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Developing Adaptive Reasoning in Teacher Preparation: Elementary Pre-service Teachers' Perspectives of Writing in Mathematics

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Abstract: A growing body of research suggests that preservice teachers (PSTs) need opportunities to engage with writing in mathematics to understand its role in promoting adaptive reasoning. This study explores how elementary PSTs perceive and implement authentic, problem-based writing tasks in mathematics classrooms. Drawing on socio-ecological theory, I examine individual, interpersonal, institutional, community, and policy influences on beliefs. Thirty-two PSTs completed pre- and post-questionnaires during a mathematics methods course; four participated in interviews, and final projects were analyzed for evidence of writing integration. Findings reveal that PSTs deepened their understanding of writing to explain and justify thinking and expressed strong support for use in elementary math. These results highlight key considerations for preparing teachers to integrate writing meaningfully into mathematics instruction.

Keywords: Writing in Mathematics, Adaptive Reasoning, Teacher Preparation, Strands of Mathematical Proficiency

INTRODUCTION

Teachers have a profound responsibility to meet grade-level standards and proficiency on assessments, prepare for the following grade, and, more importantly, ensure that the students can apply their knowledge to authentic tasks; all within one academic year. There is an ongoing debate as to whether the depth of understanding of a limited number of concepts *or* exposure to a broad range of mathematical concepts and procedures is more valuable and transferable to a student's application of knowledge (Eshetu et al., 2022; National Research Council, 2001; Rittle-Johnson & Alibali, 1999). "Mathematical understanding and procedural skill are equally important, and both are available using mathematical tasks of sufficient richness" (National Governors Association Center for Best Practices, 2010, p. 6). So, how do we ensure that both occur within the context of a single academic year?

Gaining more minutes in the day is *not* within a teacher's control; however, making the most efficient use of time for rigorous learning *is* within the manageability of teachers. Teacher preparation programs can prepare future teachers to manage their time effectively by teaching and practicing the use of writing in mathematics.

Writing is an uncommon practice in mathematics instruction and assessment (Kim et al., 2024; Teledahl et al., 2024), and in-service teachers have little experience or training using writing in the mathematics classroom (Cheung, 2013). Therefore, it is up post-secondary institutions to include writing in the teacher preparation programs actively to begin to change PSTs' and elementary students' views of mathematics. Teachers use teaching methods that they saw as a student (Cody et al., 2023), referred to as the "apprenticeship of observation," which has an impact on how teachers plan and teach their own lessons (Loewenberg Ball et al., 2008; Philipp et al., 2002). Teacher preparation programs should continue to develop ways to change mindsets with the use of high-quality models of teaching mathematics for PSTs to better understand how they can implement mathematical reasoning and procedural skill.

Writing in mathematics supports procedural fluency by encouraging students to articulate, justify, and reflect on each step they take, which deepens procedural understanding and reinforces accuracy, efficiency, and flexibility, a claim supported by findings between written reasoning and fluency (Andal & Andrade, 2022). Procedural fluency refers to students' ability to carry out mathematical procedures accurately, efficiently, and flexibly, and is strengthened when paired with adaptive reasoning through tasks that require explanation and justification (National Research Council, 2001). Adaptive reasoning is the ability to think through logically, reflect on, and explain one's solution in mathematical situations (National Research Council [NRC], 2001). Adaptive reasoning enables students to make sense of mathematical procedures, apply known procedures to new problems, and effectively communicate their reasoning to others. Despite its documented benefits in supporting mathematical thinking (Bangert-Drowns et al., 2004), writing remains underused in mathematics classrooms due to teachers' limited preparation (Wilcox & Monroe, 2011). While research has often examined in-service teachers' instructional practices, less is known about how PSTs develop beliefs and practices related to writing in math.

This study addresses that gap by exploring the impact of a mathematics methods course on PSTs' perspectives, instructional planning, and intentions related to writing in mathematics instruction.

The research will examine the experiences of PSTs and their willingness to incorporate writing into their future mathematics classrooms, regardless of their prior experiences. Specifically, it explores how participation in a mathematics methods course influences their perspectives on the role of writing in mathematics instruction. By focusing on explanatory writing for justification and reflection, the study investigates how pre-service teachers perceive writing as an essential component of conceptual understanding and adaptive reasoning. To understand the application and connection to the field, this study also analyzes participants' mathematics lesson plans for evidence of independently choosing to include writing tasks that required their *own* students to justify and explain their answers.

Research Questions:

1. To what extent do pre-service teachers independently integrate writing, particularly for explanation and justification, into their mathematics lesson plans?

2. What shifts in thinking do pre-service teachers report about how they were taught mathematics versus how they plan to teach it?

When students write explanations and justifications of their mathematical thinking, they engage in adaptive reasoning, one of the strands of mathematical proficiency (National Research Council, 2001). Such writing processes enable students to move beyond computational thinking to genuine sense-making in mathematics (National Research Council, 2001). However, despite evidence supporting the value of writing for mathematical thinking, the practice of using writing as a teaching and learning tool in the mathematics classroom is not widespread. Teachers' lack of experience, confidence, or preparation in using writing in mathematics is one of the reasons for this deficit. Teacher preparation programs are an important source of knowledge and experience for future teachers to acquire practices and pedagogical content knowledge that can be used in their future classrooms. PSTs who arrive at teacher preparation programs often have a traditional understanding of mathematics that separates it from language arts, have had limited or no exposure to writing in mathematics, and receive no or limited explicit instruction on how to integrate mathematics and writing instruction.

The current study is significant because it contributes to the existing body of research on teacher preparation by examining the effects of participating in a mathematics methods course on PSTs' beliefs, experiences, and practices related to writing in mathematics. Previous research has primarily investigated in-service teachers' instructional practices or attitudes towards writing in mathematics but has not sufficiently explored teacher preparation in this area (Kosko, 2016). Moreover, most existing research has used limited quantitative measures to study teachers' instructional practices or attitudes, such as surveys or self-report data that do not capture the details of teacher learning or the complexities of instructional decision making (Kosko, 2016). This study employs a mixed-methods quasi-experimental design, gathering information from multiple data sources (e.g., pre- and post-questionnaires, interviews, and authentic lesson plans) to explore the evolvement in participants' beliefs, experiences, and practices after participating in the methods course (Darling-Hammond, 2006; Zeichner, 2010). Moreover, by grounding the research in Bronfenbrenner's socio-ecological theory, the current study can extend knowledge on the subject by illustrating the interplay between individual characteristics, the learning environment, and the larger system in the formation of teachers' evolving conceptions of writing in mathematics. Results from this study can inform teacher preparation programs that are interested in planning and/or revising mathematics methods courses to intentionally support preservice teachers to write effectively in mathematics in ways that can benefit their students' learning of mathematics.

LITERATURE REVIEW

Mathematical Proficiency

Language Arts curricula often embrace the application of deep thinking and analysis while reading a text, a skill focused on as early as pre-kindergarten (Burbach et al., 2004). Expectations for higher-order thinking are often unconsciously minimized when it comes to learning math (Stein et al., 2009);

the tradition of how the subject has been taught is partly to blame (Shepard, 1991). Teachers often untangle difficult mathematics problems for students, rather than requiring the work to be carried out by students. Mathematics teachers frequently do cognitive heavy lifting when problems seem "hard" or time is short. For example, instead of setting up students to do the math, a teacher may decompose a problem into several steps and demonstrate how to solve it when students start to struggle (Hiebert & Wearne, 1993). This method is especially true in classrooms focused on upcoming standardized tests, as state accountability measures encourage teachers to prioritize procedural accuracy and curriculum completion over student understanding (Stigler & Hiebert, 1999). Stein et al. (1996) found that although some teachers use high cognitive demand tasks at the beginning of a unit, most eventually reduce the cognitive demand by "scaffolding down," or removing, changing, or solving the problem themselves.

The Strands of Mathematical Proficiency (National Research Council, 2001) elaborate on ways that students can demonstrate an understanding of mathematics, rather than merely conducting procedures. Pre-service teachers have been shown to lack conceptual reasoning in mathematics and frequently have below-standard self-perceptions about their mathematical abilities (Isiksal et al., 2009). Their knowledge tends to be fragmented and insecure, making it difficult to move fluidly between algorithms, verbal explanations, and representations (Kajander, 2010; Ball, 1988). According to the National Council of Teachers of Mathematics (2000), mathematically proficient students can demonstrate knowledge, skills, abilities, and beliefs based on a body of research discovered in cognitive psychology and mathematics education. The Strands of Mathematical Proficiency (National Research Council, 2001) include:

1. Conceptual Understanding
2. Procedural Fluency
3. Strategic Competence
4. Adaptive Reasoning
5. Productive Disposition

This study focuses on adaptive reasoning although the five strands of mathematical proficiency are equally important and interact with each other. The National Research Council (2001) reiterates that the strands are "mutually supportive. Each supports and is supported by the others" (para. 56). Students take the place of passive observers to active problem solvers through writing to explain and justify their ideas; reasoning and writing are closely interwoven, and concepts that may include lessons from the past, current, and future lessons as well as other modes of learning that can be used to show that they understand. Barham (2020) proposed the order of the five mathematical reasoning strands as productive disposition, adaptive reasoning, conceptual understanding, strategic competence, and procedural fluency. The proposed order of the strands was based on Barham's (2020) finding that most teachers do not feel confident and motivated enough when teaching mathematics, and as a result, a teacher must first develop a productive disposition as the foundation to implementing the other strands of mathematics-related skills and knowledge. Productive disposition creates a positive mathematical identity and a belief that students can learn mathematics. The teacher must have strong beliefs and

positive attitudes about teaching mathematics to have a deep engagement in the following cognitive abilities (Barham, 2020). This was the starting point, followed by adaptive reasoning and conceptual understanding that is used to justify and make sense of mathematics, and finally before problem solving and procedural fluency. The sequence of the cognitive strands that was used is because of a natural process of learning as beliefs and attitude that are affective components of engagement in teaching, affect the cognitive aspect. Conceptual and strategic strands are best left to be taught after a teacher's affective and reasoning competencies (National Research Council, 2001). The six cognitive strands were the framework for the interview protocol. They were also used as analytic codes to determine the shifts in participants' beliefs about mathematics and mathematics planning practices.

Mathematical writing to explain and justify includes opportunities for students to share mathematical thinking to demonstrate their understanding. By some scholars, showing a correct understanding of the concept comes in the form of writing (Kilpatrick et al., 2001). Writing can help to deepen mathematical understanding and make thinking visible, so it can be used to both encourage and assess conceptual knowledge. Mathematical writing may ask students to organize ideas, justify reasoning, and explain processes, which are aligned with dimensions of mathematical proficiency (National Research Council, 2001). However, Rittle-Johnson et al. (2015) state that "conceptual knowledge can be implicit or explicit, and thus not verbalizable" (p. 588). Teachers that work with students that have reading, writing, or language-based differences in learning must consider this. If the writing is the only or primary form of evidence that is used to determine reasoning, it can be considered construct-irrelevant variance (essentially measuring the writing and not the mathematical thinking) (Abedi, 2004; Solano-Flores & Trumbull, 2003). A teacher may choose to use writing to support reflection and reasoning but should do so in a flexible manner and with the support of other modes (e.g., visual model, oral explanation, manipulatives, gestures). Researchers found that most teachers expected their students to write in math classes at least once a week and that the teachers structure the writing tasks they assign across types of tasks (Arsenault et al., 2024; Gjøvik et al., 2024) (e.g., exploratory, argumentative), and the writing is typically assessed with indicators such as logical coherence. These types of writing tasks support what is called adaptive reasoning or "capacity for logical thought, reflection, explanation, and justification" (National Academies of Sciences, Engineering, and Medicine, 2001, p. 6). When students are asked to justify and explain their mathematical thinking, they show stronger mathematical problem-solving skills (Boaler, 2002; Hiebert & Grouws, 2007). Therefore, strategies that give students opportunities to engage in adaptive reasoning are vital to equitable instruction because the tasks and explanations that ask students to explain their mathematical strategies invite students from all backgrounds to explain and share their mathematical strategies (Kazemi & Stipek, 2001). Mathematical writing is one tool intended to facilitate students being able to explain and justify their thinking. Explaining, for instance, can be thought of as simply giving an account of the steps it takes to complete a problem, and justifying involves proving why the strategy was used (Wagganer, 2015). For this reason, students have an obligation to justify their reasoning, as it will promote further understanding (Rawding & Wills, 2012). Writing in math has also been found to deepen students' learning as it requires students to think carefully at each step in a process, instead of it being about memorizing the steps of a procedure (Alleman, 2010; Urquart, 2009).

Example:

A fourth grader answers a fill-in-the-blank question ⅘__⅔ with a > symbol. The test design allowed the student to answer several questions correctly without necessarily providing meaning (e.g., multiple choice, a key on the side), as the student was interviewed by the teacher and asked to explain his thinking, the student responded that he knew the "correct" answer because 4 is bigger than 2 (numerator) and 5 is bigger than 3 (in the denominator). The teacher would not have known a student's misconceptions without having to explain their thinking. A more authentic problem could ask:

Carson and Amelia both walk to school every day. Carson walks ⅘ of a mile. Amelia walks ⅔ of a mile. Create a number sentence using <, >, or =. Who walks farther? *Explain and justify* how you arrived at the correct answer.

Writing makes student thinking transparent. To authentically implement writing in math classrooms, it must be a part of the teacher's education experience. Kuzle (2013) identified that pre-service teachers that strongly agreed that writing and doing mathematics were related showed more positive attitudes in math instruction and learning. Colonnese & Casto (2023) also explored pre-service teachers' explanations of reasoning when selecting strategies, but to also illustrate that there needed to be more opportunities to write. Pre-service teachers should be partaking in the same writing that is expected of their students. Writing in Language Arts can show student identity and learning capacity; it can show this in mathematics as well. Lesson plans of participants and interviews were analyzed to identify PSTs' identification of the differences between explanation and justification and how it was used in the design of their instruction.

Teacher Preparation

Teachers play a significant role in the success of their students' education (Oliver & Reschly, 2007). Teacher preparation programs aim to improve the proficiency of student teachers to ensure quality education in schools (Oliver & Reschly, 2007). When colleges offer high-quality teacher preparation to pre-service teachers, these soon-to-be teachers are much more likely to provide higher-quality education to their students (Rokenes & Krumsvik, 2014). An emphasis on quality teaching must be grounded in distinguished teacher training as offered in teacher preparation programs (Darling-Hammond et al., 2005; Hattie & Yates, 2013). Although others have suggested that teachers' own experiences as learners can have a greater impact on their teaching than formal preparation (Feiman-Nemser, 2001), integrated coursework has the potential to re-shape those prior beliefs and build more effective instructional practices (Mainali, 2022).

In a teacher development study, McCarthy (2010) found that pre-service teachers could develop the capacity and foster a positive disposition toward using writing in mathematics instruction, even though these pre-service teachers had no prior experience with using writing-to-learn strategies before enrolling in the teacher preparation program. Similarly, Brownlow (2021) reported that

early childhood pre-service teachers often lacked confidence when framing and solving word problems, and that structured opportunities to engage with authentic, cognitively demanding tasks were necessary for developing their mathematical reasoning and confidence.

Furthermore, based on the National Council on Teacher Quality (2022), although the quantity of mathematics courses taken in teacher preparation programs has increased, "substantial variation between programs in the rigor and quality of their pedagogical content preparation" (p. 5), so further evidence-based reform is required. Teaching writing has had comparable evaluations. Although writing instruction is a part of the elementary curriculum, teacher preparation programs have historically provided limited coursework in the pedagogy of writing. Instead of having a course dedicated to writing instruction, it has often been integrated into a more general literacy methods framework that has focused more on reading than writing (Brehmer, 2023). Practice-based simulations have been found to increase pre-service teachers' confidence in instruction and help them develop the more specific feedback strategies that their students need (Gillespie Rouse et al., 2023). These experiences are vital, as many pre-service teachers begin their programs with limited confidence or with dated preconceptions about writing instruction (Dace, 2015).

Teacher effectiveness has been linked to teacher preparation programs (Boyd et al., 2009). Pre-service elementary teachers have views about the teaching of mathematics that have been shaped in their early classroom experiences (Lloyd, 2006). They may see mathematics as a decontextualized and static set of procedures and algorithms and then use these methods in their own teaching (Coffey, 2004; Smith, 1996). To be successful teachers of writing and mathematics, pre-service teachers need experience with the instructional strategies that they will later use in the classroom, including working with writing-to-learn approaches and open-ended tasks. Prior research shows a positive relationship between teachers' efficacy beliefs about teaching math and their effectiveness with students (Guskey, 1988; Ross, 1994). Therefore, pre-service teachers need to be aware of and unlearn previous beliefs before internalizing new ideas about mathematics instruction (Piaget, 1954). Steele (1994) argued, "If the teaching of mathematics is to change, then teachers' conceptions about mathematics and mathematics teaching and learning must change first" (p.1). Similarly, the most recent research has offered evidence supporting a positive impact of teacher preparation on pre-service teachers' ability, self-confidence, and preparedness for the classroom in elementary mathematics instruction, if that preparation is well-structured, supported, and delivered effectively (Fu & Kartal, 2023; Hood, 2023).

International work also indicates that writing instruction is needed in teacher preparation. Selek (2024) analyzed 28 pre-service teachers taking a mathematical literacy elective in Turkey. Participants successfully designed authentic problems but had issues with contextualization and originality. This work called for "richer, literacy-focused, task design" in preparing mathematics teachers (Selek, 2024, p. 4). In Australia, Colonnese, and Casto (2023) explored the written strategies of 27 pre-service teachers taking a mathematics methods course. Most participants included justifications in problem-solving explanations, but the quality of reasoning differed. The findings support this study's examination of how a mathematics methods course affects pre-service teachers' beliefs, practices, and intentions to use writing in mathematics.

CURRENT STUDY

Theoretical Framework

The present study was theoretically framed using Bronfenbrenner's (1979) socio-ecological model of human development, which comprises multiple interrelated systems. I adapted the model to focus only on the beliefs and practices of PSTs regarding writing in mathematics.

At the individual level, pre-service teachers' beliefs and prior K–12 and college mathematics experiences often shaped how they viewed writing in mathematics, with many seeing it as unrelated to math and limited to English classes. These early experiences, which typically emphasized formulas and single answers over explanation and reflection, influenced their openness to using writing later in teacher preparation, though positive exposure to writing-rich instruction could support metacognitive growth and conceptual learning (Graham et al., 2020). Interpersonally, pre-service teachers were strongly influenced by former teachers and current educators, often replicating procedural, teacher-directed practices unless provided with modeled examples and feedback that showed how writing could deepen mathematical reasoning (Wellberg, 2024; Ding et al., 2024; Cruz & Zahed, 2022). At the organizational level, program structures and coursework either constrained or encouraged writing, with evidence showing that teacher preparation programs embedding writing-to-learn practices improved pre-service teachers' efficacy and student outcomes (Zeichner & Bier, 2017; Herbert et al., 2019). Extending to the community, writing connected classroom mathematics to families and cultures, offering a way to strengthen home–school communication and promote equity through valuing students' lived experiences (Suh & Calabrese, 2025). At the societal level, although standards rarely emphasize writing explicitly, it has gained recognition as a powerful tool across STEM fields for fostering reasoning, problem-solving, and communication, reinforcing the importance of integrating writing into mathematics instruction (Graham et al., 2020; Herbert et al., 2019).

Conceptual Framework

The study is based on the Adaptive Reasoning strand from the Strands of Mathematical Profciency conceptual framework (National Research Council, 2001). Adaptive reasoning is one of the five strands of mathematical proficiency, and it "includes the capacity to think logically and reason, to justify and explain one's strategies and solutions, and to reflect on and refine one's understanding" (National Research Council, 2001, p. 57). Writing in mathematics has been shown to promote adaptive reasoning in the classroom because when students are required to write their thought processes and problem-solving strategies, as well as justify their work and solutions, they become reflective learners. The focus on pre-service teachers' perspectives of writing as a tool to help them both conceptualize and help students reach the level of adaptive reasoning, as well as explore what contexts impacted their practices, will be used to form recommendations for instruction that can be adapted to fit teacher preparation coursework.

Purpose

The purpose of the present study is to offer the field both conceptual tools and empirical evidence regarding pre-service teachers' views on using explanatory writing to support mathematics instruction. The study focuses on the explanatory genre of mathematical writing, with special attention to the use of writing that is explicit and highlights step-by-step reasoning. The teacher candidates were expected to model this kind of writing for students and were encouraged to reflect on and practice explanatory writing as a strategy for communicating mathematical ideas. The PSTs also analyzed student work. A strong emphasis was placed on preparing pre-service teachers to use the adaptive reasoning strand of mathematical proficiency (one of five strands defined by the National Research Council) as a tool to support both conceptual understanding and procedural fluency. Explanatory writing in mathematics can meet both of those goals by having students explain their reasoning while rehearsing procedural steps.

The work is important because it focuses on a relatively scarce area of scholarship regarding ways to incorporate both reasoning and procedures into mathematics instruction. The study also demonstrates the importance of mathematics methods courses in teacher preparation programs. The research findings provide insights to support teacher educators' work and inform teacher preparation curricula. K–12 policymakers may also find the results informative for standards revisions that seek to improve mathematical reasoning using writing.

METHODS

Design

I employed a mixed-methods research design for this study. I used quantitative and qualitative data to provide a more in-depth and comprehensive understanding of the participants' experiences and perspectives. I triangulated the two types of data using one type to support and corroborate the other, which helped to increase the trustworthiness of the study (Creswell & Creswell, 2017; Creswell, 2021). The study's mixed-methods design was guided by Bronfenbrenner's (1979) socio-ecological model, which accounts for multiple levels of influence on individuals. I used quantitative survey items to collect data on individual-level constructs such as personal beliefs and past experiences with writing in mathematics. I used qualitative interviews and lesson plan data to better understand the interpersonal, institutional, and societal-level factors that impacted participants' shifting beliefs and instructional choices. I used this framework to make sense of the data, interpreting individual-level data within the context of the various systems that impacted the participants.

Course Description

Participants were pre-service teachers enrolled in a one-semester, spring Mathematics Methods Course for PreK-4 Education at a small public university in Pennsylvania. The 15-week course met three times per week for 50-minute sessions. As the researcher and instructor of the course, I

conducted lectures, group work, field experience, and reflection assignments to address the mathematical content knowledge and mathematical teaching practices appropriate for the early childhood and elementary classroom. Key course objectives included:

- Strengthening mathematical knowledge aligned with PreK-4 standards
- Integrating explanatory writing as a strategy for mathematical reasoning, explanation, and communication
- Analyzing student work samples for adaptive reasoning
- Designing lesson plans that connect mathematics with literacy
- Connect the pedagogy learned within their class to their field placement

Weekly activities involved reading course texts and research articles, participating in collaborative writing and problem-solving tasks, analyzing classroom videos focusing on students' math writing, and engaging in fifteen hours of mathematics field placements in local elementary schools. The course culminated in a final lesson plan assignment, where pre-service teachers were encouraged, but not required, to embed writing into their math lessons.

Instructional content, materials, assignments, and rubrics were standardized and delivered consistently across two consecutive cohorts included in this study.

Participants

Thirty-two pre-service teachers (3% male, 97% female) enrolled in the Early Childhood Education Teacher Preparation program participated in the study. I purposefully selected participants from two cohorts enrolled in the mathematics methods course. Thirty participants reported no prior training in integrating writing into content areas, while two participants reported having received some initial training. Eight reported no previous experience writing in math as students; 24 reported some expertise.

All participants met the program admission criteria, including completion of at least 48 college credits, passing prerequisite education courses with a grade of B- or better, completing at least six credits in both college mathematics and English, and maintaining a minimum GPA of 2.8. Before this course, participants completed 90–100 hours of field placements. They were concurrently enrolled in a Mathematics and Reading Pedagogy Lab, which required an additional 30 hours of field placement.

I taught the mathematics methods course and served as the instructor for the reading, writing, and mathematics methods courses, as well as the pre-student and student teaching courses. Additionally, I am the Director of Field Placements. I have fourteen years of experience teaching elementary-level mathematics. To minimize variability and the influence of social context on participant responses, I consistently used standardized instructions and materials throughout the study. I also ensured participant anonymity to encourage honest and authentic sharing.

Data Collection

Before data collection, I obtained written informed consent from all participants and informed them of their right to withdraw from the study at any time without penalty. I told the participants that I would have access to their pre-questionnaire responses. It was also made clear to the participants that I would not use their pre-questionnaire responses to influence the course grade or award any extra credit points. I collected data during two consecutive spring semesters. There were three sources of data:

- **Pre- and post-course questionnaires** were administered on the first and last days of the course, respectively. These included both Likert-scale and open-ended questions (see Tables 2 and 3). The questionnaire was developed by the researcher, reviewed by three faculty experts for clarity and alignment, and piloted with five pre-service teachers outside the course to ensure validity and clarity. Cronbach's alpha for Likert items was .82, indicating acceptable internal consistency.
- **Semi-structured interviews:** Midway through the semester, participants received an optional invitation to an interview via email. Four participants volunteered and were selected based on availability, academic performance, and teaching interests. Interviews followed a semi-structured protocol reviewed by four expert researchers for clarity and relevance. Interviews were recorded, transcribed, and analyzed using NVivo software (see Table 4).
- **End-of-semester lesson plan assignment:** Participants created original mathematics lesson plans. Although embedding writing was not a rubric requirement, the open-ended criteria allowed participants to incorporate writing strategies at their discretion. Lesson plans were analyzed and coded to identify the inclusion of writing components (see Table 5).

I designed the study instruments with Bronfenbrenner's socio-ecological model and the Adaptive Reasoning strand of mathematical proficiency in mind. Likert-scale items addressed individual experience and belief (microsystem), whereas open-ended survey items and interview protocols gathered information about how participants' relationships with instructors, curriculum design, and field experiences (mesosystem and exosystem) influenced their conceptualizations of writing in mathematics. Items that asked participants to reflect on their state's standards or education culture addressed the macrosystem. The study utilized multiple data sources, providing information about the individual, interpersonal, organizational, and institutional levels of influence on participants' beliefs and practices. Questionnaires captured individual-level beliefs and previous learning; interviews provided insight into interpersonal and organizational factors that might have influenced the teachers' beliefs and thinking; lesson plans revealed evidence of the application and integration of the teachers' beliefs and the influences of those around them within the classroom context (institutional level).

Data Analysis

Interviews and open-ended responses from the questionnaire were analyzed with the use of thematic analysis (Clarke and Braun, 2017). The strategy for coding the data was: 1) familiarization

with the information received, 2) creating the initial codes, 3) searching for themes, 4) reviewing themes, and 5) producing the report (Braun et al., 2016). The software NVivo was used to enhance the data analysis.

Firstly, during the familiarization stage of data, 146 initial codes were produced, which reflects various aspects of the participants' perception of writing in math classrooms. Then, 32 categories were created by merging initial codes into groups with similar meanings. These groups are the pre-themes, and they offer a more structured approach to the analysis of the data. Finally, pre-themes were grouped into four major themes that are presented below:

Balancing Basic Skills and Conceptual Understanding

This theme was mostly about the issues that must be addressed by balancing basic skills and procedures with conceptual understanding. To be more specific, many participants were striving to maintain this balance in their writing activities. In a way, they were trying to blend procedural knowledge and understanding of mathematical concepts when it came to the tasks. At the same time, other participants talked about the main purpose of writing in math to improve conceptual understanding. In short, this theme demonstrates how participants have been speaking about this aspect of mathematics education either as something that needs to be addressed when assigning writing tasks or as a main goal that can be reached through writing.

Explaining and Justifying Reasoning

Another major point that was shared by many participants had to do with the ability of students to explain and justify reasoning by using writing as the medium for it. The common idea was that writing in math is not only about presenting the results but also about demonstrating an ability to think mathematically. When these participants were assigning writing tasks to students, the main purpose was the one described above.

Reconciling Previously Learned and New Methods Of Doing Math

The pre-service teachers also had issues with using writing in math to present their knowledge, and ability. Many of them had to face the challenge of learning new methods of doing math while also using those that they learned previously during the process. The use of writing in math was seen by these participants as a complicated process that needed to be thoroughly planned.

Cross-curricular Integration of Subjects

The last theme was mostly about the ways that writing in math can be a link between different subject areas. Pre-service teachers have understood that when they make their students complete writing tasks in math, they are also helping them to develop their literacy skills and potentially the knowledge in other subjects as well. In short, this theme is mostly about the benefits of cross-

curricular integration. These themes were triangulated with questionnaire and lesson plan data to enhance trustworthiness (Flick, 2018).

I conducted the quantitative data analysis in phases:

1. Likert-scale responses from pre- and post-questionnaires were analyzed in Qualtrics to identify shifts in perceptions over the course.
2. Interview transcripts were coded by research question using NVivo.
3. Lesson plan data were compiled in a spreadsheet categorizing whether writing was incorporated.

To address trustworthiness, I attempted to strengthen validity, credibility, dependability, and confirmability in several ways. I employed methodological triangulation (i.e., drawing on three sources of data: questionnaires, interviews, and lesson plans) to enhance the dependability of this study.

I also used NVivo, a software program that supports coding and thematic analysis, to encourage analytic consistency. I engaged in peer debriefing with two colleagues in teacher education to further ensure coding consistency and thematic alignment. I maintained an audit trail to document analytic decisions, thereby supporting the confirmability and dependability of this study. Member checking (Lincoln & Guba, 1985) occurred when participants were allowed to review their interview transcripts for accuracy, thereby supporting the credibility of this study.

I also considered discrepant cases, such as participants who did not report a shift in attitudes about writing in mathematics or lesson plan writing, and those who did not include writing in their mathematics lesson plans. These were explained in depth to further explore possible resistance to the course and/or limitations on the instruction's ability to influence participant lesson plans.

I grounded the coding in theory and aligned it with the study's guiding frameworks. I coded responses to interview and questionnaire data both inductively, by generating codes from the data, and deductively, by applying a priori codes drawn from Bronfenbrenner's ecological levels (e.g., individual learning history, instructor modeling, institutional structures) and aspects of adaptive reasoning (e.g., explanation, justification, procedural flexibility). This approach allowed me to identify both anticipated and emergent themes through the lens of the study's theoretical framework.

RESULTS

Data Source #1: Pre-Questionnaire/Post-Questionnaire

I asked PSTs on both the pre-questionnaire and post-questionnaire (Figure 1) whether they agreed or disagreed that writing should take place in mathematics class. The mean pre-questionnaire mean was 2.91, and the post-questionnaire mean was 4.64. Table 1 shows that ten participants agreed that writing should take place in mathematics class, eleven participants either disagreed or strongly disagreed, and twelve participants were neutral before the course. After the course, thirty-one

students agreed or strongly agreed with the statement that writing should take place in mathematics class, and no one disagreed.

Participants responded to the item that asked if they had experience writing in mathematics class. Twenty-three participants strongly disagreed that they had experience writing in mathematics class during their schooling, and six participants were neutral. Twenty-two participants agreed, and ten participants strongly agreed that they had gained experience teaching writing in mathematics after the course; one participant was neutral. Only one participant strongly agreed with the statement that they had experience teaching writing in math prior to the course. There was a significant increase in the number of participants who agreed or strongly agreed that they had experience teaching writing in mathematics after the course. There was also a small number of participants who had any college-level exposure to writing in mathematics or instruction on how to teach writing in math before the course. Six participants disagreed, and twenty participants were neutral about whether they would implement writing in their future math classrooms before the course. In contrast, two participants disagreed, while thirty-one participants agreed or strongly agreed after the course, indicating an increase in their willingness to integrate writing.

The qualitative open-ended responses (Table 2) provide context for the quantitative changes in these variables. PSTs wrote responses to a question about their experience with writing in math class and their plans to integrate writing before and after the course, which provides essential contextual information. Before the course, many participants reported having little to no experience with writing in mathematics during their schooling. They recalled completing only a few isolated and often superficial tasks, such as short writing prompts or word problems. Some pre-service teachers explained that they viewed math and writing as separate subjects during their school years. One participant noted, “Writing and math were two different subjects when I was in school.”

Post-course, participants described what they had learned about the purposes of and strategies for teaching writing in mathematics. They wrote that they learned to help students use writing to explain, justify, and think more deeply about mathematics. Comments such as “Explain and justify … it will develop a different level of thinking” and “Writing to explain and justify math ideas helps students understand better” suggest that participants changed their views about the pedagogical purposes and potential of writing to support mathematics learning. They also provided concrete examples of resources and tools they learned during the course that they could use to support their students, such as how to model writing tasks, use rubrics, or provide vocabulary posters, which suggested they were more confident and ready to teach writing in mathematics. Although some indicated that they would need more time to become completely comfortable, post-course comments suggested interest in continued learning and development in this area, with some participants commenting that the course “opened my eyes to a new idea” but they needed more practice.

Quantitative and qualitative data suggest that the course changed pre-service teachers’ beliefs, experiences, and future intentions related to writing in mathematics. The dramatic increases in participants' agreement with the importance of including writing in mathematics instruction, as well as their more detailed descriptions of specific teaching practices and rationales in the qualitative

data, suggest that they began to see writing as an important part of mathematics instruction that supported student understanding rather than as something separate or unrelated to the subject.

Data Source #2: Semi-Structured Interviews

To further investigate students' responses, I conducted semi-structured interviews with four students who responded "somewhat" to questions or had indeterminate answers on the post-questionnaires. Table 3 summarizes the key ideas that were reported by more than one participant in response to the questions on the pre- and post-questionnaires. Participants' comments focused on how their ideas about writing in mathematics had changed after the methods course. The most frequently reported idea was that basic skills and writing need to be balanced in mathematics education. Respondents reported that pre-service teachers felt that students still needed to demonstrate basic math skills and procedures, that those skills should be retained, and that they should be integrated with writing. Several mentioned the use of a problem-based context. An additional salient idea was the need for students to explain and justify through writing. The final idea in Table 3, not surprisingly, is the one with the most overlap between questions. The importance of the vocabulary word wall for scaffolding student language development is an important idea that participants reported that they had learned from the course. The final idea, that there was overlap of subjects was also strongly emphasized in the responses by the pre-service teachers who were interviewed.

Data Source #3: End of Semester Assignment

For the third source of data, I collected an end of semester assignment. I served as the professor for the course and I assigned the pre-service teachers to design a mathematics lesson. The rubric for the assignment and our class discussions about how to design it did not contain any prompts or elements about writing. This ensured that any decision to include writing in their lesson plans was self-directed, and therefore an authentic measure of internalized beliefs and intentions.

The results, summarized in Table 4, are that most participants (27/33; 81.8%) included written products or activities as part of their lesson and required students to explain and justify mathematical thinking. This means that a large majority of pre-service teachers not only placed value on using writing to learn math but also perceived, and implemented, this type of writing in meaningful and pedagogically grounded ways.

A slightly smaller group (4 participants; 12.1%) included writing tasks or products in their lessons but did not require students to both explain and justify mathematical thinking. These lessons may have included labeling, recording steps, or other forms of sentence frames but did not include the richer student thinking. This result indicates an intermediate degree of implementation or of belief and confidence in the value of such rich use of writing. Two participants (6.1%) did not include any element of written products or assessments in their lesson plans. This finding of about 6% is consistent with the results from the pre-post questionnaire and the interview data which identified

a small group of students who did not express or perceive a shift in beliefs and were ambivalent or unsure about the use of writing in math.

To conclude, this summative, performance-based assessment provides concrete evidence that the participants were able to transfer their beliefs and intentions into actual lesson design. As they had the agency to make this decision, most students created lessons that included writing explicitly to support student understanding, in particular the practices of explaining and justifying. These data, which corroborate the self-reported growth in experience and intent from the two previous sources of data, indicate that the course was successful in preparing the pre-service teachers to use writing to reason in elementary math classrooms.

Bronfenbrenner's socio-ecological framework, which emphasizes the interaction of multiple environmental systems shaping individual development (Bronfenbrenner, 1977), helps explain the shifts observed in pre-service teachers' beliefs and intentions. Individual level factors, such as prior experience with writing in math and personal attitudes, may be included as variables that experienced changed throughout the course, as reflected in the increase in mean questionnaire scores from 2.91 to 4.64. At the microsystem level, participants' classroom experiences during the methods course provided immediate learning contexts where they engaged with writing pedagogy. Interview data also showed how the participants negotiated the tension between skills and integration in writing, and how they were influenced by different educational settings and content experiences, an example of the mesosystem. Exosystemic and macrosystemic factors such as teacher preparation program standards, institutional policies, and societal beliefs about math instruction also influenced their writing intentions and beliefs.

DISCUSSION

Research Question 1: To what extent do pre-service teachers independently integrate writing, particularly for explanation and justification, into their mathematics lesson plans?

The end-of-semester assignment demonstrated that even though there was no specific directive to include writing, most of the students elected to include writing in their lessons. Of the 33 respondents, 31 included writing strategies and 27 of those included both explanation and justification.

Research Question 2: What shifts in thinking do pre-service teachers report about how they were taught mathematics versus how they plan to teach it?

Participants reported notable shifts in their thinking about mathematics instruction. Interviews revealed that they recognized differences between how they were taught mathematics, often emphasizing procedural skills, and how they plan to teach it, which now includes balancing basic skills with conceptual understanding and encouraging explanation and justification through writing. In addition to the changes in thinking and shifts in instructional approach that participants reported, these findings also support the common theme of the need to bridge previously learned and new ways of doing math. They also noted that cross-curricular integration is important and that multiple

subjects must be taught together to best support student learning. Bronfenbrenner's socio-ecological model was useful in examining participant beliefs and practices with respect to writing as mathematical communication. At the intrapersonal level, the magnitude of change in participants' attitudes and self-reported teaching experience indicate that they experienced shifts in their beliefs and confidence in writing in math during the methods course. The microsystem, or immediate learning environment of the methods course, may have offered the participants opportunities to observe and practice strategies for writing instruction and may have thus contributed to their beliefs and practices. The mesosystem, or links between environments, can be seen in the recognition of cross-curricular connections between math and language arts and the awareness that writing can be interconnected. The exosystem, or broader educational systems and policies, could have influenced participants' awareness of professional standards and expectations. The macrosystem, or cultural and societal influences, might have been reflected in the overall attitudes and values surrounding math education and the importance of writing as a tool for mathematical communication. This study is important for teacher preparation programs as it suggests that teacher educators should consider influences at different ecological levels to support pre-service teachers' instructional development.

CONCLUSIONS

Pre-service teachers require a solid understanding of mathematical content to interpret student thinking effectively. Engaging pre-service teachers in assessing student work through writing enhances their mathematics learning and assessment skills (Namakshi et al., 2022). Authentic open-response questions, writing to justify thinking, and multiple solution pathways promote more profound understanding and productive struggle. Teacher preparation programs should emphasize these practices to move away from traditional methods and foster genuine problem solvers. Rather than only teaching the procedural fluency strand of mathematical proficiency or teaching this strand before any others, educators must sequentially place adaptive reasoning after productive disposition and before conceptual understanding, strategic competence, and procedural fluency (Barham, 2020). Moreover, adopting a societal approach to mathematics education that mandates writing to explain and justify mathematical thinking within standards may better support these efforts (Bronfenbrenner, 1977).

LIMITATIONS

The researcher noted limitations within the study. A limitation was that the audience for the pre-service teachers' data collection instruments was the course instructor. In turn, the pre-service teachers may have answered in a certain way, knowing that their instructor was reading and scoring the response. Researchers should conduct further studies involving additional professors and students enrolled in mathematics methods courses.

Despite many positive changes, the study also considered cases of resistance where some participants showed difficulty in integrating writing into mathematics. These discrepancies may reflect

deeper socio-ecological influences such as mixed messages from standardized curricula or previous schooling experiences (Bronfenbrenner, 1979). The complex nature of teacher change necessitates time, repeated exposure, and support that extend beyond a single semester.

FUTURE DIRECTION

Future research should follow these cohorts into their classrooms to observe implementation practices. Additionally, integrating coursework with fieldwork and in-service teaching, alongside professional development for cooperating teachers, may foster a collaborative approach that supports writing in mathematics instruction far beyond time spent at the university (Feiman-Nemser, 2001; Weiss & Weiss, 2001).

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APPENDIX

Figure 1: Questionnaire for Pre-Service Teachers

Writing in Math Class Questionnaire for Pre-Service Teachers

When using the phrase "writing in math class", it is implied that this includes any writing that takes place in which the writer thoroughly describes the *process of thinking* when solving a math problem. This does not include short answers which require explanation.

1. I feel writing should take place in math class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

2. I have experience writing during math class from my own personal experiences when I attended school (any grade levels).

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

3. If you experienced writing during your own math classes when you attended school, please describe how writing was implemented in these classes.

_____

_____

_____

_____

4. I have experience <u>teaching</u> writing during math class.

1	2	3	4	5
Strongly	Disagree	Neutral	Agree	Strongly Agree

If you have experience teaching writing in <u>math</u> class, please describe how you have learned to implement this into your teaching and student learning.

_____

_____

_____

_____

5. I have taken a college class(es) (any institution) in which the instructor expected me to write as part of the math class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

6. I have taken a college class(es) (any institution) that taught me how to teach writing in math class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

7. I plan to implement writing in math class in my future classroom.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

8. My cooperating/mentor teachers or field placement experiences have influenced my views on using writing in mathematics instruction.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

If you agree, please describe how your mentor teacher(s) or field placement experiences influenced your thinking about writing in math.

_____

_____

_____

9. My understanding of writing in math instruction has been shaped by broader influences such as state standards, standardized assessments, or cultural expectations around math teaching.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Please explain how policies, standards, or societal views about math education have affected your perception of writing in math.

_____

_____

_____

Demographic Questions

10. What is your current year?

Freshman Sophomore Junior Senior

11. Indicate your age group:

- ❒ Under 18 years of age
- ❒ 18-24 years of age
- ❒ 25-34 years of age
- ❒ 35-44 years of age
- ❒ 45-54 years of age
- ❒ 55 years of age or older

12. Are there any additional comments you would like to include about writing in math class (please write in the space provided)?

_____

_____

_____

Table 1. Pre- and Post-Questionnaire Responses and Point Growth for Selected Items

Questionnaire Item	Response	Pre (%)	Post (%)	Change (points)
#1. Writing should take place in math class	Strongly Disagree	6.1	0.0	–6.1
	Disagree	27.3	0.0	–27.3
	Neutral	36.4	6.1	–30.3
	Agree	30.3	24.2	–6.1
	Strongly Agree	0.0	69.7	+69.7
#4. I have experience teaching writing in math class	Strongly Disagree	51.5	0.0	–51.5
	Disagree	27.3	0.0	–27.3
	Neutral	18.2	3.0	–15.2

Questionnaire Item	Response	Pre (%)	Post (%)	Change (points)
#8. I plan to implement writing in math class	Agree	3.0	66.7	+63.7
	Strongly Agree	0.0	30.3	+30.3
	Strongly Disagree	0.0	0.0	0.0
	Disagree	18.2	6.1	–12.1
	Neutral	60.6	0.0	–60.6
	Agree	21.2	69.7	+48.5
	Strongly Agree	0.0	24.2	+24.2

Table 1: Questionnaire point growth from pre-test to post-test questionnaire items

Table 2: Open-Ended Responses from Pre- and Post-Questionnaires on Writing in Mathematics

Question #3: If you experienced writing during your mathematics classes when you attended school, please describe how writing was implemented in these classes

Pre-Questionnaire Responses

I can remember sometimes writing in math class; it was usually short sentences to describe how we did a problem.

Our one junior high teacher had us write a lot in math class. We usually solved a problem and explained how we did it.

Our school had us write in most of our math classes. We always wrote about what we did in math problems.

I believe we wrote a little bit in math class when I was in elementary school, but I don't remember why.

I can remember in elementary school we had to explain some of our answers in sentences.

I don't think we had to do this in junior or senior high.

We would write to explain word problems.

Maybe writing a few sentences but just to explain what we did.

I remember writing on workbook pages, but I don't remember what I had to write.

Question #5. If you have experience teaching writing in mathematics class, please describe how you have learned to implement this into your teaching and student learning.

Post-Questionnaire Response

Explain and justify. These are two verbs that the students should learn to do early in their school years. It will develop a different level of thinking, and they can actually use and remember their skills when they leave the classroom.

Start slow and gradually increase until the students are more comfortable. It will be awkward at first, especially if previous teachers have not used it. If students see us model it, it will become like any other form of learning.

There are many things I will take from this course. The student-created vocabulary posters were most useful. The professor gave us a checklist we could use for grading, and I want to save it for my classroom.

By using real-world problems that students can relate to. This helps them enjoy math while learning. Much of our class this semester taught us how to model writing in math class before expecting students to do it. Once students learn how, they reach a deeper level of understanding.

By focusing on one well-developed, real-life problem rather than a worksheet of thirty one-step problems.

We did group work to critique student writing samples, used a rubric to grade them, and developed discussion points for student conferences. I learned that it's better to have one or a few rich questions—even if the teacher has to write them—than many procedural problems.

We used Bloom's Taxonomy to write deep-thinking questions. Writing to explain and justify math ideas helps students understand better. Vocabulary posters support this. It takes more time but leads to better learning than workbook pages.

Question #11. Are there any additional comments you would like to include about writing in mathematics class?

Pre-Questionnaire Responses

Would love to become more familiar with this idea.

I may have done some writing during school.

I don't remember a lot from my K–12 years.

Writing and math were two different subjects when I was in school. I don't really know how to put them together.

I'm glad we are going to be learning about this topic. I'd like to learn some more about it.

I don't know why math class is including writing.

Post-Questionnaire Responses

There are many good things I learned about this, but I feel like it depends on many different factors (school, other teachers, curriculum, etc.).

Yes, I enjoyed learning something new. I like the idea but wish we could talk about it in other semesters to get more practice.

It may have helped more if we were expected to complete more writing assignments ourselves for firsthand experience. We saw examples, but doing it multiple times might have helped more.

Just that this opened my eyes to a new idea. In upper-level classes, we see a lot of repetition, and professors help us practice a lot. But this was something brand new I learned.

I look forward to learning more about this topic. I don't really know how to do it yet.

I have not experienced teaching writing in math class, but I'm excited to try it. I don't have much knowledge about writing. I'd like to know more.

Table 2: Pre-Questionnaire and Post-Questionnaire Open-Ended Responses

Table 3: Frequency of Interview Statements by Key Theme

Key theme	Number of interview statements referring to key theme
Balance of Basic Skills	14
Explain and Justify	9
Vocabulary Word Walls	7
Overlap of Subjects	6

Table 3: Key themes that emerged from the interviews with participants

Table 4: End of Semester Assignment

Table 4: End of semester assignment to show voluntary inclusion of writing

Student did not include any form of writing in end-of-semester assignment	2
Student did include writing in end-of-semester assignment, but did not include both explaining and justifying of student work	4
Student did include writing in end-of-semester assignment and included both explaining and justifying of student work	27

Figures 1-7: End of Semester Assignment

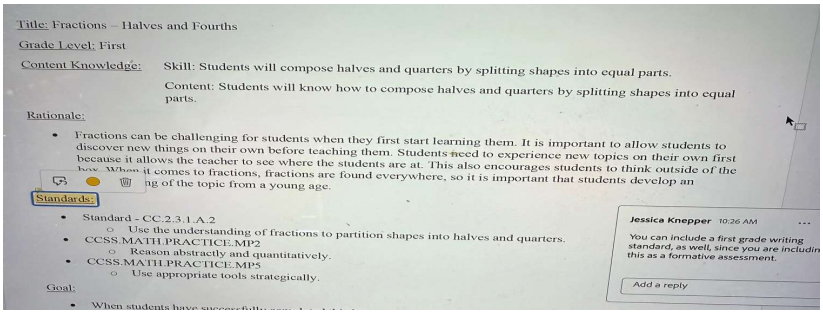


Figure 1: Example standards of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections

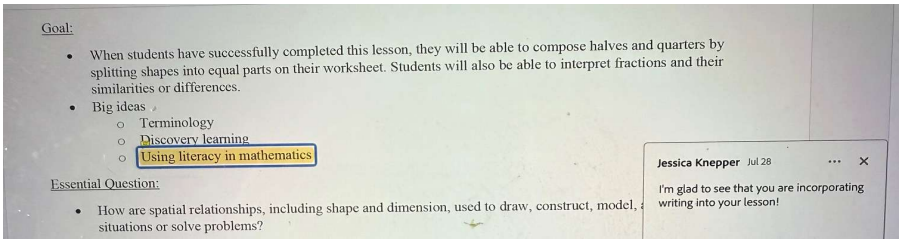


Figure 2: Example goal of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections

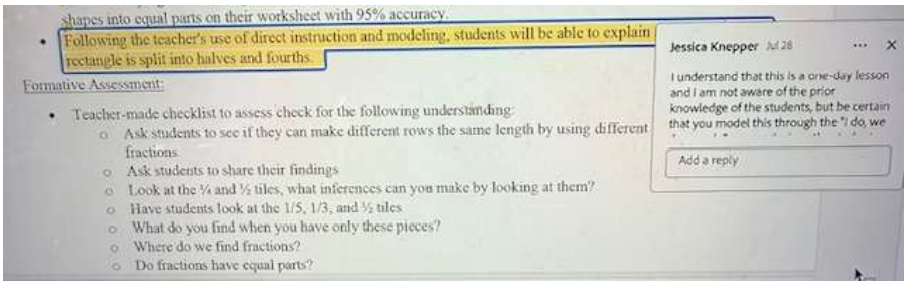


Figure 3: Example objective of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections

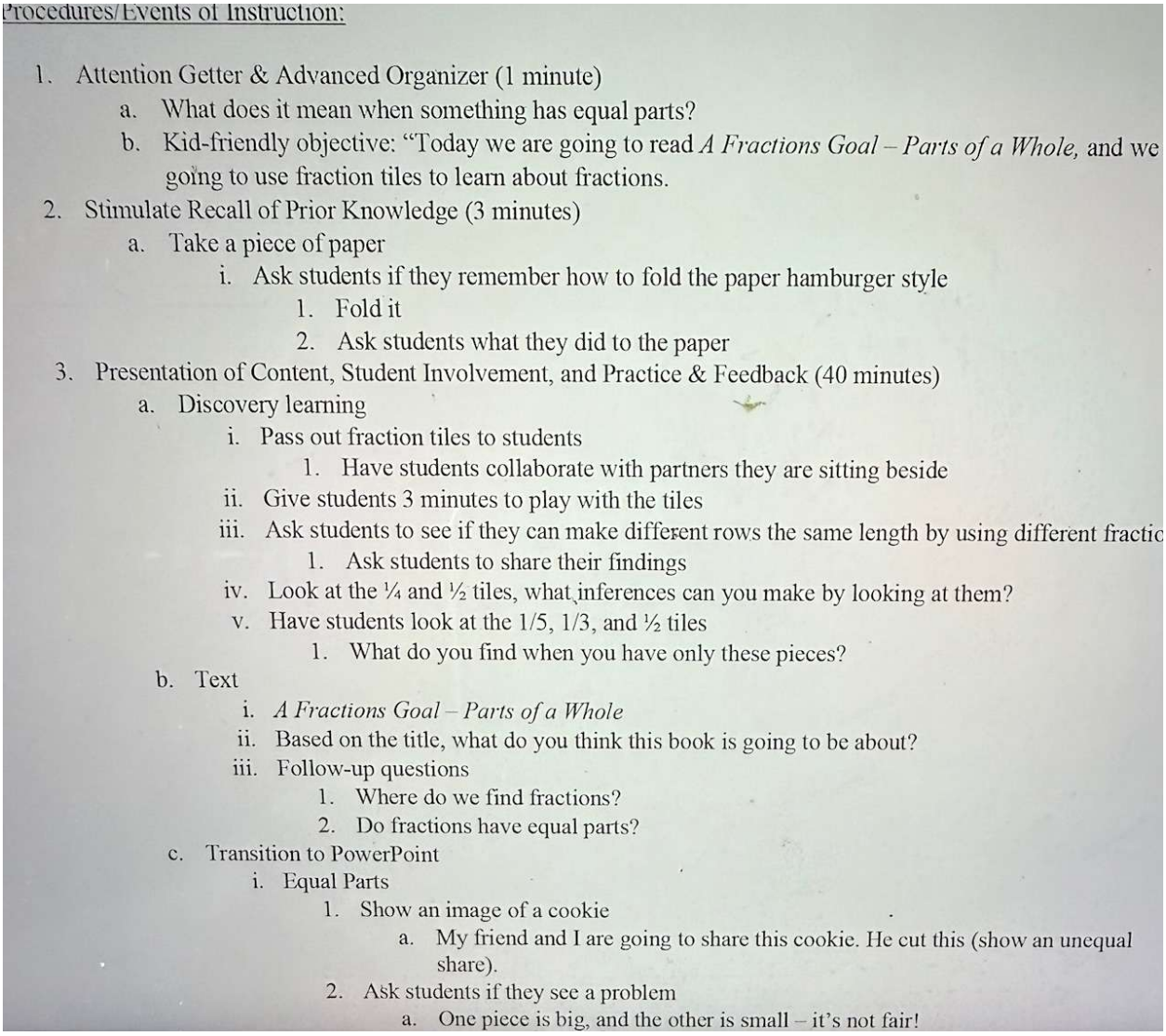
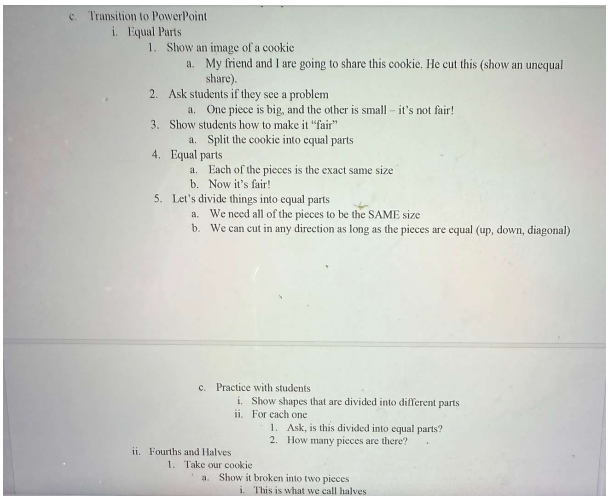


Figure 4: Example procedures section of a pre-service teacher lesson plan that integrated writing.


c. Transition to PowerPoint
i. Equal Parts
1. Show an image of a cookie
a. My friend and I are going to share this cookie. He cut this (show an unequal share).
2. Ask students if they see a problem
a. One piece is big, and the other is small – it's not fair!
3. Show students how to make it "fair"
a. Split the cookie into equal parts
4. Equal parts
a. Each of the pieces is the exact same size
b. Now it's fair!
5. Let's divide things into equal parts
a. We need all of the pieces to be the SAME size
b. We can cut in any direction as long as the pieces are equal (up, down, diagonal)

c. Practice with students
i. Show shapes that are divided into different parts
ii. For each one
1. Ask, is this divided into equal parts?
2. How many pieces are there?
ii. Fourths and Halves
1. Take our cookie
a. Show it broken into two pieces
i. This is what we call halves

Figure 5: Example procedures section of a pre-service teacher lesson plan that integrated writing

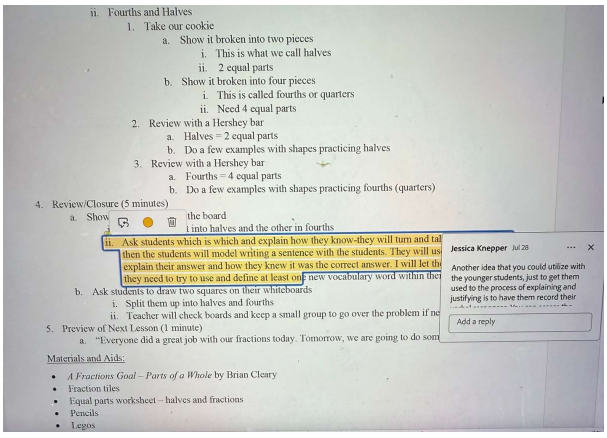
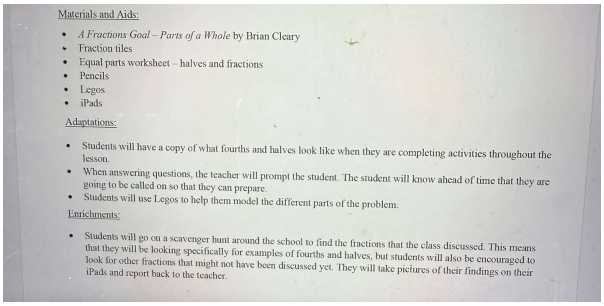

ii. Fourths and Halves
1. Take our cookie
a. Show it broken into two pieces
i. This is what we call halves
ii. 2 equal parts
b. Show it broken into four pieces
i. This is called fourths or quarters
ii. Need 4 equal parts
2. Review with a Hershey bar
a. Halves = 2 equal parts
b. Do a few examples with shapes practicing halves
3. Review with a Hershey bar
a. Fourths = 4 equal parts
b. Do a few examples with shapes practicing fourths (quarters)
4. Review/Closure (5 minutes)
a. Show
the board
into halves and the other in fourths
ii. Ask students which is which and explain how they know-they will turn and tal
then the students will model writing a sentence with the students. They will us
explain their answer and how they knew it was the correct answer. I will let th
they need to try to use and define at least one new vocabulary word within ther
b. Ask students to draw two squares on their whiteboards
i. Split them up into halves and fourths
ii. Teacher will check boards and keep a small group to go over the problem if ne
5. Preview of Next Lesson (1 minute)
a. "Everyone did a great job with our fractions today. Tomorrow, we are going to do som
Jessica Knepper Jul 28
Another idea that you could utilize with the younger students, just to get them used to the process of explaining and justifying is to have them record their
Add a reply
Materials and Aids:
- *A Fractions Goal – Parts of a Whole* by Brian Cleary
- Fraction tiles
- Equal parts worksheet – halves and fractions
- Pencils
- Legos

Figure 6: Example review/closure of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections


Materials and Aids:
- *A Fractions Goal – Parts of a Whole* by Brian Cleary
- Fraction tiles
- Equal parts worksheet – halves and fractions
- Pencils
- Legos
- iPads

Adaptations:
- Students will have a copy of what fourths and halves look like when they are completing activities throughout the lesson.
- When answering questions, the teacher will prompt the student. The student will know ahead of time that they are going to be called on so that they can prepare.
- Students will use Legos to help them model the different parts of the problem.

Enrichments:
- Students will go on a scavenger hunt around the school to find the fractions that the class discussed. This means that they will be looking specifically for examples of fourths and halves, but students will also be encouraged to look for other fractions that might not have been discussed yet. They will take pictures of their findings on their iPads and report back to the teacher.

Figure 7: Example materials, adaptations, and enrichments of a pre-service teacher lesson plan that integrated writing