all variables are gone, and you get:*

number = another number (ex. 2 = 5, 0 = 3), which is false, it means an equation has NO solution.

number = itself (ex. 2 = 2, 0 = 0), which is true, it means there are INFINITELY many solutions.

Rational Equations:

Consider an equation: $\frac{2}{x} + \frac{1}{3} = \frac{4}{x}$

We may solve it as we did before, by using inverse operations to isolate x.

Or we may use LCD method; multiply each term by LCD, reduce all denominators and continues solving resulting non-rational equation.

$\frac{2}{x} + \frac{1}{3} = \frac{4}{x}$ LCD = 3x

$\frac{3x}{1}\frac{2}{x} + \frac{3x}{1}\frac{1}{3} = \frac{3x}{1}\frac{4}{x}$

6 + x = 12

x = 6

Note: *You may also cross-multiply, but ONLY when you have a fraction = another fraction.*

Example 2.1.5: Solve and check.

a) $\frac{2}{3}x - 2 = \frac{3}{2}x + \frac{1}{6}$

LCD = 6

Multiply both sides by 6 and simplify, all denominators will be reduced.

4x – 12 = 9x + 1
– 5x = 13
$x = \frac{-13}{5}$

b) $\frac{3}{x+5} = \frac{2}{x+1}$

3(x + 1) = 2(x + 5)
3x + 3 = 2x + 10
x = 7

c) $\frac{15x}{18} + \frac{12}{54} = 3$

Multiply both sides by 54 and simplify, all denominators will be reduced.

3(15x) + 12 = 162
45x + 12 = 162
45x = 150
$x = \frac{150}{45} = \frac{10}{3}$

Example 2.1.6: A student gets an average of 65 on three exams. Let *x* be the score on the third exam. If the first two exams have scores of 54 and 68:

a) Write an equation you can use to solve for *x.*

$\frac{54+68+x}{3} = 65$

b) Find the score on the third exam.

x = 73

Example 2.1.7: A runner starts 70 meters away from the finish line. Suppose that their distance to the finish line decreases by 3 meters every second.

a) Write an expression in terms of *t* (time in seconds) representing their distance from the finish line in meters after t seconds.

70 – 3t

b) After how many seconds will the runner reach the finish line?

70 – 3t = 0 $t = \frac{70}{3}$

2.3 Solving a Formula for a Specific Variable

Literal equations are equations that contain mostly letters (variables), usually they represent formulas.

- It is specified for which letter (variable) they must be solved.

- To solve for a specific letter (variable), treat all other letters (variables) as constants and isolate a specific letter (variable) the same way as you would solve regular equations.

Example:

Regular	**vs**	**Literal**	
$3x = 12$		$wx = z$	In both examples, *x* is multiplied by something
$\frac{3x}{3} = \frac{12}{3}$		$\frac{wx}{w} = \frac{z}{w}$	To isolate the *x* we divide by the coefficient of 3 or w
$x = 4$		$x = \frac{z}{w}$	Solution

Example 2.3.1: Solve each for a specified letter.

a) $m + n = p$ for n

$n = p - m$

b) $y = mx + b$ for m

$y - b = mx$

$m = \frac{y-b}{x}$

c) $p = 2l + 2w$ for l

$p - 2w = 2l$

$\frac{p-2w}{2} = l$

$l = \frac{p}{2} - w$

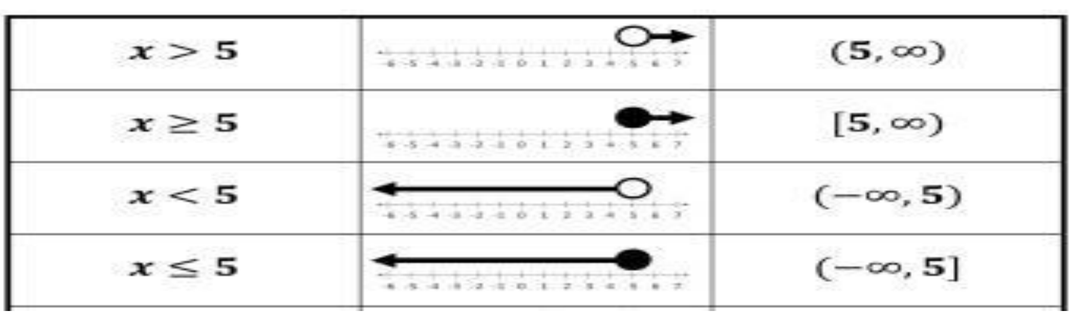
2.5 Solving Linear Inequalities

Inequalities:

Symbols: **>, ≥, <, ≤, ≠**

Example:

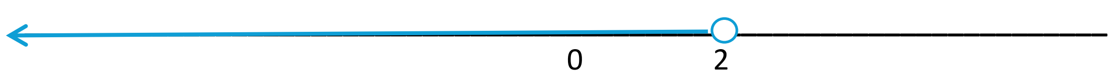
Symbol	Description	Example	Solution Set
>	Greater than, more than	x > 3	All numbers greater than 3; does not include 3
≥	Greater than or equal to, at least	x ≥ 3	All numbers greater than or equal to 3; includes 3
<	Less than	x < 3	All numbers less than 3; does not include 3
≤	Less than or equal to, at most	x ≤ 3	All numbers less than or equal to 3; includes 3
≠	Not equal to	x ≠ 3	Includes all numbers except 3



Algebraic solution	Graphic solution	Interval notation
$x > 5$		$(5, \infty)$
$x \geq 5$		$[5, \infty)$
$x < 5$		$(-\infty, 5)$
$x \leq 5$		$(-\infty, 5]$

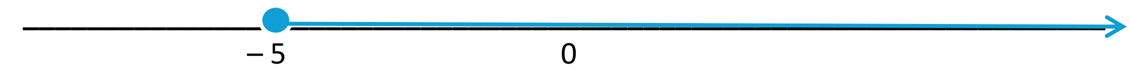
Example 2.5.1: Graph and write solution in interval notation.

a) x < 2



(– ∞, 2)

b) $y \geq -5$



[– 5, ∞)

c) $b < 0$

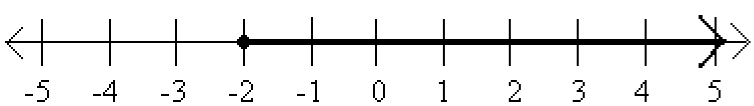
(– ∞, 0)

d) $4 \leq k$

[4, ∞)

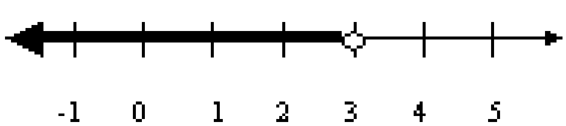
Example 2.5.2: Give the algebraic solution and interval notation solution from its graph. You may use x to represent the solution.

a)



$x \geq -2$ [– 2, ∞)

b)



$x < 3$ (– ∞, 3)

Solving Linear Inequalities:

Linear inequalities are solved the same way as linear equations, the only difference is to have inequality symbol instead of equality and when **<u>you multiply/divide by a negative number the inequality symbol changes its order</u>** (from $>$ to $<$, from $<$ to $>$, from $\geq$ to $\leq$, from $\leq$ to $\geq$).

Example 2.5.3: Solve, graph, and write solution in interval notation.

a) $5 - 2x > 11$

$-2x > 6$

$x < -3$

$(-\infty, -3)$

b) $3(2x - 4) + 4x \leq 4(3x - 7) + 8$

$6x - 12 + 4x \leq 12x - 28 + 8$

$10x - 12 \leq 12x - 20$

$-2x \leq -8$

$x \geq 4$

$[4, \infty)$

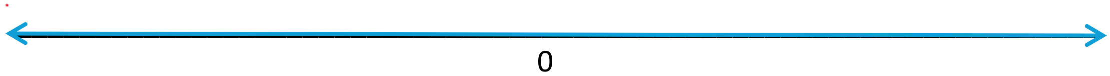
c) $-(k - 2) > -k - 20$

$-k + 2 > -k - 20$

$-k > -k - 22$

$0 > -22$ True

Infinitely many solutions



$(-\infty, \infty)$

Example 2.5.4: You can afford to spend no more than 90 dollars per month on a video game plan. If your video game plan costs 30 dollars per month, plus 4 dollars per game downloaded how many games can you afford to download?

Let *x* represent the number of games.

a) Write an inequality that represents the number of games you can afford to buy.

$$4x + 30 \leq 90$$

b) Determine how many games you can afford to buy.

$$x \leq 15$$

You can afford to buy no more than 15 games.

Example 2.5.5: A student's grade is based on the average of three exams, where an average of 60 is a passing grade. If the student's grade on the first two exams is 22 and 34, the students must get at least how many points on third exam to have an average of 60 in the course?

$$\frac{22 + 34 + x}{3} \leq 60$$

$$56 + x \leq 180$$

$$x \leq 124$$

It is impossible for the student to pass the course.