Boosting Students' Abilities in Understanding, Application, and Reasoning of Numeracy: Didactical Differentiated Learning

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Abstract: This study develops and implements a differentiated mathematics learning model integrating targeted didactical strategies and observation-based assessments to enhance numeracy skills among Indonesian elementary students. Using a Teacher Action Research (TAR) approach, 28 fifth-grade students participated in a learning intervention involving differentiated strategies. Data collection included numeracy tests across three cognitive levels and structured observations. Paired t-tests and ANOVA were used to analyze the intervention's effectiveness. The findings indicate that differentiated learning significantly improved students' numeracy skills across understanding, application, and reasoning dimensions. Normality tests confirmed that pre-test and post-test scores followed a normal distribution ($p > 0.05$). Paired samples tests showed significant improvements in understanding ($p = 0.000$), application ($p = 0.000$), and reasoning ($p = 0.000$), with ANOVA results ($F = 6.208$, $p = 0.003$) indicating varied impacts across cognitive levels. Post-test scores increased significantly (mean difference = -13.811, $p = 0.000$), meeting the success criteria for classical completeness. The study highlights the effectiveness of differentiated learning in promoting comprehensive cognitive development in mathematics. It offers valuable insights for educators in designing tailored instructional approaches to optimize student learning outcomes in numeracy, addressing both theoretical and practical gaps in mathematics education.

Keywords: Cognitive Development; Elementary; Learning Trajectory; Mathematics

INTRODUCTION

The increasing demand for 21st-century skills has led to a growing emphasis on literacy and numeracy as essential components of educational development. Numeracy, as defined by the World Economic Forum (2015), refers to the ability to understand, apply, and analyze mathematical concepts effectively in real-life contexts (Atkinson & Jackson, 2016). In the Indonesian context, the Ministry of Education and Culture defines numeracy as the capacity to reason using mathematical concepts, procedures, facts, and tools to solve everyday problems and make informed decisions (Ayuningtyas & Sukriyah, 2020). Despite its critical role in educational success and daily problem-solving, numerous studies indicate that Indonesian students face persistent challenges in this area. The 2018 PISA assessment revealed that Indonesian students ranked near the bottom globally, with a score of 379, significantly below the OECD average of 489 (Schleicher, 2019). These findings underscore the urgent need for educational interventions to enhance students' numeracy competencies and mathematical reasoning.

Interviews with teachers from Phase C classes (fifth grades and sixth grades) further highlight this concern. Teachers observed that students often struggle with basic reading comprehension and mathematical reasoning, which hampers their ability to grasp and apply mathematical concepts effectively. These challenges indicate that the current instructional approaches may not adequately support diverse student learning needs, particularly in mathematics education. As numeracy forms the foundation for academic success and critical problem-solving abilities, addressing this issue is essential to prepare students for future academic and personal endeavors.

One promising approach to addressing this challenge is differentiated learning, a pedagogical framework that emphasizes tailored instruction to meet the diverse needs, abilities, and preferences of students (Tomlinson, 2001). Differentiated learning is widely recognized for its effectiveness in fostering essential learning competencies, particularly in mathematics education. Research has demonstrated that differentiated learning helps students engage with subject matter more effectively by adapting content, process, product, and learning environments to their individual needs (Bhattacharya, 2021; Magee & Breaux, 2013; C. A. Tomlinson & Eidson, 2003). Furthermore, it has been shown to reduce frustration and prevent perceptions of failure, fostering a positive learning experience (Coubergs et al., 2017).

The concept of differentiated learning has garnered significant attention in educational research and practice as a strategy to accommodate diverse student needs. According to (Coubergs et al., 2017; Yong & Sokumaran, 2023), differentiated learning emphasizes tailoring content, processes, and products to individual student abilities and learning preferences. This approach has demonstrated positive outcomes in fostering student engagement and improving academic performance (Valiandes & Neophytou, 2018). Moreover, studies by Lindner & Schwab (2020); Smets & Struyven (2020); and van Geel et al. (2019) have highlighted its effectiveness in various educational contexts, including inclusive education settings. Despite these theoretical advancements, a notable gap exists in the application of differentiated learning specifically within mathematics education to enhance numeracy skills.

The existing body of research largely focuses on general classroom applications of differentiated instruction or its impact on literacy and cognitive engagement (Lindner & Schwab, 2020). However, empirical studies investigating the integration of differentiated learning models tailored to mathematics education remain limited (Eikeland & Ohna, 2022; Maulyda et al., 2025). Furthermore, while differentiated learning is acknowledged as a promising approach for meeting diverse learning needs, there is a paucity of research examining its role in directly enhancing foundational numeracy skills, particularly in primary school contexts where these skills are critical. Another theoretical gap pertains to the lack of comprehensive instructional models that incorporate practical, didactically sound modules for differentiated mathematics learning. Although several studies have proposed frameworks for differentiation (Coubergs et al., 2017; H. Tomlinson, 2013), few provide concrete, structured models specifically designed to address numeracy challenges. Additionally, existing models often neglect the role of classroom observation and reflection, which are essential components for assessing and refining instructional strategies.

This study addresses a critical gap in numeracy education by developing and implementing a differentiated learning model that integrates targeted didactical strategies and observation-based assessments. Despite extensive research on differentiated instruction, much of the existing literature has focused on general classroom practices without fully exploring its potential in enhancing specific cognitive dimensions of numeracy, particularly in diverse learning contexts. Furthermore, the theoretical discourse often lacks clarity on how tailored instruction can be systematically designed and evaluated to meet varying student needs. This research not only bridges that theoretical void but also offers a practical framework for educators to implement differentiation effectively. By emphasizing the nuanced relationship between instructional strategies and students' cognitive development in mathematics, this study advances both pedagogical theory and practice, offering actionable insights for addressing persistent challenges in numeracy education.

To refine the research focus and strengthen the manuscript’s contribution to the field, this study emphasizes two key areas of novelty. First, it develops and implements a differentiated mathematics learning model and teaching modules specifically designed for numeracy, addressing the distinct challenges faced by Indonesian elementary students. Unlike prior studies that often concentrate on generalized educational outcomes, this research provides a targeted instructional framework that caters to students' diverse learning needs, promoting an inclusive, student-centered environment that enhances critical numeracy skills. Second, the research uniquely focuses on identifying and analyzing students' numeracy proficiency across three cognitive levels: understanding, applying, and reasoning. By mapping students' capabilities at these levels, the study offers valuable insights into how differentiated instruction can be adapted to meet varying numeracy demands. This dual emphasis not only enriches the pedagogical landscape but also contributes practical solutions for improving mathematics education in diverse classroom settings.

LITERATURE REVIEW

Cognitive Level of Numeracy

Numeracy encompasses the ability to understand, apply, and reason with mathematical concepts, forming a fundamental component of an individual's cognitive and problem-solving abilities (OECD, 2019; Schleicher, 2019). The cognitive dimension of numeracy involves three hierarchical levels: understanding, application, and reasoning. To support the development of these cognitive levels, effective instructional strategies and practical examples should be integrated into teaching practices.

Understanding

Understanding is the foundation, enabling students to grasp mathematical concepts, recognize patterns, interpret symbols, and establish relationships among numbers (Keller et al., 2001). At this level, students develop essential skills such as decoding numerical expressions, comprehending mathematical operations, and visualizing abstract concepts. Strategies to strengthen understanding include exploration activities, discovery learning, and guided practice. For example, using manipulatives or visual aids to demonstrate place value or fractions helps students concretely grasp abstract ideas. Teachers can also employ number talks and pattern recognition exercises to deepen students' conceptual understanding. Without a strong foundation at this level, students may face challenges in progressing to higher cognitive levels (Kirschner, 2002; Lin et al., 2018).

Application

Application involves the ability to use mathematical knowledge to solve contextual problems, encompassing both routine and non-routine tasks. This stage requires students to bridge theoretical mathematical understanding with real-world situations (Maulyda et al., 2020). Strategies to enhance application skills include providing contextualized problems that relate to students' everyday experiences, incorporating project-based learning, and engaging students in hands-on activities. For instance, students might calculate the cost of items during a simulated shopping activity or measure distances in a classroom project. These experiences enable students to perform computations, apply formulas, and develop problem-solving strategies relevant to specific scenarios. Research indicates that students often struggle to apply mathematical knowledge when faced with unfamiliar contexts, emphasizing the need for authentic and practical learning experiences (Chapman, 2013; de Walle et al., 2016).

Reasoning

The reasoning level, representing the pinnacle of numeracy, involves critical thinking, analysis, synthesis, and justification of mathematical processes. At this stage, students are expected to evaluate multiple solution strategies, construct logical arguments, and defend their conclusions. Inquiry-based learning environments, classroom discussions, and problem-solving tasks that require justification of solutions are effective strategies for fostering reasoning skills (Rasmussen & Isoda, 2019). For example, students can be presented with open-ended problems where they must explore various solution methods, compare their effectiveness, and articulate their reasoning process. Teachers can also encourage peer-to-peer discussions where students critique and refine each other's solutions, thereby promoting reflective thinking and deeper understanding.

By structuring instruction to address these three cognitive levels (understanding, application, and reasoning), educators can cultivate comprehensive numeracy competencies that are essential for academic success and everyday problem-solving. The concept of fractions can be examined through the three cognitive levels of numeracy: understanding, application, and reasoning. At the understanding level, students grasp the fundamental concept of fractions as parts of a whole. This involves recognizing that a fraction represents a division of an object or quantity into equal parts. For instance, when presented with a circle divided into four equal sections, students can identify that shading one section represents the fraction $\frac{1}{4}$. Visual aids and manipulatives, such as paper folding or using physical objects like pieces of fruit or pizza models, help students concretize abstract ideas and recognize relationships between different fractions. Establishing this foundational understanding is crucial, as it enables students to interpret numerical symbols, recognize patterns, and comprehend basic mathematical operations.

Moving to the application level, students use their understanding of fractions to solve real-world problems. This stage involves bridging theoretical knowledge with practical situations that are familiar to students. For example, a student might encounter a scenario where Alya has a pizza cut into eight slices and eats three of them. The student is then tasked with determining the fraction of the pizza that has been eaten ($\frac{3}{8}$) and the portion that remains ($\frac{5}{8}$). Other practical applications include measuring ingredients for a recipe, where a student needs to measure half a cup of sugar, or distributing six candies evenly among four friends. Such contextualized tasks not only help students perform computations but also encourage them to connect mathematical concepts to everyday experiences, thereby enhancing the relevance and practicality of their learning.

At the highest cognitive level, reasoning, students are expected to analyze mathematical situations, evaluate different solution strategies, and justify their conclusions. For instance, when asked to determine which fraction is larger between $\frac{3}{4}$ and $\frac{2}{3}$, students must compare the two fractions using methods such as drawing visual models, converting fractions to decimals, or finding common denominators. A student might explain, "By dividing two rectangles into four and three equal parts respectively, I observed that shading three out of four parts covers more area than shading two out of three parts, which shows that $\frac{3}{4}$ is greater." Beyond simple comparisons, reasoning tasks may also involve challenging assumptions, such as questioning whether $\frac{1}{2}$ is always larger than $\frac{1}{4}$ when considering contexts involving differently sized wholes. Encouraging students to articulate their thought processes, engage in peer discussions, and explore multiple solutions fosters critical thinking and deepens their conceptual understanding.

Differentiated Learning

Differentiated learning is a key strategy in addressing the diverse learning needs of students while promoting equity in educational outcomes by recognizing variations in students' readiness levels, learning profiles, and interests (Hasanah et al., 2022). By tailoring instruction to meet these differences, teachers can design and deliver lessons that are both accessible and appropriately challenging, thereby fostering a more inclusive classroom environment. In mathematics education, differentiated learning has proven effective in enhancing students' problem-solving skills and conceptual understanding.

Practical implementation of differentiated learning can be achieved by adapting the content, process, product, and learning environment to suit students' varied needs. For example, in terms of content, teachers can offer materials at varying levels of complexity or present information through different modes (e.g., videos, texts, and hands-on activities). To differentiate the process, scaffolding complex mathematical concepts allows students to build on prior knowledge gradually, reducing cognitive overload and fostering deeper understanding (van Geel et al., 2019). Differentiating the product may involve allowing students to demonstrate their understanding through written assignments, presentations, or creative projects. As for the learning environment, flexible seating arrangements and collaborative group work can accommodate diverse learning preferences. Providing multiple problem-solving pathways encourages creativity and critical thinking, enabling students to approach mathematical problems from different angles and develop versatile strategies.

Technology integration further expands the possibilities for differentiated learning in mathematics classrooms. Digital tools and platforms offer interactive and customizable learning experiences, allowing students to work at their own pace and receive instant feedback on their progress (Bhattacharya, 2021; Yong & Sokumaran, 2023). Tools such as virtual manipulatives and dynamic mathematical representations have been shown to enhance engagement and comprehension, particularly among students with varying levels of mathematical proficiency.

However, effective implementation of differentiated learning requires ongoing teacher training, access to appropriate resources, and a shift toward more flexible and student-centered teaching practices (Kostadinov & Kolev, 2020). Despite its potential, empirical studies exploring practical applications of differentiated learning in mathematics education remain limited, underscoring the need for further research to develop comprehensive instructional models that support cognitive development in numeracy.

Conjecture of Study

This study addresses a gap in the literature by introducing a novel approach that integrates differentiated learning with the identification of students' numeracy proficiency levels—understanding, application, and reasoning—an area often overlooked in previous research. Unlike earlier studies that focus on general educational outcomes, this research emphasizes the alignment between instructional strategies and students' specific cognitive needs in mathematics. By focusing on differentiated teaching models tailored to individual numeracy levels, the study aims to provide a more contextually relevant and classroom-applicable framework, particularly for Indonesian elementary education. The research is guided by three key research questions designed to explore under-examined aspects of differentiated instruction in numeracy:

1. How does differentiated learning influence students' cognitive levels of numeracy (understanding, application, and reasoning)?
2. Which cognitive level benefits most from differentiated learning, and why?
3. How does the integration of differentiated teaching models contribute to students' overall numeracy proficiency?

These questions stem from three primary conjectures based on theoretical frameworks and existing literature. First, differentiated learning is hypothesized to positively impact students' cognitive levels of numeracy by providing targeted instructional support that enhances comprehension, contextual application, and reasoning skills (H1). Second, it is anticipated that the reasoning level will experience the most significant improvement (H2), considering its reliance on complex cognitive processes and higher-order thinking. This focus addresses a gap in studies that often overlook the nuanced progression from understanding to reasoning. Third, the study posits that differentiated instruction fosters holistic numeracy development, offering a practical solution to the challenges Indonesian students face in mathematics education (H3). By connecting differentiated learning to specific cognitive levels, this research presents an innovative instructional model that directly responds to classroom realities and students' diverse learning needs, thereby offering both theoretical and practical contributions to the field.

METHOD

Research Design

This research uses the Teacher Action Research (TAR) approach. The TAR approach is a research design carried out by teachers to improve the learning process in the classroom (Bell & Aldridge, 2014). The TAR approach allows teachers to improve student learning outcomes, improve the quality of learning activities, and measure the success of interventions designed and implemented by teachers in the classroom (Reeves, 2008). This approach consists of four main stages:

Step 1. Assessing the Learning Environment

At this stage, researchers conducted a comprehensive analysis of students' learning obstacles to identify specific challenges in mastering addition and subtraction story problems. This involved reviewing previous mathematics learning outcomes and gathering observational data from classroom interactions to gain insights into students' difficulties. The research team then evaluated the developmental aspects that needed to be nurtured in elementary school students, aligning these with the current elementary school curriculum to ensure that the planned interventions were curriculum relevant.

Based on these findings, researchers designed developmentally appropriate differentiated learning strategies tailored to diverse student needs. This included preparing games that matched children's various learning styles to enhance engagement and cognitive development. Additionally, researchers created comprehensive teaching modules to guide instruction and provided suitable learning media aligned with the intervention theme. Learner worksheets were meticulously developed to support differentiated learning practices, allowing students to work at their own pace and understanding levels. To ensure a systematic and effective implementation, researchers developed observation and evaluation formats. These tools enabled the accurate monitoring of students' progress, classroom dynamics, and the effectiveness of the differentiated learning interventions. The structured observation and evaluation formats also helped identify areas for improvement and refinement during subsequent instructional cycles, ensuring a continuous improvement process.

Step 2. Reflection and Discussion

Teachers used data from students' previous mathematics learning outcomes and observation notes to determine the appropriate interventions. To support this process, researchers facilitated a collaborative focus group discussion (FGD) involving teachers, lecturers, and doctoral students. The purpose of this FGD was to evaluate and refine the didactical design developed by teachers, ensuring its alignment with students' learning needs and curriculum requirements. The discussion was conducted online through the Zoom platform, enabling interactive sessions between all participants.

During the FGD, teachers presented the initial didactical design, including planned learning sequences, instructional strategies, and assessment methods. Researchers and experts provided constructive feedback by analyzing the strengths and weaknesses of the proposed design. The discussions highlighted the need to incorporate more engaging and hands-on learning activities, particularly for abstract mathematical concepts like addition and subtraction in story problems. Suggestions included incorporating more scaffolded tasks and enhancing the role of visual aids. As a result of the collaborative evaluation, it was decided to integrate Dienes Block media and carefully designed Mathematical Worksheets to help students grasp mathematical concepts through structured and interactive activities. The finalized didactical design aimed to foster better student understanding, engagement, and problem-solving skills, ensuring that learning outcomes in addition and subtraction story problems would improve significantly.

Step 3. Intervention

At this stage, researchers begin to plan, implement, monitor, and reflect on the interventions provided in the classroom. This intervention activity is carried out in several cycles depending on student needs. The learning cycle model in this research was developed from Kemmis in Bell & Aldridge (2014) which can be seen in Figure 1 below.

In this study, the intervention process was carried out in three distinct cycles: pre-cycle, Cycle 1, and Cycle 2, each designed to address specific cognitive level of numeracy based on ongoing observations and evaluations. Each cycle concluded with a reflective session, where researchers and teachers collaboratively reviewed the outcomes and refined the didactical design to better meet students' learning needs.

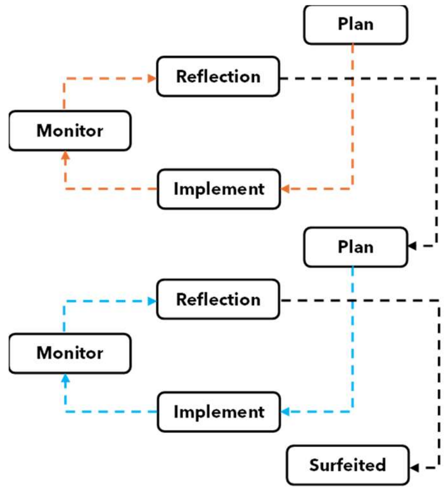


Figure 1: Kemmis's Learning Cycle in Research

Step 4. Re-assessment

The final stage involves conducting a comprehensive assessment to measure students' numeracy skills. Reflection activities include analyzing, synthesizing, interpreting, explaining, and drawing conclusions. The outcomes of this reflection process led to revisions of the implemented plan, which can be used to improve teacher performance in subsequent sessions. Reflection in action research involves examining what has or has not occurred, what outcomes have been achieved, and what remains incomplete despite corrective actions. The insights gained from this reflective process guide the determination of further steps to achieve the objectives of the research. As a result, action research is not limited to a single meeting, as the findings from reflection require time for implementation and refinement.

Participants

This research involved 28 fifth-grade students from Muhammadiyah Rambah Private School. The school is in a suburban area and is known for its commitment to integrating character education into the learning process. The students who participated were selected based on their availability and willingness to be part of the study. They represented diverse academic abilities, providing a comprehensive overview of the impact of the instructional approach on different learner profiles. Ethical considerations, including parental consent, were thoroughly addressed to ensure responsible research practices (Adams et al., 2023).

Didactical Differentiated Learning

Differentiated learning addresses the diverse needs of students by considering their varying learning styles, readiness levels, and interests. In mathematics education, this approach is vital for enhancing numeracy skills, as students often possess differing levels of prior knowledge, cognitive abilities, and engagement with mathematical concepts. By employing differentiated instruction, teachers can provide tailored learning experiences that promote inclusivity and improve student outcomes. The proposed didactic design follows a structured model that incorporates content, process, product, and learning environment differentiation. This model begins with pre-assessment to identify students' numeracy proficiency levels, ranging from understanding to applying and reasoning through mathematical problems. Based on these assessments, content differentiation is implemented by offering various materials such as visual aids, manipulatives, and real-life contextual problems catering to different learning styles. Process differentiation involves varied instructional strategies, including guided practice, peer collaboration, and individualized support, ensuring that all students engage with mathematical concepts at an appropriate pace. To differentiate the product, students demonstrate their understanding through diverse formats, such as solving practical problems, creating mathematical models, or presenting solutions verbally. Lastly, the learning environment is adapted to foster collaboration and autonomy, with flexible seating and the integration of technology to provide interactive learning experiences. The instructional model is implemented through a series of learning phases: (1) *Introduction*, activating prior knowledge and presenting learning objectives; (2) *Exploration*, engaging students with hands-on activities and group discussions; (3) *Concept Development*, guiding students to discover and understand mathematical concepts; (4) *Application*, encouraging students to apply learned concepts to solve problems; and (5) *Reflection*, allowing students to evaluate their understanding and learning process.

Data Collection

The data collection process in this research was conducted through a numeracy test designed based on three cognitive levels: understanding, application, and reasoning. The test aimed to comprehensively measure students' abilities in solving numeracy problems according to the cognitive levels specified in the curriculum. The test instruments consisted of various items covering each cognitive level to ensure comprehensive data collection. The test results were then analyzed to identify changes in students' abilities throughout the research process. Detail of test instrument show in table 1.

No	***Dimension***	***Item***	***Example***
1	*Understanding*	*3*	*Students are asked to add simple integers and explain the result.*
2	*Application*	*4*	*Students are asked to calculate the total shopping cost with a certain discount.*
3	*Reasoning*	*3*	*Students are asked to solve a word problem that requires problem-solving strategies and provide logical reasoning for their answer.*
	Total	*10*	

Table 1: Numeracy Test Instrument

Additionally, an observation sheet was used to describe the learning process in each cycle of the research. Observations were conducted to monitor student engagement, the quality of teacher-student interactions, the use of learning media, and the reflection process during learning. The observation instrument was structured based on five key dimensions that capture important aspects of cycle-based learning. Detail of observation instrument show in table 2.

No	*Dimension*	*Item*	*Example*
1	*Apperception and Motivation*	*6*	*Conveying the benefits of learning materials*
2	*Delivery of Competencies and Activity Plan*	*2*	*Explaining the competencies students are expected to achieve*
3	*Mastery of Learning Material*	*4*	*Presenting material systematically (from simple to complex, concrete to abstract)*
4	*Application of Educational Learning Strategies*	*7*	*Actively involving students in learning activities*
5	*Implementation of Independent Learning and Student-Centered Learning*	*8*	*Applying differentiated learning (Content, Process, and Product)*
	Total	*27*	

Table 2: Observation Instrument

Data Analysis

The data analysis in this research was conducted using quantitative statistical methods to evaluate the effectiveness of differentiated learning interventions on students' numeracy skills. Initially, a simple paired t-test was employed to assess the impact of the interventions on each cognitive level: understanding, application, and reasoning. This analysis aimed to determine whether there were statistically significant improvements in student performance at each level after the differentiated learning activities.

To further explore differences between cognitive levels, a One-Way ANOVA test was conducted to compare the mean scores across the three cognitive dimensions. This test provided insights into whether the learning interventions had varying effects depending on the cognitive complexity of the tasks. Post-hoc analyses were performed to identify specific pairwise differences between the cognitive levels if a significant overall difference was detected. Additionally, a Paired Sample t-Test was conducted to measure the combined impact of the interventions on overall numeracy skills by aggregating the data across all three cognitive levels. This test assessed whether the collective improvements observed in the understanding, application, and reasoning dimensions were statistically significant (Hair et al., 2019).

To complement the test results, data from the observation sheets were analyzed as supporting evidence. These observations captured key dimensions of the learning process, including student engagement, teacher-student interactions, the use of learning media, and the reflection process during each instructional cycle. The structured observation data provided valuable qualitative insights, allowing researchers to interpret the statistical findings more comprehensively. By triangulating the numeracy test outcomes with observation data, a more nuanced understanding of the effectiveness of differentiated learning interventions was achieved. This combined analysis ensured that both quantitative and qualitative evidence informed the study's conclusions, thereby strengthening its validity.

RESULTS

The research findings highlight the effectiveness of differentiated learning in enhancing students' numeracy skills across cognitive dimensions. Before conducting the experimental test, the researchers performed a normality test to assess the data distribution. This step was taken to ensure that the data could be analyzed using parametric and non-parametric statistical tests. The results of the normality test are presented in Table 3 below.

Tests of Normality

	Kolmogorov-Smirnov[a]			*Shapiro-Wilk*		
	Statistic	*df*	*Sig.*	*Statistic*	*df*	*Sig.*
PostTest Average	*0.145*	*28*	*0.137*	*0.941*	*28*	*0.117*
PreTest Average	*0.140*	*28*	*0.168*	*0.967*	*28*	*0.502*

a. Lilliefors Significance Correction

Table 3: Normality Test Source: SPSS Outputs

The normality test results using the Kolmogorov-Smirnov and Shapiro-Wilk tests indicate that both the pre-test and post-test average scores follow a normal distribution. For the post-test average, the Kolmogorov-Smirnov test yielded a statistic of 0.145 with a significance level of 0.137, while the Shapiro-Wilk test showed a statistic of 0.941 and a significance level of 0.117. Similarly, the pre-test average yielded a Kolmogorov-Smirnov statistic of 0.140 with a significance level of 0.168 and a Shapiro-Wilk statistic of 0.967 with a significance level of 0.502. Since all p-values are greater than 0.05, it can be concluded that the data for both pre-test and post-test average scores do not significantly deviate from a normal distribution, thus meeting the assumption of normality for subsequent parametric analyses (Hair et al., 2019). These findings ensure the validity of using parametric tests to investigate the impact on cognitive levels of numeracy and overall numeracy abilities, allowing for a robust analysis of the effectiveness of the interventions applied.

Impact on Cognitive Level of Numeracy ➔ H^1 & H^2

Table 4 presents the results of the Paired Samples Test, illustrating the differences in students' cognitive level scores before and after differentiated instruction across three meetings. The test compared pre-test and post-test scores for understanding, application, and reasoning levels, as shown by the mean difference values, standard deviation, and confidence intervals.

Paired Samples Test

		Paired Differences				
		Mean	Std. Deviation	95% Confidence Interval of the Difference Lower	Upper	Sig. (2-tailed)
First Meeting	Understanding PreTest - Understanding PostTest	-16.071	4.973	-18.000	-14.143	0.000
Second Meeting	Application PreTest - Application PostTest	-7.500	6.597	-10.058	-4.942	0.000
Third Meeting	Reasonable PreTest - Reasonable PostTest	-17.857	13.501	-23.092	-12.622	0.000

Table 4: Effect on Cognitive Dimensions Source: SPSS Outputs

The results indicate that differentiated instruction had a statistically significant effect on all cognitive levels, with p-values is 0.000 for each paired sample, suggesting a strong influence on students' learning outcomes. Among the three cognitive levels, the reasoning level was the most affected by differentiated learning, as demonstrated by the largest mean difference of -17.857 and a broader confidence interval range (-23.092 to -12.622). The understanding level also showed a substantial effect with a mean difference of -16.071, while the application level exhibited a relatively smaller impact, with a mean difference of -7.500 (Hair et al., 2019). These findings highlight the effectiveness of differentiated instruction in enhancing students' reasoning and comprehension skills compared to their application abilities.

The results of the ANOVA analysis, as presented in Table 5, provide compelling evidence of the significant impact of differentiated instruction on students' cognitive achievement levels in numeracy. The findings reveal a statistically significant difference between groups ($F = 6.208$, $p = 0.003$), indicating that the instructional approach did not affect all cognitive levels uniformly (Hair et al., 2019). This variation underscores the importance of analyzing specific group comparisons to identify which cognitive domains benefit most from differentiated instruction.

ANOVA

Numeracy Test	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1282.930	2	641.465	6.208	0.003
Within Groups	8369.245	81	103.324		
Total	9652.176	83			

Table 5: ANOVA Test Source: SPSS Outputs

Multiple Comparisons

Dependent Variable: Sum					
	(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.
Tukey HSD	Understanding	Application	3.150	2.717	0.481
		Reasonable	9.404*	2.717	0.002
	Application	Understanding	-3.150	2.717	0.481
		Reasonable	6.254	2.717	0.021
	Reasonable	Understanding	-9.404*	2.717	0.002
		Application	-6.254	2.717	0.021

*. The mean difference is significant at the 0.05 level.

Table 6: Multiple Comparisons Test Source: SPSS Outputs

The post hoc multiple comparisons using the Tukey HSD test (Table 6) offer further clarification. The reasoning group demonstrated the most substantial improvement compared to the understanding and application groups, with statistically significant mean differences of 9.404 ($p = 0.002$) and 6.254 ($p = 0.021$), respectively. These results suggest that differentiated learning strategies are particularly effective in enhancing higher-order cognitive skills, specifically students' reasoning abilities. In contrast, the difference between the understanding and application groups was not statistically significant ($p = 0.481$), implying that differentiated instruction may exert a similar influence on these cognitive levels (Hair et al., 2019). These findings highlight the critical role of tailored instructional methods in fostering complex cognitive development and underscore the necessity of prioritizing differentiated strategies to advance students' reasoning capabilities.

Impact on Numeracy Abilities ➔ H^3

Table 7 presents the results of the paired samples t-test, which was conducted to analyze the impact of differentiated learning on students' numeracy achievement. This test was employed to evaluate whether there was a significant difference between the pre-test and post-test scores in terms of cognitive levels. The negative mean difference (-13.811) indicates an improvement in numeracy performance after the differentiated learning intervention. A statistically significant result (Sig. = 0.000, $p < 0.05$) confirms that differentiated learning positively influenced students' cognitive development.

Paired Samples Test

		Paired Differences				Sig. (2-tailed)
				95% Confidence Interval of the Difference		
		Mean	Std. Deviation	Lower	Upper	
Numeracy Test	PreTest Average - PostTest Average	-13.811	5.776	-16.051	-11.571	0.000

Table 7: Paired Samples Test Source: SPSS Outputs

The findings in Table 7 demonstrate that differentiated learning significantly enhanced students' cognitive numeracy levels, as evidenced by the substantial decrease in the difference between pre-test and post-test scores. The mean difference of -13.811 suggests that students achieved notable progress after the learning intervention. The narrow 95% confidence interval (-16.051 to -11.571) further indicates the stability of these findings. Therefore, it can be concluded that differentiated instruction primarily contributes to higher levels of cognitive understanding and problem-solving abilities, reinforcing its effectiveness as a teaching strategy for improving numeracy skills (Hair et al., 2019). Additionally, the findings further indicate that differentiated mathematics learning models were effective in achieving classical completeness for students' numeracy skills. The action hypothesis in this study was successfully tested, as the results met the success criteria set: a minimum numeracy score of 75 in the Minimum Competencies Assessment (aligned with the School's Standar) and at least 80% of students achieving classical completeness.

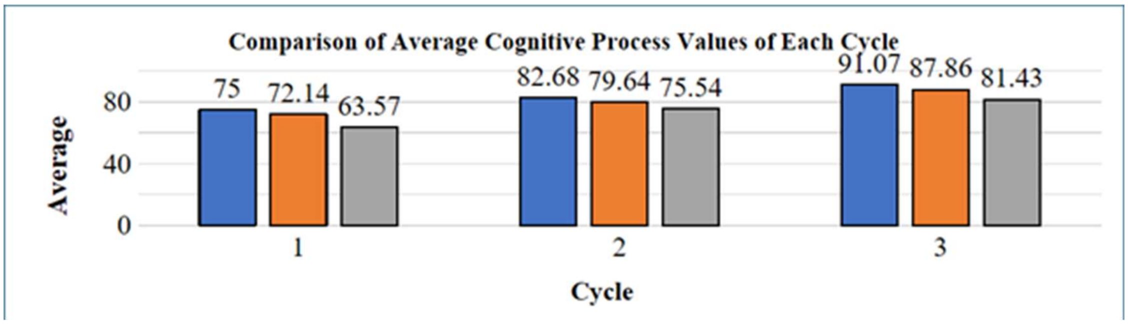


Figure 2: Comparison of Cognitive Processes of Each Cycle

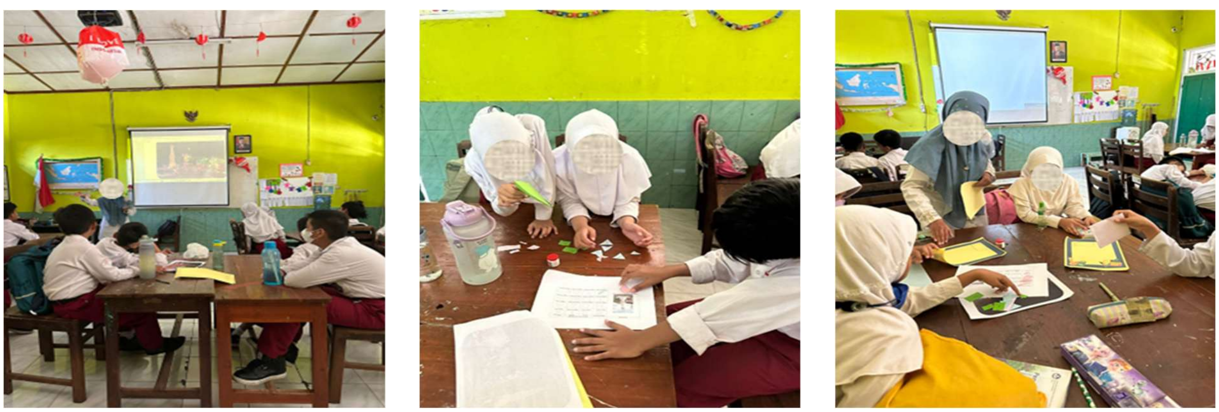
Figure 2 illustrates consistent improvements in cognitive processes across the three action research cycles. The cognitive process of understanding increased from 76.43 in Cycle I to 83.39 in Cycle II and reached 91.07 in Cycle III. Similarly, the application cognitive process improved from 73.75 in Cycle I to 80.54 in Cycle II and further increased to 87.86 in Cycle III. The reasoning cognitive process showed a remarkable increase from 63.57 in Cycle I to 75.54 in Cycle II and reached 81.43 in Cycle III. These results provide strong evidence that differentiated learning positively impacted students' cognitive numeracy abilities, fostering their understanding, application, and reasoning skills at a classical level of completeness.

Learning Description Processes

The classroom observations revealed a structured learning process aligned with the indicators on the observation sheet. In the initial phase, the teacher effectively conducted ***apperception and motivation activities*** by connecting prior knowledge with the new topic through contextual questions and real-life examples related to students' daily experiences. This approach successfully engaged students and created an enthusiastic learning atmosphere, as reflected in students' active responses during the apperception session. Motivational strategies were also evident when the teacher outlined clear learning objectives and emphasized the relevance of numeracy skills in real-life problem-solving. Documentation of this phase includes photographs capturing student engagement during the apperception activities and a video recording showing the teacher's motivational speech, both demonstrating students' attentive expressions and willingness to participate.

During the ***delivery of competencies and activity plan***, the teacher clearly articulated the learning goals and provided step-by-step instructions to guide students through the learning process. Lesson plans, aligned with the differentiated learning model, emphasized cognitive levels understanding, application, and reasoning allowing students to navigate learning activities with clear expectations. The teacher's mastery of the learning material was evident through precise explanations and the use of varied representations, such as visual aids, manipulatives, and digital media, to clarify complex concepts. Observational notes recorded instances where students asked thoughtful questions, indicating a deepening comprehension of the material. Supporting documentation includes lesson plan excerpts, worksheets completed by students, and classroom observation forms highlighting the teacher's instructional clarity.

In the application of ***educational learning strategies and the implementation*** of independent and student-centered learning, the teacher employed differentiated instruction techniques tailored to students' cognitive levels. Group work activities were structured to encourage peer collaboration, while individual tasks allowed for independent exploration of numeracy problems. The teacher provided scaffolding as needed, ensuring students could work at their own pace while being challenged appropriately. Student-centered approaches were particularly evident during problem-solving tasks, where students discussed strategies and justified their reasoning in groups before sharing with the class. Evidence of these activities includes photographs of group discussions and student activity shows in figure 3 below.



Learning video documentation:
https://youtube.com/shorts/p8dpRJCZqJs?feature=share
https://www.youtube.com/shorts/Hp_61GrDxz8?feature=share
Figure 3: Learning Process Documentation

DISCUSSION

This study highlights the significant impact of differentiated instruction on students' numeracy cognitive abilities across three dimensions: comprehension, application, and reasoning. The Paired Samples Test results from the first, second, and third sessions indicate significant improvements in these cognitive levels. These findings align with previous studies that emphasize how differentiated instruction enables students to maximize their potential at various cognitive levels (Hasanah et al., 2022; C. A. Tomlinson, 2001).

In the comprehension dimension, the study revealed a mean difference of -16.071 with a significance level of p = 0.000 or less than 0.050, confirming that differentiated instruction effectively enhances students' comprehension skills. This finding is consistent with Santrock (2016); & Slavin (2018), who argued that adaptive learning processes help students grasp mathematical concepts more deeply as they are tailored to their learning preferences. Similarly, Perso (2006); & Tout (2020) emphasized that differentiated learning fosters conceptual skills development through activities relevant to students' needs.

In the second session, the application dimension showed a mean difference of -7.500 with p = 0.000. Although this increase was smaller than in other dimensions, it remained statistically significant. This outcome supports Bhattacharya (2021); & Yong & Sokumaran (2023) research, which noted that differentiated instruction enhances students' ability to apply mathematical concepts in real-world situations. The comparatively lower improvement in this dimension may be attributed to the complexity of activities requiring more advanced problem-solving skills, as highlighted by de Walle et al. (2016). Integrating project-based learning, which emphasizes real-life application of concepts, is suggested as a potential solution.

The reasoning dimension demonstrated the most substantial impact, with a mean difference of -17.857 and a wide confidence interval range (-23.092 to -12.622). These findings suggest that differentiated instruction significantly supports the development of higher-order thinking skills. Atasoy & Özden (2020); & Kurt et al. (2023) similarly asserted that instructional strategies designed to enhance metacognitive and critical reasoning skills have a profound impact on students' conceptual understanding. The ANOVA results further confirmed significant differences across cognitive dimensions, with an F-value of 6.208 and p = 0.003. This supports the idea that instructional approaches do not affect all cognitive levels uniformly, as Chandrawati et al. (2023) noted that each cognitive level demands distinct instructional strategies.

Post hoc Tukey HSD analysis indicated that the reasoning dimension exhibited the largest improvement compared to comprehension and application, with significant mean differences of 9.404 (p = 0.002) and 6.254 (p = 0.021), respectively. These findings underscore the importance of differentiated instruction that presents cognitively challenging tasks to encourage students' critical thinking development. Choi & Kim (2017); & Coker (2010) similarly emphasized that challenge-based learning significantly enhances students' mathematical reasoning skills.

The success of differentiated instruction in fostering significant gains in students' cognitive abilities can be attributed to several key aspects of this teaching approach. One of the most critical factors is the individualized attention and the ability to tailor lessons according to students' diverse learning needs. By offering a variety of learning materials and activities that accommodate different learning styles, differentiated instruction ensures that students are not only engaged but also challenged at their respective cognitive levels (Baybayon et al., 2024). This personalization, as emphasized by Tomlinson, allows students to progress at their own pace, ensuring deeper understanding and mastery of mathematical concepts (Jakfar Shodiq & Juniati, 2024). Additionally, providing varying levels of complexity and real-world applications in the lessons supports students' development across all three cognitive dimensions, comprehension, application, and reasoning by bridging theory with practice.

Moreover, the incorporation of formative assessments and ongoing feedback played a crucial role in the experimental group's success. Through regular assessments, educators could identify individual learning gaps and adjust the instruction, accordingly, ensuring that students remained on track to achieve their cognitive potential. This continuous feedback loop, aligned with the principles of differentiated instruction, encouraged students to reflect on their learning, thus fostering metacognitive skills essential for higher-order thinking and problem-solving (Sie & Darko Agyei, 2024). As suggested by Slavin (2018), this adaptive approach not only improves immediate academic performance but also equips students with the cognitive tools necessary for tackling complex, real-world challenges.

Beyond addressing cognitive development, the instructional design emphasizes creating an inclusive and student-centered learning environment. By integrating flexible learning activities that cater to varying proficiency levels, the approach ensures that all students regardless of their initial numeracy skills are actively engaged and supported. This inclusivity not only promotes equitable learning opportunities but also fosters a classroom environment that values individual growth and collaborative learning.

Furthermore, the Paired Samples Test analysis demonstrated that differentiated instruction generally had a significant impact on students' numeracy achievements, with a mean difference of -13.811 and a significance level of $p = 0.000$ or less than 0.050. These findings align with Purpura et al. (2011); & Segers et al. (2015), who highlighted that flexible and responsive instructional strategies significantly enhance students' numeracy skills. The consistent improvements across comprehension, application, and reasoning cognitive processes throughout the three instructional cycles (from 76.43 to 91.07 for comprehension; from 73.75 to 87.86 for application; and from 63.57 to 81.43 for reasoning) reaffirm the effectiveness of differentiated instruction in holistically enhancing students' numeracy outcomes.

These findings are also consistent with Vygotsky's Zone of Proximal Development (ZPD) theory, which posits that students can achieve higher cognitive levels with appropriate guidance and interventions tailored to their specific needs (Liu & Matthews, 2005; Zhou, 2020). By implementing differentiated instruction, students receive scaffolding that enables them to progress from their actual ability to their potential capacity.

Consequently, this study underscores that differentiated instruction not only effectively enhances conceptual understanding but also promotes the development of critical thinking and problem-solving skills among students (Hidayati, 2020). As a practical implication, educators are encouraged to adopt a more systematic approach to differentiated instruction by addressing students' cognitive needs at various levels. Future research could explore how factors such as student characteristics and technology-based learning approaches can further enhance the effectiveness of differentiated instruction.

Hence, the findings demonstrate that a targeted instructional model grounded in differentiated instruction not only enhances students' conceptual understanding but also cultivates critical thinking and problem-solving skills. Educators are encouraged to systematically address students' cognitive needs by incorporating instructional strategies tailored to comprehension, application, and reasoning dimensions. Future research could investigate how student characteristics and technology-based learning tools can further optimize the effectiveness of such differentiated models in diverse educational contexts.

One of the challenges faced during the implementation of differentiated instruction was managing the diverse learning needs of students within a single classroom. Teachers had to balance varying levels of comprehension, application, and reasoning skills while ensuring that all students remained engaged and supported. To address this, educators employed flexible grouping strategies, allowing students to collaborate with peers at similar cognitive levels while also encouraging cross-group interactions for more complex tasks. Additionally, ongoing formative assessments were used to monitor progress and adjust instruction, accordingly, ensuring that all students received the necessary support. Despite these challenges, the implementation of differentiated instruction ultimately fostered a more inclusive learning environment, where each student could thrive at their own pace and level of understanding.

To demonstrate how differentiated instruction works in practice, a detailed lesson plan can highlight various strategies employed to meet the diverse needs of students. For example, in a math lesson on algebra, the teacher might start by introducing the core concept to the whole class. Afterward, students are grouped based on their readiness levels: advanced learners might work on more complex algebraic equations, while those who need more support might focus on basic problem-solving skills and practice foundational concepts. In each group, the teacher circulates to provide targeted support, offering more scaffolding to struggling students while providing enrichment opportunities for advanced learners.

Additionally, flexible grouping is used throughout the lesson to encourage peer collaboration. In one activity, students could work in mixed-ability pairs to solve problems together, allowing them to learn from one another. For more complex tasks, such as applying algebraic concepts in real-life scenarios, students are grouped by similar cognitive levels, ensuring that they can engage with the material in a way that is challenging yet achievable. Formative assessments, such as exit tickets or quick quizzes, are employed to gauge understanding and guide future instruction, ensuring that all students, regardless of their initial level, are able to progress at their own pace.

CONCLUSION

This study provides compelling evidence for the effectiveness of differentiated instruction in enhancing students' cognitive numeracy abilities, particularly in the domain of reasoning. The significant improvements observed in comprehension, application, and reasoning dimensions underscore the value of tailored instructional strategies in fostering holistic cognitive development. The statistical significance of the improvements observed in comprehension, application, and reasoning highlights the profound contribution of differentiated instruction to students' numeracy growth. By catering to individual learning needs, this approach effectively fosters enhanced cognitive abilities, particularly in complex problem-solving and higher-order reasoning skills. These findings align with cognitive development theories, such as Vygotsky's Zone of Proximal Development, which emphasizes the importance of scaffolding learning experiences. However, the relatively modest gains in the application dimension highlight the need for more targeted approaches to strengthen students' practical problem-solving skills.

Given these insights, this study recommends the adoption of scaffolded learning pathways and the integration of real-world tasks that encourage active application of mathematical concepts. Teachers should be equipped with professional development programs that enhance their capacity to design and implement differentiated instruction. Furthermore, the integration of digital tools offers significant potential to personalize learning and provide immediate feedback, making instruction more responsive to diverse learning needs. Diversified assessment methods that evaluate comprehension, application, and reasoning will also ensure a comprehensive understanding of students' cognitive progress.

The findings contribute to advancing knowledge on differentiated instruction and offer practical insights for educators seeking to optimize classroom practices. Future research could explore how contextual factors such as school environment, teacher expertise, and resource availability impact the effectiveness of differentiation strategies. Longitudinal studies are also recommended to assess the sustained impact of differentiated instruction on students' academic and cognitive development, particularly in the application of problem-solving skills. By embracing these recommendations, educators can better foster students' numeracy achievement and cognitive growth.

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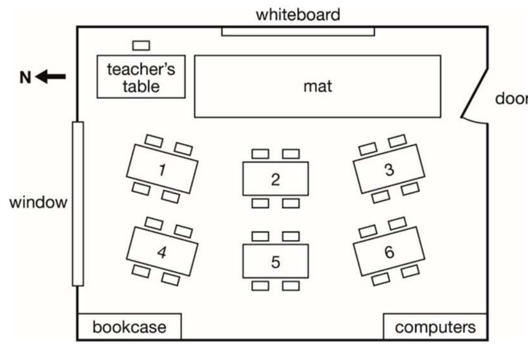
APPENDIX

Pretest and Posttest Instrument

a. **Understanding**

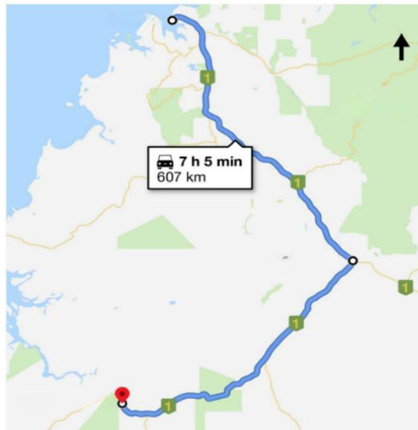
1. The following image shows the classroom layout at an elementary school in Yogyakarta. The laboratory room in this school adopts the IRIS Centre laboratory arrangement. The teacher's desk is located at the front corner of the classroom. Four students sit at each of the six tables as shown in the image:



Below are several statements based on the provided layout. Choose 'True' or 'False' for each statement:

- The door is located near the southeast corner of the classroom.
- Viewed from the teacher's desk facing the back of the classroom, the computer is in the back right corner of the room.

2. Andi lives on the coast of Depok Beach, Yogyakarta. During his work break, he wants to go on a vacation to Mount Merapi. He departs from Depok Village to Pakem Village, located at the foot of Mount Merapi, using his private car. The following map shows the distance, route, and time needed to reach Pakem Village.



Andi departs from Depok Village at 08:30 AM. After driving for 3 hours and 10 minutes, he takes a break at Ambarukmo Mall. After resting for 45 minutes at the mall, Andi continues his journey to Pakem Village. What time does Andi arrive at Pakem Village?

3. In a kindergarten class, there are 2 teachers and 22 students. The class is planning a field trip to the city park. The minimum required ratio is one adult for every 3 students. How many additional adults are needed for the trip?

b. **Application**

1. The Sekatenan celebration is a traditional festival regularly held at the Yogyakarta town square. This year, the Head of Baleharjo Village intends to participate in the festival. One of the required items is 500 traditional cakes. The price of one box containing 72 traditional cakes is Rp. 199,500. How many boxes of traditional cakes need to be purchased, and what is the total cost?
2. An elderly couple is participating in the "Steps for a Healthy Start" challenge. They must record the number of steps they take each day for 10 weeks. The grandfather's recorded steps over the last 6 weeks total 416,000, while the grandmother's total 262,000 steps. The average number of steps the grandfather and grandmother must take each day over the next 4 weeks is:
    - Grandfather: 6,000 steps and Grandmother: 8,500 steps
    - Grandfather: 3,000 steps and Grandmother: 8,500 steps
    - Grandfather: 3,000 steps and Grandmother: 8,000 steps
    - Grandfather: 6,500 steps and Grandmother: 8,500 steps
3. In 2019, the Indonesian National Team in the Triathlon event participated for the first time in the qualification championship for the 2024 Olympics. Triathlon is an endurance sport that includes three disciplines in a single competition: swimming, cycling, and running. Athletes compete to complete the race in the shortest possible time. The table below shows

the distances athletes must cover in the competition. The total distance athletes must complete is 3,575 meters.

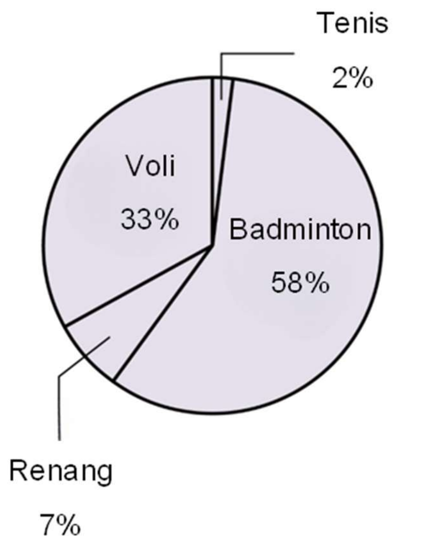
Swimming	Bicycle	Running
X	3 km	500 m

Based on the table provided, select 'True' or 'False' for each statement:

- The distance athletes must swim is 75 km.
- The distance athletes must swim is 85 m.
- The swimming distance is 0.85 kilometers.
- The running distance is the longest distance athletes must cover.

4. Using the illustration from the previous question, the two fastest athletes from Indonesia, Ardhan and Ilham, completed the race in 32 minutes and 33 minutes, respectively. Both athletes took 7 minutes to complete the swimming track. For the running track, Ardhan recorded a time of 12 minutes, while Ilham recorded a time of 10 minutes. If Ardhan's cycling track time was 13 minutes, how many minutes is the time difference between Ardhan's and Ilham's cycling track times?

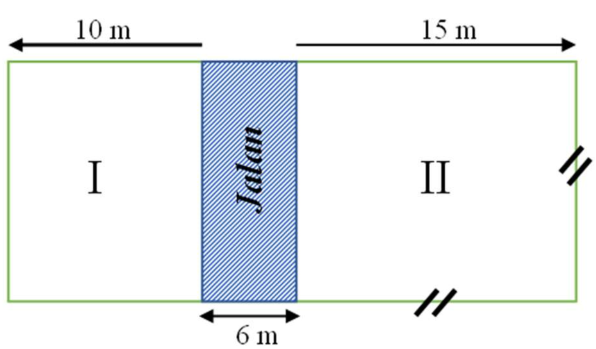
5. **Reasoning**
   1. The World Health Organization (WHO) recommends limiting sodium consumption to 2,400 mg or about one teaspoon of salt per day. The likelihood of sodium deficiency for those living in Indonesia is very low because most foods in the country are high in sodium content. Even diet menus tend to be high in sodium, such as the South Beach Diet, which includes 2,300 to 6,700 mg of sodium per day.
      Foods high in sodium, such as chips, canned soup, processed meats, salted fish, soy sauce, tomato sauce, chicken broth cubes, and cheese, should be avoided or at least reduced, as they can cause high blood pressure (hypertension), which can trigger heart disease, stroke, and kidney problems. One 100-gram bag of potato chips contains 525 mg of sodium. On average, canned soup contains 700 mg of sodium. Below are several statements based on the provided information. Choose 'True' or 'False' for each statement:
      - The amount of sodium in one bag of potato chips exceeds the "adequate intake" level.
      - Consuming 2 bags of potato chips and 3 cans of soup is sufficient for daily dietary intake.
   2. The following pie chart shows the percentage of students participating in four extracurricular activities at school.



Based on the given chart, select 'True' or 'False' for each statement:

- The number of students participating in Volleyball extracurricular activities is more than four times the number of students participating in Swimming extracurricular activities.
- The ratio of the total number of students participating in Tennis and Badminton extracurricular activities to those participating in Volleyball and Swimming extracurricular activities is 3 to 2.

3. A teacher coordinates the school gardening club. The club plans to clean the school garden, which has the following layout:



Sections I and II of the garden to be cleaned are the outermost parts.

- What is the perimeter of sections I and II?
- If one tree is planted every 2 meters, how many trees will there be?