Comparison of the Effects of Explanatory Faded Worked Examples and Correct-Incorrect Worked Examples Methods on Sixth-Grade Students' Mathematics Performance*

Özden Yavaş[1], Zeynep Çiğdem Özcan[2]

[1]Kartal Prof. Dr. Şaban Duralı Science and Art Center, İstanbul, Türkiye

[2] Department of Mathematics and Science Education, İstanbul Medeniyet University, İstanbul, Türkiye

ozdenyavas36@gmail.com, cigdem.ozcan@medeniyet.edu.tr

Abstract: Achieving high-level mathematics learning is a key goal of the mathematics curriculum; however, while some students excel, others encounter significant challenges. One strategy that has gained attention in recent years is the use of worked examples, a teaching method grounded in Cognitive Load Theory. This study compares the effects of two types of worked examples: Explanatory Faded Worked Examples (EFWE) and Correct and Incorrect Worked Examples (CIWE) on sixth-grade students' mathematics performance. A pre-posttest quasi-experimental design was used with 29 sixth-grade students from a public secondary school during the 2021–2022 academic year. Quantitative data were collected through a Mathematics Achievement Test and a Worked Examples Worksheet, while open-ended questionnaires captured student perspectives. The findings showed that the EFWE group had significant improvement in overall mathematics scores but no significant change in scores for specific subtopics like exponential notation, divisibility, or order of operations. In contrast, the CIWE group showed significant improvement in distributive property, exponential notation, and divisibility. ANCOVA results revealed that, after adjusting for pre-test scores, the CIWE group outperformed the EFWE group in order of operations and divisibility. Students from both groups found the Worked example methods useful, noting that they contributed positively to their learning experience.

Keywords: Cognitive load, worked example method, correct and incorrect Worked examples, explanatory faded Worked examples.

INTRODUCTION

Mathematics plays a crucial role in solving various problem situations encountered in daily life. Therefore, it is essential to develop students' numerical skills. One of the primary objectives of

* This study is derived from the first author's master's thesis completed at İstanbul Medeniyet University.

mathematics education is to ensure that students achieve the highest level of mathematical understanding (MoNE, 2018). However, while some students meet this goal successfully, others struggle to reach it. The abstract nature of mathematics often presents significant challenges for these students.

Research in mathematics education aims to identify effective methods to overcome learning difficulties arising from the abstract nature of mathematics and to enhance student success (Işık & Konyalıoğlu, 2005). Mathematics is integral to all stages of education, and numerous studies emphasize the importance of effective mathematics instruction (e.g., Franke & Kazemi, 2001). Recently, one of the suggested methods for students struggling with mathematics is the use of worked examples (e.g., Bokosmaty et al., 2015; Reisslein et al., 2006; Renkl & Atkinson, 2003; Van Loon-Hillen et al.,2012). Worked examples are learning activities based on Cognitive Load Theory (CLT) and involve demonstrating step-by-step problem-solving procedures. This research aims to compare the effects of two types of worked examples—explanatory faded and correct-incorrect worked examples—on the mathematics performance of sixth-grade secondary school students.

In the Turkish education system, the sixth-grade mathematics curriculum includes fundamental concepts such as the order of operations, exponents, distributive property, and divisibility rules. In other countries, these topics may be introduced at different grade levels. These concepts form the basis of many mathematical topics that students find challenging. Difficulties in these foundational topics can negatively impact students' ability to perform operations in other mathematical areas. This study aims to support learning by presenting the teaching of sixth-grade topics (order of operations, distributive property, exponents, and divisibility) using explanatory faded Worked examples and correct-incorrect Worked examples. During the pandemic period, students were left to learn independently, making the use of worked examples particularly beneficial in scenarios where students must learn material on their own.

Conceptual Background of the Study

Cognitive Load Theory and Worked Examples

Cognitive load is a learning theory that aims to learn effectively by presenting instructional design principles, based on the fact that the human mind has a limited working memory and an unlimited long-term memory (Clark et al., 2005; Wong et al., 2012). According to the Cognitive Load Theory, the information obtained by individuals during the learning process is processed in three different categories in working memory, according to the field of use: intrinsic load, extraneous load, and germane load. Intrinsic cognitive load refers to the cognitive demand imposed by the inherent complexity of the subject being taught, which is independent of the teaching methods or techniques used (Sweller, 1994). It reflects the difficulty of the content itself and is a key factor in determining the cognitive resources required for learning. Extrinsic cognitive load arises from the way content is organized and presented and depends on the teaching resources employed. This type of load places additional demands on working memory that go beyond the learning objectives and is often seen as unproductive if it does not contribute to schema formation (Sweller, 2011, 2020). Activities that do not support schema development are generally viewed as detrimental to the learning process (Sweller, 2005). Germane cognitive load is influenced by content and activities that facilitate the learning process and aid in schema formation. It is considered beneficial as it positively impacts learning by helping to build and integrate new knowledge (Sweller et al., 1998).

The Cognitive Load Theory aims to reveal teaching methods that will ensure that the cognitive load encountered by individuals who are educated in any field is presented in a way that is suitable for the cognitive capacities of the learners so that learning can take place effectively (Moreno & Park, 2010). Worked examples, which are one of the learning activities related to Cognitive Load Theory (Sweller, 1988), have the feature of reducing the external cognitive load and increasing the germane cognitive load (Paas et al., 2003).

Worked Example is a teaching method that shows the learner step by step how a problem is solved (Clark et al., 2005; Pershan, 2023). It is an effective method, especially for people who have not developed schemas related to the subject, to gain knowledge and skills. In this method, before solving the questions on his/her own, the person who will learn the subject is given solved examples one after the other. Worked examples create a model to show how the expert solves any problem, so that the learner applies the given solution first to similar problems and then to less similar problems. Thus, learning transfer takes place from similar examples to less similar examples (Sweller & Cooper, 1985). The way the worked examples are presented also plays a role in students' problem-solving performance (Bokosmaty et al., 2015; Renkl et al., 2002). A standard worked example provides students with solution steps and explanations, but what students need is to process this information mentally. Therefore, samples with different prepared solutions are needed.

Some of the types of worked examples explored in the literature include faded examples (Booth et al., 2013; İltüzer, 2016; Lee, 2013), examples with explanations (Emecen, 2020; Große, 2015; Hesser & Gregory, 2015; Özcan et al., 2018; Tepgeç, 2017), and correct-incorrect examples with solutions (Durkin & Johnson, 2012; Özcan et al., 2018). This research will focus specifically on correct-incorrect worked examples and faded worked examples with explanations. Examples with correct and incorrect solutions present a problem with both a correct solution and an incorrect solution (Siegler & Chen, 2008). By studying not only the correct solution steps but also the wrong solution steps, the learner increases his/her readiness for possible mistakes they may encounter in solving the problem (Tepgeç, 2017). In the examples with faded explanations and solutions, the question is first completely solved and then presented to the student. Then the solutions in the example are left blank step by step. The worked examples continue in this way until all steps of the solution are removed. Finally, only the question remains, and the learner is expected to solve the question (Eiriksdottir & Catrambone, 2012).

Worked examples are a learning activity grounded in Cognitive Load Theory. These examples help reduce extraneous load while increasing germane load (Cierniak et al., 2006). By incorporating worked examples at the early stages of skill acquisition, cognitive load unrelated to the subject matter is minimized. Therefore, this research aims to compare the explanatory faded worked example method with the true-false worked example method to determine which is more effective in facilitating learning.

Correct-Incorrect Worked Examples

In examples featuring correct and incorrect solutions, two approaches to a question are presented side by side—one being correct and the other incorrect. After acquiring the necessary information about the topic, learners typically analyze both solutions to identify which is correct (Siegler & Chen, 2008). This practice not only reinforces the correct steps but also prepares learners for potential mistakes they may encounter during problem-solving. Mainly, learning with incorrect solutions involves presenting these examples to enhance understanding after reviewing the correct solutions (Tepgeç, 2017).

Tennyson and Cocchiarella (1986) emphasized the benefits of using both correct and incorrect examples when learning new concepts, while Bransford and Schwartz (1999) noted that contrasting examples can highlight critical points and facilitate learning transfer. Incorrect worked examples, particularly when accompanied by explanatory prompts (Renkl, 2014), are thought to be effective in correcting misconceptions (e.g., Renkl, 2017). For instance, in the domain of decimals, where children often hold misconceptions (such as the belief that 0.6 is smaller than 0.365 because $6 < 365$), studying both correct and incorrect worked examples has been shown to improve conceptual understanding more than studying only correct examples (Durkin & Rittle-Johnson, 2012). However, the impact of prior knowledge on learning from incorrect worked examples is less definitive than with correct examples. Some studies suggest that incorrect worked examples benefit students with limited prior knowledge (Barbieri & Booth, 2016), while others indicate advantages only for those with more extensive prior knowledge (Große & Renkl, 2007). Additionally, some research finds no significant effect of prior knowledge (e.g., Durkin & Rittle-Johnson, 2012).

Große and Renkl (2007) reported that combining true and false solutions with self-explanation positively contributed to learning, particularly for students with strong mathematics performance. Özcan, Kılıç, and Obalar (2018) revealed that using true-false solution examples supported by explanatory clues improved students' mathematics performance, as they helped identify and correct mistakes when representing improper fractions on a number line. In another study, Booth et al. (2013) assessed whether true-false examples with self-explanatory prompts effectively enhanced students' understanding and procedural skills in algebra. Their findings indicated that combining correct and incorrect examples proved more beneficial than relying solely on correct examples in solving algebraic equations. In the current study, the students lack prior knowledge of the subject, as these topics have not been previously introduced. However, they possess the prerequisite knowledge necessary to understand the subject.

Fading Worked Examples with Explanations

In fading worked examples with explanations, the fully solved example is presented first. Then, the step-by-step solutions in the example are left blank, and this process continues until all steps are removed. Thus, in the end, only the problem remains, and the learner is expected to solve it (Eiriksdottir & Catrambone, 2012). In this way, guidance diminishes in the transition from solved examples to problem solving. The fading worked examples are implemented in two different ways. One approach involves increasing the gaps in the solution steps from the very end (backward fading), while the other increases the gaps from the beginning to the end (forward fading). The results of the studies conducted in the literature support that examples with fading solutions increase academic success, especially in novice students. In their study, Hesser and Gregory (2015) introduced fading worked examples to students who were determined not to be mathematically prepared enough in a university chemistry course. As a result, they reported that students' examples with reduced solutions improved their general learning and problem-solving abilities in chemistry. Emecen (2020) aimed to examine the effect of using the worked example methods of fading and explanatory clue together for students with learning disabilities on the word problem-solving skills of students who have difficulties in mathematics lessons. According to the study's results, presenting worked example types of fading and explanatory clues together contributed to the verbal problem-solving skills of all participating students. It was observed that the learned information remained permanent after 2, 3, and 4 weeks of application.

In his research, Große (2015) explored whether using faded examples would be a better alternative to presenting fully worked examples. Additionally, he sought to find out whether increasing the gaps in the solution steps from the end (backward fading) is more effective than increasing them from the beginning to the end (forward fading) in illustrative faded examples. The results of the study indicated that backward fading, where gaps in the solution steps are introduced starting from the end, is more effective than forward fading. Similarly, Tepgeç (2017) aimed to examine the effects of traditional worked example and faded worked example methods for algorithm teaching on the success and cognitive load of students studying at university. He found that the faded worked examples used for algorithm teaching turned out to be more effective than traditional solved examples. It was concluded that there was no significant difference between the cognitive load levels of the students in the experimental and control groups, but in terms of efficiency of learning, examples with faded solutions were a more effective method.

Current Study

The motivation behind this study stems from the ongoing need to explore effective instructional strategies that help students overcome learning difficulties in mathematics, particularly in foundational topics like order of operations, exponents, the distributive property, and divisibility rules. While the literature on worked examples suggests that they are a valuable tool for reducing cognitive load and supporting learning (Sweller, 1988), there remains a gap in understanding the comparative effectiveness of different types of worked examples, such as explanatory faded examples and correct-incorrect examples, particularly for younger students in a secondary school context.

This study aims to address this gap by examining the impact of teaching sixth-grade mathematics topics—such as order of operations, distributive property, exponents, and divisibility—through the use of explanatory faded examples and correct-incorrect worked examples, with the goal of identifying which method is more effective in improving students' learning outcomes. Although the students in this study have no prior knowledge of these specific topics, they possess the prerequisite knowledge necessary to engage with and understand the material. Based on the purpose of the study, the following research questions were addressed:

Quantitative Research Questions

1. Is there a statistically significant difference between the pre-test and post-test mathematics scores (exponents, order of operations, distributive property, divisibility rules) of the Explanatory Faded Worked Example (EFWE) group?
2. Is there a statistically significant difference between the pre-test and post-test mathematics scores of the Correct-Incorrect Worked Example (CIWE) group?
3. Which of the two instructional approaches (EFWE or CIWE) is more effective in improving sixth-grade students' mathematics performance after controlling for pre-test scores?

Qualitative Research Questions

4. What are the views of the EFWE group regarding the implementation of the instructional approach?
5. What are the views of the CIWE group regarding the implementation of the instructional approach?
6. What do students perceive as the benefits and challenges of EFWE and CIWE in their learning process?

METHODS

In this study, a quasi-experimental design with a pre-test and post-test was employed (Cook et al., 2002). To gain deeper insights into the impact of the intervention, after conducting the experiment, open-ended survey questions were posed to the students to gather their perspectives and opinions.

Study Group

The study group was determined in two stages. First, through convenient sampling, 40 sixth-grade students from two different classes at a public school in Istanbul, where one of the researchers worked during the 2021-2022 academic year, were selected. The participating students were, on average, around 11 years old, corresponding to the typical age for sixth-grade learners in the Turkish education system. A pre-test containing four mathematical operations questions was administered to assess the prerequisite skills for learning sixth-grade topics (order of operations, distributive property, exponents, and divisibility). Students who scored at least 80 out of 100 were chosen for the study group. The research was conducted in two classes, with one class assigned as the Explanatory Faded Worked Examples (EFWE) Experimental Group and the other as the Correct-Incorrect Worked Examples (CIWE) Experimental Group. The EFWE group consisted of 14 students (7 girls and seven boys), while the CIWE group included 15 students (7 girls and eight boys).

Data Collection Tools

Both quantitative and qualitative data collection tools were employed in the study. For the quantitative data, students were selected based on their performance on the Four Operations Questions, designed by the researcher to assess core prerequisite operational skills. The Mathematics Achievement Test was administered both as a pretest and posttest, alongside the Worked Examples Worksheet (focusing on topics like order of operations, distributive property, exponents, and divisibility), to evaluate the impact of the instructional intervention. To assess the effectiveness of the applied method, qualitative data were collected through students' responses to open-ended survey questions regarding their opinions. A detailed description of the data collection tools used in the study is provided below.

Four Operations Questions

To ensure that the participants possessed the prerequisite skills necessary for the mathematical concepts being taught, the Four Operations Questions were administered. The questions were designed to select students for the study and consisted of 10 questions: 2 addition, 2 subtraction, 3 multiplication, and 3 division questions. Each correct answer was awarded 10 points, while incorrect or unanswered questions received 0 points. Students were asked to solve the questions within one lesson hour using pencil and paper. Three sixth-grade mathematics teachers from the same school assessed the papers. In cases of differing scores for the same question, the teachers met to reach a consensus on a standard score. Students who scored 80 points or below were eliminated from the study.

Mathematics Achievement Test

The Mathematics Achievement Test was used both as a pre-test and a post-test in the study. It consisted of 10 questions aligned with the sixth-grade objectives in the numbers learning area, focusing on order of operations, distributive property, exponents, and divisibility rules. The test was initially prepared by the researchers. A table of specification was prepared, and expert opinions were obtained from four mathematics teachers to refine the test and remove repetitive question types. After obtaining the necessary permissions (ethics committee approval, research permissions, student and parental consent), a pilot study was conducted in a public school. Based on the results, any deficiencies in the test were addressed, and the final version was prepared for implementation. A scoring system was developed for the 10 questions, where the first two questions were worth 5 points each for a complete and correct answer, and the remaining questions were worth 10 points each. The remaining questions were scored as follows: 0 points for completely incorrect or unanswered questions, 3 points for partial or incorrect solutions, 6 points for selecting the correct strategy but with an incorrect solution, and 10 points for correct solutions. The maximum score for the test was 90 points. The questions were all at the application level according to Bloom's taxonomy. The study groups were asked to solve the questions within one lesson hour (40 minutes) using pencil and paper. The papers were evaluated by four mathematics teachers working in the same school and teaching sixth-grade classes. For different scores given to the same question, the answers were re-evaluated with the teachers, and a common grade was given after reaching a consensus.

Worked Example Worksheets

Two worksheets have been prepared for both experimental groups according to the types of worked examples. At the beginning of the worksheets, there was a short explanation about the subject. Students have the chance to refer back to this information while solving or examining the questions related to the worked examples. For example, at the beginning of the worksheet on order of operations, students are presented with what order of operations is and the order of precedence of operations.

In the worksheet prepared with the explanation, the EFWE method, one example was solved step by step, completely and correctly, with explanations in brackets. Then, for the questions similar to the first example, the steps were left blank starting from the end, and the student was asked to solve them. The steps of the solution of the last example were left completely blank, and the student was asked to solve the question. This information has been presented to students at the beginning of the worksheet. Examples of the intervention of examples with faded solutions with explanations are provided in Appendix 2.

In the worksheet prepared using the CIWE method, an example problem and its solution were provided. For the other problems, two solutions were given: one correct and one incorrect. Students were first asked to analyze the sample problem and its solution. Then, they were required to review solutions provided by other students, answer questions about those solutions, and finally solve their assigned problem. An example of this application is provided in Appendix 3.

The intervention worksheets with solutions, provided as examples for the EFWE and CIWE study groups, were prepared on topics such as order of operations, the distributive property, exponents, and divisibility, all of which are sub-dimensions of number learning—subsequently, four mathematics teachers from the same school as the researcher reviewed the questions for accuracy.

Open-ended Questions Regarding EFWE and CIWE Interventions

After working with the experimental groups, open-ended questionnaire questions were administered to gather students' opinions, as the aim was to conduct a more detailed analysis of the research. These questions were designed to explore students' experiences during the Worked Example intervention. Open-ended questions allow respondents to answer freely, providing the researcher with more detailed and sometimes unexpected insights (Büyüköztürk, 2005). The open-ended survey questions were developed by the researcher in collaboration with an expert. Two Turkish teachers from a public school then reviewed the questions for clarity and grammar. After necessary revisions, the questions were finalized. In the pilot study, the question 'Do you have anything to add?' was removed from the final version, as students did not respond to it.

Data Collection

Before the data collection phase, the necessary permissions (ethics committee approval, research authorization, student participation consent forms, and parental consent forms) were obtained. A pilot study was conducted, and adjustments were made to the data collection tools and experimental procedures. Following this, the main study was carried out.

Data Analysis

Quantitative data were analyzed using a statistical software package. Before seeking answers to the research questions, descriptive statistics calculated for the scores of both groups regarding the dependent variables were presented. After the descriptive analyses, a dependent samples t-test was conducted to compare the pre-test and post-test mathematics performances of the EFWE and CIWE experimental groups. An ANCOVA test was conducted to answer whether there was a significant difference in the post-test scores of the EFWE and CIWE groups. The ANCOVA test is a test that equalizes the differences determined in the pretest scores of the experimental and control groups in experimental studies (Büyüköztürk et al., 2018). In this study, the ANCOVA test was preferred because there was a statistically significant difference in pre-test scores.

The opinions of the EFWE and CIWE groups about the interventions were analyzed using thematic analysis. Thematic analysis is a method that identifies, analyses, and reports the patterns in the data. Thematic analysis enables data to be organized and a rich description to be made. The thematic analysis in this study is based entirely on the data obtained; it does not depend on a predetermined coding or the analytical prejudice of the researcher. The data coding process, which is conducted without the analytical prejudice of the research, is called the inductive approach (Boyatsiz, 1998). The themes identified in the inductive approach are tightly linked to the data (Patton, 1990). This study was evaluated by a thematic analysis method with an inductive approach.

The answers to the open-ended questionnaire questions were taken from the students in written form. In order to ensure the reliability of codes and categories, coding harmony was ensured with another researcher. In cases where there was no coding harmony, a common decision was made, and a consensus was reached.

Experimental Process

The study was conducted by the first author in a state school in the 2021-2022 academic year during mathematics class hours. The students in the study group were divided into two groups, namely, the EFWE experimental group and the CIWE experimental group. Both study groups were administered a pre-test measuring the students' four operation skills, which is a prerequisite for participating in the study of the sub-dimensions of the number learning domain (operation priority, distributive property, exponents, and divisibility). Out of 40 students who participated in the pre-test, 11 students were eliminated, and the study was continued with 29 students. A two-week study was implemented with the students using the EFWE and CIWE worksheets. The table detailing the applied study is provided below.

	EFWE Experimental Group n = 14	**CIWE Experimental Group n = 15**
One week before the interventions	Four operation questions Mathematics Achievement test	Four operation questions Mathematics Achievement test
Interventions (Two weeks)	Worksheet based on EFWE Method	Worksheet based on CIWE Method
One week after the intervention	Mathematics Achievement Test Open-ended Questionnaire Questions Regarding EFWE Intervention	Mathematics Achievement Test Open-ended Questionnaire Questions Regarding EFWE Intervention

Table 1: Experimental process of the study

Worksheets on the order of operations, the distributive property (first week), exponents, and divisibility rules (second week) were distributed to the respective groups. Each of the studies lasted approximately 2 lesson periods (80 minutes). The teacher was only the supervisor in the study. After the 2-week study, a post-test containing the same questions as the pre-test was administered. Quantitative analyses were made on the data obtained from these tests, and the extent to which the study affected the performance of the students in learning the subjects was investigated.

RESULTS

Preliminary Analysis

Before addressing the research questions, the normality of the data distribution was assessed based on skewness and kurtosis values. Considering that the skewness and kurtosis coefficients of almost all the dependent variables in Table 2 are between -1 and +1, and those not in this range do not deviate significantly from these values, it was concluded that the data have a normal distribution (George & Mallery, 2010). Following this, the pre-test results of the groups are compared as a preliminary study. Pretest results showed significant differences between the groups in favor of the CIWE group on the EN ($t(28) = -2.37$, $p < 0.05$), OO ($t(28) = -2.23$, $p < 0.05$), DR ($t(28) = -2.67$, $p < 0.05$), and total scores ($t(28) = -2.99$, $p < 0.01$), while no significant difference was found on the DP scale ($t(28) = -0.36$, $p > 0.05$) at baseline (see Table 2).

Pretest and Posttest Differences in the EFWE and CIWE Groups

		Descriptive Statistics		Comparison between the EFWE and CIWE groups
		EFWE Score mean (SD)	CIWE Score mean (SD)	
EN	**Pretest**	5.5 (5.37)	10.07 (5.01)	$t=-2.37$, $p< 0.05$
	Posttest	11.43 (4.97)	14.00 (2.80)	$F_{(1,28)}=2.68$, $p = 0.11$; partial $\eta2 = 0.09$
OO	**Pretest**	3.93 (5.61)	8.67 (5.81)	$t=-2.23$, $p< 0.05$
	Posttest	3.57 (5.35)	11.00 (6.32)	$F_{(1,67)}=6.34$, $p<0.01$; partial $\eta2 = 0.196$
DP	**Pretest**	7.86 (6.99)	8.67 (4.81)	$t=-0.36$, $p > 0.05$
	Posttest	17.14 (4.69)	18.67 (3.52)	$F_{(1,67)}=1.10$, $p>0.01$; partial $\eta2 = 0.04$
DR	**Pretest**	2.86 (4.69)	8.00 (5.61)	$t=-2.67$, $p< 0.05$
	Posttest	11.71 (10.58)	26.37 (11.13)	$F_{(1,67)}=8.9$, $p<0.01$; partial $\eta2 = 0.26$
Total	**Pretest**	20.14 (15.58)	35.40 (11.22)	$t=-2.99$, $p< 0.01$
	Posttest	57.55 (20.05)	70.33 (16.20)	$F_{(1,67)}=8.18$, $p<0.001$; partial $\eta2 = 0.12$

Table 2: Descriptive statistics and comparison of two experimental groups

Note. EFWE = Explanatory Faded Worked Example; CIWE = Correct-Incorrect Worked Example; EN = Exponential Numbers; OO = Order of Operations; DP = Distributive Property; DR = Divisibility Rules.

First, the analysis examines whether there is a significant difference between the pre-test and post-test mathematics scores of the EFWE group. According to the paired sample t-test, there is no significant difference in the EFWE group for exponential numbers (t=-2.83, p>0.05), order of operations (t=0.19, p>0.05), or divisibility (t=-2.45, p>0.05). The pre-test score for exponential numbers increased from 5.50 to 11.43, but this difference is not statistically significant. Similarly, the divisibility score increased from 2.86 to 11.71, with no significant difference. However, for the distributive property, a significant difference was found (t=-4.19, p<0.05), with scores rising from 7.86 to 17.14. A significant difference in total scores was observed in favor of the post-test (t=-11.98, p < 0.001).

Second, the difference in pre-test and post-test scores within the CIWE group is explored. Paired sample t-test results show significant differences in favor of the post-test scores for the CIWE group in exponential numbers (t=-2.82, p<0.05), distributive property (t=-1.45, p<0.05), and divisibility (t=-6.09, p<0.01). Score of exponential numbers increased from $\bar{X}$=10.07 to $\bar{X}$=14.00, distributive property from $\bar{X}$=8.67 to $\bar{X}$=18.67, and divisibility from $\bar{X}$=8.00 to $\bar{X}$=26.67. This indicates a significant improvement. No significant difference was found for order of operations (t=-1.45, p>0.01), with scores rising from $\bar{X}$=8.67 to $\bar{X}$=11.00. Total pre-test scores increased significantly from $\bar{X}$=35.40 to $\bar{X}$=70.33 (t=-11.18, p<0.01), reflecting the positive impact of the group study.

Comparison of Post-test Scores Between the EFWE and CIWE Groups

The post-test scores between the EFWE and CIWE groups are compared to assess the effectiveness of each approach, using an ANCOVA test to determine whether there is a significant difference in post-test performance between the two methods in the teaching of 6th-grade mathematics topics. Initially, it was assessed whether the "homogeneity of regression slopes" assumption, a crucial requirement for ANCOVA, was met for EN, OO, DP, DR, and total scores. To this end, the F-values for the interaction between the group type (independent variable) and the pre-test scores of the dependent variables were determined. The F-values were found to be 0.46 (p=0.50) for EN, 2.25 (p=0.15) for DP, 0.74 (p=0.40) for OO, and 0.78 (p=0.38) for DR, with all of the p-values greater than 0.05. This result is interpreted as fulfilling the condition of matching regression slopes needed for ANCOVA. To examine another important assumption of ANCOVA, Levene's test was performed; the result (F = 1.38, p > 0.05 for EN, F = 0.67, p > 0.05 for DP, F = 0.46, p > 0.05 for OO and F = 0.28, p > 0.05 for DP) of Levene's test for equality of variances specified that the assumption of the homogeneity of variances in the groups was satisfied. After these assumptions were satisfied, the ANCOVA test was conducted on the dependent variables.

Controlling for EN pre-test scores, no significant difference emerged between the groups regarding EN post-test scores (F(1,28) = 2.68, p > 0.05). The effect size, a measure of the magnitude and direction of the observed difference, was calculated as 0.094. Based on Kılıç (2014), since this value exceeds 0.06, it is classified as a medium effect, indicating a moderate impact. For DP post-test scores, after controlling for the pre-test, the ANCOVA analysis showed no significant difference between the groups ($F(1,28) = 1.10$, $p > 0.05$). The effect size was calculated at 0.041, which Kılıç (2014) identifies as a small effect due to being greater than 0.01, reflecting a low impact in this area. In the case of OO post-test scores, a significant difference was found between the groups ($F(1,28) = 6.34$, $p < 0.05$) after adjusting for OO pre-test scores. Post hoc tests indicated that this difference favored the EFWE group. The effect size, calculated at 0.196, is considered significant as it surpasses 0.14, signifying a substantial impact. Similarly, for the DR variable, with $F = 8.93$ and $p = 0.01$, the ANCOVA results indicated a significant difference between the groups, favoring the EFWE group after controlling for the pre-test. The effect size of 0.256 further qualifies as large according to Kılıç (2014), highlighting a substantial effect from the intervention. Finally, when the posttest results of the total score are examined, it is seen that the difference between the groups is significant ($F(1,67) = 8.18$, $p < 0.001$) favoring the EFWE group after controlling for the pre-test; in addition, the partial $\eta^2 = 0.12$ indicates that the effect of this difference is moderate.

Participant Views on the Implementation of EFWE and CIWE Methods

The views of both the EFWE and CIWE groups regarding the implementation are analyzed to gain insights into their experiences with the instructional methods. This study employed a thematic analysis to compare the experiences and feedback of two experimental groups. The analysis was based on responses to open-ended questions. The responses were categorized into key themes, including perceptions and evaluations of the study, challenges faced in problem-solving, and students' satisfaction with the overall learning process.

Perceptions and Evaluations of the Study by the EFWE and CIWE Groups

Of the 14 students in the EFWE group, ten reported that they found the study useful, particularly emphasizing how the structured approach to problem-solving enhanced their understanding of mathematical concepts. As one student explained, “*Seeing the answers to the questions answered in front of me really helped me to see where I went wrong*.” Another student similarly noted, “*The study helped me grasp the topics that I previously struggled with*.” On the other hand, four students indicated that they found the process boring and uninteresting. One student described the experience as follows: “*I didn’t enjoy the process; it felt repetitive and boring.*” Taken together, these findings suggest that while the majority of students benefited from the structured format of the EFWE approach, a smaller group felt disengaged, possibly due to a lack of interactivity or variation in the method.

The CIWE group showed a more positive response to the study overall. Out of the 15 students, nine indicated that the study made a significant contribution to their learning. Three students found the approach particularly explanatory, while ten described it as enjoyable, and three noted that they considered it fun. Students highlighted both the affective and cognitive benefits of the method. For instance, one student explained: “*The study helped me understand things in a way that made the topics more enjoyable*.” Another reflected: “*I found the process interesting, and I could relate it* *to real-life situations.*" The interactive and dynamic aspects of the approach were also emphasized, with one participant remarking: "*The study was not only helpful but also fun to participate in.*"

Taken together, these findings underscore the potential of dynamic and engaging methodologies such as CIWE to foster a positive learning environment and to enhance both understanding and motivation among students.

Challenges in Problem Solving

A significant portion of the EFWE group expressed difficulties in solving the provided exercises. Out of 14 students, seven reported struggling with understanding and completing the questions. As one student explained: "*I struggled with understanding the problems, especially when there were multiple steps involved*." Another similarly noted: "*Without the teacher explaining every detail, I got lost in some of the problems*." At the same time, a small number of students emphasized the importance of teacher explanations for better comprehension, while five students reported that they found the tasks easy and understandable. For instance, one participant stated that the exercises were clear and manageable without further support. These findings suggest that while the structured format of EFWE can provide clarity for some learners, others may still require additional scaffolding and direct teacher guidance to engage with problem-solving tasks confidently.

The CIWE group, by contrast, reported very few difficulties with problem-solving. Out of 15 students, only two indicated that they had trouble understanding the questions, and one mentioned difficulty in explaining the answers. For example, one student explained: "*Some problems were a bit difficult, but I managed to solve them after thinking about it for a while*." Another noted: "*I had a hard time understanding a couple of concepts, but overall it was manageable*." In contrast, the majority of students (11 out of 15) stated that they found the tasks easy and understandable. This overall pattern suggests that the dynamic and interactive nature of the CIWE approach helped most students to engage effectively with the exercises and to develop greater confidence in their problem-solving abilities.

Students' Satisfaction with the Overall Learning Process

To gather students' opinions about the learning process, they were asked whether they would like to study other mathematics subjects using the same method and to explain their reasons. In the EFWE group, 10 out of 14 students responded positively, while 4 stated that they would not prefer this method for other topics. Among those who answered "yes," five emphasized the usefulness of the method and one highlighted the benefit of having complete solutions. As one student expressed: *"I would like to because this method both explains and solves."* In contrast, less satisfied students described the lessons as repetitive, with one noting: *"I learned from the examples, but the lessons felt long and boring."*

In the CIWE group, almost all students expressed willingness to apply the method to other mathematics subjects. Fourteen students answered "yes," with ten indicating that they found the helpful method and three describing it as easy and enjoyable. Only one student reported that they would not prefer to continue with this approach. Positive evaluations included remarks such as: *“The lessons were fun, and I felt like I was learning without getting bored,”* and *“It did not feel like a regular math class. I enjoyed the process and felt like I learned a lot.”*

These findings suggest that while both groups showed a generally positive attitude toward extending the method to other topics, satisfaction was markedly higher in the CIWE group. The interactive and dynamic features of the CIWE approach appeared to resonate strongly with students, making the learning experience more enjoyable and effective.

DISCUSSION

This study aimed to evaluate the effectiveness of two instructional methods—Explanatory Faded Worked Examples (EFWE) and Correct-Incorrect Worked Examples (CIWE)—on the mathematical performance of sixth-grade students, as well as to compare their relative effectiveness. The results reveal that both methods influenced student performance, but with varying degrees of success across different mathematical concepts, such as the distributive property, exponents, divisibility, and the order of operations. These findings contribute to the ongoing discourse on the effectiveness of worked-example strategies in mathematical instruction, particularly highlighting the role of scaffolding and error correction in learning.

In the groups where EFWE methods were applied, there was no significant difference between the pre-test and post-test mathematics scores in the subdimensions of order of operations, exponents, and divisibility. However, a significant difference was found in the subdimension of distributive property and the overall math score, indicating that the application had a positive effect on the distributive property and students' overall math performance, though it did not influence the areas of exponents, divisibility, and order of operations. Of the students who participated in the EFWE study, 10 found the activity beneficial, while four expressed that they found it boring and did not enjoy it. Most students found the EFWE valuable method because it allowed them to follow the provided procedural steps in the examples, reach solutions, and learn at their own pace.

In the study, the work on order of operations, exponents, and divisibility with the group that used the EFWE method was insufficient. A longer duration of study on these topics would be beneficial. Although there was a considerable difference between pre-test and post-test scores in the subtests for exponents, divisibility, and order of operations, this difference was not statistically significant. In the literature, there are studies that comparatively test the positive impact of faded worked examples. For instance, Tepgeç (2017) investigated the effects of standard worked examples and faded worked examples on college students’ achievement and cognitive load in algorithm teaching and found that faded worked examples were more effective than standard ones. Foster et al. (2017) observed in their study on students learning principle-based concepts that those using faded worked examples showed superior performance in controlling solutions between examples and problem-solving tasks. Emecen (2020) aimed to examine the effects of combining faded worked examples with explanatory cues on students with learning difficulties in terms of their verbal problem-solving skills. This study concluded that the integration of faded worked examples and explanatory cues contributed to verbal problem-solving skills for all participating students. Recent research with underprepared college students in computational courses found that faded worked examples enhanced problem-solving skills and improved understanding of course material (Hesser & Gregory, 2015). The findings of the current study are consistent with prior research indicating that faded worked examples are more effective than standard worked examples. In the cited studies, the outcome was the overall mathematics score; using the same outcome in our data, fading was also effective and did not differ significantly from the CIWE condition. However, content-specific (topic-by-topic) analyses revealed that, for some mathematical concepts which fading showed no detectable advantage. This pattern suggests that the impact of fading may be topic dependent, underscoring the need for further research to determine when and for which mathematical topics fading should be preferred, and when alternative approaches (e.g., CIWE) may be equally or more appropriate.

The EFWE method appears to support students' procedural understanding, as they could follow step-by-step solutions in distributive property tasks. However, it was less effective for topics requiring deeper conceptual understanding, such as exponents and order of operations, where fading guidance may have left some students without sufficient conceptual support. The lack of significance may alsobe due to the sample size. Even though the distribution was normal, the small sample size may have prevented statistically significant results (Kılıç, 2014).

In the group using the CIWE method, a significant difference was observed in favor of the post-test scores for divisibility, distributive property, and exponents, though not for order of operations. This indicates that the application was largely practical in improving children's math performance. The studies on order of operations were insufficient for both groups. Additionally, when considering the total post-test mathematics scores, a significant difference was found in favor of the post-test, demonstrating the positive impact of the study on students' overall math performance. Based on student feedback, the CIWE application was found beneficial for teaching math topics. Students reported that they greatly enjoyed the study as they played an active role in the process. Furthermore, the presentation of both correct and incorrect solutions provided valuable opportunities for self-reflection, enabling students to recognize and address their own mistakes more effectively.

While most worked example studies focus on the use of correct examples, recent studies suggest that using both correct and incorrect worked examples together may be more effective for students (Durkin, Rittle, and Johnson, 2012; Rittle & Johnson, 2006; Siegler, 2002; Siegler and Chen, 2008). In their study, Durkin and Johnson (2012) examined whether working with correct and incorrect worked examples could help eliminate misconceptions and support students' understanding of decimal fractions. By presenting common misconceptions alongside correct solutions, they helped students focus on decimal concepts and improve both their conceptual and procedural knowledge. Booth et al. (2013) aimed to investigate whether self-explanation prompts given with correct and incorrect worked examples could be practical in helping students understand topics and improve their algebraic skills when combined with guided practice. Their study concluded that using both correct and incorrect examples was more beneficial than using correct examples alone for solving algebraic expressions.

In another study, Özcan et al. (2018) sought to determine the impact of correct-incorrect worked examples with explanatory hints on students' mathematical performance by identifying and addressing mistakes students made when placing improper fractions on a number line. They found that this method positively impacted students' performance. Große and Renkl (2007) also examined the impact of correct-incorrect worked examples and self-explanation on learning, noting that using both types of examples positively contributed to the learning of students who already had strong math skills.

According to the analysis results on whether post-test scores differed between groups, there was no significant difference in the subgroups of distributive property and exponents. However, a significant difference in favor of EFWE was found in the divisibility and order of operations subdimensions. Looking at the overall mathematics scores of the study, a significant difference was also found in favor of EFWE, indicating a positive impact on total mathematics performance. Based on eta-squared results, 0.01 is considered a low effect size, 0.06 a medium effect size, and 0.14 and above a large effect size (Kılıç, 2014). In this study, ANCOVA results showed that distributive property and exponents had medium effect sizes, while divisibility and order of operations had large effect sizes. Although no statistically significant difference emerged, the high eta-squared values indicate a group difference in favor of EFWE. In the literature, the fading procedure has been reported to yield higher learning outcomes compared to traditional worked example–problem pairs (e.g. Atkinson, Renkl and Merrill, 2003; Fleischmann and Jones, 2002; Renkl, Atkinson, Maier and Staley, 2002). In our study, however, the fading procedure appeared to produce lower learning outcomes when compared to true–false worked examples, suggesting that fading was less effective than the true–false approach in this context.

When reviewing the literature, no studies were found that directly compared EFWE and CIWE; thus, no directly comparable results were available. However, a recent meta-analysis examining the effectiveness of different types of worked examples highlights the effectiveness of correct-incorrect worked examples. In correct-incorrect worked examples, students are allowed to conduct a form of error analysis by comparing a wrongly solved question with a correctly solved example, which facilitates deeper conceptual learning. The approach of analyzing one's own or a peer's mistakes (Zrebiec, Uberti, Mastropieri, and Scruggs, 2004) is noted in the literature as a method that aids learning. In faded worked examples, creating multiple questions for the fading process in multi-step problems is often necessary, which, as participants have indicated, can become tedious. Instead of solving numerous questions within the same skill, comparing correct-incorrect worked examples may be more beneficial for understanding the topic. Explanatory Faded Worked Examples (EFWE) and Correct–Incorrect Worked Examples (CIWE) reflect scaffolding principles. EFWE parallels scaffolding through the gradual fading of support, while CIWE mirrors scaffolding by drawing attention to misconceptions. Similarly, research on IT-assisted scaffolding in abstract algebra showed that structured guidance helped students recognize and correct errors, improving their reasoning (Warli, Rahayu, & Cintamulya, 2025).

Theoretical and Practical Implications

These findings add to the growing body of literature on worked-example methods in education. While Tepgeç (2017) and Sweller (1999) highlight the advantages of fading guidance for procedural learning, the current study suggests that this approach is not universally applicable across all mathematical topics. The EFWE method, while effective for certain concepts, may not offer enough structured support for more abstract topics like exponents and the order of operations. Conversely, the success of the CIWE method in addressing misconceptions aligns with earlier research advocating for the use of both correct and incorrect worked examples to enhance learning (Rittle-Johnson, 2006; Siegler & Chen, 2008). This method's effectiveness in improving procedural skills, particularly in the context of divisibility and exponents, suggests that incorporating error analysis into instructional practices can significantly benefit students.

Limitations and Future Directions

One limitation of this study is the relatively small sample size, which may have affected the statistical significance of some findings. Although the observed improvements in exponents and the order of operations are promising, larger studies are necessary to confirm these results (Kılıç, 2014). Additionally, the short duration of the intervention may have limited students' engagement with complex topics, suggesting that future studies should extend the study period and provide more focused instruction on challenging concepts.

Another limitation is the absence of control groups, which would have allowed for a more rigorous comparison between the two methods. Future research should incorporate control groups and more objective measures, such as cognitive load assessments and long-term retention tests, to better understand the impact of these instructional methods.

CONCLUSIONS

Overall, this study demonstrates the differential effectiveness of EFWE and CIWE methods in teaching various mathematical topics. Educators should carefully consider which method best aligns with the specific concepts they are teaching. For higher-order thinking skills, such as exponents and the order of operations, extending the guidance provided by the EFWE method may be necessary. On the other hand, the CIWE method is particularly effective for addressing misconceptions in areas like divisibility and the distributive property. Future research should explore a hybrid approach, combining the scaffolding of EFWE with the error analysis of CIWE, to create a more comprehensive instructional framework for mathematics education.

REFERENCES

[1] Barbieri, C. A., Booth, J. L., & Chawla, K. (2023). Let's be rational: Worked examples supplemented textbooks improve conceptual and fraction knowledge. *Educational Psychology, 43*(1), 1–21. https://doi.org/10.1080/01443410.2022.2144142

[2] Bokosmaty, S., Sweller, J., & Kalyuga, S. (2015). Learning geometry problem solving by studying worked examples: Effects of learner guidance and expertise. *American Educational Research Journal, 52*(2), 307–333. https://doi.org/10.3102/0002831214549450

[3] Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24-34. https://doi.org/10.1016/j.learninstruc.2012.11.002

[4] Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education, 24* (1), 61-100. https://doi.org/10.3102/0091732X024001061

[5] Büyüköztürk, Ş. (2005). Anket geliştirme [Questionnaire development]. *Türk Eğitim Bilimleri Dergisi, 3*(2), 133-151.

[6] Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2018). *Eğitimde bilimsel araştırma yöntemleri [Scientific research methods in education].* Ankara: Pegem Akademi.

[7] Cierniak, G., Scheiter, K., & Gerjets, P. (2009). Explaining the split attention effect: Is the reduction of extraneous cognitive load accompanied by an increase in germane cognitive load? *Computers in Human Behavior, 25*(2), 315–324. https://doi.org/10.1016/j.chb.2008.12.020

[8] Clark, R., Nguyen, F., & Sweller, J. (2005). *Efficiency in learning: Evidence-based guidelines to manage cognitive load.* Sydney: Pfeiffer.

[9] Cook, T. D., Campbell, D. T., & Shadish, W. (2002). *Experimental and quasi-experimental designs for generalized causal inference.* Boston, MA: Houghton Mifflin.

[10] Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction, 22*(3), 206-214. https://doi.org/10.1016/j.learninstruc.2011.11.001

[11] Eiriksdottir, E., & Catrambone, R. (2011). Procedural instructions, principles, and examples: How to structure instructions for procedural tasks to enhance performance, learning, and transfer. *Human Factors, 53*(6), 749-770. https://doi.org/10.1177/0018720811419154

[12] Emecen, D. D. (2020). Öğrenme güçlüğü gösteren öğrencilere problem çözme becerilerinin öğretiminde çözümlü örnekler uygulamasının etkililiği [The effectiveness of using worked examples in teaching problem-solving skills to students with learning difficulties]. *Karadeniz Uluslararası Bilimsel Dergi, (48),* 81-101.

[13] Foster, N. L., Rawson, K. A., & Dunlosky, J. (2018). Self-regulated learning of principle-based concepts: Do students prefer worked examples, faded examples, or

problem solving? *Learning and Instruction, 55*, 124-138. https://doi.org/10.1016/j.learninstruc.2017.10.002

[14] Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory Into Practice, 40*(2), 102-109. https://doi.org/10.1207/S15430421TIP4002_4

[15] George, D., & Mallery, M. (2010). *SPSS for Windows step by step: A simple guide and reference, 17.0 update* (10th ed.). Pearson, Boston.

[16] Große, C. S. (2015). Fostering modeling competencies: Benefits of worked examples, problems to be solved, and fading procedures. *European Journal of Science and Mathematics Education, 3*(4), 364-375. https://doi.org/10.30935/scimath/9444

[17] Große, C. S., & Renkl, A. (2007). Finding and fixing errors in worked examples: Can this foster learning outcomes? *Learning and Instruction, 17*(6)*,* 612-634. https://doi.org/10.1016/j.learninstruc.2007.09.008

[18] Hesser, T. L., & Gregory, J. L. (2015). Exploring the use of faded worked examples as a problem solving approach for underprepared students. *Higher Education Studies, 5*(6), 36-46. https://doi.org/10.5539/hes.v5n6p36

[19] Işık, A., & Konyalıoğlu, A. C. (2010). Matematik eğitiminde görselleştirme yaklaşımı [The visualization approach in mathematics education]. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi, (11),* 462-471.

[20] İltüzer, Y. (2016). Öz açıklamalı çözümlü örneklerle desteklenmiş çevrimiçi öğrenme sürecinin karar kurallarını uygulama becerilerine etkisi [The effect of an online learning process supported by self-explanatory worked examples on the ability to apply decision rules]. *[Master's Thesis, Hacettepe University, Institute of Educational Sciences].*

[21] Kilic, S. (2014). Etki büyüklüğü. *Journal of Mood Disorders, 4*(1), 44-46. https://doi.org/10.5455/jmood.20140228010423

[22] Lee, N. (2013). The effects of self-explanation and reading questions and answers on learning computer programming language. [Doctoral Thesis, University of California].

[23] MoNE. (2018). Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar) [Mathematics curriculum (primary and middle school 1st-8th grades)]. http://mufredat.meb.gov.tr/

[24] Moreno, R. E., & Park, B. (2010). *Cognitive load theory: Historical development and relation to other theories*. In J. L. Plass, R. Moreno, & R. Brünken (Eds.), *Cognitive Load Theory* (pp. 9-28). Cambridge University Press. https://doi.org/10.1017/CBO9780511844744.003

[25] Pershan, M. (2021). Teaching math with examples. John Catt Educational.

[26] Paas, F., Tuovinen, J. E., Tabbers, H., & Van Gerven, P. W. M. (2003). Cognitive load measurement as a means to advance cognitive load theory. *Educational Psychologist, 38*, 63-71. https://doi.org/10.1207/S15326985EP3801_8

[27] Paas, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist, 38*(1), 1-4. https://doi.org/10.1207/S15326985EP3801_1

[28] Renkl, A. (2014). Toward an instructionally oriented theory of example-based learning. *Cognitive Science*, *38*(1), 1-37. https://doi.org/10.1111/cogs.12086

[29] Renkl, A. (2017). Learning from worked-examples in mathematics: Students relate procedures to principles. *ZDM–Mathematics Education, 49*(4), 571-584. https://doi.org/10.1007/s11858-017-0859-3

[30] Renkl, A., Atkinson, R. K., Maier, U. H., & Staley, R. (2002). From example study to problem solving: Smooth transitions help learning. *The Journal of Experimental Education, 70*(4), 293-315. https://doi.org/10.1080/00220970209599510

[31] Renkl, A., & Atkinson, R. K. (2003). Structuring the transition from example study to problem solving in cognitive skill acquisition: A cognitive load perspective. *Educational Psychologist, 38*(1), 15-22. https://doi.org/10.1207/S15326985EP3801_3

[32] Renkl, A., Atkinson, R. K., & Große, C. S. (2004). How fading worked solution steps works:A cognitive load perspective. *Instructional Science, 32*(1), 59 –82. https://doi.org/10.1023/B:TRUC.0000021815.74806.f6

[33] Reisslein, J., Atkinson, R. K., Seeling, P., & Reisslein, M. (2006). Encountering the expertise reversal effect with a computer-based environment on electrical circuit analysis. *Learning and Instruction, 16*(2), 92 –103. https://doi.org/10.1016/j.learninstruc.2006.02.008

[34] Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. *Child Development, 77*(1)*,* 1–29. https://doi.org/10.1111/j.1467-8624.2006.00852.x

[35] Siegler, R. S., & Chen, Z. (2008). Differentiation and integration: Guiding principles for analyzing cognitive change. *Developmental Science, 11*(4)*,* 433-448. https://doi.org/10.1111/j.1467-7687.2008.00689.x

[36] Sweller, J. (2020). Cognitive load theory and educational technology. *Educational Technology Research and Development, 68*(1), 1–16. https://doi.org/10.1007/s11423-019-09701-3

[37] Sweller, J. (2011). Cognitive load theory. In J. P. Mestre & B. H. Ross (Eds.), *Psychology of learning and motivation* (Vol. 55, pp. 37–76). Academic Press. https://doi.org/10.1016/B978-0-12-387691-1.00002-8

[38] Sweller, J. (2005). Implications of cognitive load theory for multimedia learning. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 19–30). Cambridge University Press. https://doi.org/10.1017/CBO9780511816819.003

[39] Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction, 4*(4), 295–312. https://doi.org/10.1016/0959-4752(94)90003-5

[40] Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*(2), 257-285. https://doi.org/10.1207/s15516709cog1202_4

[41] Sweller, J., & Cooper, G. A. (1985). The use of worked examples as a substitute for problem solving in learning algebra. *Cognition and Instruction, 2*(1), 59–89. https://doi.org/10.1207/s1532690xci0201_3

[42] Sweller, J., Ayres, P., & Kalyuga, S. (2011). Cognitive load theory in perspective. In *Cognitive load theory* (pp. 237-242). Springer. https://doi.org/10.1007/978-1-4419-8126-4_15

[43] Sweller, J., van Merrienboer, J. J. G., & Paas, F. (1998). Cognitive architecture and instructional design. *Educational Psychology Review, 10*(3)*,* 251–296. https://doi.org/10.1023/A:1022193728205

[44] Tennyson, R. D., & Cocchiarella, M. J. (1986). An empirically based instructional design theory for teaching concepts. *Review of Educational Research, 56*(1), 40–71. https://doi.org/10.3102/00346543056001040

[45] Tepgeç, M. (2017). Algoritma öğretiminde çözümlü örnek kullanımının öğrenci başarısına ve bilişsel yüke etkileri. [The effects of worked examples on student achievement and cognitive load in algorithm teaching] (Yüksek lisans tezi). *Hacettepe Üniversitesi Eğitim Bilimleri Enstitüsü*.

[46] Van Loon-Hillen, N., van Gog, T., & Brand-Gruwel, S. (2012). Effects of worked examples in a primary school mathematics curriculum. *Interactive Learning Environments, 20*(1), 89–99. https://doi.org/10.1080/10494820.2011.556089

[47] Yenilmez, K., & Çoksöyler, A. (2018). Altıncı sınıf öğrencilerinin işlem önceliği konusunda karşılaştığı zorluklar [Difficulties faced by sixth grade students regarding order of operations]. *Eğitim ve Öğretim Araştırmaları Dergisi, 7*(2), 155–156.

[48] Zrebiec Uberti, H., Mastropieri, M. A., & Scruggs, T. E. (2004). Check it off: Individualizing a math algorithm for students with disabilities via self-monitoring checklists. *Intervention in School and Clinic, 39*(5), 269–275. https://doi.org/10.1177/10534512040390050201

[49] Warli, W., Rahayu, P., & Cintamulya, I. (2025). Exploring the ability to prove abstract algebra problems through information technology-assisted scaffolding learning strategies. *Mathematics Teaching Research Journal, 17*(1), 28–55.

[50] Wong, A., Leahy, W., Marcus, N., & Sweller, J. (2012). Cognitive load theory, the transient information effect and e-learning. *Learning and Instruction*, *22*(6), 449- 457. https://doi.org/10.1016/j.learninstruc.2012.05.004

APPENDIX

Appendix I

The Mathematics Achievement Test

1) Find the result of the operation $10^1+2^3+1^5$.
2) Find the result of $25 + 5 \times 15 - 5$.
3) Find the result of operation $10^2 \div (4+3 \times 7)-1$.
4) Ms. İlknur wants to make bead necklaces for her 2 friends as gifts. Each necklace requires 6 blue beads and 3 red beads. Write a mathematical expression that gives the total number of beads Ms. İlknur needs to buy.
5) The prices of some products sold in a stationery store are given below:

Product	Price (TL)
Eraser	3
Pencil	4
Notebook	8
Book	24

A person buys 3 erasers, 3 pencils, 5 notebooks, and 5 books. Write a mathematical expression that gives the total cost of these items.

6) Mr. Arda will deposit 3^3 TL, Ms. Aslı will deposit 6^3 TL, and Mr. Çağrı will deposit 5^3 TL into the bank. Determine who will deposit the most money into the bank.

7) For the number 35b2 to be divisible by 9 without a remainder, find the digit that should replace 'b'. (10 points)

8) Which of the following numbers is divisible by 2, 3, and 5 without a remainder?
4742, 6501, 7450, 8340

9) The four-digit number 843? is divisible by both 2 and 3 without a remainder. Accordingly, find the sum of the possible digits that can replace '?'.

10) For the number 235a to be divisible by 6 without a remainder, find the possible digits that can replace 'a'.

Appendix II

Explanatory Faded Worked Examples (Example of order of operations)

Dear Students,

Below are questions related to the order of operations. The solution of the first example is provided step-by-step, completely and correctly, with explanations in parentheses. In the other questions, some steps are left blank. By examining the fully solved example, complete the missing steps. Finally, solve the “Your Turn” question by showing all the steps.

Example 1: Find the result of the expression $6\times4+(8^2\div4) -20$.

Step 1: 6 × 4 + (64 ÷ 4) - 20 (According to the order of operations, the value of the exponential expression is calculated first, so the value of 8^2 is calculated.)

Step 2: 6 × 4 + 16 - 20 (The operation inside the brackets will be performed according to the order of operations. Therefore, division is carried out. The number 64 is divided by 4, and its value is calculated.)

Step 3: 24 + 16 - 20 (Multiplication is performed according to the order of operations. Therefore, the value is calculated by multiplying 6 by 4.)

Step 4: 40-20 (Addition and subtraction are performed last according to the order of operations. Therefore, the addition and subtraction operations are carried out from left to right, respectively.)

Step 5: 20 (The result is obtained after performing the operations while adhering to the order of operations.)

Example 2: Find the result of the expression: 50 ÷ (5 + 2 × 10) - 2

Step 1: 50 ÷ (5 + 20) – 2 (According to the order of operations, multiplication in the brackets is performed first. Therefore, the value is calculated by multiplying 2 by 10)

Step 2: 50 ÷ 25 – 2 (The addition inside the brackets is performed next. Therefore, the value is calculated by adding 5 and 20)

Step 3: 2- 2 (Division is performed according to the order of operations. Therefore, the value is calculated by dividing 50 by 25.)

Step 4: You will solve this step.

Example 3: Find the result of the expression: (25 + 55) ÷ 5 + 3^2

Step 1: (25 + 55) ÷ 5 + 9 (According to the order of operations, the exponent is calculated first. Therefore, the value of 3^2 is calculated.)

Step 2: 80 ÷ 5 + 9 (The addition inside the brackets is performed next. Therefore, the value is calculated by adding 25 and 55.)

Step 3: You will solve this step

Step 4: You will solve this step.

Appendix III

Correct–Incorrect Worked Examples (Order of Operations)

Below are questions related to order of operations. First, carefully examine the example question and its step-by-step solution. Then, read the two solutions (one correct, one incorrect) provided by your sixth-grade classmates for the same question, determine which one is correct, explain the error and reason for the incorrect solution, and answer the related guiding questions. Finally, answer the related guiding questions and, following the same principles, solve the "Your Turn" question by showing all the steps.

Example: Find the result of the expression $7 \times 2 - (4^2 \div 2) + 8$.

Solution: $7\times 2 - (16:2) + 8$ (According to the order of operations, exponential expressions are calculated first. Therefore, by finding the value of 4^2, the result is 16, which is then substituted in.)

$= 7 \times 2 - 8 + 8$ (According to the order of operations, the division inside the parentheses is performed first. Therefore, 16 is divided by 2, and the result is written in its place.)

$= 14 - 8 + 8$ (Multiplication is performed according to the order of operations. Therefore, 7 is multiplied by 2, and the result is written in its place.)

$= 14$ (The final addition and subtraction operations are performed in sequence according to the order of operations. Therefore, 8 is subtracted from 14, and the result is added to 8. The final result is then written.)

Example 1: Find the result of the expression $6 \times 4 + 8^2 \div 4 - 20$.

Arda's solution: $6 \times 4 + (8^2 \div 4) - 20 = 6 \times 4 + (64 \div 4) - 20$

$= 6 \times 4 + 16 - 20$

$= 24 + 16 - 20$

$= 20$

Arda explained the solution to the problem as follows:

According to the order of operations, the exponential expression is evaluated first. Therefore, I found the value of 8^2 as 64 and substituted it. In the next step, the operations inside the brackets are performed according to the order of operations. So, I divided 64 by 4 and got 16. Next, multiplication is performed according to the order of operations. I multiplied 6 by 4, found 24, and wrote it down. Finally, addition and subtraction are performed according to the order of operations. I added 24 and 16 to get 40, then subtracted 20 from 40, arriving at 20.

Derin's solution: $6 \times 4 + (8^2 \div 4) - 20 = 24 + (64 \div 4) - 20$

$= 88 \div 4 - 20$

$= 22 - 20$

$= 2$

Derin explained the solution to the problem as follows:

According to the order of operations, the exponential expression is evaluated first. So, I found the value of 8^2 as 64 and substituted it. Next, multiplication is performed based on the order of operations. Therefore, I multiplied 6 by 4 and got 24. Then, I added 24 to 64 according to the order of operations, and got 88. Division is carried out next according to the order of operations. Thus, I divided 88 by 4 and found 22. Finally, addition and subtraction are performed according to the order of operations. I subtracted 20 from 22 and got 2.

Which of the solutions is correct? Why? Explain.

Which of the solutions is incorrect? Why? Explain.

Why is the method used by the student who solved it incorrectly not valid? Explain.