Exploring Pre-service Mathematics Teachers' Challenges in Understanding Intuition and Applying Scaffolding in Geometric Proofs

Sugi Hartono[1], Tatag Yuli Eko Siswono[1], Rooselyna Ekawati[1], Farah Aisyah Nafidiastri[2]

[1] Mathematics Education of Universitas Negeri Surabaya, Indonesia

[2]FMIPA, Universitas Negeri Surabaya, Indonesia

sugihartono@unesa.ac.id; tatagsiswono@unesa.ac.id; rooselynaekawati@unesa.ac.id; farahnafidiastri@unesa.ac.id

Abstract: This study aims to explore the challenges pre-service mathematics teachers face in understanding intuition in geometric proofs and how scaffolding can support their learning. A qualitative approach was employed with 105 pre-service mathematics teachers from the State University of Surabaya as participants. Data were collected through tests and interviews, then analyzed using Miyazaki's classification: Proof Types A and B (deductive methods) and Proof Types C and D (inductive methods). Four students who struggled with intuition in proof construction were selected for scaffolding interventions. The findings reveal that 71% of participants successfully applied deductive reasoning in geometric proofs, while 29% relied on inductive reasoning. Additionally, 34 students exhibited difficulties in understanding intuition, and four were given scaffolding through strategies such as suggesting and investigating, explanation and justification, conceptual discussions, negotiating meaning, making connections, coordinating problems, and developing representative tools. The results suggest that targeted scaffolding can help pre-service teachers overcome difficulties in intuitive understanding and improve their proof construction skills in geometry.

Keywords: student difficulties, pre-service mathematics teachers', understanding of intuition concepts, scaffolding, geometry tasks

INTRODUCTION

Proof holds a central position in mathematics as a fundamental means of validating and communicating mathematical ideas (Hartono et al, 2024). It is not only a method for verifying the truth of statements but also a powerful tool for fostering deep understanding and logical reasoning (Arieli et al, 2024). The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes the importance of proof in developing students' reasoning and problem-solving skills, aligning with broader educational goals to cultivate critical thinking. Therefore, cultivating strong proof abilities among mathematics students is essential to their academic development and professional readiness as future educators (Firmasari & Sulaiman, 2019).

In Indonesian universities, the Geometry course plays a vital role in developing students' proof skills by introducing them to formal mathematical language, axiomatic systems, and logical argumentation (Hartono et al, 2025; Isran et al, 2025; Isnani et al, 2025). Through this course, students are trained to verify mathematical statements and express ideas in a structured, systematic manner (Qomariyah & Rosyidah, 2022). In the context of mathematical reasoning, proof methods generally fall into two main categories: deductive with deriving conclusions from established truths, and inductive with drawing generalizations from specific patterns (Miyazaki, 2000). Miyazaki (2000) further classifies proof into four types (Proof A, Proof B, Proof C, and Proof D) based on reasoning type and the use of functional language or visual representations, providing a framework to analyze students' approaches in constructing proofs.

Despite the recognized importance of proof, many students struggle with its construction (Chin & Fu, 2021; Aisyah et al., 2023). These difficulties are often rooted in a limited intuitive understanding of mathematical concepts (Moore, 1996), which hinders their ability to develop coherent arguments and identify valid logical connections (Antonini, 2019). Mathematical intuition defined as the informal, often visual or experiential sense of a concept serves as a foundation for generating ideas prior to formalization. When students lack this intuitive base or a clear understanding of definitions, theorems, and relationships, they frequently encounter obstacles in proof construction and are more likely to misapply proof strategies.

To address these challenges, effective instructional strategies are necessary. One promising approach is scaffolding—the structured support provided by instructors to bridge the gap between what learners know and what they are expected to learn (Ding et al., 2011; Belland, 2017). Scaffolding can assist students in developing mathematical intuition by guiding them through visualization, identification of key assumptions, and analysis of simplified examples. Anghileri (2006) outlines three levels of scaffolding: (1) environmental provisions such as appropriate learning media and structured materials; (2) interactive teaching practices including explanation, questioning, and material restructuring; and (3) conceptual engagement that encourages students to articulate and connect their understanding. Research by Indrawati (2017) supports the effectiveness of scaffolding in helping students concretely visualize and grasp mathematical ideas, particularly in geometry.

Despite the established importance of mathematical proof, few studies have directly examined how pre-service teachers' intuitive understanding influences their ability to construct formal geometric proofs—particularly in the Indonesian context. Prior research has often focused on logical structures, overlooking the formative role of intuition in early mathematical reasoning (Stylianides et al., 2024; Haj-Yahya et al., 2023). Moreover, there is a lack of classroom-based interventions that specifically address intuitive misconceptions through structured scaffolding. Given the persistent challenges that pre-service mathematics teachers face in understanding intuition and constructing geometric proofs, research in this area is both timely and essential. Mastery of proof is not only vital for academic success but also critical for effective mathematics instruction in future classrooms. As future educators, pre-service teachers must be equipped to communicate the concept and importance of proof in ways that are accessible and meaningful to students. Therefore, this study aims to investigate: (1) What forms of intuitive misunderstanding are exhibited by pre-service mathematics teachers in geometric proof tasks? and (2) How can a scaffolded instructional approach address these challenges and improve students' reasoning?

LITERATURE REVIEW

Mathematical intuition plays a foundational role in initiating the reasoning process and bridging informal thinking with formal proof construction. Moore (1994) highlighted that many students struggle with constructing proofs due to their lack of intuitive understanding of mathematical concepts. Epp (2003) emphasized that intuition is not only useful but necessary for transitioning from visual or informal reasoning toward formal mathematical arguments. Nevertheless, Antonini (2019) cautioned that intuitive or visual reasoning, when not well structured, often leads to misconceptions—especially in complex topics like indirect proofs or generalizations.

Pre-service mathematics teachers, in particular, face unique challenges in proof construction. Martin and Harel (1989) observed that these future educators frequently confuse valid proofs with empirical arguments or generalizations, which indicates a fragile conceptual understanding. Stylianides (2007) added that limited exposure to diverse proof methods such as direct, indirect, or inductive proofs compounds this difficulty. Although several studies have examined students' overall struggles with proof, only a few have investigated how pre-service teachers develop, misuse, or fail to activate intuitive understanding when constructing geometric proofs (Stylianides et al., 2024).

To address these difficulties, scaffolding has emerged as a powerful instructional tool. Defined as temporary, targeted support that is gradually removed as learners gain independence, scaffolding has been shown to improve both reasoning and problem-solving performance (Belland, 2017). Anghileri (2006) categorized scaffolding into three progressive levels: environmental structuring, interactive dialogue (such as probing and prompting), and the promotion of conceptual thinking. Ding et al. (2011) demonstrated that scaffolding significantly enhances students' conceptual problem-solving, although most of these studies have focused on general mathematics tasks rather than the specific demands of geometric proof construction.

Despite the established importance of both intuition and scaffolding in mathematics education, research integrating the two remains scarce. In particular, few studies have examined the specific challenges pre-service teachers face in intuitive understanding within geometric proof tasks, especially through the lens of Miyazaki's classification of proof types. Moreover, classroom-based studies that not only diagnose intuitive misconceptions but also implement tailored scaffolding interventions are still limited. This study addresses that gap by exploring how scaffolding strategies can effectively support pre-service teachers' understanding and application of intuition across different proof styles.

METHOD

This study employed a descriptive qualitative approach using a case study design to explore the intuitive understanding difficulties faced by pre-service mathematics teachers and the effectiveness of scaffolded instructional support in overcoming these challenges. The research offers a novel perspective by classifying students' approaches to proof using Miyazaki's framework and by investigating their intuitive understanding through task-based interviews. Unlike previous studies that primarily emphasized deductive reasoning, this study highlights how targeted scaffolding can help transform students' intuitive misconceptions into structured and valid formal proofs, offering a practical instructional model for classroom implementation.

The participants in this study were 105 second-year pre-service mathematics teachers (aged 19–21) enrolled in the "Elementary Geometry" course at Surabaya State University. All participants had previously completed foundational courses in logic and mathematical structures, although they had limited exposure to formal proof construction. Representing a typical cohort in Indonesian mathematics teacher education programs, these students engaged in structured proof tasks as part of their regular coursework in a standard classroom setting. The primary data sources consisted of written test responses and semi-structured interviews. The test featured a single open-ended geometric proof task designed to elicit both deductive and inductive reasoning: *Given an arbitrary quadrilateral ABCD, let P, Q, R, and S be the midpoints of sides AB, BC, CD, and DA, respectively. Suppose PR and QS intersect at point O. Prove that PO = OR and QO = OS*. The proof test consisted of one constructed-response task adapted from classical geometry problems to assess inductive and deductive reasoning. The task was reviewed by two subject-matter experts to ensure content validity. Interview protocols included six guiding questions addressing students' reasoning, interpretation of midpoints, and generalization patterns. Questions were piloted and refined based on feedback from a small group of pre-service teachers not involved in the main study. Based on their performance in this task, four students were purposively selected for in-depth case study analysis, each representing one of the four proof types categorized by Miyazaki's framework. These individuals, labeled MA, MB, MC, and MD, participated in individual interviews that were conducted in a quiet classroom, audio-recorded with their consent, and lasted approximately 30–45 minutes each.

The students' written responses were analyzed using Miyazaki's (2000) proof classification, which distinguishes proofs based on their method (deductive or inductive) and the nature of representation (use of functional language or reliance on visual manipulation). Table 1 below outlines the classification:

Representation	Method	
	Deductive	Inductive
Using functional language according to the theorem	Proof A	Proof D
Do not use functional languages, use images, or manipulate objects	Proof B	Proof C

Table 1: Levels of proof in mathematics

Based on this framework, the researchers identified students' difficulties within each proof type, revealing limited intuitive understanding of the concept of proof. The students' written responses were analyzed using Miyazaki's (2000) proof classification. In addition to the case analysis of four representative students (MA, MB, MC, MD), a descriptive quantitative analysis of all 105 participants' answers was conducted. Frequencies and percentages of correct and incorrect responses for each proof type (A–D) were calculated. Furthermore, the 34 students who experienced difficulties were categorized into three main error patterns: (1) misinterpretation of midpoints, (2) over-generalization from special cases (squares/rectangles), and (3) assuming intersections automatically imply equal segments. This categorization aimed to provide a more comprehensive picture of students' intuitive difficulties. Interview questions were developed based on themes from Miyazaki's classification and expert feedback and were validated through a peer-review process involving two mathematics education researchers. Data were analyzed thematically, guided by Miyazaki's framework and complemented by open coding. Thematic units included misinterpretation of midpoints, over-generalization, and over-reliance on visual forms. Two researchers independently coded the responses, and discrepancies were resolved through consensus. Scaffolding strategies based on Anghileri's (2006) framework were provided to support students' reasoning, and triangulation between written responses and interview data was used to enhance the validity of the findings. Researchers provide scaffolding assistance based on Anghileri's theory (2006) which divides the scaffolding hierarchy into three levels, namely:

Types of Scaffolding		
Scaffolding level 1	*environmental provisions*	Sequencing and pacing
		Giving structured tasks
		Student grouping
Scaffolding level 2	*explaining*	
	Reviewing	Seeing, touching, stating verbally
		Probing and prompting
		Interpreting student actions
		Parallel modeling
		Student explanation and justification

	Restructuring	Identifying meaningful content
		Simplifying the problem
		Paraphrasing student conversations
		negotiation of meaning
Scaffolding level 3	Developing representative tools	
	Making a connection	
	Generating conceptual discussion	

Table 2: Scaffolding to overcome difficulties in proof

RESULTS AND DISCUSSION

The findings of the research can be categorized according to the focus established at the beginning of the research, namely:

Type	Number of pre-service teachers'	Difficulties of pre-service mathematics teachers' understanding of intuition concepts (Number of teachers)
Proof A	64	18
Proof B	10	3
Proof C	7	1
Proof D	24	12
Total	105	34

Table 3: Types of pre-service teachers' proof in geometry

Based on Table 3, the results indicate that 74 students (71%) used deductive proof, while 31 students (29%) used inductive proof. Out of the 74 students who used deductive proof, 64 employed proof type A, with 15 students providing correct answers and 49 students making errors, while 10 employed proof type B, with 7 correct and 3 incorrect. Of the 31 students who used inductive proof, 7 used proof type C (2 correct, 5 incorrect), and 24 used proof type D (6 correct, 18 incorrect). Overall, 34 students (33%) experienced difficulties in constructing proofs due to limited intuitive understanding. These 34 students' responses were further categorized into three types of intuitive errors: (1) 14 students (41%) misinterpreted midpoints by assuming any intersection is a midpoint, (2) 11 students (32%) generalized properties from special cases such as squares or rectangles, and (3) 9 students (27%) assumed that intersecting diagonals automatically divide into equal segments. Table 4 presents the distribution of these error patterns below. For Proof A, as many as 18 students, then continued with Proof B as many as 10 students, 7 students from Proof C, and 24 students from Proof D.

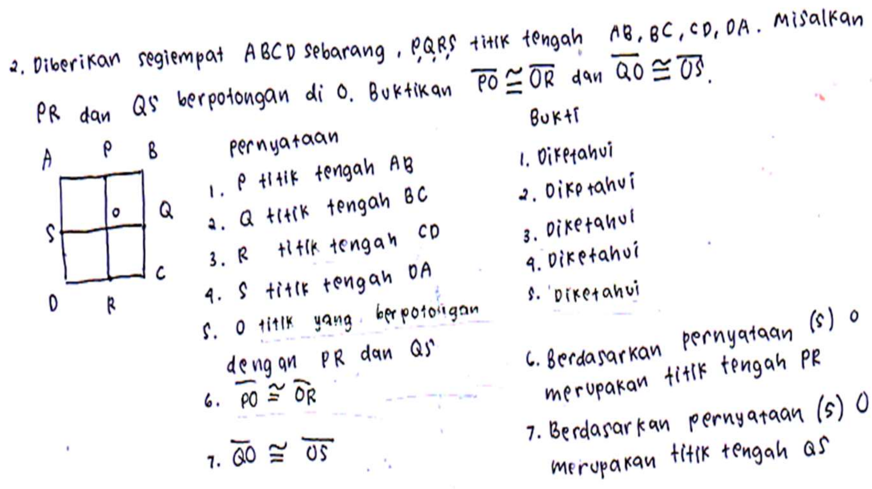
Error Category	Number of Students	Percentage	Example of Error
Misinterpreting the midpoint	14	41%	Assuming that the intersection of two lines automatically represents the midpoint
Generalization from special cases (square/rectangle)	11	32%	Using the properties of a square to conclude the properties of all quadrilaterals
Assuming intersections automatically divide equally	9	27%	Stating that PO = OR merely because line PR intersects at O

Table 4: Categories of Students' Intuitive Errors

This analysis shows that students' difficulties were not only caused by weaknesses in formal logic, but also by fundamental misconceptions in intuitive understanding. These findings also indicate the necessity of examining not only the correctness of students' formal reasoning but also the specific types of intuitive misconceptions they display. By categorizing errors such as misinterpreting midpoints, overgeneralizing from special cases, and assuming intersections automatically divide equally, educators can better identify the roots of students' difficulties. This categorization provides a framework for designing targeted scaffolding strategies that directly address the nature of students' misunderstandings, rather than merely focusing on procedural accuracy.

In this study, inductive reasoning refers to students' spontaneous inferences or justifications that are based on visual impressions or informal understanding, rather than systematic logical analysis. According to Lajos (2023), intuitive thinking in mathematics can support or hinder proof development depending on how it aligns with formal concepts. In our analysis, several students made assumptions, for example: assuming O is the midpoint simply because lines intersect) without articulating the underlying geometric properties. Such assumptions reflect pseudo-intuitive understanding or misconceptions rather than true intuitive insight. Some examples of students' answers regarding pre-service mathematics teachers' understanding of intuition concepts are as follows.

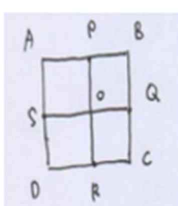
PROOF A


2. Diberikan segiempat ABCD sebarang, PQRS titik tengah AB, BC, CD, DA. Misalkan PR dan QS berpotongan di O. Buktikan $\overline{PO} \cong \overline{OR}$ dan $\overline{QO} \cong \overline{OS}$.

Pernyataan	Bukti
1. P titik tengah AB	1. Diketahui
2. Q titik tengah BC	2. Diketahui
3. R titik tengah CD	3. Diketahui
4. S titik tengah DA	4. Diketahui
5. O titik yang berpotongan dengan PR dan QS	5. Diketahui
6. $\overline{PO} \cong \overline{OR}$	6. Berdasarkan pernyataan (5) O merupakan titik tengah PR
7. $\overline{QO} \cong \overline{OS}$	7. Berdasarkan pernyataan (5) O merupakan titik tengah QS

Translation

Given an arbitrary quadrilateral ABCD, let P, Q, R, and S be the midpoints of AB, BC, CD, and DA, respectively. Suppose that PR and QS intersect at point O. Prove that PO=OR and QO=OS.



Statements	Proof
1. P is the midpoint of AB	Given
2. Q is the midpoint of BC	Given
3. R is the midpoint of CD	Given
4. S is the midpoint of DA	Given
5. O is the intersection point of PR and QS	Given
6. $\overline{PO} \cong \overline{OR}$	Based on statement (5), O is the midpoint of PR
7. $\overline{QO} \cong \overline{OS}$	Based on statement (5), O is the midpoint of QS

Figure 1. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof A

Based on Figure 1, it can be seen that pre-service teachers have used deductive methods with functional language according to the theorem but there is still a misunderstanding. Subject MA explained that O is a point that intersects PR and QS so that it causes the PO line to be congruent with OR and QO to be congruent with OS and O is the midpoint. This can be seen from the results of interviews with subject MA as follows.

Researcher : Okay, try to look at the picture of the quadrilateral, do you think this is an arbitrary quadrilateral?

MA : Yes, arbitrary, sir, because it is known to be an arbitrary quadrilateral and the proof must use a deductive method so that it cannot use examples other than arbitrary quadrilaterals.

Researcher : When is a quadrilateral said to be arbitrary?

MA : A quadrilateral that has four sides but is not specific.

Researcher : In your opinion, point O is the midpoint, right?

MA : In my opinion, point O is the midpoint, sir.

Researcher : Why can point O be called the midpoint?

MA : Because point O intersects 2 line segments, sir, so point O is the midpoint

As seen in the interview above, although Subject MA can explain the step-by-step answer in proving the theorem using the logical structure of a proof. Subject MA has difficulty in understanding the concept of the intersection of two lines and the midpoint so that he cannot prove the problem. This response is categorized as reflecting a low level of intuitive understanding. Specifically, the subject shows a limited understanding of the concepts of the intersection of two lines and the midpoint of a line segment, which leads to difficulty in constructing a valid proof.

PROOF B

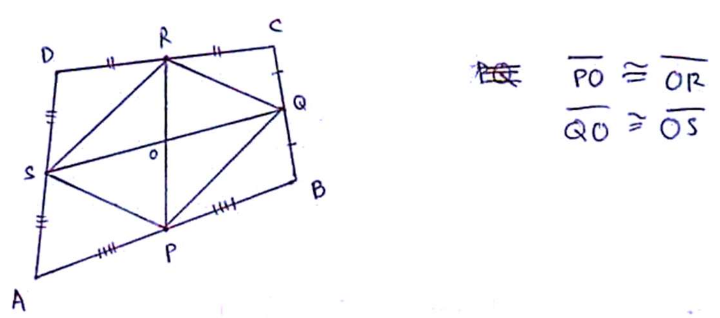


Figure 2. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof B

Based on Figure 2, it can be seen that Subject MB uses the concept of an arbitrary quadrilateral, namely that each side is different in the visualization of the image without using functional language. The subject claims that point O is definitely in the middle because SQ and PR cut the side of the arbitrary quadrilateral into two equal parts. Therefore, the results of the intersection of the two diagonals are also the same length so that PO = OR and QO = OS are obtained. This can be seen from the results of the interview with subject MB as follows.

Researcher : Please explain what proof concept you use in working on this proof problem?

MB : Let me assume an arbitrary quadrilateral and then prove the midpoint, sir.

Researcher : Are you sure about your answer? How was the proof?

MB : Given an arbitrary quadrilateral ABCD. So let me draw it first, sir. Suppose P, Q, R, and S are the midpoints of AB, BC, CD, and DA, respectively. Then draw a line P to R and Q to S. The point where the lines intersect is at O.

Researcher : Wait, from there how can you prove that PO is equal to OR and SO is equal to OQ?

MB : First, it is known that this point is in the middle. Next, this line SQ intersects BC at Q, which is at the midpoint and intersects AD at point S, which is also in the middle. So, I assume that the point of intersection at O with PR is in the middle. Therefore, I assume that PO is equal to OR.

Researcher : So from there, we immediately find that PO is equal to OR, right?

MA : Yes, sir.

As seen in the interview above, the Subject claims that point O is definitely in the middle because SQ and PR cut any side of the quadrilateral into two equal parts. Therefore, the results of the intersection of the two diagonals are also the same length so that PO = OR and QO = OS are obtained. At this stage, the subject has difficulty in understanding the concept of the midpoint so that the claim produced to prove the problem is less precise.

PROOF C

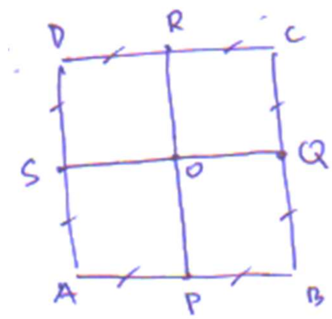


Figure 3. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof C

Based on Figure 3, it can be seen that Subject MC uses the concept of a square without using functional language, using images, or manipulating objects. Furthermore, it can be seen in the figure that Subject MC directly justifies that point O is the midpoint so that the RO and OP lines and the SO and OQ lines are the same. This can be seen from the results of the interview with the MC subject as follows.

Researcher: What is the first step?

MC: Make a square sir

Researcher: Okay after knowing that it is a square, is RO the same as OP?

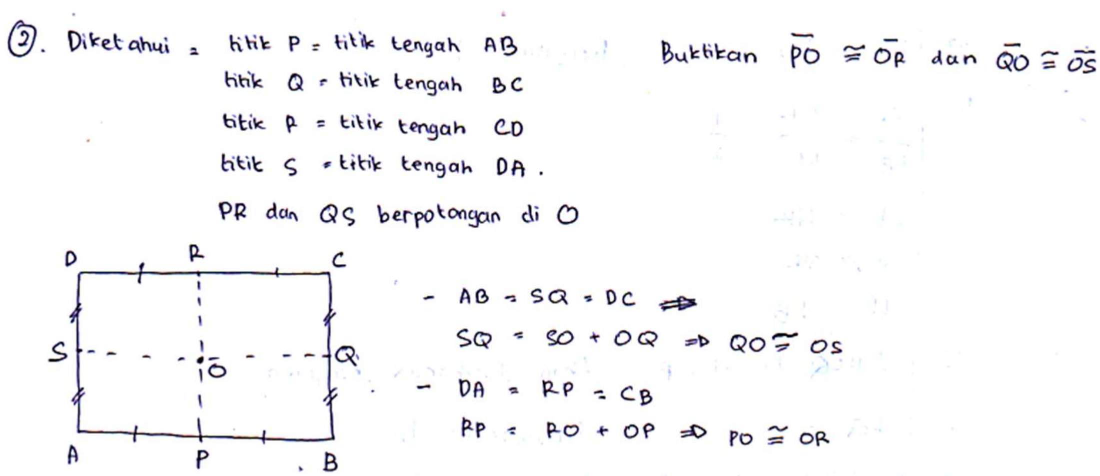
MC: Same

Researcher: Does that mean you meet it right away?

MC: Yes sir, because the square is the same length divided by the midpoint

Based on the interview, Subject MC attempts to justify the conclusion using a square—an overly specific case that does not represent all arbitrary quadrilaterals. The student immediately concludes that PO = OR and QO = OS by relying on the symmetry of a square, without attempting to generalize from multiple cases. This is not an example of intuitive understanding; rather, it reflects a misconception of the problem, particularly a misunderstanding of what constitutes an arbitrary quadrilateral. The student's reasoning lacks generalization and appears to confuse a specific shape (square) with the general category (quadrilateral), which is a conceptual error rather than an intuitive leap. Although Subject MC used a square as the base case and generalized from that one example, there is no evidence of systematic generalization across multiple cases, which is a hallmark of inductive reasoning. This shows a misinterpretation of inductive proof. Thus, the classification as 'Proof C' should be interpreted as the student attempting an empirical generalization rather than formal induction.

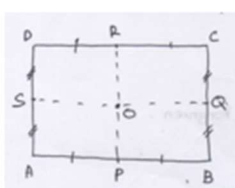
PROOF D



Translation

2. **Given:**
   - Point **P** is the midpoint of **AB**
   - Point **Q** is the midpoint of **BC**
   - Point **R** is the midpoint of **CD**
   - Point **S** is the midpoint of **DA**

PR and QS intersect at **O**



Proof:

Prove that:

$$PO \cong OR \quad \text{and} \quad QO \cong OS$$

1. $AB = SQ = DC$
2. $SQ = SO + OQ \Rightarrow QO \cong OS$
3. $DA = RP = CB$
4. $RP = PO + OP \Rightarrow PO \cong OR$

Figure 4. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof D

Based on Figure 4, it can be seen that Subject MD uses the concept of a rectangle using functional language, using images, or object manipulation. The subject answers with inductive proof steps, namely with the concept of a rectangle as an arbitrary quadrilateral and argues that SQ = SO+SQ and concludes that the lines SO and OQ are the same. The subject also argues that RP = RO+OP and concludes that the lines PO and OR are the same. This can be seen from the results of the interview with subject MD as follows.

Researcher: What is the first step?

MD: I use a rectangle sir

Researcher: Where do you think QO is congruent with OS

MD: According to my answer, this is because the rectangle so AB is the same length as SQ and the same length as DC then SQ is SO plus OQ then QO is congruent with OS

Researcher: So it meets directly, right?

MC: Yes sir, because it uses the concept of a rectangle

Based on the interview above, it can be seen that Subject MD had difficulty in continuing the proof of the problem because the subject's understanding was lacking, the subject used a rectangle as an arbitrary quadrilateral and argued that SQ = SO + OQ and concluded that lines SO and OQ were the same. The subject also argued that RP = RO + OP and concluded that lines PO and OR were the same. At this stage the subject experienced difficulties because the subject argued that one line segment was formed by two equal lines because he concluded that point O was the midline and divided it into two lines of equal length in proving this theorem so that it was categorized as difficult because he had little intuitive understanding of the concepts. Although the student uses language that appears logical, their reasoning is based entirely on properties of rectangles. This suggests the student does not differentiate between specific and general geometric figures. The use of a special case to justify a general proof without broader justification is not considered intuitive understanding; it is a surface-level strategy that bypasses the essence of proof construction. Such thinking stems from an incomplete understanding of geometric classifications rather than an intuitive insight.

Given these challenges in intuitive understanding, effective teaching interventions are needed to bridge the gap. Scaffolding provides structured support by gradually guiding students through conceptual difficulties, allowing them to develop independent problem-solving skills over time. In addition, based on Anghileri (2006) that scaffolding was implemented through three structured sessions (Table 2). Level 1 involved environmental support (e.g., use of diagrams and guiding tools). Level 2 included probing questions, prompting explanation and justification. Level 3 engaged students in generating connections and visual representations. Regarding scaffolding to overcome student difficulties in proof, as many as 4 students answered incorrectly and were then selected to be given scaffolding, namely as follows:

No	Subjects	Initial proof type	Types of scaffolding	Final proof type
1	MA	Proof A	Probing and prompting, Explanation and Justification, Generating conceptual discussion, Negotiating Meaning, Making Connections	Proof A (Correct answer)
2	MB	Proof B	Probing and prompting, Explaining and justifying, Simplifying problems, Generating conceptual discussions, Making connections	Proof A (Correct answer)
3	MC	Proof C	Probing and prompting, Explanation and Justification, Generating Conceptual Discussion, Making Connections, Negotiating Meaning	Proof A (Correct answer)
4	MD	Proof D	Probing and prompting, Explanation and Justification, Generating Conceptual Discussion, Making Connections, Negotiating Meaning, and Developing Representative Tools	Proof A (Correct answer)

Table 5: Scaffolding to overcome difficulties of pre-service mathematics teachers' understanding of intuition concepts

Based on Table 5, it shows that students apply the type of proof in constructing evidence, namely the deductive and inductive methods. The deductive method consists of two types, namely proof A and proof B, while the inductive method consists of two types including proof C and proof D. Table 5 shows that 74 students used the deductive method (71%) while 31 of them used the inductive method (29%). This means that more than half of the students' answers used the deductive method. Although some students make mistakes in constructing proofs, they have tried to construct proofs using deductive and inductive methods. This finding is consistent with research conducted by Miyazaki (2000) that most students in his research use the deductive method rather than the inductive method in constructing proofs. In addition, the results of the study showed that the main difficulty faced by students in constructing proofs of geometric theorems seemed to be closely related to the lack of intuitive understanding of the basic concepts in proofs, namely 34 students (33%). Nearly one-third of the subjects experienced this difficulty, indicating that students tend to get stuck in mechanical processes, such as following a certain proof pattern without really understanding the concepts underlying the steps. In line with this finding, research by Kögce et al. (2010)

also showed that difficulties in proofs are often caused by a lack of deep conceptual understanding. They found that most students performed proofs by following a familiar pattern without really understanding the concept or reasoning behind the steps.

Based on Table 5, it can be seen that all students experienced difficulties in various types of proofs, both using deductive methods (proofs A and B) and inductive methods (proofs C and D), which were caused by a lack of intuitive understanding related to the concepts underlying the proof. This difficulty reflects that without a strong intuitive understanding, students often have difficulty in feeling the relationship between elements in the problems they face. For example, students often get stuck in the definition of two intersecting lines, then they will divide them equally so that the point of intersection is the midpoint. This difficulty leads to incoherent proofs, where the steps do not support each other, and students have difficulty in proving the relationship between geometric elements correctly. This is in line with Mahfudy's (2017) research on students' mathematical proof strategies in geometry problems, including proving the Pythagorean theorem. It was found that students often rely on their intuition in making assumptions or guesses that lead to conclusions, but lack a deep understanding of the concepts underlying the proof.

Furthermore, the lack of intuitive understanding also affects students' ability to choose the most appropriate proof approach. Moore (1996) stated that in mathematical proofs, sometimes there is more than one way to reach the desired conclusion, and intuition helps students choose the most efficient path. The findings of the present study also resonate with earlier works emphasizing the central role of intuitive reasoning in students' proof construction. Moore (1996) highlighted that many students experience significant difficulties in constructing proofs because they rely on incomplete or flawed intuitive conceptions rather than rigorous logical structures. Similarly, Antonini (2019) noted that intuitive thinking can both support and hinder proof activity, depending on whether the intuition aligns with formal concepts. More recently, Stylianides (2024) emphasized the need to design instructional tasks that bridge the gap between informal intuitive reasoning and formal deductive proof in order to promote sustainable proof learning.

This difficulty reflects that without a strong intuitive understanding, students often fail to recognize the correct relationships between geometric elements. Each category of intuitive error can be addressed through specific scaffolding strategies: misinterpretation of midpoints can be handled through probing, prompting, and justification (Level 2 scaffolding); over-generalization from special cases requires conceptual discussion and dynamic representations with GeoGebra (Level 3 scaffolding); and assumptions about intersections dividing equally can be corrected through guided diagrams, simplified tasks, and negotiation of meaning (Level 1–2 scaffolding). The uniqueness of this research lies in combining Miyazaki's classification of students' proof types with Anghileri's scaffolding framework, implemented within the Indonesian context. This integration provides both theoretical and practical contributions: theoretically, it refines our understanding of how proof types correlate with intuitive errors, and practically, it offers context-sensitive interventions for mathematics teacher education.

Based on Table 5, all subjects succeeded in proving up to the correct deductive proof (Proof A with correct answer) and also explained that suggesting and investigating are types of scaffolding that are often used to help students and researchers identify each student's difficulties in each type of proof. With this scaffolding, researchers can ensure students' understanding of the problem through investigative questions. In addition, suggesting and investigating help guide students in determining the steps that need to be taken when working on the problem. This is in line with the research results of Siregar and Fauzi (2017) and Izzah and Azizah (2019) showing that students' problem-solving abilities can increase along with the development of mathematical reasoning through the help of scaffolding suggesting and investigating. This type of scaffolding also helps students realize their difficulties when solving proof problems (Cintamulya & Rahayu, 2020).

Researchers also often ask students to justify their answers to each type of proof in order to strengthen understanding if the answer is correct or to be aware of errors if there are any. In some cases, negotiation of meaning is carried out when students have not realized the error even though they have been asked to provide an explanation. According to Zhang, Cao, Chan, & Wan (2022) found that negotiation of meaning is effective in helping students correct errors, especially in generating relevant numbers to solve problems. This finding supports the results of research related to the effectiveness of scaffolding to overcome students' difficulties in proving the problem. In addition, there are also other types of scaffolding that are also helpful, namely generating conceptual discussions, negotiating meaning, making connections, simplifying problems, and developing representative tools in overcoming difficulties because students have little intuitive understanding of the concept of proof.

These findings suggest that a structured scaffolding design, based on Anghileri's framework, can serve as the basis for a practical instructional model to support proof instruction. Rather than being an isolated remedial tool, scaffolding can be systematically embedded in lesson design, particularly when introducing complex proof tasks. For instance, teachers may initially prepare tasks with visual aids (level 1), facilitate guided questioning and dialogue (level 2), and gradually move toward abstract conceptualization (level 3). The success of the four case participants in transitioning from incorrect to correct deductive proofs demonstrates the potential of this scaffolding framework to be applied in classroom settings. Future instructional designs can adapt this model to support a broader group of learners with varying levels of intuitive understanding.

This study offers instructional implications for mathematics educators aiming to improve students' proof construction abilities. The scaffolding strategies implemented here can be structured into a repeatable classroom model. Educators can utilize a phased approach to address intuitive misconceptions, beginning with task design and continuing through interactive explanation and conceptual generalization. Such a model not only addresses immediate student difficulties but also promotes long-term development of mathematical reasoning. Furthermore, incorporating tools like dynamic geometry software (e.g., GeoGebra) into each scaffolding phase may enhance student visualization and engagement.

These findings indicate that some students' errors stem from a fundamental misunderstanding of geometric definitions, such as failing to distinguish between squares, rectangles, and arbitrary quadrilaterals. Consequently, what may appear as intuitive understanding is often a misapplied or overly narrow understanding of shape properties. Clarifying these definitions and explicitly confronting misconceptions during instruction is essential before expecting students to reason intuitively or construct valid geometric proofs.

CONCLUSIONS

In conclusion, the results of this study indicate that the deductive method is the most commonly used by students in mathematical proofs, with 71% of participants opting for it due to its reliance on formal reasoning and the deductive-axiomatic approach. Among the four types of proofs classified by Miyazaki, Type A was the most frequently used, with 64 students favoring the use of functional language to structure their arguments systematically and logically. However, 18 pre-service teachers still experienced difficulties, not due to a complete lack of intuitive understanding, but because of their inability to accurately interpret geometric definitions and translate their intuitions into valid formal proofs. To address these challenges, scaffolding strategies, including steps such as suggesting and investigating, explaining and justifying, fostering conceptual discussions, negotiating meaning, making connections, simplifying problems, and developing representative tools, proved effective. Given the structured nature of the tasks and the clear categorization of proof types, this approach can be adapted by mathematics educators in both pre-service teacher training and secondary school settings.

As a recommendation, integrating GeoGebra technology into scaffolding strategies offers significant potential to enhance students' understanding of mathematical proofs. For instance, a GeoGebra task could involve dynamically dragging the vertices of a quadrilateral while observing how the intersection of midpoints behaves, enabling students to visually explore the conditions under which $PO = OR$ and $QO = OS$. This process promotes both intuitive insight and formal deductive reasoning, as such interactive visualizations help bridge the gap between visual understanding and formal proof construction. By providing dynamic representations, interactive explorations, and step-by-step demonstrations, GeoGebra makes abstract mathematical concepts more accessible, meaningful, and engaging. Future research should examine the effectiveness of these scaffolding strategies in other mathematical domains, such as algebra and calculus, and explore how digital tools like GeoGebra can be optimized to support proof development in various learning contexts. However, this study has several limitations. It was conducted at a single institution with a relatively homogeneous participant group, which limits the generalizability of the findings. Moreover, the qualitative case study approach involved a small number of participants, which may introduce subjectivity despite the use of validated analytical frameworks. To improve the reliability and broader applicability of future findings, subsequent studies should involve larger and more diverse samples and consider using mixed-method approaches that combine qualitative insights with quantitative validation.

ACKNOWLEDGMENTS

I would like to thank you funding for this research. This work was supported by the Beasiswa Pendidikan Indonesia (BPI) for the funding of the completion of doctoral studies, the Rector of Surabaya State University, LPPM of Surabaya State University for the basic research (2025) and the School of Postgraduate Studies, Surabaya State University, for the valuable discussions upon initial drafts of the paper.

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APPENDIX
Lesson Plan: Scaffolding-Based Instruction for Geometric Proof

1. General Information
- Subject: Geometry
- Level: Pre-service Mathematics Teachers (Undergraduate)
- Topic: Proof of Midpoint Theorem in Quadrilaterals
- Duration: 2 × 50 minutes (100 minutes)
- Learning Approach: Scaffolding (Anghileri, 2006)
- Proof Framework: Miyazaki's Proof Types (A–D)

2. Learning Objectives
By the end of the lesson, students are expected to:
1. Understand the concepts of midpoint and intersection of line segments.
2. Identify misconceptions related to geometric intuition.
3. Construct a valid geometric proof using deductive reasoning.
4. Transition from intuitive reasoning to formal proof.

3. Learning Materials
- Definition of midpoint
- Intersection of line segments
- Properties of quadrilaterals
- Problem:
  Given a quadrilateral ABCD, with midpoints P,Q,R,S on each side. If PR and QS intersect at O, prove that PO= and QO= OS.

4. Learning Media
- Whiteboard / digital board
- Worksheet (proof task)
- Dynamic geometry software (e.g., GeoGebra) *(optional but recommended)*

5. Learning Activities

Phase 1: Introduction (10 minutes)
- Lecturer presents a quadrilateral diagram.
- Ask students:
  - *What is a midpoint?*
  - *What happens when two lines intersect?*
- Identify initial intuitive responses.

Phase 2: Core Activities (80 minutes)
Level 1 Scaffolding: Environmental Provisions (20 minutes)
Activities:

- Provide students with a structured diagram of quadrilateral ABCD.
- Students identify:
    - Midpoints
    - Line segments PR and QS
- Use visual aids (diagram or GeoGebra).

Scaffolding Strategies:
- Sequencing tasks
- Providing structured worksheets
- Visual representation

Expected Outcome:
Students recognize key elements of the problem.

Level 2 Scaffolding: Interactive Support (30 minutes)
Activities:
- Lecturer guides students through probing questions:
    - *Why do you think point O is (or is not) a midpoint?*
    - *What properties define a midpoint?*
    - *Can intersection alone guarantee equal segments?*
- Students explain their reasoning.

Scaffolding Strategies:
- Probing and prompting
- Explanation and justification
- Simplifying the problem
- Negotiation of meaning

Expected Outcome:
Students begin identifying misconceptions (e.g., "intersection = midpoint").

Level 3 Scaffolding: Conceptual Development (30 minutes)
Activities:
- Students attempt to construct a formal proof.
- Lecturer encourages:
    - Connecting prior knowledge (midpoint properties, parallel lines, triangles)
    - Representing relationships symbolically

Scaffolding Strategies:
- Generating conceptual discussion
- Making connections
- Developing representations

Expected Outcome:
Students construct a valid deductive proof (Proof A).

Phase 3: Closing (10 minutes)

- Students reflect:
    - *What was your initial intuition?*
    - *How did your understanding change?*
- Lecturer summarizes:
    - Difference between intuition and formal proof
    - Importance of correct conceptual understanding

6. Assessment

a. Process Assessment

- Student participation in discussion
- Ability to respond to probing questions

b. Product Assessment

- Written proof evaluated based on:
    - Logical structure
    - Correct use of definitions
    - Valid conclusion

7. Indicators of Success

- Students no longer assume intersection implies midpoint
- Students correctly apply midpoint definition
- Students construct deductive proofs (Proof A)

8. Notes for Implementation

- Common student errors:
    - Misinterpreting midpoint
    - Overgeneralizing from special shapes (square/rectangle)
    - Assuming equal division from intersection
- Suggested support:
    - Use multiple diagrams (not only symmetric shapes)
    - Encourage justification, not assumption

9. Extension Activity (Optional)

- Use GeoGebra to dynamically move vertices of quadrilateral
- Observe whether PO= OR and QO= OS always hold
- Discuss why visual evidence must be supported by proof