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Integrating Digital and Audiovisual Resources into Problem-Based Learning: An Interdisciplinary Experience in Mathematics

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Abstract: This research explores the impact of integrating digital resources and audiovisual media within problem-based learning (PBL) in the mathematics classroom, highlighting its didactic and research potential. A teaching proposal was implemented with 11th-grade students in the science–technology track, using a TV series segment to contextualize a mathematical function and GeoGebra for dynamic visualization. A case study with an instrumental and interpretive approach was conducted, based on non-participant classroom observation, comparative analysis, and a semi-structured teacher interview. Results show that PBL enriched with digital and audiovisual tools promotes a shift from procedural to conceptual reasoning, supporting knowledge construction through exploration, collaboration, and contextualized problem solving. This approach increases student motivation, engagement, and conceptual understanding, while fostering transversal skills such as digital literacy, critical thinking, and teamwork. Findings align with constructivist and sociocultural learning theories. However, challenges were identified, including the teacher's facilitation role, the design of open-ended problems, and unequal access to technological resources. Despite these limitations, the approach demonstrates clear benefits in helping students address complex problems in a more structured and critical way.

Keywords: Problem-based learning, PBL, didactic learning situation, teaching and learning of mathematics, GeoGebra, audiovisual resources, interdisciplinary teaching

INTRODUCTION

The methods in teaching and learning mathematics have been changing significantly over the years due to the various trends in educational innovation that are increasingly being implemented in secondary education classrooms. Therefore, practicing teachers must seek methodological strategies that represent a shift in their educational practices (Carbonell et al., 2009), where the procedure takes precedence over the final result (De Guzmán, 2007) and, above all, help introduce students to

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meaningful learning that allows them to reason logically and systematically, and helps them develop their mathematical skills to understand and solve real-world problems (Jaramillo & Puga, 2016; Organization for Economic Cooperation and Development [OECD], 2023).

To achieve an effective and productive teaching-learning process in mathematics, it is necessary to create contextualized learning situations that promote the discussion and reflection of the various theoretical-practical concepts taught in the subject (Pifarré & Sanuy, 2001). In this regard, given that students today are immersed in a society where the use of audiovisual media is ubiquitous, it would be beneficial to use them as didactic tools to enhance student interest and motivation in learning mathematics (Zapatera, 2018). One way to use audiovisual resources in the classroom to develop mathematical concepts is through movies and TV shows, as they are among the most captivating media for students, along with video games and social media (Beltrán-Pellicer & Asti, 2014).

Although there are numerous compilations of movies and TV shows where mathematics is present, either from a historical or sociocultural perspective or through reasoning and problem-solving in realistic contexts, research on their didactic use in education is relatively scarce. This may be due, among other things, to the lack of training for mathematics teachers or the challenge of applying changes to traditional teaching methods (Ballesta & Guardiola, 2001; Spanhel, 2011; Beltrán-Pellicer & Asti, 2014). However, this must change, as teachers must face the challenges imposed by the educational community over time.

One of the current challenges that mathematics teachers face is providing integrated training that develops competence-based learning, allowing mathematics to be contextualized transversally with other scientific, technological, or even epistemological disciplines. According to Lehrer (2021) and Tytler et al. (2021), using interdisciplinary proposals in the classroom promotes the transfer of knowledge between mathematics and other areas of knowledge in order to build a structured and integrated system that facilitates problem-solving. In this way, the teacher can help students stop viewing mathematics as something unattainable and realize its great usefulness.

In this sense, a good strategy to achieve this is to incorporate problem-based learning (PBL) as an educational methodology in the teaching and learning of mathematics from an interdisciplinary perspective. It is essential that students actively participate in the learning process and that the teacher involves them in solving complex, real-world problems, rather than simply transmitting content passively (Bermúdez, 2021). According to Smith et al. (2022), PBL enables students to understand the relationship between what they are learning and its potential applications in real life, making it a comprehensive, constructive, and contextualized approach (Dolmans et al., 2005).

Moreover, the PBL methodology will also improve students' digital competence, particularly in terms of their skills in Information and Communication Technologies (ICT), as students will need to search for important data and extract accurate, verified information from the internet for the problem-solving process within the PBL framework to be effective (Ortiz et al., 2025). In this regard, although introducing the use of specific dynamic mathematics software for calculation or the representation of mathematical concepts is not necessary in PBL, it is highly recommended, as it

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broadens the range of competencies acquired in the learning process, while also acting as a motivational element and helping to understand the concepts involved in the problem.

In this context, this work aims to delve into a pedagogical framework that addresses the didactic use of cinema and TV series in mathematics education, but from an interdisciplinary perspective. This means selecting movie or TV show segments that introduce situations or problems within a realistic context to foster meaningful learning, critical and creative thinking, and mathematical flexibility in students. To this end, an innovative didactic situation was designed based on previous theoretical studies that allowed us to connect PBL with audiovisual resources and digital tools.

To analyze the impact of this methodology, we designed a proposal for 11th grade high school students (16-17 years old) based on the study of functions, as we consider it to be an essential mathematical concept for studying the behavior of many real-life phenomena. After its implementation, we were able to explore how this methodology influences the professional development and pedagogical skills of teachers and how it affects the teaching and learning process of mathematics, including aspects such as student motivation, conceptual understanding, problem-solving, and transversal competencies.

RESEARCH GAP

Although there is a growing body of research exploring the use of audiovisual media to motivate and engage students in mathematics education, as well as studies on the application of dynamic mathematics software like GeoGebra and the implementation of PBL methodologies individually, there is a notable lack of research that explicitly integrates all three elements within an interdisciplinary educational framework. While prior research has demonstrated the individual benefits of audiovisual resources for increasing student engagement, the effectiveness of dynamic mathematical software like GeoGebra for enhancing conceptual understanding, and the value of PBL for fostering transversal skills, these approaches have typically been explored in isolation.

A search conducted across several academic databases (including Scopus, ERIC, Web of Science, and Google Scholar) did not identify any studies that systematically combine all three elements (audiovisual media, dynamic mathematics software, and PBL) within a single interdisciplinary teaching model. Although some research has explored pairwise combinations, such as PBL with GeoGebra (e.g., Suratno & Waliyanti, 2023; Ramadhani & Narpila, 2018; Septian et al., 2020; Martín-Cudero, 2024; Martín-Cudero & Rodas, 2025; Martín-Cudero et al., 2025), or films and mathematics education (e.g., Beltrán-Pellicer & Asti, 2014; Beltrán-Pellicer, 2015; Zapatera, 2017; Beltrán-Pellicer et al., 2018; Dofková & Škutléty, 2021), an integrated pedagogical framework that unites all three to promote mathematical understanding, flexibility, and cross-disciplinary competencies remains absent. Addressing this gap is essential for advancing innovative, engaging, and context-responsive mathematics teaching practices aligned with contemporary educational demands.

RESEARCH QUESTION

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Given students' immersion in digital and audiovisual media, leveraging these resources in mathematical instruction presents an opportunity to enhance engagement and understanding (Zapatera, 2018). This study investigates how integrating audiovisual and digital resources in PBL impacts students' problem-solving skills and conceptual understanding, addressing a key teaching-research question:

Q1. How does integrating audiovisual and digital resources in PBL affect students' problem-solving skills in mathematics?

To contribute to teaching-research practices, this study:

1. Analyzes the iterative development of a PBL-based teaching methodology as well as its structure and potential.
2. Examines measurable student learning outcomes, including problem-solving ability and conceptual grasp.

PROPOSAL FOR A DIDACTIC LEARNING SITUATION

Theoretical Background

As previously mentioned, it is essential for mathematics teachers to seek alternatives to traditional instruction by incorporating motivating elements that introduce mathematical concepts in new and engaging ways (Beltrán-Pellicer & Asti, 2014). One such alternative is the use of films or TV shows that feature mathematics within their dialogue. These audiovisual resources can enhance students' perception of mathematics, increase motivation, and support the development of mathematical knowledge through diverse problem-solving strategies (Raga et al., 2009; Beltrán-Pellicer & Asti, 2014; Beltrán-Pellicer et al., 2018; Población, 2006; Dofková & Škutléty, 2021).

Obviously, screening entire films or episodes is not efficient if the only goal is to analyze a specific scene. In this sense, it is more common to use short clips embedded within a specific learning activity. In such tasks, students apply previously learned mathematical concepts to solve a problem related to the clip. As Población (2006) notes, this targeted approach allows teachers to control the clip's duration and replay key scenes as needed to aid comprehension. It also helps avoid student disengagement from unrelated content. Numerous reference materials, available online and in print, offer examples for teachers interested in adopting this approach (Polster & Ross, 2019; Raga et al., 2009; Kasman, 2012; Polster & Ross, 2012; Sorando, 2018; Sorando, n.d.; Roberts & Roberts, n.d.).

Within this broad spectrum of possibilities, Beltrán-Pellicer et al. (2018) highlight the importance of selecting clips based on their mathematical richness. Some clips offer limited content, useful for clarifying isolated concepts, while others present more substantial material suited for exploring broader mathematical ideas. The former can be used for brief in-class exercises, whereas the latter lend themselves to longer, more complex activities where students engage with real-world problems involving multiple mathematical concepts (Palmer, 1994; Meyer et al., 2001). In both cases,

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the goal is to create an optimal and meaningful mathematics teaching-learning process that synthesizes and organizes the mathematical notions explained in class. Beyond content richness, teachers must also consider how mathematical concepts are presented in the clip. Regarding format, some clips rely solely on narration (verbal), others use images, graphs, or situations (non-verbal), and some combine both. As for approach, clips may serve a theoretical purpose (conceptual discussion), a practical one (application-focused exercises), or a combination of both.

It is important to acknowledge that the mathematics portrayed in films or TV shows is typically not designed for educational use. Consequently, concepts, procedures, or calculations may lack clarity or mathematical rigor. Teachers must therefore contextualize each scene carefully, present the activity with clear objectives, and ensure the clip meets both technical and pedagogical standards. Thorough preparation is essential to avoid improvisation and maximize learning outcomes (Población, 2006). This process falls within what Verret (1975) termed *didactic transposition*, later adapted to mathematics education by Chevallard (1985). Chevallard distinguishes between *wise knowledge* (or academic knowledge), that is the formal, discipline-based understanding of mathematics; and *didactic knowledge* (or knowledge to be taught or taught knowledge), adapted by teachers to suit students' needs. For Chevallard, didactic transposition in mathematics involves transforming wise knowledge (the knowledge the teacher must understand) into content that students can meaningfully engage with. This process develops through a triangular relationship between the teacher, the content, and the student, who will construct their own knowledge (Mora, 2014).

The 21st century, marked by rapid technological and social changes, presents new challenges for education. Consequently, it is important to highlight the use of cross-disciplinary approaches among mathematics education professionals (Herce-Palomares et al., 2022). Using resources that integrate mathematics with other disciplines can help students stop viewing the subject as complicated and inaccessible and instead learn to value it and recognize its importance. As students progress, they often seek justifications for mathematical concepts and their real-world applications. Demonstrating mathematics' utility across disciplines becomes critical. However, classroom activities are frequently taught in isolation, without clear connections between subjects (Bartels et al., 2019). Mathematics has an abstract nature that causes rejection among many students, a problem that could be avoided if mathematics were taught in a cross-disciplinary way. By connecting mathematics with other fields, teachers can promote competency-based learning and enhance motivation (Herce-Palomares et al., 2022). Within this context, audiovisual resources can serve as powerful interdisciplinary tools.

While previous studies have shown that film and TV clips effectively boost student engagement and motivation, none of them use this resource as a potential interdisciplinary approach. That is, no study has systematically explored how the mathematical content of a scene might connect with other scientific, technological, artistic, or social disciplines. According to Chevallard's (1985) didactic transposition process, wise knowledge, in this context should encompass mathematical concepts applicable to other disciplines. Teachers must then transform this knowledge into teachable content using audiovisual materials. This increases the teacher's responsibility, not only must they

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select mathematically rich clips, but also ensure their interdisciplinary potential. The aim is to foster meaningful learning that stimulates both critical and creative thinking.

This is where PBL comes into play, as it is an ideal methodology to promote these skills. First, it allows students to confront problematic situations that require analysis, reasoning, and creativity to be solved (Dolmans et al., 2005). Instead of learning in a superficial or mnemonic manner, students must search for solutions to the problem in an integrated way, favoring long-term retention of knowledge (El Sayary et al., 2015). Additionally, through PBL, students become active agents in their learning process, thereby developing self-management skills and ability to independently seek out information (Domènech-Casal, 2018). It also fosters engagement and responsibility, helping students consolidate their knowledge (Pazos-Yerovi & Aguilar-Gordón, 2024).

By confronting real-world problems, students are motivated to explore and understand more deeply the practical value of the mathematical concepts they are learning, which enables them to make connections between different areas of knowledge (Tytler et al., 2021). Moreover, PBL also helps to develop transferable skills that go beyond mathematics, such as collaboration, communication, logical thinking, organization, and decision-making (Smith et al., 2022). These skills are valuable not only academically but also in future professional and personal contexts, promoting adaptability and flexible problem-solving (Savin-Baden & Major, 2004).

Finally, digital and technological competencies can be integrated into the PBL process, especially through internet research and educational software (Ortiz et al., 2025). In mathematics education, GeoGebra stands out as a versatile, free, and accessible tool that combines multiple mathematical domains in a single interactive platform. Its user-friendly design and visual learning capabilities make it suitable for diverse teaching styles and levels (Recio et al., 2019). Incorporating GeoGebra into PBL enhances conceptual understanding, as well as increasing student motivation and creativity (Yohannes & Chen, 2021).

Didactic Situation

Initially, PBL in mathematics education was often limited to text-based scenarios. However, early implementations revealed certain challenges, particularly regarding student engagement and accessibility. These findings highlighted the need to evolve towards more inquiry-based, collaborative, and real-world-oriented learning experiences (Dolmans et al., 2005).

To address these challenges, we propose a problem-based learning experience in which the didactic transposition process integrates film or TV series clips as contextual triggers for the problem, along with the use of GeoGebra to support students in understanding the mathematical concepts involved. We believe this approach can enhance student motivation and foster deeper mathematical competence from an interdisciplinary perspective.

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Phases of the Learning Process

The primary objective is to create a learning environment in which students, working in pairs or small groups, solve a real-world problem that allows them to transform prior knowledge into more structured and meaningful understanding. To achieve this, we find it appropriate to organize the learning process into the following phases:

1. Identification and understanding of the problem: Students analyze and internalize the prob lem context, fostering curiosity and ensuring a clear understanding of the task ahead.
2. Formulation: Guided by teacher-generated questions, students identify and analyze the mathematical concepts involved in the problem.
3. Practical application and information search: Students apply the identified mathematical concepts to solve the problem, engaging in information search, critical analysis, and creative reasoning. This phase reinforces knowledge transfer and promotes research, synthesis, and evaluation skills.
4. Proposing solutions, evaluation, and reflection: Students present and analyze their solutions and engage in reflective discussions to strengthen metacognitive and collaborative skills.

The introduction phase is crucial for PBL's success, as it marks the beginning of the construction of didactic knowledge. This is the moment when the teacher-student interaction initiates the didactic transposition process. As already mentioned, one of our intentions is to incorporate a clip from a movie or TV series in this phase to enhance student motivation and appropriately contextualize the problem. In this regard, Población (2006) points out that simply projecting a clip in class without adequate preparation is insufficient and may lead to confusion, disengagement, or misalignment with curricular goals. To prevent this, the teacher must carefully have clear objectives and carefully plan the activity to avoid improvisation

In line with our intervention proposal, it is also essential for the teacher to design the problem considering the use of GeoGebra during the practical application phase of PBL. GeoGebra provides an interactive platform where students can interact with content graphically, explore possible solutions, and verify their results, enhancing their understanding of the phenomenon under study.

Problem Design

For this study, we designed a PBL experience focusing on the topic of functions and their applications in real life, since it is a highly useful mathematical concept in other disciplines for drawing conclusions about the behavior of a specific phenomenon. From an educational perspective, this makes it a clear example of interdisciplinary practice. In this case, the phenomenon chosen was carbon-14 dating, as we believed it to be an interesting topic to explore with students who have a clear inclination toward experimental science, as is the case here.

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To introduce the problem, we selected a short clip (1 minute and 40 seconds) from the TV series "NUMB3RS" (Dettman, 2005), where mathematics is applied to solve complex police cases. In the selected scene, the character Charlie, a mathematics professor, explains the principles of carbon-14 dating in a clear and relatable manner using an everyday analogy (ice cubes in a refrigerator). The dialogue illustrates the key mathematical idea behind radioactive decay and introduces students to the concept of exponential functions.

Part of the transcript of the scene is as follows:

- *Charlie*: They are formulas to calculate the age of an object.
- *Partner*: Radiation is used there, right?
- *Charlie*: Look, all living beings absorb radioactive carbon-14 into their bodies: plants through photosynthesis and animals through food by eating plants or other animals. Like all radioactive material, the atoms decay and are only replenished if the organism is alive. It's like those refrigerators that make ice cubes. Let's say it is programmed to always make fifty ice cubes. If we use five cubes a day, it will make another five to compensate for the ones we've used and will always maintain fifty if the refrigerator is connected to a water supply. Death is like turning off the water; no more cubes are made. So, if we subtract the number of cubes left, say thirty-five, from the fifty that were there and divide by five cubes per day…
- *Partner*: The freezer has been off for three days…
- *Charlie*: This is a linear example, and radioactive carbon is exponential, but that's the idea!

In addition to the dialogue, the scene includes mathematical notations and formulas that serve as visual aids, supporting students in solving the problem at hand. For example, it displays the expression describing how the quantity of radioactive nuclei $N(t)$, such as carbon-14, or any substance undergoing exponential decay, decreases over time, as well as the formula for the half-life, which defines the time required for $N(t)$ to reduce to half its original value. This half-life is expressed as a function of the decay constant, which governs the rate at which the decay process unfolds.

Using this scene as a starting point, we worked collaboratively with the classroom teacher to design a problem-based learning (PBL) activity closely aligned with the intended curricular goals. The teacher placed particular emphasis on key competencies such as conceptual understanding of mathematical functions, the capacity to apply mathematical models to interpret and solve real-world phenomena, the ability to independently seek out and validate relevant scientific data, and the development of reflective problem-solving skills, including assessing the validity of solutions and justifying one's reasoning through mathematical argumentation.

The complete activity, structured in clearly defined phases and accompanied by an evaluation of its pedagogical effectiveness, is detailed in the Appendix of this article. The activity focuses on three core areas of mathematical knowledge: exponential functions, logarithmic functions, and inverse functions. These concepts are explored within an interdisciplinary framework that incorporates

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fundamental ideas from physics, specifically those that require mathematics as a tool for interpretation and understanding (see Table 1).

Wise Knowledge	Didactic Knowledge	Learning Outcomes
Exponential Function	Radioactive decay equation: $N = N_0 e^{-\lambda t}$.	Understanding the relationship between decay and time.
	Study of the evolution of the function $N(t)$.	Ability to model and understand natural phenomena using exponential functions.
	Application in radioactive decay contexts.	Identification and problem-solving involving exponential functions.
Logarithmic Function	Solving for t in the equation $N = N_0 e^{-\lambda t}$.	Skill in solving logarithmic equations applied to radioactive decay.
	Relationship between the logarithmic function and the decay time.	Understanding how logarithmic functions are useful in solving growth and decay problems.
Inverse Function	Relationship between the functions $N(t)$ y $f(N)$.	Use of the inverse function to find time t from a quantity N.
	Practical application to determine the time elapsed since the organism's death.	Understanding the inversion of exponential and logarithmic functions in problem-solving.
Limits and Behavior of Functions	Limit at infinity in the graph of $N(t)$ (relationship with the horizontal asymptote).	Understanding the long-term behavior of the decay function.
	Study of the domain, monotonicity, and curvature of the function $N(t)$ $(t \geq 0)$	Ability to analyze the increasing or decreasing behavior of a function in a physical context, as well as the rate at which it grows or decays over time.

Table 1: Classification of the knowledge and learning outcomes derived from the proposed PBL.

Moreover, these central topics naturally bring into play additional mathematical concepts that, while not always explicitly emphasized, are essential for a deep understanding of functional behavior over time. These include, among others, the concept of a function's domain and range, the behavior of a function near specific points (e.g., limits), the presence and nature of asymptotes, as well as properties such as monotonicity and curvature. All of these elements contribute to a richer comprehension of how a function evolves and how its mathematical properties reflect real-world processes, such as radioactive decay or population dynamics. In this sense, students are encouraged not only to interpret graphical representations and algebraic expressions, but also to establish meaningful connections between mathematical models and the phenomena they describe.

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METHODOLOGY

Approach

The approach adopted in this research is purely qualitative, as the objective is to understand the mathematics teacher's perception of the methodological proposal we developed. Specifically, a case study with an instrumental and interpretive approach was, serving as a means to explore, illustrate, and deepen a broader phenomenon (Stake, 2007) and to understand the meanings, perspectives, and experiences of the teacher within their particular context. This approach also allowed us to capture the depth and complexity of the methodology from a subjective and contextualized framework (Lincoln & Guba, 1985; González-Monteagudo, 2001; Bassey, 2003).

Context and Participants

The PBL was carried out in a mathematics class at a public secondary education school in Madrid, Spain, during April 2024, over two sessions of fifty-five minutes each. Specifically, the introduction and problem formulation phases took place during the first session, with the remaining phases completed in the second session. The sessions took place in a computer lab, where students worked in groups of two or three, each group using a computer to complete the exercise (8 groups of two students and 3 groups of three).

The participating teacher in the experience holds a degree in physics, with a Pedagogical Aptitude Certificate since 2005, and has 19 years of experience teaching mathematics and physics and chemistry from first-year secondary education to second-year high school. Given the teacher's background and extensive teaching experience, we considered them to be the appropriate profile for conducting this research. On the other hand, the class level was not considered highly relevant, as prior to implementation, we shared the designed PBL with the teacher, who adapted it as needed for the students' level.

Data Collection and Evaluation Instruments

The main sources of evidence were classroom observations, comparative analysis, and a semi-structured teacher interview. We used non-participant direct observation, whereby the researchers remained in the background without interfering with the teacher's work or interacting with the class (Cohen & Manion, 2002). This allowed us to analyze the behavior of both teacher and students throughout the different phases of the experience, as well as evaluate the situations that arose during the learning process. Qualitative data were collected through descriptive field notes (Lofland & Lofland, 1984), enabling identification of behavioral patterns among students and trends in the teacher's conduct.

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On the other hand, students' performance was assessed through a comparative analysis of problem-solving accuracy before and after the activity. Days before the PBL implementation, the teacher assigned an exercise requiring analytical study of the main properties of the function $f(x)=e^{-kx}$ for all $k>0$. Note that the function $N(t)=N_0 e^{-\lambda t}$ is a scaled version of $f(x)$. This allowed us to assess the students' analytical and deductive abilities by introducing the function within a contextualized, real-life applicable problem. In this way, we also evaluated whether the integration of the audiovisual resource helped students better relate to the activity and if the GeoGebra tool enhanced their understanding of the problem.

However, the primary evaluation instrument was the interview conducted with the teacher after the experience concluded. Based on their experience and training, the teacher provided objective and relevant insights regarding their perception of the methodology's structure and potential, as well as the teaching skills and implications involved in its implementation. The semi-structured interview allowed us to adapt questions according to the flow of conversation, delving into relevant topics that emerged naturally and exploring perspectives that were not initially anticipated (Adams, 2015). This enabled them to express their opinions from a more holistic perspective (Naz et al., 2022).

The questions asked were as follows:

1. What motivated you to implement this activity in one of your classes?
2. What do you think are the advantages of using PBL in secondary and high school mathematics classes?
3. Do you consider it a good idea to include clips from movies or TV series in mathematics classes? Do you think it is appropriate to integrate this into a PBL methodology?
4. Do you think it is appropriate to include ICT tools such as GeoGebra or web searches, as used in this experience, in secondary and high school education? Do you think it is appropriate to incorporate them into PBL?
5. Do you often implement interdisciplinary activities in your class? Do you think this approach is necessary in secondary and high school mathematics?
6. Please provide your overall opinion about the activity. Would you change anything in its structure or content?

Data Analysis Procedure

Due to institutional restrictions and privacy considerations, we were not permitted to record audio from the sessions or collect written responses from students. As a result, all student-related data were gathered exclusively through non-participant observation and descriptive field notes taken by the researchers during classroom implementation. These observations focused on capturing students' verbal interactions, problem-solving strategies, levels of participation, and spontaneous expressions of understanding or confusion, among other things. After each session, the researchers

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conducted reflective meetings to review and triangulate their observations, identifying recurring behavioral patterns, noticeable changes in student engagement, and the evolution of mathematical understanding throughout the activity. These reflective discussions allowed the team to consolidate their insights and ensure consistency in the interpretation of classroom dynamics. The findings related to student learning outcomes are presented in a generalized manner, derived from consensus-based interpretation of observed classroom interactions and problem-solving behaviors. This methodological decision was aligned with the study's focus on understanding the overall effectiveness and feasibility of the proposed instructional approach within a real classroom setting, taking into account the constraints and complexities typical of educational environments.

RESULTS AND DISCUSSION

Classroom Observations

To evaluate the effectiveness of this methodological approach, a comparative analysis of student performance was conducted before and after the PBL implementation, using the same group of students. Prior to the activity, students were given an analytical task to study the properties of the function $f(x)=e^{-kx}$ for all $k>0$ without any contextualization or digital tool support. Their responses reflected a procedural approach, largely focused on derivative calculations and sign analysis. From classroom observation, most students demonstrated only a superficial grasp of the function's behavior. After the PBL experience, where the problem was framed through a TV series clip and supported by GeoGebra for graphical analysis, students demonstrated a significantly deeper comprehension of the function's behavior. Most of students were able to explain key mathematical properties such as monotonicity, asymptotes, and domain more effectively and linked these aspects to the context of carbon-14 decay.

Observations indicated that students in the PBL displayed higher levels of engagement, particularly during the problem formulation and solution phases. The use of audiovisual resources appeared to generate curiosity and facilitate contextualized understanding. Additionally, GeoGebra allowed students to visualize the function dynamically, leading to better conceptual retention. In fact, we noticed that many students used GeoGebra during the problem formulation phase to graph the functions $N(t)$ and $f(N)$. This reflects an initiative to interact directly with mathematical concepts graphically rather than opting for a purely analytical study. It also demonstrates conceptual reasoning in understanding functional relationships and composition of functions, going beyond the procedural level and engaging with structural aspects of mathematics.

In this regard, Godino et al., (2012) state that many students find it easier to learn mathematical concepts through visual representations, as these make abstract concepts more concrete and easier to understand. In contrast, when students tackled the problem using traditional methods often relied on rote memorization and exhibited difficulties in conceptualizing the decay function without graphical aids. Their solutions lacked the depth of reasoning observed in the PBL. Although $f(x)$ and $N(t)$ are algebraically equivalent, their interpretation varies significantly in the context

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of the problem, revealing students' ability to distinguish between formal equivalence and contextual meaning, an essential aspect of conceptual reasoning.

Regarding the students' behavior, we observed generally positive reactions. Introducing the problem through a clip from a television series captured their attention and sparked curiosity but also created some agitation. According to Vásquez et al. (2024), doing something different from the traditional in the educational context often generates a mix of excitement, curiosity, and even initial resistance, as students may perceive it as a challenge. However, this unease also has a positive side, as it is a sign that students are engaged in learning, even if they do not express it immediately (García-Alonso et al., 2021).

As for collaborative work, we observed good communication and exchange of ideas among the members of each group, as well as the teacher's active cooperation at key moments of the activity. In general, what one member of the group did not understand, another one did, showing a high level of active listening and cooperation towards a common goal. This promotes knowledge construction through interpersonal interaction, reducing the sense of anxiety that some students may experience when working on the problem in isolation (Glinz, 2005; Cabezas et al., 2016).

On the other hand, observations also revealed the pedagogical content knowledge that the teacher used to address the activity (from now on, we will refer to the teacher as Leonhard). Following the research by Carrillo et al. (2018), elements of knowledge related to the teaching of mathematics and the characteristics of Leonhard's learning, as described in the Mathematics Teacher's Specialized Knowledge model, were identified. In particular, Leonhard effectively used the audiovisual resource to contextualize the activity and demonstrated the ability to organize the problem's information, recognizing the epistemological aspects of the phenomenon involved.

As noted by Zamorano et al. (2019) and Tytler et al. (2021), in addition to understanding the intellectual, emotional, and affective demands of students, the teacher must possess the necessary knowledge of the skills and abilities developed in interdisciplinary practice to perform their work effectively. In this regard, Leonhard displayed a solid understanding of the mathematical concepts involved and of mathematical practice, using representation registers to clarify the content and strengthen the interdisciplinary focus of the problem (D'Amore, 2004). Although after introducing the problem, Leonhard adopted a more passive role, allowing the students to build their own learning, they interacted with the workgroups to guide them and resolve doubts, especially during the problem formulation phase, where students faced the most significant difficulties.

Semi-structured Interview

The results of the interview we conducted with Leonhard turned out to be particularly significant. Leonhard states that what motivated him to implement PBL was the practical and applied focus of the problem, as, in his view, this allows students to see the usefulness of mathematics beyond the abstract and contextless exercises typically done in class. Additionally, he highlighted the originality of how the activity was introduced and the relevance of the content, considering the students' profile. According to Leonhard, involving students with a strong interest in experimental science in

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exploring scientific phenomena that require mathematical concepts and procedures is highly recommended.

This perspective aligns with other studies, such as Simms (2016), which indicate that students interested in experimental science are often more motivated when they can directly apply mathematics to their interests, which increases their engagement and curiosity. Furthermore, using mathematics to interpret experimental results teaches them to analyze, model, and validate hypotheses, improving their ability to reason logically and systematically (Godino & Burgos, 2020). This highlights how the activity fosters conceptual reasoning through the application of mathematical modeling in scientific contexts, encouraging students to go beyond procedural calculation and engage in critical and systematic thought processes. According to Bosch et al. (2006), many real-world phenomena depend on mathematical models to be understood, so actively engaging with this connection prepares students to tackle complex problems practically.

On the other hand, Leonhard also emphasized the historical connection of the activity, as he believes that this cultural component makes it even more attractive. In this regard, De Guzmán (2007) suggests that mathematics has a vast cultural dimension that can be discovered through its history, enriching it and integrating it into the body of scientific, artistic, and humanistic knowledge that makes up culture. Research on the connections between the history of mathematics and mathematics education has gained relevance, becoming an interdisciplinary field of study (Chorlay et al., 2022). For this reason, we agree with Leonhard that it would be valuable to explore the historical and sociocultural context in the introductory phase of PBL as a resource to enhance students' interest.

Regarding the methodology, Leonhard considers that PBL stands out particularly when it is integrated into a global approach that promotes the development of all the students' competencies, not just the mathematical ones. Through this approach, students not only absorb mathematical concepts, but also improve other key skills such as autonomy, digital competence, interpersonal skills, and the ability to learn how to learn. This approach enables them to tackle more complex problems than traditional exercises, strengthening their experience and confidence. In this regard, Godino et al. (2004) point out that mastering mathematics involves more than merely recognizing definitions, formulas, and properties; it also requires using language and concepts to establish connections between mathematical elements and the context in which they are developed, in order to give them meaning. In fact, one of the pillars of PBL is to avoid memorization and to introduce students to a process of reflection in which they themselves model their knowledge and interact with the content autonomously (Savin-Baden & Major, 2004).

Leonhard also positively evaluated the inclusion of film or TV show clips in PBL, as he believes this strategy captures students' attention and contextualizes learning in a fun and interesting way. Moreover, it helps connect theory with situations that students can recognize, making the problem more relatable and relevant to them. However, he pointed out that finding a scene that addresses the desired concepts in a relevant and didactic way is not an easy task, and few teachers would be willing to make that extra effort. Beltrán-Pellicer and Asti (2014), Beltrán-Pellicer et al. (2018), and Dofková and Škutléty (2021) support this view and add that the clip must also be clear enough for

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students to understand the content without requiring too much additional explanation from the teacher. Additionally, if the clip requires too much context to be understood, the focus could shift away from the mathematical content, and students might lose interest (Población, 2006; Raga et al., 2009; Sorando, 2018). Not all students respond the same to audiovisual media as a teaching tool, and some may not feel as connected to this approach (Medina, 2010). In any case, Leonhard considers it essential to seek strategies that connect mathematics with reality, and he believes this is a good way to do so, as long as the clip truly contributes to the learning process, as otherwise, it would not make sense to include it in PBL.

When Leonhard was asked about his opinion on the use of ICT tools, he stated that their integration in education is essential, as they contribute to the development of students' digital and technological competencies while simultaneously fostering research and experimentation skills, enabling students to explore and make decisions independently. He also emphasized that it is a way for students to learn how to organize the knowledge they acquire, which leads to deeper and more meaningful learning. For this reason, Leonhard adds that PBL cannot be conceived without the use of ICT tools, as the search for information is inherent in any contextualized and interdisciplinary problem. However, studies such as those by Drijvers (2013), De Witte and Rogge (2014), or Revelo-Rosero (2020) warn that simply incorporating ICT does not guarantee a transformation of educational practice; rather, it is the way in which the teacher introduces them that can enhance this resource and enrich learning. For this reason, teacher training in this area is crucial (Beneyto-Seoane & Collet-Sabé, 2018; Batiibwe, 2024).

Regarding GeoGebra, Leonhard considered it a tool with great potential, especially in the areas of functions and geometry. He also pointed out that including it in PBL was crucial for students to understand the phenomenon being studied, as without this tool, many students would not have been able to propose solutions to the problem analytically. He concluded that it is absolutely necessary to integrate GeoGebra or any other similar software in the teaching and learning of mathematics if we want our students to receive the most complete education possible. This view is shared by Furner (2024), who emphasizes that in a technology-driven world, engaging students in learning experiences like PBL supported by GeoGebra is crucial to boosting mathematical performance. Previous research (e.g., Ramadhani & Narpila, 2018; Septian et al., 2020; Suratno & Waliyanti, 2023; Martín-Cudero et al., 2025) has shown how the integration of GeoGebra improves students' analytical skills and problem-solving abilities.

According to Leonhard, another feature that should not be absent in PBL, aside from the use of ICT tools, is interdisciplinarity. He mentions that whenever possible, he tries to incorporate activities with this approach into his classes, as he believes that the learning of mathematics is greatly enriched when connected to other areas of knowledge. In this regard, Stillman et al. (2023) highlight the importance of interdisciplinary work in mathematics education, as only a clear understanding of the interrelationship between mathematics and the real world will prevent students from perceiving them as abstract or disconnected from their interests. However, as pointed out by Barrios et al. (2022), interdisciplinarity, despite being a highly beneficial approach for teaching and learning, cannot arise spontaneously but must be well planned and organized to be effective; otherwise, it may cause confusion among students.

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Finally, when Leonhard was asked for his general opinion on the implemented proposal and if he suggested changing anything in its structure or content, he mentioned that overall, it was a very complete activity, appropriate to the level, and with a lot of potential. He assured us that students were very motivated and interested in the topic, which allowed him to see that this type of methodology had a great impact on their learning. However, he indicated that it is better for a PBL not to have a single guided solution but to involve the search for alternative solutions, thus promoting mathematical flexibility.

This process encourages students to engage in conceptual reasoning by creating and evaluating their own procedures, fostering deeper understanding and critical thinking rather than rote application. In this way, a reflective and discussion-based environment can be created at the end of the activity, allowing each working group to share its solutions with the rest of the class. If the solution to the problem is unique, this could not be explored, and a bit of the essence of PBL would be lost. Therefore, he suggests addressing more open problems where students have more opportunities to create and think of alternative procedures that provide solid, valid, and coherent solutions. In any case, he highlights the originality of the proposal and points out that undoubtedly, this type of approach that connects audiovisual media, digital tools, and interdisciplinarity adds significant value to the teaching of mathematics and fosters a more positive attitude toward the subject from students.

Adaptation of the Approach

Based on the findings, teachers looking to implement or adapt this methodology can do so by integrating PBL with digital and audiovisual resources, ensuring a dynamic and engaging learning experience. The key to successful replication lies in flexible adaptation based on the specific educational context, available resources, and student needs. To begin, teachers should identify real-world problems relevant to their subject area and student interests. These problems should be open-ended, complex, and encourage critical thinking, serving as the foundation for inquiry-based learning. Once the problem is established, teachers can curate or create audiovisual materials, such as interactive simulations, instructional videos, and multimedia case studies, to support students in exploring potential solutions.

Technology integration should be purposeful and strategic. Teachers can utilize learning management systems, collaborative online tools, and adaptive learning platforms to facilitate student interaction, research, and reflection. The key is to strike a balance between digital guidance and student autonomy, ensuring that technology enhances rather than replaces critical thinking and problem-solving processes. In this case, we propose fragments of movies or TV series and provide different databases so that teachers can search for the one that best suits their educational needs, but they can also perform an internal search or even choose another resource, such as video games or podcasts, for example. As for the digital tool, you can substitute GeoGebra for another one that you master better, as long as you teach your students how to use it before implementing the activity. For teachers with limited access to technology, adaptation is still possible by using readily available digital resources and fostering creativity in audiovisual content creation. Even simple tools such as mobile

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devices, free educational platforms, and classroom recordings can significantly enhance the PBL experience. Another important aspect is facilitating collaboration and discussion. Teachers should encourage teamwork, peer feedback, and structured debates to promote deeper engagement with the subject matter.

Finally, continuous professional development and collaboration with other teachers can support the effective adaptation of this methodology. Teachers can participate in online training sessions, educational forums, and peer-learning networks to exchange best practices and refine their approach over time. By embracing a mindset of experimentation and iteration, teachers can successfully replicate and customize this methodology to fit their classroom dynamics, ensuring an enriching and impactful learning experience for their students.

CONCLUSIONS

This study provides evidence that the integration of PBL with audiovisual resources and dynamic digital tools such as GeoGebra can promote a deeper and more meaningful understanding of mathematical concepts. Unlike traditional instruction, which often leads students to apply procedures mechanically, the implemented approach facilitated the transition from procedural to conceptual reasoning. This outcome is not only consistent with the expectations of constructivist and sociocultural learning theories but also illustrates how these theories materialize in classroom practice: students constructed knowledge through exploration, social interaction, and contextualized experiences that gave meaning to abstract content.

More specifically, the success of the intervention seems rooted in the synergistic effect of three components: contextualization (through a TV series clip), visualization (via GeoGebra), and collaborative inquiry (supported by PBL structures). The audiovisual introduction served as an authentic anchor for the problem, which—according to the socio-cultural perspective—helps reduce cognitive distance and fosters engagement. Meanwhile, GeoGebra enabled students to dynamically manipulate mathematical objects, thereby strengthening conceptual understanding through multiple representations. This is consistent with the pedagogical value of representation registers, as emphasized by Godino et al. (2012). These elements served to motivate students and, at the same time, facilitated access to the structural aspects of mathematical meaning.

Moreover, the data show that the teacher's shift toward a facilitator role and the open nature of the problem supported student autonomy and reflective thinking: two core pillars of PBL. These results confirm earlier findings in the literature, yet also reveal a key nuance: while PBL is often effective, its impact can be significantly enhanced when interdisciplinary connections and media-based engagement are intentionally planned, not incidental. This suggests that the approach did not merely replicate what the literature predicted, but amplified the known benefits of PBL through thoughtful integration of cultural, technological, and pedagogical elements.

In addition, the observed increase in students' transversal skills, such as digital competence, collaboration, and critical thinking, suggests that the approach supports holistic education. However, the analysis also identified potential limitations, such as the challenge of ensuring open-endedness in

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problem design and the teacher's critical role in scaffolding digital tools for meaningful use, echoing warnings from Drijvers (2013) and De Witte and Rogge (2014). These findings highlight that successful implementation depends not only on the methodology itself but on how it is adapted to specific classroom realities.

In sum, this research contributes to the field by showing how a carefully designed PBL approach, grounded in theory and enriched with media and technology, can fulfill and even exceed its theoretical promise. Future work could explore variations of this model in other mathematical topics and educational contexts to further investigate its potential and scalability.

Final Reflection, Limitations and Future Lines of Research

This study advances teaching-research by bridging the gap between innovative teaching practice and measurable student learning outcomes, providing a model that enhances both pedagogical strategies and educational effectiveness. Specifically, it strengthens teaching-research by: (a) providing a research-backed framework for incorporating PBL, digital tools, and audiovisual media into math education; (b) offering measurable evidence that these strategies enhance conceptual understanding, problem-solving, and interdisciplinary learning; and (c) demonstrating that teachers can replicate or adapt this approach to fit different mathematical topics and student levels. Ultimately, this research contributes to innovative mathematics instruction by merging engagement-driven teaching strategies with rigorous learning outcomes, ensuring both teacher effectiveness and student success in 21st-century education.

However, while the integration of PBL with digital and audiovisual resources has demonstrated significant educational benefits, it also presents several limitations. One of the main challenges is the accessibility and availability of digital resources, as not all schools or students have equal access to technology. This digital divide can create disparities in learning experiences and outcomes, particularly in underprivileged or rural areas where internet connectivity and device availability are limited. Another limitation is the teacher's digital proficiency and workload. Effective implementation of this methodology requires educators to be familiar with digital tools such as GeoGebra, as well as to develop the competence to design meaningful, interdisciplinary PBL activities that integrate audiovisual resources in a pedagogically sound manner. Rather than extensive technical training, what teachers may need is targeted support and guidance on how to create well-structured PBL problems that promote critical thinking and real-world application of mathematical concepts. Furthermore, developing and curating high-quality digital and audiovisual content can be time-consuming, placing extra demands on teachers who may already have heavy workloads. Without adequate support and professional development, the methodology's effectiveness could be compromised.

In addition, one of the methodological limitations of this study lies in the instruments used to evaluate student learning. Due to institutional restrictions, it was not possible to collect written student responses or audio recordings during the sessions. As a result, the analysis of student learning was based exclusively on non-participant observation and descriptive field notes, which limited the

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ability to access direct evidence of student reasoning, such as explanations, justifications, or problem-solving strategies. Consequently, the results had to be presented in a generalized way, without including verbatim student responses that could have enriched the analysis. Moreover, the assessment of student learning in a PBL environment enriched with digital tools remains a challenge. Traditional evaluation methods do not always align with the skills developed in this approach, such as problem-solving, creativity, and collaboration. Developing reliable and scalable assessment frameworks that capture these competencies in digital learning environments requires further exploration.

Finally, it is important to acknowledge that this study was conducted within a single implementation cycle. While the results provide valuable insights into the integration of PBL and digital tools, future research should consider multiple cycles. A second cycle would allow for iterative refinement of the instructional design based on observed student difficulties and provide more robust evidence on the transferability of the developed skills to other problem types and contexts. Pedagogically, the second cycle would incorporate structured metacognitive reflection at the end of each phase, enabling students to articulate their reasoning and evaluate their collaborative strategies. In addition, the teacher's facilitation role would be adjusted to provide scaffolded guidance for GeoGebra exploration, encouraging greater student autonomy while maintaining conceptual rigor. The audiovisual trigger could again stem from a popular science documentary or news segment rather than a fictional series, to test whether authenticity of context influences engagement and conceptual understanding. This redesigned cycle would not only strengthen the iterative nature of the research but also offer comparative insights into how contextual variation, reflective practice, and teacher mediation contribute to deeper mathematical learning.

In terms of future lines of research, a promising direction is designing new learning experiences based on mathematical modeling that integrate clips from movies and television series. Following the emerging approaches discussed by Martín-Cudero (2025) and Martín-Cudero and Rodas (2025), these experiences could expand the scope of media-based mathematics instruction, enhancing student motivation while deepening their conceptual and critical understanding of mathematical phenomena.

Another critical area for research is the long-term impact of digital and audiovisual PBL on student outcomes. While short-term studies highlight increased engagement and motivation, more longitudinal research is needed to assess how this methodology influences knowledge retention, critical thinking, and real-world problem-solving abilities over time. Additionally, exploring cross-disciplinary applications of this methodology could provide valuable insights. Research on its effectiveness in different subjects, age groups, and educational settings, including higher education and vocational training, could help refine best practices and optimize its implementation.

Finally, future studies could investigate strategies to bridge the digital divide, ensuring that all students benefit from this innovative approach regardless of their technological access. Research on low-cost or offline digital solutions, as well as blended learning models that combine traditional and digital resources, could make PBL with digital and audiovisual tools more inclusive and

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widely accessible. By addressing these limitations and advancing research in these key areas, educators and researchers can continue to enhance the effectiveness and scalability of this methodology, ensuring its sustainable integration into diverse learning environments.

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APPENDIX

A. Identification and understanding of the problem:

All living beings absorb radioactive carbon-14 into their bodies: plants through photosynthesis and animals by consuming plants or other animals. Like all radioactive materials, the atoms decay and are only replenished if the organism is alive. The proportion of carbon-14 remaining when the remains of an organism are examined gives an indication of the time that has passed since it died. The rate at which carbon-14 decays decreases proportionally to the number of atoms remaining after a certain period of time. This is expressed with the following equation:

$$N = N_0 e^{-\lambda t} \ ,$$

where N is the number of atoms remaining after a certain time t, N_0 is the number of carbon-14 atoms at time $t = 0$ (initial moment when decay counting begins), and λ is the decay constant (probability of decay per unit of time). The decay constant is related to the average lifetime (of a nucleus or subatomic particle) which is given by the expression:

$$t_m = \frac{1}{\lambda} \ ,$$

and this, in tun, is related to the half-life period $t_{1/2}$ (time required for half of the nuclei or subatomic particles of an initial sample of carbon-14 to decay) by the following expression:

$$t_{1/2} = t_m \cdot \ln 2.$$

B. Problem statement:

In order to understand the mathematical structure of the concepts to be studied later in the practical application phase, the following questions are posed:

1. Provide an expression to calculate N as a function of time and the half-life: $N(t)$. Study the most relevant properties of the function (domain, range, intercepts with the axes, asymptotes, monotonicity, and curvature).
2. Provide an expression to calculate t as a function of N and the half-life: $f(N)$.
3. What relationship exists between the two functions?

C. Practical application:

During an exciting archaeological expedition in the “Valley of the Kings”, one of the most emblematic and fascinating sites in Ancient Egypt, a team of researchers discovered a surprisingly

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well-preserved mummy inside an ancient Egyptian tomb. Intrigued, they decided to analyze a sample of its tissues to learn more about its age. The results revealed an interesting fact: the current radioactivity of the mummy is three-quarters of that of a living being.

1. Help scientists determine how much time has passed since this person was mummified.
2. Estimate how many carbon-14 atoms a living being might approximately have and, with that data, calculate how many of these radioactive atoms have decayed since the person was mummified.
3. Use GeoGebra to represent the function $N(t)$ and study its evolution over time based on its most significant properties. Do they match those found in the problem statement phase?
4. Search for information online about this process and compare the phenomenon with your results.

Evaluation of the effectiveness of the activity:

Once the activity is introduced, the first thing to ensure is that all students have understood the concepts involved in the activity. If not, they should be explained thoroughly before proceeding with the development of the task. Once this is understood, the first part of the activity is to express N as a function of t and $t_{1/2}$ from the equation $N = N_0 e^{-\lambda t}$. At this point, students must argue that $t_{1/2}$ depends on t_m, and that t_m depends on λ, so λ can be written in terms of t_m and $t_{1/2}$ as $\lambda = \ln 2 / t_{1/2}$. Consequently, we can express N as a function of t as follows:

$$N(t) = N_0 e^{-\frac{\ln 2}{t_{1/2}} t}.$$

As can be seen, basic concepts of "isolating" and "substituting" variables have been used, which are essential for students to master at this educational stage.

The next part of the activity involves finding an expression for time as a function of N and $t_{1/2}$. That is,

$$N = N_0 e^{-\frac{\ln 2}{t_{1/2}} t} \longrightarrow e^{-\frac{\ln 2}{t_{\frac{1}{2}}} t} = \frac{N}{N_0} \longrightarrow \ln\left[e^{-\frac{\ln 2}{t_{\frac{1}{2}}} t}\right] = \ln\left[\frac{N}{N_0}\right] \longrightarrow -\frac{\ln 2}{t_{1/2}} t \cdot \underbrace{\ln e}_{1} = \ln\left[\frac{N}{N_0}\right]$$

$$\longrightarrow -\frac{\ln 2}{t_{\frac{1}{2}}} t = \ln\left[\frac{N}{N_0}\right] \longrightarrow t = -\frac{t_{1/2} \cdot \ln\left[\frac{N}{N_0}\right]}{\ln 2}$$

It can be verified that the exercise becomes more complicated here because, in this part, students need to have a solid grasp of the properties of logarithms and the steps to correctly isolate a variable in the exponent of an exponential equation. At this stage, many students might think that they have

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made a mistake in isolating t because it comes out as negative. However, they need to reason that N can only be less than N_0, and consequently,

$$\ln\left[\frac{N}{N_0}\right]=\ln(N)-\ln(N_0)<0$$

because $\ln(N)<\ln(N_0)$ when $N<N_0$ (just remembering that the function $g(x)=\ln(x)$ is increasing throughout its domain). Finally, the relationship between $N(t)$ and

$$f(N)=-\frac{t_{1/2}\cdot\ln\left[\frac{N}{N_0}\right]}{\ln 2}$$

is clear because one is the inverse of the other.

A priori, the negative sign in the exponential should already be significant because it provides valuable information about the monotonicity of the function (it is decreasing throughout its domain). Moreover, simply by evaluating $N(0)$, students should notice that the function only intersects the y-axis at the point $(0,N_0)$, regardless of the value of N_0. Additionally, noticing the existence of a horizontal asymptote at $t=0$ will help them understand that as t increases, the number of carbon-14 atoms decreases until it disappears. This can be demonstrated analytically by calculating the following limit:

$$\lim_{n\to\infty} N_0 e^{-\frac{\ln 2}{t_{1/2}}t}=N_0 e^{-\infty}=\frac{N_0}{e^{\infty}}=\frac{N_0}{\infty}=0$$

The domain of $N(t)$ is immediate since the domain of an exponential function is the set of all real numbers. To calculate the image, one just needs to calculate the domain of the function $f(N)$, which is $N>0$.

To solve the problem posed, students must search online for the half-life period of carbon-14 and estimate the number of carbon-14 atoms in a living being. Otherwise, they will not be able to obtain a final solution to the problem. The value of $t_{1/2}$ is 5,730 years, and a better estimate for N_0 is 10^{12} carbon-14 atoms. However, these values may vary depending on the source consulted. This is a very important step in the learning process, as students must verify, by consulting multiple sources, that the information they obtain is valid.

The solution to the problem is:

1. Time elapsed since the person was mummified:

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$$f\left(0.75\,N_0\right) = -\frac{t_{\frac{1}{2}} \cdot \ln\left[\frac{0.75\,N_0}{N_0}\right]}{\ln 2} = -\frac{5{,}730 \cdot \ln\left(0.75\right)}{\ln 2} = 2{,}378.16 \quad years.$$

2. Carbon-14 atoms that have decayed since the mummification: If N_0 is the number of atoms at time $t = 0$, and N is the number of atoms at $t = 2{,}378.16$, then $N_0 - N$ atoms will have decayed in that time. That is,

$$N_0 - N = N_0 - 0.75\,N_0 = 0.25\,N_0 = 0.25 \cdot 10^{12} \quad atoms.$$

3. Representation of the function $N\left(t\right)$:

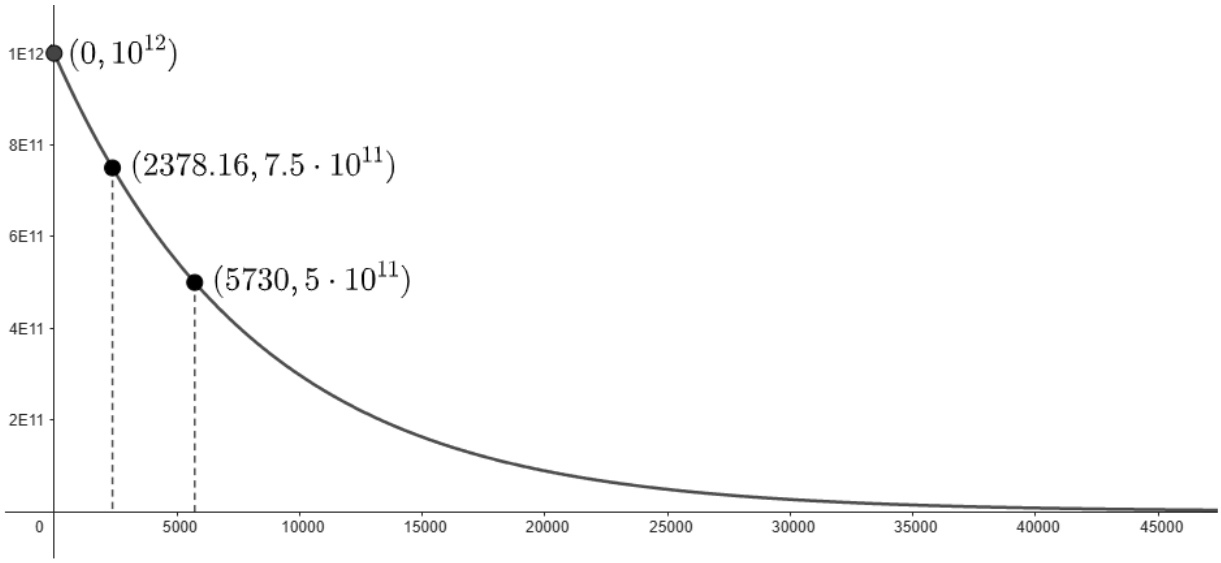

Figure 1: Representation of the function $N\left(t\right) = 10^{12}\,e^{-\frac{\ln 2}{5730}t}$ with GeoGebra.

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