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This issue of the *Mathematics Teaching Research Journal* represents a rigorous and thoughtfully curated contribution to the field of contemporary mathematics education. The fifteen studies brought together here do more than present empirical findings or theoretical developments; when read as a whole, they delineate a landscape of concerns that are currently central to the field: the nature of mathematical reasoning and its teaching, the persistent tension between abstraction and contextualization, the design of pedagogical experiences that foster deep understanding and meaningful participation, and the trajectories of those who inhabit the field as learners, teachers, and researchers.

Taken collectively, these contributions portray a field that is not only expanding its empirical and theoretical horizons, but also engaging in a necessary process of self-examination. Mathematics education, as it emerges from this issue, is no longer confined to questions of content delivery or instructional efficiency. Instead, it is approached as a domain in which knowledge is constructed through interaction, mediated by tools and representations, shaped by cultural and institutional conditions, and experienced in deeply situated ways. This repositioning is not trivial. It invites us to reconsider what counts as mathematical understanding, how such understanding is legitimized, and whose voices and experiences are made visible within educational practice.

In a global context where mathematics education is increasingly questioned in terms of its relevance, its capacity to generate meaning, and its role in shaping critically engaged citizens, this *issue* offers a nuanced and well-grounded account of how different research communities are responding to these challenges. Across the contributions, one can identify a sustained movement away from transmissive models of teaching toward perspectives that foreground reasoning, sense-making, and participation. Learning mathematics is thus reframed as a process that is inherently relational, mediated, and context-dependent, where cognitive development cannot be disentangled from affective, cultural, and social dimensions.

At the same time, the studies gathered here do not obscure the complexity of this shift. On the contrary, they bring into focus a set of enduring tensions that continue to shape the field. Among them, the challenge of supporting learners in moving from intuitive forms of thinking toward formal reasoning; the difficulty of reconciling the abstract nature of mathematics with the need for

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meaningful engagement; and the ongoing demand to design pedagogical environments that are both intellectually rigorous and responsive to diversity. These are not peripheral issues. They sit at the core of what it means to teach and learn mathematics today, and they require approaches that are as multifaceted as the problems themselves.

The organization of this *issue* follows an editorial logic that seeks to make these conversations visible, not as isolated themes, but as interconnected domains in which the articles engage with one another, complement one another, and, at times, unsettle established assumptions. What emerges is not a linear narrative, but a constellation of perspectives that, when read together, offer a richer and more layered understanding of the field. In this sense, the issue invites the reader to move beyond a fragmented engagement with individual studies and to consider instead how these contributions, in their diversity, collectively advance the ongoing reconfiguration of mathematics education as a dynamic, evolving, and deeply situated field of inquiry and practice.

Reasoning, Proof, and Representational Transitions

The issue opens with a set of contributions that address one of the most foundational concerns in mathematics education: the development of mathematical reasoning, particularly in relation to proof and transitions across representational registers. This focus is not merely technical; it speaks to how mathematical knowledge is constituted in the classroom and to the conditions under which students can meaningfully engage in disciplinary practices.

The article *"GeoGebra as an Epistemic Scaffold for Reasoning and Proof: Designing Instruction for Secondary Geometry Based on the Van Hiele Framework"* by Scristia, Tatang Herman, Kusnandi, and Jarnawi Afgani Dahlan offers a conceptually robust integration of the Van Hiele model, dynamic geometry technology, and instructional design. It demonstrates how tools such as GeoGebra can function as epistemic mediators, supporting students in moving from visual and intuitive forms of thinking toward more formal deductive structures, thereby addressing a well-documented gap in the literature.

Closely related to this concern, *"Exploring Pre-service Mathematics Teachers' Challenges in Understanding Intuition and Applying Scaffolding in Geometric Proofs"* by Sugi Hartono, Tatag Yuli Eko Siswono, Rooselyna Ekawati, and Farah Aisyah Nafidiastri examines the challenges that pre-service teachers encounter when attempting to reconcile intuition with formal proof. The study provides detailed evidence of the persistence of inductive reasoning in contexts that require formalization, and highlights how carefully designed scaffolding strategies can support cognitive restructuring.

The article *"Developing Adaptive Reasoning in Teacher Preparation: Elementary Pre-Service Teachers' Perspectives of Writing in Mathematics"* by Jessica M. McCormick introduces a significant shift by positioning writing as a constitutive practice of mathematical reasoning. From this perspective, adaptive reasoning is closely linked to the ability to explain, justify, and communicate mathematical ideas, with clear implications for teacher education.

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Finally, *"The Role of Realistic Mathematics Education with Scaffolding Teaching Model in Enhancing Mathematical Representation Ability"* by Rizki Wahyu Yunian Putra, Sugeng Sutiarso, and Nurhanurawati extends the discussion to mathematical representation. The study shows how the integration of Realistic Mathematics Education with scaffolding strategies strengthens students' capacity to flexibly coordinate multiple forms of representation.

Taken together, these contributions point toward an understanding of reasoning as a practice that requires deliberate pedagogical mediation, structured opportunities for exploration, and sustained engagement with multiple forms of mathematical expression.

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Context, Culture, and Meaningful Mathematization

The second set of contributions turns to the relationship between mathematics, context, and culture, addressing a longstanding tension between formalization and meaningful engagement. In recent scholarship, this tension has been increasingly examined in relation to the loss of meaning experienced by many learners in traditional mathematics classrooms.

The article *"Becoming (De)Contextualized Learners and Teachers of Mathematics: An Arts-Based Ethnodrama"* by Hem Lal Dhungana, Niroj Dahal, Indra Mani Shrestha, and Ruma Manandhar proposes a methodological approach that moves beyond conventional research paradigms. Through ethnodrama, the authors explore processes of (de)contextualization, situating mathematical learning within a cultural and narrative framework that challenges taken-for-granted assumptions.

In a complementary direction, *"Enhancing Student Engagement and Their Performance in Abstract Algebra Using Context Tasks"* by Sri Suryanti, Dwi Juniati, and Nia Wahyu Damayanti demonstrates that contextualization is not limited to early education. By focusing on abstract algebra, the study shows that context-based tasks can meaningfully impact both student engagement and performance, even in highly formal domains.

The article *"Integrating Ethnomathematics and GeoGebra: A Project-Based Learning Approach to Improve Mathematical Literacy"* by Siti Maysarah, Dian Armanto, and Sahat Saragih brings together cultural knowledge, technological tools, and project-based learning, offering an integrated approach to strengthening mathematical literacy.

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Finally, *"Integration of Real-Life Scenarios in Mathematical Literacy Learning in Middle Schools in Parepare Indonesia"* by Marwati Abd. Malik, Mas'ud B, Ammang Latifa, and Rafi'ah Nur provides empirical evidence on the positive impact of incorporating real-life scenarios into mathematics instruction, highlighting gains in both conceptual understanding and student participation.

These contributions invite a reconsideration of contextualization not as an auxiliary pedagogical strategy, but as a foundational principle that reshapes the relationship between mathematical knowledge and lived experience.

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Designing Pedagogies for Stronger Participation and Achievement

The third group of articles focuses on the design of pedagogical approaches aimed at strengthening learning, participation, and performance, while acknowledging the complexity of educational processes and the diversity of learners.

In *"Boosting Students' Abilities in Understanding, Application, and Reasoning of Numeracy: Didactical Differentiated Learning"*, Pariang Sonang Siregar, Hendra Sofyan, Nazurty, and Rosmiati presents differentiated instruction as an effective strategy for addressing classroom diversity, with positive effects across multiple dimensions of numeracy.

The article *"Comparison of the Effects of Explanatory Faded Worked Examples and Correct-Incorrect Worked Examples Methods on Sixth-Grade Students' Mathematics Performance"* by Özden Yavaş and Zeynep Çiğdem Özcan offers a comparative analysis grounded in cognitive load theory, showing how different approaches to worked examples lead to distinct learning outcomes.

In *"Transforming Confusion into Clarity: Employing the Feynman Technique to Overcome Mathematics Anxiety"*, Eric Machisi and Abdullah Kurudirek introduce a pedagogical approach that connects conceptual understanding with affective dimensions, demonstrating the importance of addressing mathematics anxiety as part of the learning process.

The article *"Enhancing Elementary Pre-service Teachers' STEM Integration Perceptions, Self-Efficacy, and Behavioral Intentions Through Experiential Learning in a Mathematics Methods Course"* by Jair J. Aguilar and Seokmin Kang extends the discussion to teacher education, showing how experiential learning influences preservice teachers' self-efficacy and their disposition toward interdisciplinary approaches such as STEM.

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This set of contributions foregrounds pedagogy as a domain of informed decision-making, where instructional design directly shapes both the quality of learning experiences and student outcomes.

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Profiles, Trajectories, and Professional Growth Across the Continuum

The issue concludes with a set of articles that broaden the perspective to include cognitive profiles, emerging competencies, and professional trajectories in mathematics education, situating learning within a broader developmental continuum.

In *"Secondary School Students' Critical Thinking Skills: A Comparison Between Impulsive And Reflective Students In Learning Geometry"*, Ari Suningsih, Siti Suprihatiningsih, Hidayatulloh, Ana Istiani, and Sulaiman examine the relationship between cognitive styles and critical thinking, offering insights into how different learners engage with mathematical tasks.

The article *"The Computational Thinking Profile of Vocational Students in Solving Mathematical Problems in Terms of Logical-Mathematical Intelligence"* by I Putu Pasek Suryawan, Gusti Ayu Mahayukti, I Gusti Putu Sudiarta, I Gusti Putu Suharta, and I Gusti Ngurah Pujawan explores the relationship between logical-mathematical intelligence and computational thinking, contributing to ongoing discussions about the competencies required in digitally mediated learning environments.

Finally, *"The Strategies behind the PhD in Mathematics Education Graduates' Professional Growth: Insights from a Graduate Tracer Study (2016–2023)"* by Ryan L. Cerveza offers an important perspective on doctoral education, demonstrating how pedagogical experiences during doctoral training shape professional trajectories and leadership in the field.

This closing set of contributions underscores the importance of understanding mathematics education as a continuous process that extends beyond the classroom, encompassing long-term development and professional formation.

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On behalf of the editorial team of the *Mathematics Teaching Research Journal*, we extend our sincere appreciation to all authors who contributed to this *issue*. Their work reflects not only methodological rigor and theoretical depth, but also a sustained commitment to advancing the field in intellectually responsible and pedagogically meaningful ways. Each contribution, in its own scope and orientation, participates in a broader effort to rethink what mathematics education is, what it does, and what it might yet become.

These studies do more than add to an existing body of knowledge. They actively expand the conceptual boundaries of the field, interrogate assumptions that have long remained unquestioned, and open new avenues for inquiry that are both timely and necessary. In doing so, they contribute to a more reflexive and critically engaged mathematics education—one that is attentive not only to how knowledge is produced, but also to how it is mediated, experienced, and transformed across contexts.

This issue ultimately invites readers to engage with mathematics education as a situated, critical, and evolving practice, one in which knowledge is continuously shaped through the interplay of theory, practice, and context. It also calls for a sustained commitment to navigating the tensions that define the field: between abstraction and meaning, rigor and accessibility, tradition and innovation. To take these tensions seriously is not to resolve them prematurely, but to recognize them as productive spaces from which more thoughtful, inclusive, and transformative approaches to mathematics education can emerge.

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GeoGebra as an Epistemic Scaffold for Reasoning and Proof: Designing Instruction for Secondary Geometry Based on the Van Hiele Framework

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Abstract: Constructing formal proofs remains a major hurdle for many prospective mathematics teachers, especially when their geometric thinking is confined to visualization and analysis levels. This study investigates how dynamic exploration with GeoGebra can support cognitive transitions across Van Hiele levels in structured reasoning and proof tasks on the parabola. Six pre-service teachers completed four GeoGebra-based tasks designed to link visual, symbolic, and deductive representations. Data from worksheets, think-aloud protocols, and interviews were analyzed thematically using the Van Hiele framework. Results show consistent development, with participants progressing from intuitive observations to formal deductive reasoning. GeoGebra features—such as sliders, measurement tools, and dynamic tracing-acted as epistemic scaffolds that enabled the formulation of conjectures and logically valid proofs. Based on these findings, a two-phase instructional model, Reasoning and Proof Learning (RPL), was developed, comprising Exploration and Representation, and Justification and Proof phases. This model was translated into a practical teaching plan for secondary classrooms, promoting geometric reasoning and proof habits. The study offers both theoretical and instructional contributions to mathematics teacher education and technology-enhanced geometry learning.

Keywords: dynamic geometry software; geometric thinking; pre-service mathematics teachers; reasoning and proof; van Hiele model

INTRODUCTION

Geometry is a fundamental domain in mathematics that supports the development of spatial reasoning, logical structuring, and the ability to construct deductive arguments. Within this field, geometric thinking and mathematical reasoning particularly in the context of proof construction are widely recognized as core competencies for mathematics learners (G. Stylianides et al., 2017); The National Council of Teachers of Mathematics, 2000). These competencies are essential not only for mastering mathematics as a discipline but also for fostering the higher-order thinking skills required in 21st-century learning. Despite longstanding curricular emphasis on proof, achieving advanced levels of geometric thinking remains a persistent challenge for both students and teachers.

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Empirical evidence shows that prospective mathematics teachers often struggle to reach higher levels of geometric thinking as described by the Van Hiele model, especially in constructing formal deductive proofs. According to (Van Hiele, 1999), geometric thinking progresses hierarchically from visualization to analysis, informal deduction, formal deduction, and ultimately rigor. However, many pre-service teachers operate only at the lower levels of this hierarchy. Studies in Indonesia by by Armah, (2024); Mawarsari et al., (2023); Naufal et al., (2021) revealed that most pre-service teachers remained at the visualization or analysis levels and were unable to construct formal geometric proofs. These findings are supported by Scristia et al., (2025), who reported that only a small number of students achieved the formal deduction level, indicating a significant gap in teacher education programs.

These trends align with recent international studies. (Haj Yahya & Hershkowitz, 2024) found that many pre-service teachers struggled with constructing proofs due to reliance on prototypical representations and weak logical structuring. González et al., (2024) showed that most mathematics education students operated at the informal deduction level. (Birgin & Özkan, 2024) highlighted limited abilities in hierarchical classification and deductive reasoning in plane geometry tasks. Haj-Yahya & Olsher, (2025) emphasized the need for explicit scaffolding, while Ødegaard et al., (2024) pointed to failures in transferring empirical reasoning into formal deductive arguments. Anwar et al., (2025) further emphasized these challenges, identifying three major obstacles faced by pre-service teachers when formulating conjectures in dynamic geometry environments: (1) limited proficiency with GeoGebra, (2) difficulties in identifying robust invariants, and (3) formulating conjectures without strong visual or empirical foundations. These findings suggest that while technology can open new opportunities for reasoning, its effectiveness depends heavily on how it is pedagogically integrated to support cognitive transitions toward proof construction.

In response to these challenges, scholars have advocated for the explicit integration of reasoning and proof tasks into geometry instruction. Such tasks provide structured opportunities for learners to progress through the Van Hiele levels via conjecturing, justification, and argumentation (Buchbinder & Mccrone, 2020; Hanna & de Villiers, 2021). In this context, proof is not merely a tool for verification but a dynamic process of constructing and communicating logical relationships (Ellis et al., 2012; A. J. Stylianides, 2007). Actively constructing arguments helps learners deepen their understanding of geometric structures and the principles underlying them. Aligned with this pedagogical shift is the growing adoption of Dynamic Geometry Software (DGS), such as GeoGebra, in mathematics education. DGS enables learners to manipulate geometric objects in real time, explore invariants, and formulate conjectures through visual feedback (Laborde, 2002). Several studies have shown that DGS enhances conceptual understanding and spatial reasoning (Martinovic & Manizade, 2020; Yohannes & Chen, 2021). However, most existing literature focuses on the visual and exploratory functions of DGS, with limited research exploring how such environments can be strategically used to support transitions from informal reasoning to formal deductive proof, especially among pre-service teachers (Galili et al., 2023; Hartono et al., 2024; Komatsu & Jones, 2020).

A notable study by Mariotti & Pedemonte, (2019) demonstrated that interaction with DGS allows learners to connect spatial intuition with formal logical structures through activities such as

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conjecture formulation and formal proof validation. Their findings emphasize that a "need for proof" emerges as a key component in students' mathematical reasoning processes. In other words, DGS not only helps learners *see* geometry but also fosters epistemological awareness and motivation to justify observations through deductive arguments (Thapa et al., 2022).

Although the potential of Dynamic Geometry Software (DGS) for enhancing conceptual understanding and spatial reasoning is well established, few empirical studies have examined how such environments can be systematically designed to develop formal deductive reasoning, especially among pre-service mathematics teachers. Previous research often treated DGS mainly as a visualization or exploration tool, without deeply analyzing learners' progression through the Van Hiele levels or how DGS can serve as an epistemic bridge between intuition and formal proof. This study investigates how DGS-based tasks influence the reasoning pathways and engagement of prospective mathematics teachers as they construct geometric proofs, particularly during their transitions across Van Hiele levels. The research contributes to both the theoretical understanding of deductive reasoning in digital environments and the identification of pedagogical strategies that integrate visual, symbolic, and deductive representations in geometry learning. The empirical findings inform the development of an initial instructional framework grounded in GeoGebra-based activities, designed to strengthen pre-service teachers' reasoning and proof competencies. By engaging pre-service teachers as active participants in dynamic exploration, this study provides an empirical foundation for effective teaching practices that can be applied by both mathematics educators and classroom teachers. Consequently, the results are expected to have direct implications for technology-enhanced geometry instruction and to inform curriculum development in teacher education programs emphasizing reasoning and proof.

Research Problem and Questions

Although DGS is increasingly accessible in mathematics education, many pre-service mathematics teachers still struggle to construct formal geometric reasoning and proof, especially when working with abstract concepts such as conic sections. Among these, the parabola stands out as a foundational topic that integrates visual, symbolic, and deductive reasoning. Effectively guiding learners through cognitive transitions among these representations requires well-designed instructional interventions that not only support exploration but also foster conceptual understanding and formal justification. This study aims to examine how dynamic exploration using GeoGebra can support the development of geometric reasoning among pre-service mathematics teachers, particularly in reasoning and proof tasks involving the properties of the parabola. To address this overarching aim, the following research questions are posed:

1. How does dynamic exploration within a GeoGebra-based environment support the development of pre-service mathematics teachers' geometric thinking, particularly in constructing reasoning and proof related to the properties of the parabola?
2. What features of technology-integrated tasks facilitate cognitive transitions through the Van Hiele levels among pre-service teachers?

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3. How do pre-service teachers utilize GeoGebra to move from visual exploration to formal justification in geometric proof construction?
4. How can instructional designs that incorporate DGS be structured to bridge visual, symbolic, and deductive representations in teaching and learning the concept of the parabola?

LITERATURE REVIEW

Geometric Thinking and Van Hiele Model

The Van Hiele model (Van Hiele, 1999) describes how learners' geometric thinking progresses through hierarchical stages: visualization, analysis, informal deduction, and formal deduction. This framework remains a cornerstone in mathematics education research and curriculum design (Clements & Battista, 1992; Usiskin, 1982). However, studies show that many pre-service teachers still operate at lower levels and struggle to construct formal proofs (Birgin & Özkan, 2024; González et al., 2024b). Recent international research emphasizes the need for instructional scaffolding to help learners transition between these levels (Haj-Yahya & Olsher, 2025; Ødegaard et al., 2024). Importantly, Van Hiele levels are not determined by age or grade (Škrbec & Čadež, 2015; Van Hiele, 1999). In this study, four levels were considered: (1) Visualization; (2) Analysis; (3) Informal Deduction; and (4) Formal Deduction. These levels collectively represent the developmental pathway through which students advance from perceptual understanding to rigorous deductive reasoning in geometry.

Reasoning and Proof in Mathematics Education

Reasoning is fundamentally intertwined with proof at all levels of mathematics education, serving as the process through which learners explore patterns, form conjectures, and validate them through logical argumentation (Lesseig et al., 2019; Stylianides G, 2008). Proof provides the logical structure needed to confirm or refute conjectures, ensuring the coherence and validity of mathematical reasoning (Stylianides G, 2008; Hanna & de Villiers, 2021). As emphasized by Buchbinder & McCrone, (2022); Ellis et al., (2012); The National Council of Teachers of Mathematics, (2000), this continuum of exploration, conjecturing, and proving defines what is known as *reasoning and proof*. Stylianides G, (2008) identifies reasoning and proof as inquiry-based activities involving (1) identifying patterns, (2) forming conjectures, (3) constructing logical arguments to test validity, and (4) developing formal proofs. Similarly, Thompson et al. (2012) highlight key indicators such as generating conjectures, evaluating arguments, identifying counterexamples, and correcting flawed proofs. Ellis et al., (2012) and Jeannotte & Kieran, (2017) expand this to include exploration, generalization, justification, and critical evaluation. Synthesizing these perspectives, Utari & Hartono, (2019) propose six operational indicators: (1) generating patterns to form conjectures, (2) justifying conjectures, (3) constructing proofs, (4) evaluating argument correctness, (5) finding counterexamples, and (6) revising flawed proofs. More recently, Buchbinder & McCrone, (2022) and Alsina et al., (2024) reaffirm reasoning and proof as a cyclical process

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encompassing exploration, pattern observation, conjecture formation, generalization, justification, and verification ultimately fostering the development of mathematical thinking.

Dynamic Geometry Software (DGS) as Epistemic Mediator

Dynamic Geometry Software (DGS), especially GeoGebra, allows learners to manipulate objects and visualize invariant relationships in real time. Laborde, (2002) conceptualized DGS as more than a visualization tool, it mediates mathematical reasoning by encouraging students to explore and refine their ideas. Mariotti & Pedemonte, (2019) found that interaction with DGS fosters a natural progression from empirical conjecture to formal proof, provided the tasks are epistemologically well-structured. Santos-Trigo et al., (2024) confirmed this potential by showing how DGS environments promote deeper reasoning by enabling learners to construct and test their own proofs, particularly when supported by timely teacher questioning. DGS can scaffold critical reasoning behaviors, such as conjecturing, validating, and generalizing that are foundational for proof. The integration of DGS into teacher education has been shown to not only improve conceptual understanding but also shift learners' epistemological views about mathematics. (Komatsu & Jones, 2020) argue that DGS can bridge the gap between empirical observation and deductive rigor when paired with structured reflection. (Quaresma & Janičić, 2006) extend this argument by emphasizing that exploratory environments must prompt the "need for proof," turning visual patterns into formal argumentation. Studies Gulkilik, (2023); Rizos & Gkrekas, (2023); Santos-Trigo et al., (2024) underscore that student-driven constructions using DGS lead to not only better geometric intuition, but more robust proof skills.

METHOD

All procedures and instruments in this study were strategically designed to address the four research questions that had been formulated. The analysis of participants' geometric thinking development through GeoGebra-based exploratory tasks was used to answer the first and second questions (RQ1 and RQ2), which focused on reasoning construction and cognitive transitions across Van Hiele levels. Meanwhile, verbal explorations captured through think-aloud protocols and written proofs from tasks and interviews were employed to examine the transition from visual representations to deductive justification (RQ3). The findings from this process served as the foundation for designing a two-phase GeoGebra-based instructional model that integrates visual, symbolic, and deductive representations of the parabola concept (RQ4).

Research Design and Approach

This study adopted a qualitative approach within the framework of design-based research to explore and construct a reasoning and proof-based instructional design (Reasoning and Proof Learning/RPL) using GeoGebra. This approach allowed for the iterative design, implementation, and

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reflection of instructional activities in a real classroom context, providing both theoretical insights and practical contributions.

Participants, Instruments and Indicators

Six pre-service mathematics teachers from a mathematics education program at a public university in Indonesia were purposively selected. Selection criteria included comparable academic backgrounds and prior experience with GeoGebra. All participants engaged in the full sequence of instructional tasks and provided written informed consent. The study received ethical approval from the university's ethics committee. Table 1 summarizes the instruments, indicators, and descriptors used in the study, which were developed based on theoretical frameworks such as Van Hiele's levels of geometric thinking, reasoning and proof, and dynamic geometry-based exploration.

Instruments	Indicators
Task-Based Worksheets (Van Hiele (1999); Stylianides (2007); Laborde (2002)	Van Hiele Geometric Thinking Levels
	Reasoning Structure (Formulating conjectures and justifications through dynamic observation)
	Formal Deductive Argument (Constructing proofs using definitions and theorems)
Think-Aloud Protocols (Ellis et al., 2012; G. Stylianides et al., 2013)	Real-Time Reasoning (Revealing logical thought processes during DGS-based exploration)
	Type of Reasoning (Transitioning from intuitive ideas to formal justifications)
Interview Guidelines (Buchbinder & McCrone, 2022))	Reflective Reasoning (Connecting visual representations to formal geometric concepts)
	Argumentation Quality (Constructing and evaluating logical justifications)
Observation Field Notes (Martinovic & Manizade, 2020; Santos-Trigo et al., 2024)	Engagement in Dynamic Exploration (Actively manipulating DGS tools independently)
	Independent vs. Scaffolded Reasoning (Identifying moments requiring external prompts or scaffolding)

Table 1: Indicators and Descriptors of Research Instruments Based on Theoretical Frameworks

Instructional Tasks and Learning Intervention

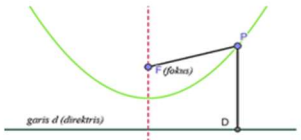
The intervention was conducted over three weeks and consisted of four instructional tasks, each designed to facilitate specific cognitive transitions in geometric thinking. Table 2 presents the topic, cognitive focus, and targeted Van Hiele level for each task.

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Task/Topic	Cognitive Focus	Targeted Level
Task 1: Definition of parabola (focus and directrix)	Foundational properties	Level 1 → Level 2
Task 2: Dynamic construction of a parabola from its focus and directrix	Invariance and symmetry	Level 2 → Level 3
Task 3: Exploring vertex form $y=a(x-h)^2+k$	Symbolic conjecturing	Level 3 → Level 4
Task 4: Formal proof construction	Deductive justification	Level4

Table 2: Task Descriptions and Targeted Van Hiele Level

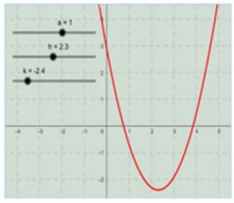
Each of these tasks was purposefully designed not only to introduce geometric content, but to provoke cognitive movement through the Van Hiele levels, from initial visualization to formal deduction. In particular, tasks 1 and 2 aim to activate spatial recognition and analytical awareness, while tasks 3 and 4 challenge students to formulate and validate conjectures using definitions and theorems, ultimately constructing structured formal proofs. This progression aligns with the research aim to trace how dynamic exploration mediates the shift from intuitive understanding to deductive argumentation.



Definition:
The locus definition of a parabola is the set of all points that are equidistant from a fixed point called the focus and a line called the directrix.

Task 1: (a) Point F is known as the focus of the parabola. What is the name of the line that passes through point F and is perpendicular to the directrix? (b) Line d is the directrix of the parabola. What is the relationship between line d and the dashed line passing through point F?
Task 2: (a) Drag point P along the shape of the parabola. How does the distance from point P to the focus compare to its distance to the directrix? What can you conclude from your observation? (b) Move the focus point (F) to different positions on the screen. Does the property you observed in Question 3 still apply? Explain your reasoning.

Figure 2: Task 1 and Task 2- Focus and Directrix



Task 3: Open the GeoGebra document titled "BENTUK PUNCAK PARABOLA", which displays the graph of a parabola in vertex form: $y = a(x - h)^2 + k$, featuring interactive sliders for the parameters a, h, and k that allow you to dynamically view the changes in the parabola's graph as the values are adjusted.
The vertex form of the parabola equation $y = a(x - h)^2 + k$ is displayed. Move the sliders to change the values of a, h, and k. Explain how changes in each parameter a, h, and k affect the shape and position of the graph.

Figure 3: Task 3- Exploring Vertex Form

Given the focus (m, n) the directrix y = d and a point P(x, y) on the parabola, use the distance formula to derive the general equation of the parabola.
Use the fact that the distance from point P to the focus is equal to its distance to the directrix. Set up an equation for the two distances and solve to find the value of y.
Using the vertex form of the parabola, $y = a(x - h)^2 + k$, and the algebraic relationship derived in Task 4, answer the following questions:

1. Explain the relationship between the focus, directrix, and the value of a.
2. What happens to the shape of the parabola as a increases? How does this relate to the distance between the focus and the directrix?"

Figure 4: Task 4 - From visual exploration to formal justification

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The instructional tasks in this study were deliberately structured to support a progressive development of deductive reasoning, guiding pre-service teachers from initial visual recognition of geometric properties to the formulation of conjectures and, ultimately, to the construction of formal deductive arguments. Each task was designed to build upon the reasoning outcomes of the previous one, thereby creating a scaffolded sequence of cognitive engagement.

These tasks functioned as the primary mechanism for addressing the study's central research question: how do pre-service mathematics teachers transition from intuitive to formal reasoning in a dynamic geometry environment? The instructional design integrated real-time dynamic manipulation via GeoGebra with structured opportunities for reflection and verbal articulation, enabling participants to externalize and evolve their reasoning processes.

Over a three-week intervention period, participants completed four sequenced GeoGebra-based tasks that required them to manipulate geometric constructions, formulate and test conjectures, and construct formal justifications. Each task was followed by a semi-structured reflective interview aimed at probing the nature and depth of participants' evolving reasoning, allowing the research team to trace their progression toward formal proof construction.

Data Collection and Analytical Focus

Data were gathered through task-based worksheets, think-aloud protocols, semi-structured interviews, classroom observations, and students' written artifacts. Each instrument was purposefully chosen to capture complementary aspects of participants' cognitive progression from intuitive reasoning to formal deductive argumentation. Think-aloud sessions allowed real-time tracing of reasoning as participants manipulated dynamic figures in DGS, revealing their transition from visual intuition to logical justification. Observation notes complemented this by recording behavioral indicators such as autonomy, engagement, and use of scaffolding. Following each task, stimulated-recall interviews explored how participants interpreted visual experiences and connected them to geometric principles, assessing the coherence and metacognitive depth of their reasoning. Written artifacts including conjectures, constructions, and proofs, provided concrete evidence of conceptual and logical development while triangulating verbal and behavioral data for analytical rigor. Assessment was integrated throughout learning. Informal assessment occurred during exploration through observation and feedback, while formal assessment within tasks required students to formulate conjectures, justify relationships, and construct deductive arguments. This structure enabled systematic evaluation of reasoning and progression across Van Hiele levels.

Data Analysis and Trustworthiness

All data were transcribed verbatim and organized using a case-based structure, treating each participant as a distinct unit of analysis. The main analytical focus was to trace individual shifts in representation, verbal justification, and deductive structure aligned with the Van Hiele levels of geometric thinking. The analysis followed Matthew Miles et al., (2014), consisting of three recursive phases: (1) data condensation: selecting, coding, and categorizing meaningful episodes from

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worksheets, interviews, and verbal protocols using deductive codes grounded in reasoning and proof theory; (2) data display: developing matrices and narrative timelines to visualize participants' reasoning progression; and (3) conclusion drawing and verification: identifying cognitive shifts and how dynamic exploration mediated transitions from informal reasoning to formal proof. This multi-layered analysis revealed how pre-service teachers evolved from visual intuition to logical deduction in geometry, directly addressing the research questions. To ensure rigor, methodological triangulation was applied across four instruments (worksheets, think-aloud protocols, interviews, and observations) and three data types (verbal, written, behavioral), enhancing validity and providing rich, cross-validated insights into learners' reasoning within the dynamic environment.

To trace reasoning development, an analytic rubric based on the Van Hiele model was used, delineating four hierarchical levels characterized by distinct cognitive behaviors and performance indicators. The rubric served as a deductive coding framework to map participants' reasoning shifts particularly transitions from visual to formal deductive thinking throughout the intervention. It was adapted from established interpretations of the Van Hiele model (Clements & Battista, 1992; Fuys, 1988; Usiskin, 1982; Van Hiele, 1999) to align with the indicators of each GeoGebra-based task in this study.

Level	Description of Cognitive Behavior	Task Performance Indicators
Visualization (V)	The student recognizes geometric shapes based on their global appearance without considering formal properties or definitions	Identifies a parabola solely by its shape ("Looks like an arch"), without mentioning properties or geometric relationships
Analysis (A)	The student mentions parts or properties of geometric figures (e.g., symmetry, vertex) but does not relate them logically	Mentions focus, directrix, axis of symmetry without constructing arguments or deductive connections between elements
Informal Deduction (ID)	The student formulates conjectures based on visual observations, using geometric terms intuitively but without formal proof	States hypotheses like "The distance to the focus and the line seems equal," but does not justify using definitions or theorems
Formal Deduction (FD)	The student proves conjectures using formal definitions, theorems, or deductive logic in a valid and structured manner	Constructs a proof that the distance from a point to the focus and to the line is equal based on the definition of a parabola

Table 3: Analytical Rubric for Tracing Cognitive Shifts Across Van Hiele Levels in Dynamic Geometry Tasks

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Expert Validation of Instruments and Task Design

Expert validation was conducted prior to implementation to ensure theoretical alignment, clarity, and relevance of the instruments and tasks. Three experts, two mathematics education lecturers specializing in geometry and reasoning and proof, and one instructional technology expert reviewed the task-based worksheets, Van Hiele-based indicators, interview and think-aloud protocols, and observation rubrics.

The validation focused on alignment with learning objectives, conceptual accuracy, appropriateness of difficulty and language, potential to elicit reasoning and deductive processes, and instructional relevance. All four structured parabola tasks met all criteria (Table 4).

Task	Learning Objectives	Concept Alignment	Difficulty & Precision	Reasoning & Deduction Potential	Instructional Relevance
Task 1	√	√	√	√	√
Task 2	√	√	√	√	√
Task 3	√	√	√	√	√
Task 4	√	√	√	√	√

Table 4: Expert Validation Checklist of Geometry Tasks

Quantitative validation using eight indicators on a 5-point Likert scale yielded an overall mean score of 4.02, classified as *Very Valid* (Qohar et al., 2021) (Table 5). Expert feedback confirmed that the tasks effectively promoted conjecturing, justification, and deductive reasoning, with GeoGebra supporting exploratory reasoning prior to formal proof. Minor revisions were made to refine descriptors and improve instrument clarity, ensuring strong content and construct validity.

No	Indicator	Average	Category
1	Clarity of task instructions	4.00	Very Valid
2	Alignment with learning objectives on reasoning and proof	4.22	Very Valid
3	Relevance to geometric thinking levels (Van Hiele framework)	4.17	Very Valid
4	Integration with Dynamic Geometry Software (GeoGebra)	4.00	Very Valid
5	Appropriateness of difficulty level for pre-service teachers	3.78	Valid
6	Stimulation of conjecture formulation and justification	4.11	Very Valid
7	Potential to elicit deductive reasoning processes	3.89	Valid
8	Language accuracy and mathematical precision	3.94	Valid
	Overall Average Score	4.02	Very Valid

Table 5: Validation of Task-Based Worksheets Instrument

RESULT

RQ1: Supporting Geometric Thinking through Dynamic Exploration

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This study found that dynamic exploration using GeoGebra significantly supported the development of geometric thinking among pre-service mathematics teachers, particularly in constructing reasoning and proof related to the properties of the parabola. As participants engaged in four structured tasks, they transitioned from intuitive and visual observations to more analytical and deductive reasoning. Their cognitive development was evaluated using the Van Hiele model, focusing on shifts in levels of reasoning.

Figure 5 illustrates that most participants progressed from lower Van Hiele levels such as visualization and analysis toward informal or formal deduction. For instance, Participant P1 advanced from the analysis level to informal deduction after using GeoGebra's measurement tool to verify the focus–directrix property by comparing distances and recognizing consistent patterns. He explained, *"I knew the definition, but seeing the distances actually equal, then measuring and trying again helped me believe it. That's when I realized I could explain it properly."* This reflects a shift from observation to structured justification—typical of movement from Van Hiele Level 2 to Level 3. Participant P2 moved from visualization to analysis while exploring symmetry using the slider parameter h in the vertex form $y = a(x - h)^2 + k$. Initially focused on movement, he later generalized: *"When I move h, the center line also moves. It's always at x = h. That must be the axis."* In a later interview, he reflected, *"At first, I just followed the movement, but then I realized I could generalize it and explain why it happens."* This shows growing analytical understanding.

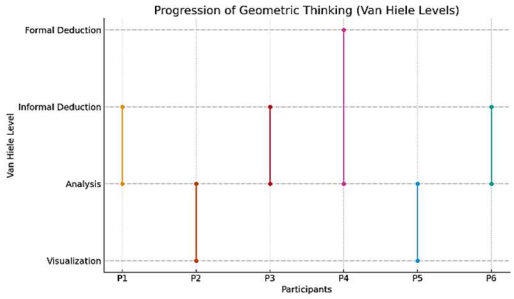


Figure 5: Van Hiele Progression of Participants

Figure 5. Each line represents a participant's progression from their initial to final Van Hiele level after the intervention.

Participant P4 exhibited the most significant improvement, reaching formal deduction by Task 4 through coordinate-based proof construction. He stated, *"Now I understand why the parabola has that shape."* Using the Euclidean distance formula, he derived the general form of the parabola and verified it algebraically: *"I set the point P(x, y), the focus F(m, n), and the directrix y = d... After simplifying, the equation matched what I had in GeoGebra."* This demonstrates full mastery at Van Hiele Level 4. Participant P6 showed meaningful growth from analysis to informal deduction. During the think-aloud protocol, he repeatedly measured distances, stating, *"Every time the point moves, the two distances stay the same. That's why it's on the parabola it has to be."* This indicates reasoning supported by pattern-based justification.

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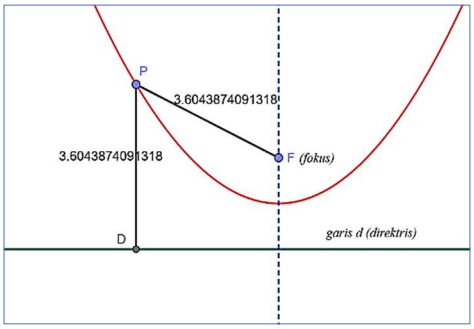


Figure 6: Participant P4's GeoGebra construction exploring the parabola's equidistance property

Finally, Participant P5, although remaining at the analysis level, showed conceptual improvement. Initially struggling with parameter interpretation, by Task 3 he explained, *"When a gets bigger, the curve is tighter because the focus and directrix get closer."* This reflects increased symbolic awareness despite not reaching formal proof.

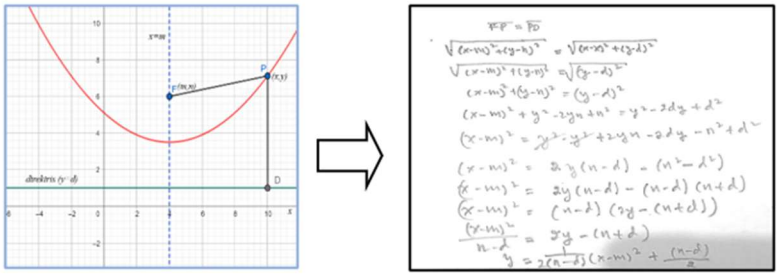


Figure 7: P4 used GeoGebra measurements to demonstrate that $\overline{FP}$=$\overline{PD}$

Overall, five of six participants advanced in their Van Hiele levels, from visualization or analysis to informal deduction, and one (P4) reaching formal deduction showing that GeoGebra-based dynamic exploration effectively supported their cognitive progression in geometric reasoning and proof.

RQ2: Technology-Based Task Features that Facilitate Van Hiele Transitions

The analysis revealed that certain GeoGebra features interactive visual manipulation, measurement tools, sliders, and dual algebraic graphical displays were instrumental in supporting participants' transitions across Van Hiele levels. These tools helped learners connect geometric objects, dynamic patterns, and symbolic representations, fostering movement from intuition to structured reasoning.

Task 1 introduced the parabola's geometric form through point plotting and tracing, which promoted initial visual awareness. Participant P5 reflected, *"At first, I just followed the curve without thinking, but when the path started showing a pattern, I wondered why it behaved that way."*

Task 2 emphasized symmetry exploration through the vertex form $y = a(x – h)^2 + k$. Using sliders, Participant P2 observed, *"When I move the slider, the center line also moves. It's always at $x = h$.*

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That must be the axis." Later he added, *"At first, I was just watching the movement, but then I realized I could explain it with the equation."* This progression indicates a shift from visualization to analysis.

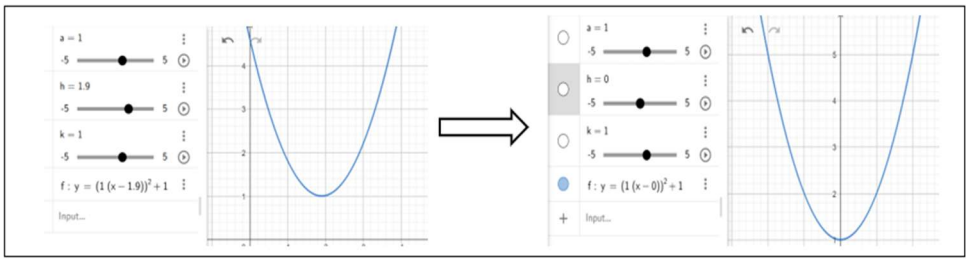


Figure 8: Dynamic Exploration of Symmetry in Vertex Form of a Parabola

Figure 8 showcases the dynamic manipulation of the vertex form of a parabola using sliders in GeoGebra. As seen, participants adjusted the value of h and observed consistent alignment of the axis of symmetry at $x = h$, fostering a connection between symbolic representation and geometric structure. Such features particularly real-time algebraic-visual interaction supported participants like P2 and P5 in recognizing mathematical invariants, even before full deductive proof was constructed.

Tasks 3 and 4 focused on the parabola's equidistance property using measurement and trace tools. Participant P1 stated, *"After measuring again and again, the distances are always equal. That's when I believed it wasn't just coincidence,"* showing the development of informal deduction. Similarly, Participant P6 shared, *"When the trace appeared, I knew that every point followed the same rule,"* reflecting a transition from intuition to rule-based reasoning. Participants responded differently to features. Trace and measurement tools encouraged pattern generalization, while sliders required reflection to deepen understanding. Participant P3, for example, initially changed parameters without clear meaning but later realized, *"Now I see that a doesn't just stretch the curve-it changes the focus and the curve at once."*

Table 7 summarizes how different GeoGebra features, linked to specific tasks, supported reasoning development across participants. Five out of six participants progressed through at least one Van Hiele level, with varied pathways depending on their interaction with technological features and reflective opportunities.

Participant	InitialLevel	Final Level	Related Task	Observed Reasoning Development
P1	A	ID	Task 3	Moved from observation to structured justification
P2	V	A	Task 2	Began linking visual changes to symbolic structure
P3	A	ID	Task 2–3	Developed language of informal justification
P4	A	FD	Task 4	Produced formal argument using definition/theorem
P5	V	A	Task 3	Improved understanding of function-graph relationships
P6	A	ID	Task 3	Transitioned from intuitive to rule-based explanation

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Table 7: Van Hiele Progressions, GeoGebra Features, and Reasoning Outcomes

These findings highlight that effective technology-based tasks must align with learners' cognitive stages. Dynamic features should be sequenced intentionally to scaffold reasoning transitions—from visualization, to identifying relationships, to constructing logical arguments. Furthermore, guided reflection and teacher scaffolding are essential for helping learners transform empirical observations into formal mathematical justifications.

RQ3: Transitioning from Visual Exploration to Formal Justification through DGS

The analysis showed a clear shift from intuitive visual exploration to structured deductive reasoning facilitated by Dynamic Geometry Software (DGS). GeoGebra enabled participants to dynamically interact with geometric objects, bridging empirical observation and formal proof.

Initially, participants relied on visual cues such as symmetry and trace patterns. Participant P1 moved from observation to informal deduction, explaining: *"At first, I just knew the definition, but after I saw the distances were always the same, then I measured and tried again, I started to believe it. That's when I felt I could explain why it works."* Participant P4 showed the most significant progress, achieving formal deduction in Task 4. He algebraically proved the equidistance property of the parabola using coordinate-based reasoning: *"Now I understand why the parabola has that shape."* He derived the vertex form from the distance formula, connecting visual intuition to symbolic proof.

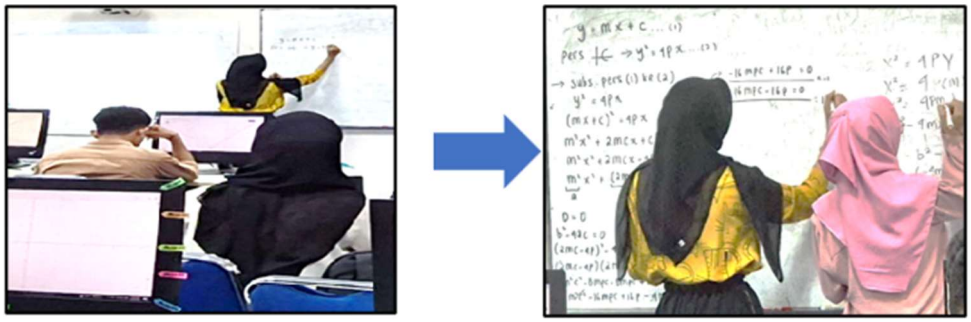


Figure 9: Transition from Dynamic Visualization to Formal Deductive Proof (*P4 and a peer moving from computer-based exploration to collaborative proof construction*)

A follow-up interview with P4 further demonstrated his conceptual integration of visual and algebraic understanding:

"After trying several times in GeoGebra, I saw that when 'a' increases, the parabola becomes narrower. Measuring the vertex-to-focus and vertex-to-directrix distances showed they get shorter because a = 1/4p". "Using the distance formula, I matched the vertex form to the geometric definition. It made the proof feel logical, not just memorized."

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Participant P6 also transitioned from intuition to generalization: *"Every time the point moves, the two distances stay the same. That's why it's on the parabola."* Participant P3 stated, *"I didn't just try; I wanted to prove it works."*

Participant P2 reflected, *"The diagrams helped me connect the movement to the formula".* Meanwhile, Participant P5, though remaining at the analysis level, reported deeper symbolic understanding: *"The real-time changes helped me think deeper."*

Collectively, these findings confirm that DGS not only promotes pattern recognition but also transforms empirical exploration into formal justification. Participants evolved from passive observers to active justifiers of geometric truths, demonstrating how dynamic visual interaction can foster symbolic and logical reasoning.

RQ4: Designing an Instructional Framework to Bridge Visual, Symbolic, and Deductive Representations in the Concept of Parabola

Building on the findings from RQ1–RQ3, an instructional framework was designed to integrate dynamic GeoGebra tasks with reasoning and proof activities. The framework aimed to help preservice mathematics teachers transition smoothly between visual, symbolic, and deductive representations of the parabola while addressing common learning challenges such as limited GeoGebra proficiency, difficulty distinguishing invariants, and reliance on memorized theory (Anwar et al., 2025).

Phase 1: Dynamic Exploration and Symbolic Representation

This phase focused on developing geometric understanding by dynamically manipulating the focus, directrix, and vertex using GeoGebra. The activities aimed to (1) build a visual understanding of the parabola as a locus of points equidistant from the focus and directrix, and (2) connect this concept to its symbolic representation, particularly the vertex form. Participants used sliders to adjust parameters and observed how graph changes reflected algebraic relationships. Participant P3 noted, *"When I moved the focus closer to the vertex, the graph became steeper, and the a value increased. That's when I realized that the distance actually affects the equation."* This phase strengthened the coordination between visual and symbolic reasoning.

Phase 2: From Symbolic Patterns to Deductive Reasoning

In the second phase, participants engaged in structured proof tasks-deriving the parabola's general equation from its geometric definition using the distance formula and constructing algebraic justifications. Participant P4 shared, *"I compared the lengths FP and PD, then simplified it to get the vertex form. That's when I really understood-it wasn't just told to me, I discovered it myself."*

As shown in Figure 9, learners transitioned from digital exploration to collaborative proof construction, linking symbolic reasoning with formal deduction.

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Transition	Learning Activity	Van Hiele Level Transition	Instructional Support	Evidence of Transition
Visual → Symbolic	Manipulating focus, directrix, and vertex using GeoGebra sliders	Level 1 → Level 2	Real-time algebraic feedback, guided questions	Students linked visual changes to the vertex form $y = a(x–h)^2 + k$, e.g., "As I moved the focus closer to the vertex, the curve became sharper."
Symbolic → Deductive	Constructing general parabola equation using distance formula	Level 3 → Level 4	Structured proof task, visual-symbolic scaffolding	Students justified the vertex form algebraically: "I proved it myself, step by step using the distance formula."
Full Integration	Project-based modeling (e.g., water fountain, projectile motion)	Across Levels 2-4	Open-ended tasks, collaborative discussion	Students integrated visual models, symbolic equations, and logical arguments coherently to explain real phenomena.

Table 8: Representational Transitions Supported by the Instructional Design in Parabola

Collectively, the instructional design promoted representational flexibility and addressed key challenges in learning reasoning and proof. The integration of GeoGebra, structured guidance, and open-ended projects empowered pre-service teachers to construct and validate their understanding of parabolic concepts through connected visual, symbolic, and deductive reasoning.

Instructional Design Model for Reasoning and Proof Learning

Findings from classroom observations, task analyses, and student interviews revealed a consistent pattern of cognitive growth supported by a systematically structured instructional approach. As pre-service mathematics teachers engaged with GeoGebra-based tasks, their reasoning evolved progressively across visual, symbolic, and deductive representations in a clear and traceable sequence.

From this evidence, a two-phase data-driven model Reasoning and Proof Learning (RPL) model was developed to represent how participants navigated the reasoning cycle during instruction. *Phase 1: Exploration and Representation*, this phase involved dynamic interaction with visual aspects of the parabola using GeoGebra and the gradual articulation of symbolic relationships. Manipulation of parameters, algebraic interpretation, and reflective dialogue were key indicators of reasoning growth. *Phase 2: Justification and Proof*, in this phase, participants constructed algebraic arguments to validate independently formed conjectures. Data from worksheets,

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whiteboard collaborations, and group discussions showed clear transitions from symbolic reasoning to formal deduction, grounded in definitions and structured proof strategies.

These two phases were identified inductively from empirical classroom data rather than imposed by prior theory. Hence, the RPL model is not a theoretical abstraction but a grounded, evidence-based framework reflecting how learners authentically experienced reasoning and proof within a technology-enhanced environment.

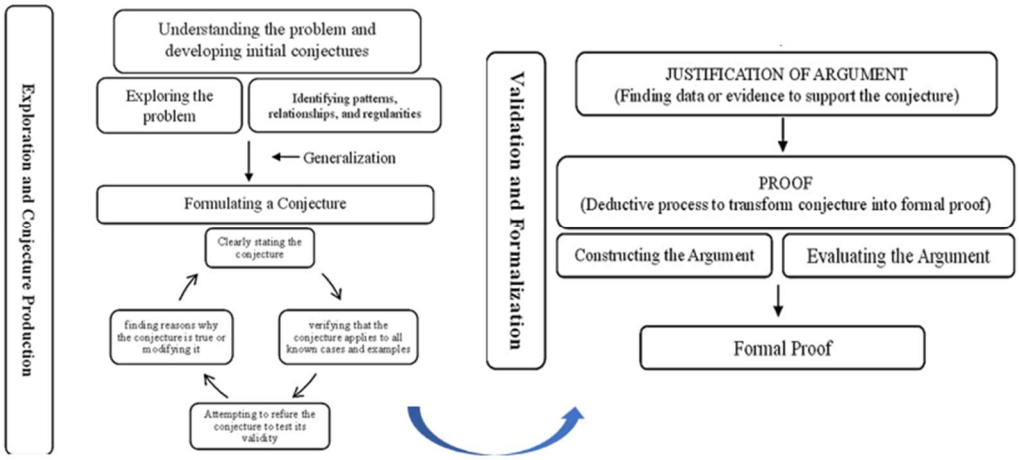


Figure 10: Conceptual Framework of the Reasoning and Proof Process

This visualization highlights how the instructional design effectively supported both the formulation of conjectures and the validation of mathematical arguments, capturing authentic learning trajectories observed throughout the study.

Implementation of the Instructional Design: A Lesson Plan for Secondary School Teachers

Drawing from the instructional design research, a practical lesson plan was developed based on the Reasoning and Proof Learning (RPL) model to help secondary school mathematics teachers bridge visual, symbolic, and deductive representations in teaching the parabola. The plan maintains the pedagogical integrity of the RPL model while ensuring classroom applicability.

The lesson plan was structured across two 120-minute sessions, each corresponding to one of the two main phases of the model: (1) *Exploration and Representation*, and (2) *Justification and Proof*. The first phase focused on dynamic visual exploration using GeoGebra, where students manipulated the focus, directrix, and vertex of a parabola to construct its shape and observe real-time algebraic feedback. The second phase emphasized the construction of formal justifications and proofs derived from the geometric definition of a parabola, using the distance formula as the basis for algebraic reasoning. Collaborative group work, discussion, and whiteboard presentations were incorporated to support deductive thinking.

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Developed through synthesis of classroom observations, task analyses, and student interviews, this plan is both flexible for teaching and grounded in cognitive transitions identified during the study.

Phase	Learning Objectives	Core Activities	Tools and Media	Expected Representational Transition
1	Construct a parabola as a locus of points- Connect visual and symbolic forms (vertex form)	GeoGebra exploration with parameter sliders- Guided questions and group discussion- Individual reflection	GeoGebra, worksheets, projector	Visual to Symbolic
2	Derive the vertex form from the geometric definition- Construct a formal proof using algebraic reasoning	Derivation using the distance formula- Collaborative proof construction- Whole-class discussion	Proof worksheets, whiteboard	Symbolic to Deductive

Table 9: Lesson Plan Based on the Two-Phase RPL Model

This lesson plan serves as a practical application of the RPL model and forms a foundation for the ongoing dissertation on implementing reasoning and proof-oriented instruction in dynamic, technology-enhanced learning environments. By aligning visual exploration, symbolic manipulation, and deductive reasoning, it offers a structured, research-based pathway for improving geometry teaching and fostering deeper mathematical understanding.

DISCUSSION

This study set out to examine how dynamic exploration through GeoGebra could support preservice mathematics teachers in developing geometric reasoning, particularly concerning the properties and proofs of the parabola. The findings provide compelling evidence that a well-designed instructional sequence can facilitate meaningful cognitive transitions across visual, symbolic, and deductive representations. These outcomes confirm the relevance of Van Hiele's geometric thinking model Van Hiele, (1999) and align with prior studies on technology-supported reasoning and proof development (Stylianides & Stylianides, 2008; Baccaglini-Frank & Mariotti, 2010).

From Dynamic Exploration to Proof Construction: A Teaching Pathway

This study demonstrates that dynamic environments, when supported by structured tasks, can move learners from visual exploration toward mathematical justification. Through parabola-based explorations, participants formed conjectures grounded in measurement and progressively articulated justifications using distance formulas and symbolic reasoning. Participant P4 exhibited Van Hiele Level 4 reasoning through an algebraic derivation of the parabola's definition, while others (e.g., P1, P2, P6) produced informal or semi-formal justifications during the transition from

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empirical observation.

These findings support prior research showing that reasoning embedded in dynamic environments promotes both conceptual understanding and logical structure (Hanna & de Villiers, 2012; Ellis et al., 2012) and highlight the epistemological role of proof in developing logical reasoning (Hanna & de Villiers, 2012; Buchbinder et al., 2020; Kokeb et al., 2025). However, consistent with Anwar et al. (2025), dynamic tools alone were insufficient; without structured reflection and teacher prompts, students tended to rely on visual intuition, faced difficulties identifying invariants, and drew on memorized theory. Participants with stronger manipulation and observation skills (e.g., P4) engaged more deeply with invariants and were more likely to construct deductive arguments.

Overall, the results provide empirical support for positioning Dynamic Geometry Software as an epistemic mediator rather than a visualization tool (Laborde, 2002; Sinclair, 2003). The observed shift from memorization and passive observation to experimentation, justification, and proof aligns with prior studies emphasizing DGS in supporting the transition from intuitive reasoning to formal thinking (Gulkilik, 2023; Komatsu & Jones, 2020). The use of traces, sliders, and interactive constructions further enabled immediate feedback in exploring symmetry and equidistance properties of the parabola, strengthening both conceptual understanding and learner confidence.

Strengthening Learning Through Representational Transition

The structured sequence of tasks in this study proved effective in guiding participants through Van Hiele levels, from visualization, to analysis, to informal and formal deduction. This aligns with (Gravemeijer, 1999) *guided reinvention* model and confirms the role of task design in reasoning development. GeoGebra, when used purposefully, enabled participants to detect invariants such as symmetry and equidistance, supporting the transition from perceptual reasoning to symbolic generalization. Nonetheless, as Anwar et al., (2025) noted, the inability to differentiate between construction-based premises and emergent conclusions (e.g., confusing given constraints with discovered properties) poses a cognitive risk in DGS environments. Teachers must therefore emphasize the logical structure of geometric reasoning, including how dynamic constructions provide premises from which conclusions must be derived, not assumed. This insight has powerful pedagogical implications. Secondary teachers can adopt the Reasoning and Proof Learning (RPL) model to structure their lessons into two phases: (1) *Exploration and Representation*, where students interact with parameters and detect patterns visually and symbolically, and (2) *Justification and Proof*, where they transition toward deductive argumentation. Without this progression, the learning risks becoming observationally rich but logically shallow.

Teaching Implications: Managing Technology-Related Challenges

The findings of Anwar et al., (2025) further caution that students' lack of familiarity with GeoGebra tools can lead to construction errors that undermine conceptual understanding. As seen in both studies, poor tool usage results in inaccurate figures, misinterpretation of measurements, and flawed reasoning. This highlights the importance of deliberate training in DGS before embedding

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it within reasoning and proof tasks. Strategies such as collaborative task design, tutorial walkthroughs, and feedback loops are essential for optimizing learning. Moreover, Anwar et al., (2025) emphasized that conjectures formed without observation often reflect passive, recall-based learning an issue also noted in our study when participants initially hesitated to test and generalize visual phenomena. As Borba et al., (2016); Mariotti, (2014) suggest, mathematics teachers must explicitly cultivate habits of active observation, pattern recognition, and generalization, especially when using dynamic technology.

Bridging Intuition and Formalism Through the RPL Model

Our proposed RPL model emerging inductively from student learning trajectories offers a practical framework for teachers to bridge empirical observation and formal reasoning. By integrating GeoGebra into classroom instruction, educators can create learning experiences where students construct understanding through dynamic manipulation and consolidate it through proof. However, consistent with Anwar's findings, our data show that students still struggle with formalizing their ideas into structured proofs. Many arguments remained intuitive or lacked logical rigor. This underscores the need to incorporate proof scaffolds such as two-column formats or Toulmin diagrams in classroom use of DGS, to support the full transition to deductive reasoning (Stylianides, 2007; Weber, 2001).

Implications for Teaching Practice

The findings of this study provide several key implications for integrating reasoning and proof into geometry instruction.

First, the results highlight the importance of a sequenced instructional design that begins with dynamic exploration and gradually guides students toward formal justification. Teachers are encouraged to adopt the two-phase Reasoning and Proof Learning (RPL) model-*Exploration and Representation* followed by *Justification and Proof*-which helps students move from intuitive visualization to deductive reasoning by experimenting, observing invariants, verbalizing patterns, and justifying their claims logically or algebraically. Second, while tools like GeoGebra and other Dynamic Geometry Software (DGS) can powerfully support cognition, their effectiveness depends on structured scaffolding. Teachers should design investigative and reflective tasks, not just demonstrations, to promote active exploration, generalization, and justification rather than passive observation. Third, students often face challenges such as limited DGS skills, difficulty identifying invariants, and overreliance on memorized theorems. Teachers should provide progressive DGS training, guided modeling, and collaborative opportunities, emphasizing the difference between premises (conditions) and conclusions (results) in constructing valid conjectures and proofs. Fourth, many students can make insightful observations but struggle to formalize them. Teachers can address this by incorporating explicit proof structures such as two-column proofs, guided worksheets, and flow diagrams to help students organize logical reasoning. Finally, the RPL model offers a practical, classroom-ready approach for teaching geometric topics such as parabolas, loci,

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or triangle centers-through dynamic, reasoning-oriented instruction. By embedding reasoning within everyday classroom practice and leveraging technology, teachers can foster deeper engagement and help students view proof not merely as an endpoint but as an integral process of mathematical thinking.

Limitations and Future Research

Although promising, this study was conducted with a small sample and focused on a single geometric topic. Moreover, the proof strategies used were emergent rather than pre-taught. Future research should explore how combining RPL with explicit proof instruction enhances student outcomes. Longitudinal studies could examine the lasting effects of such instructional models across different topics, such as triangle centers, loci, or transformations. Additionally, the complexity of navigating DGS especially for those with limited prior experience should inform professional development for both pre-service and in-service teachers. Future studies may consider how RPL can be scaled for classroom use while addressing barriers in GeoGebra fluency and proof literacy.

CONCLUSION

This study underscores the importance of targeted instructional design to help pre-service mathematics teachers develop reasoning and proof skills, particularly in geometric topics like the parabola, which are central to secondary education. Achieving Van Hiele's formal deduction level is vital not only for deepening teachers' own understanding but also for nurturing structured mathematical thinking in their future students.

The findings confirm that Dynamic Geometry Software (DGS) can effectively support the transition from intuitive visualization to formal proof when embedded within a well-sequenced instructional framework. Through dynamic exploration, participants identified patterns, formulated conjectures, and validated them using algebraic and logical reasoning. In this context, DGS served as more than a visualization tool it acted as a conceptual bridge that promoted deductive reasoning through interactive feedback and representational fluency.

The resulting instructional design provides teachers with practical, classroom-ready activities that guide learners through representational transitions from visual to symbolic to deductive reasoning enhancing students' geometric understanding of familiar concepts such as the parabola. The pedagogical implications are clear: by adopting the Reasoning and Proof Learning (RPL) model, teachers can foster meaningful cognitive transitions and cultivate students' engagement with proof as a central element of mathematical learning, rather than mere observation.

REFERENCES

[1] Alsina, Á., Pincheira, N., & Delgado-Rebolledo, R. (2024). The professional practice of designing tasks: how do pre-service early childhood teachers promote mathematical

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processes in early algebra? ZDM - Mathematics Education. https://doi.org/10.1007/s11858-024-01636-1

[2] Anwar, L., Sa'dijah, C., Listiawan, T., Dewi Utami, A., & Zulnaidi, H. (2025). Challenges of Prospective Mathematics Teachers in Formulating Geometrical Conjecture through Interaction with GeoGebra. In *MATHEMATICS TEACHING RESEARCH JOURNAL Early Spring* (Vol. 17, Issue 7).

[3] Armah, R. (2024). Geometric Thinking of Prospective Mathematics Teachers: Assessing the Foundation Built by University Undergraduate Education in Ghana. Teacher Education and Curriculum Studies, 9(2), 40–51. https://doi.org/10.11648/j.tecs.20240902.12

[4] Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practises in Cabri environments. Zentralblatt Für Didaktik Der Mathematik, 34(3), 66–72. https://doi.org/10.1007/BF02655708

[5] Baccaglini-Frank, A., & Mariotti, M. A. (2010). Generating Conjectures in Dynamic Geometry: The Maintaining Dragging Model. *International Journal of Computers for Mathematical Learning*, *15*(3), 225–253. https://doi.org/10.1007/s10758-010-9169-3

[6] Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education. *ZDM*, *48*(5), 589–610. https://doi.org/10.1007/s11858-016-0798-4

[7] Birgin, O., & Özkan, K. (2024). Comparing the concept images and hierarchical classification skills of students at different educational levels regarding parallelograms: a cross-sectional study. International Journal of Mathematical Education in Science and Technology, 55(4), 850–882. https://doi.org/10.1080/0020739X.2022.2052196

[8] Buchbinder, O., & Mccrone, S. (2020). Preservice teachers learning to teach proof through classroom implementation: Successes and challenges. The Journal of Mathematical Behavior, 58, 100779. https://doi.org/10.1016/j.jmathb.2020.100779

[9] Buchbinder, O., & McCrone, S. (2022). Guiding principles for teaching mathematics via reasoning and proving. https://hal.archives-ouvertes.fr/hal-03746878v2

[10] Cihan, F., & Akkoç, H. (2023). An Intervention Study for Improving Pre-service Mathematics Teachers' Proof Schemes. MATHEMATICS TEACHING RESEARCH JOURNAL (Vol. 15, Issue 2).

[11] Clements, D. H., & Battista, M. (1992). Geometry and spatial reasoning. https://www.researchgate.net/publication/258932007

[12] Ellis, A. B., Bieda, K., & Knuth, E. (2012). Developing essential understanding of proof and proving for teaching mathematics in grades 9-12. NCTM.

[13] Fuys, D. J. (1988). The van Hiele Model of Thinking in Geometry among Adolescents. Journal for Research in Mathematics Education Monograph Number 3. https://api.semanticscholar.org/CorpusID:118373716

[14] Galili, R., Hartono, Y., & Hiltiramartin, C. (2023). Desain Pembelajaran Identitas Trigonometri Dengan Proof Based Learning di SMA. Jurnal Cendekia : Jurnal Pendidikan Matematika, 7(3), 2287–2298. https://doi.org/10.31004/cendekia.v7i3.2677

[15] González, A., Manero, V., Arnal-Bailera, A., & Puertas, M. L. (2024a). Proof levels of graph theory students under the lens of the Van Hiele model. International Journal of

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Mathematical Education in Science and Technology, 55(8), 1938–1956. https://doi.org/10.1080/0020739X.2022.2113467

[16] Gravemeijer, K. (1999). How Emergent Models May Foster the Constitution of Formal Mathematics. *Mathematical Thinking and Learning*, *1*(2), 155–177. https://doi.org/10.1207/s15327833mtl0102_4

[17] Gulkilik, H. (2023). Analyzing preservice secondary mathematics teachers' prompts in dynamic geometry environment tasks. Interactive Learning Environments, 31(1), 22–37. https://doi.org/10.1080/10494820.2020.1758729

[18] Haj Yahya, A., & Hershkowitz, R. (2024). Interference of prototypical geometry representations in students' construction of concepts and development of proofs. Mathematical Thinking and Learning, 1–22. https://doi.org/10.1080/10986065.2024.2386619

[19] Haj-Yahya, A., & Olsher, S. (2025). Exploring proof building blocks as scaffolds for proving high school geometry theorems. International Journal of Mathematical Education in Science and Technology, 1–30. https://doi.org/10.1080/0020739X.2025.2475933

[20] Hanna, G., & de Villiers, M. (2021). Correction to: Proof and Proving in Mathematics Education (pp. C1–C1). https://doi.org/10.1007/978-94-007-2129-6_20

[21] Hartono, S., Yuli, T., Siswono, E., & Ekawati, R. (2024). From Informal to Formal Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions for Improving Preservice Teachers' Level of Proof. In MATHEMATICS TEACHING RESEARCH JOURNAL (Vol. 48, Issue 2).

[22] Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. Educational Studies in Mathematics, 96(1), 1–16. https://doi.org/10.1007/s10649-017-9761-8

[23] Kokeb, A., Atnafu, M., & Mohammed, A. (2025). The effects of visually assisted guided discovery learning strategy on tenth grade students' problem solving skill of plane geometry. In MATHEMATICS TEACHING RESEARCH JOURNAL. (Vol. 17, Issue 2).

[24] Komatsu, K., & Jones, K. (2020). Interplay between Paper-and-Pencil Activity and Dynamic-Geometry-Environment Use during Generalisation and Proving. Digital Experiences in Mathematics Education, 6(2), 123–143. https://doi.org/10.1007/s40751-020-00067-3

[25] Laborde, C. (2002). Integration of Technology in the Design of Geometry Tasks with Cabri-Geometry. International Journal of Computers for Mathematical Learning, 6(3), 283–317. https://doi.org/10.1023/A:1013309728825

[26] Lesseig, K., Hine, G., Na, G. S., & Boardman, K. (2019). Perceptions on proof and the teaching of proof: a comparison across preservice secondary teachers in Australia, USA and Korea. Mathematics Education Research Journal, 31(4), 393–418. https://doi.org/10.1007/s13394-019-00260-7

[27] Leung, F. K. S., Graf, K.-D., & Lopez-Real, F. (2006). Mathematics Education in Different Cultural Traditions-A Comparative Study of East Asia and the West. https://api.semanticscholar.org/CorpusID:53505585

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[28] Manero, V., & Arnal-Bailera. (2021). Understanding Proof Practices of pre-Service Mathematics Teachers in Geometry. In MATHEMATICS TEACHING RESEARCH JOURNAL (Vol. 99, Issue 3).

[29] Mariotti, M. A. (2014). Transforming Images in a DGS: The Semiotic Potential of the Dragging Tool for Introducing the Notion of Conditional Statement. In *Transformation - A Fundamental Idea of Mathematics Education* (pp. 155–172). Springer New York. https://doi.org/10.1007/978-1-4614-3489-4_8

[30] Mariotti, M. A., & Pedemonte, B. (2019). Intuition and proof in the solution of conjecturing problems'. ZDM, 51(5), 759–777. https://doi.org/10.1007/s11858-019-01059-3

[31] Martinovic, D., & Manizade, A. G. (2020). Teachers Using GeoGebra to Visualize and Verify Conjectures About Trapezoids. Canadian Journal of Science, Mathematics and Technology Education, 20(3), 485–503. https://doi.org/10.1007/s42330-020-00103-9

[32] Matthew Miles, Michael Hberman, & JohnnySdana. (2014). Qualitative Data Analysis A Methods (3rd ed.).

[33] Mawarsari, V. D., Waluya, St. B., & Dewi, N. R. (2023). Profile of Students' Geometric Thinking Ability in Terms of Van Hiele Level (pp. 109–117). https://doi.org/10.2991/978-2-38476-078-7_13

[34] Naufal, M. A., Abdullah, A. H., Osman, S., Abu, M. S., Ihsan, H., & Rondiyah. (2021). Reviewing the Van Hiele model and the application of metacognition on geometric thinking. International Journal of Evaluation and Research in Education, 10(2), 597–605. https://doi.org/10.11591/ijere.v10i2.21185

[35] Ødegaard, R. P., Arnesen, K. K., & Langfeldt, M. B. (2024). Tools to support learning to plan discussions that promote students' mathematical reasoning. Journal of Mathematics Teacher Education. https://doi.org/10.1007/s10857-024-09668-w

[36] Qohar, Abd., Susiswo, S., Nasution, S. H., & Wahyuningsih, S. (2021). Development of Android-Based Mathematics Learning Game on the Topic of Congruence and Similarity. International Journal of Interactive Mobile Technologies (IJIM), 15(09), 52. https://doi.org/10.3991/ijim.v15i09.20723

[37] Quaresma, P., & Janičić, P. (2006). Integrating Dynamic Geometry Software, Deduction Systems, and Theorem Repositories (pp. 280–294). https://doi.org/10.1007/11812289_22

[38] Rizos, I., & Gkrekas, N. (2023). Is there room for conjectures in mathematics? The role of dynamic geometry environments. European Journal of Science and Mathematics Education, 11(4), 589–598. https://doi.org/10.30935/scimath/13204

[39] Santos-Trigo, M., Camacho-Machín, M., & Barrera-Mora, F. (2024). Focusing on foundational Calculus ideas to understand the derivative concept via problem-solving tasks that involve the use of a Dynamic Geometry System. ZDM - Mathematics Education, 56(6), 1287–1301. https://doi.org/10.1007/s11858-024-01607-6

[40] Scristia, Herman, T., & Septy Sari Yukans. (2025). Redesigning geometry assessments to promote advanced geometric thinking: A case study on formal deduction and rigor. Jurnal Elemen, 11(1), 186–205. https://doi.org/10.29408/jel.v11i1.27723

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[41] Sinclair, M. P. (2003). Some implications of the results of a case study for the design of preconstructed, dynamic geometry sketches and accompanying materials. Educational Studies in Mathematics, 52(3), 289–317. https://doi.org/10.1023/A:1024305603330

[42] Škrbec, M., & Čadež, T. H. (2015). Identifying and fostering higher levels of geometric thinking. Eurasia Journal of Mathematics, Science and Technology Education, 11(3), 601–617. https://doi.org/10.12973/eurasia.2015.1339a

[43] Stylianides, A. J. (2007). Proof and proving in school mathematics. Journal for Research in Mathematics Education, 38(3), 289–321. https://www.scopus.com/inward/record.uri?eid=2-s2.0-34248588003&partnerID=40&md5=653faded3b3ed2b0012da31415e13b6d

[44] Stylianides G. (2008). Reasoning and proof, framework Stylianedes. FLM Publishing Association, Edmonton, Alberta, Canada.

[45] Stylianides, G., Stylianides, A., & Shilling-Stouffer, L. (2013). PROSPECTIVE TEACHERS' CHALLENGES IN TEACHING REASONING-AND-PROVING. International Journal of Science and Mathematics Education, 11. https://doi.org/10.1007/s10763-013-9409-9

[46] Stylianides, G., Stylianides, A., & Weber, K. (2017). Research on the teaching and learning of proof: Taking stock and moving forward (pp. 237–266).

[47] Thapa, R., Dahal, N., & Pant, B. P. (2022). GeoGebra Integration in High School Mathematics: An Experiential Exploration on Concepts of Circle. In MATHEMATICS TEACHING RESEARCH JOURNAL (Vol. 16, Issue 5).

[48] The National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. The National Council of Teachers of Mathematics, Inc.

[49] Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to Learn Reasoning and Proof in High School Mathematics Textbooks. Journal for Research in Mathematics Education, 43(3), 253–295. https://doi.org/10.5951/jresematheduc.43.3.0253

[50] Usiskin, Z. P. (1982). Van Hiele Levels and Achievement in Secondary School Geometry. CDASSG Project. https://api.semanticscholar.org/CorpusID:115884211

[51] Utari, T., & Hartono, H. (2019). Muatan penalaran dan pembuktian matematis pada buku teks matematika SMA kelas X Kurikulum 2013. Jurnal Riset Pendidikan Matematika, 6(1), 1–13. https://doi.org/10.21831/jrpm.v6i1.17002

[52] Van Hiele, P. M. (1999). Developing Geometric Thinking through Activities that Begin With Play. Teaching Children Mathematics. . In (pp 310-316). Reston VA: National Council of Teachers of Mathematics. (pp. 310–316). Reston VA: National Council of Teachers of Mathematics.

[53] Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. Educational Studies in Mathematics, 48(1). https://doi.org/10.1023/A:1015535614355

[54] Yohannes, A., & Chen, H.-L. (2021). GeoGebra in mathematics education: a systematic review of journal articles published from 2010 to 2020. Interactive Learning Environments, 1–16. https://doi.org/10.1080/10494820.2021.2016861

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APPENDIX

Dynamic Tasks on the Parabola

Parabola

Name: _____

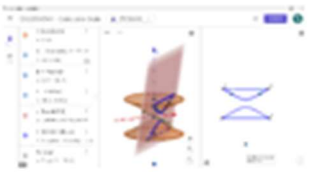
Group: _____

Course Learning Outcomes (CLO)

- Students will identify how each conic section is generated through the slicing of a cone.
- Students will understand the definition of the locus of a parabola.
- Students will describe how the values of *a*, *h*, and *k* in the vertex form of a parabola equation affect its graph.
- Students will use the locus definition of a parabola to derive its equation and explain the relationship between the focus, directrix, and the parameters in the vertex form.
- Students will strategically use appropriate tools and make use of mathematical structure

Open the document: Introduction to Conics – GeoGebra

Is there a connection between the definition of a locus and the vertex form of a parabola? In this activity, you will explore conic sections and the concept of the parabola.



Use the slider at point D to view the different types of conic sections one by one.
Based on your observations:

1. How does the position of the cutting plane relative to the cone determine the shape such as a circle, ellipse, parabola, or hyperbola? Briefly explain by referring to the position of the plane in relation to the cone's axis or side. Complete the table below.

Conic Section	Description
Circle	
Ellipse	
Parabola	
Hyperbola	

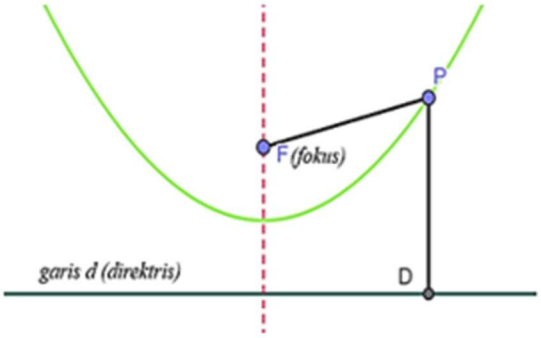
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Parabola

Name: _____

Group: _____

Definition:
The locus definition of a parabola is the set of all points that are equidistant from a fixed point called the focus and a line called the directrix.
Buka Dokumen PARABOLA – GeoGebra



Task 1:

a. Point F is known as the focus of the parabola. What is the name of the line that passes through point F and is perpendicular to the directrix?
b. Line d is the directrix of the parabola. What is the relationship between line d and the dashed line passing through point F?

Task 2:

a. Drag point P along the shape of the parabola. How does the distance from point P to the focus compare to its distance to the directrix? What can you conclude from your observation?
b. Move the focus point (F) to different positions on the screen. Does the property you observed in Question 3 still apply? Explain your reasoning.

.....
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Parabola

Name: _____

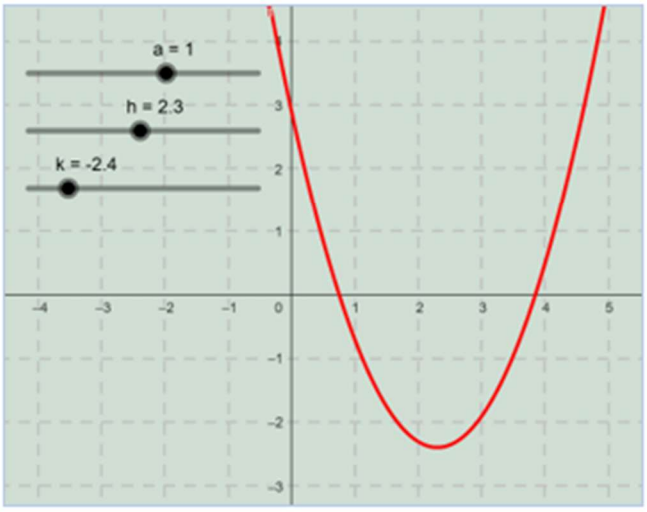
Group: _____

Task 3:

Open the GeoGebra document titled "BENTUK PUNCAK PARABOLA – GeoGebra, which displays the graph of a parabola in vertex form:

$$y = a(x - h)^2 + k$$

featuring interactive sliders for the parameters a, h, and k that allow you to dynamically view the changes in the parabola's graph as the values are adjusted.



The vertex form of the parabola equation $y = a(x - h)^2 + k$ is displayed. Move the sliders to change the values of $a, h,$ and k.

Explain how changes in each parameter $a, h,$ and k affect the shape and position of the graph.

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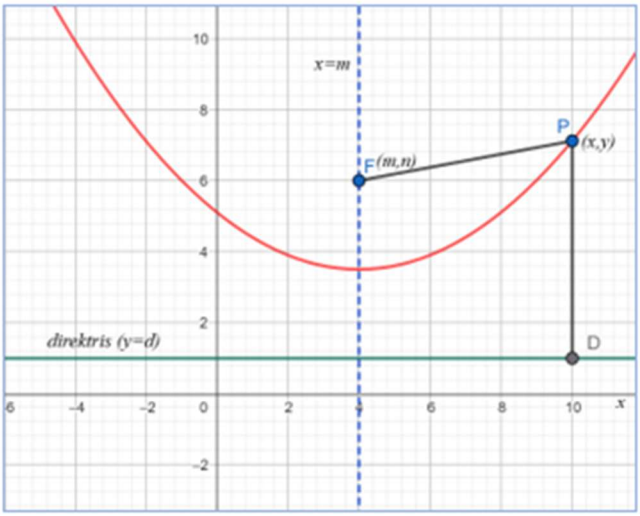
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Parabola

Name: _____

Group: _____

Given the focus (m,n) the directrix $y=d$ and a point $P(x,y)$ on the parabola, use the distance formula to derive the general equation of the parabola.



a. Find the distance between the focus and point P.

_____

b. Find the distance between point P and directrix

_____

a. Use the fact that the distance from point P to the focus is equal to its distance to the directrix. Set up an equation for the two distances and solve to find the value of y.

_____

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Parabola

Name: _____

Group: _____

Using the vertex form of the parabola,

$$y = a(x - h)^2 + k$$

and the algebraic relationship derived in Task 4, answer the following questions:

a. Explain the relationship between the focus, directrix, and the value of a

_____
_____
_____
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_____

b. What happens to the shape of the parabola as a increases? How does this relate to the distance between the focus and the directrix?"

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Exploring Pre-service Mathematics Teachers' Challenges in Understanding Intuition and Applying Scaffolding in Geometric Proofs

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Abstract: This study aims to explore the challenges pre-service mathematics teachers face in understanding intuition in geometric proofs and how scaffolding can support their learning. A qualitative approach was employed with 105 pre-service mathematics teachers from the State University of Surabaya as participants. Data were collected through tests and interviews, then analyzed using Miyazaki's classification: Proof Types A and B (deductive methods) and Proof Types C and D (inductive methods). Four students who struggled with intuition in proof construction were selected for scaffolding interventions. The findings reveal that 71% of participants successfully applied deductive reasoning in geometric proofs, while 29% relied on inductive reasoning. Additionally, 34 students exhibited difficulties in understanding intuition, and four were given scaffolding through strategies such as suggesting and investigating, explanation and justification, conceptual discussions, negotiating meaning, making connections, coordinating problems, and developing representative tools. The results suggest that targeted scaffolding can help pre-service teachers overcome difficulties in intuitive understanding and improve their proof construction skills in geometry.

Keywords: student difficulties, pre-service mathematics teachers', understanding of intuition concepts, scaffolding, geometry tasks

INTRODUCTION

Proof holds a central position in mathematics as a fundamental means of validating and communicating mathematical ideas (Hartono et al, 2024). It is not only a method for verifying the truth of statements but also a powerful tool for fostering deep understanding and logical reasoning (Arieli et al, 2024). The National Council of Teachers of Mathematics (NCTM, 2000) emphasizes the importance of proof in developing students' reasoning and problem-solving skills, aligning with

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broader educational goals to cultivate critical thinking. Therefore, cultivating strong proof abilities among mathematics students is essential to their academic development and professional readiness as future educators (Firmasari & Sulaiman, 2019).

In Indonesian universities, the Geometry course plays a vital role in developing students' proof skills by introducing them to formal mathematical language, axiomatic systems, and logical argumentation (Hartono et al, 2025; Isran et al, 2025; Isnani et al, 2025). Through this course, students are trained to verify mathematical statements and express ideas in a structured, systematic manner (Qomariyah & Rosyidah, 2022). In the context of mathematical reasoning, proof methods generally fall into two main categories: deductive with deriving conclusions from established truths, and inductive with drawing generalizations from specific patterns (Miyazaki, 2000). Miyazaki (2000) further classifies proof into four types (Proof A, Proof B, Proof C, and Proof D) based on reasoning type and the use of functional language or visual representations, providing a framework to analyze students' approaches in constructing proofs.

Despite the recognized importance of proof, many students struggle with its construction (Chin & Fu, 2021; Aisyah et al., 2023). These difficulties are often rooted in a limited intuitive understanding of mathematical concepts (Moore, 1996), which hinders their ability to develop coherent arguments and identify valid logical connections (Antonini, 2019). Mathematical intuition defined as the informal, often visual or experiential sense of a concept serves as a foundation for generating ideas prior to formalization. When students lack this intuitive base or a clear understanding of definitions, theorems, and relationships, they frequently encounter obstacles in proof construction and are more likely to misapply proof strategies.

To address these challenges, effective instructional strategies are necessary. One promising approach is scaffolding—the structured support provided by instructors to bridge the gap between what learners know and what they are expected to learn (Ding et al., 2011; Belland, 2017). Scaffolding can assist students in developing mathematical intuition by guiding them through visualization, identification of key assumptions, and analysis of simplified examples. Anghileri (2006) outlines three levels of scaffolding: (1) environmental provisions such as appropriate learning media and structured materials; (2) interactive teaching practices including explanation, questioning, and material restructuring; and (3) conceptual engagement that encourages students to articulate and connect their understanding. Research by Indrawati (2017) supports the effectiveness of scaffolding in helping students concretely visualize and grasp mathematical ideas, particularly in geometry.

Despite the established importance of mathematical proof, few studies have directly examined how pre-service teachers' intuitive understanding influences their ability to construct formal geometric proofs—particularly in the Indonesian context. Prior research has often focused on logical structures, overlooking the formative role of intuition in early mathematical reasoning (Stylianides et al., 2024; Haj-Yahya et al., 2023). Moreover, there is a lack of classroom-based interventions that specifically address intuitive misconceptions through structured scaffolding. Given the persistent challenges that pre-service mathematics teachers face in understanding intuition and constructing geometric proofs, research in this area is both timely and essential. Mastery of proof is not only

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vital for academic success but also critical for effective mathematics instruction in future classrooms. As future educators, pre-service teachers must be equipped to communicate the concept and importance of proof in ways that are accessible and meaningful to students. Therefore, this study aims to investigate: (1) What forms of intuitive misunderstanding are exhibited by pre-service mathematics teachers in geometric proof tasks? and (2) How can a scaffolded instructional approach address these challenges and improve students' reasoning?

LITERATURE REVIEW

Mathematical intuition plays a foundational role in initiating the reasoning process and bridging informal thinking with formal proof construction. Moore (1994) highlighted that many students struggle with constructing proofs due to their lack of intuitive understanding of mathematical concepts. Epp (2003) emphasized that intuition is not only useful but necessary for transitioning from visual or informal reasoning toward formal mathematical arguments. Nevertheless, Antonini (2019) cautioned that intuitive or visual reasoning, when not well structured, often leads to misconceptions—especially in complex topics like indirect proofs or generalizations.

Pre-service mathematics teachers, in particular, face unique challenges in proof construction. Martin and Harel (1989) observed that these future educators frequently confuse valid proofs with empirical arguments or generalizations, which indicates a fragile conceptual understanding. Stylianides (2007) added that limited exposure to diverse proof methods such as direct, indirect, or inductive proofs compounds this difficulty. Although several studies have examined students' overall struggles with proof, only a few have investigated how pre-service teachers develop, misuse, or fail to activate intuitive understanding when constructing geometric proofs (Stylianides et al., 2024).

To address these difficulties, scaffolding has emerged as a powerful instructional tool. Defined as temporary, targeted support that is gradually removed as learners gain independence, scaffolding has been shown to improve both reasoning and problem-solving performance (Belland, 2017). Anghileri (2006) categorized scaffolding into three progressive levels: environmental structuring, interactive dialogue (such as probing and prompting), and the promotion of conceptual thinking. Ding et al. (2011) demonstrated that scaffolding significantly enhances students' conceptual problem-solving, although most of these studies have focused on general mathematics tasks rather than the specific demands of geometric proof construction.

Despite the established importance of both intuition and scaffolding in mathematics education, research integrating the two remains scarce. In particular, few studies have examined the specific challenges pre-service teachers face in intuitive understanding within geometric proof tasks, especially through the lens of Miyazaki's classification of proof types. Moreover, classroom-based studies that not only diagnose intuitive misconceptions but also implement tailored scaffolding

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interventions are still limited. This study addresses that gap by exploring how scaffolding strategies can effectively support pre-service teachers' understanding and application of intuition across different proof styles.

METHOD

This study employed a descriptive qualitative approach using a case study design to explore the intuitive understanding difficulties faced by pre-service mathematics teachers and the effectiveness of scaffolded instructional support in overcoming these challenges. The research offers a novel perspective by classifying students' approaches to proof using Miyazaki's framework and by investigating their intuitive understanding through task-based interviews. Unlike previous studies that primarily emphasized deductive reasoning, this study highlights how targeted scaffolding can help transform students' intuitive misconceptions into structured and valid formal proofs, offering a practical instructional model for classroom implementation.

The participants in this study were 105 second-year pre-service mathematics teachers (aged 19–21) enrolled in the "Elementary Geometry" course at Surabaya State University. All participants had previously completed foundational courses in logic and mathematical structures, although they had limited exposure to formal proof construction. Representing a typical cohort in Indonesian mathematics teacher education programs, these students engaged in structured proof tasks as part of their regular coursework in a standard classroom setting. The primary data sources consisted of written test responses and semi-structured interviews. The test featured a single open-ended geometric proof task designed to elicit both deductive and inductive reasoning: *Given an arbitrary quadrilateral ABCD, let P, Q, R, and S be the midpoints of sides AB, BC, CD, and DA, respectively. Suppose PR and QS intersect at point O. Prove that PO = OR and QO = OS*. The proof test consisted of one constructed-response task adapted from classical geometry problems to assess inductive and deductive reasoning. The task was reviewed by two subject-matter experts to ensure content validity. Interview protocols included six guiding questions addressing students' reasoning, interpretation of midpoints, and generalization patterns. Questions were piloted and refined based on feedback from a small group of pre-service teachers not involved in the main study. Based on their performance in this task, four students were purposively selected for in-depth case study analysis, each representing one of the four proof types categorized by Miyazaki's framework. These individuals, labeled MA, MB, MC, and MD, participated in individual interviews that were conducted in a quiet classroom, audio-recorded with their consent, and lasted approximately 30–45 minutes each.

The students' written responses were analyzed using Miyazaki's (2000) proof classification, which distinguishes proofs based on their method (deductive or inductive) and the nature of representation (use of functional language or reliance on visual manipulation). Table 1 below outlines the classification:

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Representation	Method	
	Deductive	Inductive
Using functional language according to the theorem	Proof A	Proof D
Do not use functional languages, use images, or manipulate objects	Proof B	Proof C

Table 1: Levels of proof in mathematics

Based on this framework, the researchers identified students' difficulties within each proof type, revealing limited intuitive understanding of the concept of proof. The students' written responses were analyzed using Miyazaki's (2000) proof classification. In addition to the case analysis of four representative students (MA, MB, MC, MD), a descriptive quantitative analysis of all 105 participants' answers was conducted. Frequencies and percentages of correct and incorrect responses for each proof type (A–D) were calculated. Furthermore, the 34 students who experienced difficulties were categorized into three main error patterns: (1) misinterpretation of midpoints, (2) over-generalization from special cases (squares/rectangles), and (3) assuming intersections automatically imply equal segments. This categorization aimed to provide a more comprehensive picture of students' intuitive difficulties. Interview questions were developed based on themes from Miyazaki's classification and expert feedback and were validated through a peer-review process involving two mathematics education researchers. Data were analyzed thematically, guided by Miyazaki's framework and complemented by open coding. Thematic units included misinterpretation of midpoints, over-generalization, and over-reliance on visual forms. Two researchers independently coded the responses, and discrepancies were resolved through consensus. Scaffolding strategies based on Anghileri's (2006) framework were provided to support students' reasoning, and triangulation between written responses and interview data was used to enhance the validity of the findings. Researchers provide scaffolding assistance based on Anghileri's theory (2006) which divides the scaffolding hierarchy into three levels, namely:

Types of Scaffolding		
Scaffolding level 1	*environmental provisions*	Sequencing and pacing
		Giving structured tasks
		Student grouping
Scaffolding level 2	*explaining*	
	Reviewing	Seeing, touching, stating verbally
		Probing and prompting
		Interpreting student actions
		Parallel modeling
		Student explanation and justification

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	Restructuring	Identifying meaningful content
		Simplifying the problem
		Paraphrasing student conversations
		negotiation of meaning
Scaffolding level 3	Developing representative tools	
	Making a connection	
	Generating conceptual discussion	

Table 2: Scaffolding to overcome difficulties in proof

RESULTS AND DISCUSSION

The findings of the research can be categorized according to the focus established at the beginning of the research, namely:

Type	Number of pre-service teachers'	Difficulties of pre-service mathematics teachers' understanding of intuition concepts (Number of teachers)
Proof A	64	18
Proof B	10	3
Proof C	7	1
Proof D	24	12
Total	105	34

Table 3: Types of pre-service teachers' proof in geometry

Based on Table 3, the results indicate that 74 students (71%) used deductive proof, while 31 students (29%) used inductive proof. Out of the 74 students who used deductive proof, 64 employed proof type A, with 15 students providing correct answers and 49 students making errors, while 10 employed proof type B, with 7 correct and 3 incorrect. Of the 31 students who used inductive proof, 7 used proof type C (2 correct, 5 incorrect), and 24 used proof type D (6 correct, 18 incorrect). Overall, 34 students (33%) experienced difficulties in constructing proofs due to limited intuitive understanding. These 34 students' responses were further categorized into three types of intuitive errors: (1) 14 students (41%) misinterpreted midpoints by assuming any intersection is a midpoint, (2) 11 students (32%) generalized properties from special cases such as squares or rectangles, and (3) 9 students (27%) assumed that intersecting diagonals automatically divide into equal segments. Table 4 presents the distribution of these error patterns below. For Proof A, as many as 18 students, then continued with Proof B as many as 10 students, 7 students from Proof C, and 24 students from Proof D.

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Error Category	Number of Students	Percentage	Example of Error
Misinterpreting the midpoint	14	41%	Assuming that the intersection of two lines automatically represents the midpoint
Generalization from special cases (square/rectangle)	11	32%	Using the properties of a square to conclude the properties of all quadrilaterals
Assuming intersections automatically divide equally	9	27%	Stating that PO = OR merely because line PR intersects at O

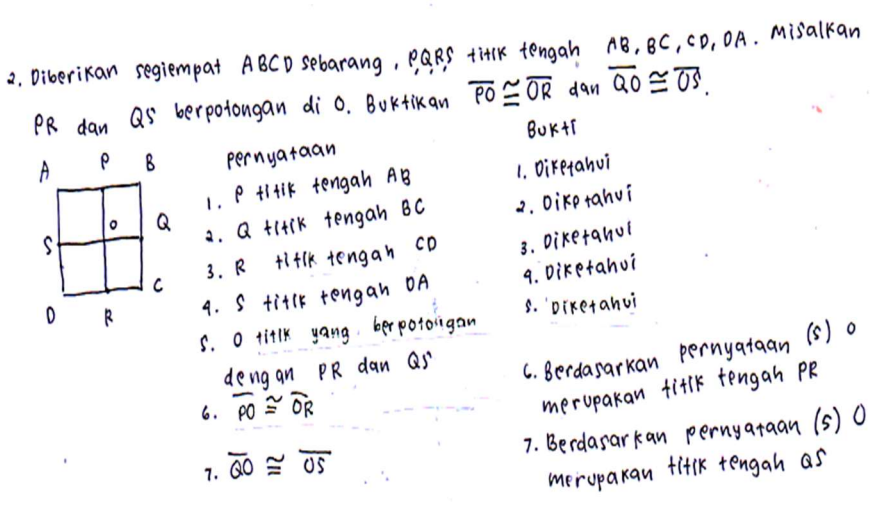
Table 4: Categories of Students' Intuitive Errors

This analysis shows that students' difficulties were not only caused by weaknesses in formal logic, but also by fundamental misconceptions in intuitive understanding. These findings also indicate the necessity of examining not only the correctness of students' formal reasoning but also the specific types of intuitive misconceptions they display. By categorizing errors such as misinterpreting midpoints, overgeneralizing from special cases, and assuming intersections automatically divide equally, educators can better identify the roots of students' difficulties. This categorization provides a framework for designing targeted scaffolding strategies that directly address the nature of students' misunderstandings, rather than merely focusing on procedural accuracy.

In this study, inductive reasoning refers to students' spontaneous inferences or justifications that are based on visual impressions or informal understanding, rather than systematic logical analysis. According to Lajos (2023), intuitive thinking in mathematics can support or hinder proof development depending on how it aligns with formal concepts. In our analysis, several students made assumptions, for example: assuming O is the midpoint simply because lines intersect) without articulating the underlying geometric properties. Such assumptions reflect pseudo-intuitive understanding or misconceptions rather than true intuitive insight. Some examples of students' answers regarding pre-service mathematics teachers' understanding of intuition concepts are as follows.

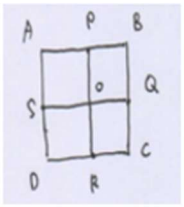
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PROOF A



Translation

Given an arbitrary quadrilateral ABCD, let P, Q, R, and S be the midpoints of AB, BC, CD, and DA, respectively. Suppose that PR and QS intersect at point O. Prove that PO=OR and QO=OS.



Statements	Proof
1. P is the midpoint of AB	Given
2. Q is the midpoint of BC	Given
3. R is the midpoint of CD	Given
4. S is the midpoint of DA	Given
5. O is the intersection point of PR and QS	Given
6. $\overline{PO} \cong \overline{OR}$	Based on statement (5), O is the midpoint of PR
7. $\overline{QO} \cong \overline{OS}$	Based on statement (5), O is the midpoint of QS

Figure 1. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof A

Based on Figure 1, it can be seen that pre-service teachers have used deductive methods with functional language according to the theorem but there is still a misunderstanding. Subject MA explained that O is a point that intersects PR and QS so that it causes the PO line to be congruent with OR and QO to be congruent with OS and O is the midpoint. This can be seen from the results of interviews with subject MA as follows.

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Researcher : Okay, try to look at the picture of the quadrilateral, do you think this is an arbitrary quadrilateral?

MA : Yes, arbitrary, sir, because it is known to be an arbitrary quadrilateral and the proof must use a deductive method so that it cannot use examples other than arbitrary quadrilaterals.

Researcher : When is a quadrilateral said to be arbitrary?

MA : A quadrilateral that has four sides but is not specific.

Researcher : In your opinion, point O is the midpoint, right?

MA : In my opinion, point O is the midpoint, sir.

Researcher : Why can point O be called the midpoint?

MA : Because point O intersects 2 line segments, sir, so point O is the midpoint

As seen in the interview above, although Subject MA can explain the step-by-step answer in proving the theorem using the logical structure of a proof. Subject MA has difficulty in understanding the concept of the intersection of two lines and the midpoint so that he cannot prove the problem. This response is categorized as reflecting a low level of intuitive understanding. Specifically, the subject shows a limited understanding of the concepts of the intersection of two lines and the midpoint of a line segment, which leads to difficulty in constructing a valid proof.

PROOF B

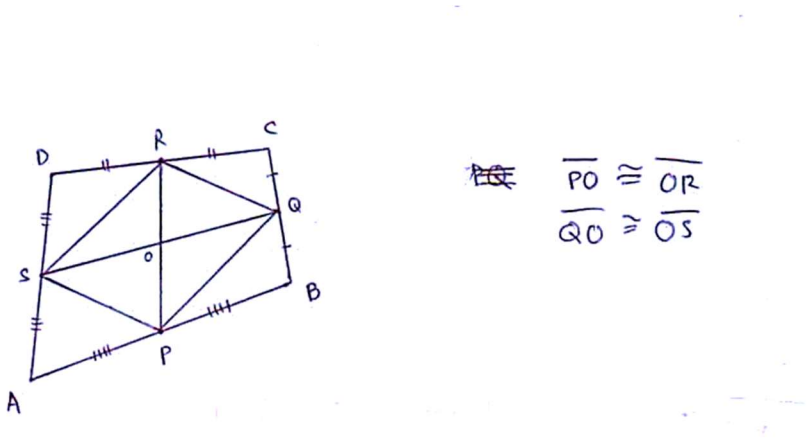


Figure 2. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof B

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Based on Figure 2, it can be seen that Subject MB uses the concept of an arbitrary quadrilateral, namely that each side is different in the visualization of the image without using functional language. The subject claims that point O is definitely in the middle because SQ and PR cut the side of the arbitrary quadrilateral into two equal parts. Therefore, the results of the intersection of the two diagonals are also the same length so that PO = OR and QO = OS are obtained. This can be seen from the results of the interview with subject MB as follows.

Researcher : Please explain what proof concept you use in working on this proof problem?

MB : Let me assume an arbitrary quadrilateral and then prove the midpoint, sir.

Researcher : Are you sure about your answer? How was the proof?

MB : Given an arbitrary quadrilateral ABCD. So let me draw it first, sir. Suppose P, Q, R, and S are the midpoints of AB, BC, CD, and DA, respectively. Then draw a line P to R and Q to S. The point where the lines intersect is at O.

Researcher : Wait, from there how can you prove that PO is equal to OR and SO is equal to OQ?

MB : First, it is known that this point is in the middle. Next, this line SQ intersects BC at Q, which is at the midpoint and intersects AD at point S, which is also in the middle. So, I assume that the point of intersection at O with PR is in the middle. Therefore, I assume that PO is equal to OR.

Researcher : So from there, we immediately find that PO is equal to OR, right?

MA : Yes, sir.

As seen in the interview above, the Subject claims that point O is definitely in the middle because SQ and PR cut any side of the quadrilateral into two equal parts. Therefore, the results of the intersection of the two diagonals are also the same length so that PO = OR and QO = OS are obtained. At this stage, the subject has difficulty in understanding the concept of the midpoint so that the claim produced to prove the problem is less precise.

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PROOF C

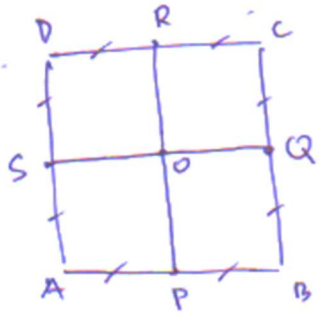


Figure 3. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof C

Based on Figure 3, it can be seen that Subject MC uses the concept of a square without using functional language, using images, or manipulating objects. Furthermore, it can be seen in the figure that Subject MC directly justifies that point O is the midpoint so that the RO and OP lines and the SO and OQ lines are the same. This can be seen from the results of the interview with the MC subject as follows.

Researcher: What is the first step?

MC: Make a square sir

Researcher: Okay after knowing that it is a square, is RO the same as OP?

MC: Same

Researcher: Does that mean you meet it right away?

MC: Yes sir, because the square is the same length divided by the midpoint

Based on the interview, Subject MC attempts to justify the conclusion using a square—an overly specific case that does not represent all arbitrary quadrilaterals. The student immediately concludes that PO = OR and QO = OS by relying on the symmetry of a square, without attempting to generalize from multiple cases. This is not an example of intuitive understanding; rather, it reflects a misconception of the problem, particularly a misunderstanding of what constitutes an arbitrary quadrilateral. The student's reasoning lacks generalization and appears to confuse a specific shape (square) with the general category (quadrilateral), which is a conceptual error rather than an intuitive leap. Although Subject MC used a square as the base case and generalized from that one example, there is no evidence of systematic generalization across multiple cases, which is a hallmark of inductive reasoning. This shows a misinterpretation of inductive proof. Thus, the classification as 'Proof C' should be interpreted as the student attempting an empirical generalization rather than formal induction.

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PROOF D

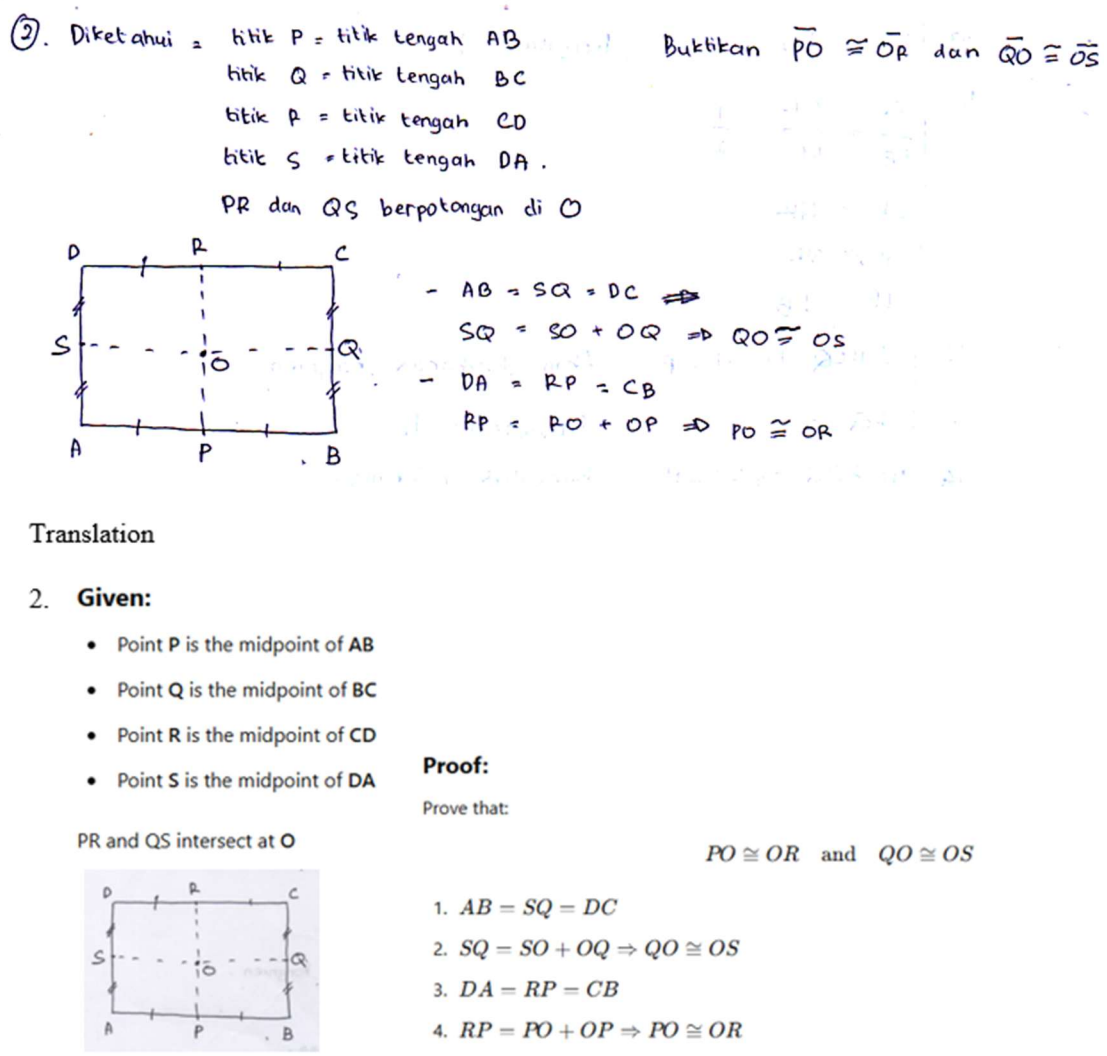


Figure 4. Difficulties of pre-service math teachers' understanding of intuition concepts in Proof D

Based on Figure 4, it can be seen that Subject MD uses the concept of a rectangle using functional language, using images, or object manipulation. The subject answers with inductive proof steps, namely with the concept of a rectangle as an arbitrary quadrilateral and argues that SQ = SO+SQ and concludes that the lines SO and OQ are the same. The subject also argues that RP = RO+OP and concludes that the lines PO and OR are the same. This can be seen from the results of the interview with subject MD as follows.

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Researcher: What is the first step?

MD: I use a rectangle sir

Researcher: Where do you think QO is congruent with OS

MD: According to my answer, this is because the rectangle so AB is the same length as SQ and the same length as DC then SQ is SO plus OQ then QO is congruent with OS

Researcher: So it meets directly, right?

MC: Yes sir, because it uses the concept of a rectangle

Based on the interview above, it can be seen that Subject MD had difficulty in continuing the proof of the problem because the subject's understanding was lacking, the subject used a rectangle as an arbitrary quadrilateral and argued that SQ = SO + OQ and concluded that lines SO and OQ were the same. The subject also argued that RP = RO + OP and concluded that lines PO and OR were the same. At this stage the subject experienced difficulties because the subject argued that one line segment was formed by two equal lines because he concluded that point O was the midline and divided it into two lines of equal length in proving this theorem so that it was categorized as difficult because he had little intuitive understanding of the concepts. Although the student uses language that appears logical, their reasoning is based entirely on properties of rectangles. This suggests the student does not differentiate between specific and general geometric figures. The use of a special case to justify a general proof without broader justification is not considered intuitive understanding; it is a surface-level strategy that bypasses the essence of proof construction. Such thinking stems from an incomplete understanding of geometric classifications rather than an intuitive insight.

Given these challenges in intuitive understanding, effective teaching interventions are needed to bridge the gap. Scaffolding provides structured support by gradually guiding students through conceptual difficulties, allowing them to develop independent problem-solving skills over time. In addition, based on Anghileri (2006) that scaffolding was implemented through three structured sessions (Table 2). Level 1 involved environmental support (e.g., use of diagrams and guiding tools). Level 2 included probing questions, prompting explanation and justification. Level 3 engaged students in generating connections and visual representations. Regarding scaffolding to overcome student difficulties in proof, as many as 4 students answered incorrectly and were then selected to be given scaffolding, namely as follows:

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No	Subjects	Initial proof type	Types of scaffolding	Final proof type
1	MA	Proof A	Probing and prompting, Explanation and Justification, Generating conceptual discussion, Negotiating Meaning, Making Connections	Proof A (Correct answer)
2	MB	Proof B	Probing and prompting, Explaining and justifying, Simplifying problems, Generating conceptual discussions, Making connections	Proof A (Correct answer)
3	MC	Proof C	Probing and prompting, Explanation and Justification, Generating Conceptual Discussion, Making Connections, Negotiating Meaning	Proof A (Correct answer)
4	MD	Proof D	Probing and prompting, Explanation and Justification, Generating Conceptual Discussion, Making Connections, Negotiating Meaning, and Developing Representative Tools	Proof A (Correct answer)

Table 5: Scaffolding to overcome difficulties of pre-service mathematics teachers' understanding of intuition concepts

Based on Table 5, it shows that students apply the type of proof in constructing evidence, namely the deductive and inductive methods. The deductive method consists of two types, namely proof A and proof B, while the inductive method consists of two types including proof C and proof D. Table 5 shows that 74 students used the deductive method (71%) while 31 of them used the inductive method (29%). This means that more than half of the students' answers used the deductive method. Although some students make mistakes in constructing proofs, they have tried to construct proofs using deductive and inductive methods. This finding is consistent with research conducted by Miyazaki (2000) that most students in his research use the deductive method rather than the inductive method in constructing proofs. In addition, the results of the study showed that the main difficulty faced by students in constructing proofs of geometric theorems seemed to be closely related to the lack of intuitive understanding of the basic concepts in proofs, namely 34 students (33%). Nearly one-third of the subjects experienced this difficulty, indicating that students tend to get stuck in mechanical processes, such as following a certain proof pattern without really understanding the concepts underlying the steps. In line with this finding, research by Kögce et al. (2010)

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also showed that difficulties in proofs are often caused by a lack of deep conceptual understanding. They found that most students performed proofs by following a familiar pattern without really understanding the concept or reasoning behind the steps.

Based on Table 5, it can be seen that all students experienced difficulties in various types of proofs, both using deductive methods (proofs A and B) and inductive methods (proofs C and D), which were caused by a lack of intuitive understanding related to the concepts underlying the proof. This difficulty reflects that without a strong intuitive understanding, students often have difficulty in feeling the relationship between elements in the problems they face. For example, students often get stuck in the definition of two intersecting lines, then they will divide them equally so that the point of intersection is the midpoint. This difficulty leads to incoherent proofs, where the steps do not support each other, and students have difficulty in proving the relationship between geometric elements correctly. This is in line with Mahfudy's (2017) research on students' mathematical proof strategies in geometry problems, including proving the Pythagorean theorem. It was found that students often rely on their intuition in making assumptions or guesses that lead to conclusions, but lack a deep understanding of the concepts underlying the proof.

Furthermore, the lack of intuitive understanding also affects students' ability to choose the most appropriate proof approach. Moore (1996) stated that in mathematical proofs, sometimes there is more than one way to reach the desired conclusion, and intuition helps students choose the most efficient path. The findings of the present study also resonate with earlier works emphasizing the central role of intuitive reasoning in students' proof construction. Moore (1996) highlighted that many students experience significant difficulties in constructing proofs because they rely on incomplete or flawed intuitive conceptions rather than rigorous logical structures. Similarly, Antonini (2019) noted that intuitive thinking can both support and hinder proof activity, depending on whether the intuition aligns with formal concepts. More recently, Stylianides (2024) emphasized the need to design instructional tasks that bridge the gap between informal intuitive reasoning and formal deductive proof in order to promote sustainable proof learning.

This difficulty reflects that without a strong intuitive understanding, students often fail to recognize the correct relationships between geometric elements. Each category of intuitive error can be addressed through specific scaffolding strategies: misinterpretation of midpoints can be handled through probing, prompting, and justification (Level 2 scaffolding); over-generalization from special cases requires conceptual discussion and dynamic representations with GeoGebra (Level 3 scaffolding); and assumptions about intersections dividing equally can be corrected through guided diagrams, simplified tasks, and negotiation of meaning (Level 1–2 scaffolding). The uniqueness of this research lies in combining Miyazaki's classification of students' proof types with Anghileri's scaffolding framework, implemented within the Indonesian context. This integration provides both theoretical and practical contributions: theoretically, it refines our understanding of how proof types correlate with intuitive errors, and practically, it offers context-sensitive interventions for mathematics teacher education.

Based on Table 5, all subjects succeeded in proving up to the correct deductive proof (Proof A with correct answer) and also explained that suggesting and investigating are types of scaffolding

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that are often used to help students and researchers identify each student's difficulties in each type of proof. With this scaffolding, researchers can ensure students' understanding of the problem through investigative questions. In addition, suggesting and investigating help guide students in determining the steps that need to be taken when working on the problem. This is in line with the research results of Siregar and Fauzi (2017) and Izzah and Azizah (2019) showing that students' problem-solving abilities can increase along with the development of mathematical reasoning through the help of scaffolding suggesting and investigating. This type of scaffolding also helps students realize their difficulties when solving proof problems (Cintamulya & Rahayu, 2020).

Researchers also often ask students to justify their answers to each type of proof in order to strengthen understanding if the answer is correct or to be aware of errors if there are any. In some cases, negotiation of meaning is carried out when students have not realized the error even though they have been asked to provide an explanation. According to Zhang, Cao, Chan, & Wan (2022) found that negotiation of meaning is effective in helping students correct errors, especially in generating relevant numbers to solve problems. This finding supports the results of research related to the effectiveness of scaffolding to overcome students' difficulties in proving the problem. In addition, there are also other types of scaffolding that are also helpful, namely generating conceptual discussions, negotiating meaning, making connections, simplifying problems, and developing representative tools in overcoming difficulties because students have little intuitive understanding of the concept of proof.

These findings suggest that a structured scaffolding design, based on Anghileri's framework, can serve as the basis for a practical instructional model to support proof instruction. Rather than being an isolated remedial tool, scaffolding can be systematically embedded in lesson design, particularly when introducing complex proof tasks. For instance, teachers may initially prepare tasks with visual aids (level 1), facilitate guided questioning and dialogue (level 2), and gradually move toward abstract conceptualization (level 3). The success of the four case participants in transitioning from incorrect to correct deductive proofs demonstrates the potential of this scaffolding framework to be applied in classroom settings. Future instructional designs can adapt this model to support a broader group of learners with varying levels of intuitive understanding.

This study offers instructional implications for mathematics educators aiming to improve students' proof construction abilities. The scaffolding strategies implemented here can be structured into a repeatable classroom model. Educators can utilize a phased approach to address intuitive misconceptions, beginning with task design and continuing through interactive explanation and conceptual generalization. Such a model not only addresses immediate student difficulties but also promotes long-term development of mathematical reasoning. Furthermore, incorporating tools like dynamic geometry software (e.g., GeoGebra) into each scaffolding phase may enhance student visualization and engagement.

These findings indicate that some students' errors stem from a fundamental misunderstanding of geometric definitions, such as failing to distinguish between squares, rectangles, and arbitrary quadrilaterals. Consequently, what may appear as intuitive understanding is often a misapplied or

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overly narrow understanding of shape properties. Clarifying these definitions and explicitly confronting misconceptions during instruction is essential before expecting students to reason intuitively or construct valid geometric proofs.

CONCLUSIONS

In conclusion, the results of this study indicate that the deductive method is the most commonly used by students in mathematical proofs, with 71% of participants opting for it due to its reliance on formal reasoning and the deductive-axiomatic approach. Among the four types of proofs classified by Miyazaki, Type A was the most frequently used, with 64 students favoring the use of functional language to structure their arguments systematically and logically. However, 18 pre-service teachers still experienced difficulties, not due to a complete lack of intuitive understanding, but because of their inability to accurately interpret geometric definitions and translate their intuitions into valid formal proofs. To address these challenges, scaffolding strategies, including steps such as suggesting and investigating, explaining and justifying, fostering conceptual discussions, negotiating meaning, making connections, simplifying problems, and developing representative tools, proved effective. Given the structured nature of the tasks and the clear categorization of proof types, this approach can be adapted by mathematics educators in both pre-service teacher training and secondary school settings.

As a recommendation, integrating GeoGebra technology into scaffolding strategies offers significant potential to enhance students' understanding of mathematical proofs. For instance, a GeoGebra task could involve dynamically dragging the vertices of a quadrilateral while observing how the intersection of midpoints behaves, enabling students to visually explore the conditions under which PO = OR and QO = OS. This process promotes both intuitive insight and formal deductive reasoning, as such interactive visualizations help bridge the gap between visual understanding and formal proof construction. By providing dynamic representations, interactive explorations, and step-by-step demonstrations, GeoGebra makes abstract mathematical concepts more accessible, meaningful, and engaging. Future research should examine the effectiveness of these scaffolding strategies in other mathematical domains, such as algebra and calculus, and explore how digital tools like GeoGebra can be optimized to support proof development in various learning contexts. However, this study has several limitations. It was conducted at a single institution with a relatively homogeneous participant group, which limits the generalizability of the findings. Moreover, the qualitative case study approach involved a small number of participants, which may introduce subjectivity despite the use of validated analytical frameworks. To improve the reliability and broader applicability of future findings, subsequent studies should involve larger and more diverse samples and consider using mixed-method approaches that combine qualitative insights with quantitative validation.

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REFERENCES

[1] Aisyah, N., Susanti, E., Meryansumayeka, M., Siswono, T. Y. E., & Maat, S. M. (2023). Proving geometry theorems: Student prospective teachers' perseverance and mathematical reasoning. *Infinity Journal*, 12(2), 377-392. https://doi.org/10.22460/infinity.v13i1.p119-138

[2] Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education,* 9(1), 33-52. https://doi.org/10.1007/s10857-006-9005-9

[3] Antonini, S. (2019). Intuitive acceptance of proof by contradiction. *ZDM*, *51*(5), 793-806.

[4] Arieli, O., van Berkel, K., & Straßer, C. (2024, March). Defeasible normative reasoning: A proof-theoretic integration of logical argumentation. In *Proceedings of the AAAI Conference on Artificial Intelligence* (Vol. 38, No. 9, pp. 10450-10458).

[5] Belland, B. R. (2017). *Instructional scaffolding in STEM education: Strategies and efficacy evidence* (p. 144). Springer Nature.

[6] Chin, K. E., & Fu, S. H. (2021). Exploring the Implementation of an Intervention for a Pupil with Mathematical Learning Difficulties: A Case Study. *Journal on Mathematics Education*, 12(3), 531-546. http://doi.org/10.22342/jme.12.3.14473.531-546

[7] Cintamulya, I., & Rahayu, P. (2020). Scaffolding process based on students diagnostic difficulties in proving group problems by using mathematics mapping. In *Journal of Physics: Conference Series* (Vol. 1422, No. 1, p. 012012). IOP Publishing.

[8] Ding, L., Reay, N., Lee, A., & Bao, L. (2011). Exploring the role of conceptual scaffolding in solving synthesis problems. *Physical Review Special Topics—Physics Education Research*, *7*(2), 020109.

[9] Firmasari, S., & Sulaiman, H. (2019). Kemampuan pembuktian matematis mahasiswa menggunakan induksi matematika. *Journal of Medives: Journal of Mathematics Education IKIP Veteran Semarang*, *3*(1), 1-9.

[10] Haj-Yahya, A., Hershkowitz, R., & Dreyfus, T. (2023). Investigating students' geometrical proofs through the lens of students' definitions. *Mathematics Education Research Journal*, 35(3), 607-633. https://doi.org/10.1007/s13394-021-00406-6

[11] Hartono, S., Siswono, T. Y. E., & Ekawati, R. (2024). From Informal to Formal Proof in Geometry: A Preliminary Study of Scaffolding-Based Interventions for Improving pre-service Teachers' Level of Proof. *Mathematics teaching Research Journal*, 16(2), 48-62.

[12] Hartono, S., Siswono, T. Y. E., Ekawati, R., & Bonyah, E. (2025). Using the Moore's

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theory to explain pre-service teachers'difficulties in proving of the triangle sum theorem. *Jurnal Ilmiah Ilmu Terapan Universitas Jambi*, 9(1), 104-118.

[13] Indrawati, I. (2017). Pengaruh Metode Scaffolding Berbasis Konstruktivisme terhadap Hasil Belajar Matematika. *Journal of Medives: Journal of Mathematics Education IKIP Veteran Semarang*, *1*(1), 9-16.

[14] Isnani, I., Waluya, S. B., Dwijanto, D., & Asih, T. S. N. (2025). Grow proof ability student through mathematical working space strategies. *Edelweiss Applied Science and Technology*, 9(4), 1240-1251.

[15] Isran, D., Susanta, A., Rahimah, D., & Syafri, F. S. (2025). Exploring Mathematical Proofs and Solutions in Higher Education: A Case Study on Real Analysis. *Jurnal Pendidikan Matematika*, 19(1), 163-180.

[16] Izzah, K. H., & Azizah, M. (2019). Analisis kemampuan penalaran siswa dalam pemecahan masalah matematika siswa kelas IV. *Indonesian journal of educational research and review*, 2(2), 210-218.

[17] Köğce, D., Aydın, M., & Yıldız, C. (2010). The views of high school students about proof and their levels of proof (The case of Trabzon*). Procedia-Social and Behavioral Sciences*, 2(2), 2544-2549.

[18] Lajos, J. (2023). An updated conceptualization of the intuition construct for mathematics education research. *The Journal of Mathematical Behavior*, 71, 101080.

[19] Mahfudy, S. (2017). Strategi pembuktian matematis mahasiswa pada soal geometri. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, 1(1), 31-40.

[20] Miyazaki, M. (2000). Levels of proof in lower secondary school mathematics. *Educational Studies in Mathematics*, 41(1), 47-68.

[21] Moore, R. C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics*, 27(3), 249-266.

[22] NCTM. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

[23] Qomariyah, S., & Rosyidah, U. (2022). Kesulitan Belajar Mahasiswa Pendidikan Matematika Pada Mata Kuliah Analisis Real. *Jurnal Educatio FKIP UNMA*, *8*(2), 396-400.

[24] Siregar, N. H., & AminFauzi, K. M. (2017). The Difference between Mathematical Reasoning Ability Improvements by Learning with Meta Cognitive Approach Aided Probing and Prompting Techniques in SMP Negeri 4 SeiSuka. *World*, 4(1).

[25] Stylianides, G. J., Stylianides, A. J., & Moutsios-Rentzos, A. (2024). Proof and proving in school and university mathematics education research: a systematic review. *ZDM–Mathematics Education*, *56*(1), 47-59.

[26] Zhang, S., Cao, Y., Chan, M. C. E., & Wan, M. E. V. (2022). A comparison of meaning negotiation during collaborative problem solving in mathematics between students in China and Australia. *ZDM–Mathematics Education*, 54(2), 287-302.

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APPENDIX
Lesson Plan: Scaffolding-Based Instruction for Geometric Proof

1. General Information
- Subject: Geometry
- Level: Pre-service Mathematics Teachers (Undergraduate)
- Topic: Proof of Midpoint Theorem in Quadrilaterals
- Duration: 2 × 50 minutes (100 minutes)
- Learning Approach: Scaffolding (Anghileri, 2006)
- Proof Framework: Miyazaki's Proof Types (A–D)

2. Learning Objectives
By the end of the lesson, students are expected to:
1. Understand the concepts of midpoint and intersection of line segments.
2. Identify misconceptions related to geometric intuition.
3. Construct a valid geometric proof using deductive reasoning.
4. Transition from intuitive reasoning to formal proof.

3. Learning Materials
- Definition of midpoint
- Intersection of line segments
- Properties of quadrilaterals
- Problem:
  Given a quadrilateral ABCD, with midpoints P,Q,R,S on each side. If PR and QS intersect at O, prove that PO= and QO= OS.

4. Learning Media
- Whiteboard / digital board
- Worksheet (proof task)
- Dynamic geometry software (e.g., GeoGebra) *(optional but recommended)*

5. Learning Activities

Phase 1: Introduction (10 minutes)
- Lecturer presents a quadrilateral diagram.
- Ask students:
  - *What is a midpoint?*
  - *What happens when two lines intersect?*
- Identify initial intuitive responses.

Phase 2: Core Activities (80 minutes)
Level 1 Scaffolding: Environmental Provisions (20 minutes)
Activities:

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- Provide students with a structured diagram of quadrilateral ABCD.
- Students identify:
  - Midpoints
  - Line segments PR and QS
- Use visual aids (diagram or GeoGebra).

Scaffolding Strategies:
- Sequencing tasks
- Providing structured worksheets
- Visual representation

Expected Outcome:
Students recognize key elements of the problem.

Level 2 Scaffolding: Interactive Support (30 minutes)
Activities:
- Lecturer guides students through probing questions:
  - *Why do you think point O is (or is not) a midpoint?*
  - *What properties define a midpoint?*
  - *Can intersection alone guarantee equal segments?*
- Students explain their reasoning.

Scaffolding Strategies:
- Probing and prompting
- Explanation and justification
- Simplifying the problem
- Negotiation of meaning

Expected Outcome:
Students begin identifying misconceptions (e.g., “intersection = midpoint”).

Level 3 Scaffolding: Conceptual Development (30 minutes)
Activities:
- Students attempt to construct a formal proof.
- Lecturer encourages:
  - Connecting prior knowledge (midpoint properties, parallel lines, triangles)
  - Representing relationships symbolically

Scaffolding Strategies:
- Generating conceptual discussion
- Making connections
- Developing representations

Expected Outcome:
Students construct a valid deductive proof (Proof A).

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Phase 3: Closing (10 minutes)

- Students reflect:
  - *What was your initial intuition?*
  - *How did your understanding change?*
- Lecturer summarizes:
  - Difference between intuition and formal proof
  - Importance of correct conceptual understanding

6. Assessment

a. Process Assessment

- Student participation in discussion
- Ability to respond to probing questions

b. Product Assessment

- Written proof evaluated based on:
  - Logical structure
  - Correct use of definitions
  - Valid conclusion

7. Indicators of Success

- Students no longer assume intersection implies midpoint
- Students correctly apply midpoint definition
- Students construct deductive proofs (Proof A)

8. Notes for Implementation

- Common student errors:
  - Misinterpreting midpoint
  - Overgeneralizing from special shapes (square/rectangle)
  - Assuming equal division from intersection
- Suggested support:
  - Use multiple diagrams (not only symmetric shapes)
  - Encourage justification, not assumption

9. Extension Activity (Optional)

- Use GeoGebra to dynamically move vertices of quadrilateral
- Observe whether PO= OR and QO= OS always hold
- Discuss why visual evidence must be supported by proof

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Developing Adaptive Reasoning in Teacher Preparation: Elementary Pre-service Teachers' Perspectives of Writing in Mathematics

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Abstract: A growing body of research suggests that preservice teachers (PSTs) need opportunities to engage with writing in mathematics to understand its role in promoting adaptive reasoning. This study explores how elementary PSTs perceive and implement authentic, problem-based writing tasks in mathematics classrooms. Drawing on socio-ecological theory, I examine individual, interpersonal, institutional, community, and policy influences on beliefs. Thirty-two PSTs completed pre- and post-questionnaires during a mathematics methods course; four participated in interviews, and final projects were analyzed for evidence of writing integration. Findings reveal that PSTs deepened their understanding of writing to explain and justify thinking and expressed strong support for use in elementary math. These results highlight key considerations for preparing teachers to integrate writing meaningfully into mathematics instruction.

Keywords: Writing in Mathematics, Adaptive Reasoning, Teacher Preparation, Strands of Mathematical Proficiency

INTRODUCTION

Teachers have a profound responsibility to meet grade-level standards and proficiency on assessments, prepare for the following grade, and, more importantly, ensure that the students can apply their knowledge to authentic tasks; all within one academic year. There is an ongoing debate as to whether the depth of understanding of a limited number of concepts *or* exposure to a broad range of mathematical concepts and procedures is more valuable and transferable to a student's application of knowledge (Eshetu et al., 2022; National Research Council, 2001; Rittle-Johnson & Alibali, 1999). "Mathematical understanding and procedural skill are equally important, and both are available using mathematical tasks of sufficient richness" (National Governors Association Center for Best Practices, 2010, p. 6). So, how do we ensure that both occur within the context of a single academic year?

Gaining more minutes in the day is *not* within a teacher's control; however, making the most efficient use of time for rigorous learning *is* within the manageability of teachers. Teacher preparation programs can prepare future teachers to manage their time effectively by teaching and practicing the use of writing in mathematics.

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Writing is an uncommon practice in mathematics instruction and assessment (Kim et al., 2024; Teledahl et al., 2024), and in-service teachers have little experience or training using writing in the mathematics classroom (Cheung, 2013). Therefore, it is up post-secondary institutions to include writing in the teacher preparation programs actively to begin to change PSTs' and elementary students' views of mathematics. Teachers use teaching methods that they saw as a student (Cody et al., 2023), referred to as the "apprenticeship of observation," which has an impact on how teachers plan and teach their own lessons (Loewenberg Ball et al., 2008; Philipp et al., 2002). Teacher preparation programs should continue to develop ways to change mindsets with the use of high-quality models of teaching mathematics for PSTs to better understand how they can implement mathematical reasoning and procedural skill.

Writing in mathematics supports procedural fluency by encouraging students to articulate, justify, and reflect on each step they take, which deepens procedural understanding and reinforces accuracy, efficiency, and flexibility, a claim supported by findings between written reasoning and fluency (Andal & Andrade, 2022). Procedural fluency refers to students' ability to carry out mathematical procedures accurately, efficiently, and flexibly, and is strengthened when paired with adaptive reasoning through tasks that require explanation and justification (National Research Council, 2001). Adaptive reasoning is the ability to think through logically, reflect on, and explain one's solution in mathematical situations (National Research Council [NRC], 2001). Adaptive reasoning enables students to make sense of mathematical procedures, apply known procedures to new problems, and effectively communicate their reasoning to others. Despite its documented benefits in supporting mathematical thinking (Bangert-Drowns et al., 2004), writing remains underused in mathematics classrooms due to teachers' limited preparation (Wilcox & Monroe, 2011). While research has often examined in-service teachers' instructional practices, less is known about how PSTs develop beliefs and practices related to writing in math.

This study addresses that gap by exploring the impact of a mathematics methods course on PSTs' perspectives, instructional planning, and intentions related to writing in mathematics instruction.

The research will examine the experiences of PSTs and their willingness to incorporate writing into their future mathematics classrooms, regardless of their prior experiences. Specifically, it explores how participation in a mathematics methods course influences their perspectives on the role of writing in mathematics instruction. By focusing on explanatory writing for justification and reflection, the study investigates how pre-service teachers perceive writing as an essential component of conceptual understanding and adaptive reasoning. To understand the application and connection to the field, this study also analyzes participants' mathematics lesson plans for evidence of independently choosing to include writing tasks that required their *own* students to justify and explain their answers.

Research Questions:

1. To what extent do pre-service teachers independently integrate writing, particularly for explanation and justification, into their mathematics lesson plans?

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2. What shifts in thinking do pre-service teachers report about how they were taught mathematics versus how they plan to teach it?

When students write explanations and justifications of their mathematical thinking, they engage in adaptive reasoning, one of the strands of mathematical proficiency (National Research Council, 2001). Such writing processes enable students to move beyond computational thinking to genuine sense-making in mathematics (National Research Council, 2001). However, despite evidence supporting the value of writing for mathematical thinking, the practice of using writing as a teaching and learning tool in the mathematics classroom is not widespread. Teachers' lack of experience, confidence, or preparation in using writing in mathematics is one of the reasons for this deficit. Teacher preparation programs are an important source of knowledge and experience for future teachers to acquire practices and pedagogical content knowledge that can be used in their future classrooms. PSTs who arrive at teacher preparation programs often have a traditional understanding of mathematics that separates it from language arts, have had limited or no exposure to writing in mathematics, and receive no or limited explicit instruction on how to integrate mathematics and writing instruction.

The current study is significant because it contributes to the existing body of research on teacher preparation by examining the effects of participating in a mathematics methods course on PSTs' beliefs, experiences, and practices related to writing in mathematics. Previous research has primarily investigated in-service teachers' instructional practices or attitudes towards writing in mathematics but has not sufficiently explored teacher preparation in this area (Kosko, 2016). Moreover, most existing research has used limited quantitative measures to study teachers' instructional practices or attitudes, such as surveys or self-report data that do not capture the details of teacher learning or the complexities of instructional decision making (Kosko, 2016). This study employs a mixed-methods quasi-experimental design, gathering information from multiple data sources (e.g., pre- and post-questionnaires, interviews, and authentic lesson plans) to explore the evolvement in participants' beliefs, experiences, and practices after participating in the methods course (Darling-Hammond, 2006; Zeichner, 2010). Moreover, by grounding the research in Bronfenbrenner's socio-ecological theory, the current study can extend knowledge on the subject by illustrating the interplay between individual characteristics, the learning environment, and the larger system in the formation of teachers' evolving conceptions of writing in mathematics. Results from this study can inform teacher preparation programs that are interested in planning and/or revising mathematics methods courses to intentionally support preservice teachers to write effectively in mathematics in ways that can benefit their students' learning of mathematics.

LITERATURE REVIEW

Mathematical Proficiency

Language Arts curricula often embrace the application of deep thinking and analysis while reading a text, a skill focused on as early as pre-kindergarten (Burbach et al., 2004). Expectations for higher-order thinking are often unconsciously minimized when it comes to learning math (Stein et al., 2009);

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the tradition of how the subject has been taught is partly to blame (Shepard, 1991). Teachers often untangle difficult mathematics problems for students, rather than requiring the work to be carried out by students. Mathematics teachers frequently do cognitive heavy lifting when problems seem "hard" or time is short. For example, instead of setting up students to do the math, a teacher may decompose a problem into several steps and demonstrate how to solve it when students start to struggle (Hiebert & Wearne, 1993). This method is especially true in classrooms focused on upcoming standardized tests, as state accountability measures encourage teachers to prioritize procedural accuracy and curriculum completion over student understanding (Stigler & Hiebert, 1999). Stein et al. (1996) found that although some teachers use high cognitive demand tasks at the beginning of a unit, most eventually reduce the cognitive demand by "scaffolding down," or removing, changing, or solving the problem themselves.

The Strands of Mathematical Proficiency (National Research Council, 2001) elaborate on ways that students can demonstrate an understanding of mathematics, rather than merely conducting procedures. Pre-service teachers have been shown to lack conceptual reasoning in mathematics and frequently have below-standard self-perceptions about their mathematical abilities (Isiksal et al., 2009). Their knowledge tends to be fragmented and insecure, making it difficult to move fluidly between algorithms, verbal explanations, and representations (Kajander, 2010; Ball, 1988). According to the National Council of Teachers of Mathematics (2000), mathematically proficient students can demonstrate knowledge, skills, abilities, and beliefs based on a body of research discovered in cognitive psychology and mathematics education. The Strands of Mathematical Proficiency (National Research Council, 2001) include:

1. Conceptual Understanding
2. Procedural Fluency
3. Strategic Competence
4. Adaptive Reasoning
5. Productive Disposition

This study focuses on adaptive reasoning although the five strands of mathematical proficiency are equally important and interact with each other. The National Research Council (2001) reiterates that the strands are "mutually supportive. Each supports and is supported by the others" (para. 56). Students take the place of passive observers to active problem solvers through writing to explain and justify their ideas; reasoning and writing are closely interwoven, and concepts that may include lessons from the past, current, and future lessons as well as other modes of learning that can be used to show that they understand. Barham (2020) proposed the order of the five mathematical reasoning strands as productive disposition, adaptive reasoning, conceptual understanding, strategic competence, and procedural fluency. The proposed order of the strands was based on Barham's (2020) finding that most teachers do not feel confident and motivated enough when teaching mathematics, and as a result, a teacher must first develop a productive disposition as the foundation to implementing the other strands of mathematics-related skills and knowledge. Productive disposition creates a positive mathematical identity and a belief that students can learn mathematics. The teacher must have strong beliefs and

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positive attitudes about teaching mathematics to have a deep engagement in the following cognitive abilities (Barham, 2020). This was the starting point, followed by adaptive reasoning and conceptual understanding that is used to justify and make sense of mathematics, and finally before problem solving and procedural fluency. The sequence of the cognitive strands that was used is because of a natural process of learning as beliefs and attitude that are affective components of engagement in teaching, affect the cognitive aspect. Conceptual and strategic strands are best left to be taught after a teacher's affective and reasoning competencies (National Research Council, 2001). The six cognitive strands were the framework for the interview protocol. They were also used as analytic codes to determine the shifts in participants' beliefs about mathematics and mathematics planning practices.

Mathematical writing to explain and justify includes opportunities for students to share mathematical thinking to demonstrate their understanding. By some scholars, showing a correct understanding of the concept comes in the form of writing (Kilpatrick et al., 2001). Writing can help to deepen mathematical understanding and make thinking visible, so it can be used to both encourage and assess conceptual knowledge. Mathematical writing may ask students to organize ideas, justify reasoning, and explain processes, which are aligned with dimensions of mathematical proficiency (National Research Council, 2001). However, Rittle-Johnson et al. (2015) state that "conceptual knowledge can be implicit or explicit, and thus not verbalizable" (p. 588). Teachers that work with students that have reading, writing, or language-based differences in learning must consider this. If the writing is the only or primary form of evidence that is used to determine reasoning, it can be considered construct-irrelevant variance (essentially measuring the writing and not the mathematical thinking) (Abedi, 2004; Solano-Flores & Trumbull, 2003). A teacher may choose to use writing to support reflection and reasoning but should do so in a flexible manner and with the support of other modes (e.g., visual model, oral explanation, manipulatives, gestures). Researchers found that most teachers expected their students to write in math classes at least once a week and that the teachers structure the writing tasks they assign across types of tasks (Arsenault et al., 2024; Gjøvik et al., 2024) (e.g., exploratory, argumentative), and the writing is typically assessed with indicators such as logical coherence. These types of writing tasks support what is called adaptive reasoning or "capacity for logical thought, reflection, explanation, and justification" (National Academies of Sciences, Engineering, and Medicine, 2001, p. 6). When students are asked to justify and explain their mathematical thinking, they show stronger mathematical problem-solving skills (Boaler, 2002; Hiebert & Grouws, 2007). Therefore, strategies that give students opportunities to engage in adaptive reasoning are vital to equitable instruction because the tasks and explanations that ask students to explain their mathematical strategies invite students from all backgrounds to explain and share their mathematical strategies (Kazemi & Stipek, 2001). Mathematical writing is one tool intended to facilitate students being able to explain and justify their thinking. Explaining, for instance, can be thought of as simply giving an account of the steps it takes to complete a problem, and justifying involves proving why the strategy was used (Wagganer, 2015). For this reason, students have an obligation to justify their reasoning, as it will promote further understanding (Rawding & Wills, 2012). Writing in math has also been found to deepen students' learning as it requires students to think carefully at each step in a process, instead of it being about memorizing the steps of a procedure (Alleman, 2010; Urquart, 2009).

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Example:

A fourth grader answers a fill-in-the-blank question ⅘__⅔ with a > symbol. The test design allowed the student to answer several questions correctly without necessarily providing meaning (e.g., multiple choice, a key on the side), as the student was interviewed by the teacher and asked to explain his thinking, the student responded that he knew the "correct" answer because 4 is bigger than 2 (numerator) and 5 is bigger than 3 (in the denominator). The teacher would not have known a student's misconceptions without having to explain their thinking. A more authentic problem could ask:

Carson and Amelia both walk to school every day. Carson walks ⅘ of a mile. Amelia walks ⅔ of a mile. Create a number sentence using <, >, or =. Who walks farther? *Explain and justify* how you arrived at the correct answer.

Writing makes student thinking transparent. To authentically implement writing in math classrooms, it must be a part of the teacher's education experience. Kuzle (2013) identified that pre-service teachers that strongly agreed that writing and doing mathematics were related showed more positive attitudes in math instruction and learning. Colonnese & Casto (2023) also explored pre-service teachers' explanations of reasoning when selecting strategies, but to also illustrate that there needed to be more opportunities to write. Pre-service teachers should be partaking in the same writing that is expected of their students. Writing in Language Arts can show student identity and learning capacity; it can show this in mathematics as well. Lesson plans of participants and interviews were analyzed to identify PSTs' identification of the differences between explanation and justification and how it was used in the design of their instruction.

Teacher Preparation

Teachers play a significant role in the success of their students' education (Oliver & Reschly, 2007). Teacher preparation programs aim to improve the proficiency of student teachers to ensure quality education in schools (Oliver & Reschly, 2007). When colleges offer high-quality teacher preparation to pre-service teachers, these soon-to-be teachers are much more likely to provide higher-quality education to their students (Rokenes & Krumsvik, 2014). An emphasis on quality teaching must be grounded in distinguished teacher training as offered in teacher preparation programs (Darling-Hammond et al., 2005; Hattie & Yates, 2013). Although others have suggested that teachers' own experiences as learners can have a greater impact on their teaching than formal preparation (Feiman-Nemser, 2001), integrated coursework has the potential to re-shape those prior beliefs and build more effective instructional practices (Mainali, 2022).

In a teacher development study, McCarthy (2010) found that pre-service teachers could develop the capacity and foster a positive disposition toward using writing in mathematics instruction, even though these pre-service teachers had no prior experience with using writing-to-learn strategies before enrolling in the teacher preparation program. Similarly, Brownlow (2021) reported that

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early childhood pre-service teachers often lacked confidence when framing and solving word problems, and that structured opportunities to engage with authentic, cognitively demanding tasks were necessary for developing their mathematical reasoning and confidence.

Furthermore, based on the National Council on Teacher Quality (2022), although the quantity of mathematics courses taken in teacher preparation programs has increased, "substantial variation between programs in the rigor and quality of their pedagogical content preparation" (p. 5), so further evidence-based reform is required. Teaching writing has had comparable evaluations. Although writing instruction is a part of the elementary curriculum, teacher preparation programs have historically provided limited coursework in the pedagogy of writing. Instead of having a course dedicated to writing instruction, it has often been integrated into a more general literacy methods framework that has focused more on reading than writing (Brehmer, 2023). Practice-based simulations have been found to increase pre-service teachers' confidence in instruction and help them develop the more specific feedback strategies that their students need (Gillespie Rouse et al., 2023). These experiences are vital, as many pre-service teachers begin their programs with limited confidence or with dated preconceptions about writing instruction (Dace, 2015).

Teacher effectiveness has been linked to teacher preparation programs (Boyd et al., 2009). Pre-service elementary teachers have views about the teaching of mathematics that have been shaped in their early classroom experiences (Lloyd, 2006). They may see mathematics as a decontextualized and static set of procedures and algorithms and then use these methods in their own teaching (Coffey, 2004; Smith, 1996). To be successful teachers of writing and mathematics, pre-service teachers need experience with the instructional strategies that they will later use in the classroom, including working with writing-to-learn approaches and open-ended tasks. Prior research shows a positive relationship between teachers' efficacy beliefs about teaching math and their effectiveness with students (Guskey, 1988; Ross, 1994). Therefore, pre-service teachers need to be aware of and unlearn previous beliefs before internalizing new ideas about mathematics instruction (Piaget, 1954). Steele (1994) argued, "If the teaching of mathematics is to change, then teachers' conceptions about mathematics and mathematics teaching and learning must change first" (p.1). Similarly, the most recent research has offered evidence supporting a positive impact of teacher preparation on pre-service teachers' ability, self-confidence, and preparedness for the classroom in elementary mathematics instruction, if that preparation is well-structured, supported, and delivered effectively (Fu & Kartal, 2023; Hood, 2023).

International work also indicates that writing instruction is needed in teacher preparation. Selek (2024) analyzed 28 pre-service teachers taking a mathematical literacy elective in Turkey. Participants successfully designed authentic problems but had issues with contextualization and originality. This work called for "richer, literacy-focused, task design" in preparing mathematics teachers (Selek, 2024, p. 4). In Australia, Colonnese, and Casto (2023) explored the written strategies of 27 pre-service teachers taking a mathematics methods course. Most participants included justifications in problem-solving explanations, but the quality of reasoning differed. The findings support this study's examination of how a mathematics methods course affects pre-service teachers' beliefs, practices, and intentions to use writing in mathematics.

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CURRENT STUDY

Theoretical Framework

The present study was theoretically framed using Bronfenbrenner's (1979) socio-ecological model of human development, which comprises multiple interrelated systems. I adapted the model to focus only on the beliefs and practices of PSTs regarding writing in mathematics.

At the individual level, pre-service teachers' beliefs and prior K–12 and college mathematics experiences often shaped how they viewed writing in mathematics, with many seeing it as unrelated to math and limited to English classes. These early experiences, which typically emphasized formulas and single answers over explanation and reflection, influenced their openness to using writing later in teacher preparation, though positive exposure to writing-rich instruction could support metacognitive growth and conceptual learning (Graham et al., 2020). Interpersonally, pre-service teachers were strongly influenced by former teachers and current educators, often replicating procedural, teacher-directed practices unless provided with modeled examples and feedback that showed how writing could deepen mathematical reasoning (Wellberg, 2024; Ding et al., 2024; Cruz & Zahed, 2022). At the organizational level, program structures and coursework either constrained or encouraged writing, with evidence showing that teacher preparation programs embedding writing-to-learn practices improved pre-service teachers' efficacy and student outcomes (Zeichner & Bier, 2017; Herbert et al., 2019). Extending to the community, writing connected classroom mathematics to families and cultures, offering a way to strengthen home–school communication and promote equity through valuing students' lived experiences (Suh & Calabrese, 2025). At the societal level, although standards rarely emphasize writing explicitly, it has gained recognition as a powerful tool across STEM fields for fostering reasoning, problem-solving, and communication, reinforcing the importance of integrating writing into mathematics instruction (Graham et al., 2020; Herbert et al., 2019).

Conceptual Framework

The study is based on the Adaptive Reasoning strand from the Strands of Mathematical Profciency conceptual framework (National Research Council, 2001). Adaptive reasoning is one of the five strands of mathematical proficiency, and it "includes the capacity to think logically and reason, to justify and explain one's strategies and solutions, and to reflect on and refine one's understanding" (National Research Council, 2001, p. 57). Writing in mathematics has been shown to promote adaptive reasoning in the classroom because when students are required to write their thought processes and problem-solving strategies, as well as justify their work and solutions, they become reflective learners. The focus on pre-service teachers' perspectives of writing as a tool to help them both conceptualize and help students reach the level of adaptive reasoning, as well as explore what contexts impacted their practices, will be used to form recommendations for instruction that can be adapted to fit teacher preparation coursework.

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Purpose

The purpose of the present study is to offer the field both conceptual tools and empirical evidence regarding pre-service teachers' views on using explanatory writing to support mathematics instruction. The study focuses on the explanatory genre of mathematical writing, with special attention to the use of writing that is explicit and highlights step-by-step reasoning. The teacher candidates were expected to model this kind of writing for students and were encouraged to reflect on and practice explanatory writing as a strategy for communicating mathematical ideas. The PSTs also analyzed student work. A strong emphasis was placed on preparing pre-service teachers to use the adaptive reasoning strand of mathematical proficiency (one of five strands defined by the National Research Council) as a tool to support both conceptual understanding and procedural fluency. Explanatory writing in mathematics can meet both of those goals by having students explain their reasoning while rehearsing procedural steps.

The work is important because it focuses on a relatively scarce area of scholarship regarding ways to incorporate both reasoning and procedures into mathematics instruction. The study also demonstrates the importance of mathematics methods courses in teacher preparation programs. The research findings provide insights to support teacher educators' work and inform teacher preparation curricula. K–12 policymakers may also find the results informative for standards revisions that seek to improve mathematical reasoning using writing.

METHODS

Design

I employed a mixed-methods research design for this study. I used quantitative and qualitative data to provide a more in-depth and comprehensive understanding of the participants' experiences and perspectives. I triangulated the two types of data using one type to support and corroborate the other, which helped to increase the trustworthiness of the study (Creswell & Creswell, 2017; Creswell, 2021). The study's mixed-methods design was guided by Bronfenbrenner's (1979) socio-ecological model, which accounts for multiple levels of influence on individuals. I used quantitative survey items to collect data on individual-level constructs such as personal beliefs and past experiences with writing in mathematics. I used qualitative interviews and lesson plan data to better understand the interpersonal, institutional, and societal-level factors that impacted participants' shifting beliefs and instructional choices. I used this framework to make sense of the data, interpreting individual-level data within the context of the various systems that impacted the participants.

Course Description

Participants were pre-service teachers enrolled in a one-semester, spring Mathematics Methods Course for PreK-4 Education at a small public university in Pennsylvania. The 15-week course met three times per week for 50-minute sessions. As the researcher and instructor of the course, I

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conducted lectures, group work, field experience, and reflection assignments to address the mathematical content knowledge and mathematical teaching practices appropriate for the early childhood and elementary classroom. Key course objectives included:

- Strengthening mathematical knowledge aligned with PreK-4 standards
- Integrating explanatory writing as a strategy for mathematical reasoning, explanation, and communication
- Analyzing student work samples for adaptive reasoning
- Designing lesson plans that connect mathematics with literacy
- Connect the pedagogy learned within their class to their field placement

Weekly activities involved reading course texts and research articles, participating in collaborative writing and problem-solving tasks, analyzing classroom videos focusing on students' math writing, and engaging in fifteen hours of mathematics field placements in local elementary schools. The course culminated in a final lesson plan assignment, where pre-service teachers were encouraged, but not required, to embed writing into their math lessons.

Instructional content, materials, assignments, and rubrics were standardized and delivered consistently across two consecutive cohorts included in this study.

Participants

Thirty-two pre-service teachers (3% male, 97% female) enrolled in the Early Childhood Education Teacher Preparation program participated in the study. I purposefully selected participants from two cohorts enrolled in the mathematics methods course. Thirty participants reported no prior training in integrating writing into content areas, while two participants reported having received some initial training. Eight reported no previous experience writing in math as students; 24 reported some expertise.

All participants met the program admission criteria, including completion of at least 48 college credits, passing prerequisite education courses with a grade of B- or better, completing at least six credits in both college mathematics and English, and maintaining a minimum GPA of 2.8. Before this course, participants completed 90–100 hours of field placements. They were concurrently enrolled in a Mathematics and Reading Pedagogy Lab, which required an additional 30 hours of field placement.

I taught the mathematics methods course and served as the instructor for the reading, writing, and mathematics methods courses, as well as the pre-student and student teaching courses. Additionally, I am the Director of Field Placements. I have fourteen years of experience teaching elementary-level mathematics. To minimize variability and the influence of social context on participant responses, I consistently used standardized instructions and materials throughout the study. I also ensured participant anonymity to encourage honest and authentic sharing.

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Data Collection

Before data collection, I obtained written informed consent from all participants and informed them of their right to withdraw from the study at any time without penalty. I told the participants that I would have access to their pre-questionnaire responses. It was also made clear to the participants that I would not use their pre-questionnaire responses to influence the course grade or award any extra credit points. I collected data during two consecutive spring semesters. There were three sources of data:

- **Pre- and post-course questionnaires** were administered on the first and last days of the course, respectively. These included both Likert-scale and open-ended questions (see Tables 2 and 3). The questionnaire was developed by the researcher, reviewed by three faculty experts for clarity and alignment, and piloted with five pre-service teachers outside the course to ensure validity and clarity. Cronbach's alpha for Likert items was .82, indicating acceptable internal consistency.
- **Semi-structured interviews:** Midway through the semester, participants received an optional invitation to an interview via email. Four participants volunteered and were selected based on availability, academic performance, and teaching interests. Interviews followed a semi-structured protocol reviewed by four expert researchers for clarity and relevance. Interviews were recorded, transcribed, and analyzed using NVivo software (see Table 4).
- **End-of-semester lesson plan assignment:** Participants created original mathematics lesson plans. Although embedding writing was not a rubric requirement, the open-ended criteria allowed participants to incorporate writing strategies at their discretion. Lesson plans were analyzed and coded to identify the inclusion of writing components (see Table 5).

I designed the study instruments with Bronfenbrenner's socio-ecological model and the Adaptive Reasoning strand of mathematical proficiency in mind. Likert-scale items addressed individual experience and belief (microsystem), whereas open-ended survey items and interview protocols gathered information about how participants' relationships with instructors, curriculum design, and field experiences (mesosystem and exosystem) influenced their conceptualizations of writing in mathematics. Items that asked participants to reflect on their state's standards or education culture addressed the macrosystem. The study utilized multiple data sources, providing information about the individual, interpersonal, organizational, and institutional levels of influence on participants' beliefs and practices. Questionnaires captured individual-level beliefs and previous learning; interviews provided insight into interpersonal and organizational factors that might have influenced the teachers' beliefs and thinking; lesson plans revealed evidence of the application and integration of the teachers' beliefs and the influences of those around them within the classroom context (institutional level).

Data Analysis

Interviews and open-ended responses from the questionnaire were analyzed with the use of thematic analysis (Clarke and Braun, 2017). The strategy for coding the data was: 1) familiarization

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with the information received, 2) creating the initial codes, 3) searching for themes, 4) reviewing themes, and 5) producing the report (Braun et al., 2016). The software NVivo was used to enhance the data analysis.

Firstly, during the familiarization stage of data, 146 initial codes were produced, which reflects various aspects of the participants' perception of writing in math classrooms. Then, 32 categories were created by merging initial codes into groups with similar meanings. These groups are the pre-themes, and they offer a more structured approach to the analysis of the data. Finally, pre-themes were grouped into four major themes that are presented below:

Balancing Basic Skills and Conceptual Understanding

This theme was mostly about the issues that must be addressed by balancing basic skills and procedures with conceptual understanding. To be more specific, many participants were striving to maintain this balance in their writing activities. In a way, they were trying to blend procedural knowledge and understanding of mathematical concepts when it came to the tasks. At the same time, other participants talked about the main purpose of writing in math to improve conceptual understanding. In short, this theme demonstrates how participants have been speaking about this aspect of mathematics education either as something that needs to be addressed when assigning writing tasks or as a main goal that can be reached through writing.

Explaining and Justifying Reasoning

Another major point that was shared by many participants had to do with the ability of students to explain and justify reasoning by using writing as the medium for it. The common idea was that writing in math is not only about presenting the results but also about demonstrating an ability to think mathematically. When these participants were assigning writing tasks to students, the main purpose was the one described above.

Reconciling Previously Learned and New Methods Of Doing Math

The pre-service teachers also had issues with using writing in math to present their knowledge, and ability. Many of them had to face the challenge of learning new methods of doing math while also using those that they learned previously during the process. The use of writing in math was seen by these participants as a complicated process that needed to be thoroughly planned.

Cross-curricular Integration of Subjects

The last theme was mostly about the ways that writing in math can be a link between different subject areas. Pre-service teachers have understood that when they make their students complete writing tasks in math, they are also helping them to develop their literacy skills and potentially the knowledge in other subjects as well. In short, this theme is mostly about the benefits of cross-

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curricular integration. These themes were triangulated with questionnaire and lesson plan data to enhance trustworthiness (Flick, 2018).

I conducted the quantitative data analysis in phases:

1. Likert-scale responses from pre- and post-questionnaires were analyzed in Qualtrics to identify shifts in perceptions over the course.
2. Interview transcripts were coded by research question using NVivo.
3. Lesson plan data were compiled in a spreadsheet categorizing whether writing was incorporated.

To address trustworthiness, I attempted to strengthen validity, credibility, dependability, and confirmability in several ways. I employed methodological triangulation (i.e., drawing on three sources of data: questionnaires, interviews, and lesson plans) to enhance the dependability of this study.

I also used NVivo, a software program that supports coding and thematic analysis, to encourage analytic consistency. I engaged in peer debriefing with two colleagues in teacher education to further ensure coding consistency and thematic alignment. I maintained an audit trail to document analytic decisions, thereby supporting the confirmability and dependability of this study. Member checking (Lincoln & Guba, 1985) occurred when participants were allowed to review their interview transcripts for accuracy, thereby supporting the credibility of this study.

I also considered discrepant cases, such as participants who did not report a shift in attitudes about writing in mathematics or lesson plan writing, and those who did not include writing in their mathematics lesson plans. These were explained in depth to further explore possible resistance to the course and/or limitations on the instruction's ability to influence participant lesson plans.

I grounded the coding in theory and aligned it with the study's guiding frameworks. I coded responses to interview and questionnaire data both inductively, by generating codes from the data, and deductively, by applying a priori codes drawn from Bronfenbrenner's ecological levels (e.g., individual learning history, instructor modeling, institutional structures) and aspects of adaptive reasoning (e.g., explanation, justification, procedural flexibility). This approach allowed me to identify both anticipated and emergent themes through the lens of the study's theoretical framework.

RESULTS

Data Source #1: Pre-Questionnaire/Post-Questionnaire

I asked PSTs on both the pre-questionnaire and post-questionnaire (Figure 1) whether they agreed or disagreed that writing should take place in mathematics class. The mean pre-questionnaire mean was 2.91, and the post-questionnaire mean was 4.64. Table 1 shows that ten participants agreed that writing should take place in mathematics class, eleven participants either disagreed or strongly disagreed, and twelve participants were neutral before the course. After the course, thirty-one

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students agreed or strongly agreed with the statement that writing should take place in mathematics class, and no one disagreed.

Participants responded to the item that asked if they had experience writing in mathematics class. Twenty-three participants strongly disagreed that they had experience writing in mathematics class during their schooling, and six participants were neutral. Twenty-two participants agreed, and ten participants strongly agreed that they had gained experience teaching writing in mathematics after the course; one participant was neutral. Only one participant strongly agreed with the statement that they had experience teaching writing in math prior to the course. There was a significant increase in the number of participants who agreed or strongly agreed that they had experience teaching writing in mathematics after the course. There was also a small number of participants who had any college-level exposure to writing in mathematics or instruction on how to teach writing in math before the course. Six participants disagreed, and twenty participants were neutral about whether they would implement writing in their future math classrooms before the course. In contrast, two participants disagreed, while thirty-one participants agreed or strongly agreed after the course, indicating an increase in their willingness to integrate writing.

The qualitative open-ended responses (Table 2) provide context for the quantitative changes in these variables. PSTs wrote responses to a question about their experience with writing in math class and their plans to integrate writing before and after the course, which provides essential contextual information. Before the course, many participants reported having little to no experience with writing in mathematics during their schooling. They recalled completing only a few isolated and often superficial tasks, such as short writing prompts or word problems. Some pre-service teachers explained that they viewed math and writing as separate subjects during their school years. One participant noted, “Writing and math were two different subjects when I was in school.”

Post-course, participants described what they had learned about the purposes of and strategies for teaching writing in mathematics. They wrote that they learned to help students use writing to explain, justify, and think more deeply about mathematics. Comments such as “Explain and justify … it will develop a different level of thinking” and “Writing to explain and justify math ideas helps students understand better” suggest that participants changed their views about the pedagogical purposes and potential of writing to support mathematics learning. They also provided concrete examples of resources and tools they learned during the course that they could use to support their students, such as how to model writing tasks, use rubrics, or provide vocabulary posters, which suggested they were more confident and ready to teach writing in mathematics. Although some indicated that they would need more time to become completely comfortable, post-course comments suggested interest in continued learning and development in this area, with some participants commenting that the course “opened my eyes to a new idea” but they needed more practice.

Quantitative and qualitative data suggest that the course changed pre-service teachers’ beliefs, experiences, and future intentions related to writing in mathematics. The dramatic increases in participants' agreement with the importance of including writing in mathematics instruction, as well as their more detailed descriptions of specific teaching practices and rationales in the qualitative

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data, suggest that they began to see writing as an important part of mathematics instruction that supported student understanding rather than as something separate or unrelated to the subject.

Data Source #2: Semi-Structured Interviews

To further investigate students' responses, I conducted semi-structured interviews with four students who responded "somewhat" to questions or had indeterminate answers on the post-questionnaires. Table 3 summarizes the key ideas that were reported by more than one participant in response to the questions on the pre- and post-questionnaires. Participants' comments focused on how their ideas about writing in mathematics had changed after the methods course. The most frequently reported idea was that basic skills and writing need to be balanced in mathematics education. Respondents reported that pre-service teachers felt that students still needed to demonstrate basic math skills and procedures, that those skills should be retained, and that they should be integrated with writing. Several mentioned the use of a problem-based context. An additional salient idea was the need for students to explain and justify through writing. The final idea in Table 3, not surprisingly, is the one with the most overlap between questions. The importance of the vocabulary word wall for scaffolding student language development is an important idea that participants reported that they had learned from the course. The final idea, that there was overlap of subjects was also strongly emphasized in the responses by the pre-service teachers who were interviewed.

Data Source #3: End of Semester Assignment

For the third source of data, I collected an end of semester assignment. I served as the professor for the course and I assigned the pre-service teachers to design a mathematics lesson. The rubric for the assignment and our class discussions about how to design it did not contain any prompts or elements about writing. This ensured that any decision to include writing in their lesson plans was self-directed, and therefore an authentic measure of internalized beliefs and intentions.

The results, summarized in Table 4, are that most participants (27/33; 81.8%) included written products or activities as part of their lesson and required students to explain and justify mathematical thinking. This means that a large majority of pre-service teachers not only placed value on using writing to learn math but also perceived, and implemented, this type of writing in meaningful and pedagogically grounded ways.

A slightly smaller group (4 participants; 12.1%) included writing tasks or products in their lessons but did not require students to both explain and justify mathematical thinking. These lessons may have included labeling, recording steps, or other forms of sentence frames but did not include the richer student thinking. This result indicates an intermediate degree of implementation or of belief and confidence in the value of such rich use of writing. Two participants (6.1%) did not include any element of written products or assessments in their lesson plans. This finding of about 6% is consistent with the results from the pre-post questionnaire and the interview data which identified

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a small group of students who did not express or perceive a shift in beliefs and were ambivalent or unsure about the use of writing in math.

To conclude, this summative, performance-based assessment provides concrete evidence that the participants were able to transfer their beliefs and intentions into actual lesson design. As they had the agency to make this decision, most students created lessons that included writing explicitly to support student understanding, in particular the practices of explaining and justifying. These data, which corroborate the self-reported growth in experience and intent from the two previous sources of data, indicate that the course was successful in preparing the pre-service teachers to use writing to reason in elementary math classrooms.

Bronfenbrenner's socio-ecological framework, which emphasizes the interaction of multiple environmental systems shaping individual development (Bronfenbrenner, 1977), helps explain the shifts observed in pre-service teachers' beliefs and intentions. Individual level factors, such as prior experience with writing in math and personal attitudes, may be included as variables that experienced changed throughout the course, as reflected in the increase in mean questionnaire scores from 2.91 to 4.64. At the microsystem level, participants' classroom experiences during the methods course provided immediate learning contexts where they engaged with writing pedagogy. Interview data also showed how the participants negotiated the tension between skills and integration in writing, and how they were influenced by different educational settings and content experiences, an example of the mesosystem. Exosystemic and macrosystemic factors such as teacher preparation program standards, institutional policies, and societal beliefs about math instruction also influenced their writing intentions and beliefs.

DISCUSSION

Research Question 1: To what extent do pre-service teachers independently integrate writing, particularly for explanation and justification, into their mathematics lesson plans?

The end-of-semester assignment demonstrated that even though there was no specific directive to include writing, most of the students elected to include writing in their lessons. Of the 33 respondents, 31 included writing strategies and 27 of those included both explanation and justification.

Research Question 2: What shifts in thinking do pre-service teachers report about how they were taught mathematics versus how they plan to teach it?

Participants reported notable shifts in their thinking about mathematics instruction. Interviews revealed that they recognized differences between how they were taught mathematics, often emphasizing procedural skills, and how they plan to teach it, which now includes balancing basic skills with conceptual understanding and encouraging explanation and justification through writing. In addition to the changes in thinking and shifts in instructional approach that participants reported, these findings also support the common theme of the need to bridge previously learned and new ways of doing math. They also noted that cross-curricular integration is important and that multiple

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subjects must be taught together to best support student learning. Bronfenbrenner's socio-ecological model was useful in examining participant beliefs and practices with respect to writing as mathematical communication. At the intrapersonal level, the magnitude of change in participants' attitudes and self-reported teaching experience indicate that they experienced shifts in their beliefs and confidence in writing in math during the methods course. The microsystem, or immediate learning environment of the methods course, may have offered the participants opportunities to observe and practice strategies for writing instruction and may have thus contributed to their beliefs and practices. The mesosystem, or links between environments, can be seen in the recognition of cross-curricular connections between math and language arts and the awareness that writing can be interconnected. The exosystem, or broader educational systems and policies, could have influenced participants' awareness of professional standards and expectations. The macrosystem, or cultural and societal influences, might have been reflected in the overall attitudes and values surrounding math education and the importance of writing as a tool for mathematical communication. This study is important for teacher preparation programs as it suggests that teacher educators should consider influences at different ecological levels to support pre-service teachers' instructional development.

CONCLUSIONS

Pre-service teachers require a solid understanding of mathematical content to interpret student thinking effectively. Engaging pre-service teachers in assessing student work through writing enhances their mathematics learning and assessment skills (Namakshi et al., 2022). Authentic open-response questions, writing to justify thinking, and multiple solution pathways promote more profound understanding and productive struggle. Teacher preparation programs should emphasize these practices to move away from traditional methods and foster genuine problem solvers. Rather than only teaching the procedural fluency strand of mathematical proficiency or teaching this strand before any others, educators must sequentially place adaptive reasoning after productive disposition and before conceptual understanding, strategic competence, and procedural fluency (Barham, 2020). Moreover, adopting a societal approach to mathematics education that mandates writing to explain and justify mathematical thinking within standards may better support these efforts (Bronfenbrenner, 1977).

LIMITATIONS

The researcher noted limitations within the study. A limitation was that the audience for the pre-service teachers' data collection instruments was the course instructor. In turn, the pre-service teachers may have answered in a certain way, knowing that their instructor was reading and scoring the response. Researchers should conduct further studies involving additional professors and students enrolled in mathematics methods courses.

Despite many positive changes, the study also considered cases of resistance where some participants showed difficulty in integrating writing into mathematics. These discrepancies may reflect

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deeper socio-ecological influences such as mixed messages from standardized curricula or previous schooling experiences (Bronfenbrenner, 1979). The complex nature of teacher change necessitates time, repeated exposure, and support that extend beyond a single semester.

FUTURE DIRECTION

Future research should follow these cohorts into their classrooms to observe implementation practices. Additionally, integrating coursework with fieldwork and in-service teaching, alongside professional development for cooperating teachers, may foster a collaborative approach that supports writing in mathematics instruction far beyond time spent at the university (Feiman-Nemser, 2001; Weiss & Weiss, 2001).

REFERENCES

[1] Abedi, J. (2004). The No Child Left Behind Act and English language learners: Assessment and accountability issues. *Educational Researcher, 33*(1), 4–14. https://doi.org/10.3102/0013189X033001004

[2] Alleman J., Brophy J., Knighton B., Ley R., Botwinski B., Middlestead S. (2010). *Homework done right: Powerful learning in real-life situations*. Thousand Oaks, CA: Corwin Press.

[3] Andal, S. G. B., & Andrade, R. R. (2022). International Journal of Science Technology Engineering and Mathematics, 2(1), 1–25. https://doi.org/10.53378/352872

[4] Arsenault, T. L., Powell, S. R., & King, S. G. (2024). Mathematics-writing synthesis: Kindergarten through grade 12 mathematics-writing outcomes and instructional methods. *Reading and Writing, 38*(6), 1039-1081. https://doi.org/10.1007/s11145-024-10530-x

[5] Ball, D.L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, 8(1), 40-48.

[6] Bangert-Drowns, R. L., Hurley, M. M., & Wilkinson, B. (2004). The Effects of School-Based Writing-to-Learn Interventions on Academic Achievement: A Meta-Analysis. *Review of Educational Research*, *74*(1), 29-58. https://doi.org/10.3102/00346543074001029

[7] Barham, A.I. (2020). Exploring In-Service Mathematics Teachers' Perceived Professional Development Needs Related to the Strands of Mathematical Proficiency, *EURASIA Journal of Mathematics, Science and Technology Education, 16*(10), 1-18. https://doi.org/10.29333/ejmste/8399

[8] Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning* (Rev. ed.). Routledge. https://doi.org/10.4324/9781410606365

[9] Boyd, D., Grossman, P., Lankford, H., Loeb, S., & Wyckoff, J. (2009). Teacher preparation and student achievement. *Education Evaluation and Policy Analysis, 31*(4),

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416–440. https://doi.org/10.3102/0162373709353129

[10] Brehmer, L. (2023). *Elementary and special-education teacher preparation for writing instruction: A mixed-methods exploration* [Doctoral dissertation, University of Missouri-Columbia]. ProQuest Dissertations Publishing.

[11] Bronfenbrenner, U. (1979). *The ecology of human development*. Harvard University Press.

[12] Brownlow, L. (2021). *Framing word problems: Task confidence in early-childhood pre-service teachers.* Mathematics Teaching-Research Journal, Vol. 13(2).

[13] Burbach, M. E., Matkin, G. S., & Fritz, S. M. (2004). Teaching critical thinking in an introductory leadership course utilizing active learning strategies: A confirmatory study. *College Student Journal, 38*(3), 482–493.

[14] Cheung, Y. L. (2013). The impact of an in-service professional development course on writing teacher attitudes and pedagogy. *Journal of Pedagogic Development, 3*(1). Retrieved from https://www.beds.ac.uk/jpd/volume-3-issue-1/the-impact-of-an-in--professional-development-course-on-writing-teacher-attitudes-and-pedagogy

[15] Clarke. V., & Braun. V. (2017) Thematic analysis. *The Journal of Positive Psychology, 12*(3), 297–298.

[16] Cody, R. A., Boldt, G. T., & Gubbins, E. J. (2023). How Teachers Make Decisions in Response to Professional Learning: A Study on Grade 3 Differentiation in Mathematics. *Journal of Advanced Academics*, *35*(1), 156-185. https://doi.org/10.1177/1932202X231221143

[17] Coffey, D. C. (2004). Using alternative assessment to affect pre-service elementary teachers' beliefs about mathematics. In T. Watanabe & D. R. Thompson (Eds.), *The work of mathematics teacher educators: Exchanging ideas for effective practice AMTE MonographSeries,1,* 49–66.

[18] Colonnese, M. & Casto, A.R. (2023). Analysis of pre-service teachers' written mathematical reasoning. *Mathematics Teacher Education and Development (25),* 1, 23-42. https://files.eric.ed.gov/fulltext/EJ1382280.pdf

[19] Cruz, M., & Zahed, S. (2022). Writing to learn mathematics: A synthesis of the empirical literature. *Journal of Mathematical Behavior, 65*, 100909. https://doi.org/10.1016/j.jmathb.2021.100909

[20] Dace, K. M. (2015). *Pre-service teachers' self-efficacy for teaching writing: A phenomenological study* [Doctoral dissertation, Liberty University]. ProQuest Dissertations Publishing.

[21] Darling-Hammond, L. (2006). Constructing 21st-Century Teacher Education. *Journal of Teacher Education*, *57*(3), 300-314. https://doi.org/10.1177/0022487105285962

[22] Darling-Hammond, L., Holtzman, D. J., Gatlin, S. J., & Heilig, J. V. (2005). Does teacher preparation matter? Evidence about teacher certification, Teach for America, and teacher effectiveness. Education Policy Analysis Archives, 13(42).

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https://doi.org/10.14507/epaa.v13n42.2005

[23] Ding, M., Mochaourab, R., & Spiro, B. (2024). Promoting teaching improvement in elementary mathematics: A case with inverse relations. *Asian Journal for Mathematics Education*, *3*(2), 143-173. https://doi.org/10.1177/27527263241256678

[24] Eshetu, E. W., Atnafu, M., & Woldemichael, M. (2022). *The effectiveness of guided inquiry-based technology integration on pre-service mathematics teachers' understanding of plane geometry. Journal of Pedagogical Research, 6*(4), 84–100. https://doi.org/10.33902/JPR.202215241

[25] Feiman-Nemser, S. (2001). From Preparation to Practice: Designing a Continuum to Strengthen and Sustain Teaching. Teachers College Record, 103(6), 1013–1055. https://www.brandeis.edu/mandel/questcase/Documents/Readings/Feiman_Nemser.pdf

[26] Flick, U. (2018). Triangulation. In N. K. Denzin, & Y. S. Lincoln (5th Eds.), The SAGE handbook of qualitative research (pp. 777–804). SAGE Publishing.

[27] Fu, Y. & Kartal, O. (2023). Developing Pre-service Teachers' Self-Efficacy and Growth Mindset for Teaching Mathematics: Practices from a Mathematics Methods Course, *Mathematics Teacher Education and Development (25)*, 2, 1-19. https://files.eric.ed.gov/fulltext/EJ1412325.pdf

[28] Gillespie Rouse, A., Young, M. K., & Gifford, D. (2023). *Exploring relationships between preservice teachers' selfefficacy for writing and instruction provided in simulated elementary writing conferences*. *Frontiers in Psychology, 14*, Article 1214086. https://doi.org/10.3389/fpsyg.2023.1214086

[29] Gjøvik, T., Clemmensen, T., & Pors, S. (2024). Defining and measuring quality in students' mathematical writing: A systematic literature review. *Mathematics Education Research Journal*. Advance online publication. https://doi.org/10.1007/s13394-024-00501-4

[30] Graham, S., Kiuhara, S. A., & MacKay, M. (2020). *The effects of writing on learning in science, social studies, and mathematics: A meta-analysis.* Review of Educational Research, 90(2), 179–226. https://doi.org/10.3102/0034654320914744

[31] Guskey, T. R. (1988). Teacher efficacy, self-concept, and attitudes toward the implementation of Instructional innovation. *Teaching and Teacher Education*, 4(1), 63–69.

[32] Hattie, J.A.C. (2003). Teachers make a difference: What is the research evidence? Paper presented at the Building Teacher Quality: What does the research tell us ACER Research Conference, Melbourne, Australia. Retrieved from http://research.acer.edu.au/research_conference_2003/4/

[33] Hattie, J. & Yates, G. (2013). Understanding learning: Lessons for learning, teaching and research [Paper presentation]. How the Brain Learns: What lessons are there for teaching? https://research.acer.edu.au/research_conference/RC2013/6august/10

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[34] Hebert, M. A., Powell, S. R., Bohaty, J. J., & Roehling, J. (2019). *Piloting a mathematics writing intervention with late elementary students at risk for learning difficulties. Learning Disabilities Research & Practice, 34*, 144–157. https://doi.org/10.1111/ldrp.12202

[35] Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Information Age Publishing.

[36] Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students'learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393-425. https://doi.org/10.3102/00028312030002393

[37] Hood, C. E. (2023). *Elementary preservice math teachers' evolving dispositions throughout a math methods course: A mixed methods study* (Unpublished doctoral dissertation). University of Denver, Digital Commons.

[38] Isiksal, M., Curran, J. M., Koc, Y., & Askun, C. S. (2009). Mathematics anxiety and mathematical self-concept: Considerations in preparing elementary-school teachers. *Social Behavior and Personality: An international journal*, *37*(5), 631-644.

[39] Kajander, A. (2010). Elementary mathematics teacher preparation in an era of reform: The development and assessment of mathematics for teaching. *Canadian Journal of Education,33*(1), 228–255. https://journals.sfu.ca/cje/index.php/cje-rce/article/view/3075

[40] Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59–80. https://doi.org/10.1086/499693

[41] Kim, Y.-S. G., Yang, D., & Hwang, J. (2024). *Are mathematics and writing skills related? Evidence from meta-analysis. Educational Psychology Review, 36*(4), 125–147. https://doi.org/10.1007/s10648-024-09960-4

[42] Kosko, K. W. (2016). Writing in mathematics: A survey of K–12 teachers' reported frequency in the classroom. *School Science and Mathematics, 116*(5), 276–285. https://doi.org/10.1111/ssm.12177

[43] Kuzle, A. (2013). Promoting writing in mathematics: Prospective teachers' experiences and perspectives on the process of writing when doing mathematics as problem solving. *Center for Educational Policy Studies Journal(3),* 4, 41-59. https://files.eric.ed.gov/fulltext/EJ1129441.pdf

[44] Lloyd, G. M. (2006). Using K-12 mathematics curriculum materials in teacher education: Rationale, strategies, and pre-service teachers' experiences. In K. Lynch-Davis & R. L. Rider (Eds.), *The work of mathematics teacher educators: Continuing the conversation (AMTE Monograph Series)* Vol. 3, pp. 11–27. San Diego , CA : Association of Mathematics Teacher Educators.

[45] Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, *59*(5), 389-407.

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https://doi.org/10.1177/0022487108324554 (Original work published 2008

[46] Mainali, B. R. (2022). Investigating pre-service teachers' beliefs towards mathematics: A case study. *European Journal of Science and Mathematics Education, 10*(4), 412–435. https://doi.org/10.30935/scimath/12103

[47] McCarthy, D.S. (2010). Communication in mathematics: Preparing pre-service teachers to include writing in mathematics teaching and learning, *School Science and Mathematics, 108*(7),334-340.doi:https://doi-org.pitt.idm.oclc.org/10.1111/j.1949-8594.2008.tb17846.x

[48] McIntyre D. J., Byrd D. M., Foxx S. M. (1996). Field and laboratory experiences. In Sikula J.,Buttery T., Guyton E.(Eds.), *Handbook of research on teacher education* (pp. 171–193). Simon & Schuster Macmillan.

[49] Merriam, S. B., & Tisdell, E. J. (2016). Qualitative research: A guide to design and implementation (4th ed.). Jossey-Bass.

[50] Namakshi,N., Warshauer, H.K, Strickland, S, & McMahon, L. (2022). Investigating pre-service teachers' assessment skills: Relating aspects of teacher noticing and content knowledge for assessing student thinking in written work. *School Science & Mathematics, 122*(3), 142-154. https://doi.org/10.1111/ssm.12522

[51] National Academies of Sciences, Engineering, and Medicine (2001). Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press. https://doi.org/10.17226/9822.

[52] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM. National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards*. National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

[53] National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

[54] National Council on Teacher Quality. (2022). *Teacher prep review: Preparation for teaching elementary mathematics*. Washington, DC: National Council on Teacher Quality.

[55] National Governors Association. (2010). Common Core State Standards for English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects. Washington DC: National Governors Association Center for Best Practices, Council of Chief State School Officers. http://www.corestandards.org/

[56] National Research Council. (2001). *Adding it up: Helping children learn mathematics* (J. Kilpatrick, J. Swafford, & B. Findell, Eds.). National Academy Press. https://doi.org/10.17226/9822

[57] Oliver, R.M. & Reschly, D.J. (2007). Effective classroom management: Teacher preparation and professional development. *National Comprehensive Center for Teacher Quality*. Washington, D.C.: Vanderbilt University. https://files.eric.ed.gov/fulltext/ED543769.pdf

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[58] Philipp, R. A., Thanheiser, E., & Clement, L. (2002). The role of a mathematics methods course in developing pre-service teachers' pedagogical content knowledge. In D. L. Chambers (Ed.), *Putting research into practice in the elementary grades: Readings from journals of the NCTM* (pp. 413–424). National Council of Teachers of Mathematics.

[59] Piaget, J. (1954). *The construction of reality in the child* (C. Margaret, Trans.). New York Basic Books. (Original work published 1937).

[60] Pope, D. C. (2001). *Doing school: How we are creating a generation of stressed-out, materialistic, and miseducated students*. New Haven: Yale University Press.

[61] Rawding, M.R. & Wills, T. (2012). Discourse: Simple moves that work. *Mathematics Teaching in the Middle School, 18*(1), 46-51. https://doi.org/10.5951/MTLT.2023.0248

[62] Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346–362. https://doi.org/10.1037/0022-0663.93.2.346

[63] Røkenes, F. M., & Krumsvik, R. J. (2014). Development of student teachers' digital competence in teacher education-a literature review. Nordic Journal of Digital Literacy, 9(4), 250-280. https://doi.org/10.18261/ISSN1891-943X-2014-04-03

[64] Ross, J. A. (1994, June). *Beliefs that make a difference: The origins and impacts of teacher efficacy* [Paper presentation]. Annual Meeting of the Canadian Association for Curriculum Studies, Calgary, Alberta, Canada. ERIC Document Reproduction Service No. ED379216.

[65] Selek, H. K. G. (2024). Preservice mathematics teachers' views on the process of mathematics literacy problem writing. *Shanlax International Journal of Education, 12*(S1),148–155. https://doi.org/10.34293/ education.v12iS1- June.7283

[66] Shepard, L. A. (1991). *Mastery learning and higher-order skills*. *Educational Leadership, 49*(8), 14–18.

[67] Smith III, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education*, 27, 387–402.

[68] Solano-Flores, G., & Trumbull, E. (2003). Examining language in context: The need for new research and practice paradigms in the testing of English-language learners. *Educational Researcher, 32*(2), 3–13. https://doi.org/10.3102/0013189X032002003

[69] Steele, D. F. (1994). *Helping pre-service teachers confront their conceptions about mathematics and mathematics teaching and learning*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA. https://www.researchgate.net/publication/253893836_Editorial_The_Components_of_Authentic_Learning
http://www.researchgate.net/publication/253893836_Editorial_The_Components_of_Authentic_Learning

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[70] Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488. https://doi.org/10.3102/00028312033002455

[71] Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (2nd ed.). Teachers College Press.

[72] Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Free Press.

[73] Suh, J.M., Calabrese, S.C. (2025). Culturally responsive mathematics engagement through a family-inspired mathematizing routine. *Educational Studies in Math*ematics https://doi.org/10.1007/s10649-025-10411-2

[74] Teledahl, A., Kilhamn, C., Ahl, L. M., & Helenius, O. (2024). Defining and measuring quality in students' mathematical writing: A systematic literature review. *Mathematics Education Research Journal, 36*(2), 187–205. https://doi.org/10.1007/s13394-024-00501-4

[75] Urquhart, V. (2009). Using writing in mathematics to deepen student learning. *Mid-Continent Research for Education and Learning.* https://files.eric.ed.gov/fulltext/ED544239.pdf

[76] Wagganer, E.L. (2015). Creating mathematics talk communities. *Teaching Children Mathematics, 22*(4), 248-254. doi: 10.5951/teacchilmath.22.4.0248

[77] Weiss E. M., Weiss S. (2001). Doing reflective supervision with student teachers in professional development school culture. *Reflective Practice*, 2, 125–154. doi:10.1080/14623940120071343

[78] Wellberg, S. An examination of pre-service mathematics teachers' course-taking, beliefs, and preferred assessment practices. *J Math Teacher Educ* (2024). https://doi.org/10.1007/s10857-024-09640-8

[79] Wilcox, B., & Monroe, E. E. (2011). Integrating writing and mathematics. *The Reading Teacher, 64*(7), 521–529. doi: 10.2307/41203443

[80] Zeichner, K. (2010). Rethinking the Connections Between Campus Courses and Field Experiences in College- and University-Based Teacher Education. *Journal of Teacher Education*, *61*(1-2), 89-99. https://doi.org/10.1177/0022487109347671

[81] Zeichner, K. M., & Bier, M. (2017). Universities and teacher education. In D. J. Clandinin & J. Husu (Eds.), *The SAGE handbook of research on teacher education* (Vol. 2, pp. 51–64). SAGE Publications. https://doi.org/10.4135/9781526402042.n4

APPENDIX

Figure 1: Questionnaire for Pre-Service Teachers

Writing in Math Class Questionnaire for Pre-Service Teachers

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When using the phrase "writing in math class", it is implied that this includes any writing that takes place in which the writer thoroughly describes the *process of thinking* when solving a math problem. This does not include short answers which require explanation.

1. I feel writing should take place in math class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

2. I have experience writing during math class from my own personal experiences when I attended school (any grade levels).

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

3. If you experienced writing during your own math classes when you attended school, please describe how writing was implemented in these classes.

_____

_____

_____

_____

4. I have experience <u>teaching</u> writing during math class.

1	2	3	4	5
Strongly	Disagree	Neutral	Agree	Strongly Agree

If you have experience teaching writing in <u>math</u> class, please describe how you have learned to implement this into your teaching and student learning.

_____

_____

_____

_____

5. I have taken a college class(es) (any institution) in which the instructor expected me to write as part of the math class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

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6. I have taken a college class(es) (any institution) that taught me how to teach writing in math class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

7. I plan to implement writing in math class in my future classroom.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

8. My cooperating/mentor teachers or field placement experiences have influenced my views on using writing in mathematics instruction.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

If you agree, please describe how your mentor teacher(s) or field placement experiences influenced your thinking about writing in math.

_____

_____

_____

9. My understanding of writing in math instruction has been shaped by broader influences such as state standards, standardized assessments, or cultural expectations around math teaching.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Please explain how policies, standards, or societal views about math education have affected your perception of writing in math.

_____

_____

_____

Demographic Questions

10. What is your current year?

Freshman Sophomore Junior Senior

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11. Indicate your age group:

- ❒ Under 18 years of age
- ❒ 18-24 years of age
- ❒ 25-34 years of age
- ❒ 35-44 years of age
- ❒ 45-54 years of age
- ❒ 55 years of age or older

12. Are there any additional comments you would like to include about writing in math class (please write in the space provided)?

_____

_____

_____

Table 1. Pre- and Post-Questionnaire Responses and Point Growth for Selected Items

Questionnaire Item	Response	Pre (%)	Post (%)	Change (points)
#1. Writing should take place in math class	Strongly Disagree	6.1	0.0	–6.1
	Disagree	27.3	0.0	–27.3
	Neutral	36.4	6.1	–30.3
	Agree	30.3	24.2	–6.1
	Strongly Agree	0.0	69.7	+69.7
#4. I have experience teaching writing in math class	Strongly Disagree	51.5	0.0	–51.5
	Disagree	27.3	0.0	–27.3
	Neutral	18.2	3.0	–15.2

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Questionnaire Item	Response	Pre (%)	Post (%)	Change (points)
#8. I plan to implement writing in math class	Agree	3.0	66.7	+63.7
	Strongly Agree	0.0	30.3	+30.3
	Strongly Disagree	0.0	0.0	0.0
	Disagree	18.2	6.1	–12.1
	Neutral	60.6	0.0	–60.6
	Agree	21.2	69.7	+48.5
	Strongly Agree	0.0	24.2	+24.2

Table 1: Questionnaire point growth from pre-test to post-test questionnaire items

Table 2: Open-Ended Responses from Pre- and Post-Questionnaires on Writing in Mathematics

Question #3: If you experienced writing during your mathematics classes when you attended school, please describe how writing was implemented in these classes

Pre-Questionnaire Responses

I can remember sometimes writing in math class; it was usually short sentences to describe how we did a problem.

Our one junior high teacher had us write a lot in math class. We usually solved a problem and explained how we did it.

Our school had us write in most of our math classes. We always wrote about what we did in math problems.

I believe we wrote a little bit in math class when I was in elementary school, but I don't remember why.

I can remember in elementary school we had to explain some of our answers in sentences.

I don't think we had to do this in junior or senior high.

We would write to explain word problems.

Maybe writing a few sentences but just to explain what we did.

I remember writing on workbook pages, but I don't remember what I had to write.

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Question #5. If you have experience teaching writing in mathematics class, please describe how you have learned to implement this into your teaching and student learning.

Post-Questionnaire Response

Explain and justify. These are two verbs that the students should learn to do early in their school years. It will develop a different level of thinking, and they can actually use and remember their skills when they leave the classroom.

Start slow and gradually increase until the students are more comfortable. It will be awkward at first, especially if previous teachers have not used it. If students see us model it, it will become like any other form of learning.

There are many things I will take from this course. The student-created vocabulary posters were most useful. The professor gave us a checklist we could use for grading, and I want to save it for my classroom.

By using real-world problems that students can relate to. This helps them enjoy math while learning. Much of our class this semester taught us how to model writing in math class before expecting students to do it. Once students learn how, they reach a deeper level of understanding.

By focusing on one well-developed, real-life problem rather than a worksheet of thirty one-step problems.

We did group work to critique student writing samples, used a rubric to grade them, and developed discussion points for student conferences. I learned that it's better to have one or a few rich questions—even if the teacher has to write them—than many procedural problems.

We used Bloom's Taxonomy to write deep-thinking questions. Writing to explain and justify math ideas helps students understand better. Vocabulary posters support this. It takes more time but leads to better learning than workbook pages.

Question #11. Are there any additional comments you would like to include about writing in mathematics class?

Pre-Questionnaire Responses

Would love to become more familiar with this idea.

I may have done some writing during school.

I don't remember a lot from my K–12 years.

Writing and math were two different subjects when I was in school. I don't really know how to put them together.

I'm glad we are going to be learning about this topic. I'd like to learn some more about it.

I don't know why math class is including writing.

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Post-Questionnaire Responses

There are many good things I learned about this, but I feel like it depends on many different factors (school, other teachers, curriculum, etc.).

Yes, I enjoyed learning something new. I like the idea but wish we could talk about it in other semesters to get more practice.

It may have helped more if we were expected to complete more writing assignments ourselves for firsthand experience. We saw examples, but doing it multiple times might have helped more.

Just that this opened my eyes to a new idea. In upper-level classes, we see a lot of repetition, and professors help us practice a lot. But this was something brand new I learned.

I look forward to learning more about this topic. I don't really know how to do it yet.

I have not experienced teaching writing in math class, but I'm excited to try it. I don't have much knowledge about writing. I'd like to know more.

Table 2: Pre-Questionnaire and Post-Questionnaire Open-Ended Responses

Table 3: Frequency of Interview Statements by Key Theme

Key theme	Number of interview statements referring to key theme
Balance of Basic Skills	14
Explain and Justify	9
Vocabulary Word Walls	7
Overlap of Subjects	6

Table 3: Key themes that emerged from the interviews with participants

Table 4: End of Semester Assignment

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Table 4: End of semester assignment to show voluntary inclusion of writing

Student did not include any form of writing in end-of-semester assignment	2
Student did include writing in end-of-semester assignment, but did not include both explaining and justifying of student work	4
Student did include writing in end-of-semester assignment and included both explaining and justifying of student work	27

Figures 1-7: End of Semester Assignment

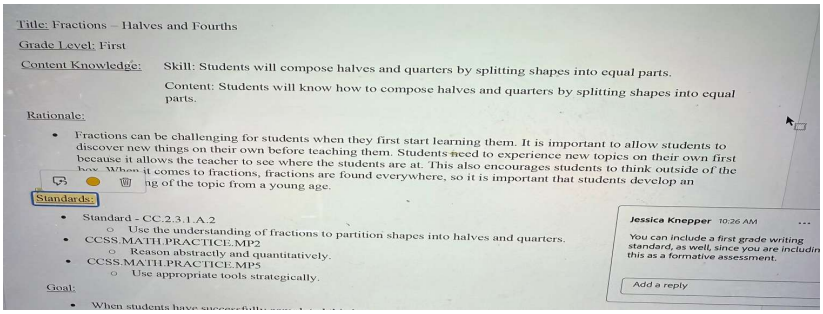


Figure 1: Example standards of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections

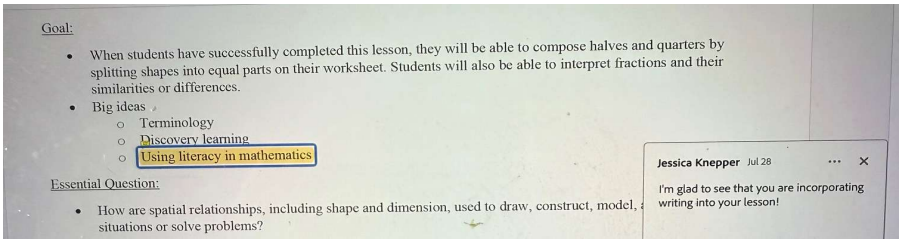


Figure 2: Example goal of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections

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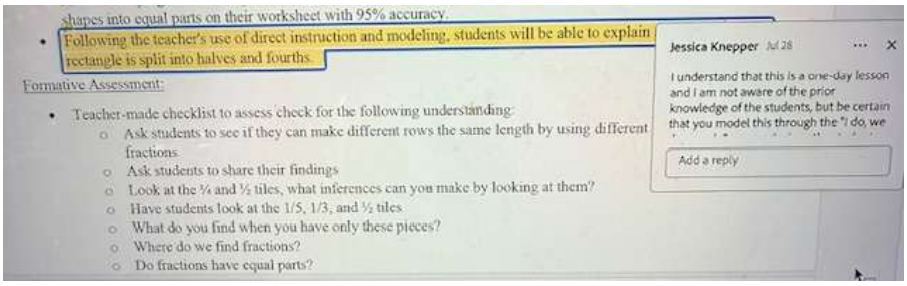


Figure 3: Example objective of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections

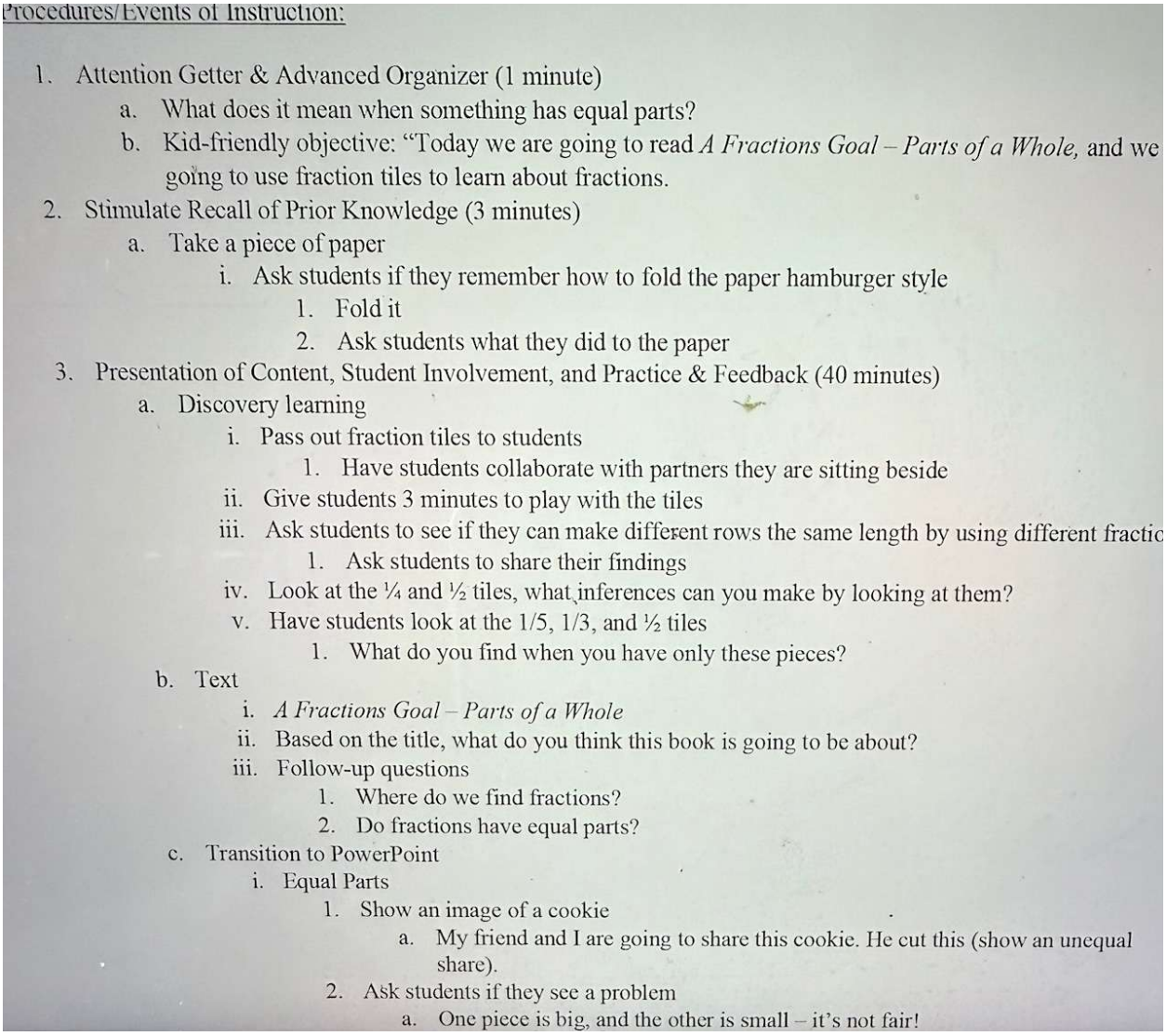
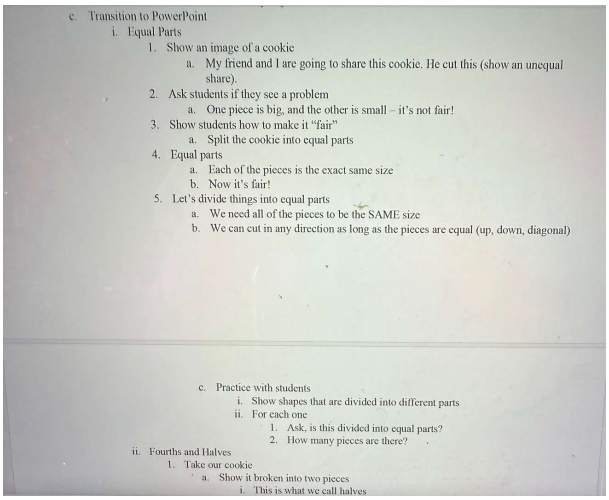


Figure 4: Example procedures section of a pre-service teacher lesson plan that integrated writing.

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c. Transition to PowerPoint
i. Equal Parts
1. Show an image of a cookie
a. My friend and I are going to share this cookie. He cut this (show an unequal share).
2. Ask students if they see a problem
a. One piece is big, and the other is small – it's not fair!
3. Show students how to make it "fair"
a. Split the cookie into equal parts
4. Equal parts
a. Each of the pieces is the exact same size
b. Now it's fair!
5. Let's divide things into equal parts
a. We need all of the pieces to be the SAME size
b. We can cut in any direction as long as the pieces are equal (up, down, diagonal)

c. Practice with students
i. Show shapes that are divided into different parts
ii. For each one
1. Ask, is this divided into equal parts?
2. How many pieces are there?
ii. Fourths and Halves
1. Take our cookie
a. Show it broken into two pieces
i. This is what we call halves

Figure 5: Example procedures section of a pre-service teacher lesson plan that integrated writing

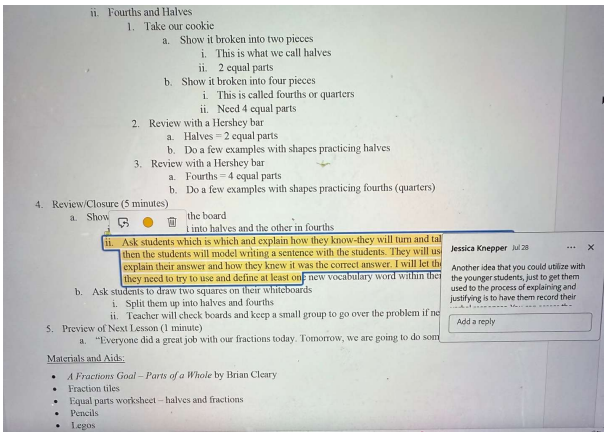
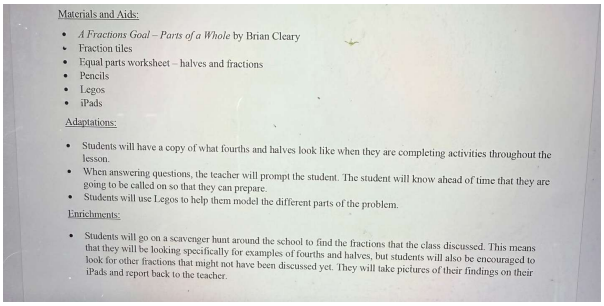

ii. Fourths and Halves
1. Take our cookie
a. Show it broken into two pieces
i. This is what we call halves
ii. 2 equal parts
b. Show it broken into four pieces
i. This is called fourths or quarters
ii. Need 4 equal parts
2. Review with a Hershey bar
a. Halves = 2 equal parts
b. Do a few examples with shapes practicing halves
3. Review with a Hershey bar
a. Fourths = 4 equal parts
b. Do a few examples with shapes practicing fourths (quarters)
4. Review/Closure (5 minutes)
a. Show
the board
into halves and the other in fourths
ii. Ask students which is which and explain how they know-they will turn and ta
then the students will model writing a sentence with the students. They will us
explain their answer and how they knew it was the correct answer. I will let th
they need to try to use and define at least one new vocabulary word within ther
b. Ask students to draw two squares on their whiteboards
i. Split them up into halves and fourths
ii. Teacher will check boards and keep a small group to go over the problem if ne
5. Preview of Next Lesson (1 minute)
a. "Everyone did a great job with our fractions today. Tomorrow, we are going to do som
Jessica Knepper Jul 28
Another idea that you could utilize with the younger students, just to get them used to the process of explaining and justifying is to have them record their
Add a reply
Materials and Aids:
- *A Fractions Goal – Parts of a Whole* by Brian Cleary
- Fraction tiles
- Equal parts worksheet – halves and fractions
- Pencils
- Legos

Figure 6: Example review/closure of a pre-service teacher lesson plan that integrated writing, indicated by highlighted sections


Materials and Aids:
- *A Fractions Goal – Parts of a Whole* by Brian Cleary
- Fraction tiles
- Equal parts worksheet – halves and fractions
- Pencils
- Legos
- iPads

Adaptations:
- Students will have a copy of what fourths and halves look like when they are completing activities throughout the lesson.
- When answering questions, the teacher will prompt the student. The student will know ahead of time that they are going to be called on so that they can prepare.
- Students will use Legos to help them model the different parts of the problem.

Enrichments:
- Students will go on a scavenger hunt around the school to find the fractions that the class discussed. This means that they will be looking specifically for examples of fourths and halves, but students will also be encouraged to look for other fractions that might not have been discussed yet. They will take pictures of their findings on their iPads and report back to the teacher.

Figure 7: Example materials, adaptations, and enrichments of a pre-service teacher lesson plan that integrated writing

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The Role of Realistic Mathematics Education with Scaffolding Teaching Model in Enhancing Mathematical Representation Ability

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Abstract: Mathematics education in higher learning often faces difficulties in developing students' mathematical representation skills, with many struggling in verbal, visual, and symbolic expressions. This study explored the impact of the Realistic Mathematics Education (RME) approach combined with scaffolding on improving students' ability to represent mathematical concepts. A quasi-experimental design was employed, involving 50 students across three experimental groups and 16 students in a control group, all selected at random. The data were analyzed using both inferential and descriptive statistics, with mathematical representation assessed through a specific test. Hypothesis testing was conducted using one-way ANOVA and post hoc comparisons. The results revealed significant differences between the experimental and control groups, demonstrating that the RME model with scaffolding effectively enhanced students' mathematical representation skills. These findings highlight that integrating RME can lead to substantial improvements in mathematical understanding within higher education.

Keywords: Realistic Mathematics Education, Scaffolding, Mathematical Representation Ability

INTRODUCTION

Mathematics is a core subject taught from elementary to higher education levels (Langoban & Langoban, 2020). Its primary purpose is to develop essential mathematical competencies. According to Ferrini-Mundy (2001) these include: (1) problem-solving, (2) reasoning and proof, (3) connections, (4) communication, and (5) representation. Among these, mathematical representation is crucial for helping students understand, communicate, and apply mathematical ideas effectively.

Mathematical representation refers to the ability to express mathematical concepts using various forms such as graphs, symbols, and written language (Mainali, 2021). Tang (2023) classifies representation into three types: visual (e.g., diagrams, graphs), symbolic (e.g., formulas, notations), and verbal (e.g., written explanations). Vincent-Lancrin (2023) highlight that this ability allows students to translate concepts across different forms, enabling deeper understanding. McCoy et al. (1996) emphasize the importance of encouraging students to construct their own representations as tools for

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thinking and communication. This skill helps students concretize abstract concepts, making mathematics more accessible and meaningful (Santiago & Alves, 2022). However, many university students struggle with developing mathematical representations, particularly in translating between verbal, visual, and symbolic forms. Studies by Hwang & Ham (2021), students tend to demonstrate lower levels of mathematical literacy when they are more frequently exposed to applied mathematics reasoning tasks. Krawec (2014) suggests that this approach might have unintentionally hindered some students. For instance, certain students may have been capable of solving several problems without relying on visual representations, leading to incomplete visual outputs and, in turn, weakening the correlation between visual representation and problem-solving accuracy.

To address this, various instructional models have been explored. One promising approach is Realistic Mathematics Education (RME), which emphasizes contextual learning through real-life situations (Fredriksen, 2021). RME is a learning approach that people naturally apply in everyday life, often unconsciously (Fredriksen, 2021). It situates mathematical concepts within contexts that are closely connected to students' real-world experiences and environments. According to Van Den Heuvel-Panhuizen & Drijvers (2020), one of the seven key principles that guide RME design is the interactivity principle. This principle emphasizes that mathematics learning should be a collaborative process, involving group work and class discussions, rather than being solely an individual activity. A study by Kaba & Şengül (2017) demonstrated that using RME-based instruction significantly outperformed traditional methods in improving students' measurement estimation skills, underscoring the practical benefits of applying RME principles in mathematics education.

Despite its benefits, RME in higher education still faces challenges, particularly in supporting students with diverse learning needs. To address this gap, scaffolding can be introduced. Scaffolding, introduced by Wood et al. (1976), is a method of providing tailored, temporary support to help students master new concepts. As a result, the scaffolding construct is now being broadly used, becoming "a proxy for any cultural practices associated with performance, knowledge and skills" (Pea, 2018).

While both RME and scaffolding have independently shown effectiveness in improving learning outcomes, limited research has explored their integration in the context of higher education, especially regarding mathematical representation skills. Previous studies (Fredriksen, 2021; Laurens et al., 2017; Yuanita et al., 2018) have validated RME's effectiveness in enhancing conceptual understanding, while scaffolding has supported structured learning (Brush & Saye, 2002; Kim & Lim, 2019). However, a combined model remains underexplored.

Therefore, integrating RME with scaffolding presents a novel approach to address the limitations of each model. This study investigates the effectiveness of combining RME and scaffolding to improve students' mathematical representation skills in Mathematics Education at Universitas Islam Negeri Raden Intan Lampung, Indonesia. It is expected to contribute to more effective instructional practices and serve as a reference for educators and curriculum developers aiming to enhance the quality of mathematics education.

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We formulate the research questions: Tow hat extent does the integration of RME with scaffolding improve undergraduate students' mathematical representation abilities—verbal, visual, and symbolic—compared to conventional teaching methods in higher education?

LITERATURE REVIEW

Realistic Mathematics Education

RME is an instructional approach that focuses on connecting mathematical concepts with real-life contexts to foster a deeper understanding of mathematics among students. RME, originally developed in the Netherlands by Hans Freudenthal in the 1970s, is based on the idea that mathematics is an activity that humans engage in to understand and solve problems related to their environment (Kösece & Doğanay, 2023). RME views mathematics as a human activity that should be rooted in problem-solving related to students' everyday experiences, making abstract concepts more accessible and meaningful (Fitria et al., 2022; Susanto et al., 2024). A key feature of RME is the distinction between horizontal and vertical mathematization.

Horizontal mathematization involves translating real-world problems into mathematical models, while vertical mathematization refines these models into formal mathematical reasoning (Hermandra et al., 2022; Izzah & Ekawati, 2023). This progression helps students develop deeper problem-solving skills by moving from concrete to abstract understanding. Collaboration and interaction are essential in RME, fostering a learning environment where students discuss, share strategies, and build upon each other's ideas. This social interaction deepens cognitive engagement and enhances understanding (Nermin & Kapucu, 2022; Umar & Zakaria, 2022). Additionally, RME encourages students to develop their own models, serving as bridges between informal reasoning and formal mathematics, enabling a smoother transition to abstract thought (Nurfadilah et al., 2021). RME aligns well with 21st-century educational goals, promoting critical thinking, creativity, and collaboration. Research shows that students taught through RME demonstrate greater proficiency in mathematical representation and practical problem-solving (Ningsi et al., 2024). By anchoring learning in real-world contexts, RME prepares students to apply mathematics both academically and effectively in everyday life (Altiner et al., 2023).

Previous studies have shown that RME can enhance students' mathematical understanding and problem-solving skills, though challenges remain in aligning real-world contexts with formal models (Laurentiz, 2021; Payadnya et al., 2023; Sembiring et al., 2008). These mixed outcomes indicate that the success of RME depends on factors such as instructional design, scaffolding, and learner context. Therefore, while RME holds strong potential for higher education, its effective implementation requires well-prepared teachers, equitable learning opportunities, and contextually adapted assessment strategies.

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Scaffolding

Scaffolding was introduced by educational psychologist Jerome Bruner in the 1970s (Bruner, 1978). This concept is closely related to constructivist learning theory and refers to the way educators provide support to students during the learning process to help them develop new understanding and skills (Wood et al., 1976). In an educational context, scaffolding is a strategy where teachers offer assistance tailored to students' needs as they engage with new or challenging material (Hogan & Pressley, 1997). This support may include hints, encouragement, guiding questions, or additional explanations (Van de Pol et al., 2010). As students begin to grasp the content or skills being taught, the level of assistance is gradually reduced until they are able to complete tasks independently (Pea, 2004).

Scaffolding is an instructional strategy that offers temporary, structured support to learners as they acquire new skills or understanding. Originally introduced by Jerome Bruner and later expanded upon by Lev Vygotsky, the concept is closely tied to Vygotsky's theory of the Zone of Proximal Development (ZPD) (see Figure 1)—the range of tasks a learner can perform with guidance but not yet independently (Fokong, 2023; Rokhmat et al., 2019). This assistance may take the form of breaking down complex tasks into manageable steps, posing guiding questions, offering strategic hints, or giving clarifying explanations (Richardson et al., 2021). The ultimate goal is to bridge the gap between what students currently know and what they can achieve with temporary support, fostering independence over time (Safa & Motaghi, 2021).

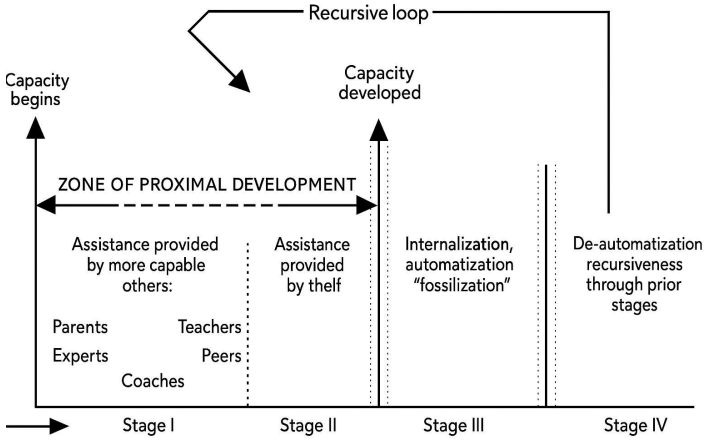


Figure 1: Model of four stages in the ZPD (Gallimore, 1990)

The process begins with Stage I, where learners rely on guidance from more capable individuals. In Stage II, they begin to support themselves after internalizing the help received. By Stage III, knowledge becomes automatic—often called "fossilization." In Stage IV, learners may revisit earlier stages to adjust their understanding when facing new challenges.

In classroom settings, scaffolding is especially valuable in helping students master difficult concepts. In mathematics education, for example, scaffolding supports learners in linking abstract ideas to familiar, real-world contexts, thereby deepening their conceptual understanding (Johnson et al., 2022).

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The gradual release of responsibility from teacher to student ensures that learners gain confidence and internalize essential skills, eventually becoming capable of completing tasks independently. Scaffolding can be implemented through various forms, including modeling (demonstrating how to perform a task), questioning (encouraging critical thinking), and feedback (offering corrections and suggestions). Each type of support plays a crucial role in advancing cognitive development and promoting self-regulated learning. In essence, scaffolding is a powerful educational tool that supports learners as they transition from dependence to independence. By providing appropriate, responsive assistance and gradually removing it, educators can help students achieve higher levels of understanding and academic success across disciplines.

Realistic Mathematics Education Supported Scaffolding

The integration of RME and scaffolding provides a powerful instructional approach for strengthening students' mathematical understanding and representation skills. RME, pioneered by Freudenthal (2002), emphasizes connecting mathematical concepts to real-life contexts, making learning meaningful and relevant to students' experiences. However, engaging with such authentic problems often requires support, which aligns with Vygotsky (1978) Sociocultural Theory of Cognitive Development. Central to this theory is the Zone of Proximal Development (ZPD), defined as the gap between what a learner can accomplish independently and what they can achieve with guidance from adults or collaboration with more capable peers. The ZPD highlights not only completed abilities but also emerging skills that are "in the process of maturing," emphasizing learning as a dynamic, forward-looking process.

Within the Zone of Proximal Development (ZPD), scaffolding serves as a vital instructional strategy that provides temporary, structured support to help learners tackle tasks they cannot yet complete independently (Wood et al., 1976). While distinct from the ZPD itself, scaffolding operates within it by enabling gradual engagement with complex problems and reducing support as learners gain autonomy (Smagorinsky, 2018). In the context of Realistic Mathematics Education (RME), scaffolding facilitates both horizontal mathematization—translating real-world problems into mathematical models—and vertical mathematization—refining these into formal reasoning. Strategies such as modeling, prompting, and feedback foster confidence, active engagement, and deeper understanding, ultimately enhancing students' mathematical representation skills (Ariska et al., 2021; Bostic et al., 2020). By integrating RME with scaffolding, educators can provide a coherent framework that nurtures independence, problem-solving ability, and meaningful application of mathematics in real contexts.

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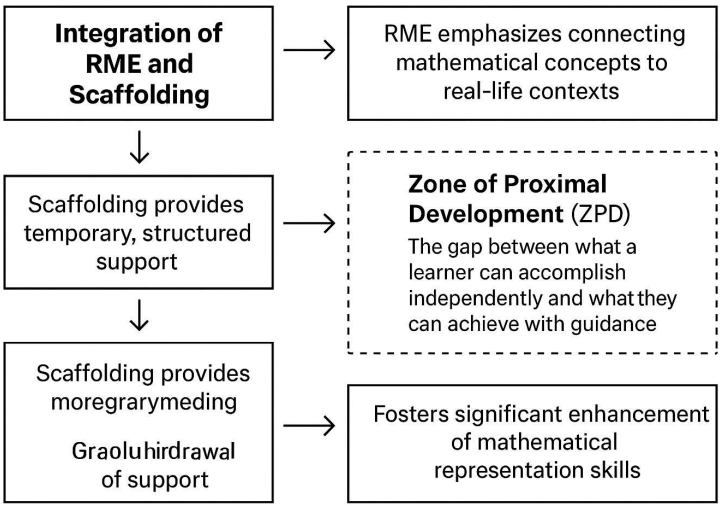


Figure 2: RME with Scaffolding Theory of Cognitive Development for ZPD (create by authors)

The RME with Scaffolding approach differs from traditional instruction by emphasizing real-world problem contexts, where students engage in horizontal mathematization (modeling everyday situations) and vertical mathematization (developing formal reasoning). Scaffolding strategies—such as guided questioning, modeling, collaboration, and gradual release of responsibility—support learners in moving from informal to formal understanding. Unlike conventional teacher-centered methods that stress procedures, rote practice, and abstract problems with little connection to students' experiences, RME with Scaffolding fosters conceptual understanding, multiple representations, and stronger problem-solving skills.

METHODS

Participants

The participants in this study were undergraduate students from the Mathematics Education Study Program at Universitas Islam Negeri Raden Intan Lampung, enrolled in courses covering secondary school mathematics topics such as probability, arithmetic sequences and series, and three-dimensional geometry. This study was carried out in collaboration with several mathematics education lecturers from Universitas Islam Negeri Raden Intan Lampung, Indonesia, who teach high school mathematics courses. Each lecturer was responsible for supervising a different class, which was assigned to either the experimental group or the control group. The experimental groups included Group A with 16 students (2 male, 14 female), Group B with 15 students (all female), and Group C with 19 students (2 male, 17 female). These groups received instruction using the Realistic Mathematics Education (RME) model integrated with scaffolding. The control group consisted of Group

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D, which included 16 students (6 male, 10 female) and followed conventional teaching methods without the RME-scaffolding integration.

Research Model

This study employed a quasi-experimental design within the Mathematics Education Study Program at Universitas Islam Negeri Raden Intan Lampung, involving undergraduate students enrolled in secondary mathematics courses. The intervention covered probability, arithmetic sequences and series, three-dimensional geometry, and combinatorics, all contextualized through real-life problems such as drawing marbles, savings patterns, calculating water tank volumes, and arranging books or forming committees. These tasks were designed to link abstract concepts with practical applications and strengthen students' mathematical reasoning. The research model is presented in Table 1.

Class	*Pretest*	Treatment	*Posttest*
Group A	O_1	XR_1	O_2
Group C	O_3	XR_2	O_4
Group D	O_5	XR_3	O_6
Group B	O_7	C	O_8

Table 1: Research Design. *Note:* XR = RME model with scaffolding; C = Control; O = test

This study employs a five-step instructional model integrating RME with scaffolding—contextual problem presentation, guided questioning, collaborative discussion, multi-representation exploration, and gradual release of support—to enhance students' conceptual understanding and representation skills. In contrast, the traditional model follows a teacher-centered sequence of concept explanation, formula memorization, worked examples, routine practice, and answer correction, emphasizing procedural fluency over meaningful engagement. The instructional syntax is summarized in Appendix A (modified from).

Research Procedures

The research was conducted in three experimental classes and one control class. Each experimental class followed the steps of the Realistic Mathematics Education (RME) model integrated with scaffolding, while the control class used traditional teaching. The learning process began with a preparation phase, in which students in the experimental groups took a pretest to assess their initial mathematical representation skills in verbal, visual, and symbolic forms. Lesson plans were carefully designed to include real-life problems relevant to the students' local context, with a focus on developing mathematical representation abilities.

The intervention was carried out over 12 class meetings from July to September 2024. During this implementation phase, students participated in weekly 90-minute sessions centered on problem-

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solving activities aligned with RME and scaffolding principles. In the first two weeks, students were introduced to the learning context and objectives, with teachers presenting problems rooted in local culture, such as traditional craft patterns. From weeks three to nine, students engaged in guided explorations of these problems, working collaboratively in groups with ongoing teacher support, gradually progressing toward formal mathematical understanding. Weeks 10 and 11 were dedicated to reflection and presentations, during which students shared their work and discussed their insights. In the final week, a posttest was administered to measure improvements in students' mathematical representation skills.

Instrument

The study employed an essay-based test to evaluate students' mathematical representation skills across three forms: verbal, visual, and symbolic. The test comprised five items in total both pre-post test (see Appendix B and C). For verbal representation, students were asked to articulate their reasoning in written form. Visual representation required them to interpret problems through graphs or diagrams, while symbolic representation involved translating real-life situations into mathematical expressions. The following is the scoring rubric for mathematical representation skills.

Score	Verbal	Visual	Expression
0	No answer, or if there is, it only shows misunderstanding of the concept so the information provided is meaningless		
1	Only a small part of the explanation is correct	Only a small part of the diagram or drawing is correct	Only a small part of the mathematical model is correct
2	The mathematical explanation makes sense but is only partially complete and correct	Draws a diagram or picture, but it is incomplete and correct	Finds the mathematical model correctly, but makes mistakes in obtaining the solution
3	The mathematical explanation makes sense, although not logically structured or contains minor language errors	Draws a diagram or picture completely and correctly	Finds the mathematical model correctly, then performs calculations or obtains the solution correctly and completely
4	The mathematical explanation makes sense, is clear, and logically and systematically structured	Draws a diagram or picture completely, correctly, and systematically	Finds the mathematical model correctly, then performs calculations or obtains the solution correctly, completely, and systematically

Table 2: The scoring rubric for mathematical representation ability

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An example of the test item is illustrated.

Given a rectangular prism ABCD.EFGH as shown in the figure, the dimensions are as follows: AB = 10 cm, BC = 4 cm, CG = 8 cm, AS = 2 cm, and GM = 3 cm. A small ant starts at point S and moves along the surface of the prism toward a piece of food located at point M. What is the shortest distance the ant must travel from S to M? Draw the route the ant would take to reach the food.



The validity and reliability of the test instrument were evaluated to ensure it met standard criteria. The construct validity of the five test items ranged from 0.49 to 0.86, indicating moderate to high validity. Reliability analysis using Cronbach's alpha yielded a coefficient of 0.75, which suggests that the instrument had acceptable internal consistency. The difficulty level of the items varied from difficult to easy, with values ranging between 0.30 and 0.81. Meanwhile, the item discrimination indices ranged from 0.34 to 0.50, indicating a fair to good ability to distinguish between high- and low-performing students. Overall, these results confirm that the test instrument is both valid and reliable for assessing students' mathematical representation skills.

Data Analysis

This study utilized SPSS 25 to conduct descriptive statistical analyses and examine the correlations among variables. To evaluate the impact of the RME learning model integrated with a scaffolding approach, a one-way ANOVA was employed, along with the N-Gain Multiple Comparison Test to analyze the results in greater detail. To better understand the practical significance of the findings, Cohen's d was calculated to determine the effect size, indicating the strength of the RME model with scaffolding on students' mathematical representation abilities. Additionally, R software was used to generate visualizations of the relationships between scale variables, offering deeper insight into students' performance.

The normalized gain (N-Gain): $N\text{-}Gain = \frac{Posttest\ score - Pretest\ score}{Maximum\ score - Pretest\ score}$

Based on the resulting scores, N-Gain was categorized as low (N-Gain < 0.3), medium (0.3 ≤ N-Gain < 0.7), or high (N-Gain ≥ 0.7).

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RESULTS

Statistical Description

An initial pretest was administered before the intervention to gather baseline data from both the experimental and control groups. Following the implementation of the treatment, a posttest was conducted to assess students' progress. A detailed summary of the test scores is presented in Table 3 below.

Class	Pretest					Posttest				
	n	***M***	**SD**	**Min**	**Max**	***n***	***M***	**SD**	**Min**	**Max**
Experiment 1	16	38.75	17.81	5	75	16	76.40	13.03	55	97.5
Experiment 2	15	38	15.06	15	62.5	15	75	12.46	55	97.5
Experiment 3	19	38.55	11.55	7.5	55	19	75.26	11.42	55	97.5
Control	16	38.59	10.56	12.5	57.5	16	51.40	12.64	32.5	75
Total	66	38.48	13.60	5	75	66	69.69	15.95	32.5	97.5

Table 3: Description of observed Pretest and posttest scores

Table 3 presents the descriptive statistics of pretest and posttest scores for three experimental classes and one control class. In the pretest, all groups had similar average scores, with means around 38, indicating a comparable baseline in mathematical representation skills. However, notable differences emerged in the posttest results. The experimental groups, which received the RME learning model with scaffolding, showed substantial improvements. Experiment 1 increased from a mean of 38.75 to 76.40, Experiment 2 from 38 to 75, and Experiment 3 from 38.55 to 75.26, with corresponding increases in minimum and maximum scores across the board. By contrast, the control group taught through conventional instruction exhibited only a slight improvement, with the mean rising from 38.59 to 51.40 and a narrower distribution of scores. The results show a noticeable difference between the experimental and control groups.

Normality and Homogeneity Data

A normality test was conducted to assess whether the data from the four classes were normally distributed. This analysis is essential to determine the appropriate statistical tests for further data interpretation. The Lilliefors test was employed for this purpose, using a significance level of 5%. The results of the normality test for each class are presented in Table 4 below.

Class	Kolmogorov-smirnov		
	Statistic	***df***	**Sig.**
Experiment 1	0.154	16	0.200*
Experiment 2	0.116	15	0.200*
Experiment 3	0.176	19	0.125
Control	0.199	16	0.089

Table 4: The normality results data.

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Table 4 displays the results of the normality test using the Kolmogorov-Smirnov method for each class. The significance values (Sig.) for all groups are greater than the 0.05 threshold, indicating that the data are normally distributed. Specifically, Experiment 1 and Experiment 2 both yielded a significance value of 0.200, which is the maximum value reported by SPSS, suggesting strong evidence of normality. Experiment 3 had a slightly lower significance value of 0.125, and the control group showed a value of 0.089—both still above the 0.05 level. Overall, these results indicate that the assumption of normality was met for all four groups, supporting the use of parametric statistical tests in subsequent analyses.

The homogeneity test was conducted to determine whether the four classes had similar or different variances. The test was performed using Levene's test for homogeneity of variances, and the results are presented in Table 5. For all conditions—based on the mean, median, adjusted degrees of freedom, and trimmed mean—the significance values (sig.) were all well above the 0.05 threshold. Specifically, the significance values were 0.937, 0.930, 0.930, and 0.928, respectively. These results indicate that there is no significant difference in the variances between the groups, suggesting that the assumption of homogeneity of variances was met for the data.

Data	Levene Statistic	*df1*	*df2*	sig.
Based on Mean	0.138	3	62	0.937
Based on Median	0.148	3	62	0.930
Based on Median and with adjusted *df*	0.148	3	55	0.930
Based on Trimmed mean	0.152	3	62	0.928

Table 5: Homogeneity test results

Hypothesis Analysis

This analysis was conducted following the normality and homogeneity tests, which confirmed that the N-gain data from the four classes were normally distributed and homogeneous. The test was carried out using a one-way ANOVA, and the results are presented in Table 6. The ANOVA test showed a significant difference between the groups, with an F-value of 17.442 and a significance value (Sig.) of 0.000, which is below the 0.05 threshold. This indicates that there is a statistically significant difference in the N-gain scores between the four groups. The sum of squares between groups was 1.901, and within groups, it was 2.252. The results suggest that the treatment applied in the experimental groups had a meaningful effect on the students' performance compared to the control group.

Data	Sum of Squares	*df*	Mean Square	F	Sig.
Between Groups	1.901	3	0.634	17.442	0.000
Within Groups	2.252	62	0.036		
Total	4.153	65			

Table 6: One-way Anova test result data

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Multiple Comparison Data

The analysis was conducted following the results of the one-way ANOVA, which indicated a significant difference between the groups. To further explore these differences, a multiple comparison test was performed using the Bonferroni method in SPSS. The results of the multiple comparison test are presented in Table 7.

Class (I)	Group (J)	Mean Difference (I-J)	S.E.	Sig.	96% CI	
					Lower Bound	Upper Bound
Experiment 1	Experiment 2	0.043	0.068	< 0.001	-0.143	0.229
	Experiment 3	0.044	0.065	1	-0.131	0.221
	Control	0.424*	0.067	< 0.05	0.240	0.607
Experiment 2	Experiment 1	-0.043	0.068	1	-0.229	0.143
	Experiment 3	0.002	0.065	1	-0.177	0.181
	Control	0.381*	0.068	< 0.05	0.194	0.567
Experiment 3	Experiment 1	-0.045	0.064	1	-0.221	0.131
	Experiment 2	-0.002	0.065	1	-0.181	0.177
	Control	0.378*	0.064	< 0.05	0.202	0.555
Control	Experiment 1	-0.424	0.067	< 0.001	-0.607	-0.240
	Experiment 2	-0.381	0.068	< 0.001	-0.567	-0.194
	Experiment 3	-0.379*	0.064	< 0.05	-0.555	-0.202

Table 7: Multiple comparison test result data. *Note:* * = significance level at $p < 0.05$

The table shows the mean differences between the groups. For example, in the comparison between Experiment 1 and the Control group, the mean difference was 0.424, with a significance value of $p < 0.05$, suggesting a significant difference between these two groups. Similar significant differences were found between the Control group and each of the experimental groups (Experiment 1, Experiment 2, and Experiment 3), with all comparisons showing $p < 0.05$. However, no significant differences were found within the experimental groups themselves, as all comparisons within the experimental groups (Experiment 1 vs. Experiment 2, Experiment 1 vs. Experiment 3, etc.) had $p > 0.05$. These results highlight that the RME model with scaffolding had a significant positive effect compared to the control group, but the differences between the experimental groups were not statistically significant.

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Students Performance in Mathematical Representation Indicators

Figure 3 presents the final results of students' mathematical representation abilities, measured through three key indicators: verbal, visual, and symbolic expression. A clear upward trend from pretest to posttest is evident across all experimental groups.

In verbal representation, the most substantial gains were recorded, with Experiment 1 showing the highest increase from 38.54 to 93.23 (N-Gain = 0.890), followed closely by Experiment 3 (39.91 to 91.23; N-Gain = 0.854) and Experiment 2 (42.22 to 91.11; N-Gain = 0.846). This suggests a strong impact of the RME-scaffolding approach in developing students' ability to articulate mathematical reasoning verbally.

The visual representation indicator also exhibited a positive trend, though to a lesser degree. Experiment 1 improved from 62.5 to 87.5 (N-Gain = 0.667), Experiment 2 from 70 to 86.67 (N-Gain = 0.556), and Experiment 3 from 84.21 to 89.47 (N-Gain = 0.333), indicating that students were better able to translate problems into graphical or diagrammatic forms after the intervention.

In symbolic expression, while the pretest scores were initially lower across all groups, noticeable improvements were achieved. Experiment 1 increased from 29.69 to 61.88 (N-Gain = 0.458), Experiment 2 from 22.67 to 60.67 (N-Gain = 0.491), and Experiment 3 from 19.47 to 60.00 (N-Gain = 0.503), suggesting moderate gains in students' ability to convert real-world problems into mathematical symbols.

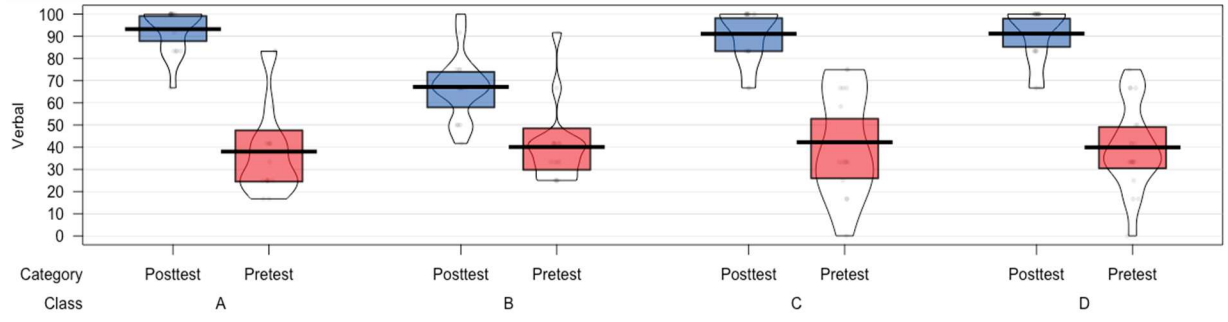


Figure 3a: Verbal

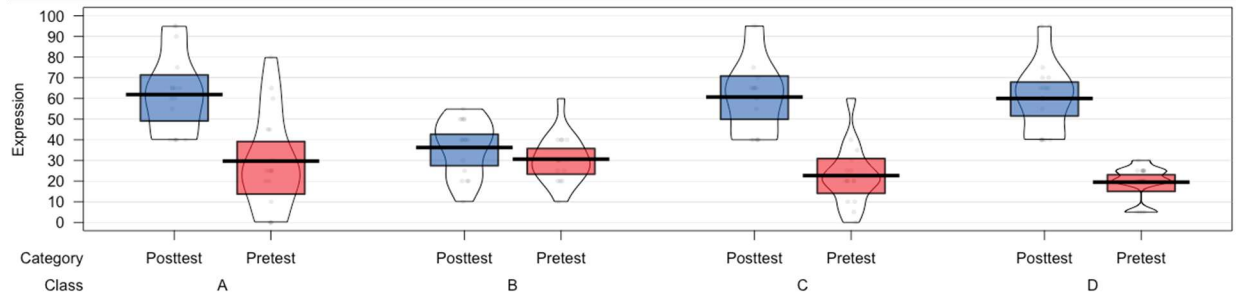


Figure 3b: Expression

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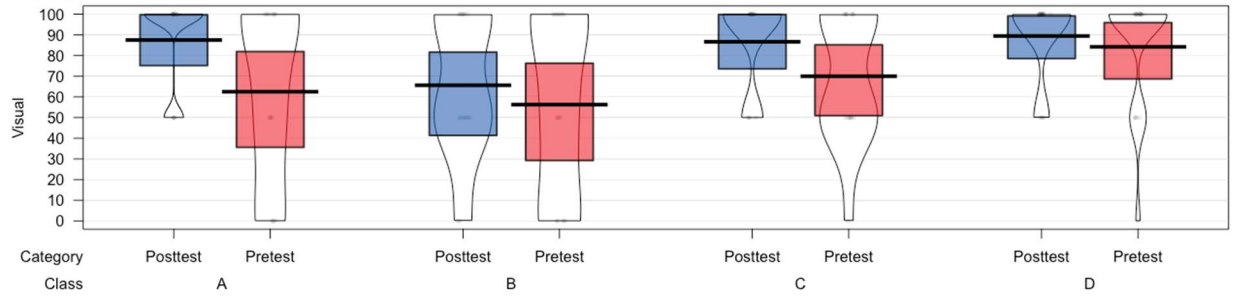


Figure 3c: Visual

Figure 3: Visualization of the students' performance (R software output)

In contrast, the control group showed limited progress. Verbal representation increased modestly from 40.10 to 67.19 (N-Gain = 0.452), visual representation saw a small rise from 56.25 to 65.63 (N-Gain = 0.214), and symbolic expression remained unchanged at 36.25 (N-Gain = 0.081). Overall, the data show a strong positive trend in posttest performance for the experimental groups, highlighting the effectiveness of the RME-scaffolding model in enhancing students' mathematical representation skills across all indicators. The students' answer is presented in Figure 4 and Figure 5.

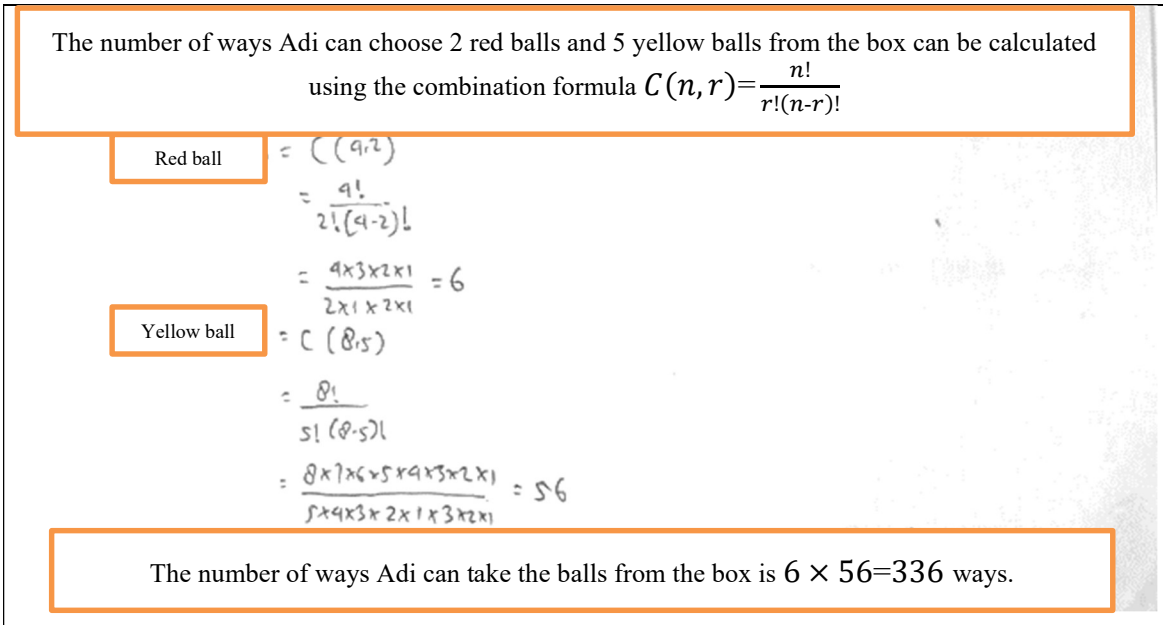


Figure 4a: Verbal Representation–Experimental Class

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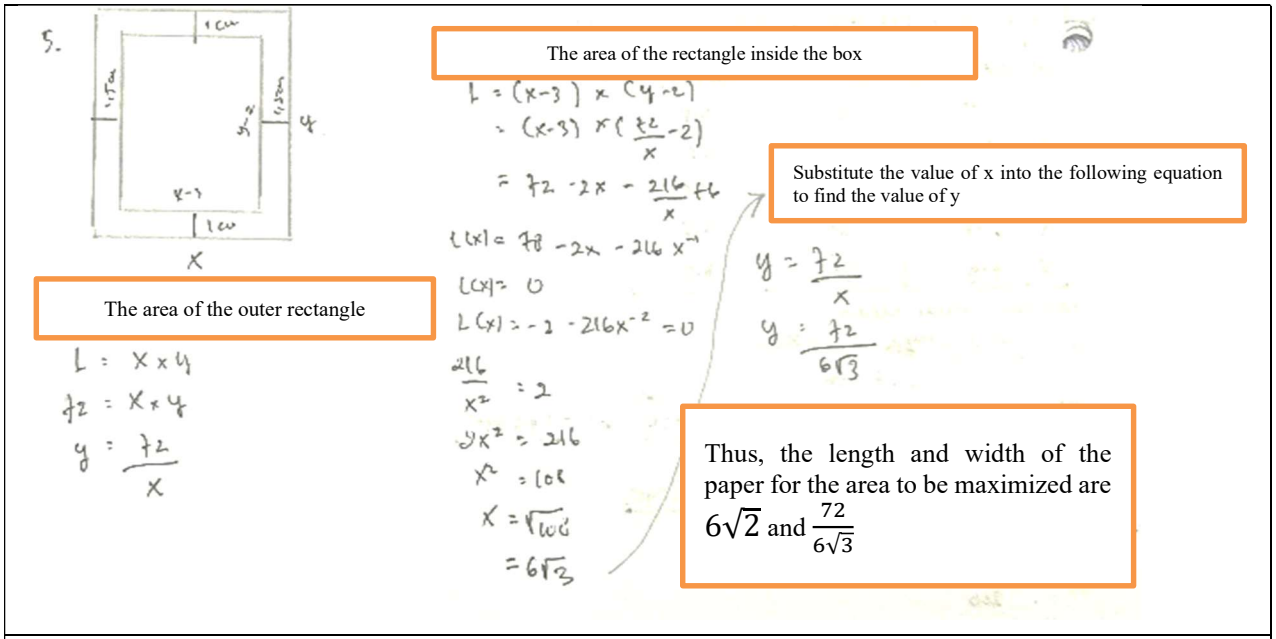


Figure 4b: Visual Representation–Experimental Class

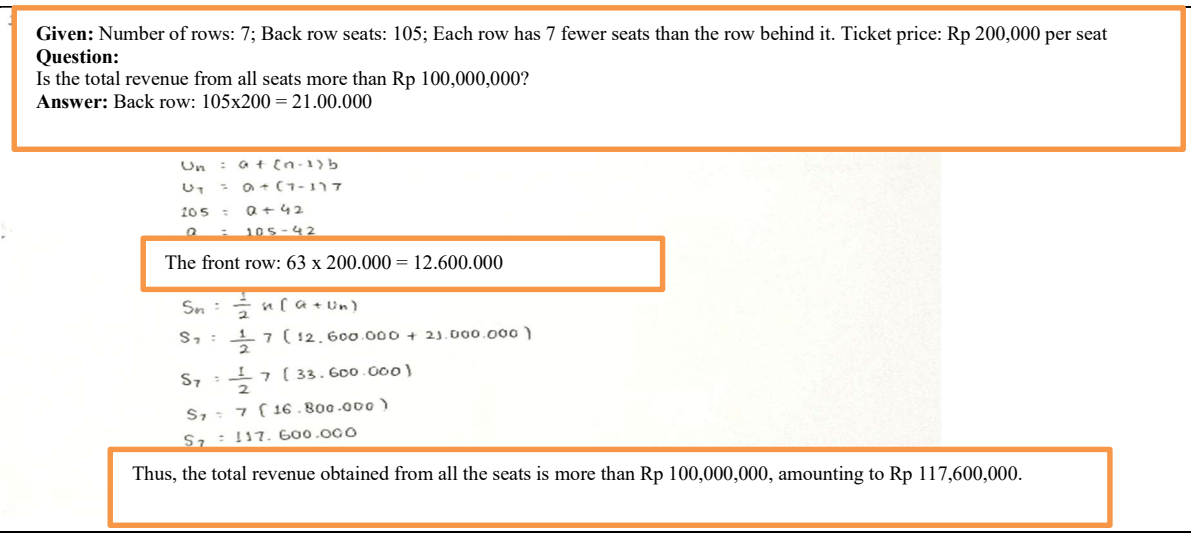


Figure 4c: Expressive Representation–Experimental Class

Figure 4: Students' Responses in Mathematical Representations Categories in experiment class (students answer)

Figure 4a, 4b, and 4c illustrate students in the experimental class engaging with verbal, visual, and expressive representation tasks, respectively. In Figure 4a, students articulate mathematical ideas through discussion, demonstrating their ability to translate abstract concepts into verbal explanations. Figure 4b shows students using diagrams and models to solve problems, reflecting the use of visual

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tools to enhance comprehension. Figure 4c highlights students creating expressive representations, such as written solutions, sketches, or symbolic formulations, to communicate their understanding.

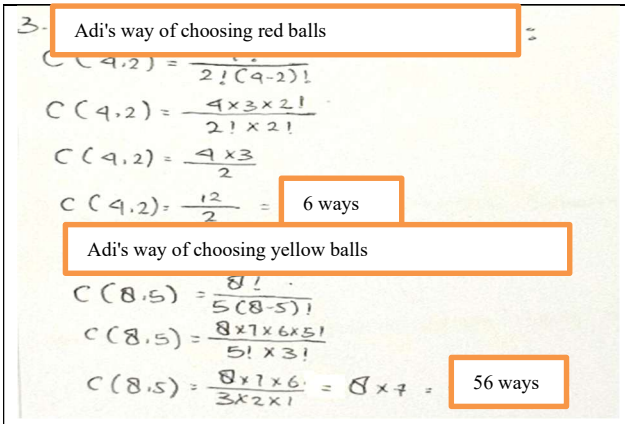


Figure 5a: Visual Representation–Control Class

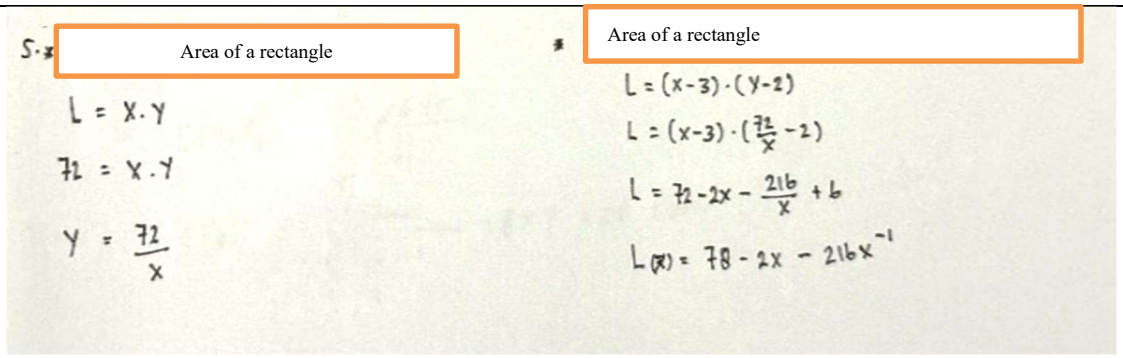
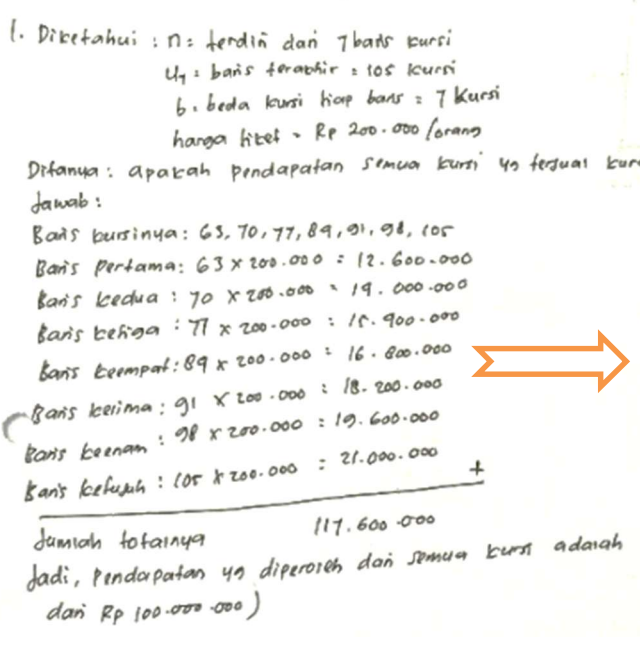


Figure 5b: Visual Representation–Control Class

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Given:
- n consists of 7 rows of seats
- U_1 = number of seats in the first row = 105 seats
- The difference in the number of seats between each row = 7 seats
- Ticket price = Rp 200,000 (per seat)

Question:
Is the revenue from all seats less than Rp 100,000,000?

Answer:
Number of seats per rows: 63, 70, 77, 84, 91, 98, 105
- First row: 63 × 200,000 = 12,600,000
- Second row: 70 × 200,000 = 14,000,000
- Third row: 77 × 200,000 = 15,400,000
- Fourth row: 84 × 200,000 = 16,800,000
- Fifth row: 91 × 200,000 = 18,200,000
- Sixth row: 98 × 200,000 = 19,600,000
- Seventh row: 105 × 200,000 = 21,000,000

Total revenue = 117,600,000
Thus, the total revenue from all the seats is Rp 117,600,000; which is more than Rp 100,000,000

Figure 5c: Expressive Representation–Control Class

Figure 5: Students' Responses in Mathematical Representations Categories in control class (students answer)

Figure 5 presents the representation performance of the control class across different tasks. In the control class, students' answers reflected variations in how they represented their understanding. For the visual representation–control class, learners tended to use the mathematical symbol as a representation of visual. In the second visual representation–control class, responses were also delivered through mathematical symbol, but the ideas appeared to be more structured and clearer and sometimes incomplete. On the other hand, in the expressive representation–control class, students relied more on mathematical symbol or verbal explanations rather than visual forms, which highlighted their ability to describe concepts but often without strong visual support.

DISCUSSIONS

The integration of RME with scaffolding has been shown to be an effective instructional approach for enhancing students' mathematical representation skills. This approach situates mathematical learning in real-world contexts (Vos, 2018), helping students relate abstract concepts to everyday experiences (Farida et al., 2023). Scaffolding, in this context, provides graduated support that enables students to gradually develop their reasoning and mathematical argumentation skills (Belland et al., 2008; Khairunnisak et al., 2021). Initially guided by lecturers, students learn to recognize patterns, formulate problem-solving strategies, and draw logical conclusions independently over time. The instructional design includes structured scaffolding through contextual problems, reflective

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questioning, collaborative learning, and exploration of multiple representations (Belland et al., 2011). This structure facilitates deeper conceptual understanding and encourages the application of knowledge across diverse contexts. Students in the experimental groups exhibited superior abilities in articulating mathematical ideas, converting visual data into mathematical expressions, and generalizing patterns when compared to their peers in the control group.

In the experimental class, students engaged directly with real-world problems using RME combined with scaffolding. Lecturers provided step-by-step guidance, posed reflective questions, and facilitated group discussions, allowing students to progressively take ownership of their learning and represent mathematical concepts through visual, symbolic, and written forms. In contrast, the control class followed a conventional teaching approach, relying mainly on lectures and routine exercises. Students in the control group tended to focus on memorizing procedures and solving problems mechanically, with limited opportunities to explore the relationships between concepts or use multiple forms of representation. As a result, students in the experimental class demonstrated greater flexibility in thinking, problem-solving, and expressing mathematical ideas compared to their peers in the control group.

The effectiveness of this approach is statistically supported by the results of a one-way ANOVA conducted on the N-Gain scores of the posttest data, which revealed a significant difference between the experimental and control groups. The Bonferroni multiple comparison test further confirmed that students in all three experimental classes outperformed those in the control group. Notably, more than 80% of students in the experimental groups achieved scores above the minimum threshold of 56, a clear indication of learning gains not observed in the control group. These findings affirm the advantage of using an RME-based scaffolding model over conventional teaching strategies.

Visual representation, a critical component of mathematical understanding, also showed notable differences. Experimental group students consistently created accurate and contextually relevant diagrams, graphs, or schematics that complemented their problem-solving processes. The control group, however, displayed limited use of visuals, and when attempted, the representations were often disconnected from the problem context, reducing their effectiveness in communicating understanding.

Similarly, in terms of symbolic or mathematical expression, students exposed to the RME-scaffolding model demonstrated a higher degree of precision and fluency in using mathematical symbols and equations. Their expressions were clearly linked to conceptual understanding and problem context. In contrast, control group students used symbols inconsistently and often failed to connect them meaningfully to underlying concepts, reflecting a more superficial understanding.

The RME approach with scaffolding fosters the gradual development of mathematical reasoning, moving students from basic comprehension to autonomous application of concepts. As teacher support is reduced, students begin to demonstrate more complex and independent uses of mathematical representation, particularly in visual forms (Gravemeijer, 1994). This transition highlights the effectiveness of the approach in promoting self-regulated learning and higher-order thinking.

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The contribution of this study extends beyond empirical results, offering theoretical insights into mathematics education. It validates the integration of RME and scaffolding as a powerful model for teaching representation skills and supports the constructivist paradigm, which holds that learners construct knowledge through meaningful experiences (De Lange, 1995). By embedding mathematical concepts within real-life contexts, students not only build deeper theoretical understanding but also develop practical competencies for real-world problem-solving. Moreover, the study aligns with Vygotsky's theory of the ZPD, illustrating the role of scaffolding in advancing students' learning beyond their current level of competence. As students receive tailored support and feedback, they are enabled to perform tasks that would otherwise be beyond their reach (Suherman & Vidakovich, 2022). The gradual removal of support leads to the internalization of concepts, resulting in more autonomous and sophisticated mathematical thinking (Anggoro et al., 2024; Suherman & Vidákovich, 2024; Supriadi et al., 2024; Zakaria et al., 2024).

The integration of RME and scaffolding presents a robust pedagogical model for enhancing mathematical representation in higher education. It effectively supports students' development across verbal, visual, and symbolic domains, enabling them to construct, apply, and generalize mathematical ideas with confidence and depth. This approach contributes meaningfully to the refinement of mathematics education theory and practice, advocating for instructional designs that are both contextually rich and cognitively supportive.

CONCLUSIONS

The findings of this study indicate that integrating Realistic Mathematics Education (RME) with scaffolding effectively enhances students' mathematical representation skills, with consistent outcomes across classes underscoring its reliability. This approach supports students in connecting abstract concepts to real-world contexts, fostering deeper understanding compared to conventional methods. However, the study's limited sample, single-institution scope, and short intervention duration restrict the generalizability of the results. Future research should extend this work by exploring long-term effects, application across diverse contexts, and its impact on broader mathematical competencies such as problem-solving and reasoning.

REFERENCES

[1] Altiner, E. Ç., Önal, H., & Yorulmaz, A. (2023). An Analysis of Realistic Mathematics Education Activities of Pre-Service Teachers Trained with a Constructivist Approach. *Mathematics Teaching Research Journal*, *15*(4), 26–44.

[2] Anggoro, B. S., Dewantara, A. H., Suherman, S., Muhammad, R. R., & Saraswati, S. (2024). Effect of game-based learning on students' mathematics high order thinking skills: A meta-analysis. *Revista de Psicodidáctica (English Ed.)*, 500158. https://doi.org/10.1016/j.psicoe.2024.500158

[3] Ariska, R., Gustine, G. G., & Setyarini, S. (2021). Promoting Students' Higher-Order Thinking Skills Through Teacher's Feedback in an EFL Classroom. *Proceedings of the*

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Thirteenth Conference on Applied Linguistics (CONAPLIN 2020). https://doi.org/10.2991/assehr.k.210427.072

[4] Belland, B. R., Glazewski, K. D., & Richardson, J. C. (2008). A scaffolding framework to support the construction of evidence-based arguments among middle school students. *Educational Technology Research and Development*, *56*, 401–422. https://doi.org/10.1007/s11423-007-9074-1

[5] Belland, B. R., Glazewski, K. D., & Richardson, J. C. (2011). Problem-based learning and argumentation: Testing a scaffolding framework to support middle school students' creation of evidence-based arguments. *Instructional Science*, *39*, 667–694. https://doi.org/10.1007/s11251-010-9148-z

[6] Bostic, J. D., Clark, Q., Vo, T., Esters, L. T., & Knoblach, N. A. (2020). A design process for developing agricultural life science-focused model eliciting activities. *School Science and Mathematics*, *121*(1). https://doi.org/10.1111/ssm.12444

[7] Bruner, J. S. (1978). *The Role of Dialogue in Language Acquisition." In A. Sinclair, R. J. Jarvella, & W. J. M. Levelt (Eds.), The Child's Concept of Language*. Springer Verlag.

[8] Brunner, L. G., Peer, R. A. M., Zorn, C., Paulik, R., & Logan, T. M. (2024). Understanding cascading risks through real-world interdependent urban infrastructure. *Reliability Engineering & System Safety*, *241*, 109653. https://doi.org/10.1016/j.ress.2023.109653

[9] Brush, T. A., & Saye, J. W. (2002). A summary of research exploring hard and soft scaffolding for teachers and students using a multimedia supported learning environment. *The Journal of Interactive Online Learning*, *1*(2), 1–12.

[10] De Lange, J. (1995). *Assessment: No change without problems. In T.A. Romberg (Ed.), Reform in school mathematics and authentic assessment*. State University of New York.

[11] Farida, F., Alamsyah, Y. A., & Suherman, S. (2023). Assessment in educational context: The case of environmental literacy, digital literacy, and its relation to mathematical thinking skill. *Revista de Educación a Distancia (RED)*, *23*(76), 1–26. https://doi.org/10.6018/red.552231

[12] Ferrini-Mundy, J. (2001). Introduction: Perspectives on principles and standards for school mathematics. *School Science and Mathematics*, *101*(6), 277. https://doi.org/10.1111/j.1949-8594.2001.tb17957.x

[13] Fitria, K. N., Mastur, Z., & Suyitno, A. (2022). Relationship of Mathematical Disposition with Student Problem Solving Construction Using Realistic Mathematics Education (RME) Model. *AlphaMath: Journal of Mathematics Education*, *8*(2). https://doi.org/10.30595/alphamath.v8i2.13612

[14] Fokong, N. H. (2023). Comparative Effects of Maternal Educational Attainment and Instructional Scaffolding on a Child's Educational Outcomes in Cameroon. *British Journal of Education, Learning and Development Psychology*, *6*(2). https://doi.org/10.52589/bjeldp-ewmvvoct

[15] Fredriksen, H. (2021). Exploring Realistic Mathematics Education in a Flipped Classroom Context at the Tertiary Level. *International Journal of Science and Mathematics Education*, *19*(2), 377–396. https://doi.org/10.1007/s10763-020-10053-1

[16] Freudenthal, H. (2002). *Revisiting mathematics education: China lectures*. Springer.

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[17] Gallimore, R. (1990). Teaching mind in society: Teaching schooling, and literate discourse. *Vygotsky and Education. Instructional Implications and Application of Sociohistorical Psychology*. https://cir.nii.ac.jp/crid/1573105975464002560

[18] Gravemeijer, K. (1994). *Developing realistic mathematics education*. https://research.tue.nl/en/publications/developing-realistic-mathematics-education

[19] Hermandra, R. H., Lestari, Z. S., & Qudratuddarsi, H. (2022). Achievement Goals, Metacognition and Horizontal Mathematization: A Mediational Analysis. *TEM Journal*. https://doi.org/10.18421/tem114-14

[20] Hogan, K., & Pressley, M. (1997). *Scaffolding Student Learning: Instructional Approaches and Issues*. Brookline Books.

[21] Hwang, J., & Ham, Y. (2021). Relationship between mathematical literacy and opportunity to learn with different types of mathematical tasks. *Journal on Mathematics Education*, *12*(2), 199–222. https://doi.org/10.22342/jme.12.2.13625.199-222

[22] Izzah, R., & Ekawati, R. (2023). Horizontal and Vertical Mathematization Processes of Junior High School Students in Solving Open-Ended Problems. *MATHEdunesa*, *12*(2). https://doi.org/10.26740/mathedunesa.v12n2.p400-413

[23] Johnson, P., Almuna, F., & Silva, M. (2022). The role of problem context familiarity in modelling first-order ordinary differential equations. *Journal on Mathematics Education*, *13*(2). https://doi.org/10.22342/jme.v13i2.pp323-336

[24] Kaba, Y., & Şengül, S. (2017). The relationship between middle school students' mathematical understanding and math anxiety-apprehensions. *Global Conference on Education and Research (GLOCER 2017)*, 449. https://glocer.org/wp-content/uploads/GLOCER_2017_Conference_Proceedings.pdf#page=464

[25] Khairunnisak, C., Johar, R., Zubainur, C. M., & Sasalia, P. (2021). Learning Trajectory of Algebraic Expression: Supporting Students' Mathematical Representation Ability. *Mathematics Teaching Research Journal*, *13*(4), 27–41.

[26] Kim, J. Y., & Lim, K. Y. (2019). Promoting learning in online, ill-structured problem solving: The effects of scaffolding type and metacognition level. *Computers & Education*, *138*, 116–129. https://doi.org/10.1016/j.compedu.2019.05.001

[27] Kösece, P., & Doğanay, A. (2023). Developing the Skill of Associating Mathematics with Real Life Through Realistic Mathematics Education: An Action Research. *Kuramsal Eğitimbilim*, *16*(2). https://doi.org/10.30831/akukeg.1214339

[28] Krawec, J. L. (2014). Problem representation and mathematical problem solving of students of varying math ability. *Journal of Learning Disabilities*, *47*(2), 103–115. https://doi.org/10.1177/0022219412436

[29] Langoban, M. A., & Langoban, M. A. (2020). What makes mathematics difficult as a subject for most students in higher education. *International Journal of English and Education*, *9*(3), 214–220.

[30] Laurens, T., Batlolona, F. A., Batlolona, J. R., & Leasa, M. (2017). How does realistic mathematics education (RME) improve students' mathematics cognitive achievement? *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(2), 569–578. https://doi.org/10.12973/ejmste/76959

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[31] Laurentiz, S. (2021). ART IN THE CONTEXT OF ALGORITHMIC LOGIC PROCEDURES. *ARBOR-CIENCIA PENSAMIENTO Y CULTURA*, *197*(800). (WOS:000690360500006). https://doi.org/10.3989/arbor.2021.800005

[32] Mainali, B. (2021). Representation in Teaching and Learning Mathematics. *International Journal of Education in Mathematics, Science and Technology*, *9*(1), 1–21. https://doi.org/10.46328/ijemst.1111

[33] McCoy, L. P., Baker, T. H., & Little, L. S. (1996). Using multiple representations to communicate: An algebra challenge. *Communication in Mathematics, K-12 and Beyond*, *58*.

[34] Nermin, B. a. L., & Kapucu, M. S. (2022). The Effect of Realistic Mathematics Education Activities Applied in Secondary School 7th Grade Mathematics Education on the Development of Life Skills. *The Eurasia Proceedings of Educational and Social Sciences*, *25*. https://doi.org/10.55549/epess.1218207

[35] Ningsi, G., Nendi, F., Pantaleon, K., Jehadus, E., & Bahagia, M. (2024). Application Of Realistic Mathematic Education to Improve Mathematical Creative Thinking Ability. *Proceedings of the 3rd International Conference on Education, Humanities, Health and Agriculture, ICEHHA 2023, 15-16 December 2*. https://doi.org/10.4108/eai.15-12-2023.2345633

[36] Nurfadilah, I., Nindiasari, H., & Fatah, A. (2021). Using Realistic Mathematics Education in Mathematical Problem-Solving Ability Based on Students' Mathematical Initial Ability. *Prima: Jurnal Pendidikan Matematika*, *5*(1). https://doi.org/10.31000/prima.v5i1.3166

[37] Payadnya, I. P. A. A., Wena, I. M., Noviantari, P. S., Palgunadi, I. M. P. K., & Pradnyanita, A. D. C. (2023). Development of RME Learning Media Based on Virtual Exhibition to Improve Students' High Order Thinking Skills (HOTS). *Mathematics Teaching Research Journal*, *15*(5), 129–156.

[38] Pea, R. D. (2004). The Social and Technological Dimensions of Scaffolding and Related Theoretical Concepts for Learning, Education, and Human Activity. *The Journal of the Learning Sciences*, *13*(3), 423–451.

[39] Pea, R. D. (2018). The social and technological dimensions of scaffolding and related theoretical concepts for learning, education, and human activity. In *Scaffolding* (pp. 423–451). Psychology Press. https://doi.org/10.1207/s15327809jls1303_6

[40] Richardson, J., Caskurlu, S., Castellanos-Reyes, D., Duan, S., Duha, M. S. U., Fiock, H., & Long, Y. (2021). Instructors' conceptualization and implementation of scaffolding in online higher education courses. *Journal of Computing in Higher Education*, *34*(1). https://doi.org/10.1007/s12528-021-09300-3

[41] Rokhmat, J., Marzuki, M., Wahyudi, W., & Putrie, S. D. (2019). A Strategy of Scaffolding Development to Increase Students Problem-Solving Abilities: The Case of Physics Learning with Causalitic-Thinking Approach. *Turkish Journal of Science Education*, *16*(4). https://doi.org/10.36681/tused.2020.8

[42] Safa, M. A., & Motaghi, F. (2021). Cognitive vs. Metacognitive scaffolding strategies and EFL learners' listening comprehension development. *Language Teaching Research*, *28*(3). https://doi.org/10.1177/13621688211021821

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[43] Santiago, P. V. da S., & Alves, F. R. V. (2022). Math for olympiad: A didactic proposal for high school from the perspective of the international mathematical olympiad. *Journal of Advanced Sciences and Mathematics Education*, *2*(2), Article 2. https://doi.org/10.58524/jasme.v2i2.135

[44] Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM*, *40*(6), 927–939. https://doi.org/10.1007/s11858-008-0125-9

[45] Smagorinsky, P. (2018). Deconflating the ZPD and instructional scaffolding: Retranslating and reconceiving the zone of proximal development as the zone of next development. *Learning, Culture and Social Interaction*, *16*, 70–75. https://doi.org/10.1016/j.lcsi.2017.10.009

[46] Suherman, S., & Vidakovich, T. (2022). Tapis patterns in the context of cthnomathematics to assess students' creative thinking in mathematics: A rasch measurement. *Mathematics Teaching Research Journal*, *14*(4), 56–72.

[47] Suherman, S., & Vidákovich, T. (2024). Role of creative self-efficacy and perceived creativity as predictors of mathematical creative thinking: Mediating role of computational thinking. *Thinking Skills and Creativity*, *53*, 101591. https://doi.org/10.1016/j.tsc.2024.101591

[48] Suherman, S., Vidákovich, T., Mujib, M., Hidayatulloh, H., Andari, T., & Susanti, V. D. (2025). The role of STEM teaching in education: An empirical study to enhance creativity and computational thinking. *Journal of Intelligence*, *13*(7), 88. https://doi.org/10.3390/jintelligence13070088

[49] Supriadi, N., Jamaluddin Z, W., & Suherman, S. (2024). The role of learning anxiety and mathematical reasoning as predictor of promoting learning motivation: The mediating role of mathematical problem solving. *Thinking Skills and Creativity*, *52*, 101497. https://doi.org/10.1016/j.tsc.2024.101497

[50] Susanto, E., Susanta, A., Aliyyah Irsal, N., & Stanggo, P. D. (2024). Developing RME-based module in statistics to improve problem-solving skills for higher education students. *Jurnal Elemen*, *10*(2). https://doi.org/10.29408/jel.v10i2.25157

[51] Tang, K. (2023). The characteristics of diagrams in scientific explanations: Multimodal integration of written and visual modes of representation in junior high school textbooks. *Science Education*, *107*(3), 741–772. https://doi.org/10.1002/sce.21787

[52] Umar, U. P. S., & Zakaria, Z. (2022). The effectiveness of the realistic math education (RME) learning method based on manipulative media in improving the problem-solving abilities of elementary school students. *Ekspose: Jurnal Penelitian Hukum Dan Pendidikan*, *21*(1). https://doi.org/10.30863/ekspose.v21i1.3405

[53] Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in Teacher–Student Interaction: A Decade of Research. *Educational Psychology Review*, *22*(3), 271–296.

[54] Van Den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 713–717). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_170

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[55] Vincent-Lancrin, S. (2023). Fostering and assessing student critical thinking: From theory to teaching practice. *European Journal of Education*, *58*(3), 354–368. https://doi.org/10.1111/ejed.12569

[56] Vos, P. (2018). "How real people really need mathematics in the real world"—Authenticity in mathematics education. *Education Sciences*, *8*(4), 195. https://doi.org/10.3390/educsci8040195

[57] Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (Vol. 86). Harvard university press.

[58] Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17*(2), 89–100.

[59] Yuanita, P., Zulnaidi, H., & Zakaria, E. (2018). The effectiveness of Realistic Mathematics Education approach: The role of mathematical representation as mediator between mathematical belief and problem solving. *PloS One*, *13*(9), e0204847. https://doi.org/10.1371/journal.pone.0204847

[60] Zakaria, M. I., Nasran, N. A. H. N., Abdullah, A. H., Alhassora, N. S. A., Pairan, R., & Yanuarto, W. N. (2024). Unlocking the Future: Mathematics Teachers' Insight into Combination of M-Learning with Problem-Based Learning Teaching Activities. *Mathematics Teaching Research Journal*, *16*(3), 196–216.

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APPENDIX A

Syntax teaching

Teaching period	Lesson Plan
Week 1	Experiment and Control Activities Research design Course overview and giving pre-test
Week 2 – Week 11	**Control group** 1. Concept Explanation I; teacher presents definitions, rules, and procedures in probability. 2. Formula Memorization I; students memorize arithmetic sequence formulas and solve simple examples. 3. Worked Examples I; teacher demonstrates arithmetic series problem-solving while students observe. 4. Routine Practice I; students solve similar problems individually; teacher provides limited guidance. 5. Concept Explanation II; teacher presents combinatorics concepts and formulas; students take notes. 6. Formula Memorization II; students memorize formulas and solve exercises in mixed topics. 7. Worked Examples II; teacher demonstrates multi-topic problems; students observe and replicate. 8. Routine Practice II; students solve additional exercises individually; minimal teacher guidance. 9. Answer Correction I; teacher provides correct solutions and explains errors; students review their work. 10. Answer Correction II; continued review and error correction under teacher guidance.
Week 2 – Week 11	**Experiment group** 1. Contextual Problem Presentation; introduce real-life scenarios (e.g., traditional fabrics, local crafts) to anchor learning. 2. Guided & Reflective Questioning I; probability problems (drawing marbles); teacher scaffolds open-ended questioning. 3. Guided & Reflective Questioning II; arithmetic sequences (weekly savings patterns); structured guidance from teacher. 4. Collaborative Discussion I; arithmetic series (total savings) in groups; co-construct understanding. 5. Multi-Representation Exploration I; 3D geometry (volume of cylinder); express solutions verbally, visually, symbolically. 6. Collaborative Discussion II; probability & combinatorics (arranging books, forming committees); peer interaction and reasoning. 7. Guided & Reflective Questioning III; mixed topics (integrating probability & sequences); teacher gradually reduces guidance. 8. Multi-Representation Exploration II; mixed topics; students express understanding using verbal, visual, and symbolic forms; teacher monitors progress. 9. Reflection & Presentation I; students present solutions, reflect on learning strategies, receive teacher feedback. 10. Reflection & Presentation II; peer review, discussion of insights, refinement of solutions.
Week 12	Wrap-up: Conclude the course by measuring improvements in students' verbal, visual, and symbolic representation skills (experimental) or by evaluating overall learning and summarizing key concepts (control). Post-test

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APPENDIX B

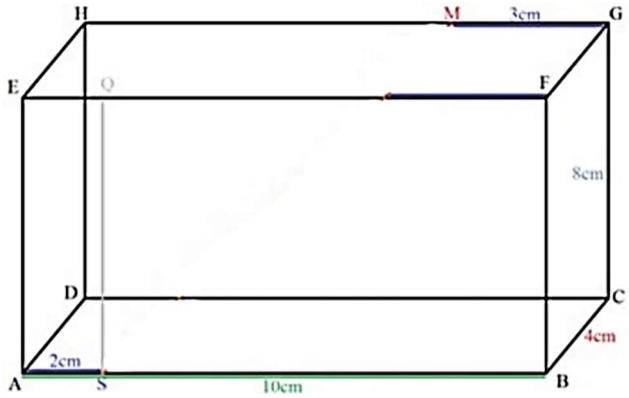
Mathematical Representation Skills Pretest

1. At the premiere screening of a film, the cinema provides a room for the audience that is divided into 6 rows of seats. The back row consists of 90 seats, and each row toward the front has 6 fewer seats than the row behind it. The ticket price for each seat is Rp100.000,00 and all tickets are sold out. If the total revenue from the front row is Rp6.000.000,00 determine whether the total revenue obtained from all the rows in the cinema is less than Rp50.000.000,00. Provide reasons and calculations!
2. In a survey, 200 adults were asked about the number of books they read in a year. The results of the survey are as follows:

Number	Time Spent Reading Books	Number of People
1	0–1	30
2	2–3	50
3	4–5	60
4	6–7	40
5	8–9	20

Determine the average number of books read per person!

3. In a soccer team, there are 11 players consisting of 3 goalkeepers and 8 field players. How many ways can the coach select 2 goalkeepers and 4 field players for the next match?
4. A cuboid ABCD.EFGH is shown in the figure below.



The cuboid has dimensions AB = 10 cm, BC = 4 cm, CG = 8 cm, AS = 2 cm, GM = 3 cm. If an ant walks on the surface of the cuboid from point S to food located at point M, what is the shortest distance from the ant (S) to the food (M)? Draw the route from the ant to the food!

5. A sheet of HVS paper has an area of 54 cm². Sukardi will use the paper to type an invitation letter. If the typing margins are 1 cm for the top and bottom, and 1.5 cm for the left and right sides, what is the maximum length and width of the typing area? Illustrate your answer!

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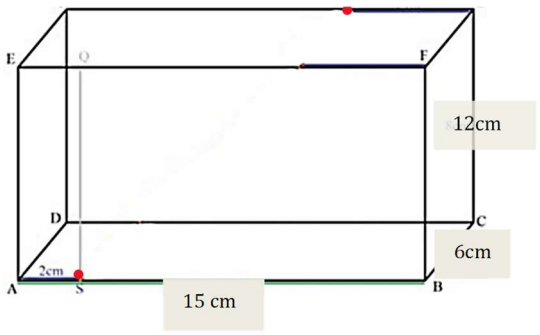
APPENDIX C

Mathematical Representation Skills Posttest

1. At a music concert, the committee provided a room for the audience which was divided into 7 rows of seats. The back row consisted of 105 seats and each row decreased by 7 seats until the front row. The ticket price for each seat was Rp200,000.00 and all seats were sold out. If the total money obtained from the front row was Rp12,600,000.00, was the total revenue from all seats in the concert hall less than Rp100,000,000.00? Provide reasons along with the calculation!
2. In a survey, 200 adults were asked about how many hours per day they use their mobile phones. The results are as follows:

No.	Time (hours)	Number of People
1	0–1	15
2	2–3	20
3	4–5	35
4	6–7	60
5	8–9	70

   Determine the average daily duration of mobile phone usage!
3. In a box, there are 12 balls consisting of 4 red balls and 8 yellow balls. How many ways can Adi choose 2 red balls and 5 yellow balls?
4. Given a cuboid ABCD.EFGH as shown in the figure:



   The cuboid has dimensions AB = 15 cm, BC = 6 cm, CG = 12 cm, AS = 2 cm, GM = 3 cm. If an ant walks on the surface of the cuboid from point S to the food located at point M, what is the shortest distance from the ant's starting point S to the food at M? Illustrate the ant's path to the food!
5. A sheet of HVS paper has an area of 72 cm². Doni will use this paper to type an invitation letter. If the typing margin is 1 cm for the top and bottom, and 1.5 cm for the left and right sides, what are the maximum length and width of the typing area? Illustrate your answer!

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Becoming (De)Contextualized Learners and Teachers of Mathematics: An Arts-Based Ethnodrama

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Abstract: Grounded in living educational theory, socio-cultural theory, and transformative learning theory, this inquiry investigates the professional growth of the first author, Hem, from a traditional practitioner to a critical and culturally aware mathematics educator. This inquiry examines critically (de)contextualized mathematics learning and teaching to envision a pathway toward becoming a transformative learner and teacher, while also investigating the mathematical concepts learners develop through contextually grounded instruction. The study is structured around the research question: How has Hem developed as a contextualized learner and mathematics teacher? The research employs an auto-ethnographic methodology, drawing upon Hem's personal experiences as a learner, teacher, and educational researcher. The study employs an arts-based approach to facilitate this investigation, subscribing to an ethnodrama. This method is a distinct form of dramatic literary expression, constructing a theatrical setting across three acts where the authors and participants engage in collaborative performance. We critically reflect on past experiences and analyze the applications of cultural heritage as a contextualized framework for mathematics instruction. The study presents a critical self-investigation for integrating personal narrative and cultural context in mathematics education. This inquiry calls for curricular reform and teacher development in Nepal to foster more inclusive and empowering mathematics education.

Keywords: ethnodrama, transformative learning, ethnomathematics, mathematics education

INTRODUCTION

This article originates from a section of the first author's MPhil dissertation in mathematics education, with contributions from the second, third, and fourth authors. The third and fourth authors, Indra and Ruma, served as the second and first supervisors, offering various perspectives from inception to the finalization of the article. The second author, Niroj, helped me shape and reshape the article at the publication level by offering critical comments throughout the article. The article explores methods for teaching and learning mathematics both inside and outside the classroom, aiming to help teachers become contextualized, culturally aware, and responsible mathematics educators. Using a narrative inquiry technique, we have concentrated on Hem's experiences and opinions regarding the nature of mathematics while working as a math teacher in Surkhet Valley, Nepal. Hereafter, we have used the first person

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'I, me, my' to embody Hem's reflections and perspective in the article. We begin this article with the following vignette:

> "I practiced it many times, but it did not get into my head. Sir, is memorization a good technique for understanding mathematics?"

Reflecting on the vignette above, it continues to resonate in my mind. I have taught mathematics at the school level for over ten years, yet I still consider myself a novice in mathematics education. I work as a researcher, educator, and teacher at a school and university in Nepal. Throughout my career, I have encountered various teaching and learning techniques. In this context, Dhungana et al. (2023) noted that "teaching and learning mathematics in Nepali schools is not as fruitful as expected" (p. 186). This observation prompted me to reflect critically on my beliefs and experiences regarding mathematics education. My postgraduate studies have also exposed me to various research approaches, including multi-paradigmatic and critical autoethnographic methods, which can enhance my teaching and learner practices. These experiences have motivated me to envision better alternatives for achieving more effective outcomes (Dhungana, 2023). Inspired by my life experiences, I am committed to transforming from a decontextualized mathematics teacher to a culturally contextualized one.

So far, as a learner, from my earliest academic endeavours, my pursuit has been unwaveringly focused on attaining superior grades in school mathematics. As a teacher, I initially held the conviction that teaching mathematics involved delivering information to my students, who were seen as passive learners. I held onto this viewpoint until 2010, when I enrolled in a Master of Education (Mathematics) program. As a novice researcher, I was eager to explore my previously undisclosed personal experiences and contradictions (Whitehead, 2008), aspects of school mathematics teaching and learning, and to share them with the world. I came to realize, as a passive recipient of knowledge, that what my teachers conveyed as they transmitted from the mathematics textbook was a collection of culturally decontextualized knowledge that did not encompass my own cultural experiences. My practices of decontextualization turn a real situation into abstract symbols, thereby working with those symbols as if they are separate from their original context. Contextualizing means pausing during this process to consider what those symbols represent in the real world (Lee, 2025; Yeea & Bostic, 2014). I have actively participated in research, which has led to a restructuring of my teaching perspective. I understand my responsibility to promote a meaningful understanding of learning mathematics by encouraging my students to incorporate their cultural knowledge into the classroom.

With all of the above, this article aims to demonstrate my investigation into the enduring issue of culturally contextualized mathematics in promoting meaningful classroom learning, incorporating the ethnomathematics perspective. Hallden (1999) explains that contextualizing a problem can involve connecting it to a real-life scenario and linking it to other concepts or ideas. Utilizing ethnodrama, the authors aim to enhance readers' critical awareness of the importance of crafting culturally contextualized mathematics education. Additionally, we aim to encourage the development of readers' critical reflective thinking to bring about transformations in their pedagogical and curricular practices. This inquiry addresses the following research question:

- How have I (Hem) been developing myself as a contextualized learner and mathematics teacher?

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LITERATURE REVIEW

As a practitioner-researcher, teacher-educator, and educator, I have initiated a thorough examination of accepted theories in my profession. I have learned that there is not just one "royal road" to transformative teaching. I find value in locally produced frameworks that address my research needs as well as the needs of others, rather than relying exclusively on broad theories. Furthermore, it makes sense for me to select Vygotsky's Socio-cultural Theory (Vygotsky, 1978), Transformative Learning Theory (Mezirow, 1997), and Living Theory Methodology (Whitehead, 2008). Transformative research adopts a comprehensive perspective, enabling researchers to formulate inquiries that span multiple paradigms, with the goal of reshaping educational policy and practice (Dahal, 2024; Paudel et al., 2023; Taylor et al., 2012). Living Educational Theory is a potent instrument for self-awareness and career advancement in this research study. It prompts me to consider the essential question, "How can I improve my practice?" I explore my journey as a mathematics teacher and educational researcher through reflective narratives, drawing on Living Education Theory. I describe my current teaching and learning methods as I become more conscious, fusing my goals, experiences, and insights. It enables me to develop, refine, and make significant contributions to the field of education through my academic work.

This study used socio-cultural theory (Vygotsky, 1978) to perceive teachers' conceptual understanding of contextualization and how they implement it in the teaching-learning process through social interactions and the culture of learning. In addition, I applied socio-cultural theory to my study to improve students' comprehension of teaching and learning activities and leveraged insights into how students learn. Understanding that students learn differently from one another is crucial. I have chosen socio-cultural theory as one of the theoretical frameworks for my study to delve more into this. My stories familiarized me with the Zone of Proximal Development (ZPD) (Vygotsky, 1978). I used it frequently to enhance my understanding of mathematical concepts. Transformative Learning Theory supplied new ontological, epistemological, and axiological underpinnings for research orientation (Luitel & Taylor, 2007; Mezirow, 1997; Pant, 2019). From an ontological perspective, it made it easier for me to incorporate various worldviews into my own, resulting in a shift from my existing state of being to a more pleasant state through critical self-reflection. Epistemologically, my instrumental knowledge shifted to communicative knowledge, aligning with the principles of transformative learning theory. Axiologically, this broadened my understanding, combining instrumental and communicative knowing, ultimately promoting contextualization in mathematics education through transformative pedagogy.

METHODS

This study used a qualitative research design based on an arts-based auto-ethnographic approach aligned with Hem's experiences as a learner, teacher, teacher educator, and educational researcher from an early career. The primary methodological framework is ethnodrama, which serves as both the process of inquiry and the mode of representation. This approach is chosen to critically and evocatively explore the first author's (Hem's) transformative journey from a decontextualized to a contextualized perspective as a mathematics educator.

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Research Paradigms

The inquiry is situated within a multi-paradigmatic research space, guided by interpretivism, criticalism, and postmodernism. The interpretive paradigm facilitates a deep understanding of the meanings Hem assigns to his experiences as a learner, teacher, and researcher (Shrestha, 2018). The critical paradigm challenges dominant, transmission-based models of mathematics education and questions the assumptions underlying decontextualized pedagogy (Brookfield, 2000). Ultimately, a postmodern lens enables the acceptance of multiple, subjective truths and alternative forms of expression, such as narrative and metaphor, to represent the complexity of lived experience (Denzin & Lincoln, 2005).

Auto-Ethnographic Approach as Research Method

Subscribing to auto-ethnography (Ellis et al., 2011; Spry, 2001), this method involves a systematic (self-)study of Hem's personal and professional experiences within the cultural context of mathematics education in Nepal, particularly in the Surkhet Valley. As an auto-ethnographer, Hem critically reflects on his own "living contradictions" (Whitehead, 2008)—the tensions between his past practices as a traditional teacher and his evolving identity as a culturally responsive educator. This process treats the self as the primary data source, with personal narratives serving as the evidentiary base for analyzing broader cultural patterns and pedagogical possibilities.

Ethnodrama as Methodology and Representation

To operationalize the auto-ethnographic inquiry, we subscribed to an arts-based approach by crafting an ethnodrama (Saldana, 2011). Ethnodrama transforms ethnographic data—in this case, autobiographical memories, reflective journals, and dialogue from Hem's teaching practice—into a dramatic script. This method was selected for its capacity to engage readers emotionally and intellectually, moving beyond a traditional analytical report to a performative, evocative representation of the research findings. The ethnodrama is structured in three acts, each corresponding to a key theme in Hem's developmental journey:

1. *Act I: Step-by-Step Memorization* dramatizes the limitations of rote learning and decontextualized mathematics instruction.
2. *Act II: Mathematics in My Religious/Cultural Texts* explores the discovery of sophisticated mathematical concepts within Hindu epics (e.g., the Ramayana, Mahabharata) and scriptures, challenging the dichotomy between academic mathematics and cultural knowledge.
3. *Act III: Mathematics in Religious Places* illustrates the practical application of local cultural artifacts (e.g., temple symbols, Pipal leaves) to teach abstract mathematical concepts like symmetry.

Each act presents dramatized vignettes constructed from Hem's lived experiences. The script format includes stage directions, dialogue, and the internal monologue of the "Storyteller Researcher" (Hem), creating a theatrical setting that invites the audience (readers) into a collaborative performance of meaning-making.

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Data Collection and Analysis

Data for this self-study were drawn from Hem's extensive personal archive, which includes:

- Autobiographical memories from his time as a student and teacher.
- Critical incident journals documenting specific classroom interactions and moments of professional dissonance.
- Reflective narratives written during his MPhil studies catalyzed a transformative perspective.
- Dialogues with students, colleagues, and community members (e.g., a Pandit) were reconstructed for the dramatic script.

The analytical process was iterative and reflective, aligning with the principles of living educational theory (Whitehead, 2008). Data were analyzed through critical self-reflection, focusing on the central research question: *How have I (Hem) been developing myself as a contextualized learner and mathematics teacher?* Thematic analysis was used to identify recurring patterns and key turning points in Hem's understanding, which were then organized into the three thematic acts of the ethnodrama. The scripting process served as a form of analysis, as selecting, structuring, and dramatizing the narratives required deep interpretation and crystallization of the core insights.

Ensuring Rigor and Addressing Ethical Issues

As an auto-ethnographic study employing an arts-based methodology, this inquiry operates outside the traditional criteria of validity and reliability used in positivist research. Instead, we have prioritized trustworthiness, authenticity, and ethical reflexivity throughout the research process (Lincoln & Guba, 1985). Ensuring rigor in such subjective and performative study requires deliberate and continuous engagement with specific practices.

Ensuring Rigor

We employed several strategies aligned with arts-based and qualitative research paradigms to establish the credibility and authenticity of the narratives and dramatic representations. The research process was grounded in critical self-reflection, a core tenet of Living Educational Theory (Whitehead, 2008). As the primary author and subject, Hem continuously interrogated his assumptions, biases, and positions of privilege as a Brahmin male, a teacher, and a researcher. This involved maintaining a detailed reflective journal to trace the evolution of his understanding, ensuring that the narratives presented in the ethnodrama were not merely recollections but critically examined accounts of his lived experiences (Dahal, 2023). While the study is auto-ethnographic, it is not solipsistic. The research was shaped through dialogue and critical feedback from the co-authors, who served as supervisors and critical friends. Their diverse perspectives—offering "various perspectives from inception to the finalization of the article"—acted as a form of triangulation, challenging interpretations and enriching the analysis. Furthermore, the narratives incorporate the voices of students, a Pandit, and a friend, providing multiple vantage points on the phenomena of (de)contextualized learning. The ethnodrama offers a "thick description" (Geertz, 1973) of the cultural contexts, including detailed accounts of the Hindu epics, religious practices, and classroom interactions. This rich-

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ness allows readers to vicariously experience the settings and judge the transferability of the insights to their own contexts. For instance, the detailed dialogue with the student Prakriti about symmetry offers a concrete example of the pedagogical shift being advocated.

A key goal of this research is to explore its transformative potential, aligning with Transformative Learning Theory (Mezirow, 1997). Therefore, the criterion for success is accurate representation and catalytic authenticity—the ability to provoke reflection and inspire action in the reader (Dahal, 2023; Saldana, 2011). The ethnodrama format is specifically chosen for its *verisimilitude*, or "lifelikeness," aiming to create an emotional and intellectual resonance that encourages educators to critically examine their own practices. The question posed at the end of Act II—"Is it contextualized mathematics?"—is deliberately left open to stimulate this reader-response.

Addressing Ethical Issues

Conducting research that delves into personal memory, cultural traditions, and pedagogical practice necessitates a vigilant approach to ethics (Dahal & Luitel, 2022). Although this is an auto-ethnography, it involves other participants. The students, the Pandit, and the friend mentioned in the acts are anonymized using pseudonyms (e.g., Prakriti, Dinesh, Mahesh) to protect their identities and confidentiality. While the specific school in Surkhet Valley is not named, the cultural context is essential to the study. We have striven to present this context in a way that honors its integrity without exposing individuals or institutions to any risk. A significant ethical consideration involves the representation of Hindu religious texts (Ramayana, Mahabharata, Upanishads) and practices. We have approached these cultural elements not as objects of exotic scrutiny but as legitimate sources of mathematical knowledge (ethnomathematics). The intent is to affirm and integrate cultural heritage into academic discourse, thereby challenging the hegemony of Western, decontextualized curricula. We have described these elements accurately and respectfully, drawing on established interpretations. auto-ethnography requires a significant degree of vulnerability from the researcher (Adams & Holman Jones, 2008; Dahal & Luitel, 2022; Ellis et al., 2011). Hem's narrative exposes past pedagogical shortcomings and personal struggles with mathematics. This self-exposure is a deliberate ethical and methodological choice, intended to present an honest, authentic account of professional growth. The risk of self-indulgence is mitigated by framing the personal within broader theoretical and social concerns about mathematics education, ensuring the work contributes to a larger scholarly and practical conversation. Thus, this research aims to improve mathematics education for Nepali students by making it more inclusive and meaningful. We have been mindful to avoid any representation that could cause harm, misrepresent cultural practices, or reinforce stereotypes. The critique of rote learning is directed at systemic pedagogical practices, not individual teachers or students, who are often products of the same system.

DISCUSSIONS

Based on the formulated problem, purpose, and methodological roadmap, we have attempted to discuss acts aligned with the themes of culturally (de)contextualized mathematics in subsequent acts.

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Act I: Step-by-Step Memorization

The director addresses the audience, informing them that Act I will commence shortly and briefly explaining the nature of the upcoming drama.

Whistle sounds, background music, and stage lights on

Storyteller researcher: "I practiced it many times, but it did not get into my head.

Sir, is just memorization a good understanding technique …?"

This was my voice on the first day of my master's degree in the Algebra class. After that, my teacher smiled at me and said that understanding and memorizing are two different things. Sometimes, understanding comes by memorizing. Go this way. You may get your aim. It was the year 2010. I hold a master's degree in mathematics education from the Central Campus of Education, Kirtipur. I would fear Abstract Algebra. My seniors told me that algebra is the most challenging subject. My teacher starts that course from "Group Theory". He teaches the definition of a theorem in each chapter. I remember that day, my Algebra teacher taught us about "Cyclic Group" and he moved to this content/chapter as a definition of the theorem. When I wrote the definition of a cyclic group in the classroom, I memorized that definition but did not understand. I questioned myself: How can I know that definition? Is this definition learned in a real-life situation? After that, my teacher taught a theorem. He wrote a theorem on the whiteboard and proved it. He used that definition of a cyclic group to prove the related theorem.

I am thinking and realize a couple of minutes…

> "…understanding a definition is more effective than rote memorization when proving related theorems. In my experience, I wouldn't fully grasp a theorem without first understanding its definition. Moreover, learning mathematics with an emphasis on understanding involves making connections among different ideas. These connections help facilitate the transfer of prior knowledge to novel situations. I argue that rote memorization does not reflect true understanding."

It was in July 2016, when I was a new teacher at a reputed private boarding school in the Surkhet Valley. It was like a paradigm shift for me when I changed schools. During that time, I would teach the *Matrix of Basic Mathematics* class. I started this chapter with a definition of a Matrix and its examples. After that, I defined various matrices with examples, including Row Matrix, Column Matrix, Square Matrix, Diagonal Matrix, etc. At the end of that class, I assigned the task of memorizing the definition of a Matrix and its various types. The next day I asked them some definitions I had already taught yesterday. Some students find it easy to define. Most students did not explain their points clearly; they often skipped one or two sentences or words. For example, one student describes a Diagonal Matrix: A matrix is considered a diagonal matrix if all elements except those on the main diagonal are zero. However, the correct definition is that of a diagonal matrix: A square matrix is considered a diagonal matrix if all elements except those on the main diagonal are zero. Here, the *word "square" is skipped i*n that definition. Similarly, some students incorrectly define Upper Triangular and

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Lower Triangular Matrices. They were wrong, such as the incorrect use of "above instead of below" and "below instead of above" in that definition.

For an upper triangular matrix, a student's response is 'A square matrix in which all elements above the main diagonal are zero is called an upper triangular matrix.' However, the correct response is, 'A square matrix in which all elements below the main diagonal are zero is called an upper triangular matrix.' Likewise, for a lower triangular matrix, a student responds as 'A square matrix in which all elements below the main diagonal are zero is called an upper triangular matrix. Though the correct response is 'A square matrix in which all elements above the main diagonal are zero is called a lower triangular matrix.' Most students made conceptual errors when defining upper- and lower-triangular matrices. Although some students attempted to state the definitions, they often confused the positions of the zero elements relative to the main diagonal or mislabeled the matrix type. In addition, several students provided definitions without supporting examples, which suggests that their understanding may have relied largely on rote memorization rather than conceptual comprehension. As researchers and teachers, this observation raises several reflective questions: Why do students tend to memorize definitions mechanically? Why do they struggle to interpret the meaning behind these definitions? These questions highlight a broader pedagogical issue. When definitions are learned only through memorization, students may reproduce the wording but fail to understand the structural idea underlying the concept. Therefore, teaching mathematical definitions through visual representations, examples, and contextual or real-life connections may help students develop deeper conceptual understanding rather than merely recalling statements. This approach can support meaningful learning definitions and other mathematical concepts that are often taught through memorization alone.

Now, I believe that memorization alone is not a good understanding technique. We could have understood the matter more conceptually if it were taught in relation to a daily life problem in mathematics. In the past, the learning process was teacher-centric; students were rarely motivated to learn through the concept of the learning-by-doing approach. According to modern learning innovations, teachers are facilitators, and students are their own teachers. As a facilitator, they facilitate the learning process, enabling students to be more critical of it.

Typically, there are two primary methods of learning: the surface learning strategy and the deep learning strategy. Surface learning involves memorizing information without delving into deeper comprehension, often referred to as rote learning. On the other hand, the deep approach involves immersing students in the study process to thoroughly understand the subject matter. Rote memorization is a prevalent practice across educational levels, emphasizing the memorization of content rather than its comprehensive understanding and application to real-life situations. Rote learning is simply storing data in the brain; it doesn't require understanding the data being stored. Rote learning is often employed when there is an inadequate level of foundational knowledge. This approach to learning is highly effective when dealing with information of temporary urgency, such as memorizing phone numbers or addresses. However, relying solely on rote learning is highly ineffective in mathematics. Mathematics involves abstract concepts, necessitating a profound understanding rather than mere memorization. Mastery of mathematical concepts is crucial, with a focus on comprehension rather than memorization. The result of applying the rote learning method is that students formally know a concept, but because they do not understand it, they will be unable to use it to solve their

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daily problems. Additionally, the rote learning method does not involve any cognitive activities at all.

I would learn and teach using the traditional decontextualized method as a learner and teacher (see Dhungana et al., 2024; Dhungana et al., 2025). I added that the mnemonic technique stifles creativity (this memory technique is primarily used when memorizing lists, names, numbers, or vocabulary in a foreign language) when we teach in a decontextualized way in the classroom. In addition, I would learn and teach by rote learning methods, such as memorizing step by step. Sometimes I would memorize the definition of a mathematical topic by learning and reciting many formulae one by one. In my view, rote learning kills creativity in mathematics content. From what I have observed, I believe that decontextualized learning helps students memorize mathematical formulae for tests; however, they will likely forget formulae memorized from lists rapidly. These ideas seem to be accepted by rote memorization.

Although Shrestha (2019) concludes that rote memorization and practice methods have created students as mechanistic learners disadvantaged of conceptual and meaningful learning due in no small way to culturally decontextualized mathematics syllabuses, from my teaching-learning experiences, now I believe that the rote memorization method did not develop creativity; it develops some skills to memorize formulae, definitions, or some steps of theorems. Therefore, I conclude that rote memorization produces students who are mechanistic learners, much like parrots. The first session is successful. 15-minute break!

Background music off, curtain comes down.

Act II: Mathematics in My Religious/Cultural Texts

The director addresses the audience, informing them that Act II will commence shortly and providing a brief explanation of the upcoming drama.

Whistle sounds, background music, and stage lights on.

Storyteller researcher: I remember my childhood when I belonged to a Brahmin family. My father had a daily routine of prayer. He recited the Mantras of Hindu epics, such as the Mahabharata, Ramayana, and Gita. These Ślōka and Mantras are still striking on my mind. I still remember some of the Mantras. One of the Mantras that I remember is from the Ramayana. In the Sundarakanda of the Ramayana, in Sarga 35, Verses 17-21, Hanuman comprehensively describes various aspects of Shri Rama's physique and character, as shown in Figure 1. In addition, Hanuman elaborates on Rama's eyes, his qualities, intellect, reputation, adherence to social norms based on the four castes (Brahmana, Kshatriya, Vaishya, and Shudra), and his dedication to upholding righteousness. He recounts Rama's assistance to ascetics, his promotion of righteous actions, and his exceptional prowess in archery, among other attributes. Hanuman, the expert scholar, as he was knowledgeable in the Samudrikalakshanas or characteristics and traits of great men, briefly described the bodily features by merely indicating them using numbers such as 4, 3, 14, 5, 10, 2, 6, and 9 (M.M., 2020). Similarly, that day I re-

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alized that we do not have a television in our home. During my childhood, the television serials Ramayan and Mahabharat were broadcast on the Indian television channel Doordarshan. My parents watched these shows every evening, and I often joined them. Together we watched almost all the episodes, and these stories gradually became part of my early cultural and religious understanding. These early experiences shaped my curiosity about the philosophical and moral lessons embedded in these epics. Years later, as a researcher, I found myself reflecting on these childhood memories during a visit to Padam Prakaseshower Mahadev Temple, where I met a pandit. Our conversation naturally turned to the very epics I had watched as a child. The pandit shared many stories from the Ramayana, Mahabharat, and the Bhagavad Gita, elaborating on their meanings and moral teachings. He also gave me a small book containing narratives and summaries of these texts, including a summary of the Ramayana that I mentioned earlier. Through this interaction, I revisited the stories that had once been part of my childhood television experience. Our discussion deepened my understanding of these epics, and I realized that I was already familiar with several ślokas and bhajans related to the Ramayan, Mahabharat, and the Bhagavad Gita, which had become part of my cultural memory over time.

This *Sloka* is a better example of a number system in the Ramayana. Hanuman cautiously describes the abstract divine form of Rama using a number system. According to the general understanding of devotees, it is impossible to describe the manifestation of God in its divine form. But in reality, Human, a major character in Ramayana, describes Rama's characteristics in its divine form.



Figure 1: Hanuman

As a researcher, I believe that religious scripture is not different from the mathematical rules. These can be better articulated through mathematical concepts. I still remember one of the scriptures that the father frequently recited during prayer. The Svetaštatara Upanisad (5.9):

> *"balägra-sata-bhagasya śatadhā kalpitasya ca bhago fiuah sampeyah sacdnannya kalpate"*

This scripture explains the nature of the soul, which is eternal and never dies. Translated it says, "When the upper point of a hair is divided into one hundred parts and again each of such parts is further divided into one hundred parts. Could we do it? No. That is not possible. But each such part measures the aspect of the spirit soul. There are uncountable particles of spiritual atoms, which are measured as one ten-thousandth of the upper portion of the hair."

Hence, the spiritual essence within each living being exists as a minute spiritual particle, smaller in size than a material atom, and these spiritual particles are countless. This tiny spiritual spark serves as the fundamental element of the physical body, with its influence permeating throughout the body, much like the effects of a medicinal active principle spread throughout the body. As a researcher, I believe that the rules of mathematics are the ultimate way to understand every mystery in human life. As a novice educator, I question whether school mathematics can be easy for students according to the above scripture. During those days, I was unaware of the meaning of this verse. While I began searching for mathematics in

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religion, I found this Ślōkā to be very amazing in describing the measurement of the soul. I am so surprised and wonder how the religious scripture scaled the soul in its tiny form? It prevents a person from understanding religious arguments in a superstitious way. One can understand religion logically by applying mathematics. The divisions, multiplication, and subtraction are the general mathematical rules used to understand the spiritual affairs of human life.

I realized that religious content should be included in the school curriculum to broaden the concept of mathematics and explore the intersection of science, religion, and culture. My daughter often watches the Mahabharata serial on television. She oversees it and shares it with her family. I realized that if the level curriculum incorporated the story of this Hindu epic, illustrating the mathematical concept, it would facilitate students' learning of mathematics. Therefore, religious scripture should be connected to mathematics teaching. Furthermore, expanding the scope of mathematics would help distance people from superstitious beliefs in religion.



Figure 2: Chakravyuh

While studying in class 10, my father told me about the '*Chakravyuh'* in the *Mahabharat*. My father and mother would discuss it interestingly, but I couldn't understand the concept. I knew that the '*Chakravyuh'* was created by a great teacher of the Mahabharata era, Dronacharya, to imprison the Samrāṭa *Yudhishthira* on the battlefield, distracting Arjun from the battlefield, as shown alongside in Figure 2. Arjun was the only one who knew all about *Chakravyuh*. He was skilled enough to enter Chakravyuh and exit it successfully. On the day of the creation of *Chakravyuh,* Samrāṭa Yudhishthir was in tension. Abhimanyu sought permission to enter the battlefield, revealing that he had learned half the strategy of penetrating the Chakravyuh while still in his mother's womb. Although he knew how to join the Chakravyuh, he was unaware of the exit strategy.

Knowing this, Bhīma and the other Pāṇḍava prepared to support him, and he penetrated the Chakravyuh on the battlefield and entered it. During the war, Abhimanyu showed great valor and defeated many soldiers of the Kauravas' army. He defeated mighty warriors like Karna, Duryodhan, Kripacharya, and Ashwathama. Here, Bhīma and the other Pāṇḍavas attempted to enter the Chakravyūha, but Jayadratha did not let them in. Unfortunately, he was killed so mercilessly by great warriors, violating the rules of war. Moreover, I knew the story of *Chakravyuh*, but I could not have realized its mathematical significance in my life.

However, one day while researching my MPhil program, I met my friend and enquired about the mathematical concepts in *Mahabharata*. In response to my question, he suddenly suggested that I explore Chakravyuh in the Mahabharata epics. Upon receiving that suggestion, I began to research it, and I found it too interesting and inspiring to study.

Chakra means "spinning wheel," and "vyuan" means formation. Hence, chakravyuh refers to the intricate plan of soldiers that involves moving in a circular formation, similar to a spinning wheel, as illustrated in Figure 3. In mathematics, this concept is represented by concen-

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tric circles. Additionally, the rotation of soldiers is similar to the helix of a screw, commonly seen in watches. The Chakravyuha is a multi-tiered protective plan that resembles a disc (chakra) (TMR, 2020).

The mathematical concept of the *Chakravyuha* is a circle of proportion or concentric ripple circles. A circle of proportion is a slide rule that helps calculate additions, subtractions, divisions, multiplication, exponential, logarithmic, and, to some extent, even trigonometry. It is composed of concentric circles, which are divided proportionally using arcs and tangents, and the pieces rotate in unison to facilitate calculation.

As a researcher, I realized that Chakravyuh is one of the most effective mathematical concepts applied in ancient wartime periods, particularly in the Mahabharata. Mathematics helps solve problems; it has also been an inherent part of a decisive, war-prone world, helping to establish truth in a devil-ridden world. The Mahabharata war was fought for truth and justice, and mathematics was crucial in developing strategies to defeat the enemy.

In addition, mathematics is universal. Be it business, architecture, design, engineering, nature, or even our religious texts. The Vedas, Upanishads, and Puranas provide evidence for the use of large numbers, fractions, infinity, and zero. Advanced mathematical concepts are discussed in the two great Hindu epics, the Ramayana and the Mahabharata. Besides, many instances in the Bible, the Quran, and other scriptures speak volumes about their richness (Thakur, 2016). In Jaina mathematics, many topics on mathematics are discussed in the Sthananga Sutra, such as arithmetic operations like multiplication, division, subtraction, addition; Number Theory; Geometry; Mensuration of solid objects like spheres, cylinders, etc.; fractions; Solving linear, quadratic, cubic, biquadratic equations, laws of indices, and Permutation & Combinations. Like people from the Vedic period, Jaina mathematicians were also involved in cosmology in large numbers (Bhangale, 2013).

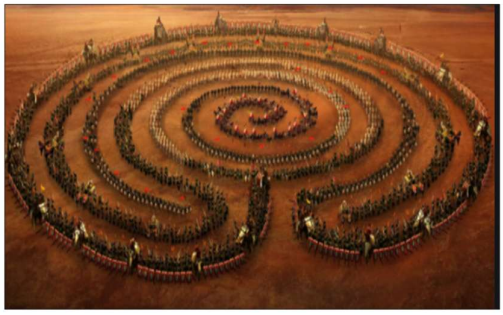


Figure 3: Chakravyuh

I agree that religion is not separate from rigorous mathematical concepts. It has carefully applied these concepts to its authenticity and legitimacy. The religious texts convince the followers that they hardly think critically about it. It could not have been possible in the absence of mathematics. Therefore, it would be beneficial if both school and university curricula incorporate mathematics in relation to religion and cultures to culturally contextualize academic mathematics, which may help make mathematics learning more authentic, empowering, inclusive, and justifiable. I believe that including such contextual mathematics also helps students feel a sense of ownership over their learning. As a novice mathematics teacher, I question myself: Is it contextualized mathematics?

The second session is successful. 15-minute break!

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Background music off, curtain comes down.

Act III: Mathematics in Religious Places

The director addresses the audience, informing them that Act III will commence shortly and providing a brief explanation of the upcoming drama.

Whistle sounds, background music, and stage lights on

Storyteller researcher: As a member of a Hindu family, I am well-versed in the practices of Hindu culture. Going back to my childhood, I would accompany my father to the temple and other Hindu religious sites. I recall that I would observe the religious monuments with great curiosity. At the same time, several questions would arise in my mind regarding the objects surrounding these religious sites. I would question the nature, characteristics, and meanings of these objects.

It could be any day of October 2019. I was a mathematics teacher at a secondary-level school in Surkhet. I recall when I was teaching the curve sketching chapter, one of my students expressed confusion about symmetry. Her main problem was the conceptual understanding of symmetry for curve sketching. I present the brief dialogue below:

> Student: *Sir! Your teaching is nice to me, but I am still getting confused about the symmetry of curve sketching.*
>
> Teacher: *It's ok, Prakriti! Let me make it clear to you.*
>
> Student: *Thank you, sir!*
>
> Teacher: *Ok! Listen here…..* (I described the definition of symmetry with an appropriate example.)
>
> Teacher: *Symmetry is a characteristic of curve sketching. A curve represented by the function is symmetric about the y-axis if no change occurs in f(x) when x is replaced by –x, if f (x) is an even function. In this case, the parts of the curve lying on either side of the y-axis are the same.*

Similarly, a curve represented by the equation $f(x, y)=0$ is symmetric about the x-axis if no change occurs in the equation when y is replaced by –y. At last, a curve represented by the equation $y=f(x)$ is symmetric about the origin if $f(-x)=-f(x)$ that is an odd function.

For example: Test the symmetry of the function $f(x)=x^2$.

For the solution, I used the definition of symmetry about the x-axis, symmetry about the y-axis, and symmetry about the origin as given below:

Given function is $f(x)=x^2$

Here, $f(-x)=(-x)^2=x^2=f(x) \Rightarrow f(-x)=f(x)$

So that is symmetric about the y-axis.

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After that, I finished my class. When I return home in the evening, I ask myself: How can I teach those students who do not understand easily? At midnight that day, I found an idea of how to teach about the symmetry of a function as a leaf of the *Pipal*. The next day was Monday. My wife fasted on that day. I was going to the temple with my wife in the morning. After that, my wife starts to pray to Lord Mahadeva. At that time, my eyes found a picture of God Mahadeva and Parbati, two in one. And I uttered myself: *Wow… this may be the best example of symmetry*, shown in Figure 4.

I think ……. After that…….

Me: *Good morning, all of you.* (I take a leaf of Pīpala and a picture of Lord Mahadeva and Parbati)

Students: *Good morning, Sir!*

Dinesh: *What is in your hand, sir?*

Me: *Yes! I have two things to share with you. Please look at me attentively.*

Dinesh: *Wow! What a nice sir!*

Me: *Yes!*

Me: *Symmetry is the balanced distribution of duplicate shapes. The word 'symmetry' means dividing an object into two equal parts so that one part coincides. When examining plants and animals, we observe that they possess symmetrical body parts or shapes.*

Symmetry is a significant geometrical concept often observed in nature and applied *in every field of our daily life. Artists, manufacturers, designers, architects, and others use the idea of symmetry. Leaves, flowers, clothing, mats, handkerchiefs, and utensils often feature symmetrical designs.* (After showing a leaf and a picture of Lord Mahadeva and Parbati….)

Me: *Look at this leaf, if we divide the leaf in half, we often find that one half has the same shape as the other. Symmetry exists all around us, and many people find it beautiful.*

Me: *Likewise, look at the picture of Lord Mahadeva and Parbati as one in the same. From it we conclude that there is symmetry about the vertical line, and the line divides the picture into two parts.*



Figure 4: Bisymmetry in Pipala Leaf

Mahesh: *Woo… its really interesting sir!*

Prakriti: *Sir! Now, I understand.*

Me: *Yes! Prakriti.*: *Tell me, what is symmetry about the x-axis, symmetry about the y-axis, and symmetry about the origin*

After that, she said all the definitions of

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symmetry.

(after ……..her answer about symmetry)

Me: .....*Good! Prakriti.*

After that I take an example which is given below:

Example: Test for symmetry of the function $f(x)=x^2$.

For solution, at first I start for testing as, $f(x)=x^2$

Here, it is clear that f(x) is symmetric about the y-axis, shown below in Figure 5.

$$f(-x)=(-x)^2=x^2=f(x)$$

$$\Longrightarrow\Longrightarrow f(-x)=f(x)$$

Now, look at the following table how f(x) seems symmetric about the y-axis.

X	1	-1	2	-2	3	-3
Y	1	1	4	4	9	9

We have to plot six points: (1, 1), (-1, 1), (2, 4), (-2, 4), (-2, 4), (3, 9), and (-3, 9).

If X = 0, then y = 0, so f(x) goes through the origin. We need to plot the point (0, 0).

Now, I have realized that learners find it easy if we teach with the help of locally available materials. I think the learner is well-versed in local materials and hence feels it is easy to learn mathematics meaningfully. Therefore, as a researcher, I argue that students need to be familiar with the mathematical concepts found in various objects and materials available in their surroundings. As I discussed above, certain objects and materials that facilitate grasping the concept of mathematics, such as geometry and algebra, are crucial for conceptually learning mathematics. Contextualization refers to the teaching of mathematics problems that highlight real-life situations. For example, a teacher could develop a contextualized task on exponential functions and equations by having students work on an Internet-based task to determine the monthly payment for a new car at a given price and interest rate for a specific loan duration. In this example, teachers would teach students how to compute with exponential formulas to calculate the monthly amount. Additionally, contextualization is defined as the development of mathematics problems that are authentic and related to real-world applications that are connected to students' future careers (Bottge & Cho, 2013). It is true that for the sustainability of learning, it needs to be linked with daily life experiences. That is why the contextualized

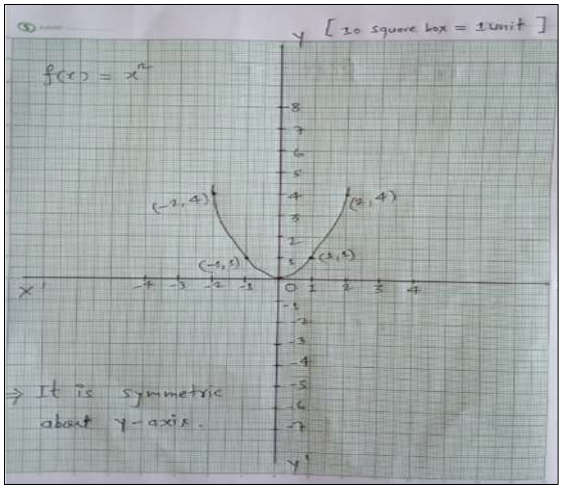


Figure 5: Curve with Symmetry about y-axis

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mathematics will be decisive for permanent learning. Ok, this is enough today! The workshop concludes.

Background music off, curtain comes down.

REFLECTIONS AND INSIGHTS

This study investigates the limitations of culturally decontextualized mathematics education in Nepali schools and advocates for a transformative, ethnomathematical approach. Traditional mathematics instruction in Nepal often focuses on rote memorization and algorithmic problem-solving, failing to engage students meaningfully or reflect their lived experiences. As classrooms grow increasingly diverse, many students arrive unprepared to learn due to socio-emotional and economic challenges (Barbara, 2000). From a Vygotskian perspective, this decontextualized approach ignores the fundamental role of culture and social interaction in learning, failing to build upon students' unique "funds of knowledge."

So, as defined by Paris (2011), culturally contextualized pedagogy leverages students' existing knowledge and community experiences to make learning more relevant. Ethnomathematics offers a framework for integrating cultural practices into mathematical instruction, fostering understanding and critical awareness (Rosa & Orey, 2007). As Stigler and Baraneb (2014) note, mathematics is a body of knowledge shaped by diverse cultural contributions over time. Research and classroom experiences reveal that contextualized instruction enhances student engagement and comprehension. Tomlinson et al. (2003) emphasize the importance of aligning instruction with students' readiness, interests, and learning styles. However, in Nepal, mathematics is still taught as a rigid, abstract discipline, disconnected from real-life applications. Nilsson and Ryve (2010) found that teachers often interpret contextualization narrowly, focusing only on using local materials rather than integrating broader cultural narratives.

This study emphasized the need to reform the mathematics curriculum in Nepal to incorporate ethnomathematical principles. Such reform would improve conceptual understanding and challenge the dominance of reductionist pedagogies that prioritize exam performance over meaningful learning. The prevailing belief among parents and schools—that mathematics is solely about calculation and exam success—further entrenches this issue. This research, grounded in my Living Theory methodology, illustrates through reflective practice and ethnodrama how culturally relevant pedagogy can transform mathematics education (Olawale, 2025). By asking, "How can I improve my practice?" I have documented a journey of critical self-reflection that aligns with Mezirow's Transformative Learning Theory. This process involved a fundamental shift in my ontological and axiological stance—from viewing mathematics as a static, instrumental body of knowledge to understanding it as a communicative, culturally-grounded practice. This personal transformation is a necessary precursor to broader change.

The study, therefore, calls for gradual curricular reform and teacher preparation programs that support inclusive, empowering, and authentic learning experiences. Such programs must be designed with Vygotsky's ZPD in mind, scaffolding teachers to navigate the challenging tran-

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sition from traditional to transformative pedagogy. As Billett (2001) describes, transformative education involves connecting new knowledge to prior understanding, enabling holistic professional growth for educators—a process that resonates deeply with the transformative learning journey this study embodies. Thus, this study is a personal and professional inquiry that contributes to a "living theory" of culturally contextualized mathematics education in Nepal. As teachers, educators, and researchers, we aim to inspire others to adopt approaches that make mathematics more accessible, meaningful, and enduring for all learners by explicitly connecting pedagogy to the socio-cultural realities of the classroom and embracing the transformative potential of critical reflection.

LIMITATIONS AND FUTURE RESEARCH

This inquiry offers valuable insights into culturally contextualized mathematics education. Primarily, its reliance on auto-ethnography and first-person narratives centres the subjective experiences of a single individual (Hem), which may constrain the broader applicability of its insights. As this method allows for introspection, it also introduces the risk of personal bias overshadowing empirical rigor. The use of arts-based ethnodrama is innovative but raises concerns about its replicability and methodological robustness. The study's focus on Nepali cultural and religious elements, such as Hindu epics and temple symbolism, further narrows its relevance to more secular or culturally diverse educational environments. Moreover, the critique of rote learning and the call for contextualized pedagogy are compelling.

Future research should pursue several key directions to build on these findings or insights and address the outlined limitations. Comparative studies across varied cultural, linguistic, and socio-economic contexts could help assess the broader applicability of ethno-mathematics principles. Empirical investigations into the use of arts-based methods, such as ethnodrama, in mathematics classrooms are also needed to evaluate their impact on student engagement, understanding, and critical thinking. Developing teacher training programs that support the integration of cultural and community knowledge into math instruction, while navigating curricular constraints, could further enhance pedagogical effectiveness. Aligning contextualized teaching approaches with standardized assessment systems would help ensure students are not disadvantaged in exam-driven environments. These efforts can strengthen the global theoretical and practical foundations of culturally responsive mathematics education.

Data Availability Statement

The raw data supporting the conclusion of this article will be made available by the authors without undue reservation.

Ethics Statement

The research committee at Nepal Open University, Lalitpur, Nepal, granted ethical approval for this research. The study was conducted in accordance with the local legislation and institutional requirements.

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Author Contributions

HLD: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Resources, Validation, Visualization, Writing – original draft, Writing – review & editing. **ND:** Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Resources, Writing – original draft, Writing – review & editing. **IMS:** Conceptualization, Data curation, Methodology, Resources, Visualization, Writing – original draft, Writing – review & editing. **RM:** Conceptualization, Data curation, Formal Analysis, Writing – original draft, Writing – review & editing.

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Conflict of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

REFERENCES

[1] Adams, T. E., & Holman Jones, S. (2008). Autoethnography is queer. In N. K. Denzin, Y. S. Lincoln, & L. T. Smith (Eds.), *Handbook of critical and indigenous methodologies* (pp. 373–390). Sage Publications.

[2] Barbara, L. (2000). Transforming Teaching Practice: Becoming the critically reflective teacher. *International and Multidisciplinary Perspectives, 1*(3), 293–307. https://doi.org/10.1080/713693162

[3] Bhangale, R. (2013). *Post-Vedic period*. http://mathlearners.com/history-of-mathematics/1500-bc-400-ad/post-vedic-period/

[4] Billett, S. (2001). *Learning in the workplace: Strategies for effective practice*: Eric.

[5] Bottge, B. A., & Cho, S.-J. (2013). Effects of enhanced anchored instruction on skills aligned to common core math standards. *A Multidisciplinary Journal, 19*(2), 73–83.

[6] Brookfield, S. (2000). The concept of critical reflective practice. In A. L. Wilson & E. R. Hayes (Eds.), *Handbook of adult and continuing education* (pp. 33–49). Jossey-Bass.

[7] Dahal, N. (2023). Ensuring quality in qualitative research: A researcher's reflections. *The Qualitative Report, 28*(8), 2298-2317. https://doi.org/10.46743/2160-3715/2023.6097

[8] Dahal, N. (2024). Transformative visions of qualitative inquiry: Performative, philosophical, and artistic transformations. *The Qualitative Report, 29*(1), 312-318. https://doi.org/10.46743/2160-3715/2024.6982

[9] Dahal, N., & Luitel, B. C. (2022). Understanding and encountering the ethics of self and others in autoethnography: Challenging the extant and exploring possibilities. *The Qualitative Report, 27*(12), 2671-2685. https://doi.org/10.46743/2160-3715/2022.5572

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[10] Denzin, N. K., & Lincoln, Y. S. (2005). *SAGE handbook of qualitative research* (3rd ed.). Sage.

[11] Dhungana, H. L. (2023). Journeying towards a transformative mathematics teacher through critical self-reflection. *Academic Journal of Mathematics Education*, *6*(1), 71–80.

[12] Dhungana, H. L., Dhakal, G. R., & Dahal, N. (2025). Evolving myself as a De/contextualized Learner and Teacher of Mathematics. *Siddhajyoti Interdisciplinary Journal*, *6*(1), 37–52. https://doi.org/10.3126/sij.v6i1.78007

[13] Dhungana, H. L., Manandhar, R., Dahal, N., & Shrestha, I. M. (2024). *Developing Culturally Contextualized Mathematics: The Journey to Becoming a Transformative Teacher*. *International Journal of Research and Innovation in Social Science, 8*(8), 1554–1566. https://doi.org/10.47772/IJRISS.2024.8080113

[14] Dhungana, S., Pant, B. P., & Dahal, N. (2023). Students' experience in learning trigonometry in high school mathematics: A phenomenological study. *Mathematics Teaching Research Journal*, *15*(4), 184–201.

[15] Ellis, C., Adams, T. E., & Bochne, A. P. (2011). *Autoethnography: An Overview*. Excellent Education.

[16] Geertz, C. (1973). Thick description: Toward an interpretive theory of culture. In *The interpretation of cultures: Selected essays* (pp. 3–30). Basic Books.

[17] Hallden, O. (1999). Conceptual change and contextualization. In W. Schnotz, S. Vosniadou, & M. Carretero (Eds.), *New perspectives on conceptual change* (pp. 53–65). Elsevier.

[18] Lee, Y. J. (2025). The effects of functional moves in teacher questioning on students' contextualization of mathematical word problem solving. *Journal of Mathematics Teacher Education*, *28*(1), 73–101.

[19] Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry* (pp. 289–331). Sage Publications.

[20] Luitel, B. C., & Taylor, P. C. (2007). The Shanai, the pseudosphere, and other imaginings: envisioning. *Cultural Study of Science Education, 2*, 621-655.

[21] M.M. (2020). Rama's bodily features in numbers. https://sites.google.com/site/mathematicsmiscellany/rama-s-bodily-features-in-numbers.

[22] Mezirow, J. (1997). Transformative learning: Theory to practice. *New directions for adult and continuing education, 1997*(74), 5-12.

[23] Nilsson, P., & Ryve, A. (2010). Focal event, contextualization, and effective communication in the classroom. Educational Studies in Mathematics, 74(3), 241–258.

[24] Olawale, B. E. (2025). Decolonising mathematics education: Towards a culturally relevant pedagogy. *Interdisciplinary Journal of Education Research*, *7*(1), a26-a26.

[25] Pant, B. P. (2019). An integral perspective on research: Methodological and theoretical journey of a teacher educator. In P. C. Taylor & B. C. Luitel (Eds.), *Research as transformative learning for sustainable futures: Global voices and visions* (pp.75–87). Brill Sense.

[26] Paris, D. (2011). Culturally sustaining pedagogy: A needed change in stance, terminology, and practice. *Educational Researcher, 41*(3), 93–97.

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[27] Paudel, T., Luitel, B. C., & Dahal, N. (2023). Journey of realization and adaptation through auto-ethnography: A shift to transformative educational research. *The Qualitative Report, 28*(4), 1017-1037. https://doi.org/10.

[28] Rosa, M., & Orey, D. C. (2006). Abordagens atuais do programa etnomatemática: delineando-se um caminho para a ação pedagógica [Current approaches in the ethnomathematics as a program: Delineating a path toward pedagogical action]. BOLEMA, 19(26), 19-48.

[29] Rosa, M., & Orey, D. C. (2007). Cultural assertions and challenges towards pedagogical action of an ethnomathematics program. *For the Learning of Mathematics, 27*(1), 10-16.

[30] Saldana, J. (2011). Ethnotheatre: Research from page to page. In N. K. Denzin & Y. S. Lincoln (Eds.), *Qualitative inquiry and social justice*. Left Coast Press.

[31] Shrestha, I. M. (2018). My pedagogical sensitization towards holistic Mathematics education: A practitioner's inquiry. [Unpublished M. Phil. dissertation, Kathmandu University]. Nepal.

[32] Shrestha, I. M. (2019). Facilitating culturally de/contextualised mathematics education: An arts-based ethnodrama. In P. C. Taylor & B. C. Luitel (Eds.), *Research as transformative learning for sustainable futures: Global voices and visions* (pp. 225–238). Brill Sense.

[33] Spry, T. (2001). Performing autoethnography: An embodied methodological praxis. *Qualitative Inquiry*, 7(6), 706-732.

[34] Stigler, J., & Baraneb, R. (2014). *Culture and mathematics knowledge*. Review of research in education. https://doi.org/10.3102/0091732X015001253.

[35] Taylor, P. C., Taylor, E., Luitel, B. C. (2012). Multi-paradigmatic transformative research as/for teacher education: An integral perspective. In K. Tobin, B. Fraser & C. McRobbie (Eds), *International handbook of science education* (pp. 373-387). Dordrecht: Springer.

[36] Thakur, R. K. (2016). *Mathematics in religion*. Rupa Publications.

[37] TMR, (2020). *The invincible chakravyuha: Advanced war techniques of Mahabharata period.* http://mahabharataresearch.com/military%20academy/the%20mysterious%20chakrayuha.html

[38] Tomlinson, C. A., Brigthon, C., Hertberg, H., Moon, T., Brimijoin, K., Conover, L., & Reynolds, T. (2003). Differentiating Instruction in Response to Students' Readiness, Interest and Learning Profile in Academically Diverse Classrooms: A Review of Literature. *Journal for the Education of Gifted*. http://journals.sagepub.com/doi/abs/10.1177/016235320302700203.

[39] Vygotsky, L. S. (1978). *Mind in social: The development of higher psychological processes.* Harvard University Press.

[40] Whitehead, J. (2008). Using a living theory methodology in improving practice and generating educational knowledge in living theories. *Educational Journal of Living Theories 1*(1), 103–126.

[41] Yeea, S. P., & Bostic, J. D. (2014). Developing a contextualization of students' mathematical problem solving. In the National Governors Association, Council of Chief State School Officers. Common core standards for mathematics. http://www.corestandards.org/the-standards/mathematics

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Enhancing Student Engagement and Their Performance in Abstract Algebra Using Context Tasks

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Abstract: One of the biggest challenges in learning Abstract Algebra lies in the low level of student engagement caused by its high abstraction and lack of real-life context. This study aims to assess the effectiveness of contextual tasks in fostering students' cognitive, affective, and behavioral engagement, and its impact on academic performance. Through a qualitative case study approach with 35 students, the data were collected through observation, reflective journals, interviews, and pre-/post-tests. Thematic analysis demonstrated a substantial improvement in students' emotive and reflective engagement, as well as a critical reflection in learning abstract algebra. Students revealed their learning improvement in task focus, engagement, and a variety of problem-solving strategies, with an average increase of 27.3%. Additionally, students contended that context-based exercises enhanced their motivation, significance, and real learning experience. The findings validated that a context-based learning strategy effectively enhanced students' engagement and achievement in learning abstract algebra.
Keywords: *student engagement; context tasks; abstract algebra learning, students' performance*

INTRODUCTION

Student engagement has become a major focus in higher education research, given its crucial role in supporting academic achievement, retention, and long-term learning success (Miller et al., 2021; Fokkens-Bruinsma et al., 2021). Sustaining student engagement becomes a significant challenge in mathematics, particularly in esoteric disciplines such as Algebra. Emotional and cognitive detachment between pupils and abstract mathematical concepts is often a result of the subject's high level of abstraction and its limited connection to real-world contexts (Jurdak, 2016; Núñez, 2020). OECD (2023) also underscores the need for making mathematics education more meaningful through the connection between mathematical knowledge and real-world situations. Similarly, Pires et al., (2022), the failure to make connections between abstract mathematical ideas and practical experiences constitutes an important predictor of disengagement and poor conceptual knowledge among students. On the other hand, traditional Abstract Algebra teaching approaches often focus exclusively on the formal development of mathematical structures without placing sufficient emphasis on a real-world connection (Stewart et al., 2019). This issue often hinders students' ability to comprehend the relevance of their learning. as a result, students often perceive

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their studies in Abstract Algebra as disconnected from professional practice and everyday social life (Kieran, 2020; Sheppard and Wieman, 2020).

The challenges in 21st-century mathematics education can be divided into two primary aspects: the necessity to develop 21st-century learning skills, such as collaboration, communication, creativity, and problem-solving, and insufficient conceptual understanding and disparities in learning achievement (Kim et al., 2019). The problems are further exacerbated by the overwhelming prevalence of traditional mathematics education methods that prioritize procedural knowledge and rote memory over the development of conceptual comprehension and critical thinking skills. The abstract nature of the material in Abstract Algebra exacerbates these issues, as it frequently lacks direct relevance to students' real-world experiences, resulting in reduced engagement and motivation to study (Barbieri and Miller-Cotto, 2021; Asada et al., 2024). Carson and Rowlands (2007) noted that the instructional failure to establish a connection between formal symbolism and conceptual meaning leads to a significant number of students solely "memorizing definitions" rather than developing a profound conceptual understanding. As a result, a new approach is necessary to establish a connection between the practical experiences of students and abstract mathematical theory. The contextual tasks approach holds significant potential as an effective instructional strategy. This approach emphasizes the execution of context-driven tasks that are significant and relevant, thereby facilitating the practical development of conceptual comprehension in students (Treagust and Duit, 2008). In this research, for Indonesian mathematics education undergraduate students, the use of a context that is culturally relevant to them, namely batik designs, provided a tangible starting point to delve into abstract notions in algebra, such as group theory and symmetry structure (Mamolo and Glynn-Adey, 2023).

Additionally, contextual task-based learning can enhance students' cognitive, affective, and behavioral engagement by emphasizing active exploration, collaborative discourse, and real-world problem-solving (Wu et al., 2015; Jurdak, 2016; Pambudi et al., 2025). This methodology is designed to meet the needs of 21st-century education, which emphasizes interactive, reflective, and sophisticated problem-solving learning experiences (Antonenko et al., 2014; Tan and Nie, 2015). While contextual task-based learning has been extensively validated in elementary and secondary mathematics education, research explicitly examining its application in Abstract Algebra at the higher education level remains limited. This gap underscores the urgent need to investigate how the contextual task approach can enhance students' multidimensional engagement and foster deeper conceptual understanding in advanced mathematics.

Referring to the gaps identified above, the following research questions are proposed:

1. How can context-based tasks enhance students' behavioral, emotional, and cognitive engagement in Abstract Algebra learning?
2. How can context-based tasks foster students' performance in Abstract Algebra learning?

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Literature Review

Challenges in Learning Abstract Algebra

Abstract Algebra learning often faces fundamental challenges stemming from its high level of abstraction and lack of connection to students' real-life contexts. Many students struggle to understand structures such as groups, rings, and fields because teaching approaches tend to be formalistic and symbolic, without the support of visual representations or contextual applications (Stewart et al., 2019). This creates a gap between students' mathematical concepts and conceptual understanding, which in turn results in low motivation and active participation (Barbieri and Miller-Cotto, 2021). The disconnect between formal symbols and intuitive meanings not only hinders deep understanding but also encourages mechanistic and rote-based learning (Carson and Rowlands, 2007). Further, conventional approaches to teaching Abstract Algebra are considered inadequate to meet the pedagogical demands of the 21st century. Furthermore, these challenges are compounded by students' lack of emotional and cognitive engagement with abstract material. Data from the (OECD, 2023) shows that more than 60% of students consider advanced mathematics irrelevant to their lives, especially in abstract areas such as Algebra. When concepts are not connected to students' social, cultural, or real-world experiences, the potential for critical and reflective engagement decreases drastically (Jurdak, 2016). This has direct implications for learning outcomes, as emotional and cognitive engagement have indicated strong predictors of conceptual understanding and academic achievement (Fredricks et al., 2019). Consequently, it is important to develop learning approaches that can bridge formal abstraction and concrete experiences for students in order to increase engagement and effectiveness in learning Abstract Algebra.

Students Engagement and Its Measurement

The concept of student engagement has evolved into one of the key constructs in modern educational research. Fredricks et al. (2004) suggest that engagement is multidimensional, encompassing behavioral, emotional, and cognitive engagement. Behavioral engagement is characterized by attentiveness, attendance, social collaboration, and active participation in academic activities (Nguyen et al., 2018; Gomes et al., 2023). Emotional engagement refers to students' affective responses to learning, encompassing feelings of interest, satisfaction, and a sense of belonging within the learning community (Luo et al., 2017). Cognitive engagement refers to the intellectual commitment, application of metacognitive methods, and perseverance of students when they encounter academic challenges (Park and Yun, 2018; Pohl, 2020).

Recent study posits that cognitive engagement is the most significant feature of engagement for the advancement of deep learning and critical thinking (Sakeef et al., 2025). A study by Fredricks et al. (2019) showed that behavioral and emotional engagement serve as important motivational bases to support meaningful cognitive engagement. In other words, students who feel emotionally connected to their learning environment and are behaviorally engaged are more likely to demonstrate persistence in analytical and reflective thinking, which ultimately drives long-term academic achievement (Tas, 2016). These three dimensions of participation are interrelated within the social

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environment of learning, as per Vygotsky and Cole's (1978) theory of social constructivism. Cognitive engagement is enhanced by collaborative dialogues, affective engagement is enhanced by nurturing emotional environments, and behavioral engagement is promoted by robust social interactions (Pineda-Báez et al., 2019). Research by Gillies (2019) also confirmed that productive dialogue in the classroom acts as a primary means of building deep cognitive engagement. Thus, in a contextual task-based learning approach, activity design must consciously integrate these three forms of engagement to create authentic and empowering learning experiences.

Student engagement is a multifaceted concept that includes behavioral, cognitive, and emotional dimensions (Fredricks et al., 2019). Engagement is not a singular theoretical construct that can be easily quantified with precision. Various methodologies, including self-reporting, observation, and task analysis, have been employed; nevertheless, each possesses inherent limitations. Surveys represent the predominant methodology (Bond et al., 2020), however their validity is frequently scrutinized (Hodgson et al., 2017). A triangulation strategy integrating quantitative and qualitative data is strongly advised to more correctly and comprehensively capture student participation (Phillips and O'Flaherty, 2019). Within the domain of context-based tasks the evaluation of participation is essential in the field of Abstract Algebra, despite the fact that it is not frequently investigated in current research. The majority of previous research has either focused on a single aspect of interaction or has examined it in a generalized manner without differentiation (Lo and Hew, 2021; Carmichael et al., 2017). This study adopts the classification of indicators from Fredricks et al., (2019) enriched by Cevikbas and Kaiser (2022), to examine in depth how context tasks affect student engagement in each of its dimensions—behavioral, cognitive, and emotional. Table 1 below presents the indicators of student engagement used in this study. These indicators are designed to comprehensively capture students' responses during the learning process involving contextual tasks and serve as the foundation for developing observation instruments and in-depth interview guides.

Engagement	Indicator
Behavioral	Persistence, focus on task, effort to learn, high grades, preparation before class, asking and answering questions, seeking support/teacher/friends, discussion and group work, avoiding disruptive behavior
Cognitive	Thinking aloud, planning learning, choosing challenging tasks, proving & disproving, independent working style, giving evaluation, solving problems differently, self-monitoring, becoming an expert, being aware of learning goals, searching for information, correcting mistakes, linking topics, remembering & applying knowledge
Emotional	Interest, comfort, happiness, enthusiasm, pleasure, satisfaction, willingness, sense of responsibility, self-confidence, motivation and concentration in learning

Table 1. Student engagement indicators

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Context Task in Mathematics Learning

The use of context tasks in mathematics learning has developed as a pedagogical strategy designed to connect abstract concepts to real-world situations that are meaningful to students. Unlike simply presenting real-world problems, context tasks involve the construction of complex meanings between the description of the situation in the task (figurative context) and the conditions of student interaction while working on the task (interactive context). Wernet (2017) emphasizes that it is important to distinguish between the context offered by the classroom task and the personal context that students construct through that interaction. The interaction between these two dimensions significantly impacts the level of student engagement in the task. As noted by Lozano (2017), context is not fixed or universal in its interpretation. That is, not all students will engage equally with the figurative context of a task. Factors such as lack of prior experience (Liang and Moore, 2021), uninteresting context, or the expectation that context is only used to "wrap" the mathematical problem (Lester Jr and Cai, 2016) can inhibit student engagement. In fact, some students may feel compelled to quickly leave the context after identifying the mathematical aspects of the task. However, it is important to recognize that low contextual engagement does not necessarily correspond directly to poor performance, just as high engagement does not automatically guarantee improved outcomes (Shimazu et al., 2015). In this regard, context tasks present a pedagogical challenge in balancing context load with students' cognitive load.

In contrast, environments that are structured in a pedagogical manner can serve as catalysts for significant learning experiences. Chevallard and Bosch (2020) proposed that the acquisition of mathematical knowledge should begin with contextual issues that students can authentically envision and experience, as a result of the Realistic Mathematics Education theory. Reinke and Casto (2022) substantiated this viewpoint by asserting that contextual assignments promote conceptual reinvention through exploration and social discourse, in addition to engaging students. Context functions as an epistemic framework that facilitates the development of mathematical meaning. The intricate abstraction of concepts such as groups, rings, and fields frequently presents obstacles to students' comprehension in advanced academic subjects like Abstract Algebra. Studies by Halldén et al., (2013) underline that the gap between formal symbolism and conceptual intuition can be overcome through the integration of context tasks. For example, the use of symmetry contexts in art, cryptography in digital security systems, and social network structures provide more accessible conceptual starting points. This is in line with the findings of Carruthers and Worthington (2005) which show that abstract understanding can be strengthened by building mental representations of symbolized concrete experiences.

Additionally, contextual activities contribute to the improvement of cognitive flexibility and adaptive reasoning, as demonstrated by Siqi-Liu and Egner (2020). Voigt (2013) argues that effective learning occurs through the negotiation, discussion, and interaction of meaning within a social environment, as a result of a social constructivist perspective. In a context-driven Abstract Algebra course, students generate meaning through collaborative discourse that connects the actual world to formal mathematical frameworks, as well as independently. Nathan and Alibali (2021) posited that the transition from tangible experience to abstract comprehension is a dialectical process that

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improves the mathematical cognition of students. The literature suggests that, despite the challenges associated with interpretative variability, cognitive load, and inconsistent engagement that contextual tasks present, this methodology has the potential to significantly improve student engagement and performance, particularly in advanced mathematical disciplines such as Abstract Algebra. As a result, the efficacy of contextual activities in higher mathematics education is contingent upon the explicit integration of context into mathematical comprehension and the pedagogical design that recognizes the diversity of students' backgrounds.

METHOD

Research Design

This study employed a qualitative exploratory case study approach, which was very suitable for exploring students' behavioral, emotional, and cognitive engagement in the context of contextual task-based Abstract Algebra learning. We employed a case study approach because it can reveal how engagement works in real life in great detail (Toogood, 2023). This perspective on engagement viewed it as a complex process influenced by social interactions, task design, and real-life learning experiences."

Participants

The participants in this study involved 35 students in the Mathematics Education study program at a state university in Indonesia. The selection of participants was carried out through a purposive sampling technique by considering several criteria, namely students who have completed elementary linear algebra courses, demonstrated readiness to participate in contextual task-based learning, and have access to basic technological devices such as smartphones or laptops to support online learning activities. The purposive sampling technique was chosen because it enabled the researchers to strategically select participants most relevant to the study's focus, who could provide rich and in-depth information (Etikan et al., 2016). Additionally, third-year students were selected because, at this stage, they had already acquired a sufficient theoretical foundation in abstract mathematics while remaining in an active phase of conceptual development, allowing for clearer observation of their responses to innovative learning interventions. Before participating in the study, all participants were provided with detailed information regarding the objectives, procedures, and their rights as subjects, and they gave their informed consent in accordance with the ethical principles governing educational research (Christenson et al., 2014).

Research Setting

This study was conducted for eight weeks at the same state university in Indonesia where the participants were enrolled. The research setting was designed to facilitate the implementation of flexible and meaningful context-based tasks. In the first two weeks, data were collected using a

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pre-test and non-intervention observations of student engagement in conventional Abstract Algebra learning. Task-based learning in context took place from weeks three to six using real-life examples such as analyzing symmetry in batik and wallpaper patterns using the concept of symmetry groups. These tasks were varied based on the degree of contextualization, namely the extent to which contextual and mathematical elements are integrated with each other (Son and Goldstone, 2009; Juniati and Budayasa, 2017). In class, students worked on group discussions and assignments that were relevant to the lesson. Outside of class, they wrote daily notes to reflect on what they had learned and worked on micro projects that were also relevant to the lesson. We obtained post-intervention data in the seventh and eighth weeks through a post-test, in-depth interviews, and an examination of reflection notes to see how students' behavioral, emotional, and cognitive engagement had changed. This environment was constructed based on the concepts of social constructivism, emphasizing the necessity of collaborative contact, contextual mediation, and critical reflection in creating conceptual knowledge (Vygotsky and Cole, 2018).

Learning Procedures

The objective of this study's learning approach was to enhance student engagement and performance in grasping abstract topics by incorporating contextual assignments directly into classroom activities in the Abstract Algebra course. The learning design was divided into two primary segments: in-class activities and post-class activities. These phases were designed to establish a reflective and contextual learning cycle through collaboration. During the in-class phase, the learning activities were centered on the investigation of context-specific issues that were formulated with varying degrees of difficulty, familiarity, and contextualization—the extent to which the mathematical model and the context were interwoven. During these sessions, students worked in small groups to complete contextual assignments and engage in discussions regarding abstract algebraic structures, such as rings, fields, and groups. For example, in one assignment, students were directed to analyze symmetry patterns in geometric art in order to identify group structures (Figure 1). In another, they evaluated relationship models in social networks to understand the concept of homomorphism. The teacher functions as a scaffolder and facilitator of conceptual discourse, encouraging open discussion, the elucidation of concepts, and the contemplation of the relationship between mathematical representation and context. Through intellectual challenge, collaboration, and the exploration of personally meaningful content, this approach enhanced students' cognitive, behavioral, and affective engagement.

The post-class phase aimed to reinforce internalization of concepts and encourage metacognitive reflection. Students were requested to create digital diaries documenting their thinking processes, problem-solving strategies, and how they could interpret and relate them to the algebraic concepts. In addition, they completed context-based mini-projects that incorporated key concepts—such as building a simple encryption model based on group structures or analyzing the geometric symmetry of physical structures—to demonstrate their applied understanding. The entire procedure was designed in line with (Vygotsky and Cole, 2018) principle of the Zone of Proximal Development (ZPD), in which tasks were structured to be slightly beyond students' actual abilities but still

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manageable through collaboration and social support. This context-based approach not only provided realistic cognitive challenges but also encouraged students to think flexibly, reflectively, and transformatively in relating abstract mathematics to real-world phenomena.

Task-1: Batik Frieze Design

Batik is one of Indonesia's cultural heritages that is highly valued by the community; besides being beautiful, batik also contains deep meaning and strong cultural identity. Many batik patterns are actually composed of mathematical transformations. One interesting type of pattern is the frieze pattern — a pattern of paths that repeats continuously in one direction.

In this assignment, you will explore local cultural motifs from your respective regions, such as the clove flower motif from the Tulungagung area, or the damar kurung motif from Gresik. Choose one cultural motif from your region and one mathematical object. Use the motif of your choice to design Batik, with a frieze pattern that contains symmetry operations such as translation, rotation, reflection, and glide reflection.

Choose one basic motif that has the potential for symmetry. Then, arrange the motif in the form of a repeating frieze pattern, ensuring that all patterns meet the properties of a group: closed, associative, identity, and each element has an inverse. Use the following application http://math.hws.edu/eck/js/symmetry/frieze.html, to help your design.

Figure 1. Context tasks- Batik Frieze Design

Instruments

This study used four different types of data gathering tools that were meant to work together to get a whole picture of how engaged students were in contextual task-based Abstract Algebra learning. First, the observation sheets were prepared based on the engagement behavior, emotion, and cognition markers as outlined in Fredricks et al., (2019), which were used to conduct non-participatory classroom observations. These observations looked at how students interacted with each other, how they participated in conversations, how they expressed their feelings, and the critical thinking skills they used while learning. Second, student diaries were collected to look at how students thought about their learning experiences, how their motivation changed, and how they saw the importance of contextual activities. This method is based on diary studies, which are good at showing how students feel and think on a daily basis (Zimmerman and Schunk, 2011). Third, a specific group of 12 students were chosen to take part in semi-structured interviews. The goal of these interviews was to have a better understanding of how students feel about their participation, working together, and how adding context might help them understand abstract ideas. The interview protocol was developed based on the principles of exploring affective and cognitive experiences in higher education (Anderson et al., 2022). Fourth, the post-test was in the form of a written test to measure changes in student performance. Each instrument was designed to meet the principles of validity and reliability, with content validity testing by three mathematics education experts and instrument trials prior to the implementation of the main study. With a combination of data from observations, written reflections, in-depth interviews, and post-tests, this study attempted to build strong source triangulation to increase credibility and confirm the findings (Cresswell, 2013).

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Student Interviews

We did semi-structured interviews with stimulated recollection just after the students finished a contextual activity that used modulo arithmetic to check the legitimacy of products and analyze the symmetry of traditional batik patterns in groups. Students were asked to watch videos of the task being done along with written scripts and then redraw relevant diagrams or patterns, like verification codes or batik symmetry shapes, to show that they understood the problem and how to use the algebraic ideas that were used. The interviewer could see and hear how students' comprehension changed by using video as a stimulus. For example, they could see how students connected the congruence class structure to the digital coding system or how they recognized symmetry transformations as parts of the group structure. This process showed how well students could build meaningful conceptual understandings and actively engage with the figurative context of the task. It also tested how well the contextual approach worked to improve their engagement and performance in learning Abstract Algebra.

Data Analysis Techniques

The systematic discovery of meaning patterns within qualitative data was facilitated by the thematic analysis method established by (Braun and Clarke, 2025), which was employed in the data analysis of this study. The analytical process consisted of six stages: (1) Familiarity with the data is achieved through the iterative examination of interview transcripts, observational notes, and student reflections. (2) Preliminary codes are formulated from indicators of behavioral, emotional, and cognitive engagement. (3) Initial themes are identified from the clusters of codes. (4) Themes are evaluated to confirm internal consistency and differentiation. (5) Themes are delineated and designated to convey essential meanings. (6) An interpretive report is compiled that associates themes with the theoretical framework of student engagement. Two independent analysts participated in the data coding procedure to improve the credibility and reliability of the analysis, and Cohen's Kappa coefficient was employed to evaluate intercoder reliability. McHugh, (2012) interprets a Kappa score of 0.91 as indicating a very high degree of agreement. Additionally, source triangulation techniques (interview, diary, observation, and conceptual testing) were implemented to verify the consistency of the results across a variety of data types. Birt et al., (2016) further validated the analysis results by conducting member verification, which involved a review of the interpretation of the findings by a large number of participants to verify the veracity of the portrayal of their experiences. The exhaustive study was conducted using NVivo to systematically manage extensive data, thereby improving the efficiency of monitoring codes, themes, and interrelations among data.

RESULTS

This study aims to explore how a contextual task-based learning approach affects students' behavioral, emotional, and cognitive engagement in Abstract Algebra learning. Data were collected from four sources: classroom engagement observations, student reflection diaries, semi-structured interviews, and pre-test and post-test results of conceptual understanding. The triangulation process between data sources produced a holistic picture of changes in student engagement patterns during

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the implementation of context-based tasks. In general, there was a consistent increase in all three dimensions of engagement, with variations in patterns across individuals and groups.

Enhancing Student Engagement in Abstract Algebra through Context-Based Tasks

Behavioral Engagement

The implementation of context-based learning significantly increased students' behavioral engagement in the Abstract Algebra learning process. Observation data, daily reflections, and in-depth interviews indicated that students were not only physically present, but also showed increased active participation, responsibility, and initiative in collaborative learning. At the beginning of the intervention, only 43% of students were actively involved in group discussions. However, within two weeks of the integration of contextual tasks, participation increased drastically to 82%. This involvement included the initiative to ask questions, answer questions, provide feedback, complete tasks collaboratively, and show enthusiasm in contributing ideas. One real example occurred in the "Symmetry in Traditional Batik" task session, where previously passive students began to take strategic roles in group discussions.

"S1-asked critical questions to his peers, encouraging discussion about how the symmetry pattern in batik can be modeled as a mathematical group" (Observation, Week 4).

This phenomenon marked a shift in attitude from passive presence to meaningful active involvement. Not only did participation increase, but the commitment to attendance also showed a positive trend—from 74% full attendance before the intervention to 91% during the contextual task period.

Student reflections revealed a transformation in attitudes toward attendance and learning responsibility. One student wrote,

"Usually I just come to fill in the attendance. But when the assignment about batik symmetry was given, I really wanted to come and discuss, because I felt my ideas were also important" (S2, Diary, Week 5).

In-depth interviews reinforced these findings; students felt that the context-based assignments gave them a "real reason" to engage. Most stated that this approach made learning more relevant, challenging, and encouraged equal interaction within the group. Teachers also noted that students were more proactive in asking for clarification and expressing ideas, and avoided unproductive behaviors that were previously common in traditional classes.

Behavioral Engagement Indicator	Increase		Decrease		Do not change	
	n	%	n	%	n	%
Persistence	26	74%	1	3%	8	23%
Focus on the task	27	77%	2	6%	6	17%
Learning effort	26	74%	1	3%	8	23%
High scores (indication of group assignment results)	24	69%	2	6%	9	26%
Preparation before class	25	71%	1	3%	9	26%

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Ask and answer	26	74%	3	9%	6	17%
Asking for support from teachers or friends	23	66%	3	9%	9	26%
Discussion and group work	27	77%	1	3%	7	20%
Avoiding disruptive behavior	25	71%	2	6%	8	23%

Table 2. Changes in Student Behavioral Engagement Through Context Tasks Intervention

Emotional Engagement

The results showed that context-based learning generally increased students' emotional engagement in Abstract Algebra lessons. Of the 35 participants, most students reported increased enthusiasm (n=30), enjoyment while learning (n=24), concentration in learning (n=23), and sense of responsibility for learning (n=23). This was supported by active interaction during problem solving, the use of contextual media, and group work that allowed students to learn more flexibly and meaningfully. In addition, students stated that they felt more comfortable (n=24), more confident (n=20), and more competent (n=20) in understanding abstract mathematical concepts. Activities such as real-world context exploration, group discussions, and task reflections helped them feel valued and cared for individually. As expressed by one student, "I used to be shy to ask questions, but now I feel more comfortable answering even if the answer is wrong." However, a small number of students (around 9–12%) found it difficult to adjust at first, mainly because they were used to passive lecture methods and were not ready to take full responsibility for their learning.

Emotional Engagement Indicator	Increase		Decrease		Do not change	
	n	%	n	%	n	%
Enthusiasm	30	86%	1	3%	4	11%
The joy of learning	24	69%	2	6%	9	26%
Concentration on studying	23	66%	3	9%	9	26%
Sense of responsibility for learning	23	66%	1	3%	11	31%
Happiness	22	63%	3	9%	10	29%
The spirit of learning mathematics	22	63%	3	9%	10	29%
Motivation to learn	22	63%	3	9%	12	34%
Convenience when learning mathematics	24	69%	5	14%	8	23%
Satisfaction with mathematics learning	21	60%	1	3%	13	37%
Self-confidence in mathematics	20	57%	3	9%	12	34%
Self-confidence in mathematics	20	57%	5	14%	10	29%
Interest in learning mathematics	19	54%	3	9%	13	37%

Table 3. Changes in Students' Emotional Engagement through Context Tasks Intervention

S1 – From class observation notes and student journals: "I feel more interested when learning algebra with assignments that are related to real situations. When the teacher gives questions related to transportation systems or social patterns, I understand and enjoy the learning process better. I no longer just memorize formulas, but feel like I am solving real problems. That makes me more enthusiastic about participating in discussions."

S2 – During individual interviews: "I used to be rarely active in class, afraid of making mistakes and being embarrassed. But when we worked in groups and were asked to create solutions with

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friends, I became more confident. Even though my answers were sometimes wrong, the teacher still appreciated them and my friends also helped me improve them. It felt comfortable learning like that, I was no longer afraid."

S3 – In group discussions: "I used to get bored when learning algebra because there were many difficult terms. But now it's different. When the assignments are related to real life, such as questions about asset division or encryption systems, I actually become curious. I even started to like learning mathematics, even though I used to avoid it."

S4 – From a female student interview: "In a regular class, I just listen to the teacher talk and take notes. Now, I have to be active, sometimes presenting in front of my friends. At first it was stressful, but after a while I started to like it because I could express my opinion and it turns out I can. This makes me more confident."

S5 – From a student journal: "When we were asked to make a small project from a math problem we found at the market, I felt like I was doing real research. It was fun and made me feel important. I usually don't like math, but now I look forward to algebra class."

Cognitive Engagement

Students' cognitive engagement indicated a very significant increase during the implementation of context-based Abstract Algebra learning. Data from daily reflections, classroom observations, and interviews revealed that most students actively invested their intellectual efforts to understand, evaluate, and relate abstract concepts to contextual representations. Students not only memorized definitions and repeated procedural algorithms but also began to develop higher-order thinking skills, including constructing proofs, modeling, and logically mapping relationships between concept. As many as 83% of students reported an increase in their ability to solve problems with various approaches. One student noted,

"I can solve one problem in several ways, because I learned to see the problem from different sides" (S1, reflective journal).

Prior to the intervention, their learning approach was predominantly mechanistic, characterized by memorizing axioms and algebraic properties without fostering deep understanding. However, after several weeks of engagement in contextual tasks, a marked cognitive transformation occurred. Students began to ask critical questions such as:

"If the symmetry operation in this batik satisfies the associative and identity properties, do all batik patterns necessarily form a group?" (Group Discussion, Week 5).

Another student also stated, *"I realize this concept is not just a theory. I can relate it to art and culture" (S2, group interview)*, indicating that they began to actively transfer abstract concepts into the local cultural context. This process was reinforced through mathematical visualization activities, such as the frieze symmetry design based on the clove motif. Students stated that they integrated rotations and reflections to ensure that the patterns satisfied the group axioms. In a reflective note, one student wrote,

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"I arranged the rotation and reflection sequences as part of the group, and tried to ensure that the entire batik pattern still fulfilled the associative properties and had elements of visual identity" (S5, diary).

This narrative showed that students not only understood the concept formally, but also presented it creatively and contextually.

One form of implementation of cognitive involvement could be viewed from the results of students' work in designing frieze batik patterns based on symmetry groups. In this assignment, students were asked to develop visual designs by combining local cultural elements and mathematical transformation notations. One student produced a design that contained a repeating frieze pattern, integrated rotation and reflection as elements of the group, and explained that the structure follows associative properties and had the elements of visual identity.



Figure 2. Frieze design by integrating rotation and reflection elements in symmetry groups.

Furthermore, students' creativity in linking mathematical concepts with local culture enriched their cognitive engagement. One of the prominent results was the frieze design based on the clove motif typical of the *Tulungagung* region. Students identified translation and reflection patterns in the repetition of the clove flower motif, while maintaining the nature of the group structure in their design.



Figure 3. Frieze design with clove motif from the Tulungagung (Indonesia).

Context-based learning also enhanced metacognition and self-reflection. Many students realized that true understanding was not just about memorizing, but constructing meaning.

"I thought I understood because I could answer the questions. But when I was asked to explain why the structure was a group, I had difficulty. Now I am more reflective and start to test my own understanding" (S3, class discussion).

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Another student added,

"I started to look for my own references because the assignment challenged me to understand more than just the formula" (S5, end-of-semester interview).

This is in line with the increase in indicators such as self-assessment, evaluation of ideas, and seeking additional information (each increasing by 58–70% of students). Support from teachers also strengthens this finding. The teacher stated, *"Students ask 'why' and 'how' more often than just 'what.' They make connections between topics independently, which I rarely see in regular classes" (G1, class observation).* Even students who were previously passive began to actively engage in group discussions and correct each other's understanding.

"I can see how my friends solve problems, then compare it with my way. I also often help correct if there is something wrong" (S6, interview).

This approach not only built deeper cognitive understanding, but also strengthened students' collaborative and reflective skills.

Overall, 74% of students showed an increase in the use of higher-order thinking strategies, such as conceptual analysis, logical argumentation, and mapping relationships between topics. This contextual task-based learning effectively shifted students' learning orientation from a surface approach to a deep learning approach, as idealized in advanced mathematics education.

Cognitive Engagement Indicator	Positive		Negative		Do not change	
	n	%	n	%	n	%
Solving problems in a different way	29	83%	0	0%	6	17%
Thinking aloud	26	74%	3	9%	6	17%
Remembering and applying previous learning	28	80%	0	0%	7	20%
Planning learning	25	71%	3	9%	7	20%
Trying to be an expert in mathematics	25	71%	1	3%	9	26%
Proving and refuting	25	71%	1	3%	9	26%
Looking for additional information	24	69%	5	14%	6	17%
Independent working style	24	69%	5	14%	6	17%
Giving evaluation to ideas	22	63%	3	9%	10	29%
Realizing the purpose of learning	22	63%	0	0%	13	37%
Connecting between topics	22	63%	0	0%	13	37%
Monitoring and self-assessment	21	60%	0	0%	14	40%
Recognizing and correcting errors	19	54%	0	0%	16	46%
Choose challenging questions	15	43%	1	3%	19	54%

Table 4: Changes in Students' Cognitive Engagement through Context Tasks Intervention

Results of Improving Student Performance

To evaluate the development of students' conceptual understanding in Abstract Algebra material, quantitative measurements were carried out through a pre-test before the intervention and a post-

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test after context-based learning tasks. The average results of students' scores on three main conceptual aspects—Symmetry Group, Frieze Group, and Dihedral Group—showed consistent and significant improvements. Details of the data are presented in Table 5 below:

Topic	Skor Pre-test (Mean ± SD)	Skor Post-test (Mean ± SD)	Improvement
Symmetry Group	48.5 ± 12.4	76.2 ± 10.3	27.4%
Freeze Group	44.7 ± 14.1	72.5 ± 11.5	27.4%
Dihedral Group	41.2 ± 13.7	68.9 ± 12.2	27.3%

Table 5. Comparison of Pre-test and Post-test Scores of Abstract Algebra

Comparative analysis between pre-test and post-test scores indicated substantial conceptual improvement across all aspects. The average improvement of ±27.7% reflects a profound transformation in students' understanding, which is not limited to procedural mastery or memorization of formal definitions, but includes functional meanings of algebraic structures in real-world contexts. This finding is supported by qualitative data from post-intervention interviews. Many students reported that they only "saw" the real relevance of concepts such as symmetry groups or frieze transformations after working on tasks related to cultural artifacts, architecture, and everyday visual patterns. One student stated,

"Before, I only knew the definition of a group from a book, but now I can explain why rotations and reflections in batik can form groups. This is not just mathematics, it is also art." (S7, interview).

Furthermore, students reported that they were able to relate mathematical operations to visual structures, evaluate group properties based on axioms, and explain the relationships between different concepts (such as identity and inverse) within a local cultural context. This not only reflected deeper conceptual mastery but also demonstrated strong knowledge transferability—a key indicator of meaningful learning in higher mathematics education. Overall, both quantitative and qualitative data suggested that the integration of contextual tasks successfully improved students' academic performance and fostered a conceptual understanding that was durable, reflective, and contextually grounded. These findings confirmed that context-based learning design played a critical role in bridging the gap between mathematical abstraction and authentic, meaningful learning experiences.

Summary of Key Findings

The results indicated that context-based learning significantly increased student engagement in three main dimensions—behavioral, emotional, and cognitive—and had a direct impact on improving academic performance in the Abstract Algebra course. In terms of behavior, students showed a clear increase in attendance, discussion participation, questioning initiative, and responsibility for group assignments, with a clear transition from passive to proactive learning behavior. Emotionally, students reported increased intrinsic motivation, self-confidence, and engagement

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with the material, especially due to the relevance of the task context to their experiences and cultural identities, such as in the exploration of symmetry in batik motifs. In terms of cognitive, students showed a shift from a surface learning approach to a deep approach, with increased reflective, evaluative, and conceptual thinking skills that allowed them to flexibly link abstract theories to real-world situations. These qualitative findings are supported by quantitative data in the form of an average post-test score increase of 27.7% across all aspects of the material, reflecting a clear strengthening of students' academic performance. Thus, this study confirms that the integration of context-based tasks not only expands the space for student engagement multidimensionally, but is also an effective pedagogical strategy in shaping deeper, more meaningful, and sustainable mathematical understanding and performance in higher education.

DISCUSSION

The findings of this study clearly showed that context-based learning tasks significantly contributed to increasing student engagement in Abstract Algebra learning in three main dimensions: behavioral, emotional, and cognitive. These findings strengthen existing literature, such as those expressed by Podschuweit and Bernholt, (2018); Rubel and McCloskey (2021); Juniati and Budayasa (2021); and Juniati and Budayasa (2023), that context-based learning increases relevance, motivation, and builds a strong bridge between abstract concepts and students' concrete experiences. The increase in the average post-test score of 27.3%, as well as changes in learning behavior observed and reported through reflective journals, indicate that context tasks are not only an innovative approach, but also have a direct impact on academic performance. In addition, this finding suggests that the issues associated with mathematics learning pointed out by OECD (2023), particularly the inability of students to see the practical value of mathematics in real-life scenarios, could continue to be a problem even during university studies. The transformation of students' learning approaches from procedural to conceptual, as reflected in the increase in critical thinking skills, the use of metacognitive reflection, and concept mapping, is in line with the findings of Entwistle (2023) on the importance of encouraging deep learning approaches in higher education. Students not only memorize formal definitions of groups, rings, and fields, but also begin to build flexible conceptual schemes, transfer knowledge to new situations, and evaluate mathematical structures in the context of local culture.

Cognitively, students experience a transition from simply memorizing formal definitions to being able to construct flexible conceptual structures. They demonstrated critical thinking skills, evaluating mathematical structures in the context of local culture, and transferring knowledge to new situations—achievements that align with the deep learning framework proposed by Entwistle (2023). For example, in the frieze batik design task, students not only identified rotation and reflection as group elements, but also consciously arranged them to satisfy the group axioms—a concrete demonstration of high-level cognitive engagement. This activity reinforces the principle in Coles and Sinclair (2019), that mathematics learning becomes more meaningful when rooted in students' own visual, cultural, and social representations. Emotionally, student engagement was

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evident through increased self-confidence, comfort in expressing opinions, and pride in mathematics learning that is relevant to local culture. These findings indicate that contextual learning not only touches on cognitive aspects, but also creates an emotional bond to mathematical content—which was previously often considered dry and far from everyday life. In this case, the role of emotional affiliation to the task becomes a trigger for active participation, as confirmed by students in reflective notes and in-depth interviews. This dimension directly supports the expectancy-value model of Wang and Eccles (2013), which emphasizes that academic engagement is strongly influenced by an individual's perception of the value of the task and expectations of success. In terms of behavior, increased attendance, consistency in completing tasks, and increased participation in group discussions indicate a more internalized learning engagement. Students actively take responsibility for the learning process, show initiative, and develop academic persistence. These findings enrich Fredricks et al., (2019) multidimensional engagement model with empirical evidence that all three dimensions of engagement—behavioral, emotional, and cognitive—can be facilitated simultaneously through a context-based approach. In addition, this approach is in line with Vygotsky and Cole (2018) social constructivism theory, where social interaction, negotiation of meaning, and collective task-based scaffolding are important foundations for the formation of advanced conceptual understanding.

However, these positive changes were not homogeneous. Variations in students' engagement levels suggest that contextual factors such as perceived task value, clarity of learning goals, and social support in the classroom play an important role. Students who viewed the task as relevant and meaningful showed higher engagement, while students who questioned the relevance of the task showed behavioral resistance, although they still showed cognitive reflection in their diary. These findings support Wang and Eccles (2013) Expectancy-Value model, which asserts that perceived value and expectations of success jointly shape students' levels of academic motivation. From an engagement theory perspective, this study enriches Fredricks et al., (2019) multidimensional engagement model by adding empirical evidence on how contextual task-based learning can facilitate behavioral, emotional, and cognitive engagement simultaneously. Furthermore, these findings reinforce Vygotsky and Cole (2018) social constructivism principles, where social interaction, scaffolding, collective negotiation of meaning, and culturally based experiences have been shown to be important in the formation of advanced mathematical understanding. Limitations of this study include the limited scope of participants at one state university in Indonesia, which limits the generalizability of the results to other contexts. The intervention duration of only eight weeks may also not be sufficient to monitor long-term changes in learning patterns. Therefore, further research is recommended to conduct a longitudinal study based on mixed-methods design to see the dynamics of changes in student engagement over time.

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CONCLUSION

This study shows that a contextual task-based learning approach can significantly increase students' behavioral, emotional, and cognitive engagement in Abstract Algebra. By connecting abstract mathematical concepts to relevant real-world contexts, such as local cultural symmetry patterns and cryptographic applications, students experienced a transformation in their learning approach, shifting from surface learning to deep learning based on critical reflection. This increased engagement was reflected not only in active class participation and pre- and post-test results, but also in assignment products that demonstrated conceptual internalization, such as a frieze design based on symmetry groups that integrated local cultural aspects. Students' reflections indicated metacognitive development, where they recognized the need to construct mathematical meaning through applying concepts in real contexts, rather than simply memorizing formal definitions. These findings add to the literature on student engagement and mathematics education, emphasizing the importance of meaningful, contextual, and flexible task design in building motivation, learning relevance, and higher-order thinking skills. Theoretically, this study provides strong support for the multidimensional engagement model (Fredricks et al., 2019) and the principles of social constructivism (Vygotsky and Cole, 2018), which posit social interaction, negotiation of meaning, and use of cultural experiences as the primary foundations of cognitive development in higher education. The practical implications of this study are the importance for educators to consciously link abstract theory to students' lived experiences, through tasks that encourage exploration, creativity, and reflection. In the context of advanced mathematics learning, contextual tasks can increase students' motivation to learn, deepen conceptual understanding, and strengthen students' academic identities as active and reflective learners. As a future direction, this study encourages further exploration of the adaptation of context-based tasks across disciplines and cultures, as well as the importance of longitudinal research to understand how student engagement develops and transforms over time. Thus, contextual task-based learning can be positioned as a transformative pedagogical strategy that not only enhances academic engagement, but also builds the foundation for continuous learning oriented towards meaning, relevance, creativity, and critical reflection in 21st century mathematics education.

REFERENCES

[1] Anderson, K. M., Haynes, J. D., Ilesanmi, I., & Conner, N. E. (2022). Teacher professional development on trauma-informed care: Tapping into students' inner emotional worlds. *Journal of Education for Students Placed at Risk (JESPAR)*, *27*(1), 59–79.

[2] Antonenko, P. D., Jahanzad, F., & Greenwood, C. (2014). Fostering collaborative problem solving and 21st century skills using the DEEPER scaffolding framework. *Journal of College Science Teaching*, *43*(6), 79–88.

[3] Asada, M., Fukawa-Connelly, T., & Weber, K. (2024). What do university mathematics students value in advanced mathematics courses? *ZDM–Mathematics Education*, 1–12.

[4] Barbieri, C. A., & Miller-Cotto, D. (2021). The importance of adolescents' sense of belonging to mathematics for algebra learning. *Learning and Individual Differences*, *87*, 101993.

[5] Birt, L., Scott, S., Cavers, D., Campbell, C., & Walter, F. (2016). Member checking: a tool to enhance trustworthiness or merely a nod to validation? *Qualitative Health Research*, *26*(13),

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1802–1811.
[6] Bond, M., Buntins, K., Bedenlier, S., Zawacki-Richter, O., & Kerres, M. (2020). Mapping research in student engagement and educational technology in higher education: A systematic evidence map. *International Journal of Educational Technology in Higher Education*, *17*, 1–30.
[7] Braun, V., & Clarke, V. (2025). Reporting guidelines for qualitative research: A values-based approach. *Qualitative Research in Psychology*, *22*(2), 399-438.
[8] Carmichael, C., Muir, T., & Callingham, R. (2017). The impact of within-school autonomy on students' goal orientations and engagement with mathematics. *Mathematics Education Research Journal*, *29*(2), 219–236.
[9] Carruthers, E., & Worthington, M. (2005). Making sense of mathematical graphics: The development of understanding abstract symbolism. *European Early Childhood Education Research Journal*, *13*(1), 57–79.
[10] Carson, R. N., & Rowlands, S. (2007). Teaching the conceptual revolutions in geometry. *Science & Education*, *16*, 921–954.
[11] Cevikbas, M., & Kaiser, G. (2022). Student engagement in a flipped secondary mathematics classroom. *International Journal of Science and Mathematics Education*, *20*(7), 1455–1480.
[12] Chevallard, Y., & Bosch, M. (2020). Didactic transposition in mathematics education. *Encyclopedia of Mathematics Education*, 214–218.
[13] Christenson, N., Chang Rundgren, S.-N., & Zeidler, D. L. (2014). The relationship of discipline background to upper secondary students' argumentation on socioscientific issues. *Research in Science Education*, *44*, 581–601.
[14] Coles, A., & Sinclair, N. (2019). Re-thinking 'concrete to abstract'in mathematics education: Towards the use of symbolically structured environments. *Canadian Journal of Science, Mathematics and Technology Education*, *19*, 465–480.
[15] Cresswell, J. (2013). *Qualitative inquiry & research design: Choosing among five approaches.*
[16] Entwistle, N. (2023). Promoting deep learning through teaching and assessment. In *Assessment to promote deep learning* (pp. 9–19). Routledge.
[17] Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American Journal of Theoretical and Applied Statistics*, *5*(1), 1–4.
[18] Fokkens-Bruinsma, M., Vermue, C., Deinum, J.-F., & van Rooij, E. (2021). First-year academic achievement: the role of academic self-efficacy, self-regulated learning and beyond classroom engagement. *Assessment & Evaluation in Higher Education*, *46*(7), 1115–1126.
[19] Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research*, *74*(1), 59–109.
[20] Fredricks, J. A., Reschly, A. L., & Christenson, S. L. (2019). Interventions for student engagement: Overview and state of the field. *Handbook of Student Engagement Interventions*, 1–11.
[21] Gillies, R. M. (2019). Promoting academically productive student dialogue during collaborative learning. *International Journal of Educational Research*, *97*, 200–209.
[22] Gomes, S., Costa, L., Martinho, C., Dias, J., Xexéo, G., & Moura Santos, A. (2023). Modeling

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students' behavioral engagement through different in-class behavior styles. *International Journal of STEM Education*, *10*(1), 21.

[23] Halldén, O., Scheja, M., & Haglund, L. (2013). The contextuality of knowledge: An intentional approach to meaning making and conceptual change. In *International handbook of research on conceptual change* (pp. 71–95). Routledge.

[24] Hodgson, T. R., Cunningham, A., McGee, D., Kinne, L. J., & Murphy, T. J. (2017). Assessing behavioral engagement in flipped and non-flipped mathematics classrooms: teacher abilities and other potential factors. *International Journal of Education in Mathematics, Science and Technology*, *5*(4), 248–261.

[25] Juniati, D., & Budayasa, K. (2017). Construction of learning strategies to combine culture elements and technology in teaching group theory. *World Transactions on Engineering and Technology Education*, *15*(3), 206-211.

[26] Juniati, D., & Budayasa, I. K. (2021). Field-based tasks with technology to reduce mathematics anxiety and improve performance. *World Transactions on Engineering and Technology Education*, *19*(1), 58-64.

[27] Juniati, D., & Budayasa, I. K. (2023). Geometry learning strategies with optimized technology to improve the performance of undergraduate mathematics students. *World Transactions on Engineering and Technology Education*, *21*(1), 26-31.

[28] Jurdak, M. (2016). Learning and teaching real world problem solving in school mathematics. *Learning and Teaching Real World Problem Solving in School Mathematics. Https://Doi. Org/10.1007/978-3-319-08204-2*.

[29] Kieran, C. (2020). Algebra teaching and learning. *Encyclopedia of Mathematics Education*, 36–44.

[30] Kim, S., Raza, M., & Seidman, E. (2019). Improving 21st-century teaching skills: The key to effective 21st-century learners. *Research in Comparative and International Education*, *14*(1), 99–117.

[31] Lester Jr, F. K., & Cai, J. (2016). Can mathematical problem solving be taught? Preliminary answers from 30 years of research. *Posing and Solving Mathematical Problems: Advances and New Perspectives*, 117–135.

[32] Liang, B., & Moore, K. C. (2021). Figurative and operative partitioning activity: Students' meanings for amounts of change in covarying quantities. *Mathematical Thinking and Learning*, *23*(4), 291–317.

[33] Lo, C. K., & Hew, K. F. (2021). Student engagement in mathematics flipped classrooms: Implications of journal publications from 2011 to 2020. *Frontiers in Psychology*, *12*, 672610.

[34] Lozano, M.-D. (2017). Investigating task design, classroom culture and mathematics learning: an enactivist approach. *ZDM*, *49*(6), 895–907.

[35] Luo, N., Zhang, M., & Qi, D. (2017). Effects of different interactions on students' sense of community in e-learning environment. *Computers & Education*, *115*, 153–160.

[36] Mamolo, A., & Glynn-Adey, P. (2023). Tangible connections within the mathematical horizon: Exploring the Dihedral Calculator. *ZDM–Mathematics Education*, *55*(4), 793–805.

[37] McHugh, M. L. (2012). Interrater reliability: the kappa statistic. *Biochemia Medica*, *22*(3), 276–282.

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[38] Miller, A. L., Fassett, K. T., & Palmer, D. L. (2021). Achievement goal orientation: A predictor of student engagement in higher education. *Motivation and Emotion*, *45*(3), 327–344.
[39] Nathan, M. J., & Alibali, M. W. (2021). An embodied theory of transfer of mathematical learning. *Transfer of Learning: Progressive Perspectives for Mathematics Education and Related Fields*, 27–58.
[40] Nguyen, T. D., Cannata, M., & Miller, J. (2018). Understanding student behavioral engagement: Importance of student interaction with peers and teachers. *The Journal of Educational Research*, *111*(2), 163–174.
[41] Núñez, R. (2020). The cognitive science of mathematics: why is it relevant for mathematics education? In *Foundations for the future in mathematics education* (pp. 127–154). Routledge.
[42] OECD. (2023). *PISA 2022 Results*. Https://Www.Oecd.Org/Publication/Pisa-2022-Results.
[43] Pambudi, D. ., Kurniati, D., & Lestari, N. D. . (2025). Development of a Mathematical Cognitive Test with an Agricultural Context to Improve Students' Mathematical Connection Ability. *Mathematics Teaching Research Journal*, *17*(2), 169–192.
[44] Park, S., & Yun, H. (2018). The influence of motivational regulation strategies on online students' behavioral, emotional, and cognitive engagement. *American Journal of Distance Education*, *32*(1), 43–56.
[45] Phillips, C., & O'Flaherty, J. (2019). Evaluating nursing students' engagement in an online course using flipped virtual classrooms. *Student Success*, *10*(1), 59–71.
[46] Pineda-Báez, C., Manzuoli, C. H., & Sánchez, A. V. (2019). Supporting student cognitive and agentic engagement: Students' voices. *International Journal of Educational Research*, *96*, 81–90.
[47] Pires, A. C., Bakala, E., González-Perilli, F., Sansone, G., Fleischer, B., Marichal, S., & Guerreiro, T. (2022). Learning maths with a tangible user interface: Lessons learned through participatory design with children with visual impairments and their educators. *International Journal of Child-Computer Interaction*, *32*, 100382.
[48] Podschuweit, S., & Bernholt, S. (2018). Composition-effects of context-based learning opportunities on students' understanding of energy. *Research in Science Education*, *48*, 717–752.
[49] Pohl, A. J. (2020). Strategies and interventions for promoting cognitive engagement. In *Student engagement: Effective academic, behavioral, cognitive, and affective interventions at school* (pp. 253–280). Springer.
[50] Reinke, L. T., & Casto, A. R. (2022). Motivators or conceptual foundation? Investigating the development of teachers' conceptions of contextual problems. *Mathematics Education Research Journal*, *34*(1), 113–137.
[51] Rubel, L. H., & McCloskey, A. V. (2021). Contextualization of mathematics: which and whose world? *Educational Studies in Mathematics*, *107*(2), 383–404.
[52] Sakeef, N., Dewan, M. A. A., Lin, F., & Parmar, D. (2025). Detecting cognitive engagement in online course forums: A review of frameworks and methodologies. *Natural Language Processing Journal*, 100146.
[53] Sheppard, M. E., & Wieman, R. (2020). What do teachers need? Math and special education

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teacher educators' perceptions of essential teacher knowledge and experience. *The Journal of Mathematical Behavior*, *59*, 100798.

[54] Shimazu, A., Schaufeli, W. B., Kamiyama, K., & Kawakami, N. (2015). Workaholism vs. work engagement: The two different predictors of future well-being and performance. *International Journal of Behavioral Medicine*, *22*, 18–23.

[55] Siqi-Liu, A., & Egner, T. (2020). Contextual adaptation of cognitive flexibility is driven by task-and item-level learning. *Cognitive, Affective, & Behavioral Neuroscience*, *20*(4), 757–782.

[56] Son, J. Y., & Goldstone, R. L. (2009). Contextualization in perspective. *Cognition and Instruction*, *27*(1), 51–89.

[57] Stewart, S., Andrews-Larson, C., & Zandieh, M. (2019). Linear algebra teaching and learning: themes from recent research and evolving research priorities. *ZDM*, *51*, 1017–1030.

[58] Tan, J. P.-L., & Nie, Y. (2015). The role of authentic tasks in promoting twenty-first century learning dispositions. *Authentic Problem Solving and Learning in the 21st Century: Perspectives from Singapore and Beyond*, 19–39.

[59] Tas, Y. (2016). The contribution of perceived classroom learning environment and motivation to student engagement in science. *European Journal of Psychology of Education*, *31*, 557–577.

[60] Toogood, C. (2023). Supporting students to engage with case studies: a model of engagement principles. *Educational Review*, 1–15.

[61] Treagust, D. F., & Duit, R. (2008). Conceptual change: A discussion of theoretical, methodological and practical challenges for science education. *Cultural Studies of Science Education*, *3*, 297–328.

[62] Voigt, J. (2013). Negotiation of mathematical meaning in classroom processes: Social interaction and learning mathematics. In *Theories of mathematical learning* (pp. 21–50). Routledge.

[63] Vygotsky, L., & Cole, M. (2018). Lev Vygotsky: Learning and social constructivism. *Learning Theories for Early Years Practice. UK: SAGE Publications Inc*, 68–73.

[64] Vygotsky, L. S., & Cole, M. (1978). *Mind in society: Development of higher psychological processes*. Harvard university press.

[65] Wang, M.-T., & Eccles, J. S. (2013). School context, achievement motivation, and academic engagement: A longitudinal study of school engagement using a multidimensional perspective. *Learning and Instruction*, *28*, 12–23.

[66] Wernet, J. L. W. (2017). Classroom interactions around problem contexts and task authenticity in middle school mathematics. *Mathematical Thinking and Learning*, *19*(2), 69–94.

[67] Wu, I.-C., Pease, R., & Maker, C. J. (2015). Students' perceptions of real engagement in active problem solving. *Gifted and Talented International*, *30*(1–2), 106–121.

[68] Zimmerman, B. J., & Schunk, D. H. (2011). Self-regulated learning and performance: An introduction and an overview. *Handbook of Self-Regulation of Learning and Performance*, 15–26.

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Integrating Ethnomathematics and GeoGebra: A Project-Based Learning Approach to Improve Mathematical Literacy

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Abstract: Mathematical literacy—the ability to formulate, apply, and interpret mathematics in diverse contexts—is essential for addressing real-life challenges in the 21st century. However, many students struggle with this competency due to traditional procedural teaching methods that lack cultural relevance and technological integration. This study examines the impact of an Ethno-Project-Based Learning (Ethno-PjBL) model supported by GeoGebra on students' mathematical literacy. Using a quasi-experimental design, 212 senior high school students from three accredited public schools in Karo Regency, North Sumatra, Indonesia, were divided equally into an experimental group (Ethno-PjBL with GeoGebra) and a control group (conventional instruction). Mathematical literacy assessment utilized a test based on PISA indicators. After confirming non-normal data distribution, the Mann–Whitney U test revealed a significant difference between the groups ($p = 0.015$), with the experimental group outperforming the control group. These results suggest that integrating cultural contexts with technology through the Ethno-PjBL model effectively enhances students' mathematical literacy—even during a brief intervention period, as demonstrated in this study.

Keywords: mathematical literacy, GeoGebra, ethnomathematics, project-based learning, quasi-experimental

INTRODUCTION

Mathematical literacy is one of the essential competencies students must possess to face the challenges of the 21st century (Muhaimin et al., 2024). It encompasses computational skills and the ability to understand, interpret, and apply mathematics in various real-life contexts. Mathematical literacy plays a vital role in everyday life, enabling individuals to apply mathematical knowledge in practical situations such as managing personal finances, interpreting data, and making informed

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decisions (Serin, 2023). Research indicates that low mathematical literacy leads to difficulties in problem-solving, potentially hindering participation in high-skilled jobs and exacerbating socio-economic disparities (Sitopu et al., 2023). Therefore, integrating mathematical literacy into primary and secondary education curricula is essential for improving education quality and global competitiveness (Nurmasari et al., 2023; OECD, 2018).

Globally, the 2022 Programme for International Student Assessment (PISA) survey conducted by the OECD revealed that Indonesian students' mathematical literacy remains low, with average scores significantly below the OECD average (OECD, 2023a). This finding suggests a gap between school curricula and the practical needs of everyday mathematical problem-solving. The report further highlights Indonesian students' weaknesses in reasoning, problem-solving, and applying mathematical concepts in real-life situations (OECD, 2023b). These outcomes emphasize improving mathematics teaching approaches to be more contextual, meaningful, and supportive of developing mathematical literacy.

In practice, mathematics instruction in Indonesia often relies on conventional, procedural, and mechanistic methods. Students tend to memorize formulas without understanding underlying concepts and seldom have opportunities to explore mathematics through real-world contexts. This approach significantly impedes the development of mathematical literacy (Wijaya et al., 2024). Consequently, adopting learning approaches that connect mathematics to students' real-life experiences becomes necessary, and Project-Based Learning (PjBL) offers a promising solution.

Project-Based Learning emphasizes active student engagement in solving authentic, contextual problems through projects. Research shows that this model enhances critical thinking, collaboration, and the application of mathematical concepts in daily life (Bell, 2010). However, PjBL implementation rarely incorporates students' local culture, although integrating cultural elements can strengthen students' connection to the material.

Ethnomathematics links mathematical concepts with local cultural practices. It views mathematics as a universal discipline shaped and developed through human culture. Studies have demonstrated that incorporating ethnomathematics in teaching increases students' motivation and understanding because they perceive the material as more relevant and familiar (Rosa & Orey, 2016). Combining ethnomathematics with Project-Based Learning presents a promising instructional approach. This model engages students in authentic projects and elevates local wisdom and cultural practices within mathematics learning. Through this approach, students learn mathematics from their environment, fostering a stronger internalization of concepts (Maulina et al., 2024).

Despite these benefits, students often need visual tools to understand mathematical ideas better. GeoGebra, an interactive software, supports dynamic visualization and has proven effective in improving comprehension, particularly in geometry and algebra (Zengin et al., 2012). Integrating GeoGebra into the Ethno-Project-Based Learning model represents an innovative advancement in mathematics education. GeoGebra enables students to explore culture-based mathematical projects

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visually and interactively, helping them grasp the relationship between mathematics, culture, and its applications in daily life. This approach could strengthen mathematical literacy across conceptual, procedural, and applicative dimensions.

Research on the combined use of Ethno-PjBL and GeoGebra in Indonesian mathematics education is limited. This study aims to fill that gap by investigating how this integrated model influences students' mathematical literacy, offering a culturally responsive and technologically enriched approach to improve learning outcomes. This study aims to inform curriculum development and foster innovative instructional strategies that address current educational challenges.

However, implementing the Ethno-Project-Based Learning model with GeoGebra support poses challenges, including teacher readiness to integrate local culture, project-based methods, and digital technology simultaneously. Hence, teacher training and support are critical for practical application. This model offers novelty in mathematics education by integrating local cultural elements through ethnomathematics, project-based learning strategies, and GeoGebra's visual technology. The combination of these three approaches has been underexplored, especially in Indonesia, indicating its significant potential as a leading model to enhance students' mathematical literacy.

Considering this approach's urgency, challenges, and potential, this study on using an Ethno-Project-Based Learning model supported by GeoGebra remains timely and important. Based on the discussion above, this study addresses the following research question: "Is there a difference in mathematical literacy between students who participate in project-based learning using GeoGebra and those who follow traditional learning models?" The study aims to improve students' literacy and enrich research and practical knowledge related to contextual, relevant, and innovative mathematics education in Indonesia.

LITERATURE REVIEW

Mathematical Literacy Skills

Mathematical literacy refers to an individual's ability to understand, use, and interpret mathematical concepts and procedures in various real-life contexts (OECD, 2019b). Mathematical literacy encompasses the ability to formulate, employ, and interpret mathematics across different situations, solve problems, reason logically, communicate, and explain mathematics in everyday and shared contexts.

Mathematical literacy through three key processes: formulate, employ, and interpret (OECD, 2019a). The formulation process involves translating real-world problems into mathematical form, including recognizing the mathematical aspects of a given context. In the employ process, students actively employ mathematical knowledge and procedures to solve mathematically formulated problems. Lastly, the interpretation process involves understanding and interpreting mathematical

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outcomes within real-world contexts, including the ability to evaluate and communicate the results logically.

Various factors influence mathematical literacy, including learning motivation, teaching approaches, and students' cognitive readiness. Research by Hamdiyanti et al. (2023) shows that students with high intrinsic motivation tend to possess stronger mathematical literacy skills. Meanwhile, Yusuf et al. (2024) highlight the role of cognitive styles in shaping students' thinking processes when formulating and interpreting mathematical problems. Educators also play a critical role in encouraging reflective thinking and promoting contextual applications of mathematics in the classroom.

This study employs three primary indicators of mathematical literacy: formulating, employing, and interpreting. These indicators encompass students' abilities to identify, apply, and comprehend mathematical concepts in various real-world contexts. The indicators aim to assess students' mathematical literacy skills comprehensively.

Project-Based Learning Model

Project-Based Learning (PjBL) is an instructional model that positions students as active participants in their learning by engaging them in real and meaningful projects (Bell, 2010). Through PjBL, students do not merely receive information passively. However, they are encouraged to develop critical thinking, collaboration, and problem-solving skills by working on projects that are relevant to real-world contexts (Thomas, 2000). PjBL encourages students to conduct in-depth investigations, plan, implement, and reflect on their learning processes individually and in groups.

According to Holm (2011), one of the key strengths of PjBL lies in its ability to integrate various disciplines within a single project, enabling students to perceive the interconnections between concepts and their applications in everyday life. Maysarah (2015) found that project-based learning can enhance students' mathematical communication skills and creativity. Students taught using a project-based learning model demonstrate higher levels of mathematical literacy than those taught through conventional instruction (Maysarah et al., 2023). Implementing PjBL typically begins with presenting a problem or challenge that students must address through a specific project. Students then design a work plan, collect data, and produce a final product, which is later presented to the teacher and peers (Bell, 2010). This process involves collaborative Learning and the development of soft skills such as responsibility, time management, and effective communication. The final product serves as tangible evidence of students' competence. Nevertheless, PjBL is not without its challenges. According to Thomas (2000), teachers must manage time and resources effectively while guiding students to remain focused and avoid irrelevant activities. The learning environment and student readiness are also crucial factors that determine the success of PjBL. However, PjBL can be an effective and engaging learning model when implemented appropriately.

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Research by Krajcik & Blumenfeld (2006) shows that the PjBL model provides a robust framework for integrating multiple disciplines and supports contextual Learning focused on developing 21st-century skills such as creativity and problem-solving. Furthermore, Larmer et al. (2015) emphasized that effective PjBL implementation requires thorough planning, systematic execution, and in-depth reflection and evaluation by students and teachers. The study also highlights the importance of the teacher's role as a facilitator throughout the learning process.

The fundamental principles in the development of instructional models are, (1) *Syntax*, referring to the systematic steps of the learning process; (2) *Reaction Principles*, concerning how the teacher responds to students' behaviors to guide them toward learning objectives; (3) *Social System*, referring to the relational structure between teacher and students during instruction; (4) *Support System*, which includes tools, infrastructure, and resources that support model implementation; and (5) *Instructional and Nurturant Effects*, which encompass both the direct learning outcomes and indirect results such as the development of attitudes, values, or social skills (Joyce et al., 2009). These principles serve as a conceptual framework for designing practical and student-centered instruction.

The Project-Based Learning (PjBL) model positions students at the center of the learning process by engaging in authentic and meaningful projects. This study's PARR (Plan, Action, Report, Reward) syntax provides a systematic framework for implementing PjBL, from project planning to recognizing students' achievements. Applying this syntax makes the learning process more structured, effectively promoting student engagement and optimizing competency attainment.

Ethnomathematics

Ethnomathematics studies the relationship between mathematics and culture, emphasizing how different cultural groups develop and utilize mathematical concepts in their daily lives (D'Ambrosio, 2006). Ethnomathematics is a concept first introduced by D'Ambrosio (1999), who described it as a form of mathematical literacy that evolves within specific cultural contexts and recognizes that every community possesses unique ways of solving problems involving mathematical ideas, such as measurement, patterns or number systems. Thus, this approach highlights mathematics education's cognitive aspects and social and cultural dimensions.

According to D'Ambrosio (2001), ethnomathematics is not merely the study of traditional or local mathematics but an acknowledgment of diverse global mathematical thinking. Furthermore, ethnomathematics plays a role in preserving cultural heritage and fostering students' cultural identity through mathematics instruction (Powell & Frankenstein, 1997). Research by Sari et al. (2024) and Wulandari et al. (2024) indicates that integrating ethnomathematics into education can enhance students' understanding of mathematical concepts by connecting the material to local cultural contexts. A study by Mulenga & Folokwe (2024) demonstrates that teaching geometry

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through local cultural practices can deepen conceptual understanding, preserve cultural traditions, and promote an inclusive approach to mathematics education.

One of the main strengths of the ethnomathematical approach is its ability to bridge the gap between formal mathematics and the local knowledge systems students possess. This approach is highly effective in helping students from indigenous communities grasp mathematical concepts through familiar cultural practices (Nur et al., 2023). Such an approach increases student engagement in learning and supports preserving local culture. Moreover, such an approach enriches the learning process by embedding local values that shape students' character and attitudes toward knowledge.

Beyond conceptual understanding, ethnomathematics also fosters critical thinking and problem-solving skills. Ethnomathematics-based mathematics instruction can enhance students' ability to creatively analyze and solve mathematical problems (Imswatama, 2018). Teachers encourage students to memorize formulas and reflect on how they apply mathematical concepts within their cultural contexts. Such an approach provides a strong foundation for developing a more holistic and functional form of mathematical literacy.

In conclusion, ethnomathematics is an innovative approach significantly contributing to mathematics education. By emphasizing cultural relevance and local context, this approach strengthens students' conceptual understanding while enriching their cultural and social values. Therefore, integrating ethnomathematics into the mathematics curriculum should be continuously promoted as a strategic effort to improve the quality of education in multicultural societies. The cultural context highlighted in this study is the Karo culture, originating from North Sumatra Province, Indonesia.

GeoGebra

GeoGebra is an open-source dynamic mathematics software that integrates geometry, algebra, calculus, and statistics into a single interactive platform. Users can access it both online and offline, and it supports active and constructivist learning. Through interactive graphics, GeoGebra enables users to create mathematical visualizations that help students meaningfully connect visual and symbolic representations (Kovács et al., 2023).

Initially developed by Markus Hohenwarter, GeoGebra has been widely adopted across the globe in mathematics education, from primary schools to university-level instruction. Due to its free accessibility, user-friendly interface, and multilingual support, GeoGebra has become a popular tool in technology-enhanced mathematics education (Awaji et al., 2025).

GeoGebra has been proven effective in enhancing mathematical comprehension, problem-solving skills, and student motivation. A study by Awaji et al. (2025), which reviewed two decades of GeoGebra-related publications, identified three main areas of application: as a teaching aid in

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geometry, a tool for integrating technology into Learning, and a means to improve mathematics achievement. The growing body of research on GeoGebra highlights its significant influence on mathematics education (Kovács et al., 2023).

In geometry instruction, GeoGebra enables students and teachers to visualize abstract ideas through interactive exploration, fostering deeper Learning and encouraging discovery-based approaches (Triet et al., 2024). Combined with the 5E instructional model, its use significantly improved students' grasp of the area formula for trapezoids (Triet et al., 2024). The result demonstrates how technology-enhanced methods can boost engagement and understanding in math classrooms.

GeoGebra is an open-source dynamic mathematics software that provides an interactive platform for the real-time exploration and visualization of mathematical concepts, accessible online and offline. This software supports a more active and constructivist learning process by utilizing visual media to facilitate understanding of abstract concepts. The study explores geometric transformations using various GeoGebra features to deepen students' conceptual understanding.

METHOD

Research Design

This study employed a quantitative approach using a quasi-experimental research design. Quasi-experimental research involves manipulating the independent variable without randomly assigning participants into groups (Jhangiani et al., 2019). The design was appropriate to examine the effect of the Ethno-Project-Based Learning (Ethno-PjBL) model supported by GeoGebra on students' mathematical literacy. This approach suits researchers without complete control over the independent variable, such as randomizing subjects (Creswell, 2012).

This study employed a quasi-experimental nonequivalent control group pre-test and post-test design. The researcher non-randomly assigned participants to the experimental and control groups and measured their performance before and after the intervention (Creswell, 2012; Fraenkel et al., 2012; Jhangiani et al., 2019). The researcher purposively selected two groups: the experimental group received the Ethno-PjBL model supported by GeoGebra (X), while the control group followed conventional instruction. The researcher assessed students' mathematical literacy using pre-tests and post-tests. O_1 and O_2 correspond to the pre-test and post-test of the experimental group, whereas O_3 and O_4 correspond to the pre-test and post-test of the control group. Table 1 summarizes the research design.

Group	Pre-test	Treatment	Post-test
Experimental	O_1	X	O_2
Control	O_3	—	O_4

Table 1: Research Design

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Population and Sample

The study population consisted of all students from public senior high schools (SMAN) across Karo Regency, North Sumatra Province, Indonesia. The study used cluster random sampling by dividing the population into natural clusters (i.e., schools) and randomly selecting several clusters as the sample (Creswell, 2012; Lohr, 2019). The study grouped accredited public high schools (SMAN) within the regency into clusters. The team randomly selected schools with accreditation A to ensure comparable education quality among the chosen clusters. The total sample comprised 212 students distributed as shown in Table 2:

School	Experimental Group (Students)	Control Group (Students)
SMAN 1 Kabanjahe	36	35
SMAN 1 Berastagi	34	36
SMAN 2 Kabanjahe	36	35
Total	106	106

Table 2: Sample Distribution

Data Collection

The instructional process comprised five modules, each delivered over five 45-minute sessions. Each module focused on a geometric transformation and included a project-based task involving the design of batik motifs inspired by the Karo cultural heritage: module 1 emphasized translation, module 2 reflection, module 3 rotation, module 4 dilation, and module 5 composition of transformations. The study implemented the modules following the Ethno-Project-Based Learning (Ethno-PjBL) framework, which follows the PARR cycle. The cycle begins with Plan, in which instructors design learning activities and project assignments; continues with Action, where students actively engage in learning and complete the projects; proceeds with Report, in which students present outcomes and reflect on their experiences; and concludes with Reward, providing feedback and incentives to enhance motivation. Student projects contributed 40% of the final course grade, with the remaining 60% coming from quizzes, assignments, and class participation.

The primary instrument used in this study was a mathematical literacy test consisting of four items, each encompassing the three core PISA indicators: Formulate, Employ, and Interpret (OECD, 2023). The researcher independently developed the items, guided by the PISA mathematical literacy framework, and adapted them to align with the local cultural context and school curriculum. The instrument met both validity and reliability standards, with a Cronbach's Alpha coefficient exceeding 0.7 indicating acceptable reliability.

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Data Analysis

Quantitative data analysis employed inferential statistical tests to evaluate the hypotheses. Normality and homogeneity of variances were assessed using the Shapiro-Wilk and Levene's tests, respectively, following established protocols (Field, 2018). Because the data violated the normality and homogeneity assumptions, the study applied the Mann–Whitney U test to compare the groups (Pallant, 2020; Saragih, 2015). All analyses employed SPSS software, with a significance level of 5% ($\alpha = 0.05$) set as the criterion for interpreting the results (Larson-Hall, 2016).

In addition to other methods, this study applies a descriptive quantitative approach to evaluate students' and teachers' responses to implementing the Ethno-Project-Based Learning (Ethno-PjBL) model supported by GeoGebra in mathematics education. The researchers collect data using a 4-point Likert scale questionnaire focusing on learning interest, motivation, and perceived usefulness. They analyze the data by calculating students' and teachers' mean response scores for each aspect.

The research hypothesis tested whether mathematical literacy significantly differed between the two groups exposed to different instructional models. The null hypothesis (H_0) stated that no significant difference existed between students taught using the Ethno-Project-Based Learning model supported by GeoGebra and those receiving conventional instruction. Conversely, the alternative hypothesis (H_1) posited a significant difference in mathematical literacy between the two groups.

RESULTS

The validity test results indicate that all instrument items have correlation coefficients exceeding the critical r-table value, confirming that all items are valid and appropriate for use in this study. Pearson's Product-Moment correlation assessed the validity by comparing the calculated correlation coefficient (r-calculated) with the critical r-table value. An item is valid if r-calculated > r-table at a specified significance level (e.g., $\alpha = 0.05$). The analysis shows that all instrument items demonstrated r-calculated values greater than r-table, confirming their validity and suitability for this research (Arikunto, 2013; Creswell, 2012). The summary of the validity test results is presented in Table 3 below:

Item Number	r-calculated	r-table	Validity
1a	0.829	0.381	Valid
1b	0.867		Valid
1c	0.732		Valid
2a	0.683		Valid
2b	0.848		Valid
2c	0.489		Valid

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Item Number	r-calculated	r-table	Validity
3a	0.834		Valid
3b	0.748		Valid
3c	0.682		Valid
4a	0.858		Valid
4b	0.816		Valid
4c	0.531		Valid

Table 3: Summary of Test Instrument Validity Results

The table above shows that the item-total correlation coefficients (r values) range from 0.489 to 0.867, all exceeding the critical r value of 0.381 at the 5% significance level with a sample size of 27 students. The findings demonstrate that each test item significantly correlates with the total score, empirically confirming the instrument's construct validity. The following table presents the reliability test results for the mathematical literacy instrument:

Case Processing Summary			
		N	%
Cases	Valid	27	100.0
	Excluded[a]	0	0.0
	Total	27	100.0
a. Listwise deletion based on all variables in the procedure.			
Reliability Statistics			
Cronbach's Alpha		N of Items	
0.924		12	

Table 4: Summary of Test Reliability Results

The analysis shows that Cronbach's Alpha coefficient was 0.924, obtained from 12 test items with 27 valid respondents. This value far exceeds the minimum recommended reliability threshold of 0.70 (Nunnally & Bernstein, 1994), demonstrating that the instrument is highly reliable and allows researchers to use it in this study confidently. Therefore, the mathematical literacy test instrument proves reliable, with each test item exhibiting strong internal consistency in measuring students' mathematical literacy ability. After establishing the reliability of the mathematical literacy test instrument, the researchers analyzed the pre-test and post-test results to measure students' mathematical literacy skills in both the experimental and control groups.

Group	Pre-Test	Post-Test	N-Gain Score
Experimental	34	82	0.72
Control	38	78	0.63

Table 5. Average Scores of Students' Mathematical Literacy Skills

Based on the table, the average pre-test score of students in the experimental class was 34, which increased to 82 on the post-test, resulting in an N-Gain score of 0.72, categorized as high. In

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comparison, the control class increased from 38 to 78, with an N-Gain score of 0.63, falling into the medium–high category. These results indicate that the learning intervention implemented in the experimental class improved students' mathematical literacy more effectively than the conventional approach used in the control class.

The N-Gain scores obtained from the experimental and control groups were analyzed to assess improvements in mathematical literacy. The study employed N-Gain scores instead of post-test scores alone. N-Gain provides a more accurate measure of students' improvement by accounting for differences in pre-test performance (Hake, 1998). The study then conducted normality tests to determine whether the N-Gain scores from both groups followed a normal distribution and employed two methods for the tests: the Kolmogorov–Smirnov and Shapiro–Wilk tests. The decision was primarily based on the significance value (Sig.) from the Shapiro-Wilk test, as the sample size exceeded 50 (Ghasemi & Zahediasl, 2012). The following table presents the results of the normality tests:

Group	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
N-Gain Experimental Class	0.179	106	0.000	0.926	106	0.000
N-Gain Control Class	0.119	106	0.001	0.934	106	0.000
a. Lilliefors Significance Correction						

Table 6: Normality Test Results

The results of the normality tests using Kolmogorov–Smirnov and Shapiro–Wilk indicate that the N-Gain scores for both the experimental and control groups do not follow a normal distribution. The significance values were p = 0.000 for the experimental group and p = 0.001 / 0.000 for the control group. The data did not follow a normal distribution because all significance values were below 0.05.

Test of Homogeneity of Variances			
N-Gain Score			
Levene Statistic	df1	df2	Sig.
38.105	1	210	0.000

Table 7: Homogeneity of Variance Test

The Levene's test for homogeneity of variances on the N-Gain scores yielded a statistic of 38.105 with a significance value of 0.000 (< 0.05), indicating that the variances between the experimental and control groups were not homogeneous. Because both the normality and homogeneity assumptions did not hold, the study conducted hypothesis testing using a non-parametric test, specifically the Mann–Whitney U test (Ghasemi & Zahediasl, 2012; Pallant, 2020), which does not require normally distributed or homogenous data.

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Test Statistics	Value
Mann-Whitney U	4529.000
Wilcoxon W	10200.000
Z	-2.440
Asymp. Sig. (2-tailed)	0.015

Table 8. Mann–Whitney U Test Result

The Mann–Whitney U test examined the difference in N-Gain scores between the experimental and control groups. The test produced a U value of 4529.000 and a significance level of $p = 0.015$. Since $p < 0.05$, the difference between the two groups is statistically significant. This finding confirms that the learning intervention in the experimental group had a significantly greater impact on improving students' learning outcomes than the control group. The conclusion is that a statistically significant difference exists between the post-test scores of students in the experimental and control groups. Hence, reject the null hypothesis (H_0) and accept the alternative hypothesis (H_1). These results support the conclusion that the Ethno-Project-Based Learning model assisted by GeoGebra is more efficacious in improving students' mathematical literacy skills than conventional learning models.

After implementing the Ethno-Project-Based Learning (Ethno-PjBL) model supported by GeoGebra, we collected student and teacher responses using a questionnaire that evaluated three aspects: interest in the learning process, motivation towards learning, and perceived usefulness of the learning activities.

Aspect Evaluated	Average Student Response Score	Average Teacher Response Score
Interest in the learning process	3.61	3.80
Motivation towards learning	3.62	3.67
Perceived usefulness	3.58	3.75
Overall Average Score	3.60	3.74

Table 9: Presents the Average Scores from Students and Teachers

Students reported that the Ethno-PjBL modules supported by GeoGebra powerfully captured their interest, with an average score of 3.61, while teachers rated student interest slightly higher at 3.80. Many students explained that designing batik motifs inspired by the Karo cultural heritage made mathematics more tangible and relevant. Students felt curious and engaged by connecting abstract geometric transformations to culturally meaningful patterns throughout the learning activities.

Motivation scores were similarly high, with students scoring 3.62 and teachers 3.67. Students indicated that combining collaborative projects, interactive GeoGebra tools, and culturally relevant tasks encouraged them to persist in solving mathematical problems and completing projects thoroughly. Teachers observed increased participation, initiative, and enthusiasm, suggesting that the Ethno-PjBL approach effectively sustained student engagement and promoted active learning.

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Regarding perceived usefulness, students rated the learning activities at 3.58, while teachers provided a slightly higher score of 3.75. Both groups agreed that the modules were relevant and beneficial for enhancing mathematical understanding. Teachers highlighted that students could apply geometric transformations meaningfully within real-world and cultural contexts, demonstrating that integrating culture, technology, and project-based tasks fostered both learning effectiveness and appreciation of mathematics.

Overall, the feedback indicates that the Ethno-PjBL model supported by GeoGebra received strong acceptance and motivated students to perform better by connecting mathematics to their culture, providing hands-on problem-solving experiences, and fostering a stimulating and meaningful learning environment.

DISCUSSION

The results of this study indicate a significant difference in mathematical literacy skills between students who participated in Learning using the Ethno-Project-Based Learning (Ethno-PjBL) model assisted by GeoGebra and those who experienced conventional learning methods. The Mann–Whitney U test supports this finding, yielding a significance value of 0.015 (< 0.05), which leads to rejecting the null hypothesis and accepting the alternative hypothesis. Therefore, the Ethno-PjBL model is proven more effective in enhancing students' mathematical literacy than traditional instructional approaches.

These learning activities clearly reflect the three core dimensions of mathematical literacy defined by the OECD (2019a): formulating, employing, and interpreting. In the formulating phase, students translated traditional cultural patterns into mathematical problems by identifying geometric elements and abstracting them into formal models. In the employing phase, students applied mathematical concepts using GeoGebra, such as transformation techniques (translation, reflection, rotation), to explore and analyze the patterns. Finally, during the interpreting phase, students connected their mathematical findings back to the cultural context, demonstrating an ability to explain their results' significance mathematically and culturally. The figure below presents a more detailed analysis of students' mathematical literacy based on individual indicators.

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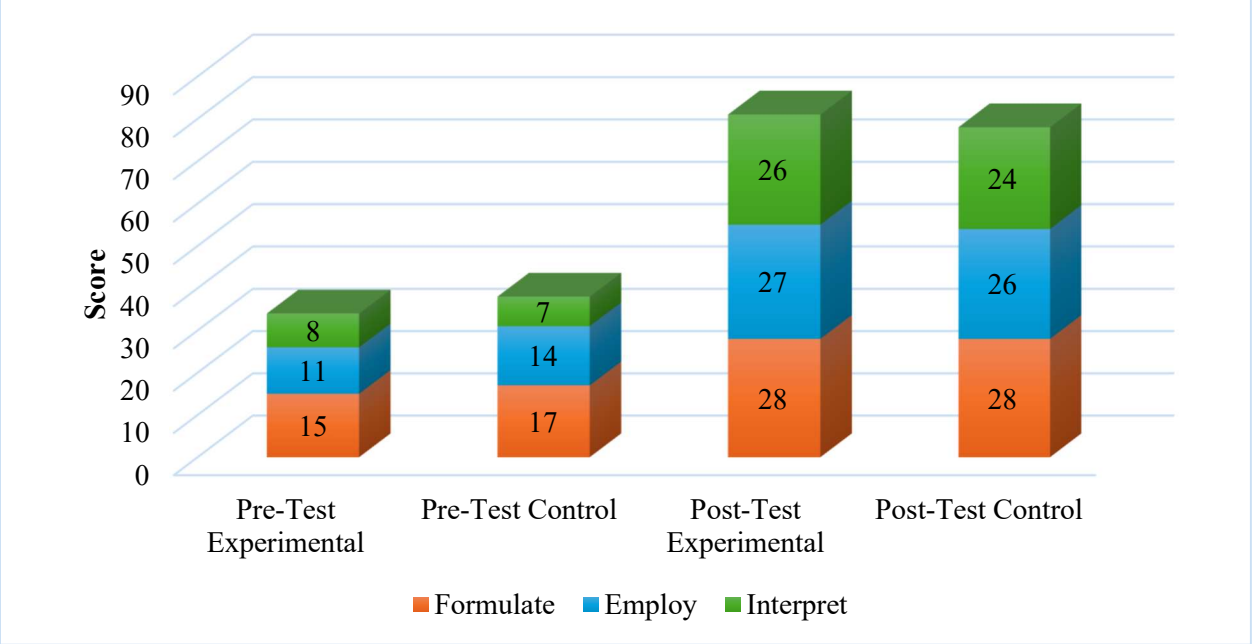


Figure 1. Average Scores of Students' Mathematical Literacy Skills by Indicator

The figure illustrates students' average scores in three components of mathematical literacy: Formulate, Employ, and Interpret, measured during the pre-test and post-test in both experimental and control groups. During the pre-test, the highest average scores appeared in the *Formulate* component, with 15 in the experimental group and 17 in the control group. The *Employ* scores were moderate (11 in the experimental group and 14 in the control group), while the *Interpret* component showed the lowest averages: 8 and 7, respectively. Overall, these results suggest that both groups had relatively comparable initial abilities, with the control group slightly ahead in *Formulate* and *Employ*, and the experimental group slightly stronger in *Interpret*.

After the intervention, all components showed substantial improvement in both groups, with greater gains in the experimental class. The *Formulate* component increased to 28 in both groups, while the *Employ* and *Interpret* components reached 27 and 26 in the experimental group, respectively, compared to 26 and 24 in the control group. Although *Formulate* improved equally, the experimental group demonstrated greater gains in *Employ* and *Interpret*. This finding suggests that the instructional approach applied in the experimental group was particularly effective in enhancing students' higher-order thinking skills compared to the conventional method used in the control group.

Ethnomathematics offers a meaningful bridge between students' cultural heritage and formal mathematics instruction. Students can interpret mathematical content through familiar representations by incorporating traditional patterns and local cultural contexts into learning activities. This relevance enriches the learning experience and affirms students' identities, fostering a stronger

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connection between their culture and mathematical knowledge. Using culturally rooted problems in the project-based model supports contextual Learning and encourages deeper engagement with mathematical concepts.

The strength of the Ethno-PjBL model lies in its integration of cultural context (ethnomathematics) with a project-based learning approach, encouraging students to construct their understanding actively. During the planning phase, students design projects based on contextual problems related to local culture, fostering strong initial engagement. This phase significantly activates prior knowledge and links mathematical concepts to real-life experiences (Sriraman & English, 2010). In the action phase, students engage in exploratory activities using GeoGebra, which enables dynamic visualization of mathematical concepts. GeoGebra increases students' engagement in Learning and helps them construct more accurate mathematical representations. This observation aligns with findings from previous studies, which emphasize that integrating technology into mathematics education strengthens mathematical literacy and deepens students' understanding of fundamental concepts (Zbiek et al., 2007).

The reporting phase in the Ethno-PjBL model encourages students to communicate their project results through presentations or written reports. This activity corresponds with the communication and reflection components of mathematical literacy outlined by OECD (2019a), which states that mathematically literate students can express mathematical ideas in various forms. Students in the experimental group improved their abilities, as reflected in their higher average post-test ranks compared to the control group.

Furthermore, the reward phase is positive reinforcement, fostering students' intrinsic motivation. Social constructivist learning theory posits that meaningful Learning occurs through social interaction, collaboration, and reinforcement (Vygotsky, 1978). Recognizing students' project outcomes makes learning more enjoyable and meaningful, enhancing engagement and achievement. These findings are consistent with prior research by Nugraheni et al. (2021), which showed that project-based Learning with cultural contexts can improve students' literacy skills and critical thinking abilities. Similarly, Surya & Sabandar (2020) found that project-based Learning combined with digital technology such as GeoGebra effectively enhances mathematical achievement and problem-solving skills.

GeoGebra provides students dynamic visual tools that transform abstract mathematical ideas into interactive experiences. In this study, students used GeoGebra to explore geometric transformations such as translation, reflection, and rotation embedded in traditional motifs. These visualizations facilitated a better understanding of spatial relationships and transformation properties. The real-time manipulation of objects in GeoGebra allows students to observe mathematical patterns, test hypotheses, and draw conclusions independently—skills essential for developing mathematical literacy.

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The course included five student projects in which students created Karo-inspired batik motifs using GeoGebra. Project 1 focused on translation, enabling students to produce repeated geometric patterns without altering shape or size. Project 2 applied reflection to create symmetrical designs across vertical and horizontal axes. Project 3 involved rotation, arranging motifs around fixed points to explore orientation changes. Project 4 utilized dilation and varying motif sizes while maintaining proportions. Project 5 integrated all four transformations into a comprehensive final motif reflecting cultural aesthetics and geometric principles. These projects allowed students to formulate mathematical problems, employ GeoGebra tools, and interpret solutions in real-world and cultural contexts, fostering all three dimensions of mathematical literacy while enhancing motivation, engagement, and appreciation of local heritage.

Below is an example of a student project developed using the Ethno-Project-Based Learning (Ethno-PjBL) model supported by GeoGebra. The project integrates cultural heritage into mathematics education by applying geometric transformation concepts to traditional motifs. By leveraging GeoGebra's dynamic tools, students actively investigated and visualized mathematical ideas, such as translation and reflection, within culturally significant contexts. The study classifies these activities according to the three core dimensions of mathematical literacy defined by the OECD (2019a): *formulating* (selecting and modeling geometric motifs from the Siwaluh Jabu house, such as triangles, rhombuses, and zigzag patterns), *employing* (applying translation and reflection to replicate patterns while preserving shape, size, and orientation), and interpreting (analyzing and explaining the results within cultural contexts, understanding the connection between mathematics and heritage). This approach strengthens students' understanding of geometric principles and their connection to local culture, making learning more engaging and meaningful.

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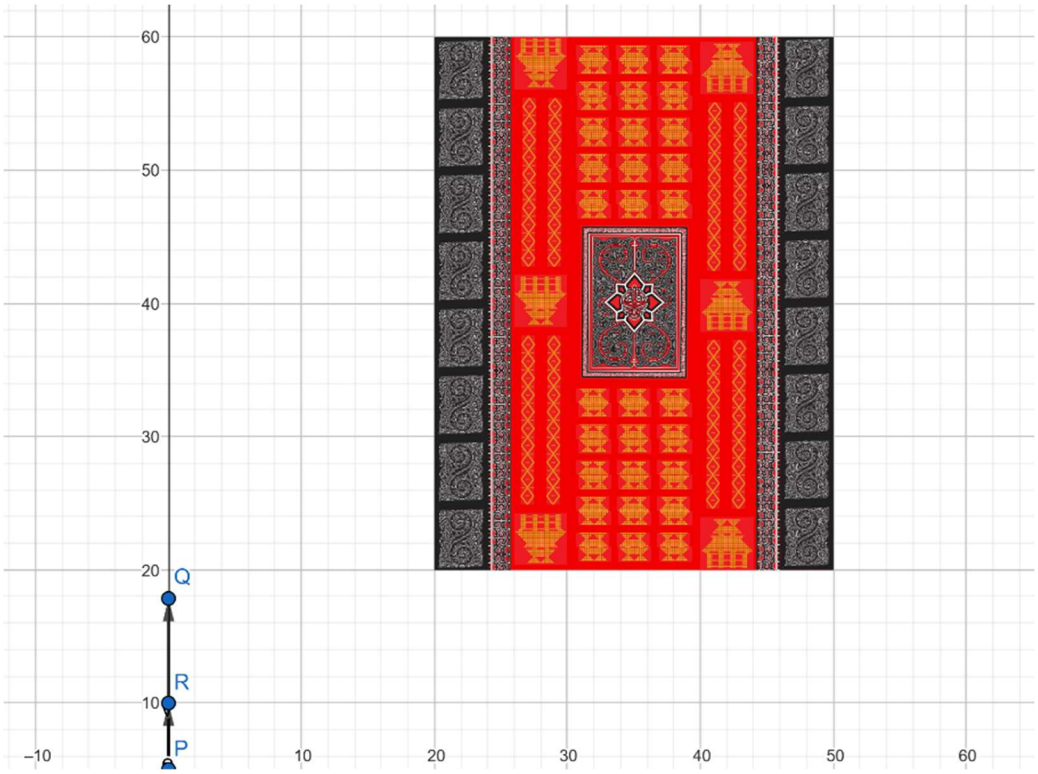


Figure 2: Student Project Output: Applying GeoGebra to Translation and Reflection

The student project illustrates how GeoGebra represents traditional architectural elements of the Karo ethnic group's Siwaluh Jabu house through geometric transformations. Students selected geometric motifs found on the house—such as triangles, rhombuses, and zigzag patterns—as the basis for applying the concepts of translation and reflection. This project demonstrates that mathematics instruction should not be separated from culture but can integrate meaningfully with rich, contextual local values.

In their diagrams, students modeled decorative elements located on the walls and roof frames of the Siwaluh Jabu, which exhibit horizontally repeating patterns. They recreated these patterns in GeoGebra and applied the concept of translation by shifting the motifs along specific vectors. Students demonstrated that the same motif could repeat consistently through translation without altering its shape, size, or orientation. This approach reinforced their understanding of translation as an isometric transformation that preserves the essential properties of geometric figures.

Beyond translation, students also applied reflection to specific symmetrical components of the traditional house, particularly the ornaments on the left and right sides of the structure. They performed reflections across vertical and horizontal axes, which correspond with the architectural symmetry of the building. Students used GeoGebra's tools to illustrate how a pattern on one side

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of the house reflects to produce a mirror image on the opposite side, providing a clear visual understanding of geometric reflection. This project deepened students' comprehension of geometric transformations and enriched their learning experience by incorporating a culturally responsive approach. When students explored cultural artifacts like the Siwaluh Jabu house, they viewed mathematics as an integral part of real life. The relevance of the content to their cultural background enhanced their motivation and engagement, highlighting how contextual and culture-based learning bridges academic concepts and local identity.

Overall, the student-generated diagrams provide compelling evidence of how integrating GeoGebra with cultural objects such as the Siwaluh Jabu house can enhance the quality of mathematics education. Students gained technical mastery of geometric concepts like translation and reflection and a meaningful understanding of their real-world applications. Moreover, the project fostered cultural awareness and appreciation, helping students connect mathematics with their heritage. Such an approach positions mathematics as not an abstract discipline but a dynamic and relevant body of knowledge.

Integrating cultural elements with digital tools enhanced students' intrinsic motivation. Students reported that representing their cultural heritage within mathematical tasks made the learning experience more meaningful and personally relevant. Interactive technology empowered students to take greater control of their learning, which increased their engagement and sustained interest. The Ethno-Project-Based Learning (Ethno-PjBL) model promoted collaboration through culturally grounded projects, making students feel more connected to the content and motivated to persist when solving complex problems. These results indicate that the model effectively fosters deeper learning and sustained motivation.

Teachers observed a marked increase in student enthusiasm and active participation during lessons that utilized the Ethno-PjBL approach. They noted improved student communication and cooperation, contributing to a positive classroom environment conducive to learning. Furthermore, teachers recognized that incorporating local cultural content helped demystify abstract mathematical concepts, making them more accessible and relevant to students.

Implementing the Ethno-PjBL model supported by GeoGebra significantly improved students' mathematical literacy. Beyond cognitive gains, this approach also enhanced student motivation and interest and teacher engagement with the instructional process. This study demonstrates the potential of culturally responsive pedagogy and digital tools to improve learning outcomes and learner motivation in mathematics education.

Overall, this study demonstrates that the Ethno-Project-Based Learning (Ethno-PjBL) model supported by GeoGebra effectively enhances students' mathematical literacy in a contextual and

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meaningful manner. The model integrates culture, technology, and collaboration within a structured learning sequence grounded in constructivist and sociocultural theories, providing an innovative approach for 21st-century mathematics education. Students formulate mathematical problems, employ appropriate strategies and tools such as GeoGebra, and interpret solutions in real-world and cultural contexts. Feedback from students and teachers indicated that integrating Karo cultural motifs with interactive projects increased interest, motivation, and engagement, supporting the meaningfulness of the learning activities. By fostering all three dimensions of mathematical literacy defined by the OECD (2019a): formulating, employing, and interpreting, this study positions its findings clearly within international mathematics education standards. It emphasizes their relevance and potential for broader application.

CONCLUSIONS

The data analysis and discussion confirm that implementing the Ethno-Project-Based Learning (Ethno-PjBL) model assisted by GeoGebra significantly enhances students' mathematical literacy skills. The Mann–Whitney U test supports this conclusion, yielding a significance value of 0.015 (< 0.05), which indicates a statistically significant difference in learning outcomes between students taught with the Ethno-PjBL model using GeoGebra and those who received conventional instruction. The higher mean rank of the experimental group suggests that this approach effectively encourages students to understand better, interpret, and apply mathematical concepts in real-life contexts. The phases of the Ethno-PjBL model—plan, action, report, and reward—have proven effective in fostering a contextual, meaningful, and participatory learning process.

These findings encourage educators to actively integrate the Ethno-PjBL model supported by GeoGebra into mathematics teaching. This model enhances mathematical literacy and enriches students' learning experiences by blending local culture with interactive technology. For schools and education policymakers, this study serves as a valuable reference for developing instructional policies that prioritize project-based, digital, and culturally relevant learning strategies. Moreover, this research paves the way for further advancements in technology-enhanced and ethnomathematics-informed mathematics education.

However, this study has certain limitations. It was conducted only in schools with excellent accreditation status, which may limit the generalizability of the findings to institutions with different educational conditions. Additionally, the research focused primarily on the cognitive domain—specifically mathematical literacy—without addressing students' affective and social aspects, which are also crucial in project-based Learning. Future studies are encouraged to involve schools with varying accreditation levels and examine the broader impact of the Ethno-PjBL model on other learning dimensions, such as collaboration, creativity, and student attitudes.

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Beyond its impact on mathematical literacy scores, the Ethno-PjBL model with GeoGebra enriched students' learning experiences. Combining cultural elements and technology fostered a deeper engagement, strengthened students' conceptual understanding, and promoted a stronger connection between mathematics and students' identities. These findings highlight the potential of culturally responsive, technology-supported learning models for 21st-century mathematics education.

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REFERENCES

[1] Arikunto, S. (2013). *Prosedur Penelitian: Suatu Pendekatan Praktik* (Revisi). Rineka Cipta.

[2] Awaji, B. M., Khalil, I., & Al-Zahrani, A. (2025). A Bibliometrics Study of Two Decades of GeoGebra Research in Mathematics Education. *Journal of Educational and Social Research*, *15*(1), 130–140. https://doi.org/10.36941/jesr-2025-0011

[3] Bell, S. (2010). Project-Based Learning for the 21st Century: Skills for the Future. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *83*(2), 39–43. https://doi.org/10.1080/00098650903505415

[4] Creswell, J. W. (2012). *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research* (4th ed.). Pearson Education.

[5] D'Ambrosio, U. (1999). Literacy, Matheracy, and Technoracy: A Trivium for Today. *Mathematical Thinking and Learning*, *1*(2), 131–153. https://doi.org/10.1207/s15327833mtl0102_2

[6] D'Ambrosio, U. (2001). Ethnomathematics and its Place in The History and Pedagogy of Mathematics. *For the Learning of Mathematics*, *21*(1), 44–48.

[7] D'Ambrosio, U. (2006). *Ethnomathematics: Link Between Traditions and Modernity*. Sense Publishers.

[8] Field, A. (2018). *Discovering Statistics Using IBM SPSS Statistics* (5th ed.). Sage Publications.

[9] Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2012). *How to Design and Evaluate Research in Education* (8th ed.). McGraw-Hill Education.

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[10] Ghasemi, A., & Zahediasl, S. (2012). Normality Tests for Statistical Analysis: A Guide for Non-Statisticians. *International Journal of Endocrinology and Metabolism*, *10*(2), 486–489. https://doi.org/10.5812/ijem.3505

[11] Hake, R. R. (1998). Interactive-Engagement Versus Traditional Methods: A Six-Thousand-Student Survey of Mechanics Test Data for Introductory Physics Courses. *American Journal of Physics*, *66*(1), 64–74. https://doi.org/10.1119/1.18809

[12] Hamdiyanti, M., Rodiana, I., Laelasari, L., & Subroto, T. (2023). Systematic Literature Review: Mathematical Literacy Skills in Terms of Mathematics Learning Motivation. *International Journal of Chemistry Education Research*, *8*(2), 34725. https://doi.org/10.20885/ijcer.vol8.iss2.art3

[13] Holm, M. (2011). Project-Based Instruction: A Review of the Literature on Effectiveness in Prekindergarten through 12th Grade Classrooms. *River Academic Journal*, *7*(2), 1–13.

[14] Imswatama, A. (2018). The Effectiveness of Mathematics Teaching Material Based on Ethnomathematics. *International Journal of Trends in Mathematics Education Research*, *1*(1), 35–38. https://doi.org/10.33122/ijtmer.v1i1.11

[15] Jhangiani, R. S., Chiang, I. C. A., Cuttler, C., & Leighton, D. C. (2019). *Research Methods in Psychology* (4th ed.). KPU Press. https://kpu.pressbooks.pub/psychmethods4e/part/quasi-experimental-research/

[16] Joyce, B., Weil, M., & Calhoun, E. (2009). *Models of Teaching* (8th ed.). Pearson Education.

[17] Kovács, Z., Recio, T., Botana, F., & Hohenwarter, M. (2023). Some Reflections About the Success and Bibliographic Impact of The Dynamic Geometry System GeoGebra. *Mathematics in Computer Science*, *17*(2), 253–271. https://doi.org/10.1007/s11786-023-00564-9

[18] Krajcik, J. S., & Blumenfeld, P. C. (2006). Project-Based Learning. *The Cambridge Handbook of the Learning Sciences*, 317–334. https://doi.org/10.1017/CBO9780511816833.020

[19] Larmer, J., Mergendoller, J. R., & Boss, S. (2015). *Setting the Standard for Project Based Learning: A Proven Approach to Rigorous Classroom Instruction*. ASCD.

[20] Larson-Hall, J. (2016). *A Guide to Doing Statistics in Second Language Research Using SPSS and R* (2nd ed.). Routledge.

[21] Lohr, S. L. (2019). *Sampling: Design and Analysis* (2nd ed.). Chapman and Hall/CRC. https://doi.org/10.1201/9780429296284

[22] Maulina, S., Junaidi, J., Nufus, M., & Maulida, N. R. (2024). Students' Mathematical Literacy through Ethnomathematics-Based Learning. *Jurnal Didaktik Matematika*, *11*(1), 59–74. https://doi.org/10.24815/jdm.v11i1.35676

[23] Maysarah, S. (2015). *Peningkatan Kemampuan Komunikasi Matematik dan Kemampuan Kreativitas Matematik Siswa Melalui Model Project Based Learning Berbantuan Ms.Excel di Kelas XI SMA Asy-Syafi'iyah Internasional Medan*. Program Pascasarjana Program Studi Pendidikan Matematika Universitas Negeri Medan.

[24] Maysarah, S., Saragih, S., & Napitupulu, E. (2023). Peningkatan Kemampuan Literasi Matematik dengan Menggunakan Model Project-Based Learning. *AKSIOMA: Jurnal*

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Program Studi Pendidikan Matematika, *12*(1), 1536. https://doi.org/10.24127/ajpm.v12i1.6627

[25] Muhaimin, L. H., Sholikhakh, R. A., Yulianti, S., Ardani, A. H., & Sahara, S. (2024). Unlocking the Secrets of Students' Mathematical Literacy to Solve Mathematical Problems: A Systematic Literature Review. *EURASIA Journal of Mathematics, Science and Technology Education*, *20*(4), em2428. https://doi.org/10.29333/ejmste/14404

[26] Mulenga, E. M., & Folokwe, K. (2024). Exploring Indigenous Teaching Approaches For Geometry Instruction in Three Secondary Schools Within Zambia. *Mathematics Teaching-Research Journal*, *16*(5), 153–192. https://mtrj.commons.gc.cuny.edu/volume-16-n-5/

[27] Nugraheni, F., Putri, R. I. I., & Fajariska, A. (2021). Ethnomathematics in Project-Based Learning to Enhance Students' Mathematical Literacy and Critical Thinking. *Journal of Physics: Conference Series*, *1776*(1), 012028. https://doi.org/10.1088/1742-6596/1776/1/012028

[28] Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric Theory* (3rd ed.). McGraw-Hill.

[29] Nur, A. S., Waluya, S. B., Kartono, & Rochmad. (2023). Ethnomathematics Perspective and Challenge as A Tool of Mathematical Contextual Learning For Indigenous People. *International Journal on Emerging Mathematics Education*, *5*(1), 48–58. https://doi.org/10.12928/ijeme.v5i1.17072

[30] Nurmasari, L., Budiyono, J., & Ramli, M. (2023). *Mathematical Literacy in Primary Schools: A Systematic Literature Review*. *2540*(1), 070011. https://doi.org/10.1063/5.0105855

[31] OECD. (2018). PISA 2018 Assessment and Analytical Framework: Mathematics, Reading, Science and Global Competence. *OECD Publishing*. https://doi.org/10.1787/b25efab8-en

[32] OECD. (2019a). PISA 2018 Assessment and Analytical Framework: Mathematics, Reading, Science and Financial Literacy. *OECD Publishing*. https://doi.org/10.1787/b25efab8-en

[33] OECD. (2019b). PISA 2018 Results (Volume I): What Students Know and Can Do. *OECD Publishing*. https://doi.org/10.1787/5f07c754-en

[34] OECD. (2023). PISA 2022 Assessment and Analytical Framework: Mathematics, Reading, Science and Creative Thinking. *OECD Publishing*. https://doi.org/10.1787/dfe0bf9c-en

[35] OECD. (2023a). PISA 2022 Results (Volume I): The State of Learning and Equity in Education. *OECD Publishing*. https://doi.org/10.1787/53f23881-en

[36] OECD. (2023b). PISA 2022 Results (Volume II): Learning During – and From – Disruption. *OECD Publishing*. https://doi.org/10.1787/a97db61c-en

[37] Pallant, J. (2020). *PSS Survival Manual: A Step by Step Guide to Data Analysis Using IBM SPSS* (7th ed.). Routledge.

[38] Powell, A. B., & Frankenstein, M. (1997). *Ethnomathematics: Challenging Eurocentrism in Mathematics Education*. SUNY Press.

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[39] Rosa, M., & Orey, D. C. (2016). Ethnomathematics and its Pedagogical Action: A cultural Perspective on Mathematics Education. *Springer*.
[40] Saragih, S. (2015). *Aplikasi SPSS dalam Statistik Penelitian Pendidikan*. Perdana Publishing.
[41] Sari, A., Putri, R. I., Zulkardi, & Prahmana, R. C. I. (2024). Ethnomathematics in Indonesian Woven Fabric: The Promising Context in Learning Geometry. *Mathematics Teaching-Research Journal*, *16*(5), 157–185. https://mtrj.commons.gc.cuny.edu/volume-16-n-5/
[42] Serin, H. (2023). The Significance of Mathematical Literacy in Today's Society. *International Journal of Social Sciences & Educational Studies*, *10*(2), 396–402. https://doi.org/10.23918/ijsses.v10i2p396
[43] Sitopu, J. W., Khairani, M., Roza, M., Judijanto, L., & Aslan, A. (2023). The Importance of Integrating Mathematical Literacy in The Primary Education Curriculum: A Literature Review. *International Journal of Teaching and Learning*, *12*(54). https://injotel.org/index.php/12/article/view/54
[44] Sriraman, B., & English, L. D. (2010). *Theories of Mathematical Learning*. In C. B. Linder.
[45] Surya, E., & Sabandar, J. (2020). GeoGebra-Assisted Project-Based Learning to Improve Mathematical Problem-Solving Ability. *International Journal of Instruction*, *13*(1), 365–376. https://doi.org/10.29333/iji.2020.13124a
[46] Thomas, J. W. (2000). *A Review of Research on Project-Based Learning*. Autodesk Foundation.
[47] Triet, L. V. M., Thanh, N. N., & Loc, N. P. (2024). Effect of GeoGebra–Supported 5E Learning Model on Students' Understanding of The Area of A Trapezium: A Quasi-Experimental Study. *Mathematics Teaching-Research Journal*, *16*(6), 190–213. https://mtrj.commons.gc.cuny.edu/volume-16-n-6/
[48] Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press.
[49] Wijaya, T. T., Hidayat, W., Hermita, N., & Alim, J. A. (2024). Exploring Contributing Factors to PISA 2022 Mathematics Achievement: Insights from Indonesian Teachers. *Infinity Journal*, *13*(1), 139–156. https://doi.org/10.22460/infinity.v13i1.p139-156
[50] Wulandari, G. A. P. A., Payadnya, I. P. A., Puspadewi, K. R., & Saelee, S. (2024). The Role of Ethnomathematics in South-East Asian Learning: A Perspective of Indonesian and Thailand Educators. *Mathematics Teaching-Research Journal*, *16*(3), 101–120. https://mtrj.commons.gc.cuny.edu/volume-16-n-3/
[51] Yusuf, Y., Nurfitriani, Y., & Koswara, U. (2024). Mathematical Literacy Ability of Students from A Cognitive Style Perspective on Rational Numbers. *EduMatSains: Jurnal Pendidikan, Matematika Dan Sains*, *9*(1), 319–331. https://doi.org/10.33541/edumatsains.v9i1.5985
[52] Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. (2007). *Research on Technology in Mathematics Education: A Perspective of Constructs. In F. K. Lester Jr. (Ed.), Second*

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Handbook of Research on Mathematics Teaching and Learning. Information Age Publishing.

[53] Zengin, Y., Furkan, H., & Kutluca, T. (2012). The effect of dynamic mathematics software geogebra on student achievement in teaching of trigonometry. *Procedia - Social and Behavioral Sciences*, *31*, 183–187. https://doi.org/10.1016/j.sbspro.2011.12.038

APPENDIX

Pre-Test and Post-Test Items on Mathematical Literacy Ability

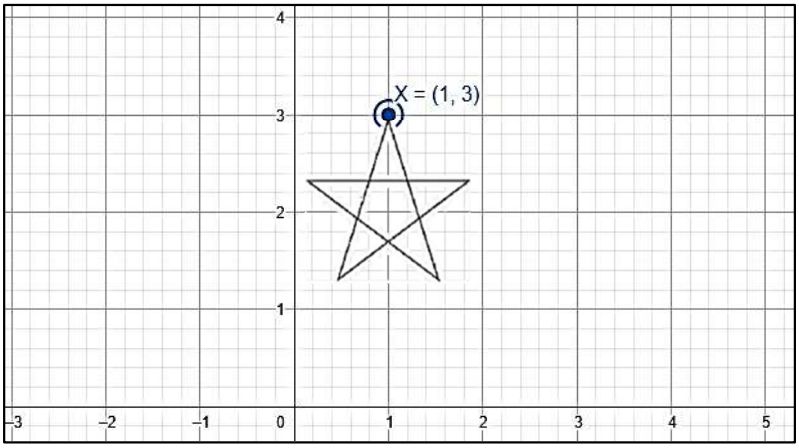
Grade	: 11th Grade Senior High School
Time Allocation	: 90 minutes
Topic	: Geometric Transformations

Instructions:

a. Fill in personal information correctly on the provided answer sheet.
b. Solve each question using complete and detailed steps on the answer sheet provided.
c. Write answers clearly and neatly using a pen.
d. Do not access notebooks or other learning resources during the test.
e. Collaboration or discussion with other students is strictly prohibited.

---

1. A batik artisan intends to create a uis fabric with a "tupak salah silima-lima" motif, with one of the peak points X located on the Cartesian coordinate system, as shown in the figure below.



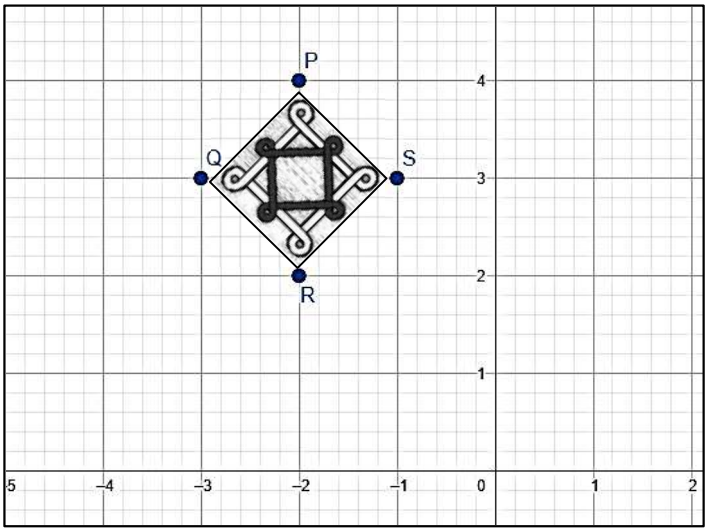
Each subsequent motif element is positioned three units to the right and four units upward from the previous one.

a. Determine the coordinates of the third peak point starting from the initial coordinate.

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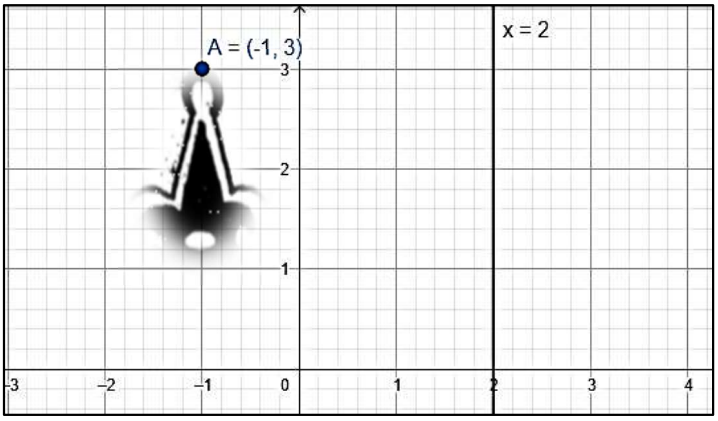
b. If the initial peak point X (1, 3) is translated three units to the left and four units downward, where will the third peak point be located from the new starting coordinate?
c. Suppose the artisan wants to create the motif on a single piece of fabric measuring 2 meters in length and 1 meter in width. How many motifs can the artisan produce at most, assuming one unit equals one decimeter?

2. Observe the following figure! Assume that a segment of carving art is taken from the Tapak Raja Sulaiman motif, as shown below.



The second motif is the result of a reflection over the y-axis
a. Determine the coordinates of points P′, Q′, R′, and S′ for the second motif.
b. Reflect the motif (from the original coordinate points) over the x-axis, and determine the coordinates of points P′, Q′, R′, and S′ for the second motif.
c. What are the coordinates of P″, Q″, R″, and S″ if the next motif is the result of a reflection over the y-axis followed by a reflection over the x-axis?

3. A batik artisan intends to design a uis fabric featuring the Teger Tudung motif, with one

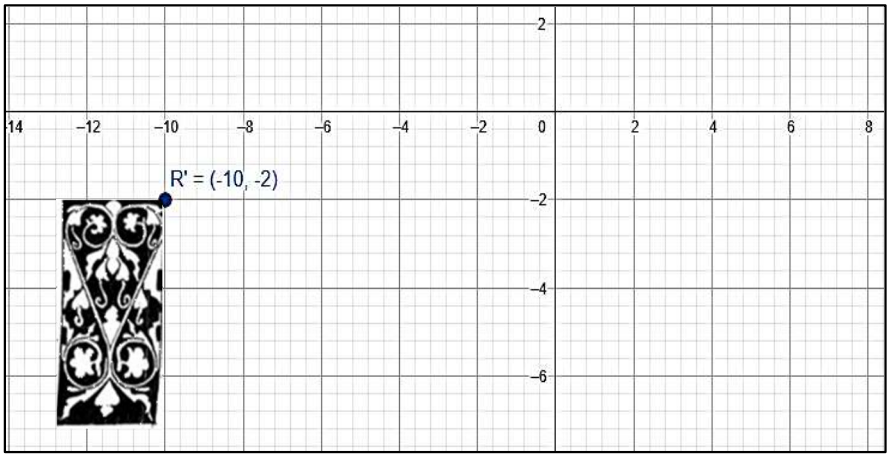


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of the motif's peak points located at point A, as shown in the figure below.

a. If the point is reflected across the line x = 2 to form the uis fabric pattern with the Teger Tudung motif, what is the image of point A (denoted as A′)?
b. If the image of point A is then reflected across the line x = –2 to create the following Teger Tudung motif pattern, what is the image of point A′ (denoted as A″)?
c. If the images A′ and A″ are then translated two units to the right and two units upward, and this translation is repeated three times, determine the resulting image of each point at every step. If a batik artisan uses the resulting images to determine the placement of the motifs on the fabric, illustrate the motif placement on the Cartesian coordinate plane.

4. A batik artisan plans to design a uis fabric pattern featuring the Puncak Tenggiang motif by rotating point R(x, y) about the origin O(0, 0) by 90° counterclockwise, as described below:



a. If the result of the rotation is R′(–10, –2), determine the original coordinates of point R in the first pattern that a batik artisan needs to create.
b. Determine the coordinates of the second and third motifs generated from the original point R in the first pattern through successive 90° counterclockwise rotations.
c. Determine the images of each point R, R′, R″, and R‴ from items (a) and (b) after being dilated concerning the origin O(0, 0) using a scale factor of 2. If a batik artisan uses these resulting images to decide the placement of the motifs on the fabric, illustrate the motif positions on the Cartesian coordinate plane.

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Appendix

Student Response Sheet After Participating in Learning Activities Using The Project-Based Learning Model with The Ethnomathematics Approach Assisted By GeoGebra

Student Name: _____
Class : _____
School name : _____

Objective

Purpose of using the questionnaire sheet: This is to collect response data from students after following the activity learning using a project-based learning model with an ethnomathematics-assisted GeoGebra in context system, kinship, bronze rakut sitelu at the wedding customs of the Karo people.

Instruction

Participants filled out the questionnaire as follows:

1) Provide a check mark (√) in the column that you consider appropriate to the existing assessment aspects, and
2) If necessary, provide constructive comments /suggestions.

Assessment aspect criteria:

Score 4 = Strongly Agree/Very Interesting /Very Happy /Very Understandable /Very Helpful /Very Training
Score 3 = Agree/ Interesting / Enjoyable / Understood / Helpful / Training
Score 2 = Less Agree / Less Interesting / Less Enjoyable / Less Understandable / Less Helpful / Less Training
Score 1 = Disagree/Not Interesting /Not Understandable /Not Helpful /Not Training

No.	Rated aspect	Rating Scale			
		1	2	3	4
1	**Interest in the Learning Process**				
	1. The study emphasizes this learning approach due to the selective use of GeoGebra software when required.				

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No.	Rated aspect	Rating Scale			
		1	2	3	4
	2. This learning approach is appealing because it allows students to study the material in advance using GeoGebra software, before the teacher discusses it in class.				
	3. This learning approach attracts interest because it allows students to study in groups customized to their ability level in beginning mathematics, which increases their confidence.				
	4. This learning approach is appealing because it allows students to express their opinions when solving informal group problems.				
	5. This learning approach allows students to reflect independently and apply appreciation-based cultural practices, including Karo traditional cloth and carvings, to address informal problems effectively.				
2	**Motivation Towards Learning**				
	1. The present study focuses on activity-based learning that incorporates cultural context.				
	2. This learning approach is appealing because the teacher facilitates learning through an effective system application.				
	3. This learning approach encourages students to assist peers with difficulties understanding and solving informal problems.				
3	**Perceived Usefulness**				
	1. The project-based learning model, incorporating an ethnomathematics approach assisted by GeoGebra, helps students enhance their understanding of the material by connecting geometric concepts to everyday life.				
	2. Informal problems in the material transformation geometry are easy to imagine and solve.				
	3. The learning activities enable students to understand and appreciate their cultural values, making the material relevant to everyday life.				
	4. Through these learning activities, students engage in self-directed discussion groups, which foster the development of positive attitudes, empathy, and cooperation with peers.				

Notes:
Completing this questionnaire does not affect grades. Respond honestly, reflecting personal opinions. Write any comments or suggestions about the implemented learning activities in the space below.

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Karo, …….
Respondents

(_____)

Appendix

Teacher Response Sheet After Participating in Learning Activities Using The Project-Based Learning Model with The Ethnomathematics Approach Assisted By GeoGebra

Teacher Name : _____
School name : _____

Objective
Purpose of using the questionnaire sheet: This is to collect teacher response data after following the activity learning using a project-based learning model with an ethnomathematics-assisted GeoGebra in context system, kinship, bronze rakut, sitelu, at the wedding customs of the Karo people.

Instruction
Participants filled out the questionnaire as follows:
1) Provide a check mark (√) in the column that you consider appropriate to the existing assessment aspects, and
2) If necessary, provide constructive comments /suggestions.

Assessment aspect criteria:
Score 4 = Strongly Agree/Very Interesting /Very Happy /Very Understandable /Very Helpful /Very Training
Score 3 = Agree/ Interesting / Enjoyable / Understood / Helpful / Training
Score 2 = Less Agree / Less Interesting / Less Enjoyable / Less Understandable / Less Helpful / Less Training
Score 1 = Disagree/Not Interesting /Not Understandable /Not Helpful /Not Training

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No.	Rated aspect	Rating Scale			
		1	2	3	4
1.	**Interest in the Learning Process**				
	1. This learning approach engages students by having them use GeoGebra software only when necessary.				
	2. This learning approach is appealing because it allows students to study the material in advance through GeoGebra software, before the teacher discusses it in class.				
	3. This learning approach is appealing because students work in groups customized to their ability level in beginning mathematics, which enhances their confidence.				
	4. This learning approach is appealing because it allows students to express their opinions and collaboratively solve informal group problems.				
	5. This learning approach is appealing because it allows students to reflect independently and engage in appreciation-based cultural practices, such as using Karo traditional cloth (UIS) and carvings as a form of gratitude, effectively addressing informal problems.				
2	**Motivation Towards Learning**				
	1. This learning approach is appealing because it incorporates the cultural context of students.				
	2. This learning approach is appealing because it facilitates learning through the user-friendly GeoGebra software.				
	3. This learning approach encourages students to assist peers with difficulties understanding and solving informal problems.				
3	**Perceived Usefulness**				
	1. The project-based learning model, incorporating an ethnomathematics approach assisted by GeoGebra, helps students enhance their understanding of the material by connecting geometric concepts to daily life.				
	2. Informal problems in the material transformation geometry are easy for students to imagine and solve.				
	3. Student learning activities help students understand the values close to daily life.				
	4. Activity student learning follows practice student. Believe in active discussion groups and improve attitude, concern, and cooperation with colleagues and friends.				

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Notes:
If there are any comments or suggestions regarding the implemented learning activities, please write them in the space provided below.

_____ _____ _____

Karo, …….
Respondents

(_____)

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Integration of Real-Life Scenarios in Mathematical Literacy Learning in Middle Schools in Parepare, Indonesia

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Abstract: The study investigates the integration of real-life scenarios into mathematical literacy education in the Middle school in Parepare, Indonesia. The study's objectives are to examine the effectiveness of integrating real-life scenarios into mathematical literacy learning at middle schools and evaluate the students' activeness during the interventions. Using one group pre-test post-test design, this study involved 32 students who participated in pre-tests, interventions, and post-test. Two instruments were used: a test to measure mathematical literacy and an observation sheet to assess the students' activeness during the interventions. Descriptive and inferential data analysis were applied to test a two-tailed hypothesis set at alpha level 0.05. The results showed a significant improvement in students' mathematical literacy scores from pre-test to post-test. Pair t-test analysis confirmed the effectiveness of this pedagogical approach, highlighting its potential to overcome gaps in traditional mathematics teaching. Based on the observation sheet, this study found that students have a high level of active participation, demonstrating their engagement and interest in the subject, and a good ability to use representations. The findings support constructivist theory, emphasize the importance of contextual learning, and provide empirical evidence for educators and policymakers to develop more effective, inclusive, and engaging math curricula.

Keywords: Mathematics Education, Mathematical Literacy, Pedagogical Approach, Real Life Scenarios

INTRODUCTION

Mathematical literacy is an essential competency that every student in middle school must have to make sure they can understand, apply, and communicate mathematical concepts in various contexts of everyday life. This ability is important for academic achievement and for the development of creative thinking and problem-solving skills needed in professional and personal life (Parker et al., 2020). However, various studies show that the mathematical literacy of middle school students in many countries, including Indonesia, is still alarmingly low (Powell et al., 2021). Therefore,

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there is an urgent need to implement learning strategies that are not only effective but also innovative in order to improve students' mathematical literacy skills (Komarudin et al., 2024). These strategies must accommodate students' various learning styles, motivate them to learn mathematics more meaningfully, and increase their engagement in the learning process.

Integration of real-life scenarios in mathematical literacy learning in middle school is one of the innovative approaches that has been proven effective in improving mathematical literacy. Integration of real-life scenarios into the learning process is an approach that allows students to see the relevance and real-life applications of the mathematical concepts they learn in class. Through real-life scenarios, students can understand how mathematics is used in everyday situations, such as financial planning, measurement of household chores, and data analysis in simple research (Wisenöcker et al., 2024). The use of real-life contexts not only makes mathematics learning more interesting and meaningful for students and helps them develop better creative and innovative thinking skills (Carl et al., 2023; Putri et al., 2023). Thus, integrating of real-life scenarios in mathematical literacy learning in middle school is a very important strategy to consider in an effort to improve the quality of mathematics education and prepare students for future challenges.

Indonesian students' low mathematical literacy ability is a serious problem that needs special attention in the world of education. Based on the results of several studies, Indonesian students often need help understanding basic mathematical concepts, which results in difficulties in applying mathematical knowledge to real situations (Johar et al., 2021). This lack of understanding is further exacerbated by ineffective teaching methods that continue to rely on conventional approaches and fail to actively engage students in the learning process. In addition, the use of digital media in mathematics learning is still very limited, even though this media has great potential to increase student interactivity and involvement in learning (Hillmayr et al., 2020). As a result of these limitations in instructional practices and the underutilization of interactive and contextual learning tools, students not only have difficulty in understanding mathematical concepts, but are also less motivated to learn mathematics meaningfully. This study aims to address these problems by developing and implementing innovative and effective learning strategies, including the integration of digital media, to improve students' mathematical literacy in middle schools.

Research on mathematical literacy among junior high school students shows that classroom activities often lack authentic real-world contexts, which affects students' ability to understand and apply mathematical concepts effectively (Abaño et al., 2024; Agustina et al., 2021; Uegatani et al., 2024; Uyen et al., 2021). Although many studies have been conducted to observe classroom activities and conduct interviews with students and teachers, these studies tend to be limited to descriptive approaches and do not sufficiently explore how the integration of real-life scenarios can improve students' understanding and application of mathematical concepts. For example, a study by Costa (Costa et al., 2022) highlighted the importance of professional development for teachers but did not focus on observing the integration of real-life scenarios in improving the application of

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mathematical concepts. The study by Attard and Holmes (Attard & Holmes, 2022) focused on the perception of teachers and students of mixed learning but did not observe the integration of real-life scenarios. Abaño et al. (2024) examined the integration of real-life contexts into mathematical concepts using a questionnaire to measure how well mathematics connects to the real world. Their findings suggest that integrating real-life contexts can improve students' performance. Existing studies focus on surface observations without exploring more deeply the potential of real-world context-based pedagogies that are more interactive and relevant to students. Thus, there is a significant gap in the literature on the effectiveness of integrating real-life scenarios in learning mathematical literacy in junior high schools, underscoring the urgent need for more comprehensive and experimental research in this area.

This study offers a significant new contribution by combining a real-context-based pedagogical approach that has not been widely explored in the mathematics education literature. This study seeks to fill the gap in previous studies that tend to be descriptive and less in-depth regarding the practical application of mathematical concepts through real-life scenarios. By integrating real-world scenarios into learning, this study focuses not only on improving students' conceptual understanding, but also on developing creative and innovative thinking skills that are relevant to everyday life.

This study aims to investigate the effectiveness of integrating real-life scenarios in learning mathematics literacy in middle schools and to explore and analyze how real-life scenarios can motivate students, improve understanding of mathematical concepts, and apply the knowledge in everyday contexts. The questions to be explored in this study are: (1) Is there a significant difference in students' mathematical literacy skills before and after the use of real-life scenarios in classroom activities? and (2) How did the students engage in the real-scenario activities? To answer these questions, the researchers evaluated various aspects of learning, including students' conceptual understanding, and their ability to apply mathematical concepts to real-world situations. This study also used observational data to see students' activities during the learning process and measure improvements in mathematical literacy through standardized tests. With a quantitative approach, this study aims to provide strong empirical evidence on the impact of real-life scenarios on students' mathematical literacy and offer practical recommendations for implementing more effective and relevant learning strategies as well as providing practical recommendations that educators can use to improve the quality of mathematics learning in middle schools in Indonesia. Thus, this study is expected to significantly contribute to educational literature and innovative pedagogical practices, especially in mathematics education at the junior high school level.

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LITERATURE REVIEW

Mathematical literacy is defined as a person's ability to formulate, apply, and interpret mathematics in a variety of real-life contexts (Çakıroğlu et al., 2024). It encompasses essential competencies such as reasoning, representation, communication, and problem-solving (Malik et al., 2023). These skills are not only fundamental for academic success but also critical for everyday decision-making in an increasingly data-driven world. At the junior high school level, fostering mathematical literacy is especially important, as students are at a developmental stage where complex cognitive skills begin to emerge (Malik et al., 2018). In this context, innovative approaches like using visual tools to explain mathematical proofs have been shown to deepen students' conceptual understanding, ensuring the long-term sustainability of mathematics education (Sevcikova et al., 2023; Ulfatun et al., 2023). The use of holography-based visual learning applications in mathematics, particularly for shape and space topics, has also been found to be easy to use and engaging for students (Shiang Tyng et al., 2023).

Integrating real-life scenarios into mathematics education has been shown to enhance students' engagement and academic outcomes. These contextualized learning experiences allow students to see the relevance of mathematics in daily life, which can improve motivation and participation. Simulation-based interprofessional education, for instance, has demonstrated success in helping students build deeper understanding and problem-solving capabilities (Elsayed et al., 2024). Likewise, augmented reality has emerged as a promising technology to enrich learning environments, influencing students' attitudes and achievements positively (Loeb et al., 2023). However, designing authentic and meaningful tasks remains a challenge. As Wisenöcker et al. (Wisenöcker et al., 2024) emphasize, problem-solving strategies can be limited when students fail to connect mathematical problems with real-world logic. Therefore, careful instructional planning is necessary to ensure that real-life tasks are inclusive, realistic, and accessible for all learners.

Several studies have explored the impact of real-life scenario-based learning in mathematics and highlighted both benefits and implementation barriers. Wisenöcker et al. (Wisenöcker et al., 2024) reported that students who are presented with realistic problem situations tend to use more analytical and context-sensitive approaches compared to those who work on decontextualized problems. In line with this, (Thuy & Le Phuoc, 2023) developed a framework for assessing students' statistical reasoning in solving real-life medical problems, highlighting the importance of domain-specific and context-based assessment in mathematical learning. Augmented reality-based simulations in high-risk scenarios also show potential in improving learners' attentiveness and behavioral responses (Loeb et al., 2023). While technological tools like augmented reality can enhance achievement and attitudes, their effect on intrinsic motivation may vary depending on design and context (Cao & Yu, 2023). Moreover, tools that assess student self-regulation—such as combining aptitude and event-based measurements—can be instrumental in understanding and improving academic performance (Zhidkikh et al., 2023). Kirsten & Greefrath (Kirsten & Greefrath, 2024) also notes

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that affective factors and the learning environment for example on-campus versus distance tutorials significantly influence mathematics learning gains, thus reinforcing the importance of designing learning experiences that actively involve students.

In summary, embedding real-life scenarios into mathematical instruction fosters greater student engagement, deeper conceptual understanding, and the development of critical problem-solving skills. Realistic contexts promote authentic problem-solving strategies and enhance students' ability to transfer mathematical knowledge to practical situations (Wisenöcker et al., 2024). At the same time, innovative tools such as AR further enrich these learning experiences by creating interactive and immersive environments (Cao & Yu, 2023). Assessing students' self-regulated learning behaviors can help educators better tailor instruction to individual needs (Zhidkikh et al., 2023). However, successful integration of real-life scenarios requires overcoming challenges related to accessibility, teacher competence, and curriculum design (Naidoo & Reddy, 2023). Addressing these issues is vital for maximizing student participation and ensuring the long-term effectiveness of mathematical literacy instruction in junior high school classrooms.

RESEARCH METHOD

The pre-experimental study design at the First Secondary School, integrating real-life scenarios into mathematical literacy learning, aims to assess the effectiveness of interventions without a control group. By utilizing pre-tests and post-tests, researchers can directly evaluate changes in student understanding and application of mathematical concepts (Björkhammer et al., 2024). The interventions were conducted in five meetings. Statistical analysis, such as the t-test for paired samples, was employed to determine the significance of improvements resulting from the interventions. This approach is expected to offer initial valuable insights into the impact of real-life scenarios on enhancing mathematical literacy, serving as a foundation for more intricate research designs in the future.

The study was conducted at SMPN 1 Parepare, South Sulawesi, Indonesia. the research population comprised the eighth-grade students of SMPN 1 Parepare in the academic year 2023-2024. There were 320 students distributed across ten classes. The sample was selected using a cluster random sampling technique. One class was randomly selected as the sample, namely class VIIIC, which consisted of 32 students. Cluster random sampling is a method where clusters of individuals are randomly selected as the sampling units, rather than individual subjects within those clusters. This technique is known to be less efficient than completely randomized design due to the randomization being applied at the cluster level (Ozturk et al., 2023).

The research instruments utilized in the study encompass a student's mathematical literacy test and an observation sheet of student activity, aiming to quantitatively evaluate the enhancement of student mathematical literacy through real-life scenario approaches. The mathematical literacy tests

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focus on middle school statistics and practical applications, assessing students' grasp of statistical concepts in everyday contexts before and after the intervention. This test was a teacher-made test developed based on the content measured in this study. The test was evaluated by two independent PhD-level experts in mathematics education using Gregory's content validity procedure (Gregory, 1992), yielding a validity score of 1 and a reliability score of 0.85. Based on the criteria of content validity by Gregory, the tests are valid if the calculated value is equal to or more that 75%. There was no change made from the initial draft because the tests were deemed valid.

The observation sheets capture positive student engagement, teacher and peer interactions, and the integration of real-life scenarios into classroom activities, providing a holistic view of the learning process. Through the integration of these methodologies, the research guarantees the accuracy, dependability, and inclusiveness of the gathered information, thereby enabling a meticulous evaluation of the efficacy of the applied pedagogical approaches. Combining these two instruments also ensures comprehensive quantitative data collection to assess the effectiveness of the applied learning methods (Wisenöcker et al., 2024).

In the learning process of real-life scenarios, the teachers implemented materials relevant to the student's real life, such as family monthly spending, favorite colors, and school buildings. An example of conducting real-life scenarios in teaching statistics for the eighth-grade students of middle school in Parepare is described in Table 1. The real-life scenario described in Table 1 is designed to teach middle school students in Indonesia basic statistical concepts through the analysis of family monthly expenditures. The objective is to help students understand and apply these concepts by collecting, organizing, analyzing, and interpreting real-world data on household spending. The scenario involves students working with their parents or guardians to collect data on their family's monthly expenditures.

In the classroom, students organize the data into tables and create visual representations such as charts and graphs. They learn to calculate basic statistics like total monthly expenditure, mean expenditure per category, and the percentage of total expenditure for each category. Students identify spending trends and suggest areas where spending could be reduced. They present their findings to the class, discussing the implications of their data and comparing it with their classmates to understand different spending patterns and lifestyles. The activity concludes with a class discussion on the importance of budgeting and financial planning, and students explore external factors such as inflation or unexpected expenses on family budgets. An extension activity involves students creating a hypothetical monthly budget, considering various financial scenarios. This scenario teaches statistical concepts and equips students with practical financial literacy skills, making learning practical and relevant.

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No	Indicator	Descriptions
1	Topic:	Analyzing Family Monthly Expenditures
2	Objective:	To help students understand and apply basic statistical concepts such as data collection, organization, analysis, and interpretation by examining and analyzing family monthly expenditures.
3	Learning Outcomes:	- Students will learn how to collect, organize, analyze, and interpret real-world data. - They will understand the importance of budgeting and financial planning. - Students will develop skills in teamwork, communication, and critical thinking. - They will gain insights into economic concepts and the cost of living.
4	Scenario:	Understanding household expenditures is essential for budgeting and financial planning. Students will collect and analyze data on their family's monthly expenditures to gain insight into budgeting and the cost of living.
5	Activities in the classroom:	
	Data Collection:	Ask the students to work with their parents or guardians to collect data on their family expenditures. The students then categorize expenses into categories: groceries, utilities, transportation, education, entertainment, and savings
	Data Organization:	Once the data is collected, students organize their data into tables. They can create a table with those categories. They can also add charts or graphs to represent the distribution of their family' monthly expenses visually.
	Data Analysis:	Students learn how to calculate the basic statistics of total monthly expenditure, mean expenditure per category, and the percentage of total expenditure for each category. Students also identify any trends or patterns in their family's spending, such as which category has the highest expenditure. In addition, they can suggest what categories spending should be reduced.
	Data Interpretation:	Each student presents their findings to the class, discussing the implications of their data. They may discuss how their family can adjust their budget to save more money. The students compare their data with their classmates to understand different spending patterns and lifestyles.
	Discussion:	The students are facilitated to have a class discussion on the importance of budgeting and financial planning. They may also explore external factors such as inflation or unexpected expenses on family budgets.
6	Extension Activity:	The students are asked to create a monthly hypothetical budget, considering different scenarios such as an increase in utility bills or a special event requiring extra spending

Table 1. An example of real-life scenarios

During the learning session, quantitative data were collected through observation sheets to record student activity, reflecting indicators like involvement, interaction with material and peers, and participation level (Ketonen et al., 2025). These indicators were scored on specific scales for

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quantitative processing. After the intervention, students underwent a post-test to assess improvements in mathematical literacy skills. A paired-sample t-test was employed to determine the statistical significance of the difference between pre-test and post-test scores. The use of this test is justified as it is appropriate for comparing the means of two related groups to evaluate the effect of an intervention. The observation data were also analyzed by calculating average scores and frequency of engagement indicators, ensuring the validity and the reliability of the collected data (Zhidkikh et al., 2023). This comprehensive approach provides insights into the effectiveness of real-life scenario interventions in enhancing student mathematical literacy (Ruiz-Palmero et al., 2023).

RESULTS

Mathematical Literacy Data

The purpose of this study was to examine the effectiveness of using real-life scenarios in classroom activities to improve the mathematical literacy of middle school students in Parepare. The researchers, therefore, applied pretest and post-test to evaluate the students' mathematical literacy before after the intervention of the real-life scenarios. In addition, the researchers conducted observations during the interventions in the classroom. Based on the results of the statistical analysis of the pre-test and post-test, Table 2 presents the descriptions.

DESCRIPTIVE STATISTICS	PRE-TEST	POST-TEST
Count	32	32
Mean	54.69	77.81
Std	12.01	6.84
Min	45	65
25%	45	75
50%	50	80
75%	60	80
Max	80	90

Table 2: Descriptive statistics of pre-test and post-test data

The descriptive statistics in Table 2 show that the participants' performance improved significantly from the pre-test to the post-test. The mean score increased from 54.69 to 77.81, and the range of scores (as indicated by the minimum and maximum values) shifted upwards. The standard deviation decreased, indicating that the scores in the post-test were more consistent and less spread out compared to the pre-test scores. The fact suggests an overall improvement and reduced participant performance variability after the intervention or instructional period. Based on the data in Table 2

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above, a graph can be made to show a quantitative comparison of the pre-test and post-test results. As for the chart, it is as follows.

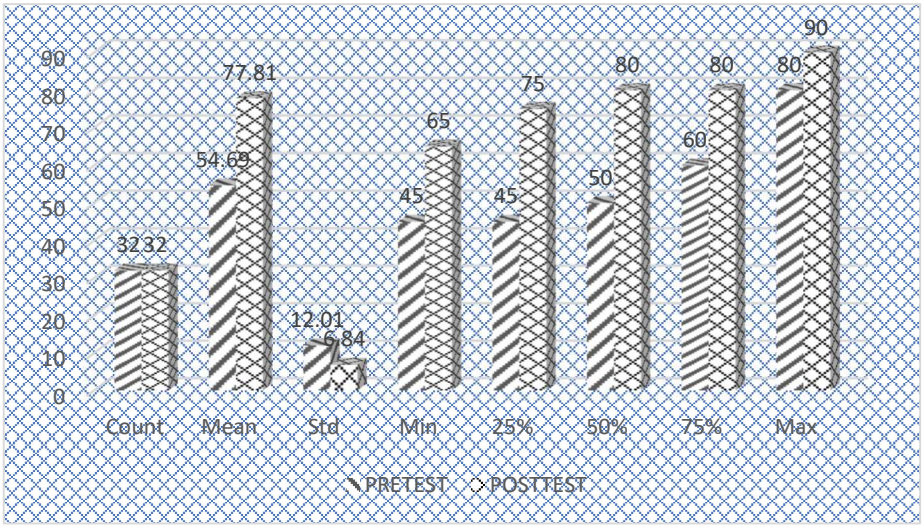


Figure 1: Pre-test and Post-test Comparison Graph

Figure 1 shows a significant improvement from the average score of the pre-test to the post-test for the mathematical literacy of middle school students. This means that there is a substantial improvement in respondent performance after following the integration of real-life scenarios. The decrease in the standard deviation from pre-test to post-test also indicates that post-test values are more consistent among respondents, which may indicate more uniform intervention effectiveness. The quarterly and median values also support the findings that most of the above respondents provide a comprehensive frame on the results of the descriptive analysis of pre-test and post-test data, which can be used as a basis for further interpretation in the context of research for significant improvements in their score after the post-test.

Results of Inferential Statistical Analysis

To measure the effectiveness of using real-life scenarios to improve the students' mathematical literacy, the researcher applied a paired t-test as an inferential data analysis. A two-tailed hypothesis was set at α=0.05. The results of the analysis were presented in Table 3.

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		Paired Differences					t	F	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper			
Pair 1	before being treated - after having been treated	-22.063	9.857	1.742	-25.616	-18.509	-12.662	31	<0.001

Table 3: Paired Samples Test

The paired t-test results reveal a significant and substantial effect of the treatment on the participants' scores, as evidenced by a mean difference of -22.063 with a standard deviation of 9.857. The negative mean difference indicates a notable decrease in scores post-treatment, suggesting that the intervention had a measurable impact. The 95% confidence interval for the difference, ranging from -25.616 to -18.509, reinforces the reliability of this finding, as the interval does not include zero, thereby confirming the statistical significance of the observed difference. Furthermore, the t-value of -12.662 and a p-value of <0.001 indicate a highly significant result, well below the conventional threshold of $p < 0.001$, signifying that the likelihood of this outcome occurring by chance is extremely low. These results strongly suggest that the treatment effectively reduced the measured outcome, underscoring the practical importance of the intervention in altering the participants' performance or condition. The consistency of these findings, as shown by the narrow confidence interval and large t-value, provides robust evidence supporting the treatment's efficacy, making a valuable contribution to the relevant academic discourse.

The practical implications of these findings are significant. The substantial reduction in scores after the intervention suggests that the treatment could be a valuable tool for improving outcomes in similar populations or settings. This has potential applications in clinical practice, educational programs, or other areas where such interventions could be implemented to achieve measurable improvements. The robustness of the statistical significance ($p < 0.001$) also implies that future implementations of this treatment are likely to yield consistent results, thereby supporting its broader adoption in real-world contexts.

Theoretical implications of the findings are also significant. The study contributes to the existing body of literature by providing empirical evidence of the treatment's efficacy, thus supporting its underlying conceptual framework. This study could inform future research by encouraging the exploration of similar interventions or the development of enhanced treatments based on the demonstrated effectiveness. Additionally, the clear and significant impact of the treatment highlights the importance of using evidence-based approaches, reinforcing the need for rigorous evaluations in program development. Overall, these results open pathways for further research and

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practical applications, emphasizing the potential to scale the intervention across different settings and populations.

Students' Activity During Intervention

To complement the descriptive and inferential data analyses, the researchers also analyzed observation data collected during the interventions involving real-life scenarios. The descriptive statistical analysis of this observation encompasses 14 indicators and includes responses from 32 respondents. The finding provides additional insights into students' activities during the interventions.

Indicators	Mean	Std Dev	Min	25%	Median	75%	Max
Active Participation	4.13	0.61	3.00	4.00	4.00	4.25	5.00
Emotional involvement	3.97	0.18	3.00	4.00	4.00	4.00	4.00
Concept Understanding	4.03	0.18	4.00	4.00	4.00	4.00	5.00
Problem Identification	3.69	0.64	3.00	3.00	4.00	4.00	5.00
Troubleshooting Strategy	3.72	0.68	3.00	3.00	4.00	4.00	5.00
Justification of Solution	3.88	0.66	3.00	3.00	4.00	4.00	5.00
Writing ability	3.53	0.62	3.00	3.00	3.00	4.00	5.00
Verbal ability	3.78	0.75	3.00	3.00	4.00	4.00	5.00
Representation Usage	4.03	0.31	3.00	4.00	4.00	4.00	5.00
Interactions in Groups	4.00	0.44	3.00	4.00	4.00	4.00	5.00
Individual Contribution	3.75	0.51	3.00	3.00	4.00	4.00	5.00
Conflict Resolution	3.75	0.44	3.00	3.75	4.00	4.00	4.00
Learning Reflection	3.97	0.18	3.00	4.00	4.00	4.00	4.00
Metacognitive consciousness	3.50	0.51	3.00	3.00	3.50	4.00	4.00

Table 4: Descriptive statistics of student activity observation data

The data reveal several distinct trends in the participants' engagement, skills, and performance across different indicators. First, there is a clear trend of higher scores in areas related to *active participation* (M = 4.13), *concept understanding* (M = 4.03), and *representation usage* (M = 4.03). This fact suggests that participants are highly engaged in learning and effectively utilize conceptual and representational tools to enhance their understanding. The relatively low standard deviations for these indicators, especially *concept understanding* (SD = 0.18), point to a consistent level of performance across the group, indicating that most participants are well-engaged and competent in these areas.

A second trend emerges in areas where variability is higher, such as *troubleshooting strategy* (M = 3.72, SD = 0.68) and *justification of solutions* (M = 3.88, SD = 0.66). The higher standard

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deviations suggest that while some participants perform well in these areas, others may struggle with problem-solving and providing clear, logical justifications for their solutions. This trend indicates a need for more targeted support to help students develop consistent critical thinking and problem-solving skills.

Thirdly, areas like *writing ability* (M = 3.53, SD = 0.62) and *metacognitive consciousness* (M = 3.50, SD = 0.51) reflect a downward trend compared to other indicators. These lower mean scores and moderate standard deviations suggest that participants may require additional instruction or practice in articulating their thoughts in written form and becoming more aware of their cognitive processes during learning. This trend points to potential gaps in skills that could impact students' ability to engage fully with more advanced academic tasks.

Lastly, the consistent performance across indicators related to group dynamics, such as *interactions in groups* (M = 4.00, SD = 0.44) and *conflict resolution* (M = 3.75, SD = 0.44), suggests that participants are generally proficient in collaborative settings. However, while group engagement is strong, there is some variability in *individual contribution* (M = 3.75, SD = 0.51), hinting that some participants may not contribute equally within group tasks.

In summary, the data reveal high engagement in active learning and conceptual understanding while highlighting areas such as problem-solving, writing, and metacognitive awareness where more individualized support may be necessary. These trends provide critical insights into participants' strengths and challenges, informing future instructional strategies that can enhance overall learning outcomes.

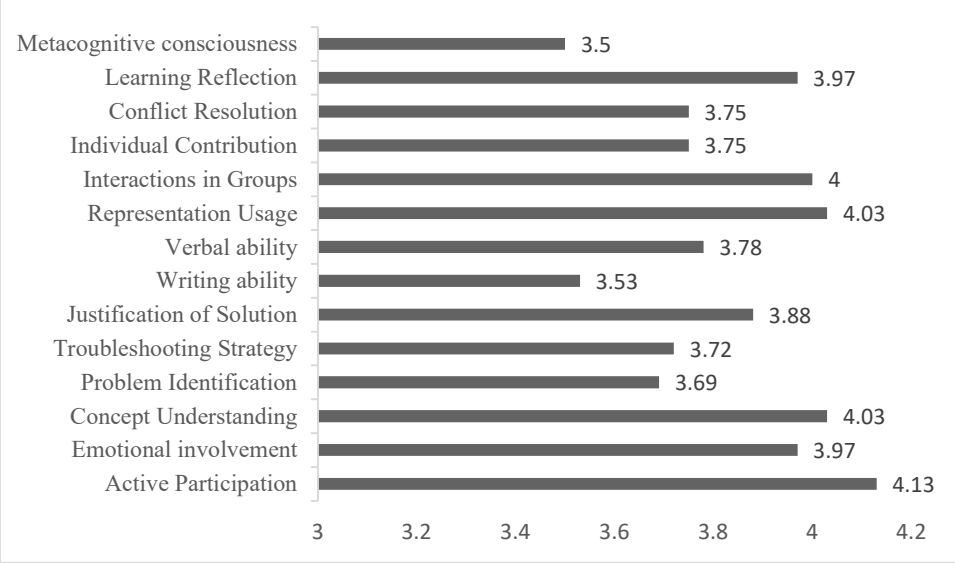


Figure 2. Student Activity Observation Outcome Graph for Each Indicator

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The graph highlights the areas where participants excel, such as active participation and conceptual understanding, and areas that may benefit from further instructional support, particularly in metacognition, writing, and problem-solving skills. These findings can inform future pedagogical approaches to enhance student performance across broader skills.

The graph offers more comprehensive insights as follows:

1. *Balance Between Group and Individual Performance*. While participants show strong performance in *group interactions* (M = 4.00) and collaborative learning environments, the slightly lower scores in *individual contribution* (M = 3.75) suggest a discrepancy between group and individual effort. These data indicate that some participants may rely more on the group dynamic, potentially contributing less individually. Highlighting this insight could inform the need for balanced group work that encourages equitable individual contributions.

2. *Disparity Between Cognitive and Metacognitive Skills*. The high scores in cognitive-based skills like *concept understanding* (M = 4.03) and *representation usage* (M = 4.03) contrast sharply with the lower score in *metacognitive consciousness* (M = 3.50). This disparity implies that while students are proficient at understanding and applying concepts, they may need more awareness of their learning processes, which is crucial for self-regulated learning and long-term academic success. This fact points to the need for integrating metacognitive strategies into the curriculum, encouraging students to reflect on their thinking and problem-solving approaches (Tuononen et al., 2023).

3. *Consistency in Emotional and Reflective Engagement*. The consistently high scores in *emotional involvement* (M = 3.97) and *learning reflection* (M = 3.97) indicate that students are emotionally invested in their learning and actively reflect on their experiences. This condition is a positive indicator of student engagement, which often correlates with improved learning outcomes. Encouraging further emotional engagement and structured reflection activities could reinforce these aspects, leading to deeper, more meaningful learning experiences.

4. *Underperformance in Problem-Solving and Writing Skills*. Areas such as *problem identification* (M = 3.69) and *troubleshooting strategy* (M = 3.72), alongside *writing ability* (M = 3.53), suggest underperformance in critical thinking and written communication. These skills are essential for higher-order cognitive tasks and academic success. Highlighting this insight could emphasize the need for pedagogical adjustments to strengthen these competencies, such as problem-based learning approaches and writing-intensive assignments to build problem-solving abilities and improve articulation in written form.

5. *Addressing Variability in Scores*. Certain indicators, such as *justification of solution* (M = 3.88) and *verbal ability* (M = 3.78), show moderate performance but exhibit variability. This variability might suggest a gap in how consistently students perform across cohorts. Identifying the underlying factors contributing to this variability, such as differences in prior knowledge,

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confidence in problem-solving, or communication skills, could help devise tailored interventions to address these inconsistencies.

Integration and Support for Student Activity Data Analysis with Pre-test and Post-test Statistical Results

The results of student activity observations and statistical analysis showed a strong correlation between student engagement in learning and improvements in students' mathematical literacy skills. The increase in average scores from the pre-test to the post-test indicates the effectiveness of the learning approach using real-life scenarios. In addition, the distribution of scores that were more concentrated around the average on the post-test than the pre-test indicated homogeneity of results after the intervention, reflecting that this method successfully reached students with various levels of initial ability.

The t-test results showed a significant difference between pre-test and post-test scores, strengthening the evidence that integrating real-life scenarios in mathematics learning significantly improved students' mathematical literacy skills. This result aligns with the findings that active participation and the use of good representations contribute to deeper understanding and better performance in mathematics. However, metacognitive awareness, which had the lowest mean score, suggests a need to improve students' awareness of their learning strategies. However, the increase in homogeneity of post-test scores suggests that this intervention is effective across a range of students' initial ability levels.

DISCUSSION

The results of this study indicate that learning with real-life scenario integration significantly improves students' mathematical literacy skills. Descriptive statistical analysis revealed an increase in the mean score from the pre-test to the post-test, with a decrease in the standard deviation from the pre-test to the post-test, indicating consistency of post-test results among respondents. Increased minimum and maximum scores and higher quartiles and medians on the post-test indicated overall performance improvements. Furthermore, inferential analysis using paired t-tests strengthened these findings by showing a significant difference between pre-test and post-test scores, confirming that intervention through real-life scenario integration in mathematics learning effectively improve students' mathematical literacy skills. These findings provide strong empirical evidence that a contextual learning approach that is relevant to students' real-life experiences can improve their understanding and performance in mathematics, and demonstrate the efficacy of this intervention across a range of students' initial ability levels.

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The second finding is based on the observation sheet. Observational data collected during the intervention showed high levels of student engagement and participation. Students demonstrated a strong capacity for conceptual understanding and frequently used various representations to express their mathematical thinking. However, while the students were active in classroom discussions and tasks, certain areas, such as problem-solving, writing skills, and metacognitive awareness, still required further development. Despite the overall high engagement, these areas highlight the need for continued emphasis on reflection and articulation in mathematical literacy to ensure well-rounded skill development.

The findings of this study show significant improvements in students' mathematical literacy skills through the integration of real-life scenarios, which align with several recent studies and offer extensions in a broader context. For example, a study by Palinussa et al. (Palinussa et al., 2021) found that realistic mathematical education improved mathematical literacy such as reasoning and communication skills in the island-based rural context. The findings also support a study by Samuelsson et al. which demonstrated that a fraction intervention focusing on real-world applications improved students' conceptual fraction knowledge, particularly in measurement aspects (Björkhammer et al., 2024). In addition, a study by Nurlinda et al. (2024) confirmed that contextual learning can improve understanding of mathematical concepts and applications in everyday life. Similarly, a study by Smith et al. (2023) published in KnE Social Science supported that mathematical literacy can be improved by problem-based learning, in which real-world contexts in learning can be applied to improve student engagement and understanding. However, these findings also showed a significant reduction in post-test score variation, indicating higher homogeneity of results after the intervention, furthering the knowledge of the effectiveness of contextual learning in reaching different levels of students' initial abilities. In contrast to what Hu (Hu, 2024) explains that conventional methods often fail to address disparities in student ability, the results of this study showed that the integration of real-life scenarios not only improved average performance but also narrowed the gap between students. Thus, this study strengthens and extends the understanding of the benefits of contextual learning in mathematics education, demonstrating its relevance and effectiveness in modern educational contexts.

The findings of this study support and extend constructivist theories in mathematics education, particularly those of (Piaget, 1972; Vygotsky & Cole, 1978) by demonstrating that real-life-relevant learning enhances student understanding and engagement. The integration of real-life scenarios increased average mathematical literacy skills and reduced performance disparities among students, challenging the assumption that traditional approaches are less effective in reaching all students equally. This study contributes to theory development in mathematics education by offering empirical evidence on the effectiveness of contextual approaches, creating inclusive and equitable learning environments (Wijaya et al., 2015). It also emphasizes the importance of integrating real-life scenarios into the mathematics curriculum, providing a basis for further theory development on the implementation of real-world contexts in mathematics instruction to improve student

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learning outcomes (Björkhammer et al., 2024; Nurlinda et al., 2024; Palinussa et al., 2021; Smith et al., 2023).

However, unlike some previous studies that focused on merely improving problem-solving skills, the current findings also emphasize the increased *engagement* of students during the intervention. For instance, a study by Ingram (Ingram et al., 2024) pointed out the challenge of promoting active learning in mathematics, especially when incorporating real-life contexts. This study's findings expand the literature by highlighting improvement in mathematical literacy and students' heightened participation. This finding aligns with the work of (Elgrably & Leikin, 2021), who argued that active engagement is a crucial, yet often under-reported outcome of real-life problem scenarios in math education. These current findings demonstrate that with proper guidance, students from varying levels can benefit from such interventions. This study, therefore, expands on previous research by suggesting that even without prior high levels of mathematical proficiency, students can still exhibit both improved literacy and active engagement when provided with carefully designed and supported real-life scenarios.

The findings from this study offer valuable contributions to existing theories on mathematical literacy and active learning, with notable implications for theory support and expansion. One of the key theoretical frameworks related to mathematical literacy is the socio-constructivist theory of learning (Gravemeijer, 2020), which emphasizes that students construct knowledge through interaction with real-life contexts and active engagement. This study's results strongly support the socio-constructivist perspective by demonstrating that real-life scenarios significantly improve students' mathematical literacy. Moreover, the heightened student engagement observed during the real-life scenario interventions challenges traditional views that passive, lecture-based approaches are sufficient for teaching mathematics. Instead, these findings support theories of active learning, such as Piaget's theory of cognitive development, which posits that learning is an active process of discovery. The increased activeness among students in this study reinforces the argument that hands-on, contextualized activities encourage deeper cognitive processing and active participation, thus contributing to the theoretical discourse around constructivist learning environments.

In terms of theory development, this research expands on existing frameworks by emphasizing the dual impact of real-life scenarios, improving both mathematical literacy and student engagement. While prior studies have primarily focused on the cognitive outcomes of mathematical literacy, this study contributes to the broader theoretical understanding by incorporating the affective and participatory dimensions of learning. It suggests that real-life scenarios not only enhance cognitive outcomes but also foster a more active, student-centered classroom environment, which could be integrated into theories that explore the relationships between cognition, affect, and active participation in learning.

Several limitations to this study that should be acknowledged. First, the sample size was relatively small which may limit the generalizability of the findings to a larger population of middle school

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students. A larger, more diverse sample might produce different results and provide a more comprehensive understanding of the effectiveness of real-life scenarios in improving mathematical literacy. Second, the study was conducted in a specific geographic location (Parepare), which may have cultural or contextual factors that influenced the results. The impact of real-life scenarios in mathematical literacy learning might vary in different educational contexts or with students from different backgrounds.

Another limitation is the reliance on pre-test and post-test scores as the primary measure of improvement. While these scores provide valuable quantitative data, they may capture a partial range of cognitive, emotional, and social benefits that real-life scenarios might offer. Additional qualitative data, such as student interviews or classroom observations, could provide richer insights into how these scenarios influence learning processes and student engagement. Lastly, the duration of the intervention was relatively short, and the study did not assess the long-term retention of mathematical literacy skills. Future studies could examine whether the improvements observed are sustained over time, offering a more complete picture of the intervention's lasting effects.

Based on the identified limitations, further research is suggested to involve larger and more diverse samples to increase the generalizability of the findings and expand the geographic scope to test the effectiveness of contextual learning in different cultural and educational settings. In addition, research with a longer intervention duration is needed to evaluate the long-term impact of this method. It is also important to control for external variables such as the quality of instruction and students' socio-economic background to obtain more accurate results. More comprehensive assessment methods can provide a more in-depth picture of students' mathematical literacy abilities. Finally, further research should consider and mitigate the Hawthorne effect to ensure more valid and reliable results.

This study makes significant contributions to the field of mathematics education, particularly through its methodology and theoretical and practical implications. Its methodology, which integrates real-life scenarios in mathematics learning, proves the effectiveness of contextual approaches in improving students' mathematical literacy skills. The findings support and extend constructivist theories that emphasize the importance of hands-on experience and relevance in learning, and suggest that this approach can reduce performance disparities among students with varying levels of initial ability. Practically, this study provides empirical evidence that can be used to inform the development of more inclusive and effective curricula and teaching strategies, assisting educators and policymakers in designing more relevant and applicable learning programs. Overall, this study strengthens the argument for the importance of contextual learning and provides a strong foundation for further research in an effort to improve the quality of mathematics education at various levels of education.

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CONCLUSIONS

This research reveals that integrating real-life scenarios into mathematics learning significantly improves students' mathematical literacy. The increase in the average score from the pre-test to the post-test, accompanied by a decrease in the standard deviation, suggests that this method is effectively increase student understanding and performance consistently. Inferential analysis supports these findings, with t-tests in pairs showing significant differences between pre-test and post-test scores, confirming the success of these interventions. This finding supports a constructivist theory that emphasizes the importance of contextual learning and the relevance of real experiences in education. In addition, the study provides empirical evidence of the benefits of contextual learning approaches and offers guidance to educators and policymakers in designing more effective and inclusive curricula and teaching strategies.

Based on the findings and conclusions of this study, several recommendations can be made for future research. First, it is recommended that the sample size be enlarged and demographic diversity increased to extend the generalization of results to a wider population. Research in different geographical and cultural locations is also needed to test the consistency and effectiveness of contextual learning approaches in different educational contexts. Second, it is recommended to extend the duration of the intervention in order to evaluate the long-term impact of integrating real-life scenarios into mathematics learning. In addition, it is important to control external variables such as the quality of teaching and students' socio-economic background to obtain more accurate and reliable results. More comprehensive assessment methods are also recommended better to capture the spectrum of students' mathematical literacy skills. Finally, further research should consider and reduce the Hawthorne effect to ensure the validity and reliability of the results. By following these recommendations, future research is expected to strengthen and broaden the understanding of the effectiveness of contextual learning and make a greater contribution to improving the quality of mathematics education.

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REFERENCES

[1] Abaño, J. T., Binondo, J. N., Fortuna, C. M., Guinang, H., Medora, J. C., & Puyod, S. J. P. (2024). Effectiveness of Integrating Real-Life Problems into Mathematical Concepts. *International Multidisciplinary Journal of Research for Innovation, Sustainability, and Excellence (IMJRISE)*, *1*(5), 90-96. https://doi.org/https://doi.org/10.5281/zenodo.11144435

[2] Agustina, E. N. S., Widadah, S., & Nisa, P. A. (2021). Developing Realistic Mathematics Problems Based on Sidoarjo Local Wisdom. *Mathematics Teaching Research Journal*, *13*(4), 181-201.

[3] Attard, C., & Holmes, K. (2022). An exploration of teacher and student perceptions of blended learning in four secondary mathematics classrooms. *Mathematics Education Research Journal*, *34*(4), 719-740.

[4] Björkhammer, C., Samuelsson, J., Träff, U., & Östergren, R. (2024). The effects of a whole-class mathematics intervention on students' fraction knowledge in primary school. *Scandinavian Journal of Educational Research*, *68*(6), 1275-1289. https://doi.org/https://doi.org/10.1080/00313831.2023.2228818

[5] Çakıroğlu, Ü., Güler, M., Dündar, M., & Coşkun, F. (2024). Virtual reality in realistic mathematics education to develop mathematical literacy skills. *International Journal of Human–Computer Interaction*, *40*(17), 4661-4673. https://doi.org/https://doi.org/10.1080/10447318.2023.2219960

[6] Cao, W., & Yu, Z. (2023). The impact of augmented reality on student attitudes, motivation, and learning achievements—a meta-analysis (2016–2023). *Humanities and Social Sciences Communications*, *10*(1), 1-12. https://doi.org/https://doi.org/10.1057/s41599-023-01852-2

[7] Carl, J., Schmittwilken, L., & Pöppel, K. (2023). Development and evaluation of a school-based physical literacy intervention for children in Germany: protocol of the PLACE study. *Frontiers in sports and active living*, *5*, 1155363. https://doi.org/https://doi.org/10.3389/fspor.2023.1155363

[8] Costa, M. C., Domingos, A. M. D., Teodoro, V. D., & Vinhas, É. M. R. G. (2022). Teacher professional development in STEM education: An integrated approach with real-world scenarios in Portugal. *Mathematics*, *10*(21), 3944.

[9] Elgrably, H., & Leikin, R. (2021). Creativity as a function of problem-solving expertise: Posing new problems through investigations. *ZDM–Mathematics Education*, *53*(4), 891-904. https://doi.org/https://doi.org/10.1007/s11858-021-01228-3

[10] Elsayed, H., Nivala, M., & Carlzon, L. (2024). Students' and instructors' perspectives on learning and professional development in the context of interprofessional simulation. *Teaching and Learning in Medicine*, *36*(4), 454-469. https://doi.org/https://doi.org/10.1080/10401334.2023.2230562

[11] Gravemeijer, K. (2020). A socio-constructivist elaboration of realistic mathematics education. In *National reflections on the Netherlands didactics of mathematics: Teaching and learning in the context of realistic mathematics education* (pp. 217-233). Springer

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International Publishing Cham. https://doi.org/https://doi.org/10.1007/978-3-030-33824-4_12

[12] Gregory, R. J. (1992). *Psychological Testing: History, Principles, and Applications*. Allyn and Bacon.

[13] Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. *Computers & Education*, *153*, 103897. https://doi.org/https://doi.org/10.1016/j.compedu.2020.103897

[14] Hu, J. (2024). The challenge of traditional teaching approach: A study on the path to improve classroom teaching effectiveness based on secondary school students' psychology. *Lecture Notes in Education Psychology and Public Media*, *50*(1), 213-219. https://doi.org/https://doi.org/10.54254/2753-7048/50/20240945

[15] Ingram, J., Abbott, A., Smith, K., Planas, N., & Erath, K. (2024). Experienced teachers talking about their mathematics teaching with linguistically disadvantaged learners. *Journal of Mathematics Teacher Education*, *27*(5), 785-808. https://doi.org/https://doi.org/10.1007/s10857-024-09628-4

[16] Johar, R., Zubainur, C. M., Ikhsan, M., & Zubaidah, T. (2021). Pedagogical Content Knowledge of Teachers in Teaching Decimals through Realistic Mathematics Education. *Mathematics Teaching Research Journal*, *13*(4), 150-169.

[17] Ketonen, L., Körkkö, M., & Pöysä, S. (2025). Authentic assessment as a support for student teachers' reflection. *European Journal of Teacher Education*, *48*(3), 467-488. https://doi.org/https://doi.org/10.1080/02619768.2023.2229004

[18] Kirsten, K., & Greefrath, G. (2024). On-campus vs distance tutorials in preparatory courses for mathematics student teachers–performance gains and influencing factors. *International Journal of Research in Undergraduate Mathematics Education*, *10*(2), 547-576. https://doi.org/https://doi.org/10.1007/s40753-023-00221-3

[19] Komarudin, K., Suherman, S., & Vidákovich, T. (2024). The RMS teaching model with brainstorming technique and student digital literacy as predictors of mathematical literacy. *Heliyon*, *10*(13). https://doi.org/ https://doi.org/10.1016/j.heliyon.2024.e33877

[20] Loeb, D., Shoemaker, J., Parsons, A., Schumacher, D., & Zackoff, M. (2023). How augmenting reality changes the reality of simulation: ethnographic analysis. *JMIR Medical Education*, *9*(1), e45538. https://doi.org/https://doi.org/10.2196/45538

[21] Malik, A. R., Pratiwi, Y., Andajani, K., Numertayasa, I. W., Suharti, S., Darwis, A., & Marzuki. (2023). Exploring Artificial Intelligence in Academic Essay: Higher Education Student's Perspective. *International Journal of Educational Research Open*, *5*, 100296. https://doi.org/10.1016/j.ijedro.2023.100296

[22] Malik, M. A., Badolo, M. s., & Badaruddin. (2018). Internet-Based Learning Management Model To Improve The Student's Mathematic Problem Solving Ability. Journal of Physics: Conference Series,

[23] Naidoo, J., & Reddy, S. (2023). Embedding sustainable mathematics higher education in the fourth industrial revolution era post-COVID-19: Exploring technology-based teaching methods. *Sustainability*, *15*(12), 9692. https://doi.org/https://doi.org/10.3390/su15129692

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[24] Nurlinda, E., Azis, Z., & Nasution, M. D. (2024). Students' Mathematical Reasoning Ability and Self-Efficacy Viewed from the Application of Problem Based Learning and Contextual Teaching and Learning Models Assisted. *JMEA: Journal of Mathematics Education and Application*, *3*(2), 54-61.

[25] Ozturk, O., Kravchuk, O., & Jarrett, R. (2023). Models for cluster randomized designs using ranked set sampling. *Statistics in Medicine*, *42*(15), 2692-2710. https://doi.org/https://doi.org/10.1002/sim.9743

[26] Palinussa, A. L., Molle, J. S., & Gaspersz, M. (2021). Realistic Mathematics Education: Mathematical Reasoning and Communication Skills in Rural Contexts. *International Journal of Evaluation and Research in Education*, *10*(2), 522-534. https://doi.org/https://doi.org/10.11591/ijere.v10i2.20640

[27] Parker, N., Breitenstein, J., & D'On Jones, C. (2020). Literacy-Based Instructional Techniques for the Middle School Mathematics Teacher. *Journal of Curriculum and Teaching*, *9*(2), 91-94. https://doi.org/https://doi.org/10.5430/jct.v9n2p91

[28] Piaget, J. (1972). Intellectual evolution from adolescence to adulthood1. In *Cognitive and moral development, academic achievement in adolescence* (pp. 1-12). Routledge. https://doi.org/https://doi.org/10.1159/000271225

[29] Powell, S. R., Lembke, E. S., Ketterlin-Geller, L. R., Petscher, Y., Hwang, J., Bos, S. E., Cox, T., Hirt, S., Mason, E. N., & Pruitt-Britton, T. (2021). Data-based individualization in mathematics to support middleschool teachers and their students with mathematics learning difficulty. *Studies in Educational Evaluation*, *69*, 100897. https://doi.org/https://doi.org/10.1016/j.stueduc.2020.100897

[30] Putri, A. S., Prasetyo, Z. K., Purwastuti, L. A., Prodjosantoso, A. K., & Putranta, H. (2023). Effectiveness of STEAM-based blended learning on students' critical and creative thinking skills. *Int J Eval & Res Educ ISSN*, *2252*(8822), 8822. https://doi.org/https://doi.org/10.11591/ijere.v12i1.22506

[31] Ruiz-Palmero, J., Guillén-Gámez, F. D., Colomo-Magaña, E., & Sánchez-Vega, E. (2023). Effectiveness of the Flipped Classroom in the Teaching of Mathematics in an Online Environment: Identification of Factors Affecting the Learning Process. *Online Learning*, *27*(2), 304-323. https://doi.org/https://doi.org/10.24059/olj.v27i2.3239

[32] Sevcikova, A., Milkova, E., Moldoveanu, M., & Konvicka, M. (2023). Graph theory: Enhancing understanding of mathematical proofs using visual tools. *Sustainability*, *15*(13), 10536. https://doi.org/https://doi.org/10.3390/su151310536

[33] Shiang Tyng, K., Badioze Zaman, H., Hanan Mohamad, U., & Ahmad, A. (2023). Perceived ease of use on visual learning application for mathematics using holography display for the topic on shape and space [version 1; peer review: 1 approved with reservations, 1 not approved]. *F1000Research*, *12*(636). https://doi.org/https://doi.org/10.12688/f1000research.133177.1

[34] Smith, G., Liowardani, A. P., Permadi, H., & Anita, Y. (2023). Application of Problem-based Learning in Efforts to Build Mathematical Literacy Skills. *KnE Social Sciences*, *8*(10), 96–105-196–105. https://doi.org/10.18502/kss.v8i10.13435

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[35] Thuy, H. T., & Le Phuoc, S. (2023). Building Framework for Assessing Students' Statistical Reasoning in Solving Real-Life Medical Problems. *Mathematics Teaching Research Journal*, *15*(4), 45-68.
[36] Uegatani, Y., Otani, H., Shirakawa, S., & Ito, R. (2024). Real and illusionary difficulties in conceptual learning in mathematics: comparison between constructivist and inferentialist perspectives. *Mathematics Education Research Journal*, *36*(4), 895-915. https://doi.org/https://doi.org/10.1007/s13394-023-00478-6
[37] Ulfatun, T., Suyatmini, S., Kusumaningtyas, A., & Setiyawan, Y. A. (2023). Teacher's understanding of teaching models and students' human literacy. *International Journal of Evaluation and Research in Education (IJERE)*, *12*(4), 1925. https://doi.org/https://doi.org/10.11591/ijere.v12i4.25618 (https://doi.org/10.11591/ijere.v12i4.25618)
[38] Uyen, B. P., Tong, D. H., Loc, N. P., & Thanh, L. N. P. (2021). The Effectiveness of Applying Realistic Mathematics Education Approach in Teaching Statistics in Grade 7 to Students' Mathematical Skills. *Journal of Education and E-Learning Research*, *8*(2), 185-197. https://doi.org/https://doi.org/10.20448/journal.509.2021.82.185.197
[39] Vygotsky, L. S., & Cole, M. (1978). *Mind in society: Development of higher psychological processes*. Harvard university press.
[40] Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational studies in Mathematics*, *89*(1), 41-65. https://doi.org/https://doi.org/10.1007/s10649-015-9595-1
[41] Wisenöcker, A. S., Binder, S., Holzer, M., Valentic, A., Wally, C., & Große, C. S. (2024). Mathematical problems in and out of school: The impact of considering mathematical operations and reality on real-life solutions. *European Journal of Psychology of Education*, *39*(2), 767-783.
[42] Zhidkikh, D., Saarela, M., & Kärkkäinen, T. (2023). Measuring self-regulated learning in a junior high school mathematics classroom: Combining aptitude and event measures in digital learning materials. *Journal of Computer Assisted Learning*, *39*(6), 1834-1851. https://doi.org/https://doi.org/10.1111/jcal.12842

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APPENDIX

Sample Questions from Pre-test

1. The average exam score for a class with 40 students is 75. One of the students in that group got a score of 25 and was not included in the average calculation. Calculate the average exam score for that class!
2. Bu Mus recorded cloth purchases at her shop for 9 consecutive days and obtained the following data: 8, 11, 13, 9, 16, 9, 10, 11, 7. Determine the median value of the data!

Sample Questions from Post-test

1. The average exam score for a class with 40 students is 62. One of the students in that group got a score of 23 and was not included in the average calculation. Calculate the average exam score for that class!
2. Pak Dani recorded fabric purchases at his shop for 9 consecutive days and obtained the following data: 9, 10, 15, 7, 17, 8, 9, 11, 8. Determine the median value of the data!

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Boosting Students' Abilities in Understanding, Application, and Reasoning of Numeracy: Didactical Differentiated Learning

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Abstract: This study develops and implements a differentiated mathematics learning model integrating targeted didactical strategies and observation-based assessments to enhance numeracy skills among Indonesian elementary students. Using a Teacher Action Research (TAR) approach, 28 fifth-grade students participated in a learning intervention involving differentiated strategies. Data collection included numeracy tests across three cognitive levels and structured observations. Paired t-tests and ANOVA were used to analyze the intervention's effectiveness. The findings indicate that differentiated learning significantly improved students' numeracy skills across understanding, application, and reasoning dimensions. Normality tests confirmed that pre-test and post-test scores followed a normal distribution ($p > 0.05$). Paired samples tests showed significant improvements in understanding ($p = 0.000$), application ($p = 0.000$), and reasoning ($p = 0.000$), with ANOVA results ($F = 6.208$, $p = 0.003$) indicating varied impacts across cognitive levels. Post-test scores increased significantly (mean difference = -13.811, $p = 0.000$), meeting the success criteria for classical completeness. The study highlights the effectiveness of differentiated learning in promoting comprehensive cognitive development in mathematics. It offers valuable insights for educators in designing tailored instructional approaches to optimize student learning outcomes in numeracy, addressing both theoretical and practical gaps in mathematics education.

Keywords: Cognitive Development; Elementary; Learning Trajectory; Mathematics

INTRODUCTION

The increasing demand for 21st-century skills has led to a growing emphasis on literacy and numeracy as essential components of educational development. Numeracy, as defined by the World Economic Forum (2015), refers to the ability to understand, apply, and analyze mathematical concepts effectively in real-life contexts (Atkinson & Jackson, 2016). In the Indonesian context, the Ministry of Education and Culture defines numeracy as the capacity to reason using mathematical concepts, procedures, facts, and tools to solve everyday problems and make informed decisions (Ayuningtyas & Sukriyah, 2020). Despite its critical role in educational success and daily problem-solving, numerous studies indicate that Indonesian students face persistent challenges in this area. The 2018 PISA assessment revealed that Indonesian students ranked near the bottom globally,

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with a score of 379, significantly below the OECD average of 489 (Schleicher, 2019). These findings underscore the urgent need for educational interventions to enhance students' numeracy competencies and mathematical reasoning.

Interviews with teachers from Phase C classes (fifth grades and sixth grades) further highlight this concern. Teachers observed that students often struggle with basic reading comprehension and mathematical reasoning, which hampers their ability to grasp and apply mathematical concepts effectively. These challenges indicate that the current instructional approaches may not adequately support diverse student learning needs, particularly in mathematics education. As numeracy forms the foundation for academic success and critical problem-solving abilities, addressing this issue is essential to prepare students for future academic and personal endeavors.

One promising approach to addressing this challenge is differentiated learning, a pedagogical framework that emphasizes tailored instruction to meet the diverse needs, abilities, and preferences of students (Tomlinson, 2001). Differentiated learning is widely recognized for its effectiveness in fostering essential learning competencies, particularly in mathematics education. Research has demonstrated that differentiated learning helps students engage with subject matter more effectively by adapting content, process, product, and learning environments to their individual needs (Bhattacharya, 2021; Magee & Breaux, 2013; C. A. Tomlinson & Eidson, 2003). Furthermore, it has been shown to reduce frustration and prevent perceptions of failure, fostering a positive learning experience (Coubergs et al., 2017).

The concept of differentiated learning has garnered significant attention in educational research and practice as a strategy to accommodate diverse student needs. According to (Coubergs et al., 2017; Yong & Sokumaran, 2023), differentiated learning emphasizes tailoring content, processes, and products to individual student abilities and learning preferences. This approach has demonstrated positive outcomes in fostering student engagement and improving academic performance (Valiandes & Neophytou, 2018). Moreover, studies by Lindner & Schwab (2020); Smets & Struyven (2020); and van Geel et al. (2019) have highlighted its effectiveness in various educational contexts, including inclusive education settings. Despite these theoretical advancements, a notable gap exists in the application of differentiated learning specifically within mathematics education to enhance numeracy skills.

The existing body of research largely focuses on general classroom applications of differentiated instruction or its impact on literacy and cognitive engagement (Lindner & Schwab, 2020). However, empirical studies investigating the integration of differentiated learning models tailored to mathematics education remain limited (Eikeland & Ohna, 2022; Maulyda et al., 2025). Furthermore, while differentiated learning is acknowledged as a promising approach for meeting diverse learning needs, there is a paucity of research examining its role in directly enhancing foundational numeracy skills, particularly in primary school contexts where these skills are critical. Another theoretical gap pertains to the lack of comprehensive instructional models that incorporate practical, didactically sound modules for differentiated mathematics learning. Although several studies have proposed frameworks for differentiation (Coubergs et al., 2017; H. Tomlinson, 2013), few

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provide concrete, structured models specifically designed to address numeracy challenges. Additionally, existing models often neglect the role of classroom observation and reflection, which are essential components for assessing and refining instructional strategies.

This study addresses a critical gap in numeracy education by developing and implementing a differentiated learning model that integrates targeted didactical strategies and observation-based assessments. Despite extensive research on differentiated instruction, much of the existing literature has focused on general classroom practices without fully exploring its potential in enhancing specific cognitive dimensions of numeracy, particularly in diverse learning contexts. Furthermore, the theoretical discourse often lacks clarity on how tailored instruction can be systematically designed and evaluated to meet varying student needs. This research not only bridges that theoretical void but also offers a practical framework for educators to implement differentiation effectively. By emphasizing the nuanced relationship between instructional strategies and students' cognitive development in mathematics, this study advances both pedagogical theory and practice, offering actionable insights for addressing persistent challenges in numeracy education.

To refine the research focus and strengthen the manuscript's contribution to the field, this study emphasizes two key areas of novelty. First, it develops and implements a differentiated mathematics learning model and teaching modules specifically designed for numeracy, addressing the distinct challenges faced by Indonesian elementary students. Unlike prior studies that often concentrate on generalized educational outcomes, this research provides a targeted instructional framework that caters to students' diverse learning needs, promoting an inclusive, student-centered environment that enhances critical numeracy skills. Second, the research uniquely focuses on identifying and analyzing students' numeracy proficiency across three cognitive levels: understanding, applying, and reasoning. By mapping students' capabilities at these levels, the study offers valuable insights into how differentiated instruction can be adapted to meet varying numeracy demands. This dual emphasis not only enriches the pedagogical landscape but also contributes practical solutions for improving mathematics education in diverse classroom settings.

LITERATURE REVIEW

Cognitive Level of Numeracy

Numeracy encompasses the ability to understand, apply, and reason with mathematical concepts, forming a fundamental component of an individual's cognitive and problem-solving abilities (OECD, 2019; Schleicher, 2019). The cognitive dimension of numeracy involves three hierarchical levels: understanding, application, and reasoning. To support the development of these cognitive levels, effective instructional strategies and practical examples should be integrated into teaching practices.

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Understanding

Understanding is the foundation, enabling students to grasp mathematical concepts, recognize patterns, interpret symbols, and establish relationships among numbers (Keller et al., 2001). At this level, students develop essential skills such as decoding numerical expressions, comprehending mathematical operations, and visualizing abstract concepts. Strategies to strengthen understanding include exploration activities, discovery learning, and guided practice. For example, using manipulatives or visual aids to demonstrate place value or fractions helps students concretely grasp abstract ideas. Teachers can also employ number talks and pattern recognition exercises to deepen students' conceptual understanding. Without a strong foundation at this level, students may face challenges in progressing to higher cognitive levels (Kirschner, 2002; Lin et al., 2018).

Application

Application involves the ability to use mathematical knowledge to solve contextual problems, encompassing both routine and non-routine tasks. This stage requires students to bridge theoretical mathematical understanding with real-world situations (Maulyda et al., 2020). Strategies to enhance application skills include providing contextualized problems that relate to students' everyday experiences, incorporating project-based learning, and engaging students in hands-on activities. For instance, students might calculate the cost of items during a simulated shopping activity or measure distances in a classroom project. These experiences enable students to perform computations, apply formulas, and develop problem-solving strategies relevant to specific scenarios. Research indicates that students often struggle to apply mathematical knowledge when faced with unfamiliar contexts, emphasizing the need for authentic and practical learning experiences (Chapman, 2013; de Walle et al., 2016).

Reasoning

The reasoning level, representing the pinnacle of numeracy, involves critical thinking, analysis, synthesis, and justification of mathematical processes. At this stage, students are expected to evaluate multiple solution strategies, construct logical arguments, and defend their conclusions. Inquiry-based learning environments, classroom discussions, and problem-solving tasks that require justification of solutions are effective strategies for fostering reasoning skills (Rasmussen & Isoda, 2019). For example, students can be presented with open-ended problems where they must explore various solution methods, compare their effectiveness, and articulate their reasoning process. Teachers can also encourage peer-to-peer discussions where students critique and refine each other's solutions, thereby promoting reflective thinking and deeper understanding.

By structuring instruction to address these three cognitive levels (understanding, application, and reasoning), educators can cultivate comprehensive numeracy competencies that are essential for academic success and everyday problem-solving. The concept of fractions can be examined through the three cognitive levels of numeracy: understanding, application, and reasoning. At the understanding level, students grasp the fundamental concept of fractions as parts of a whole. This

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involves recognizing that a fraction represents a division of an object or quantity into equal parts. For instance, when presented with a circle divided into four equal sections, students can identify that shading one section represents the fraction $\frac{1}{4}$. Visual aids and manipulatives, such as paper folding or using physical objects like pieces of fruit or pizza models, help students concretize abstract ideas and recognize relationships between different fractions. Establishing this foundational understanding is crucial, as it enables students to interpret numerical symbols, recognize patterns, and comprehend basic mathematical operations.

Moving to the application level, students use their understanding of fractions to solve real-world problems. This stage involves bridging theoretical knowledge with practical situations that are familiar to students. For example, a student might encounter a scenario where Alya has a pizza cut into eight slices and eats three of them. The student is then tasked with determining the fraction of the pizza that has been eaten ($\frac{3}{8}$) and the portion that remains ($\frac{5}{8}$). Other practical applications include measuring ingredients for a recipe, where a student needs to measure half a cup of sugar, or distributing six candies evenly among four friends. Such contextualized tasks not only help students perform computations but also encourage them to connect mathematical concepts to everyday experiences, thereby enhancing the relevance and practicality of their learning.

At the highest cognitive level, reasoning, students are expected to analyze mathematical situations, evaluate different solution strategies, and justify their conclusions. For instance, when asked to determine which fraction is larger between $\frac{3}{4}$ and $\frac{2}{3}$, students must compare the two fractions using methods such as drawing visual models, converting fractions to decimals, or finding common denominators. A student might explain, “By dividing two rectangles into four and three equal parts respectively, I observed that shading three out of four parts covers more area than shading two out of three parts, which shows that $\frac{3}{4}$ is greater.” Beyond simple comparisons, reasoning tasks may also involve challenging assumptions, such as questioning whether $\frac{1}{2}$ is always larger than $\frac{1}{4}$ when considering contexts involving differently sized wholes. Encouraging students to articulate their thought processes, engage in peer discussions, and explore multiple solutions fosters critical thinking and deepens their conceptual understanding.

Differentiated Learning

Differentiated learning is a key strategy in addressing the diverse learning needs of students while promoting equity in educational outcomes by recognizing variations in students' readiness levels, learning profiles, and interests (Hasanah et al., 2022). By tailoring instruction to meet these differences, teachers can design and deliver lessons that are both accessible and appropriately challenging, thereby fostering a more inclusive classroom environment. In mathematics education, differentiated learning has proven effective in enhancing students' problem-solving skills and conceptual understanding.

Practical implementation of differentiated learning can be achieved by adapting the content, process, product, and learning environment to suit students’ varied needs. For example, in terms of

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content, teachers can offer materials at varying levels of complexity or present information through different modes (e.g., videos, texts, and hands-on activities). To differentiate the process, scaffolding complex mathematical concepts allows students to build on prior knowledge gradually, reducing cognitive overload and fostering deeper understanding (van Geel et al., 2019). Differentiating the product may involve allowing students to demonstrate their understanding through written assignments, presentations, or creative projects. As for the learning environment, flexible seating arrangements and collaborative group work can accommodate diverse learning preferences. Providing multiple problem-solving pathways encourages creativity and critical thinking, enabling students to approach mathematical problems from different angles and develop versatile strategies.

Technology integration further expands the possibilities for differentiated learning in mathematics classrooms. Digital tools and platforms offer interactive and customizable learning experiences, allowing students to work at their own pace and receive instant feedback on their progress (Bhattacharya, 2021; Yong & Sokumaran, 2023). Tools such as virtual manipulatives and dynamic mathematical representations have been shown to enhance engagement and comprehension, particularly among students with varying levels of mathematical proficiency.

However, effective implementation of differentiated learning requires ongoing teacher training, access to appropriate resources, and a shift toward more flexible and student-centered teaching practices (Kostadinov & Kolev, 2020). Despite its potential, empirical studies exploring practical applications of differentiated learning in mathematics education remain limited, underscoring the need for further research to develop comprehensive instructional models that support cognitive development in numeracy.

Conjecture of Study

This study addresses a gap in the literature by introducing a novel approach that integrates differentiated learning with the identification of students' numeracy proficiency levels—understanding, application, and reasoning—an area often overlooked in previous research. Unlike earlier studies that focus on general educational outcomes, this research emphasizes the alignment between instructional strategies and students' specific cognitive needs in mathematics. By focusing on differentiated teaching models tailored to individual numeracy levels, the study aims to provide a more contextually relevant and classroom-applicable framework, particularly for Indonesian elementary education. The research is guided by three key research questions designed to explore under-examined aspects of differentiated instruction in numeracy:

1. How does differentiated learning influence students' cognitive levels of numeracy (understanding, application, and reasoning)?
2. Which cognitive level benefits most from differentiated learning, and why?
3. How does the integration of differentiated teaching models contribute to students' overall numeracy proficiency?

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These questions stem from three primary conjectures based on theoretical frameworks and existing literature. First, differentiated learning is hypothesized to positively impact students' cognitive levels of numeracy by providing targeted instructional support that enhances comprehension, contextual application, and reasoning skills (H1). Second, it is anticipated that the reasoning level will experience the most significant improvement (H2), considering its reliance on complex cognitive processes and higher-order thinking. This focus addresses a gap in studies that often overlook the nuanced progression from understanding to reasoning. Third, the study posits that differentiated instruction fosters holistic numeracy development, offering a practical solution to the challenges Indonesian students face in mathematics education (H3). By connecting differentiated learning to specific cognitive levels, this research presents an innovative instructional model that directly responds to classroom realities and students' diverse learning needs, thereby offering both theoretical and practical contributions to the field.

METHOD

Research Design

This research uses the Teacher Action Research (TAR) approach. The TAR approach is a research design carried out by teachers to improve the learning process in the classroom (Bell & Aldridge, 2014). The TAR approach allows teachers to improve student learning outcomes, improve the quality of learning activities, and measure the success of interventions designed and implemented by teachers in the classroom (Reeves, 2008). This approach consists of four main stages:

Step 1. Assessing the Learning Environment

At this stage, researchers conducted a comprehensive analysis of students' learning obstacles to identify specific challenges in mastering addition and subtraction story problems. This involved reviewing previous mathematics learning outcomes and gathering observational data from classroom interactions to gain insights into students' difficulties. The research team then evaluated the developmental aspects that needed to be nurtured in elementary school students, aligning these with the current elementary school curriculum to ensure that the planned interventions were curriculum relevant.

Based on these findings, researchers designed developmentally appropriate differentiated learning strategies tailored to diverse student needs. This included preparing games that matched children's various learning styles to enhance engagement and cognitive development. Additionally, researchers created comprehensive teaching modules to guide instruction and provided suitable learning media aligned with the intervention theme. Learner worksheets were meticulously developed to support differentiated learning practices, allowing students to work at their own pace and understanding levels. To ensure a systematic and effective implementation, researchers developed observation and evaluation formats. These tools enabled the accurate monitoring of students' progress, classroom dynamics, and the effectiveness of the differentiated learning interventions. The

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structured observation and evaluation formats also helped identify areas for improvement and refinement during subsequent instructional cycles, ensuring a continuous improvement process.

Step 2. Reflection and Discussion

Teachers used data from students' previous mathematics learning outcomes and observation notes to determine the appropriate interventions. To support this process, researchers facilitated a collaborative focus group discussion (FGD) involving teachers, lecturers, and doctoral students. The purpose of this FGD was to evaluate and refine the didactical design developed by teachers, ensuring its alignment with students' learning needs and curriculum requirements. The discussion was conducted online through the Zoom platform, enabling interactive sessions between all participants.

During the FGD, teachers presented the initial didactical design, including planned learning sequences, instructional strategies, and assessment methods. Researchers and experts provided constructive feedback by analyzing the strengths and weaknesses of the proposed design. The discussions highlighted the need to incorporate more engaging and hands-on learning activities, particularly for abstract mathematical concepts like addition and subtraction in story problems. Suggestions included incorporating more scaffolded tasks and enhancing the role of visual aids. As a result of the collaborative evaluation, it was decided to integrate Dienes Block media and carefully designed Mathematical Worksheets to help students grasp mathematical concepts through structured and interactive activities. The finalized didactical design aimed to foster better student understanding, engagement, and problem-solving skills, ensuring that learning outcomes in addition and subtraction story problems would improve significantly.

Step 3. Intervention

At this stage, researchers begin to plan, implement, monitor, and reflect on the interventions provided in the classroom. This intervention activity is carried out in several cycles depending on student needs. The learning cycle model in this research was developed from Kemmis in Bell & Aldridge (2014) which can be seen in Figure 1 below.

In this study, the intervention process was carried out in three distinct cycles: pre-cycle, Cycle 1, and Cycle 2, each designed to address specific cognitive level of numeracy based on ongoing observations and evaluations. Each cycle concluded with a reflective session, where researchers and teachers collaboratively reviewed the outcomes and refined the didactical design to better meet students' learning needs.

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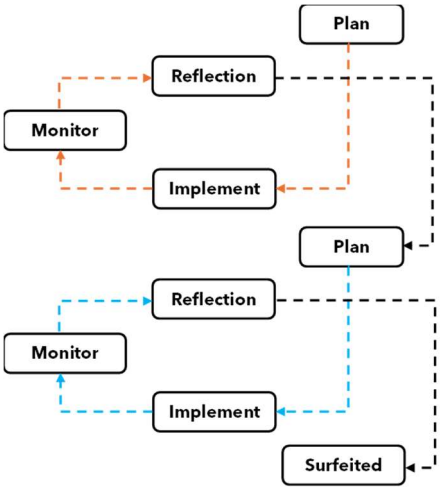


Figure 1: Kemmis's Learning Cycle in Research

Step 4. Re-assessment

The final stage involves conducting a comprehensive assessment to measure students' numeracy skills. Reflection activities include analyzing, synthesizing, interpreting, explaining, and drawing conclusions. The outcomes of this reflection process led to revisions of the implemented plan, which can be used to improve teacher performance in subsequent sessions. Reflection in action research involves examining what has or has not occurred, what outcomes have been achieved, and what remains incomplete despite corrective actions. The insights gained from this reflective process guide the determination of further steps to achieve the objectives of the research. As a result, action research is not limited to a single meeting, as the findings from reflection require time for implementation and refinement.

Participants

This research involved 28 fifth-grade students from Muhammadiyah Rambah Private School. The school is in a suburban area and is known for its commitment to integrating character education into the learning process. The students who participated were selected based on their availability and willingness to be part of the study. They represented diverse academic abilities, providing a comprehensive overview of the impact of the instructional approach on different learner profiles. Ethical considerations, including parental consent, were thoroughly addressed to ensure responsible research practices (Adams et al., 2023).

Didactical Differentiated Learning

Differentiated learning addresses the diverse needs of students by considering their varying learning styles, readiness levels, and interests. In mathematics education, this approach is vital for enhancing numeracy skills, as students often possess differing levels of prior knowledge, cognitive

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abilities, and engagement with mathematical concepts. By employing differentiated instruction, teachers can provide tailored learning experiences that promote inclusivity and improve student outcomes. The proposed didactic design follows a structured model that incorporates content, process, product, and learning environment differentiation. This model begins with pre-assessment to identify students' numeracy proficiency levels, ranging from understanding to applying and reasoning through mathematical problems. Based on these assessments, content differentiation is implemented by offering various materials such as visual aids, manipulatives, and real-life contextual problems catering to different learning styles. Process differentiation involves varied instructional strategies, including guided practice, peer collaboration, and individualized support, ensuring that all students engage with mathematical concepts at an appropriate pace. To differentiate the product, students demonstrate their understanding through diverse formats, such as solving practical problems, creating mathematical models, or presenting solutions verbally. Lastly, the learning environment is adapted to foster collaboration and autonomy, with flexible seating and the integration of technology to provide interactive learning experiences. The instructional model is implemented through a series of learning phases: (1) *Introduction*, activating prior knowledge and presenting learning objectives; (2) *Exploration*, engaging students with hands-on activities and group discussions; (3) *Concept Development*, guiding students to discover and understand mathematical concepts; (4) *Application*, encouraging students to apply learned concepts to solve problems; and (5) *Reflection*, allowing students to evaluate their understanding and learning process.

Data Collection

The data collection process in this research was conducted through a numeracy test designed based on three cognitive levels: understanding, application, and reasoning. The test aimed to comprehensively measure students' abilities in solving numeracy problems according to the cognitive levels specified in the curriculum. The test instruments consisted of various items covering each cognitive level to ensure comprehensive data collection. The test results were then analyzed to identify changes in students' abilities throughout the research process. Detail of test instrument show in table 1.

No	*Dimension*	*Item*	*Example*
1	*Understanding*	*3*	*Students are asked to add simple integers and explain the result.*
2	*Application*	*4*	*Students are asked to calculate the total shopping cost with a certain discount.*
3	*Reasoning*	*3*	*Students are asked to solve a word problem that requires problem-solving strategies and provide logical reasoning for their answer.*
	Total	*10*	

Table 1: Numeracy Test Instrument

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Additionally, an observation sheet was used to describe the learning process in each cycle of the research. Observations were conducted to monitor student engagement, the quality of teacher-student interactions, the use of learning media, and the reflection process during learning. The observation instrument was structured based on five key dimensions that capture important aspects of cycle-based learning. Detail of observation instrument show in table 2.

No	*Dimension*	*Item*	*Example*
1	*Apperception and Motivation*	*6*	*Conveying the benefits of learning materials*
2	*Delivery of Competencies and Activity Plan*	*2*	*Explaining the competencies students are expected to achieve*
3	*Mastery of Learning Material*	*4*	*Presenting material systematically (from simple to complex, concrete to abstract)*
4	*Application of Educational Learning Strategies*	*7*	*Actively involving students in learning activities*
5	*Implementation of Independent Learning and Student-Centered Learning*	*8*	*Applying differentiated learning (Content, Process, and Product)*
	Total	*27*	

Table 2: Observation Instrument

Data Analysis

The data analysis in this research was conducted using quantitative statistical methods to evaluate the effectiveness of differentiated learning interventions on students' numeracy skills. Initially, a simple paired t-test was employed to assess the impact of the interventions on each cognitive level: understanding, application, and reasoning. This analysis aimed to determine whether there were statistically significant improvements in student performance at each level after the differentiated learning activities.

To further explore differences between cognitive levels, a One-Way ANOVA test was conducted to compare the mean scores across the three cognitive dimensions. This test provided insights into whether the learning interventions had varying effects depending on the cognitive complexity of the tasks. Post-hoc analyses were performed to identify specific pairwise differences between the cognitive levels if a significant overall difference was detected. Additionally, a Paired Sample t-Test was conducted to measure the combined impact of the interventions on overall numeracy skills by aggregating the data across all three cognitive levels. This test assessed whether the collective improvements observed in the understanding, application, and reasoning dimensions were statistically significant (Hair et al., 2019).

To complement the test results, data from the observation sheets were analyzed as supporting evidence. These observations captured key dimensions of the learning process, including student

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engagement, teacher-student interactions, the use of learning media, and the reflection process during each instructional cycle. The structured observation data provided valuable qualitative insights, allowing researchers to interpret the statistical findings more comprehensively. By triangulating the numeracy test outcomes with observation data, a more nuanced understanding of the effectiveness of differentiated learning interventions was achieved. This combined analysis ensured that both quantitative and qualitative evidence informed the study's conclusions, thereby strengthening its validity.

RESULTS

The research findings highlight the effectiveness of differentiated learning in enhancing students' numeracy skills across cognitive dimensions. Before conducting the experimental test, the researchers performed a normality test to assess the data distribution. This step was taken to ensure that the data could be analyzed using parametric and non-parametric statistical tests. The results of the normality test are presented in Table 3 below.

Tests of Normality

	Kolmogorov-Smirnov[a]			*Shapiro-Wilk*		
	Statistic	*df*	*Sig.*	*Statistic*	*df*	*Sig.*
PostTest Average	*0.145*	*28*	*0.137*	*0.941*	*28*	*0.117*
PreTest Average	*0.140*	*28*	*0.168*	*0.967*	*28*	*0.502*

a. Lilliefors Significance Correction

Table 3: Normality Test Source: SPSS Outputs

The normality test results using the Kolmogorov-Smirnov and Shapiro-Wilk tests indicate that both the pre-test and post-test average scores follow a normal distribution. For the post-test average, the Kolmogorov-Smirnov test yielded a statistic of 0.145 with a significance level of 0.137, while the Shapiro-Wilk test showed a statistic of 0.941 and a significance level of 0.117. Similarly, the pre-test average yielded a Kolmogorov-Smirnov statistic of 0.140 with a significance level of 0.168 and a Shapiro-Wilk statistic of 0.967 with a significance level of 0.502. Since all p-values are greater than 0.05, it can be concluded that the data for both pre-test and post-test average scores do not significantly deviate from a normal distribution, thus meeting the assumption of normality for subsequent parametric analyses (Hair et al., 2019). These findings ensure the validity of using parametric tests to investigate the impact on cognitive levels of numeracy and overall numeracy abilities, allowing for a robust analysis of the effectiveness of the interventions applied.

Impact on Cognitive Level of Numeracy ➔ H^1 & H^2

Table 4 presents the results of the Paired Samples Test, illustrating the differences in students' cognitive level scores before and after differentiated instruction across three meetings. The test

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compared pre-test and post-test scores for understanding, application, and reasoning levels, as shown by the mean difference values, standard deviation, and confidence intervals.

Paired Samples Test

		Paired Differences				
		Mean	*Std. Deviation*	*95% Confidence Interval of the Difference* *Lower*	*Upper*	*Sig. (2-tailed)*
First Meeting	*Understanding PreTest - Understanding PostTest*	*-16.071*	*4.973*	*-18.000*	*-14.143*	*0.000*
Second Meeting	*Application PreTest - Application PostTest*	*-7.500*	*6.597*	*-10.058*	*-4.942*	*0.000*
Third Meeting	*Reasonable PreTest - Reasonable PostTest*	*-17.857*	*13.501*	*-23.092*	*-12.622*	*0.000*

Table 4: Effect on Cognitive Dimensions Source: SPSS Outputs

The results indicate that differentiated instruction had a statistically significant effect on all cognitive levels, with p-values is 0.000 for each paired sample, suggesting a strong influence on students' learning outcomes. Among the three cognitive levels, the reasoning level was the most affected by differentiated learning, as demonstrated by the largest mean difference of -17.857 and a broader confidence interval range (-23.092 to -12.622). The understanding level also showed a substantial effect with a mean difference of -16.071, while the application level exhibited a relatively smaller impact, with a mean difference of -7.500 (Hair et al., 2019). These findings highlight the effectiveness of differentiated instruction in enhancing students' reasoning and comprehension skills compared to their application abilities.

The results of the ANOVA analysis, as presented in Table 5, provide compelling evidence of the significant impact of differentiated instruction on students' cognitive achievement levels in numeracy. The findings reveal a statistically significant difference between groups ($F = 6.208$, $p = 0.003$), indicating that the instructional approach did not affect all cognitive levels uniformly (Hair et al., 2019). This variation underscores the importance of analyzing specific group comparisons to identify which cognitive domains benefit most from differentiated instruction.

ANOVA

Numeracy Test

	Sum of Squares	*df*	*Mean Square*	*F*	*Sig.*
Between Groups	*1282.930*	*2*	*641.465*	*6.208*	*0.003*
Within Groups	*8369.245*	*81*	*103.324*		
Total	*9652.176*	*83*			

Table 5: ANOVA Test Source: SPSS Outputs

Multiple Comparisons

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Dependent Variable: Sum					
	(I) Group	*(J) Group*	*Mean Difference (I-J)*	*Std. Error*	*Sig.*
Tukey HSD	*Understanding*	*Application*	*3.150*	*2.717*	*0.481*
		Reasonable	*9.404**	*2.717*	*0.002*
	Application	*Understanding*	*-3.150*	*2.717*	*0.481*
		Reasonable	*6.254*	*2.717*	*0.021*
	Reasonable	*Understanding*	*-9.404**	*2.717*	*0.002*
		Application	*-6.254*	*2.717*	*0.021*

**. The mean difference is significant at the 0.05 level.*

Table 6: Multiple Comparisons Test Source: SPSS Outputs

The post hoc multiple comparisons using the Tukey HSD test (Table 6) offer further clarification. The reasoning group demonstrated the most substantial improvement compared to the understanding and application groups, with statistically significant mean differences of 9.404 ($p = 0.002$) and 6.254 ($p = 0.021$), respectively. These results suggest that differentiated learning strategies are particularly effective in enhancing higher-order cognitive skills, specifically students' reasoning abilities. In contrast, the difference between the understanding and application groups was not statistically significant ($p = 0.481$), implying that differentiated instruction may exert a similar influence on these cognitive levels (Hair et al., 2019). These findings highlight the critical role of tailored instructional methods in fostering complex cognitive development and underscore the necessity of prioritizing differentiated strategies to advance students' reasoning capabilities.

Impact on Numeracy Abilities ➔ H^3

Table 7 presents the results of the paired samples t-test, which was conducted to analyze the impact of differentiated learning on students' numeracy achievement. This test was employed to evaluate whether there was a significant difference between the pre-test and post-test scores in terms of cognitive levels. The negative mean difference (-13.811) indicates an improvement in numeracy performance after the differentiated learning intervention. A statistically significant result (Sig. = 0.000, $p < 0.05$) confirms that differentiated learning positively influenced students' cognitive development.

Paired Samples Test

		Paired Differences				
				95% Confidence Interval of the Difference		*Sig. (2-tailed)*
		Mean	*Std. Deviation*	*Lower*	*Upper*	
Numeracy Test	*PreTest Average - PostTest Average*	*-13.811*	*5.776*	*-16.051*	*-11.571*	*0.000*

Table 7: Paired Samples Test Source: SPSS Outputs

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The findings in Table 7 demonstrate that differentiated learning significantly enhanced students' cognitive numeracy levels, as evidenced by the substantial decrease in the difference between pre-test and post-test scores. The mean difference of -13.811 suggests that students achieved notable progress after the learning intervention. The narrow 95% confidence interval (-16.051 to -11.571) further indicates the stability of these findings. Therefore, it can be concluded that differentiated instruction primarily contributes to higher levels of cognitive understanding and problem-solving abilities, reinforcing its effectiveness as a teaching strategy for improving numeracy skills (Hair et al., 2019). Additionally, the findings further indicate that differentiated mathematics learning models were effective in achieving classical completeness for students' numeracy skills. The action hypothesis in this study was successfully tested, as the results met the success criteria set: a minimum numeracy score of 75 in the Minimum Competencies Assessment (aligned with the School's Standar) and at least 80% of students achieving classical completeness.

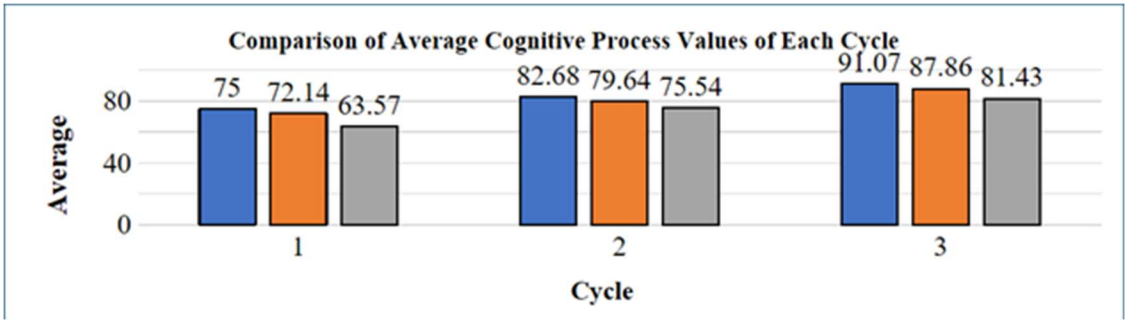


Figure 2: Comparison of Cognitive Processes of Each Cycle

Figure 2 illustrates consistent improvements in cognitive processes across the three action research cycles. The cognitive process of understanding increased from 76.43 in Cycle I to 83.39 in Cycle II and reached 91.07 in Cycle III. Similarly, the application cognitive process improved from 73.75 in Cycle I to 80.54 in Cycle II and further increased to 87.86 in Cycle III. The reasoning cognitive process showed a remarkable increase from 63.57 in Cycle I to 75.54 in Cycle II and reached 81.43 in Cycle III. These results provide strong evidence that differentiated learning positively impacted students' cognitive numeracy abilities, fostering their understanding, application, and reasoning skills at a classical level of completeness.

Learning Description Processes

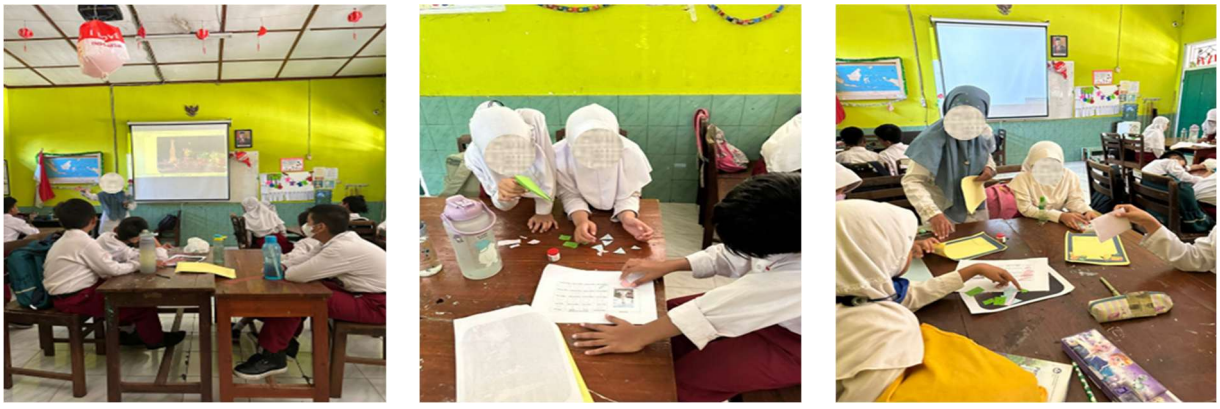
The classroom observations revealed a structured learning process aligned with the indicators on the observation sheet. In the initial phase, the teacher effectively conducted ***apperception and motivation activities*** by connecting prior knowledge with the new topic through contextual questions and real-life examples related to students' daily experiences. This approach successfully engaged students and created an enthusiastic learning atmosphere, as reflected in students' active responses during the apperception session. Motivational strategies were also evident when the teacher outlined clear learning objectives and emphasized the relevance of numeracy skills in real-life problem-solving. Documentation of this phase includes photographs capturing student engagement

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during the apperception activities and a video recording showing the teacher's motivational speech, both demonstrating students' attentive expressions and willingness to participate.

During the ***delivery of competencies and activity plan***, the teacher clearly articulated the learning goals and provided step-by-step instructions to guide students through the learning process. Lesson plans, aligned with the differentiated learning model, emphasized cognitive levels understanding, application, and reasoning allowing students to navigate learning activities with clear expectations. The teacher's mastery of the learning material was evident through precise explanations and the use of varied representations, such as visual aids, manipulatives, and digital media, to clarify complex concepts. Observational notes recorded instances where students asked thoughtful questions, indicating a deepening comprehension of the material. Supporting documentation includes lesson plan excerpts, worksheets completed by students, and classroom observation forms highlighting the teacher's instructional clarity.

In the application of ***educational learning strategies and the implementation*** of independent and student-centered learning, the teacher employed differentiated instruction techniques tailored to students' cognitive levels. Group work activities were structured to encourage peer collaboration, while individual tasks allowed for independent exploration of numeracy problems. The teacher provided scaffolding as needed, ensuring students could work at their own pace while being challenged appropriately. Student-centered approaches were particularly evident during problem-solving tasks, where students discussed strategies and justified their reasoning in groups before sharing with the class. Evidence of these activities includes photographs of group discussions and student activity shows in figure 3 below.



Learning video documentation:
https://youtube.com/shorts/p8dpRJCZqJs?feature=share
https://www.youtube.com/shorts/Hp_61GrDxz8?feature=share
Figure 3: Learning Process Documentation

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DISCUSSION

This study highlights the significant impact of differentiated instruction on students' numeracy cognitive abilities across three dimensions: comprehension, application, and reasoning. The Paired Samples Test results from the first, second, and third sessions indicate significant improvements in these cognitive levels. These findings align with previous studies that emphasize how differentiated instruction enables students to maximize their potential at various cognitive levels (Hasanah et al., 2022; C. A. Tomlinson, 2001).

In the comprehension dimension, the study revealed a mean difference of -16.071 with a significance level of $p = 0.000$ or less than 0.050, confirming that differentiated instruction effectively enhances students' comprehension skills. This finding is consistent with Santrock (2016); & Slavin (2018), who argued that adaptive learning processes help students grasp mathematical concepts more deeply as they are tailored to their learning preferences. Similarly, Perso (2006); & Tout (2020) emphasized that differentiated learning fosters conceptual skills development through activities relevant to students' needs.

In the second session, the application dimension showed a mean difference of -7.500 with $p = 0.000$. Although this increase was smaller than in other dimensions, it remained statistically significant. This outcome supports Bhattacharya (2021); & Yong & Sokumaran (2023) research, which noted that differentiated instruction enhances students' ability to apply mathematical concepts in real-world situations. The comparatively lower improvement in this dimension may be attributed to the complexity of activities requiring more advanced problem-solving skills, as highlighted by de Walle et al. (2016). Integrating project-based learning, which emphasizes real-life application of concepts, is suggested as a potential solution.

The reasoning dimension demonstrated the most substantial impact, with a mean difference of -17.857 and a wide confidence interval range (-23.092 to -12.622). These findings suggest that differentiated instruction significantly supports the development of higher-order thinking skills. Atasoy & Özden (2020); & Kurt et al. (2023) similarly asserted that instructional strategies designed to enhance metacognitive and critical reasoning skills have a profound impact on students' conceptual understanding. The ANOVA results further confirmed significant differences across cognitive dimensions, with an F-value of 6.208 and $p = 0.003$. This supports the idea that instructional approaches do not affect all cognitive levels uniformly, as Chandrawati et al. (2023) noted that each cognitive level demands distinct instructional strategies.

Post hoc Tukey HSD analysis indicated that the reasoning dimension exhibited the largest improvement compared to comprehension and application, with significant mean differences of 9.404 ($p = 0.002$) and 6.254 ($p = 0.021$), respectively. These findings underscore the importance of differentiated instruction that presents cognitively challenging tasks to encourage students' critical thinking development. Choi & Kim (2017); & Coker (2010) similarly emphasized that challenge-based learning significantly enhances students' mathematical reasoning skills.

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The success of differentiated instruction in fostering significant gains in students' cognitive abilities can be attributed to several key aspects of this teaching approach. One of the most critical factors is the individualized attention and the ability to tailor lessons according to students' diverse learning needs. By offering a variety of learning materials and activities that accommodate different learning styles, differentiated instruction ensures that students are not only engaged but also challenged at their respective cognitive levels (Baybayon et al., 2024). This personalization, as emphasized by Tomlinson, allows students to progress at their own pace, ensuring deeper understanding and mastery of mathematical concepts (Jakfar Shodiq & Juniati, 2024). Additionally, providing varying levels of complexity and real-world applications in the lessons supports students' development across all three cognitive dimensions, comprehension, application, and reasoning by bridging theory with practice.

Moreover, the incorporation of formative assessments and ongoing feedback played a crucial role in the experimental group's success. Through regular assessments, educators could identify individual learning gaps and adjust the instruction, accordingly, ensuring that students remained on track to achieve their cognitive potential. This continuous feedback loop, aligned with the principles of differentiated instruction, encouraged students to reflect on their learning, thus fostering metacognitive skills essential for higher-order thinking and problem-solving (Sie & Darko Agyei, 2024). As suggested by Slavin (2018), this adaptive approach not only improves immediate academic performance but also equips students with the cognitive tools necessary for tackling complex, real-world challenges.

Beyond addressing cognitive development, the instructional design emphasizes creating an inclusive and student-centered learning environment. By integrating flexible learning activities that cater to varying proficiency levels, the approach ensures that all students regardless of their initial numeracy skills are actively engaged and supported. This inclusivity not only promotes equitable learning opportunities but also fosters a classroom environment that values individual growth and collaborative learning.

Furthermore, the Paired Samples Test analysis demonstrated that differentiated instruction generally had a significant impact on students' numeracy achievements, with a mean difference of -13.811 and a significance level of $p = 0.000$ or less than 0.050. These findings align with Purpura et al. (2011); & Segers et al. (2015), who highlighted that flexible and responsive instructional strategies significantly enhance students' numeracy skills. The consistent improvements across comprehension, application, and reasoning cognitive processes throughout the three instructional cycles (from 76.43 to 91.07 for comprehension; from 73.75 to 87.86 for application; and from 63.57 to 81.43 for reasoning) reaffirm the effectiveness of differentiated instruction in holistically enhancing students' numeracy outcomes.

These findings are also consistent with Vygotsky's Zone of Proximal Development (ZPD) theory, which posits that students can achieve higher cognitive levels with appropriate guidance and interventions tailored to their specific needs (Liu & Matthews, 2005; Zhou, 2020). By implementing differentiated instruction, students receive scaffolding that enables them to progress from their actual ability to their potential capacity.

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Consequently, this study underscores that differentiated instruction not only effectively enhances conceptual understanding but also promotes the development of critical thinking and problem-solving skills among students (Hidayati, 2020). As a practical implication, educators are encouraged to adopt a more systematic approach to differentiated instruction by addressing students' cognitive needs at various levels. Future research could explore how factors such as student characteristics and technology-based learning approaches can further enhance the effectiveness of differentiated instruction.

Hence, the findings demonstrate that a targeted instructional model grounded in differentiated instruction not only enhances students' conceptual understanding but also cultivates critical thinking and problem-solving skills. Educators are encouraged to systematically address students' cognitive needs by incorporating instructional strategies tailored to comprehension, application, and reasoning dimensions. Future research could investigate how student characteristics and technology-based learning tools can further optimize the effectiveness of such differentiated models in diverse educational contexts.

One of the challenges faced during the implementation of differentiated instruction was managing the diverse learning needs of students within a single classroom. Teachers had to balance varying levels of comprehension, application, and reasoning skills while ensuring that all students remained engaged and supported. To address this, educators employed flexible grouping strategies, allowing students to collaborate with peers at similar cognitive levels while also encouraging cross-group interactions for more complex tasks. Additionally, ongoing formative assessments were used to monitor progress and adjust instruction, accordingly, ensuring that all students received the necessary support. Despite these challenges, the implementation of differentiated instruction ultimately fostered a more inclusive learning environment, where each student could thrive at their own pace and level of understanding.

To demonstrate how differentiated instruction works in practice, a detailed lesson plan can highlight various strategies employed to meet the diverse needs of students. For example, in a math lesson on algebra, the teacher might start by introducing the core concept to the whole class. Afterward, students are grouped based on their readiness levels: advanced learners might work on more complex algebraic equations, while those who need more support might focus on basic problem-solving skills and practice foundational concepts. In each group, the teacher circulates to provide targeted support, offering more scaffolding to struggling students while providing enrichment opportunities for advanced learners.

Additionally, flexible grouping is used throughout the lesson to encourage peer collaboration. In one activity, students could work in mixed-ability pairs to solve problems together, allowing them to learn from one another. For more complex tasks, such as applying algebraic concepts in real-life scenarios, students are grouped by similar cognitive levels, ensuring that they can engage with the material in a way that is challenging yet achievable. Formative assessments, such as exit tickets or quick quizzes, are employed to gauge understanding and guide future instruction, ensuring that all students, regardless of their initial level, are able to progress at their own pace.

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CONCLUSION

This study provides compelling evidence for the effectiveness of differentiated instruction in enhancing students' cognitive numeracy abilities, particularly in the domain of reasoning. The significant improvements observed in comprehension, application, and reasoning dimensions underscore the value of tailored instructional strategies in fostering holistic cognitive development. The statistical significance of the improvements observed in comprehension, application, and reasoning highlights the profound contribution of differentiated instruction to students' numeracy growth. By catering to individual learning needs, this approach effectively fosters enhanced cognitive abilities, particularly in complex problem-solving and higher-order reasoning skills. These findings align with cognitive development theories, such as Vygotsky's Zone of Proximal Development, which emphasizes the importance of scaffolding learning experiences. However, the relatively modest gains in the application dimension highlight the need for more targeted approaches to strengthen students' practical problem-solving skills.

Given these insights, this study recommends the adoption of scaffolded learning pathways and the integration of real-world tasks that encourage active application of mathematical concepts. Teachers should be equipped with professional development programs that enhance their capacity to design and implement differentiated instruction. Furthermore, the integration of digital tools offers significant potential to personalize learning and provide immediate feedback, making instruction more responsive to diverse learning needs. Diversified assessment methods that evaluate comprehension, application, and reasoning will also ensure a comprehensive understanding of students' cognitive progress.

The findings contribute to advancing knowledge on differentiated instruction and offer practical insights for educators seeking to optimize classroom practices. Future research could explore how contextual factors such as school environment, teacher expertise, and resource availability impact the effectiveness of differentiation strategies. Longitudinal studies are also recommended to assess the sustained impact of differentiated instruction on students' academic and cognitive development, particularly in the application of problem-solving skills. By embracing these recommendations, educators can better foster students' numeracy achievement and cognitive growth.

REFERENCES

[1] Adams, C., Pente, P., Lemermeyer, G., & Rockwell, G. (2023). Ethical principles for artificial intelligence in K-12 education. Computers and Education: Artificial Intelligence, 4, 100131. https://doi.org/10.1016/j.caeai.2023.100131

[2] Atasoy, R., & Özden, C. (2020). The Effect of Coding on Students' Visual-spatial Reasoning Skills. Croatian Journal of Education - Hrvatski Časopis Za Odgoj i Obrazovanje, 22(3). https://doi.org/10.15516/cje.v22i3.3522

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[3] Atkinson, T., & Jackson, N. S. (2016). Beyond Economic Interests: Critical Perspectives on Adult Literacy and Numeracy in a Globalised World. Literacy and Numeracy Studies, 24(2), 64–67. https://doi.org/10.5130/lns.v24i2.5305

[4] Ayuningtyas, N., & Sukriyah, D. (2020). Analisis pengetahuan numerasi mahasiswa matematika calon guru. Delta-Pi: Jurnal Matematika Dan Pendidikan Matematika, 9(2). https://doi.org/10.33387/dpi.v9i2.2299

[5] Baybayon, G. G., Rose, M., & Lapinid, C. (2024). Effects of Differentiated Instruction in Flipped Classrooms on Students' Mastery Level and Performance in Quadratic Equations. In MATHEMATICS TEACHING RESEARCH JOURNAL 213 EARLY SPRING (Vol. 16, Issue 1).

[6] Bell, Lisa. M., & Aldridge, Jill. M. (2014). Student Voice, Teacher Action Research and Classroom Improvement (Barry J. Fraser, Ed.; 1st ed., Vol. 6). Sense Publishers.

[7] Bhattacharya, A. (2021). Advancing Teacher Candidates' Capabilities for Differentiated Instruction (pp. 77–99). https://doi.org/10.4018/978-1-7998-6816-3.ch004

[8] Chandrawati, A. E., Johar, R., & Elizar, E. (2023). Spatial reasoning skills of year 5 elementary school students through realistic mathematics education. 020020. https://doi.org/10.1063/5.0154629

[9] Chapman, O. (2013). Mathematical-task knowledge for teaching. Journal of Mathematics Teacher Education, 16(1), 1–6. https://doi.org/10.1007/s10857-013-9234-7

[10] Choi, H. H., & Kim, M. J. (2017). The effects of analogical and metaphorical reasoning on design thinking. Thinking Skills and Creativity, 23, 29–41. https://doi.org/10.1016/j.tsc.2016.11.004

[11] Coker, P. (2010). Effects of an experiential learning program on the clinical reasoning and critical thinking skills of occupational therapy students. Journal of Allied Health, 39(4), 280–286.

[12] Coubergs, C., Struyven, K., Vanthournout, G., & Engels, N. (2017). Measuring teachers' perceptions about differentiated instruction: The DI-Quest instrument and model. Studies in Educational Evaluation, 53, 41–54. https://doi.org/10.1016/j.stueduc.2017.02.004

[13] de Walle, J. A. Van, Karp, K. S., & Bay-Williams, J. M. (2016). Elementary and middle school mathematics: teaching developmentally (9th ed.). Pearson Education Limited.

[14] Eikeland, I., & Ohna, S. E. (2022). Differentiation in education: a configurative review. Nordic Journal of Studies in Educational Policy, 8(3), 157–170. https://doi.org/10.1080/20020317.2022.2039351

[15] Hair, J. F., Black, W. C., Babin, B. J., & Anderson, R. E. (2019). Multivariate Data Analysis (Eighth Edition). Annabel Ainscow. www.cengage.com/highered

[16] Hasanah, E., Suyatno, S., Maryani, I., Badar, M. I. Al, Fitria, Y., & Patmasari, L. (2022). Conceptual Model of Differentiated-Instruction (DI) Based on Teachers' Experiences in Indonesia. Education Sciences, 12(10), 650. https://doi.org/10.3390/educsci12100650

[17] Hidayati, V. R., Maulyda, M. A., Gunawan, G., Rahmatih, A. N., & Erfan, M. (2020). System of Linear Equation Problem Solving: Descriptive-Study about Students' Mathematical Connection Ability. Journal of Physics: Conference Series, 1594, 012042. https://doi.org/10.1088/1742-6596/1594/1/012042

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[18] Jakfar Shodiq, L., & Juniati, D. (2024). Differentiated Instruction Based on Lateral Thinking Techniques for Enhancing Mathematical Creativity. In MATHEMATICS TEACHING RESEARCH JOURNAL (Vol. 240, Issue 5).
[19] Keller, B. A., Hart, E. W., & Martin, W. G. (2001). Illuminating NCTM's Principles and Standards for School Mathematics. School Science and Mathematics, 101(6), 292–304. https://doi.org/10.1111/j.1949-8594.2001.tb17960.x
[20] Kirschner, P. A. (2002). Cognitive load theory: Implications of cognitive load theory on the design of learning. In Learning and Instruction. https://doi.org/10.1016/S0959-4752(01)00014-7
[21] Kostadinov, A. N., & Kolev, K. I. (2020). Application of a Differentiated Instruction Approach in Computer Systems Education. 2020 XI National Conference with International Participation (ELECTRONICA), 1–4. https://doi.org/10.1109/ELECTRONICA50406.2020.9305107
[22] Kurt, G., Önel, F., & Çakıoğlu, Ö. (2023). An investigation of Middle School Students Spatial Reasoning Skills. Lnternational Electronic Journal of Elementary Education. https://doi.org/10.26822/iejee.2023.319
[23] Lin, P., Lu, H., & Lin, Y. (2018). A Study of Knowledge Dimension and Cognitive Process Pattern of Cognitive Style Differences in STEM Cooperative Learning Environment. International Journal of Information and Education Technology, 8(10), 720–724. https://doi.org/10.18178/ijiet.2018.8.10.1128
[24] Lindner, K.-T., & Schwab, S. (2020). Differentiation and individualisation in inclusive education: a systematic review and narrative synthesis. International Journal of Inclusive Education, 1–21. https://doi.org/10.1080/13603116.2020.1813450
[25] Liu, C. H., & Matthews, R. (2005). Vygotsky's philosophy: Constructivism and its criticisms examined. In International Education Journal. BMC Palliative Care.
[26] Magee, M., & Breaux, E. (2013). How the Best Teachers Differentiate Instruction. Routledge. https://doi.org/10.4324/9781315855257
[27] Maulyda, M. A., Rahmatih, A. N., Gunawan, G., Hidayati, V. R., & Erfan, M. (2020). Retroactive Thinking Interference of Grade VI Students: A Study on the Topics of PISA Literacy Lessons. Journal of Physics: Conference Series, 1471(1), 1–7. https://doi.org/10.1088/1742-6596/1471/1/012037
[28] Maulyda, M. A., Sugiman, & Wuryandani, W. (2025). Mathematical Problem Solving, Representation, and Math Primary School Teachers. Revista Colombiana de Educacion, 96. https://doi.org/10.17227/rce.num96-19787
[29] OECD. (2019). PISA Result 2018: Combined Executive Summaries. www.oecd.org/about/publishing/corrigenda.htm.
[30] Perso, T. (2006). Teachers of Mathematics or Numeracy? Australian Mathematics Teacher, 62(2).
[31] Purpura, D. J., Hume, L. E., Sims, D. M., & Lonigan, C. J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy development. Journal of Experimental Child Psychology, 4(1), 145–156. https://doi.org/10.1016/j.jecp.2011.07.004

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[32] Rasmussen, K., & Isoda, M. (2019). The intangible task – a revelatory case of teaching mathematical thinking in Japanese elementary schools. Research in Mathematics Education, 21(1), 43–59. https://doi.org/10.1080/14794802.2018.1555714
[33] Reeves, Douglas. B. (2008). Reframing Teacher Leadership to Improve Your School (1st ed.). Association for Supervision and Curriculum Development.
[34] Santrock, J. W. (2016). Educational Psychology (6th ed.). McGraw-Hill Education.
[35] Schleicher, A. (2019). PISA 2018 insights and interpretations. OECD Publishing.
[36] Segers, E., Kleemans, T., & Verhoeven, L. (2015). Role of Parent Literacy and Numeracy Expectations and Activities in Predicting Early Numeracy Skills. Mathematical Thinking and Learning, 7(3), 1–18. https://doi.org/10.1080/10986065.2015.1016819
[37] Slavin, R. E. (2018). Educational psychology. In Psychological Bulletin (Vol. 25, Issue 7). Pearson. https://doi.org/10.1037/h0074121
[38] Smets, W., & Struyven, K. (2020). A teachers' professional development programme to implement differentiated instruction in secondary education: How far do teachers reach? Cogent Education, 7(1). https://doi.org/10.1080/2331186X.2020.1742273
[39] Sie, C. K., & Darko Agyei, D. (2024). Harmony in Teaching: Unraveling the Interplay between Pre-Service Teachers' Mathematical Knowledge Fractions and Classroom Practices. In MATHEMATICS TEACHING RESEARCH JOURNAL 59 Golden Fall (Vol. 16, Issue 4).
[40] Tomlinson, C. A. (2001). How to differentiate instruction in mixed-ability classrooms (2nd ed.). Association for Supervision and Curriculum Development.
[41] Tomlinson, C. A., & Eidson, C. C. (2003). Differentiation in practice: a resource guide for differentiating curriculum, grades 5-9 (1st ed.). Association for Supervision and Curriculum Development.
[42] Tomlinson, H. (2013). Educational Management. Routledge. https://doi.org/10.4324/9780203463383
[43] Tout, D. (2020). Evolution of adult numeracy from quantitative literacy to numeracy: Lessons learned from international assessments. International Review of Education, 23(3), 456–478. https://doi.org/10.1007/s11159-020-09831-4
[44] Valiandes, S., & Neophytou, L. (2018). Teachers' professional development for differentiated instruction in mixed-ability classrooms: investigating the impact of a development program on teachers' professional learning and on students' achievement. Teacher Development, 22(1), 123–138. https://doi.org/10.1080/13664530.2017.1338196
[45] van Geel, M., Keuning, T., Frèrejean, J., Dolmans, D., van Merriënboer, J., & Visscher, A. J. (2019). Capturing the complexity of differentiated instruction. School Effectiveness and School Improvement, 30(1), 51–67. https://doi.org/10.1080/09243453.2018.1539013
[46] Yong, T. H., & Sokumaran, N. (2023). Exploring self-regulated learning through differentiated feedback. Cogent Education, 10(1). https://doi.org/10.1080/2331186X.2023.2212113
[47] Zhou, J. (2020). A Critical Discussion of Vygotsky and Bruner's Theory and Their Contribution to Understanding of the Way Students Learn. Review of Educational Theory, 3(4), 82. https://doi.org/10.30564/ret.v3i4.2444

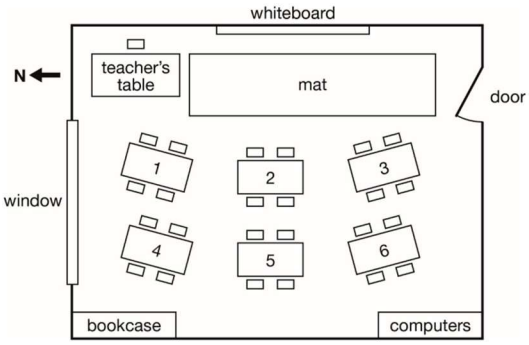
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APPENDIX

Pretest and Posttest Instrument

a. **Understanding**

1. The following image shows the classroom layout at an elementary school in Yogyakarta. The laboratory room in this school adopts the IRIS Centre laboratory arrangement. The teacher's desk is located at the front corner of the classroom. Four students sit at each of the six tables as shown in the image:



Below are several statements based on the provided layout. Choose 'True' or 'False' for each statement:

- The door is located near the southeast corner of the classroom.
- Viewed from the teacher's desk facing the back of the classroom, the computer is in the back right corner of the room.

2. Andi lives on the coast of Depok Beach, Yogyakarta. During his work break, he wants to go on a vacation to Mount Merapi. He departs from Depok Village to Pakem Village, located at the foot of Mount Merapi, using his private car. The following map shows the distance, route, and time needed to reach Pakem Village.

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Andi departs from Depok Village at 08:30 AM. After driving for 3 hours and 10 minutes, he takes a break at Ambarukmo Mall. After resting for 45 minutes at the mall, Andi continues his journey to Pakem Village. What time does Andi arrive at Pakem Village?

3. In a kindergarten class, there are 2 teachers and 22 students. The class is planning a field trip to the city park. The minimum required ratio is one adult for every 3 students. How many additional adults are needed for the trip?

b. **Application**

1. The Sekatenan celebration is a traditional festival regularly held at the Yogyakarta town square. This year, the Head of Baleharjo Village intends to participate in the festival. One of the required items is 500 traditional cakes. The price of one box containing 72 traditional cakes is Rp. 199,500. How many boxes of traditional cakes need to be purchased, and what is the total cost?
2. An elderly couple is participating in the "Steps for a Healthy Start" challenge. They must record the number of steps they take each day for 10 weeks. The grandfather's recorded steps over the last 6 weeks total 416,000, while the grandmother's total 262,000 steps. The average number of steps the grandfather and grandmother must take each day over the next 4 weeks is:
    - Grandfather: 6,000 steps and Grandmother: 8,500 steps
    - Grandfather: 3,000 steps and Grandmother: 8,500 steps
    - Grandfather: 3,000 steps and Grandmother: 8,000 steps
    - Grandfather: 6,500 steps and Grandmother: 8,500 steps
3. In 2019, the Indonesian National Team in the Triathlon event participated for the first time in the qualification championship for the 2024 Olympics. Triathlon is an endurance sport that includes three disciplines in a single competition: swimming, cycling, and running. Athletes compete to complete the race in the shortest possible time. The table below shows

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the distances athletes must cover in the competition. The total distance athletes must complete is 3,575 meters.

Swimming	Bicycle	Running
X	3 km	500 m

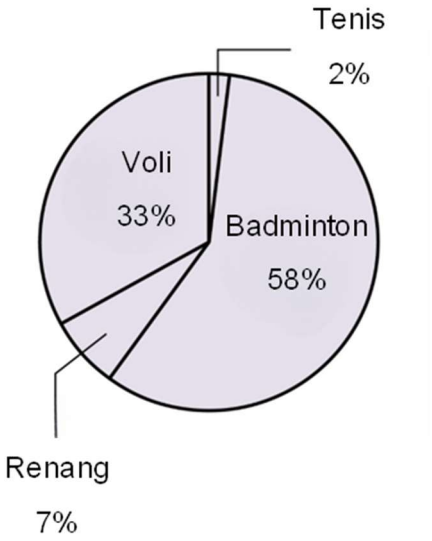
Based on the table provided, select 'True' or 'False' for each statement:

- The distance athletes must swim is 75 km.
- The distance athletes must swim is 85 m.
- The swimming distance is 0.85 kilometers.
- The running distance is the longest distance athletes must cover.

4. Using the illustration from the previous question, the two fastest athletes from Indonesia, Ardhan and Ilham, completed the race in 32 minutes and 33 minutes, respectively. Both athletes took 7 minutes to complete the swimming track. For the running track, Ardhan recorded a time of 12 minutes, while Ilham recorded a time of 10 minutes. If Ardhan's cycling track time was 13 minutes, how many minutes is the time difference between Ardhan's and Ilham's cycling track times?

5. **Reasoning**
   1. The World Health Organization (WHO) recommends limiting sodium consumption to 2,400 mg or about one teaspoon of salt per day. The likelihood of sodium deficiency for those living in Indonesia is very low because most foods in the country are high in sodium content. Even diet menus tend to be high in sodium, such as the South Beach Diet, which includes 2,300 to 6,700 mg of sodium per day.
      Foods high in sodium, such as chips, canned soup, processed meats, salted fish, soy sauce, tomato sauce, chicken broth cubes, and cheese, should be avoided or at least reduced, as they can cause high blood pressure (hypertension), which can trigger heart disease, stroke, and kidney problems. One 100-gram bag of potato chips contains 525 mg of sodium. On average, canned soup contains 700 mg of sodium. Below are several statements based on the provided information. Choose 'True' or 'False' for each statement:
      - The amount of sodium in one bag of potato chips exceeds the "adequate intake" level.
      - Consuming 2 bags of potato chips and 3 cans of soup is sufficient for daily dietary intake.
   2. The following pie chart shows the percentage of students participating in four extracurricular activities at school.

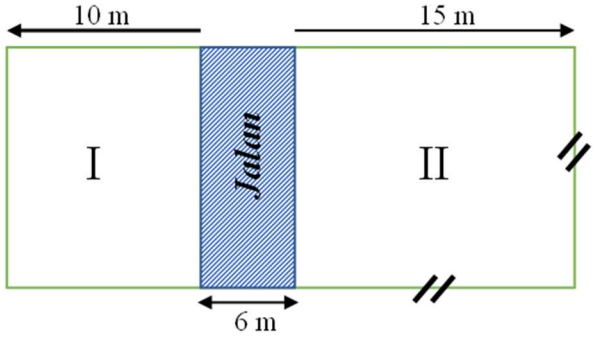
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Based on the given chart, select 'True' or 'False' for each statement:

- The number of students participating in Volleyball extracurricular activities is more than four times the number of students participating in Swimming extracurricular activities.
- The ratio of the total number of students participating in Tennis and Badminton extracurricular activities to those participating in Volleyball and Swimming extracurricular activities is 3 to 2.

3. A teacher coordinates the school gardening club. The club plans to clean the school garden, which has the following layout:



Sections I and II of the garden to be cleaned are the outermost parts.

- What is the perimeter of sections I and II?
- If one tree is planted every 2 meters, how many trees will there be?

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Comparison of the Effects of Explanatory Faded Worked Examples and Correct-Incorrect Worked Examples Methods on Sixth-Grade Students' Mathematics Performance*

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Abstract: Achieving high-level mathematics learning is a key goal of the mathematics curriculum; however, while some students excel, others encounter significant challenges. One strategy that has gained attention in recent years is the use of worked examples, a teaching method grounded in Cognitive Load Theory. This study compares the effects of two types of worked examples: Explanatory Faded Worked Examples (EFWE) and Correct and Incorrect Worked Examples (CIWE) on sixth-grade students' mathematics performance. A pre-posttest quasi-experimental design was used with 29 sixth-grade students from a public secondary school during the 2021–2022 academic year. Quantitative data were collected through a Mathematics Achievement Test and a Worked Examples Worksheet, while open-ended questionnaires captured student perspectives. The findings showed that the EFWE group had significant improvement in overall mathematics scores but no significant change in scores for specific subtopics like exponential notation, divisibility, or order of operations. In contrast, the CIWE group showed significant improvement in distributive property, exponential notation, and divisibility. ANCOVA results revealed that, after adjusting for pre-test scores, the CIWE group outperformed the EFWE group in order of operations and divisibility. Students from both groups found the Worked example methods useful, noting that they contributed positively to their learning experience.

Keywords: Cognitive load, worked example method, correct and incorrect Worked examples, explanatory faded Worked examples.

INTRODUCTION

Mathematics plays a crucial role in solving various problem situations encountered in daily life. Therefore, it is essential to develop students' numerical skills. One of the primary objectives of

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mathematics education is to ensure that students achieve the highest level of mathematical understanding (MoNE, 2018). However, while some students meet this goal successfully, others struggle to reach it. The abstract nature of mathematics often presents significant challenges for these students.

Research in mathematics education aims to identify effective methods to overcome learning difficulties arising from the abstract nature of mathematics and to enhance student success (Işık & Konyalıoğlu, 2005). Mathematics is integral to all stages of education, and numerous studies emphasize the importance of effective mathematics instruction (e.g., Franke & Kazemi, 2001). Recently, one of the suggested methods for students struggling with mathematics is the use of worked examples (e.g., Bokosmaty et al., 2015; Reisslein et al., 2006; Renkl & Atkinson, 2003; Van Loon-Hillen et al.,2012). Worked examples are learning activities based on Cognitive Load Theory (CLT) and involve demonstrating step-by-step problem-solving procedures. This research aims to compare the effects of two types of worked examples—explanatory faded and correct-incorrect worked examples—on the mathematics performance of sixth-grade secondary school students.

In the Turkish education system, the sixth-grade mathematics curriculum includes fundamental concepts such as the order of operations, exponents, distributive property, and divisibility rules. In other countries, these topics may be introduced at different grade levels. These concepts form the basis of many mathematical topics that students find challenging. Difficulties in these foundational topics can negatively impact students' ability to perform operations in other mathematical areas. This study aims to support learning by presenting the teaching of sixth-grade topics (order of operations, distributive property, exponents, and divisibility) using explanatory faded Worked examples and correct-incorrect Worked examples. During the pandemic period, students were left to learn independently, making the use of worked examples particularly beneficial in scenarios where students must learn material on their own.

Conceptual Background of the Study

Cognitive Load Theory and Worked Examples

Cognitive load is a learning theory that aims to learn effectively by presenting instructional design principles, based on the fact that the human mind has a limited working memory and an unlimited long-term memory (Clark et al., 2005; Wong et al., 2012). According to the Cognitive Load Theory, the information obtained by individuals during the learning process is processed in three different categories in working memory, according to the field of use: intrinsic load, extraneous load, and germane load. Intrinsic cognitive load refers to the cognitive demand imposed by the inherent complexity of the subject being taught, which is independent of the teaching methods or techniques used (Sweller, 1994). It reflects the difficulty of the content itself and is a key factor in determining the cognitive resources required for learning. Extrinsic cognitive load arises from the way content is organized and presented and depends on the teaching resources employed. This type of load places additional demands on working memory that go beyond the learning objectives and is often seen as unproductive if it does not contribute to schema formation (Sweller, 2011, 2020). Activities

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that do not support schema development are generally viewed as detrimental to the learning process (Sweller, 2005). Germane cognitive load is influenced by content and activities that facilitate the learning process and aid in schema formation. It is considered beneficial as it positively impacts learning by helping to build and integrate new knowledge (Sweller et al., 1998).

The Cognitive Load Theory aims to reveal teaching methods that will ensure that the cognitive load encountered by individuals who are educated in any field is presented in a way that is suitable for the cognitive capacities of the learners so that learning can take place effectively (Moreno & Park, 2010). Worked examples, which are one of the learning activities related to Cognitive Load Theory (Sweller, 1988), have the feature of reducing the external cognitive load and increasing the germane cognitive load (Paas et al., 2003).

Worked Example is a teaching method that shows the learner step by step how a problem is solved (Clark et al., 2005; Pershan, 2023). It is an effective method, especially for people who have not developed schemas related to the subject, to gain knowledge and skills. In this method, before solving the questions on his/her own, the person who will learn the subject is given solved examples one after the other. Worked examples create a model to show how the expert solves any problem, so that the learner applies the given solution first to similar problems and then to less similar problems. Thus, learning transfer takes place from similar examples to less similar examples (Sweller & Cooper, 1985). The way the worked examples are presented also plays a role in students' problem-solving performance (Bokosmaty et al., 2015; Renkl et al., 2002). A standard worked example provides students with solution steps and explanations, but what students need is to process this information mentally. Therefore, samples with different prepared solutions are needed.

Some of the types of worked examples explored in the literature include faded examples (Booth et al., 2013; İltüzer, 2016; Lee, 2013), examples with explanations (Emecen, 2020; Große, 2015; Hesser & Gregory, 2015; Özcan et al., 2018; Tepgeç, 2017), and correct-incorrect examples with solutions (Durkin & Johnson, 2012; Özcan et al., 2018). This research will focus specifically on correct-incorrect worked examples and faded worked examples with explanations. Examples with correct and incorrect solutions present a problem with both a correct solution and an incorrect solution (Siegler & Chen, 2008). By studying not only the correct solution steps but also the wrong solution steps, the learner increases his/her readiness for possible mistakes they may encounter in solving the problem (Tepgeç, 2017). In the examples with faded explanations and solutions, the question is first completely solved and then presented to the student. Then the solutions in the example are left blank step by step. The worked examples continue in this way until all steps of the solution are removed. Finally, only the question remains, and the learner is expected to solve the question (Eiriksdottir & Catrambone, 2012).

Worked examples are a learning activity grounded in Cognitive Load Theory. These examples help reduce extraneous load while increasing germane load (Cierniak et al., 2006). By incorporating worked examples at the early stages of skill acquisition, cognitive load unrelated to the subject

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matter is minimized. Therefore, this research aims to compare the explanatory faded worked example method with the true-false worked example method to determine which is more effective in facilitating learning.

Correct-Incorrect Worked Examples

In examples featuring correct and incorrect solutions, two approaches to a question are presented side by side—one being correct and the other incorrect. After acquiring the necessary information about the topic, learners typically analyze both solutions to identify which is correct (Siegler & Chen, 2008). This practice not only reinforces the correct steps but also prepares learners for potential mistakes they may encounter during problem-solving. Mainly, learning with incorrect solutions involves presenting these examples to enhance understanding after reviewing the correct solutions (Tepgeç, 2017).

Tennyson and Cocchiarella (1986) emphasized the benefits of using both correct and incorrect examples when learning new concepts, while Bransford and Schwartz (1999) noted that contrasting examples can highlight critical points and facilitate learning transfer. Incorrect worked examples, particularly when accompanied by explanatory prompts (Renkl, 2014), are thought to be effective in correcting misconceptions (e.g., Renkl, 2017). For instance, in the domain of decimals, where children often hold misconceptions (such as the belief that 0.6 is smaller than 0.365 because $6 < 365$), studying both correct and incorrect worked examples has been shown to improve conceptual understanding more than studying only correct examples (Durkin & Rittle-Johnson, 2012). However, the impact of prior knowledge on learning from incorrect worked examples is less definitive than with correct examples. Some studies suggest that incorrect worked examples benefit students with limited prior knowledge (Barbieri & Booth, 2016), while others indicate advantages only for those with more extensive prior knowledge (Große & Renkl, 2007). Additionally, some research finds no significant effect of prior knowledge (e.g., Durkin & Rittle-Johnson, 2012).

Große and Renkl (2007) reported that combining true and false solutions with self-explanation positively contributed to learning, particularly for students with strong mathematics performance. Özcan, Kılıç, and Obalar (2018) revealed that using true-false solution examples supported by explanatory clues improved students' mathematics performance, as they helped identify and correct mistakes when representing improper fractions on a number line. In another study, Booth et al. (2013) assessed whether true-false examples with self-explanatory prompts effectively enhanced students' understanding and procedural skills in algebra. Their findings indicated that combining correct and incorrect examples proved more beneficial than relying solely on correct examples in solving algebraic equations. In the current study, the students lack prior knowledge of the subject, as these topics have not been previously introduced. However, they possess the prerequisite knowledge necessary to understand the subject.

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Fading Worked Examples with Explanations

In fading worked examples with explanations, the fully solved example is presented first. Then, the step-by-step solutions in the example are left blank, and this process continues until all steps are removed. Thus, in the end, only the problem remains, and the learner is expected to solve it (Eiriksdottir & Catrambone, 2012). In this way, guidance diminishes in the transition from solved examples to problem solving. The fading worked examples are implemented in two different ways. One approach involves increasing the gaps in the solution steps from the very end (backward fading), while the other increases the gaps from the beginning to the end (forward fading). The results of the studies conducted in the literature support that examples with fading solutions increase academic success, especially in novice students. In their study, Hesser and Gregory (2015) introduced fading worked examples to students who were determined not to be mathematically prepared enough in a university chemistry course. As a result, they reported that students' examples with reduced solutions improved their general learning and problem-solving abilities in chemistry. Emecen (2020) aimed to examine the effect of using the worked example methods of fading and explanatory clue together for students with learning disabilities on the word problem-solving skills of students who have difficulties in mathematics lessons. According to the study's results, presenting worked example types of fading and explanatory clues together contributed to the verbal problem-solving skills of all participating students. It was observed that the learned information remained permanent after 2, 3, and 4 weeks of application.

In his research, Große (2015) explored whether using faded examples would be a better alternative to presenting fully worked examples. Additionally, he sought to find out whether increasing the gaps in the solution steps from the end (backward fading) is more effective than increasing them from the beginning to the end (forward fading) in illustrative faded examples. The results of the study indicated that backward fading, where gaps in the solution steps are introduced starting from the end, is more effective than forward fading. Similarly, Tepgeç (2017) aimed to examine the effects of traditional worked example and faded worked example methods for algorithm teaching on the success and cognitive load of students studying at university. He found that the faded worked examples used for algorithm teaching turned out to be more effective than traditional solved examples. It was concluded that there was no significant difference between the cognitive load levels of the students in the experimental and control groups, but in terms of efficiency of learning, examples with faded solutions were a more effective method.

Current Study

The motivation behind this study stems from the ongoing need to explore effective instructional strategies that help students overcome learning difficulties in mathematics, particularly in foundational topics like order of operations, exponents, the distributive property, and divisibility rules. While the literature on worked examples suggests that they are a valuable tool for reducing cognitive load and supporting learning (Sweller, 1988), there remains a gap in understanding the comparative effectiveness of different types of worked examples, such as explanatory faded examples and correct-incorrect examples, particularly for younger students in a secondary school context.

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This study aims to address this gap by examining the impact of teaching sixth-grade mathematics topics—such as order of operations, distributive property, exponents, and divisibility—through the use of explanatory faded examples and correct-incorrect worked examples, with the goal of identifying which method is more effective in improving students' learning outcomes. Although the students in this study have no prior knowledge of these specific topics, they possess the prerequisite knowledge necessary to engage with and understand the material. Based on the purpose of the study, the following research questions were addressed:

Quantitative Research Questions

1. Is there a statistically significant difference between the pre-test and post-test mathematics scores (exponents, order of operations, distributive property, divisibility rules) of the Explanatory Faded Worked Example (EFWE) group?
2. Is there a statistically significant difference between the pre-test and post-test mathematics scores of the Correct-Incorrect Worked Example (CIWE) group?
3. Which of the two instructional approaches (EFWE or CIWE) is more effective in improving sixth-grade students' mathematics performance after controlling for pre-test scores?

Qualitative Research Questions

4. What are the views of the EFWE group regarding the implementation of the instructional approach?
5. What are the views of the CIWE group regarding the implementation of the instructional approach?
6. What do students perceive as the benefits and challenges of EFWE and CIWE in their learning process?

METHODS

In this study, a quasi-experimental design with a pre-test and post-test was employed (Cook et al., 2002). To gain deeper insights into the impact of the intervention, after conducting the experiment, open-ended survey questions were posed to the students to gather their perspectives and opinions.

Study Group

The study group was determined in two stages. First, through convenient sampling, 40 sixth-grade students from two different classes at a public school in Istanbul, where one of the researchers worked during the 2021-2022 academic year, were selected. The participating students were, on average, around 11 years old, corresponding to the typical age for sixth-grade learners in the Turkish education system. A pre-test containing four mathematical operations questions was adminis-

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tered to assess the prerequisite skills for learning sixth-grade topics (order of operations, distributive property, exponents, and divisibility). Students who scored at least 80 out of 100 were chosen for the study group. The research was conducted in two classes, with one class assigned as the Explanatory Faded Worked Examples (EFWE) Experimental Group and the other as the Correct-Incorrect Worked Examples (CIWE) Experimental Group. The EFWE group consisted of 14 students (7 girls and seven boys), while the CIWE group included 15 students (7 girls and eight boys).

Data Collection Tools

Both quantitative and qualitative data collection tools were employed in the study. For the quantitative data, students were selected based on their performance on the Four Operations Questions, designed by the researcher to assess core prerequisite operational skills. The Mathematics Achievement Test was administered both as a pretest and posttest, alongside the Worked Examples Worksheet (focusing on topics like order of operations, distributive property, exponents, and divisibility), to evaluate the impact of the instructional intervention. To assess the effectiveness of the applied method, qualitative data were collected through students' responses to open-ended survey questions regarding their opinions. A detailed description of the data collection tools used in the study is provided below.

Four Operations Questions

To ensure that the participants possessed the prerequisite skills necessary for the mathematical concepts being taught, the Four Operations Questions were administered. The questions were designed to select students for the study and consisted of 10 questions: 2 addition, 2 subtraction, 3 multiplication, and 3 division questions. Each correct answer was awarded 10 points, while incorrect or unanswered questions received 0 points. Students were asked to solve the questions within one lesson hour using pencil and paper. Three sixth-grade mathematics teachers from the same school assessed the papers. In cases of differing scores for the same question, the teachers met to reach a consensus on a standard score. Students who scored 80 points or below were eliminated from the study.

Mathematics Achievement Test

The Mathematics Achievement Test was used both as a pre-test and a post-test in the study. It consisted of 10 questions aligned with the sixth-grade objectives in the numbers learning area, focusing on order of operations, distributive property, exponents, and divisibility rules. The test was initially prepared by the researchers. A table of specification was prepared, and expert opinions were obtained from four mathematics teachers to refine the test and remove repetitive question types. After obtaining the necessary permissions (ethics committee approval, research permissions, student and parental consent), a pilot study was conducted in a public school. Based on the

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results, any deficiencies in the test were addressed, and the final version was prepared for implementation. A scoring system was developed for the 10 questions, where the first two questions were worth 5 points each for a complete and correct answer, and the remaining questions were worth 10 points each. The remaining questions were scored as follows: 0 points for completely incorrect or unanswered questions, 3 points for partial or incorrect solutions, 6 points for selecting the correct strategy but with an incorrect solution, and 10 points for correct solutions. The maximum score for the test was 90 points. The questions were all at the application level according to Bloom's taxonomy. The study groups were asked to solve the questions within one lesson hour (40 minutes) using pencil and paper. The papers were evaluated by four mathematics teachers working in the same school and teaching sixth-grade classes. For different scores given to the same question, the answers were re-evaluated with the teachers, and a common grade was given after reaching a consensus.

Worked Example Worksheets

Two worksheets have been prepared for both experimental groups according to the types of worked examples. At the beginning of the worksheets, there was a short explanation about the subject. Students have the chance to refer back to this information while solving or examining the questions related to the worked examples. For example, at the beginning of the worksheet on order of operations, students are presented with what order of operations is and the order of precedence of operations.

In the worksheet prepared with the explanation, the EFWE method, one example was solved step by step, completely and correctly, with explanations in brackets. Then, for the questions similar to the first example, the steps were left blank starting from the end, and the student was asked to solve them. The steps of the solution of the last example were left completely blank, and the student was asked to solve the question. This information has been presented to students at the beginning of the worksheet. Examples of the intervention of examples with faded solutions with explanations are provided in Appendix 2.

In the worksheet prepared using the CIWE method, an example problem and its solution were provided. For the other problems, two solutions were given: one correct and one incorrect. Students were first asked to analyze the sample problem and its solution. Then, they were required to review solutions provided by other students, answer questions about those solutions, and finally solve their assigned problem. An example of this application is provided in Appendix 3.

The intervention worksheets with solutions, provided as examples for the EFWE and CIWE study groups, were prepared on topics such as order of operations, the distributive property, exponents, and divisibility, all of which are sub-dimensions of number learning—subsequently, four mathematics teachers from the same school as the researcher reviewed the questions for accuracy.

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Open-ended Questions Regarding EFWE and CIWE Interventions

After working with the experimental groups, open-ended questionnaire questions were administered to gather students' opinions, as the aim was to conduct a more detailed analysis of the research. These questions were designed to explore students' experiences during the Worked Example intervention. Open-ended questions allow respondents to answer freely, providing the researcher with more detailed and sometimes unexpected insights (Büyüköztürk, 2005). The open-ended survey questions were developed by the researcher in collaboration with an expert. Two Turkish teachers from a public school then reviewed the questions for clarity and grammar. After necessary revisions, the questions were finalized. In the pilot study, the question 'Do you have anything to add?' was removed from the final version, as students did not respond to it.

Data Collection

Before the data collection phase, the necessary permissions (ethics committee approval, research authorization, student participation consent forms, and parental consent forms) were obtained. A pilot study was conducted, and adjustments were made to the data collection tools and experimental procedures. Following this, the main study was carried out.

Data Analysis

Quantitative data were analyzed using a statistical software package. Before seeking answers to the research questions, descriptive statistics calculated for the scores of both groups regarding the dependent variables were presented. After the descriptive analyses, a dependent samples t-test was conducted to compare the pre-test and post-test mathematics performances of the EFWE and CIWE experimental groups. An ANCOVA test was conducted to answer whether there was a significant difference in the post-test scores of the EFWE and CIWE groups. The ANCOVA test is a test that equalizes the differences determined in the pretest scores of the experimental and control groups in experimental studies (Büyüköztürk et al., 2018). In this study, the ANCOVA test was preferred because there was a statistically significant difference in pre-test scores.

The opinions of the EFWE and CIWE groups about the interventions were analyzed using thematic analysis. Thematic analysis is a method that identifies, analyses, and reports the patterns in the data. Thematic analysis enables data to be organized and a rich description to be made. The thematic analysis in this study is based entirely on the data obtained; it does not depend on a predetermined coding or the analytical prejudice of the researcher. The data coding process, which is conducted without the analytical prejudice of the research, is called the inductive approach (Boyatsiz, 1998). The themes identified in the inductive approach are tightly linked to the data (Patton, 1990). This study was evaluated by a thematic analysis method with an inductive approach.

The answers to the open-ended questionnaire questions were taken from the students in written form. In order to ensure the reliability of codes and categories, coding harmony was ensured with

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another researcher. In cases where there was no coding harmony, a common decision was made, and a consensus was reached.

Experimental Process

The study was conducted by the first author in a state school in the 2021-2022 academic year during mathematics class hours. The students in the study group were divided into two groups, namely, the EFWE experimental group and the CIWE experimental group. Both study groups were administered a pre-test measuring the students' four operation skills, which is a prerequisite for participating in the study of the sub-dimensions of the number learning domain (operation priority, distributive property, exponents, and divisibility). Out of 40 students who participated in the pre-test, 11 students were eliminated, and the study was continued with 29 students. A two-week study was implemented with the students using the EFWE and CIWE worksheets. The table detailing the applied study is provided below.

	EFWE Experimental Group n = 14	**CIWE Experimental Group n = 15**
One week before the interventions	Four operation questions Mathematics Achievement test	Four operation questions Mathematics Achievement test
Interventions (Two weeks)	Worksheet based on EFWE Method	Worksheet based on CIWE Method
One week after the intervention	Mathematics Achievement Test Open-ended Questionnaire Questions Regarding EFWE Intervention	Mathematics Achievement Test Open-ended Questionnaire Questions Regarding EFWE Intervention

Table 1: Experimental process of the study

Worksheets on the order of operations, the distributive property (first week), exponents, and divisibility rules (second week) were distributed to the respective groups. Each of the studies lasted approximately 2 lesson periods (80 minutes). The teacher was only the supervisor in the study. After the 2-week study, a post-test containing the same questions as the pre-test was administered. Quantitative analyses were made on the data obtained from these tests, and the extent to which the study affected the performance of the students in learning the subjects was investigated.

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RESULTS

Preliminary Analysis

Before addressing the research questions, the normality of the data distribution was assessed based on skewness and kurtosis values. Considering that the skewness and kurtosis coefficients of almost all the dependent variables in Table 2 are between -1 and +1, and those not in this range do not deviate significantly from these values, it was concluded that the data have a normal distribution (George & Mallery, 2010). Following this, the pre-test results of the groups are compared as a preliminary study. Pretest results showed significant differences between the groups in favor of the CIWE group on the EN ($t(28) = -2.37$, $p < 0.05$), OO ($t(28) = -2.23$, $p < 0.05$), DR ($t(28) = -2.67$, $p < 0.05$), and total scores ($t(28) = -2.99$, $p < 0.01$), while no significant difference was found on the DP scale ($t(28) = -0.36$, $p > 0.05$) at baseline (see Table 2).

Pretest and Posttest Differences in the EFWE and CIWE Groups

		Descriptive Statistics		Comparison between the EFWE and CIWE groups
		EFWE Score mean (SD)	CIWE Score mean (SD)	
EN	**Pretest**	5.5 (5.37)	10.07 (5.01)	t=-2.37, p< 0.05
	Posttest	11.43 (4.97)	14.00 (2.80)	$F_{(1,28)}$=2.68, p = 0.11; partial η2 = 0.09
OO	**Pretest**	3.93 (5.61)	8.67 (5.81)	t=-2.23, p< 0.05
	Posttest	3.57 (5.35)	11.00 (6.32)	$F_{(1,67)}$=6.34, p<0.01; partial η2 = 0.196
DP	**Pretest**	7.86 (6.99)	8.67 (4.81)	t=-0.36, p > 0.05
	Posttest	17.14 (4.69)	18.67 (3.52)	$F_{(1,67)}$=1.10, p>0.01; partial η2 = 0.04
DR	**Pretest**	2.86 (4.69)	8.00 (5.61)	t=-2.67, p< 0.05
	Posttest	11.71 (10.58)	26.37 (11.13)	$F_{(1,67)}$=8.9, p<0.01; partial η2 = 0.26
Total	**Pretest**	20.14 (15.58)	35.40 (11.22)	t=-2.99, p< 0.01
	Posttest	57.55 (20.05)	70.33 (16.20)	$F_{(1,67)}$=8.18, p<0.001; partial η2 = 0.12

Table 2: Descriptive statistics and comparison of two experimental groups

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Note. EFWE = Explanatory Faded Worked Example; CIWE = Correct-Incorrect Worked Example; EN = Exponential Numbers; OO = Order of Operations; DP = Distributive Property; DR = Divisibility Rules.

First, the analysis examines whether there is a significant difference between the pre-test and post-test mathematics scores of the EFWE group. According to the paired sample t-test, there is no significant difference in the EFWE group for exponential numbers ($t=-2.83$, $p>0.05$), order of operations ($t=0.19$, $p>0.05$), or divisibility ($t=-2.45$, $p>0.05$). The pre-test score for exponential numbers increased from 5.50 to 11.43, but this difference is not statistically significant. Similarly, the divisibility score increased from 2.86 to 11.71, with no significant difference. However, for the distributive property, a significant difference was found ($t=-4.19$, $p<0.05$), with scores rising from 7.86 to 17.14. A significant difference in total scores was observed in favor of the post-test ($t=-11.98$, $p < 0.001$).

Second, the difference in pre-test and post-test scores within the CIWE group is explored. Paired sample t-test results show significant differences in favor of the post-test scores for the CIWE group in exponential numbers ($t=-2.82$, $p<0.05$), distributive property ($t=-1.45$, $p<0.05$), and divisibility ($t=-6.09$, $p<0.01$). Score of exponential numbers increased from $\bar{X}=10.07$ to $\bar{X}=14.00$, distributive property from $\bar{X}=8.67$ to $\bar{X}=18.67$, and divisibility from $\bar{X}=8.00$ to $\bar{X}=26.67$. This indicates a significant improvement. No significant difference was found for order of operations ($t=-1.45$, $p>0.01$), with scores rising from $\bar{X}=8.67$ to $\bar{X}=11.00$. Total pre-test scores increased significantly from $\bar{X}=35.40$ to $\bar{X}=70.33$ ($t=-11.18$, $p<0.01$), reflecting the positive impact of the group study.

Comparison of Post-test Scores Between the EFWE and CIWE Groups

The post-test scores between the EFWE and CIWE groups are compared to assess the effectiveness of each approach, using an ANCOVA test to determine whether there is a significant difference in post-test performance between the two methods in the teaching of 6th-grade mathematics topics. Initially, it was assessed whether the "homogeneity of regression slopes" assumption, a crucial requirement for ANCOVA, was met for EN, OO, DP, DR, and total scores. To this end, the F-values for the interaction between the group type (independent variable) and the pre-test scores of the dependent variables were determined. The F-values were found to be 0.46 ($p=0.50$) for EN, 2.25 ($p=0.15$) for DP, 0.74 ($p=0.40$) for OO, and 0.78 ($p=0.38$) for DR, with all of the p-values greater than 0.05. This result is interpreted as fulfilling the condition of matching regression slopes needed for ANCOVA. To examine another important assumption of ANCOVA, Levene's test was performed; the result ($F = 1.38$, $p > 0.05$ for EN, $F = 0.67$, $p > 0.05$ for DP, $F = 0.46$, $p > 0.05$ for OO and $F = 0.28$, $p > 0.05$ for DP) of Levene's test for equality of variances specified that the assumption of the homogeneity of variances in the groups was satisfied. After these assumptions were satisfied, the ANCOVA test was conducted on the dependent variables.

Controlling for EN pre-test scores, no significant difference emerged between the groups regarding EN post-test scores ($F(1,28) = 2.68$, $p > 0.05$). The effect size, a measure of the magnitude and direction of the observed difference, was calculated as 0.094. Based on Kılıç (2014), since this value exceeds 0.06, it is classified as a medium effect, indicating a moderate impact. For DP post-

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test scores, after controlling for the pre-test, the ANCOVA analysis showed no significant difference between the groups ($F(1,28) = 1.10$, $p > 0.05$). The effect size was calculated at 0.041, which Kılıç (2014) identifies as a small effect due to being greater than 0.01, reflecting a low impact in this area. In the case of OO post-test scores, a significant difference was found between the groups ($F(1,28) = 6.34$, $p < 0.05$) after adjusting for OO pre-test scores. Post hoc tests indicated that this difference favored the EFWE group. The effect size, calculated at 0.196, is considered significant as it surpasses 0.14, signifying a substantial impact. Similarly, for the DR variable, with $F = 8.93$ and $p = 0.01$, the ANCOVA results indicated a significant difference between the groups, favoring the EFWE group after controlling for the pre-test. The effect size of 0.256 further qualifies as large according to Kılıç (2014), highlighting a substantial effect from the intervention. Finally, when the posttest results of the total score are examined, it is seen that the difference between the groups is significant ($F(1,67) = 8.18$, $p < 0.001$) favoring the EFWE group after controlling for the pre-test; in addition, the partial $\eta^2 = 0.12$ indicates that the effect of this difference is moderate.

Participant Views on the Implementation of EFWE and CIWE Methods

The views of both the EFWE and CIWE groups regarding the implementation are analyzed to gain insights into their experiences with the instructional methods. This study employed a thematic analysis to compare the experiences and feedback of two experimental groups. The analysis was based on responses to open-ended questions. The responses were categorized into key themes, including perceptions and evaluations of the study, challenges faced in problem-solving, and students' satisfaction with the overall learning process.

Perceptions and Evaluations of the Study by the EFWE and CIWE Groups

Of the 14 students in the EFWE group, ten reported that they found the study useful, particularly emphasizing how the structured approach to problem-solving enhanced their understanding of mathematical concepts. As one student explained, "*Seeing the answers to the questions answered in front of me really helped me to see where I went wrong*." Another student similarly noted, "*The study helped me grasp the topics that I previously struggled with*." On the other hand, four students indicated that they found the process boring and uninteresting. One student described the experience as follows: "*I didn't enjoy the process; it felt repetitive and boring.*" Taken together, these findings suggest that while the majority of students benefited from the structured format of the EFWE approach, a smaller group felt disengaged, possibly due to a lack of interactivity or variation in the method.

The CIWE group showed a more positive response to the study overall. Out of the 15 students, nine indicated that the study made a significant contribution to their learning. Three students found the approach particularly explanatory, while ten described it as enjoyable, and three noted that they considered it fun. Students highlighted both the affective and cognitive benefits of the method. For instance, one student explained: "*The study helped me understand things in a way that made the topics more enjoyable*." Another reflected: "*I found the process interesting, and I could relate it*

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to real-life situations." The interactive and dynamic aspects of the approach were also emphasized, with one participant remarking: "*The study was not only helpful but also fun to participate in.*"

Taken together, these findings underscore the potential of dynamic and engaging methodologies such as CIWE to foster a positive learning environment and to enhance both understanding and motivation among students.

Challenges in Problem Solving

A significant portion of the EFWE group expressed difficulties in solving the provided exercises. Out of 14 students, seven reported struggling with understanding and completing the questions. As one student explained: "*I struggled with understanding the problems, especially when there were multiple steps involved*." Another similarly noted: "*Without the teacher explaining every detail, I got lost in some of the problems*." At the same time, a small number of students emphasized the importance of teacher explanations for better comprehension, while five students reported that they found the tasks easy and understandable. For instance, one participant stated that the exercises were clear and manageable without further support. These findings suggest that while the structured format of EFWE can provide clarity for some learners, others may still require additional scaffolding and direct teacher guidance to engage with problem-solving tasks confidently.

The CIWE group, by contrast, reported very few difficulties with problem-solving. Out of 15 students, only two indicated that they had trouble understanding the questions, and one mentioned difficulty in explaining the answers. For example, one student explained: "*Some problems were a bit difficult, but I managed to solve them after thinking about it for a while*." Another noted: "*I had a hard time understanding a couple of concepts, but overall it was manageable*." In contrast, the majority of students (11 out of 15) stated that they found the tasks easy and understandable. This overall pattern suggests that the dynamic and interactive nature of the CIWE approach helped most students to engage effectively with the exercises and to develop greater confidence in their problem-solving abilities.

Students' Satisfaction with the Overall Learning Process

To gather students' opinions about the learning process, they were asked whether they would like to study other mathematics subjects using the same method and to explain their reasons. In the EFWE group, 10 out of 14 students responded positively, while 4 stated that they would not prefer this method for other topics. Among those who answered "yes," five emphasized the usefulness of the method and one highlighted the benefit of having complete solutions. As one student expressed: *"I would like to because this method both explains and solves."* In contrast, less satisfied students described the lessons as repetitive, with one noting: *"I learned from the examples, but the lessons felt long and boring."*

In the CIWE group, almost all students expressed willingness to apply the method to other mathematics subjects. Fourteen students answered "yes," with ten indicating that they found the helpful

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method and three describing it as easy and enjoyable. Only one student reported that they would not prefer to continue with this approach. Positive evaluations included remarks such as: *"The lessons were fun, and I felt like I was learning without getting bored,"* and *"It did not feel like a regular math class. I enjoyed the process and felt like I learned a lot."*

These findings suggest that while both groups showed a generally positive attitude toward extending the method to other topics, satisfaction was markedly higher in the CIWE group. The interactive and dynamic features of the CIWE approach appeared to resonate strongly with students, making the learning experience more enjoyable and effective.

DISCUSSION

This study aimed to evaluate the effectiveness of two instructional methods—Explanatory Faded Worked Examples (EFWE) and Correct-Incorrect Worked Examples (CIWE)—on the mathematical performance of sixth-grade students, as well as to compare their relative effectiveness. The results reveal that both methods influenced student performance, but with varying degrees of success across different mathematical concepts, such as the distributive property, exponents, divisibility, and the order of operations. These findings contribute to the ongoing discourse on the effectiveness of worked-example strategies in mathematical instruction, particularly highlighting the role of scaffolding and error correction in learning.

In the groups where EFWE methods were applied, there was no significant difference between the pre-test and post-test mathematics scores in the subdimensions of order of operations, exponents, and divisibility. However, a significant difference was found in the subdimension of distributive property and the overall math score, indicating that the application had a positive effect on the distributive property and students' overall math performance, though it did not influence the areas of exponents, divisibility, and order of operations. Of the students who participated in the EFWE study, 10 found the activity beneficial, while four expressed that they found it boring and did not enjoy it. Most students found the EFWE valuable method because it allowed them to follow the provided procedural steps in the examples, reach solutions, and learn at their own pace.

In the study, the work on order of operations, exponents, and divisibility with the group that used the EFWE method was insufficient. A longer duration of study on these topics would be beneficial. Although there was a considerable difference between pre-test and post-test scores in the subtests for exponents, divisibility, and order of operations, this difference was not statistically significant. In the literature, there are studies that comparatively test the positive impact of faded worked examples. For instance, Tepgeç (2017) investigated the effects of standard worked examples and faded worked examples on college students' achievement and cognitive load in algorithm teaching and found that faded worked examples were more effective than standard ones. Foster et al. (2017) observed in their study on students learning principle-based concepts that those using faded worked examples showed superior performance in controlling solutions between examples and problem-solving tasks. Emecen (2020) aimed to examine the effects of combining faded worked

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examples with explanatory cues on students with learning difficulties in terms of their verbal problem-solving skills. This study concluded that the integration of faded worked examples and explanatory cues contributed to verbal problem-solving skills for all participating students. Recent research with underprepared college students in computational courses found that faded worked examples enhanced problem-solving skills and improved understanding of course material (Hesser & Gregory, 2015). The findings of the current study are consistent with prior research indicating that faded worked examples are more effective than standard worked examples. In the cited studies, the outcome was the overall mathematics score; using the same outcome in our data, fading was also effective and did not differ significantly from the CIWE condition. However, content-specific (topic-by-topic) analyses revealed that, for some mathematical concepts which fading showed no detectable advantage. This pattern suggests that the impact of fading may be topic dependent, underscoring the need for further research to determine when and for which mathematical topics fading should be preferred, and when alternative approaches (e.g., CIWE) may be equally or more appropriate.

The EFWE method appears to support students' procedural understanding, as they could follow step-by-step solutions in distributive property tasks. However, it was less effective for topics requiring deeper conceptual understanding, such as exponents and order of operations, where fading guidance may have left some students without sufficient conceptual support. The lack of significance may alsobe due to the sample size. Even though the distribution was normal, the small sample size may have prevented statistically significant results (Kılıç, 2014).

In the group using the CIWE method, a significant difference was observed in favor of the post-test scores for divisibility, distributive property, and exponents, though not for order of operations. This indicates that the application was largely practical in improving children's math performance. The studies on order of operations were insufficient for both groups. Additionally, when considering the total post-test mathematics scores, a significant difference was found in favor of the post-test, demonstrating the positive impact of the study on students' overall math performance. Based on student feedback, the CIWE application was found beneficial for teaching math topics. Students reported that they greatly enjoyed the study as they played an active role in the process. Furthermore, the presentation of both correct and incorrect solutions provided valuable opportunities for self-reflection, enabling students to recognize and address their own mistakes more effectively.

While most worked example studies focus on the use of correct examples, recent studies suggest that using both correct and incorrect worked examples together may be more effective for students (Durkin, Rittle, and Johnson, 2012; Rittle & Johnson, 2006; Siegler, 2002; Siegler and Chen, 2008). In their study, Durkin and Johnson (2012) examined whether working with correct and incorrect worked examples could help eliminate misconceptions and support students' understanding of decimal fractions. By presenting common misconceptions alongside correct solutions, they helped students focus on decimal concepts and improve both their conceptual and procedural knowledge. Booth et al. (2013) aimed to investigate whether self-explanation prompts given with correct and incorrect worked examples could be practical in helping students understand topics and improve their algebraic skills when combined with guided practice. Their study concluded that

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using both correct and incorrect examples was more beneficial than using correct examples alone for solving algebraic expressions.

In another study, Özcan et al. (2018) sought to determine the impact of correct-incorrect worked examples with explanatory hints on students' mathematical performance by identifying and addressing mistakes students made when placing improper fractions on a number line. They found that this method positively impacted students' performance. Große and Renkl (2007) also examined the impact of correct-incorrect worked examples and self-explanation on learning, noting that using both types of examples positively contributed to the learning of students who already had strong math skills.

According to the analysis results on whether post-test scores differed between groups, there was no significant difference in the subgroups of distributive property and exponents. However, a significant difference in favor of EFWE was found in the divisibility and order of operations subdimensions. Looking at the overall mathematics scores of the study, a significant difference was also found in favor of EFWE, indicating a positive impact on total mathematics performance. Based on eta-squared results, 0.01 is considered a low effect size, 0.06 a medium effect size, and 0.14 and above a large effect size (Kılıç, 2014). In this study, ANCOVA results showed that distributive property and exponents had medium effect sizes, while divisibility and order of operations had large effect sizes. Although no statistically significant difference emerged, the high eta-squared values indicate a group difference in favor of EFWE. In the literature, the fading procedure has been reported to yield higher learning outcomes compared to traditional worked example–problem pairs (e.g. Atkinson, Renkl and Merrill, 2003; Fleischmann and Jones, 2002; Renkl, Atkinson, Maier and Staley, 2002). In our study, however, the fading procedure appeared to produce lower learning outcomes when compared to true–false worked examples, suggesting that fading was less effective than the true–false approach in this context.

When reviewing the literature, no studies were found that directly compared EFWE and CIWE; thus, no directly comparable results were available. However, a recent meta-analysis examining the effectiveness of different types of worked examples highlights the effectiveness of correct-incorrect worked examples. In correct-incorrect worked examples, students are allowed to conduct a form of error analysis by comparing a wrongly solved question with a correctly solved example, which facilitates deeper conceptual learning. The approach of analyzing one's own or a peer's mistakes (Zrebiec, Uberti, Mastropieri, and Scruggs, 2004) is noted in the literature as a method that aids learning. In faded worked examples, creating multiple questions for the fading process in multi-step problems is often necessary, which, as participants have indicated, can become tedious. Instead of solving numerous questions within the same skill, comparing correct-incorrect worked examples may be more beneficial for understanding the topic. Explanatory Faded Worked Examples (EFWE) and Correct–Incorrect Worked Examples (CIWE) reflect scaffolding principles. EFWE parallels scaffolding through the gradual fading of support, while CIWE mirrors scaffolding by drawing attention to misconceptions. Similarly, research on IT-assisted scaffolding in abstract algebra showed that structured guidance helped students recognize and correct errors, improving their reasoning (Warli, Rahayu, & Cintamulya, 2025).

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Theoretical and Practical Implications

These findings add to the growing body of literature on worked-example methods in education. While Tepgeç (2017) and Sweller (1999) highlight the advantages of fading guidance for procedural learning, the current study suggests that this approach is not universally applicable across all mathematical topics. The EFWE method, while effective for certain concepts, may not offer enough structured support for more abstract topics like exponents and the order of operations. Conversely, the success of the CIWE method in addressing misconceptions aligns with earlier research advocating for the use of both correct and incorrect worked examples to enhance learning (Rittle-Johnson, 2006; Siegler & Chen, 2008). This method's effectiveness in improving procedural skills, particularly in the context of divisibility and exponents, suggests that incorporating error analysis into instructional practices can significantly benefit students.

Limitations and Future Directions

One limitation of this study is the relatively small sample size, which may have affected the statistical significance of some findings. Although the observed improvements in exponents and the order of operations are promising, larger studies are necessary to confirm these results (Kılıç, 2014). Additionally, the short duration of the intervention may have limited students' engagement with complex topics, suggesting that future studies should extend the study period and provide more focused instruction on challenging concepts.

Another limitation is the absence of control groups, which would have allowed for a more rigorous comparison between the two methods. Future research should incorporate control groups and more objective measures, such as cognitive load assessments and long-term retention tests, to better understand the impact of these instructional methods.

CONCLUSIONS

Overall, this study demonstrates the differential effectiveness of EFWE and CIWE methods in teaching various mathematical topics. Educators should carefully consider which method best aligns with the specific concepts they are teaching. For higher-order thinking skills, such as exponents and the order of operations, extending the guidance provided by the EFWE method may be necessary. On the other hand, the CIWE method is particularly effective for addressing misconceptions in areas like divisibility and the distributive property. Future research should explore a hybrid approach, combining the scaffolding of EFWE with the error analysis of CIWE, to create a more comprehensive instructional framework for mathematics education.

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REFERENCES

[1] Barbieri, C. A., Booth, J. L., & Chawla, K. (2023). Let's be rational: Worked examples supplemented textbooks improve conceptual and fraction knowledge. *Educational Psychology, 43*(1), 1–21. https://doi.org/10.1080/01443410.2022.2144142

[2] Bokosmaty, S., Sweller, J., & Kalyuga, S. (2015). Learning geometry problem solving by studying worked examples: Effects of learner guidance and expertise. *American Educational Research Journal, 52*(2), 307–333. https://doi.org/10.3102/0002831214549450

[3] Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24-34. https://doi.org/10.1016/j.learninstruc.2012.11.002

[4] Bransford, J. D., & Schwartz, D. L. (1999). Rethinking transfer: A simple proposal with multiple implications. *Review of Research in Education, 24* (1), 61-100. https://doi.org/10.3102/0091732X024001061

[5] Büyüköztürk, Ş. (2005). Anket geliştirme [Questionnaire development]. *Türk Eğitim Bilimleri Dergisi, 3*(2), 133-151.

[6] Büyüköztürk, Ş., Kılıç Çakmak, E., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2018). *Eğitimde bilimsel araştırma yöntemleri [Scientific research methods in education].* Ankara: Pegem Akademi.

[7] Cierniak, G., Scheiter, K., & Gerjets, P. (2009). Explaining the split attention effect: Is the reduction of extraneous cognitive load accompanied by an increase in germane cognitive load? *Computers in Human Behavior, 25*(2), 315–324. https://doi.org/10.1016/j.chb.2008.12.020

[8] Clark, R., Nguyen, F., & Sweller, J. (2005). *Efficiency in learning: Evidence-based guidelines to manage cognitive load.* Sydney: Pfeiffer.

[9] Cook, T. D., Campbell, D. T., & Shadish, W. (2002). *Experimental and quasi-experimental designs for generalized causal inference.* Boston, MA: Houghton Mifflin.

[10] Durkin, K., & Rittle-Johnson, B. (2012). The effectiveness of using incorrect examples to support learning about decimal magnitude. *Learning and Instruction, 22*(3), 206-214. https://doi.org/10.1016/j.learninstruc.2011.11.001

[11] Eiriksdottir, E., & Catrambone, R. (2011). Procedural instructions, principles, and examples: How to structure instructions for procedural tasks to enhance performance, learning, and transfer. *Human Factors, 53*(6), 749-770. https://doi.org/10.1177/0018720811419154

[12] Emecen, D. D. (2020). Öğrenme güçlüğü gösteren öğrencilere problem çözme becerilerinin öğretiminde çözümlü örnekler uygulamasının etkililiği [The effectiveness of using worked examples in teaching problem-solving skills to students with learning difficulties]. *Karadeniz Uluslararası Bilimsel Dergi, (48),* 81-101.

[13] Foster, N. L., Rawson, K. A., & Dunlosky, J. (2018). Self-regulated learning of principle-based concepts: Do students prefer worked examples, faded examples, or

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problem solving? *Learning and Instruction, 55*, 124-138. https://doi.org/10.1016/j.learninstruc.2017.10.002

[14] Franke, M. L., & Kazemi, E. (2001). Learning to teach mathematics: Focus on student thinking. *Theory Into Practice, 40*(2), 102-109. https://doi.org/10.1207/S15430421TIP4002_4

[15] George, D., & Mallery, M. (2010). *SPSS for Windows step by step: A simple guide and reference, 17.0 update* (10th ed.). Pearson, Boston.

[16] Große, C. S. (2015). Fostering modeling competencies: Benefits of worked examples, problems to be solved, and fading procedures. *European Journal of Science and Mathematics Education, 3*(4), 364-375. https://doi.org/10.30935/scimath/9444

[17] Große, C. S., & Renkl, A. (2007). Finding and fixing errors in worked examples: Can this foster learning outcomes? *Learning and Instruction, 17*(6), 612-634. https://doi.org/10.1016/j.learninstruc.2007.09.008

[18] Hesser, T. L., & Gregory, J. L. (2015). Exploring the use of faded worked examples as a problem solving approach for underprepared students. *Higher Education Studies, 5*(6), 36-46. https://doi.org/10.5539/hes.v5n6p36

[19] Işık, A., & Konyalıoğlu, A. C. (2010). Matematik eğitiminde görselleştirme yaklaşımı [The visualization approach in mathematics education]. *Atatürk Üniversitesi Kazım Karabekir Eğitim Fakültesi Dergisi, (11),* 462-471.

[20] İltüzer, Y. (2016). Öz açıklamalı çözümlü örneklerle desteklenmiş çevrimiçi öğrenme sürecinin karar kurallarını uygulama becerilerine etkisi [The effect of an online learning process supported by self-explanatory worked examples on the ability to apply decision rules]. *[Master's Thesis, Hacettepe University, Institute of Educational Sciences].*

[21] Kilic, S. (2014). Etki büyüklüğü. *Journal of Mood Disorders, 4*(1), 44-46. https://doi.org/10.5455/jmood.20140228010423

[22] Lee, N. (2013). The effects of self-explanation and reading questions and answers on learning computer programming language. [Doctoral Thesis, University of California].

[23] MoNE. (2018). Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar) [Mathematics curriculum (primary and middle school 1st-8th grades)]. http://mufredat.meb.gov.tr/

[24] Moreno, R. E., & Park, B. (2010). *Cognitive load theory: Historical development and relation to other theories*. In J. L. Plass, R. Moreno, & R. Brünken (Eds.), *Cognitive Load Theory* (pp. 9-28). Cambridge University Press. https://doi.org/10.1017/CBO9780511844744.003

[25] Pershan, M. (2021). Teaching math with examples. John Catt Educational.

[26] Paas, F., Tuovinen, J. E., Tabbers, H., & Van Gerven, P. W. M. (2003). Cognitive load measurement as a means to advance cognitive load theory. *Educational Psychologist, 38*, 63-71. https://doi.org/10.1207/S15326985EP3801_8

[27] Paas, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist, 38*(1), 1-4. https://doi.org/10.1207/S15326985EP3801_1

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[28] Renkl, A. (2014). Toward an instructionally oriented theory of example-based learning. *Cognitive Science*, *38*(1), 1-37. https://doi.org/10.1111/cogs.12086

[29] Renkl, A. (2017). Learning from worked-examples in mathematics: Students relate procedures to principles. *ZDM–Mathematics Education, 49*(4), 571-584. https://doi.org/10.1007/s11858-017-0859-3

[30] Renkl, A., Atkinson, R. K., Maier, U. H., & Staley, R. (2002). From example study to problem solving: Smooth transitions help learning. *The Journal of Experimental Education, 70*(4), 293-315. https://doi.org/10.1080/00220970209599510

[31] Renkl, A., & Atkinson, R. K. (2003). Structuring the transition from example study to problem solving in cognitive skill acquisition: A cognitive load perspective. *Educational Psychologist, 38*(1), 15-22. https://doi.org/10.1207/S15326985EP3801_3

[32] Renkl, A., Atkinson, R. K., & Große, C. S. (2004). How fading worked solution steps works:A cognitive load perspective. *Instructional Science, 32*(1), 59 –82. https://doi.org/10.1023/B:TRUC.0000021815.74806.f6

[33] Reisslein, J., Atkinson, R. K., Seeling, P., & Reisslein, M. (2006). Encountering the expertise reversal effect with a computer-based environment on electrical circuit analysis. *Learning and Instruction, 16*(2), 92 –103. https://doi.org/10.1016/j.learninstruc.2006.02.008

[34] Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. *Child Development, 77*(1)*,* 1–29. https://doi.org/10.1111/j.1467-8624.2006.00852.x

[35] Siegler, R. S., & Chen, Z. (2008). Differentiation and integration: Guiding principles for analyzing cognitive change. *Developmental Science, 11*(4)*,* 433-448. https://doi.org/10.1111/j.1467-7687.2008.00689.x

[36] Sweller, J. (2020). Cognitive load theory and educational technology. *Educational Technology Research and Development, 68*(1), 1–16. https://doi.org/10.1007/s11423-019-09701-3

[37] Sweller, J. (2011). Cognitive load theory. In J. P. Mestre & B. H. Ross (Eds.), *Psychology of learning and motivation* (Vol. 55, pp. 37–76). Academic Press. https://doi.org/10.1016/B978-0-12-387691-1.00002-8

[38] Sweller, J. (2005). Implications of cognitive load theory for multimedia learning. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (pp. 19–30). Cambridge University Press. https://doi.org/10.1017/CBO9780511816819.003

[39] Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction, 4*(4), 295–312. https://doi.org/10.1016/0959-4752(94)90003-5

[40] Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*(2), 257-285. https://doi.org/10.1207/s15516709cog1202_4

[41] Sweller, J., & Cooper, G. A. (1985). The use of worked examples as a substitute for problem solving in learning algebra. *Cognition and Instruction, 2*(1), 59–89. https://doi.org/10.1207/s1532690xci0201_3

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[42] Sweller, J., Ayres, P., & Kalyuga, S. (2011). Cognitive load theory in perspective. In *Cognitive load theory* (pp. 237-242). Springer. https://doi.org/10.1007/978-1-4419-8126-4_15

[43] Sweller, J., van Merrienboer, J. J. G., & Paas, F. (1998). Cognitive architecture and instructional design. *Educational Psychology Review, 10*(3), 251–296. https://doi.org/10.1023/A:1022193728205

[44] Tennyson, R. D., & Cocchiarella, M. J. (1986). An empirically based instructional design theory for teaching concepts. *Review of Educational Research, 56*(1), 40–71. https://doi.org/10.3102/00346543056001040

[45] Tepgeç, M. (2017). Algoritma öğretiminde çözümlü örnek kullanımının öğrenci başarısına ve bilişsel yüke etkileri. [The effects of worked examples on student achievement and cognitive load in algorithm teaching] (Yüksek lisans tezi). *Hacettepe Üniversitesi Eğitim Bilimleri Enstitüsü*.

[46] Van Loon-Hillen, N., van Gog, T., & Brand-Gruwel, S. (2012). Effects of worked examples in a primary school mathematics curriculum. *Interactive Learning Environments, 20*(1), 89–99. https://doi.org/10.1080/10494820.2011.556089

[47] Yenilmez, K., & Çoksöyler, A. (2018). Altıncı sınıf öğrencilerinin işlem önceliği konusunda karşılaştığı zorluklar [Difficulties faced by sixth grade students regarding order of operations]. *Eğitim ve Öğretim Araştırmaları Dergisi, 7*(2), 155–156.

[48] Zrebiec Uberti, H., Mastropieri, M. A., & Scruggs, T. E. (2004). Check it off: Individualizing a math algorithm for students with disabilities via self-monitoring checklists. *Intervention in School and Clinic, 39*(5), 269–275. https://doi.org/10.1177/10534512040390050201

[49] Warli, W., Rahayu, P., & Cintamulya, I. (2025). Exploring the ability to prove abstract algebra problems through information technology-assisted scaffolding learning strategies. *Mathematics Teaching Research Journal, 17*(1), 28–55.

[50] Wong, A., Leahy, W., Marcus, N., & Sweller, J. (2012). Cognitive load theory, the transient information effect and e-learning. *Learning and Instruction*, *22*(6), 449- 457. https://doi.org/10.1016/j.learninstruc.2012.05.004

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APPENDIX

Appendix I

The Mathematics Achievement Test

1) Find the result of the operation $10^1+2^3+1^5$.
2) Find the result of $25 + 5 \times 15 - 5$.
3) Find the result of operation $10^2 \div (4+3 \times 7)-1$.
4) Ms. İlknur wants to make bead necklaces for her 2 friends as gifts. Each necklace requires 6 blue beads and 3 red beads. Write a mathematical expression that gives the total number of beads Ms. İlknur needs to buy.
5) The prices of some products sold in a stationery store are given below:

Product	Price (TL)
Eraser	3
Pencil	4
Notebook	8
Book	24

A person buys 3 erasers, 3 pencils, 5 notebooks, and 5 books. Write a mathematical expression that gives the total cost of these items.
6) Mr. Arda will deposit 3^3 TL, Ms. Aslı will deposit 6^3 TL, and Mr. Çağrı will deposit 5^3 TL into the bank. Determine who will deposit the most money into the bank.
7) For the number 35b2 to be divisible by 9 without a remainder, find the digit that should replace 'b'. (10 points)
8) Which of the following numbers is divisible by 2, 3, and 5 without a remainder?
4742, 6501, 7450, 8340
9) The four-digit number 843? is divisible by both 2 and 3 without a remainder. Accordingly, find the sum of the possible digits that can replace '?'.
10) For the number 235a to be divisible by 6 without a remainder, find the possible digits that can replace 'a'.

Appendix II

Explanatory Faded Worked Examples (Example of order of operations)

Dear Students,

Below are questions related to the order of operations. The solution of the first example is provided step-by-step, completely and correctly, with explanations in parentheses. In the other questions, some steps are left blank. By examining the fully solved example, complete the missing steps. Finally, solve the “Your Turn” question by showing all the steps.

Example 1: Find the result of the expression $6\times4+(8^2\div4) -20$.

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Step 1: 6 × 4 + (64 ÷ 4) - 20 (According to the order of operations, the value of the exponential expression is calculated first, so the value of 8^2 is calculated.)

Step 2: 6 × 4 + 16 - 20 (The operation inside the brackets will be performed according to the order of operations. Therefore, division is carried out. The number 64 is divided by 4, and its value is calculated.)

Step 3: 24 + 16 - 20 (Multiplication is performed according to the order of operations. Therefore, the value is calculated by multiplying 6 by 4.)

Step 4: 40-20 (Addition and subtraction are performed last according to the order of operations. Therefore, the addition and subtraction operations are carried out from left to right, respectively.)

Step 5: 20 (The result is obtained after performing the operations while adhering to the order of operations.)

Example 2: Find the result of the expression: 50 ÷ (5 + 2 × 10) - 2

Step 1: 50 ÷ (5 + 20) – 2 (According to the order of operations, multiplication in the brackets is performed first. Therefore, the value is calculated by multiplying 2 by 10)

Step 2: 50 ÷ 25 – 2 (The addition inside the brackets is performed next. Therefore, the value is calculated by adding 5 and 20)

Step 3: 2- 2 (Division is performed according to the order of operations. Therefore, the value is calculated by dividing 50 by 25.)

Step 4: You will solve this step.

Example 3: Find the result of the expression: (25 + 55) ÷ 5 + 3^2

Step 1: (25 + 55) ÷ 5 + 9 (According to the order of operations, the exponent is calculated first. Therefore, the value of 3^2 is calculated.)

Step 2: 80 ÷ 5 + 9 (The addition inside the brackets is performed next. Therefore, the value is calculated by adding 25 and 55.)

Step 3: You will solve this step

Step 4: You will solve this step.

Appendix III

Correct–Incorrect Worked Examples (Order of Operations)

Below are questions related to order of operations. First, carefully examine the example question and its step-by-step solution. Then, read the two solutions (one correct, one incorrect) provided by your sixth-grade classmates for the same question, determine which one is correct, explain the error and reason for the incorrect solution, and answer the related guiding questions. Finally,

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answer the related guiding questions and, following the same principles, solve the "Your Turn" question by showing all the steps.

Example: Find the result of the expression $7 \times 2 - (4^2 \div 2) + 8$.

Solution: $7\times 2 - (16:2) + 8$ (According to the order of operations, exponential expressions are calculated first. Therefore, by finding the value of 4^2, the result is 16, which is then substituted in.)

$= 7 \times 2 - 8 + 8$ (According to the order of operations, the division inside the parentheses is performed first. Therefore, 16 is divided by 2, and the result is written in its place.)

$= 14 - 8 + 8$ (Multiplication is performed according to the order of operations. Therefore, 7 is multiplied by 2, and the result is written in its place.)

$= 14$ (The final addition and subtraction operations are performed in sequence according to the order of operations. Therefore, 8 is subtracted from 14, and the result is added to 8. The final result is then written.)

Example 1: Find the result of the expression $6 \times 4+ 8^2 \div 4 - 20$.

Arda's solution: $6 \times 4 + (8^2 \div 4) - 20 = 6 \times 4 +(64 \div 4) - 20$

$$= 6 \times 4 + 16 - 20$$

$$= 24 + 16 - 20$$

$$=20$$

Arda explained the solution to the problem as follows:

According to the order of operations, the exponential expression is evaluated first. Therefore, I found the value of 8^2 as 64 and substituted it. In the next step, the operations inside the brackets are performed according to the order of operations. So, I divided 64 by 4 and got 16. Next, multiplication is performed according to the order of operations. I multiplied 6 by 4, found 24, and wrote it down. Finally, addition and subtraction are performed according to the order of operations. I added 24 and 16 to get 40, then subtracted 20 from 40, arriving at 20.

Derin's solution: $6 \times 4+ (8^2 \div 4) -20 = 24+ (64 \div 4) -20$

$$=88 \div 4 - 20$$

$$= 22 - 20$$

$$=2$$

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Derin explained the solution to the problem as follows:

According to the order of operations, the exponential expression is evaluated first. So, I found the value of 8^2 as 64 and substituted it. Next, multiplication is performed based on the order of operations. Therefore, I multiplied 6 by 4 and got 24. Then, I added 24 to 64 according to the order of operations, and got 88. Division is carried out next according to the order of operations. Thus, I divided 88 by 4 and found 22. Finally, addition and subtraction are performed according to the order of operations. I subtracted 20 from 22 and got 2.

Which of the solutions is correct? Why? Explain.

Which of the solutions is incorrect? Why? Explain.

Why is the method used by the student who solved it incorrectly not valid? Explain.

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Transforming Confusion into Clarity: Employing the Feynman Technique to Overcome Mathematics Anxiety

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Abstract: This study examines the effectiveness of the Feynman Learning Technique (FLT) in improving mathematics performance and reducing math anxiety among tenth-grade students learning about parabolas. A quasi-experimental design was employed, involving 67 students from Erbil, Iraq, who were divided into an experimental and a control group. The experimental group received instruction through FLT, which focuses on learning by teaching, while the control group followed conventional teaching methods. Data were collected using a pretest, post-test, and a modified version of the Abbreviated Math Anxiety Scale. Analysis conducted with Quade's nonparametric ANCOVA revealed that students taught using FLT significantly outperformed their peers in the control group. Additionally, a Wilcoxon signed-rank test indicated a statistically significant reduction in mathematics anxiety within the experimental group, while anxiety levels increased in the control group. These findings suggest that FLT not only enhances conceptual understanding but also effectively reduces math-related stress. This study recommends that teachers adopt the FLT in mathematics classrooms to promote self-directed learning and enhance students' conceptual understanding and retention of knowledge. Future research should expand on this work by investigating the effectiveness of FLT across various mathematical topics and educational levels.

Keywords: Feynman Learning Technique, mathematics anxiety, mathematics education

INTRODUCTION

Education is one of the most important elements shaping societies' future (Iksal et al., 2024). Education is not merely about transmitting knowledge; it also instills high ideals in students. Furthermore, it is essential to recognize that education is a process of character development, and schools play a pivotal role in this journey (Kurudirek & Berdieva, 2024). Educators are uniquely positioned to transform society. To improve the educational system's performance, it is essential to identify existing shortcomings and make the necessary adjustments, especially in mathematics instruction (Lotfi et al., 2012).

Many students around the world experience mathematics anxiety. Approximately 17% of the

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United States population is reported to suffer from mathematics anxiety (Ashcraft & Moore, 2009). Across the 34 countries that participated in the 2012 Programme for International Student Assessment (PISA), 59% of the students reported experiencing mathematics anxiety (Luttenberger et al., 2018). Several other cases have been reported in Sub-Saharan Africa (Luneta & Sunzuma, 2022), Belgium (Demedts et al., 2022), Finland, and Korea (Fan et al., 2019). Lee (2024) identifies mathematics anxiety as a global issue.

Mathematics anxiety involves stress and decreased performance during mathematics tasks, particularly affecting high school students. Research shows a correlation between higher anxiety levels and lower mathematics achievement (Baral et al., 2024; Zanabazar et al., 2023; Zhang et al., 2019). Addressing anxiety is vital in mathematics education, as its origins often stem from personal experiences, lack of confidence, fear of failure, and ineffective teaching methods (Ambion et al., 2021). Conventional pedagogical approaches and rote learning further increase anxiety. Therefore, employing methodologies that enhance learning and foster student autonomy can help reduce mathematics anxiety.

This study evaluates the impact of the Feynman Learning Technique (FLT) on students' performance and mathematics anxiety. This technique, developed by physicist Richard Feynman in the twentieth century and popularized in the early 2000s, emphasizes the concept of "learning by teaching." Feynman believed that explaining concepts to others enhances understanding and helps identify knowledge gaps, creating a positive feedback loop that ensures accuracy and thoroughness (Feynman et al., 2006). Figure 1 visually represents the FLT.

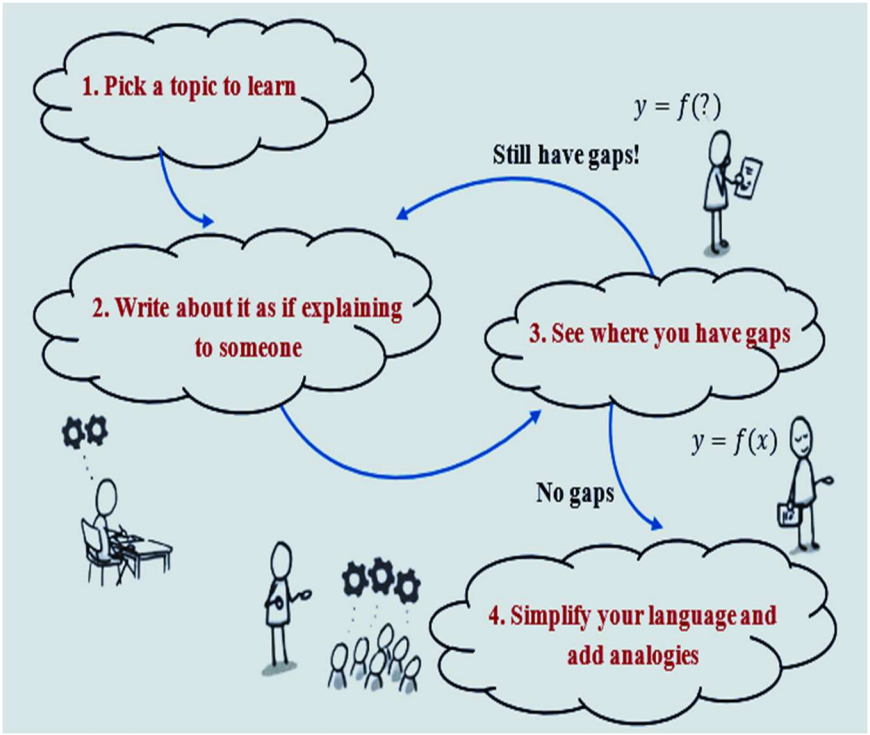


Figure 1: The Feynman Learning Technique (Adapted from Substack, 2023)

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The Feynman Technique has four phases, which encompass the subsequent steps (see Feynman et al., 2006):

- Phase 1: Pick a topic to learn.

  Select a concept that you would like to explore and study independently.

- Phase 2: Write about it as if explaining it to yourself or someone else.

  Teach the concept to yourself or someone else, as if you were explaining it to a child.

- Phase 3: See where you have gaps.

  If you find yourself confused, revisit the original material, whether a book, lesson notes, or videos, and clarify any misunderstandings.

- Phase 4: Simplify your language and add analogies.

  Simplify your notes and explanations until the ideas become clear (Feynman et al., 2006)

The Feynman Learning Technique has improved student performance in subjects like English (Bsharat et al., 2024; Reyes et al., 2021) and Physics (Wea et al., 2023). However, its impact on mathematics education is largely unexplored. Our literature review found no studies on its application in this field, prompting us to investigate its potential to enhance students' performance in mathematics and decrease mathematics anxiety. Our research objectives and questions are detailed in the following subsections.

Research Objectives

This study seeks to:

(1) Assess whether the Feynman Learning Technique leads to a statistically significant improvement in tenth-grade students' performance on concepts related to parabolas.

(2) Investigate the impact of the Feynman Learning Technique on the mathematics anxiety levels of tenth-grade students.

Research Questions

The following research questions guide the study:

(1) Does the Feynman Technique lead to a statistically significant improvement in tenth-grade students' performance on parabolas in the coordinate plane?

(2) Do tenth-grade students employing the Feynman Technique exhibit a reduction in mathematics anxiety over time?

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Hypotheses

The study seeks to test the following hypotheses:

Hypothesis 1: Mathematics Achievement

H_0: There is no significant difference in students' performance between those taught with the Feynman approach and those taught with conventional methods on the topic of parabolas in the coordinate plane.

H_a: Students using the Feynman approach achieve significantly better results than those taught with conventional methods on parabolas in the coordinate plane.

Hypothesis 2: Anxiety Levels

H_0: There is no statistically significant difference in students' mathematics anxiety levels before and after employing the Feynman Learning Technique.

H_a: Students experienced a significant reduction in their mathematics anxiety levels after using the Feynman Learning Technique.

The Significance of the Study

This study aims to enhance students' understanding of mathematics and reduce anxiety, particularly regarding parabolas, among tenth-grade students. By employing the Feynman Learning Technique, the study aims to enhance retention and make abstract ideas more comprehensible. Employing clarity-based strategies can help students appreciate mathematics as an art (Gordon, 2019), especially when traditional teaching methods fall short.

This study aims to provide empirical evidence on the effectiveness of the Feynman Technique as a teaching and learning strategy to reduce mathematics anxiety among tenth-grade students. It addresses the emotional and psychological barriers that hinder engagement in mathematics (Luttenberger et al., 2018). By identifying factors that contribute to mathematics anxiety, the research aims to support students more effectively. If a positive correlation is found between the Feynman Technique and reduced anxiety, it could enhance the teaching and learning environment, boosting students' confidence and motivation.

The implications of this study extend beyond the classroom, offering valuable insights for educators, curriculum developers, and policymakers seeking to enhance mathematics education. If the Feynman Technique effectively enhances mathematics performance and reduces anxiety, it could be incorporated into teacher training programs and educational resources, equipping educators with a practical tool to support student learning. Additionally, the study's findings could inform the development of targeted interventions for students who experience high levels of mathematics anxiety, providing personalized support tailored to their specific needs.

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THEORETICAL FRAMEWORK AND LITERATURE REVIEW

This study is grounded in constructivism and self-directed learning, which emphasize that students should actively create their own understanding of the subject matter rather than depend solely on teachers (Karagiorgi & Symeou, 2005; Magas, 2025). It challenges the passive acceptance of ideas typical of traditional teaching methods. In contemporary mathematics education, traditional "chalk-and-talk" methods are considered inadequate, as they cover excessive content while promoting insufficient critical thinking (Tshering, 2024; Wittmann, 2019). Instead, educators are encouraged to use instructional scaffolding, providing support that is gradually removed as students develop independent learning skills.

Scaffolding is the targeted support that helps students achieve their educational goals. According to Van de Pol et al. (2010), this involves techniques that assist learners in bridging cognitive gaps, enabling them to progress in their studies. Scaffolding encompasses various teaching strategies that guide students toward a deeper understanding and increased independence in their learning journey. When using the Feynman Learning Technique, teachers must first explain the strategy to students, providing essential scaffolding.

Vygotsky's social learning theory posits that learning primarily occurs through social interactions (Vygotsky & Cole, 2018). Individuals develop knowledge and cognitive skills through collaborative experiences within their cultural contexts. This theory emphasizes the importance of dialogue in the learning process. The Feynman Learning Technique exemplifies this by encouraging peer teaching, where students explain concepts to each other. This interaction deepens their understanding and enhances critical thinking and communication skills, highlighting the value of collaboration in effective learning.

Bandura's self-efficacy theory emphasizes the significance of students' self-belief in their abilities for academic success (Tessényi et al., 2024). High self-efficacy leads students to embrace challenges, persist through difficulties, and achieve better results (Syahbana & Nopriyanti, 2024). The Feynman Learning Technique is an effective method for boosting self-efficacy, as it encourages students to break down complex concepts and clearly explain them, thereby reinforcing their understanding and confidence, ultimately enhancing their academic performance. The following section reviews empirical studies on the Feynman Learning Technique in educational contexts.

Empirical Studies on the Feynman Learning Technique

The Feynman Learning Technique has gained significant attention as a potentially effective learning strategy in various educational contexts (Reyes et al., 2021). Although there is anecdotal evidence and theoretical support for the Feynman Technique, a thorough review of recent empirical studies is essential to evaluate its impact on student learning outcomes in specific subjects.

The study by Wea et al. (2023) explores the Feynman Learning Technique in an Indonesian physics classroom. It was discovered that this method enhances student engagement and promotes independent learning. Students actively participated by explaining concepts in their own words and identifying gaps in their understanding. However, challenges arose as some students memorized

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facts without grasping concepts, and many viewed physics as complex, which affected their use of the technique. The study recommends that teachers foster positive attitudes towards physics and provide support to help students overcome anxieties.

Bsharat et al. (2024) conducted a quasi-experimental study using a mixed-methods approach to examine the effects of the Feynman Learning Technique on students' English language proficiency at Al-Quds Open University in Jenin, Palestine. The study involved 44 students and 10 educators from the university's English language program. Quantitative data were collected through standardized English proficiency tests administered before and after a 12-week intervention. Additionally, qualitative data were obtained through interviews, self-report questionnaires, and observational notes to assess participants' confidence, engagement, and comfort levels. The results indicated a significant improvement in students' English language proficiency, confidence, and engagement after using the Feynman Learning Technique. The study concluded that this technique can potentially transform language education by fostering greater student autonomy and global competence.

Reyes et al. (2021) conducted an experimental study in the Philippines that involved randomly assigning participants to treatment and control groups. The research aimed to evaluate the effectiveness of the Feynman Technique as a heutagogical strategy for online learning among students in grades 4, 7, and 11. Students in the experimental group were taught using the Feynman Technique, while students in the control group received traditional instruction. Although the specific sample sizes for each grade were not provided, statistical analyses using paired t-tests showed that the experimental group had significantly higher post-test scores. The findings of the study support a causal relationship regarding the effectiveness of the Feynman Technique and demonstrate its applicability across various educational levels.

Adeoye (2023) conducted a systematic literature review that compiled findings from various sources to identify key patterns regarding the impact of the Feynman Technique on academic achievement and learner confidence among slow learners. The study emphasizes the positive effects associated with the Feynman Technique, including improved academic performance and increased self-confidence among slow learners.

In a study conducted in the Philippines, Ambion et al. (2020) investigated the impact of the Feynman Technique and team teaching on students' understanding of evolution in a Grade 10 adult night high school biology class. Data analyzed using a t-test indicated insufficient evidence to support a significant difference in the outcomes between the experimental and control groups. These findings suggest that the effectiveness of the Feynman Learning Technique may vary depending on the subject matter and environmental context. Therefore, it is necessary to further explore the effectiveness of the Feynman Learning Technique across different subject areas and educational settings (Ambion et al., 2020; Wea et al., 2023).

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The Research Gap

A review of the existing literature reveals a growing body of research investigating the application of the Feynman Learning Technique in subjects such as English, Biology, and Physics. However, research specifically addressing the use of the Feynman Learning Technique in mathematics education remains limited. Besides, no study has investigated the impact of the Feynman Learning Technique on students' mathematics anxiety levels. This study is intended to address these research gaps.

METHODS

Research Design

This study employed a quasi-experimental design with non-equivalent intact groups to assess the impact of the Feynman Learning Technique on students' achievement and levels of mathematics anxiety. Practically, random assignment of students to experimental and control groups was practically impossible.

Research Sample

This study involved a convenience sample of 67 tenth-grade students from two secondary schools in Erbil, Iraq. The students were enrolled in a mathematics course based on the Iraqi national curriculum. At the time of the study, they were learning about *parabolas in the coordinate plane*, which is a required part of the Grade 10 syllabus.

In their regular classes, the students typically experienced conventional, teacher-centered instruction that emphasized explanations, note-taking, and practice exercises. In this study, two intact classes comprising a total of 43 students were randomly assigned to the experimental group, while one intact class with 24 students served as the control group. This random assignment ensured an unbiased distribution between the experimental and control groups, despite the use of intact classes as units of selection. Participation in the study was voluntary and based on the students' willingness to take part.

Although the overall sample was defined by convenience, the assignment of intact classes to the experimental and control groups was determined through random selection to avoid systematic bias. This approach ensured that any observed differences could be attributed to the instructional method with greater confidence, rather than to pre-existing characteristics of the groups.

Instruments

Data were collected using a pretest, a posttest, and a modified Abbreviated Mathematical Anxiety Scale (AMAS) developed by the researchers. The pretest consisted of ten multiple-choice questions that assessed students' basic knowledge of parabolas (see Appendix A). The prior knowledge examined included identifying parabolas from given shapes, recognizing the axis of symmetry and

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vertices, and explaining how the sign of the leading coefficient affects the shape of the parabola. The internal consistency of the pretest was assessed using Kuder-Richardson Formula 20 (KR-20).

$$KR20=\frac{n}{n\text{-}1}\times\left[1\text{-}\frac{\sum pq}{Var}\right] \quad (1)$$

In this formula, KR20 is the score that estimates the instrument's internal reliability. n is the number of items that make up the test; p is the proportion of students who passed each item, and q is the proportion of students who failed each item; Var is the variance across all items. The pretest's reliability based on the KR20 formula was 0.91, above 0.7, the minimum requirement for acceptance (Setyaedhi, 2024).

The post-test included four open-ended questions that assessed students' understanding of concepts related to parabolas, such as vertex, focus, directrix, symmetry, finding equations, and determining the direction in which the parabola opens, as outlined in the grade 10 mathematics curriculum (see Appendix B). The questions spanned various levels of Bloom's taxonomy, from basic recall to application.

Expert judgment was used to evaluate the test instrument to ensure content validity. Eight grade 10 mathematics teachers in Iraq, each with at least 10 years of teaching experience, were asked to rate the relevance and clarity of the test items. The item-level content validity index (I-CVI) was calculated by dividing the number of raters who scored each item as 3 or 4 by the total number of raters on the panel. The item CVIs were adjusted using the modified kappa statistic to account for chance agreement. The overall scale-level content validity index was found to be 0.99, which exceeds the acceptable threshold of 0.78 for content validity assessments involving at least nine experts (Yusoff, 2019). The reliability of the post-test instrument was evaluated through a test-retest method with a sample of 30 students who were not part of the targeted group. A Pearson's r value of 0.92 was obtained, indicating a strong correlation between the scores from the first test and the subsequent retest. Therefore, the test instrument was deemed reliable.

A nine-item modified Anxiety Management Assessment Scale (AMAS), adapted from Hopko et al. (2003), was utilized to gauge participants' anxiety levels both before and after the intervention (see Appendix C). The AMAS employed a five-point rating scale, with scores ranging from 1 to 5: 1 indicating low anxiety, 2 representing some anxiety, 3 denoting moderate anxiety, 4 signifying a significant amount of anxiety, and 5 reflecting a high anxiety level. The Cronbach's alpha reliability coefficient for the modified AMAS is 0.76, exceeding the acceptable minimum of 0.7 (Arof et al., 2018).

Ethical Considerations

The study was approved by the Tishk International University Human Research Ethics Committee (Date: 13 April 2025, Protocol number: 17). All prospective participants received information about the research, including its purpose, duration, and activities involved, prior to data collection. Students were informed that participation was voluntary and based on informed consent. They were made aware of their right to decline participation in the study without penalty. No names or

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identifiable information were gathered from the participants. Participants in the experimental group were assigned random codes starting with the letter "E" (e.g., E11092), while those in the control group were assigned random codes starting with the letter "C" (e.g., C11251). Additionally, the study did not disrupt normal school operations.

Data Collection Procedures

Data were collected in the period April-May 2025. Data collection procedures were implemented in three phases: pre-intervention, intervention, and post-intervention.

Pre-Intervention Stage

In the pre-intervention stage, participants were required to sign informed consent forms, indicating that they voluntarily agreed to participate in the study without being coerced by the researchers. The researchers administered the pretest and AMAS to both the experimental and control groups. The pretest lasted for 20 minutes. To reduce concerns about researcher bias, the pre-test scripts were graded by a teacher who was not involved in the research. The pretest scores and students' pre-intervention mathematics anxiety levels were recorded.

Intervention Stage: Enactment of the Feynman Learning Technique

Over the course of two weeks, students received eight 40-minute lessons on parabolas. In the experimental classes, one researcher implemented the FLT, while in the control class, another teacher covered the same content using conventional instruction. All worksheets, class tasks, and homework were identical across groups, and experimental-group students received a one-page FLT handout.

Experimental lessons followed the FLT's four phases:

1. *Pick a topic.* Students (individually or in pairs) selected a micro-concept from the unit (e.g., orientation by the sign of a; vertex; axis of symmetry; focus/directrix; standard versus vertex form).
2. *Teach it.* Students prepared a concise, plain-language explanation intended for their peers, optionally incorporating everyday analogies.
3. *Find gaps.* While presenting, their peers and the teacher asked clarifying questions; any uncertainty was logged on a "knowledge gap slip."
4. *Refine and analogize.* Students consulted the textbook/notes (sources cited by the teacher) and returned with a more concise explanation in the next session, minimizing jargon and incorporating helpful analogies.

The teacher's facilitation emphasized scaffolding without overt instruction, prompting students to specific textbook pages or examples, and summarizing key takeaways at the end of each

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presentation cycle. Each lesson concluded with a brief practice session to reinforce the ideas. To support implementation quality, the same lesson outline was used throughout the intervention period (present → question → gap log → resource consult → re-teach → summarize → practice). The materials used in the experimental group matched those of the control group, and a facilitator checklist was used to ensure that all aspects of the FLT were incorporated in the implementation process. Here is an example of how the FLT was applied in the sixth session.

Step 1: Topic Selection

A volunteer chose the quadratic function $f(x)=ax^2+bx+c$ and its graph (a parabola) as the focus.

Step 2: Initial Explanation (student-led)

On the whiteboard, the student stated the general form $y=ax^2+bx+c$ and explained orientation: if $a>0$, the parabola opens upward, and if $a<0$, it opens downward. To aid intuition, the student used the "smile/frown" metaphor and a soccer-ball trajectory analogy.

Step 3: Gap Identification (peer/teacher prompts)

Peer: "What's the special name for the highest point on the soccer-ball path?"

Presenter: "The vertex."

Peer: "How do we find it?"

The presenter was uncertain and recorded this as a gap to refine. The presenter promised to bring a definite response in the next session. The facilitator referred the student to a textbook page where information about how to find the coordinates of the vertex could be found.

Step 4: Refined Re-teach (next session)

The student returned with a definite explanation: the vertex is the function's maximum (if $a<0$) or minimum (if $a<0$). The student presented the following worked example:

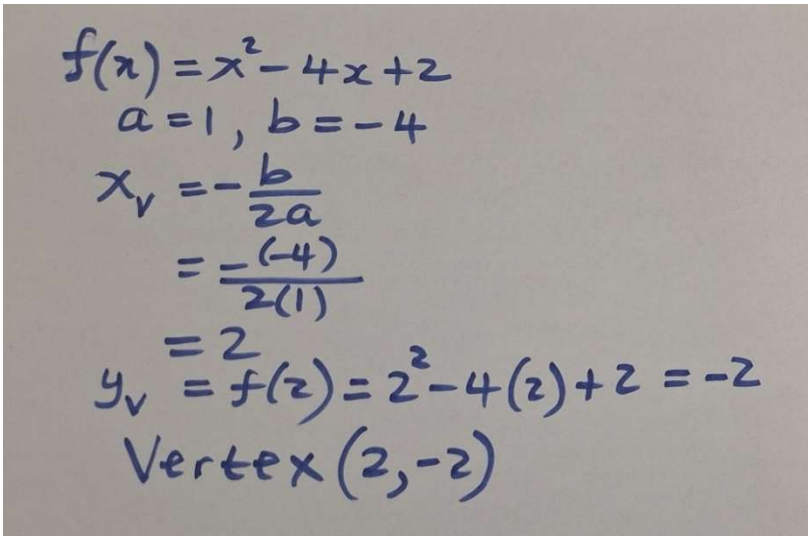


Figure 2: Student presentation sample

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The student then posed guiding checks to peers: "First, what is the sign of a? Is the parabola a 'smile' or a 'frown'? Does the vertex give a max or a min?" The peers were able to answer these questions correctly.

Synthesis and Consolidation

The session concluded with the facilitator synthesizing the three key points from the presentation:

- If $a>0$, the parabola opens upward.
- If $a<0$, the parabola opens downward.
- The vertex can be determined using $x_v=-\frac{b}{2a}$, then $y_v=f(x_v)$.

To enhance learning, students were assigned a short exercise in which they needed to state the orientation and calculate the vertices of the functions $y=2x^2+4x+5$ and $y=-x^2+6x-5$ homework.

Post-intervention Stage

In the post-intervention stage, both the post-test and the AMAS were administered to the experimental and control groups. The post-test duration was 45 minutes. The post-test scripts were graded by the same teacher who evaluated the pre-test scripts. Additionally, a second marker, an experienced Grade 10 mathematics teacher from Stirling Schools in Iraq, moderated the grading of the post-test scripts. The results of the post-test, along with the AMAS survey results collected after the intervention, were recorded for analysis. The following section explains how the data were analyzed.

Data Analysis

Students' pretest scores were analyzed using descriptive statistics and the Kruskal-Wallis test to assess differences between the experimental and control groups prior to intervention. The Kruskal-Wallis test was used because the pretest scores violated the assumption of normality, as determined by the Shapiro-Wilk test. Post-test scores were analyzed using descriptive statistics, effect size comparisons, and Quade's nonparametric ANCOVA, following violations of the normality assumption. Quade's nonparametric ANCOVA was selected to incorporate pretest scores as covariates in the analysis, as significant differences were observed between the control and experimental groups prior to the intervention. Effect sizes on mathematics anxiety levels were computed by dividing the mean gain by the pooled standard deviation across all pre- and post-survey responses. The Wilcoxon signed-rank test was employed to evaluate whether the intervention had a statistically significant effect on students' mathematics anxiety levels.

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RESULTS

A comparison of the descriptive statistics for the pretest scores revealed similar variations between the two groups in terms of minimum, maximum, range, and interquartile range. However, there were significant differences in the measures of central tendency between the groups. The experimental group exhibited higher mean and median values than the control group (Table 1).

	Group	Mean	Med.	Std. Dev.	Min.	Max.	Range	IQR
Pretest	Control	56.25	55.00	21.226	20	100	80	30
	Experimental	73.72	70.00	19.883	20	100	80	30

Notes. Std. Dev: Standard Deviation, Min.: Minimum, Max.: Maximum, IQR: Interquartile Range

Table 1: Pretest descriptive statistics

To assess whether there was a statistically significant difference in performance between students taught using the Feynman approach and those taught through conventional methods on parabolas in the coordinate plane, we first evaluated the students' prior knowledge of the topic with a multiple-choice pretest. The results indicated that the pretest scores of the experimental group did not meet the normality assumption, as determined by the Shapiro-Wilk test ($p < 0.05$) (Table 2). Consequently, parametric statistical measures were abandoned in favor of non-parametric methods.

		Kolmogorov-Smirnov[b]			Shapiro-Wilk		
	Group	Statistic	df	Sig.	Statistic	df	Sig.
Pretest	Control	0.153	24	0.151	0.946	24	0.223
	Experimental	0.170	43	0.003	0.929	43	0.011

b. Lilliefors Significance Correction

Table 2: Test of normality on pretest scores

The Kruskal-Wallis test revealed significant differences in pretest scores between the experimental and control groups ($\chi^2(1,67)=10.225$, $p=0.001$). This result indicates that pretest scores should be considered covariates when analyzing post-test scores.

Analysis of the posttest scores reveals substantial differences between the experimental and control groups. The mean and median values of the experimental group were higher than those of the control group. The standard deviation of the experimental group's post-test scores indicates a greater variation in student performance than in the control group. These results do not assert that students in the experimental group performed significantly better than those in the control group, as the two groups had differences before the intervention. Additional statistical analyses are needed

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to determine if the differences in performance between the two groups resulted from the interventions they received.

	Group	Mean	Median	Std. Dev	Min.	Max.	Range	IQR
Post-test	Control	0.83	0.00	2.036	0	8	8	0
	Experimental	46.05	44.00	26.994	0	92	92	48

Table 3: Post-test descriptive statistics

The posttest mean scores for both groups were significantly lower than their pretest mean scores due to differences in test formats. The pretest consisted of multiple-choice questions, while the posttest included free-response questions.

The Shapiro-Wilk test on posttest scores indicated a violation of the normality assumption in the control group (p<0.001). Using parametric statistics to compare the scores of the experimental and control groups was inappropriate, so nonparametric statistical measures were utilized instead.

	Group	Kolmogorov-Smirnov[b]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Posttest	Control	0.492	24	<0.001	0.472	24	<0.001
	Experimental	0.093	43	0.200*	0.955	43	0.090

b. Lilliefors Significance Correction

Table 4: Normality tests on post-test scores

The post-test scores were analyzed using Quade's nonparametric analysis of covariance (ANCOVA) in SPSS, with pretest scores as covariates (Table 5). The between-subjects effects of Quade's ANCOVA analysis revealed a statistically significant difference in students' post-test scores, after adjusting means (averages) for pre-intervention differences ($F(1,67)=69.066$, $p<0.001$). The differences in performance between the experimental and control groups could be attributed to the impact of the different learning techniques employed in the two groups – Feynman Learning Technique versus Conventional approaches.

Tests of Between-Subjects Effects					
Dependent Variable: Unstandardized Residual					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	11619.985[a]	2	5809.993	34.533	<0.001
Intercept	514.889	1	514.889	3.060	0.085
RPretest	1800.276	1	1800.276	10.700	0.002
Group	11619.985	1	11619.985	69.066	<0.001

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Error	10767.663	64	168.245
Total	22387.648	67	
Corrected Total	22387.648	66	

a. R Squared = 0.519 (Adjusted R Squared = 0.504)

Table 5: Quade's nonparametric ANCOVA

The null hypothesis, which stated that there is no significant difference in the performance of students taught parabolas using the Feynman learning approach compared to those taught with conventional methods, was rejected. Consequently, we accepted the alternative hypothesis: students who learn about parabolas with the Feynman approach achieve significantly better results than those taught using traditional methods.

Most students in the control group did not attempt the majority of the questions on the posttest. For the few control group students who attempted the posttest questions, their solutions demonstrated a complete lack of understanding of the concepts related to parabolas. Figure 3 illustrates the contrast between the responses from a control group student (C11251) and an experimental group student (E13129) when solving question 2 of the posttest. The samples of student work presented in Figure 3 highlight a significant difference in the approaches taken by students in tackling the posttest items. Student C11251 seems to be answering by randomly selecting values from the given function. The student's responses do not follow the expected format. For example, the vertex and the focus should be provided in coordinate form.

To address the study's second objective, we compared the pre- and post-intervention survey responses of each group, focusing on changes in means, medians, and effect sizes. The results in Table 6 indicate negative effect sizes for all survey items, suggesting a decrease in anxiety levels among students in the experimental group regarding parabolas. However, these findings alone are insufficient to conclude that the intervention led to a statistically significant reduction in students' anxiety levels. Additional statistical tests are needed to obtain conclusive results.

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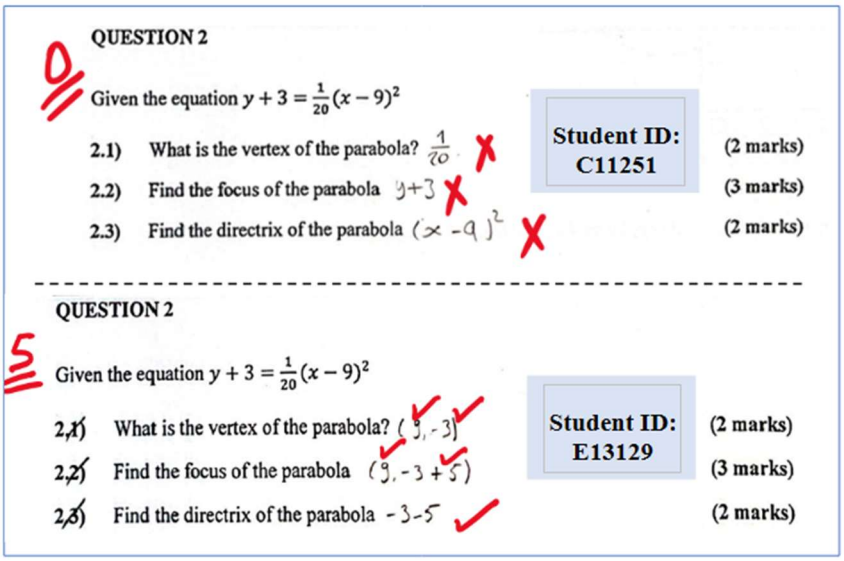


Figure 3: Samples of Students' Solutions

Item	Pre-intervention		Post-intervention		Comparison		Effect Sizes
	M	Med.	M	Med.	M Diff.	Med. Diff.	
1	2.42	2	1.74	1	-0.68	-1	-0.5
2	3.72	4	2.72	2	-1.00	-2	-0.7
3	2.72	3	2.02	1	-0.70	-2	-0.5
4	2.95	3	2.60	2	-0.35	-1	-0.3
5	2.56	2	2.23	2	-0.33	0	-0.2
6	2.44	2	2.33	2	-0.11	0	-0.1
7	2.00	2	1.98	2	-0.02	0	-0.01
8	2.02	1	1.84	2	-0.18	1	-0.1
9	3.30	3	2.74	3	-0.56	0	-0.4

Notes: M = Mean; Med. = Median; Diff. = Difference

Table 6: Experimental group comparison of pre- and post-intervention survey responses

In contrast, comparisons of pre- and post-learning survey responses in the control group, focusing on means, medians, and effect sizes, reveal increases in students' anxiety levels across seven out of nine survey items (see Table 7). Interestingly, the control group survey data showed two negative effect sizes (items 6 and 7), indicating a decrease in students' anxiety levels for these two items.

Item	Pre-intervention		Post-intervention		Comparison		Effect Sizes
	M	Med.	M	Med.	M Diff.	Med. Diff.	

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1	2.38	2	2.79	3	+0.41	+1	+0.3
2	2.63	2	3.29	3	+0.66	+1	+0.5
3	2.13	1.5	2.88	3	+0.75	+1.5	+0.6
4	2.75	3	3.17	3	+0.42	0	+0.3
5	2.46	2	3.25	3	+0.79	+1	+0.6
6	2.83	3	2.54	2.5	-0.29	-0.5	-0.21
7	2.63	3	2.5	2.5	-0.13	-0.5	-0.1
8	2.29	2	2.88	3	+0.59	+1	+0.4
9	3.25	3	3.63	4	+0.38	+1	+0.3

Notes: M = Mean; Med. = Median; Diff. = Difference

Table 7: Control group comparison of pre- and post-learning survey responses

To assess whether the decrease in anxiety levels among the students in the experimental group was statistically significant, 11 outlier cases from the experimental group survey data were removed. The remaining data was tested to see if it met the normality assumption. Despite removing the outliers, the experimental group's pre- and post-intervention survey data still violated the normality assumption ($p < .05$), which meant using parametric statistics to analyze the differences between the two datasets was inappropriate. The Wilcoxon signed-rank test in SPSS was performed on the remaining 32 valid cases from the experimental group survey data, yielding a statistically significant result ($Z = -3.378$, $p < 0.001$) (Table 8). The conclusion was that implementing the Feynman Learning Technique significantly reduced anxiety levels among students in the experimental group.

		N	Mean Rank	Sum of Ranks
Post Mean – Pre-Mean	Negative Ranks	26[a]	17.10	444.50
	Positive Ranks	6[b]	13.92	83.50
	Ties	0[c]		
	Total	32		
a. Post Mean < Pre-Mean				
b. Post Mean > Pre-Mean				
c. Post Mean = Pre-Mean				
Test Statistics[a]				
	Post Mean – Pre-Mean			
Z	-3.378[b]			
Asymp. Sig. (2-tailed)	<0.001			
a. Wilcoxon Signed Ranks Test				
b. Based on positive ranks.				

Table 8: Wilcoxon signed-rank test results

A Wilcoxon signed-rank test was conducted on the survey data from the control group to assess whether the changes in students' anxiety levels were statistically significant. The results indicated

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that the differences in anxiety levels before and after learning about parabolas in this group were statistically significant ($Z = 2.050$, $p < 0.05$). These findings, along with the positive effect sizes presented in Table 7, suggest that conventional teaching methods may increase students' anxiety levels in certain mathematics.

DISCUSSION

Improved Academic Performance

The study found that students taught using the Feynman Learning Technique performed significantly better on post-tests about parabolas than those taught using conventional methods. Quade's nonparametric ANCOVA test confirmed a significant difference in performance between the experimental and control groups. This supports earlier findings from studies in other disciplines, such as those by Reyes et al. (2021), Bsharat et al. (2024), and Wea et al. (2023), which demonstrated the effectiveness of FLT in enhancing academic outcomes. The findings of this study contribute to existing literature by demonstrating that the FLT, which has proven effective in subjects such as English and Physics, is also effective in the context of mathematics education.

Reduction in Mathematics Anxiety

A Wilcoxon signed-rank test revealed a statistically significant reduction in mathematics anxiety among students in the experimental group. In contrast, the control group experienced increased anxiety levels. Research has shown that increased mathematics anxiety affects students' attitudes toward various mathematics topics (Ferrer et al., 2025). The findings of this study fill a gap in the literature by empirically linking FLT to reduced mathematics anxiety. This is a notable contribution given the global prevalence of this issue (Ashcraft & Moore, 2009; Luttenberger et al., 2018). The increase in mathematics anxiety levels among students in the control group underscores the urgent need for mathematics educators to move away from traditional teaching methods. Instead, they should adopt innovative, student-centered approaches that foster conceptual understanding rather than merely memorizing mathematics concepts.

Implications for Mathematics Teaching in the Classroom

Based on the findings of this study, teachers should incorporate the FLT into mathematics lessons to promote student autonomy and active engagement with mathematical concepts. This approach aligns with heutagogical principles (Reyes et al., 2021) and self-directed learning theories (Magas, 2025), which emphasize the importance of learners taking initiative in their own educational process. Mathematics teachers can support this by allowing time for individual exploration and encouraging peer teaching. Students should be motivated to explain concepts to their peers, write simplified explanations, and identify areas where they have knowledge gaps. This approach is consistent with constructivist learning theory, which advocates for students constructing their own understanding (Karagiorgi & Symeou, 2005).

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Encouraging students to simplify and explain mathematical concepts through the FLT enhances their conceptual understanding. This approach shifts the focus from rote memorization to meaningful learning, addressing one of the primary sources of math anxiety (Ambion et al., 2021). Research shows that the FLT significantly reduces anxiety among learners, while traditional methods can increase it. This highlights the need to move from teacher-centered approaches to student-centered ones. Bandura's theory of self-efficacy supports this shift; as students succeed with the FLT, their confidence in their abilities grows, which in turn helps to reduce anxiety (Tessényi et al., 2024).

The FLT promotes formative assessment by encouraging peer explanation and reflection. Teachers can create assessments that evaluate students' abilities to teach and articulate mathematical reasoning, in addition to traditional summative assessments. Peer teaching among students is a highly regarded strategy in mathematics education (Kimbrough et al., 2022; Moliner & Alegre, 2020). While the study focused on parabolas, the principles of FLT can also be applied to other topics and grade levels. Future classroom practices should test FLT across various mathematical topics to investigate its broader applicability.

It is essential to recognize that implementing the FLT does not occur automatically. It requires initial scaffolding where teachers model the technique and support students as they practice. This aligns with Vygotsky's Zone of Proximal Development and the concept of instructional scaffolding (Van de Pol et al., 2010). As students gain confidence over time, this support can be gradually reduced.

Given its positive outcomes, education stakeholders should consider integrating FLT into teacher training programs and curriculum guides. This would provide a systematic approach to addressing mathematics under-performance and anxiety. FLT, based on strong educational theories, offers a practical and effective framework for enhancing both mathematical understanding and emotional well-being. Teachers, curriculum developers, and policymakers are encouraged to adopt and promote this approach to improve mathematics education.

CONCLUSION

The Feynman Learning Technique has proven to be a transformative approach in enhancing mathematics performance while simultaneously alleviating anxiety associated with the subject. By breaking down complex concepts into simple, understandable language, this method effectively addresses both the cognitive challenges students face and the emotional barriers that often hinder their learning in mathematics education. Although there are some limitations to the technique, its strong potential makes it an invaluable resource for educators who are committed to improving learning outcomes and fostering a more positive educational experience for their students.

Limitations

This study focused on parabolas and involved students in the 10th grade. It included a small sample of students from a single geographical location, which limited the generalizability of the findings. Therefore, readers are cautioned against applying the results to contexts beyond the scope of this

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study. Additionally, the research utilized self-report surveys with Likert scales, which may introduce bias. The non-random selection of participants also presents the possibility of pre-existing differences between groups. Lastly, the study was conducted over a short-term period. A long-term follow-up on students' performance or levels of mathematics anxiety may be necessary to further validate the findings.

Recommendations

Considering these limitations, future researchers can expand this study to cover additional mathematics topics and different age groups. Future studies could explore the application of the Feynman technique in specific high school mathematics topics such as quadratic equations, circle theorems, trigonometric identities, conditional probability, and introductory calculus, where students often struggle with abstraction and conceptual understanding. Longitudinal studies could also be beneficial in exploring the long-term impacts of the Feynman Learning Technique on retention and anxiety levels. Additionally, teacher training is essential to provide educators with the skills to implement the Feynman Learning Technique effectively. Moreover, future studies should incorporate qualitative research methods to gain a deeper understanding of the research topic.

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REFERENCES

[1] Adeoye, M. A. (2023). From Struggle to Success: The Feynman Techniques' Revolutionary Impact on Slow Learners. *Thinking Skills and Creativity Journal, 6(2)*, 125-133. https://doi.org/10.23887/tscj.v6i2.69681

[2] Ambion, R. I., De Leon, R. S., Mendoza, A. P., & Navarro, R. M. (2020). The Utilization of the Feynman Technique in Paired Team Teaching Towards Enhancing Grade 10 ANHS Students' Academic Achievement in Science. 2020 IEEE Integrated STEM Education Conference (ISEC) (pp. 1-3). Princeton, NJ, USA: IEEE. https://doi.org/10.1109/ISEC49744.2020.9397848

[3] Arof, K. Z., Ismail, S., & Saleh, A. L. (2018). Contractor's performance appraisal system in the Malaysian construction industry: Current practice, perception and understanding.

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International Journal of Engineering & Technology, 7(3.9), 46-51. https://doi.org/10.14419/ijet.v7i3.9.15272

[4] Ashcraft, M. H., & Moore, A. M. (2009). Mathematics anxiety and the affective drop in performance. Journal of Psychoeducational Assessment, 27(3), 197–205. https://doi.org/10.1177/0734282908330580

[5] Baral, D., Dasgupta, N., McLeman, L., Bista, R., Jones, J., & Biswakarma, M. (2024). Exploring the predictive relationship between students' math anxiety and their mathematical performance. American Journal of STEM Education, 3, 1–15. https://doi.org/10.32674/c9zg4c28

[6] Bsharat, T. R., Febrianingrum, L., & Habibullah, M. (2024). Empowering Language Learners: Unleashing the Potential of The Feynman Technique in English Language Acquisition. PANYONARA: Journal of English Education, 6(2), 67-78. https://doi.org/10.19105/panyonara.v6i2.14936

[7] Demedts, F., Reynvoet, B., Sasanguie, D., & Depaepe, F. (2022). Unraveling the role of math anxiety in students' math performance. Frontiers Psychology, 13, 979113. https://doi.org/10.3389/fpsyg.2022.979113

[8] Fan, X., Hambleton, R., & Zhang, M. (2019). Profiles of Mathematics Anxiety Among 15-Year-Old Students: A Cross-Cultural Study Using Multi-Group Latent Profile Analysis. Frontiers in Psychology, 10, 1217. https://doi.org/10.3389/fpsyg.2019.01217

[9] Ferrer, C. J., Janeo, J. C., & Iglesias, J. C. (2025). Learning Experiences of Students on the Least Preferred Topics in Mathematics. Mathematics Teaching Research Journal, 17(1), 264-279. https://eric.ed.gov/?id=EJ1470641

[10] Feynman, R. P., Leighton, R. B., & Sands, M. (2006). The Feynman lectures on physics: The definitive edition (Vols. 1–3). San Francisco, CA: Addison-Wesley Longman. https://www.feynmanlectures.caltech.edu

[11] Kimbrough, E. O., McGee, A. D., & Shigeoka, H. (2022). How do peers impact learning? An experimental investigation of peer-to-peer teaching and ability tracking. Journal of Human Resources, 57(1), 304-339. https://doi.org/10.3368/jhr.57.1.0918-9770R2

[12] Gordon, M. (2019). Mathematics Students as Artists: Broadening the Mathematics Curriculum. Journal of Humanistic Mathematics, 9(2), 192-210. https://doi.org/10.5642/jhummath.201902.13

[13] Hopko, D. R., Mahadevan, R., Bare, R. L., & Hunt, M. K. (2003). The abbreviated math anxiety scale (AMAS) construction, validity, and reliability. Assessment, 10(2), 178-182. https://doi.org/10.1177/1073191103010002008

[14] Iksal, I., Hayani, R. A., & Aslan, A. (2024). Strengthening character education as a response to the challenges of the times. Indonesian Journal of Education (INJOE), 4(3), 761-774. https://injoe.org/index.php/INJOE/article/view/116

[15] Karagiorgi, Y., & Symeou, L. (2005). Translating Constructivism into Instructional Design: Potential and Limitations. Educational Technology & Society, 8(1), 17-27. http://www.jstor.org/stable/jeductechsoci.8.1.17

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[16] Kurudirek, A., & Berdieva, O. (2024). The more empowered schools, the more fruitful students. Educenter : Jurnal Ilmiah Pendidikan, 3(1), 1-9. https://doi.org/10.55904/educenter.v3i1.921

[17] Lee, C. (2024). Mathematics anxiety as a global problem. In S. Johnston-Wilder & C. Lee (Eds.), The Mathematical Resilience Book (pp. 193–219). London: Routledge. https://doi.org/10.4324/9781003334354

[18] Lotfi, F., Dehkordi, S., & Vaez-Ghasemi, M. (2012). Teaching Method Survey Based on Constructivist Approach in Permanent Learning Geometry. Applied Mathematical Sciences, 6(6), 259-267. https://m-hikari.com/ams/ams-2012/ams-5-8-2012/lotfiAMS5-8-2012-5.pd

[19] Luneta, K., & Sunzuma, G. (2022). Instructional Interventions to Address Mathematics Anxiety in Sub-Saharan Africa: A Systematic Review (1980-2020). Africa Education Review, 19(1), 103-119. https://hdl.handle.net/10520/ejc-educare-v19-n1-a6

[20] Luttenberger, S., Wimmer, S., & Paechter, M. (2018). Spotlight on math anxiety. Psychology research and behavior management, 11, 311–322. https://doi.org/10.2147/PRBM.S141421

[21] Magas, J. C. (2025). Construction and Evaluation of Self-Directed Learning Material and Its Effect on STEM Students' Academic Performance in Mathematics. Mathematics Teaching Research Journal, 17(1), 232–263. https://eric.ed.gov/?id=EJ1470615

[22] Moliner, L., & Alegre, F. (2020). Effects of Peer Tutoring on Middle School Students' Mathematics Self-Concepts. Plos one, 15(4), e0231410. https://doi.org/10.3389/fpsyg.2020.01610

[23] Reyes, E. P., Blanco, R. M., Doroon, D. R., Limana, J. L., & Torcende, A. M. (2021). Feynman technique as a heutagogical learning strategy for independent and remote learning. Recoletos Multidisciplinary Research Journal, 9(2), 1-13. https://doi.org/10.32871/rmrj2109.02.06

[24] Setyaedhi, H. S. (2024). Comparative Test of Cronbach's Alpha Reliability Coefficient, Kr-20, Kr-21, And Split-Half Method. Journal of Education Research and Evaluation, 8(1), 47–57. https://doi.org/10.23887/jere.v8i1.68164

[25] Substack. (2023, May 28). The Feynman Learning Technique. https://sketchplanations.com/feynman-learning-technique

[26] Syahbana, A., & Nopriyanti, T. D. (2024). Development of Student Self-Efficacy for Mathematics Learning in Indonesia. Mathematics Teaching Research Journal, 16(3), 265-291. https://eric.ed.gov/?id=EJ1442292

[27] Tessényi, J., Katona, N., & Kun, Z. (2024). Exploring entrepreneurial inclination: Schwartz's values theory and Bandura's self-efficacy scale applied to Hungarian university students. International Journal of Entrepreneurship and Innovation Management, 28, 224-239. https://doi.org/10.1504/IJEIM.2024.144397

[28] Tshering, G. (2024). Teaching Algebra to a Grade 7 Student: Action Research Intervention. Mathematics Teaching Research Journal, 16(4), 132–153. https://eric.ed.gov/?id=EJ1442417

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[29] Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in Teacher–Student Interaction: A Decade of Research. Educational Psychology Review, 22(3), 271-296. https://doi.org/10.1007/s10648-010-9127-6

[30] Vygotsky, L., & Cole, M. (2018). Lev Vygotsky: Learning and social constructivism. In S. MacBlain, Learning Theories for Early Years Practice (pp. 68–73). London: SAGE.

[31] Wea, K. N., Dua, Y. S., & Elizabeth, A. (2023). An exploratory study to investigate the implementation of Feynman learning method in a physics classroom setting. Journal of Innovative Science Education, 12(3), 331–339. https://doi.org/10.15294/jise.v12i3.78672

[32] Wittmann, E. (2019). Understanding and Organizing Mathematics Education as a Design Science–Origins and New Developments. Hiroshima Journal of Mathematics Education, 12, 13–32. https://doi.org/10.24529/hjme.1202

[33] Yusoff, M. S. (2019). ABC of Content Validation and Content Validity Index Calculation. Education in Medicine Journal, 11(2), 49–54. https://doi.org/10.21315/eimj2019.11.2.6

[34] Zanabazar, A., Deleg, A., Ravdan, M., & Tsogt-erdene, E. (2023). The Relationship between Mathematics Anxiety and Mathematical Performance among Undergraduate Students. Jurnal Ilmiah Peuradeun, 11(1), 309-322. https://doi.org/10.26811/peuradeun.v11i1.780

[35] Zhang, J., Zhao, N., & Kong, Q. P. (2019). The relationship between math anxiety and math performance: A meta-analytic investigation. Frontiers in Psychology, 10, 1613. https://doi.org/10.3389/fpsyg.2019.01613

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APPENDIX A: PRE-TEST INSTRUMENT

PARTS AND FEATURES OF A PARABOLA

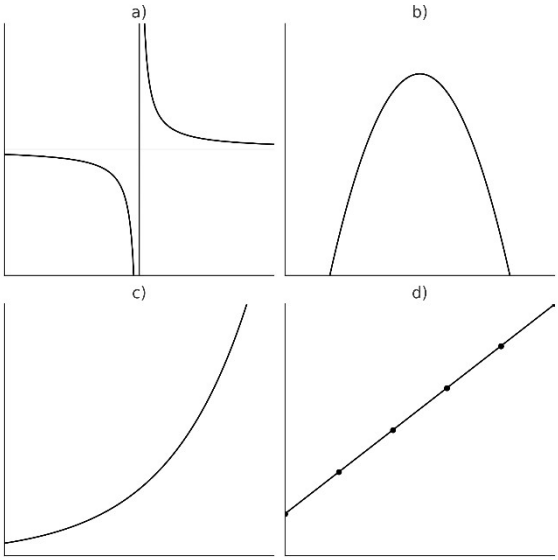
(PRE-TEST)

TOTAL MARKS: 10 **TIME ALLOWED: 20 MINUTES**

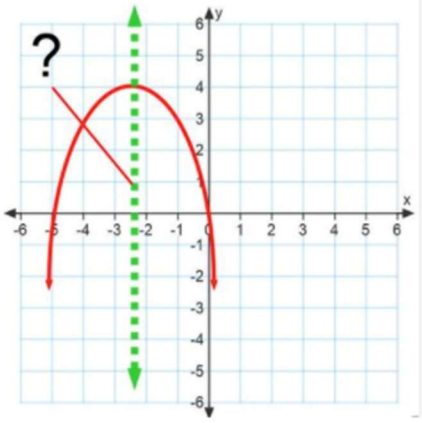
INSTRUCTIONS TO THE CANDIDATE:

1. This question paper consists of **10 multiple-choice questions**.
2. Please answer all questions by encircling the letter of the correct answer.

1. Which graph is a parabola?



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2. 

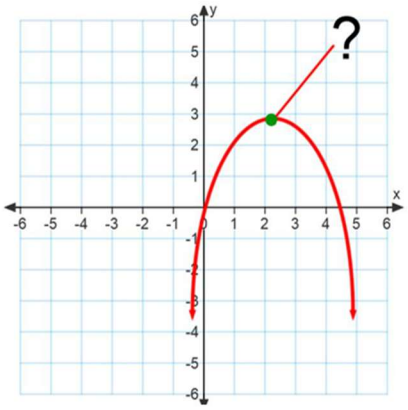
What is the green dashed line referred to as?

a) parabola

b) line of dashes

c) x-intercepts

d) axis of symmetry

3. 

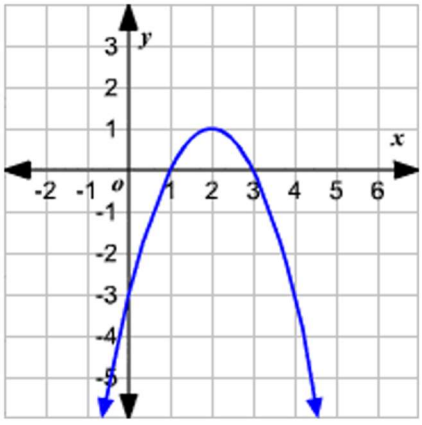
What is the green dot on this parabola called?

a) solution

b) root

c) zero

d) vertex

4. 

What is the symmetry axis for this graph?

a) $y = -3$

b) $x = 2$

c) $y = 2$

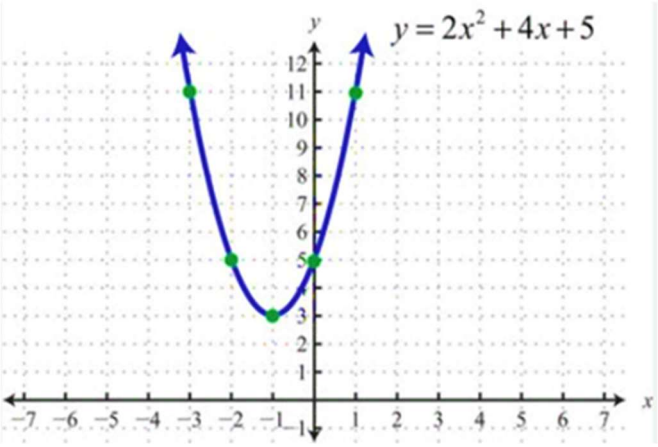
d) $x = 3$

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5. Which of the following best describes the vertex of a parabola?

a) Any point on the parabola b) A point on the x-axis

c) A point on the y-axis d) The highest or lowest point of a parabola

6. 

What is the vertex of this parabola?

a) (-2,5)

b) (0,5)

c) (-1,3)

d) (-1,-3)

7. In which direction does the parabola $y=x^2-2x+4$ open?

a) up

b) down

c) right

d) left

8. The parabola $y=x^2-2x+4$ will have …

a) a maximum vertex.

b) a minimum vertex.

c) no vertex

d) many vertices

9. In which direction does the parabola $y=-2x^2-4x+1$ open?

a) down

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b) up

c) left

d) right

10. How can you determine if a parabola $y=ax^2+bx+c$ opens upward or downward by simply looking at the equation?

a) All parabolas open upwards.

b) All parabolas open downwards.

c) The value of "a" tells you the direction of the opening.

d) It is impossible to determine simply by looking at the equation.

APPENDIX B: POST-TEST INSTRUMENT

PARABOLAS IN THE COORDINATE PLANE

TOTAL MARKS: 25 **TIME ALLOWED: 45 MINUTES**

INSTRUCTIONS TO THE CANDIDATE:

1. This question paper consists of 4 questions.
2. Answer ALL questions.
3. Write neatly and clearly.
4. The use of a nonprogrammable scientific calculator is allowed

QUESTION 1:

The set of points equidistant from (3,5) and the line $y=9$ is a parabola.

1.1) What is the vertex of the parabola? **(2 marks)**

1.2) Describe the graph of the parabola in terms of the **direction of opening** and the **line of symmetry.** **(2 marks)**

QUESTION 2:

Given the equation $y+3=\frac{1}{20}(x-9)^2$

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2.1) What is the vertex of the parabola? **(2 marks)**

2.2) Find the focus of the parabola **(3 marks)**

2.3) Find the directrix of the parabola **(2 marks)**

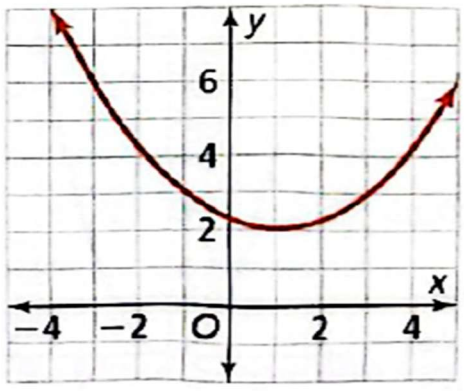
QUESTION 3:

3.1) What equation represents the parabola with focus (-1,4) and directrix y=-2? **(6 marks)**

3.2) What equation represents the parabola with focus (3,5) and vertex (3,-1)? **(5 marks)**

QUESTION 4:

Study the diagram and answer the question that follows:



Using the vertex, any other point on the parabola, and the equation

$y\text{-}k=\frac{1}{4p}(x\text{-}h)^2$, what is the value of p? **(3 marks)**

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APPENDIX C: MODIFIED AMAS

*** Please rate each item below in terms of how you would feel in each situation. Please indicate your choice by circling the corresponding number.

ITEM NO.	STATEMENTS	Low anxiety	Some anxiety	Moderate anxiety	A significant amount of anxiety	High anxiety
1	Your math teacher tells you that your next topic in math is parabolas in the coordinate plane.	1	2	3	4	5
2	You find out that you are going to write a surprise math test on parabolas when you start your math lesson.	1	2	3	4	5
3	Your math teacher tells you that you will write a text on parabolas the next day.	1	2	3	4	5
4	Your math teacher asks you to complete a worksheet on parabolas independently.	1	2	3	4	5
5	You are given some homework with challenging questions on parabolas that you must complete and submit the next day.	1	2	3	4	5
6	Your math teacher talks about parabolas in the coordinate plane for a long time.	1	2	3	4	5
7	You are listening to a student in your class explaining a solution to a problem on parabolas in the coordinate plane.	1	2	3	4	5
8	You are watching your math teacher solving a problem on parabolas on the board.	1	2	3	4	5
9	You are currently taking a math test focused on parabolas in the coordinate plane.	1	2	3	4	5

Adapted from Hopko et al. (2003)
AMAS: Abbreviated Math Anxiety Scale

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Enhancing Elementary Pre-service Teachers' STEM Integration: Perceptions, Self-Efficacy, and Behavioral Intentions Through Experiential Learning in a Mathematics Methods Course.

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Abstract: The increasing demand for STEM-literate citizens requires a systematic preparation of elementary teachers who can effectively integrate science, technology, engineering, and mathematics from the earliest grades. In this study we address the critical gap in elementary STEM teacher preparation by implementing and evaluating STEM activities within a mathematics methods course in a US teacher preparation program. We used an experiential learning approach to expose and engage Elementary Pre-Service Teachers (EPSTs) in structured STEM activities designed to enhance their confidence and pedagogical readiness. Results indicate that after engaging in STEM activities, EPSTs demonstrated significantly higher levels of confidence and preparedness to integrate STEM activities in their future lessons. Specifically, our goal was to contribute new knowledge about how experiential learning transforms elementary pre-service teachers' acceptance of STEM integration, revealing that structured hands-on experiences are more effective than traditional lecture-based approaches in developing STEM self-efficacy. EPSTs recognized the value of integrating STEM activities within their lessons as a strategy to enhance conceptual understanding in mathematics and science topics.

Keywords: Elementary Pre-service Teachers; STEM; Experiential Learning; Teacher Preparation; Self-Efficacy.

INTRODUCTION

STEM education represents an integrated approach to teaching science, technology, engineering, and mathematics that mirrors real-world problem-solving and innovation processes (Kelley & Knowles, 2016). In today's rapidly evolving technological society, STEM literacy has become essential for informed citizenship, economic competitiveness, and addressing complex global challenges such as climate change, healthcare innovation, and sustainable development (NSF, 2020). The importance of STEM education is particularly relevant during elementary years, as students' attitudes toward STEM fields and self-efficacy beliefs are largely formed during these

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foundational years (Master et al., 2017). Early exposure to integrated STEM experiences capitalizes on children's natural curiosity, establishes crucial conceptual foundations, and fosters critical thinking skills, spatial reasoning, and positive dispositions toward mathematics and science that persist throughout their academic trajectories (Bers et al., 2021; Hebebci & Usta, 2022).

Despite the recognized importance of early STEM education, elementary teachers face significant challenges in implementing integrated STEM instruction. Many teachers in grades K–6 report feeling underprepared and lacking confidence in both STEM content knowledge and pedagogical approaches (Margot & Kettler, 2019). These challenges are compounded by limited disciplinary exposure during their own K–12 education and insufficient preparation in teacher education programs that traditionally prioritize literacy and general pedagogy over disciplinary integration (Stohlmann et al., 2012).

Existing research reveals that traditional preparation approaches relying heavily on theoretical coursework without hands-on experiences are insufficient for developing the confidence and competence needed for effective STEM integration (Nadelson et al., 2013). While studies have examined professional development benefits for in-service teachers (Tomas et al., 2019), limited research addresses how experiential learning within pre-service preparation programs can enhance STEM self-efficacy and integration skills. For example, Bursal and Paznokas (2006) found that inquiry-based science experiences increased pre-service teachers' science teaching self-efficacy, and McKinnon and Lamberts (2014) reported that engineering design challenges strengthened understanding of integrated STEM pedagogies and willingness to implement them in future classrooms.

Our study addresses these gaps by examining how experiential learning within a teacher preparation program can enhance elementary pre-service teachers' (EPSTs) attitudes, self-efficacy, and behavioral intentions toward STEM integration. Using a convergent parallel mixed-methods design, we investigated changes in EPSTs' confidence, value perceptions, and implementation intentions following participation in structured STEM activities, guided by the following research question: How does EPSTs' participation in structured STEM activities within an experiential learning framework influence their attitudes, self-efficacy, and intentions to integrate STEM in their future classroom instruction? This overarching question was examined by (a) analyzing changes in EPSTs' confidence and perceived ability to implement STEM activities, (b) investigating shifts in value perceptions and expectations regarding STEM integration, and (c) exploring the development of behavioral intentions to incorporate STEM approaches in future teaching practice.

LITERATURE REVIEW AND THEORETICAL PERSPECTIVES

Our study is grounded in an integrated framework combining experiential learning theory, technology acceptance models, and self-efficacy theory. This multi-theoretical approach provides a comprehensive lens for understanding how structured hands-on experiences influence pre-service teachers' attitudes and behavioral intentions toward STEM integration.

Experiential Learning Theory

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Experiential Learning (EL) represents a holistic educational philosophy that emphasizes the central role of experience in knowledge construction and skill development (Kolb, 2014). Grounded in social constructivist principles (Vygotsky, 1978) and Piaget's (1976) constructivist theory, meaningful learning occurs when learners actively engage with materials, ideas, and social contexts through authentic participation. Kolb's (1984) four-stage cycle—concrete experience, reflective observation, abstract conceptualization, and active experimentation—provides the pedagogical framework for the STEM activities implemented in this study.

The integration of experiential learning with social learning theory (Bandura, 1977) further explains how EPSTs develop STEM teaching competencies through observational learning, modeling, and guided practice, acquiring both technical skills and the self-efficacy beliefs necessary for effective STEM integration. This social dimension is particularly relevant in STEM education, where collaborative problem-solving and peer interaction are essential components of authentic practice. Research supports the effectiveness of such approaches: Tomas et al. (2019) demonstrated that hands-on STEM experiences increased EPSTs' teaching confidence and deepened their understanding of scientific practices, while Bursal and Paznokas (2006) found that experiential approaches improved both content knowledge and pedagogical understanding. Collectively, these experiences were designed to develop what Shulman (1987) conceptualized as pedagogical content knowledge—the intersection of content knowledge, pedagogical knowledge, and knowledge of students and curriculum.

Technology Acceptance Model Applied to STEM Integration

The Technology Acceptance Model (TAM) provides valuable frameworks for understanding how individuals adopt and integrate new technologies and pedagogical approaches in educational contexts (Venkatesh et al., 2003). Applying the acceptance theory to STEM integration offers a novel perspective for understanding how EPSTs develop intentions to use STEM approaches in their future practice, contributing new knowledge about the psychological and social factors that influence STEM adoption in elementary education. This theoretical lens brings a unique perspective to elementary STEM teacher education by focusing not just on skill development, but on the acceptance processes that ultimately determine whether pre-service teachers will implement STEM approaches in their classrooms. Unlike previous research that has primarily examined STEM teacher education through self-efficacy or content knowledge frameworks, the acceptance theory perspective highlights the complex interplay of performance expectations, effort expectations, social influences, and facilitating conditions that shape EPSTs' behavioral intentions–i.e., understood as the motivation or wiliness to implement or integrate STEM activities. Venkatesh et al.'s (2003) Unified Theory of acceptance and use of technology identify four key determinants of acceptance and use: performance expectancy, effort expectancy, social influence, and facilitating conditions.

In the context of STEM education and this study, performance expectancy relates to EPSTs' beliefs about how STEM integration will enhance their teaching effectiveness, while effort expectancy concerns their perceptions of the ease of implementing STEM activities. Social influence

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encompasses the degree to which EPSTs believe important others expect them to use STEM approaches, and facilitating conditions refer to the organizational and resource support available for STEM implementation.

Self-Efficacy Theory

Self-efficacy, defined as individuals' beliefs in their capabilities to organize and execute courses of action required to produce given attainments (Bandura, 1997), represents a fundamental construct in understanding human motivation and behavior. In education, self-efficacy beliefs significantly influence teachers' instructional decisions, persistence in facing challenges, and overall effectiveness in implementing innovative pedagogical approaches (Emiru & Gedifew, 2024). Self-efficacy is particularly crucial for elementary teachers implementing STEM education because they must feel confident in their ability to integrate multiple disciplines while maintaining age-appropriate instruction and managing diverse learning needs. Research consistently demonstrates that teachers with higher self-efficacy are more likely to attempt innovative teaching methods, persist through implementation challenges, and create engaging learning environments for their students (Tschannen-Moran & Hoy, 2001).

Elementary STEM education presents unique self-efficacy challenges because teachers must feel competent across multiple disciplines while understanding how to create meaningful connections between science, technology, engineering, and mathematics concepts. Teachers with low STEM self-efficacy may avoid implementing interdisciplinary activities, thereby limiting their students' exposure to integrated STEM experiences during critical developmental years (Palmer et al., 2015). The development of STEM self-efficacy among EPSTs is therefore essential for addressing the persistent gap in elementary STEM education quality and accessibility. Bandura (1997) identified four primary sources of self-efficacy: mastery experiences, indirect experiences, verbal persuasion, and physiological and emotional states. Our study's experiential learning approach primarily targets mastery experiences—the most powerful source of self-efficacy—by providing EPSTs with successful hands-on STEM implementation experiences.

The integration of these three theoretical frameworks provides a comprehensive understanding of how experiential learning interventions can systematically develop EPSTs' self-efficacy beliefs and acceptance of STEM integration, ultimately preparing more confident and competent elementary STEM educators.

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METHODS

Research Design and Rationale

We considered a convergent parallel mixed-methods design (Creswell & Plano Clark, 2017) to comprehensively examine changes in EPSTs' STEM attitudes, self-efficacy, and behavioral intentions. This design was chosen because quantitative measures alone cannot capture a holistic experiences and meaning-making processes that occur during experiential learning. Similarly, if we relied solely on qualitative data, we would lack sufficient evidence to demonstrate systematic changes across the participant population. The integration of quantitative surveys with qualitative interviews allowed for triangulation of findings and provided both breadth and depth in understanding the intervention's impact. We recognize that in our study, we only considered a single-group pre-post design without a control group, which limit our ability to generalize and make strong causal inferences. However, we considered our study as an opportunity to obtain preliminary evidence about the potential impact of experiential learning approaches in STEM teacher preparation.

Participants and Demographics

The participant of our study consisted of 75 undergraduate EPSTs enrolled in a teacher preparation program at a Hispanic-Serving Institution (HSI) in south-central United States. A power analysis in G*Power indicated that this sample size was adequate for detecting medium effect sizes ($d = 0.50$) with 80% power at $\alpha = 0.05$ for paired t-tests. Participants EPSTs ranged in age from 19 to 25+ years, with 67 females (89%) and 8 males (11%), as considered by their biological sex. Many participants (53%) were enrolled in bilingual education programs, reflecting the institution's focus on serving diverse student populations. Notably, 89% of participants reported no prior STEM coursework experience, and 64% identified as first-generation college students. EPSTs were recruited through convenience sampling (Creswell & Guetterman, 2024) from required mathematics methods courses across two academic semesters.

Data Collection

The measurement instrument used was systematically developed through adaptation and validation of existing instruments (Abbad, 2021; Ensign, 2017; Gado et al., 2006). The final 22-item survey measured six specific constructs: self-confidence and ability, value and enjoyment of STEM, performance expectancy, effort expectancy, social influence, and behavioral intention. Items were measured using a five-point Likert scale (1 = completely disagree, 5 = completely agree). Cronbach's alpha coefficients for the six constructs ranged from .78 to .91, indicating acceptable to excellent internal consistency reliability.

In addition, semi-structured interviews were conducted with 8 volunteer EPSTs, lasting approximately 20-30 minutes each and video-recorded for analysis. Interview questions explored participants' experiences with the STEM activities, perceptions of STEM's value in elementary

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education, and intentions for future implementation (see Table 8 for complete interview protocol). Data were collected between Spring 2023 and Spring 2024 across two academic semesters to ensure consistency and replication of findings. Pre-intervention surveys were administered during the first week of each semester, with post-intervention surveys collected during the final week after completion of all STEM activities.

STEM Activities

In this study, we focused on developing EPSTs' pedagogical skills for teaching mathematics within STEM contexts that they could promote in their future classrooms. Since pre-service teachers often exhibit mathematics anxiety (Itter & Meyers, 2017), we designed activities to address these concerns by demonstrating the usefulness and accessibility of STEM integration for elementary students. Five structured STEM activities were systematically implemented across each 14-week semester, spaced approximately 2-3 weeks apart to allow for reflection and integration of learning. Each activity incorporated pre- and post-activity discussions designed to facilitate explicit connections between hands-on experiences and mathematical concepts. All activities were aligned with Kolb's (1984) experiential learning cycle and elementary mathematics standards.

Corn tortilla making (Haciendo tortillas de maíz)

This activity required EPSTs to make corn tortillas from scratch using a mechanical tortilla-maker machine while applying mathematical concepts of ratios and proportions (See Figure 1). The use of the machine was modeled by the instructor to demonstrate proper procedures and safety protocols. Mathematical learning objectives included understanding ratios, proportional reasoning, and measurement concepts through authentic application. To create quality tortilla dough, participants needed to calculate precise ingredient ratios. However, to increase mathematical challenge and engagement, EPSTs were not provided with measuring cups, requiring them to apply proportional reasoning and sensory feedback to achieve proper dough consistency. This approach connected abstract mathematical concepts to concrete, culturally relevant experiences (Brown, 2021). EPSTs worked in collaborative teams of 3-4 members (see Figure 1), promoting social learning and peer support. The activity concluded with reflection discussions connecting the mathematical concepts to elementary teaching strategies.



Figure 1: Example of the Process of Making Corn Tortilla by First Mixing the Ingredients, then Using the Tortilla-Maker Machine.

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Finch Robots Mathematical Applications

Finch Robots© are educational programmable robots designed specifically for teaching mathematics, computer science, and coding concepts to elementary students. They can also be effectively used with middle and high school students. For this study, the robots were integrated into the mathematics methods course to provide EPSTs with a hands-on technology integration experience. The pedagogical focus was on developing EPSTs' technological pedagogical content knowledge (TPACK) for elementary mathematics. Specific learning activities included programming robots to explore geometric concepts through movement patterns, investigating measurement through distance and angle calculations, and developing spatial reasoning through problem-solving challenges (see figure 2). Based on our implementation experience, Finch Robots provided the crucial connection between mathematical theory and practical application, supporting EPSTs in developing confidence to integrate technology meaningfully into their mathematics instruction while addressing elementary curriculum standards.



Figure 2: EPST Practicing with Finch Robot

Engineering-Based Measurement Tool

This activity engaged EPSTs in constructing a clinometer (see figure 3), an engineering measurement tool used for calculating object heights and determining angles of elevation. The construction and application required EPSTs to apply several mathematics topics including angles, measurements, and distances, while working in small teams. The activity targeted specific elementary mathematics standards related to geometry and measurement. The main reason for exposing the EPSTs to build and use the clinometer was for them to experience firsthand the intersection between mathematics and engineering through applied trigonometry and measurements. In addition, to model how an activity could take place outside the classroom, that can later be used as part of a lesson in the classroom. Although that a clinometer could be used to calculate heights of trees or buildings (or any other object) using any angle, for this particular activity with EPST, only angles of 30º, 45º, and 60º degrees were considered to align with their future grade-level teaching responsibilities (i.e., from first to fifth grade). In figure 6 below, an EPSTs is shown measuring the university library.

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For us as teacher educators, focusing on STEM integration, this activity bridges multiple STEM disciplines by combining creativity, teamwork, engineering design principles, geometric understanding, and mathematical application skills. The use of accessible materials demonstrates how abstract mathematical concepts can be transformed into concrete ideas that EPSTs could undoubtedly integrate in their teaching.

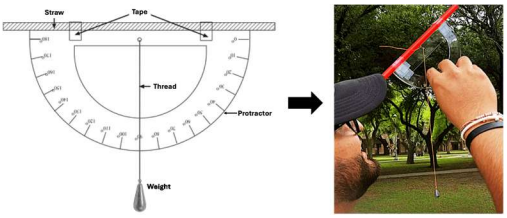


Figure 3: Clinometer description and a EPST using it.

Mixed-Reality Simulations

The EPSTs were exposed to mixed-reality simulations (MRS) to support the development of their mathematical pedagogical skills in the classroom. Studies have found that EPSTs greatly benefit from being exposed to MRS to practice and develop productive mathematical talk moves to orchestrate classroom discourse (Aguilar & Flores, 2022; Aguilar & Telese, 2018; Zhengtao & Hidayat, 2025).

During an MRS session, the EPSTs interact in a Zoom-type session with an avatar that it is controlled by an educational specialist, who have previously been prepared into the simulated scenario to be practiced (see figure 7). In the case of the EPSTs that participated in this study, they were exposed to three different problem-solving mathematical activities that followed a cognitive guide instruction framework developed by Carpenter et al. (1999). Following the exposure, the EPSTs had to write a guided self-reflection essay about their experiences and provide feedback to 3-4 peers. In addition, several EPTS's recorded videos were randomly selected to be discussed in class, as a whole-classroom discussion. The durations of each exposure lasted about 9-15 minutes, which is considered to be similar to an approximately 45 minutes of a real-life setting experience. It is relevant to mention that the exposure to MRS provided the EPSTs an approximation to the practice (Grossman, 2021), that they would not have until their last year of their teacher preparation program, when they are required to participate in clinical teaching practices. This serves as a critical connection between the teacher preparation coursework and field experiences, which allows the EPSTs to start practicing theoretical knowledge in contextual settings while developing confidence in mathematics pedagogies (Aguilar & Flores, 2022).

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Figure 4: Example of an EPST Interacting with an Avatar in a Mixed-Reality Session.

Makey-Makey Electrical Circuits

Makey-Makey© is an innovative educational set tool intended to provide a STEM experience related to electrical circuits by using a circuit board. The board use a simple USB cable that connects to a computer. The EPSTs were modeled how to use and integrate Makey-Makey within their future lessons (See figure 5) by exploring how a circuit works in a real setting. The use of Makey-Makey served as a conceptual-experienced between mathematics and STEM by supporting the exploration of circuits, conductivity, and computational thinking, while reinforcing mathematical concepts. The EPSTs could design interactive mathematics games where conductive materials (e.g., like fruit, play-dough, aluminum foil, or pencil granite) could become manipulatives for different topics like measuring, counting, geometric figures, or patterns (see figure 5).

As a hands-on STEM tool, Makey-Makey enabled EPSTs to experience how abstract mathematical concepts could be physically represented through circuits. This supported their preparation by demonstrating how to foster spatial reasoning and pattern recognition while developing technological pedagogical content knowledge. The activities were designed to show how interdisciplinary lessons could naturally integrate mathematics, science, and engineering principles at the elementary level.



Figure 5: EPSTs creating a math related lesson with Makey-Makey.

Data Analysis

Our quantitative analysis included descriptive statistics and paired-samples t-tests to examine pre-post changes. We verified normality using Shapiro-Wilk tests and examined outliers using boxplots. Analysis of covariance (ANCOVA) was conducted controlling for the EPSTs sex,

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program of study, and prior STEM experience to examine potential moderating effects. Effect sizes were calculated using Cohen's d to determine practical significance of observed changes, with 95% confidence intervals reported for all effect sizes. For the qualitative data, all interviews were transcribed and analyzed using a systematic two-cycle coding process (Saldaña & Omasta, 2016). Initial in-vivo coding captured participants' own language and expressions, followed by pattern coding to identify broader themes. For example, initial codes such as "makes math more fun," "students would be more engaged," and "hands-on is better" were categorized under the broader theme of "enhanced student engagement through experiential learning." Qualitative data coding's inter-rater agreement was established at 88% between two independent coders.

In the end, all the quantitative and qualitative data were integrated during the interpretation phase, which is depicted in the results section below. We used a joint display analysis (Creswell & Plano Clark, 2017), where statistical findings were compared with qualitative themes to identify convergent, divergent, or complementary results.

RESULTS

In this section we are depicting the findings of EPST's attitudes and self-efficacy after been exposed to a series of STEM-related activities during a mathematics methods course. We have divided the section in two parts: The qualitative and the quantitative outcomes.

Quantitative outcomes

In table 1 below we depict the descriptive statistics of the data divided by sex, age, program of study within the teacher preparation program, student's classification (understood as first or non-first generation in college), and the level of experience in STEM.

Characteristics	*n*	%	Total
Sex			
Female	67	89%	75
Male	8	11%	
Age			
19-20	20	26%	75
21-22	32	43%	
23-24	10	13%	
25+	13	17%	
Program of Study			
EC-3 (Early Childhood - 3rd Grade)	9	13%	75

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EC-6 (Early Childhood - 6th grade) generalist	13	17%	
Bilingual Education	40	53%	
ESL (English as a Second Language)	7	9%	
Special Education	6	8%	
Student Classification			
First Generation College	48	64%	75
Non-First Generation	27	36%	
STEM Experience			
Minimum (2 courses or less)	8	11%	75
None	67	89%	

Table 1: Demographic Characteristics of Elementary Pre-Service Teachers

From table 1, there are several important aspects that can be highlighted. First, most of the EPTSs participating in this study were females, as identified by their biological sex. This is not surprising since historically there has been more females in teacher preparation programs than males (NCES, 2022; Ingersoll et al., 2021). However, there are mindsets and gaps differences between males and females in terms of STEM. For example, when the EPSTs were first asked about if they find challenging integrating STEM activities within their instruction, 40% of males disagree with this statement, where only 13% of females disagree. In addition, when we asked the EPTSs if they felt confident in integrating STEM in their lessons, about 50% females agree or completely agree, where 63% of males completely agree with this statement. This could be potentially important, since there has been a shortage of females in STEM, mainly in minorities populations (Fry et al., 2021).

We were particularly interested in finding evidence that support our assumption that by exposing the EPSTs to an experiential learning in STEM their confident, interest, and value toward it would develop. Second, most of the EPSTs were in the bilingual program of study and had never taken a course related to STEM. These could potentially impact how the EPSTs responded to the questions in the survey, since they did not have any previous experience with STEM activities. Third, most of the EPSTs that participated in the study have not experience with STEM (89%). This is worrisome, since to develop a STEM mindset in elementary students, teachers need be ready and well prepared in how to integrate STEM ideas within their lessons, which is part we decided to conduct this study.

In regard to the EPTS's responses to the pre- and post-test, we have decided to depict a sample of the characteristics that were measured with the intention of providing a holistic approach of the instrument. To this end, we randomly selected two items in each measured characteristic (See table 2), and then organized the responses for each item (see table 3), depicting a combined responds for answers as "completely agree or agree" and "completely disagree or disagree" for the pre and post survey responses, recalling the survey was a liker-scale from 1 to 5, where 1 represented completely disagree, and 5 completely agree. See appendix B for the complete responses.

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Characteristic	ITEM Question
Self-confident and ability	2. I am confident I can integrate STEM/STEM (e.g., robotics) for teaching in elementary.
	6. With my current experience in STEM/STEM, I feel prepared to integrate related concepts and tools (e.g., robotics) within my lesson plans and future teaching.
Value and enjoyment of STEM	7. Children should learn STEM/STEM related concepts (e.g., Robotics) in Elementary.
	10.Students would conceptually learn mathematics if STEM/STEM related concepts (e.g., Robotics) are used for teaching elementary mathematics.
Performance and effort Expectancy	13. Using STEM/STEM (e.g., robotics) in teaching and learning elementary mathematics would help me to accomplish my teaching goals quickly.
	15. If I use STEM/STEM (e.g., robotics) in teaching and learning elementary mathematics, I will increase my chances of getting a raise once I get a teaching position.
Effort Expectancy	17. My interaction with STEM/STEM related tools like robotics would be easy for me to become skillful at using it.
	18. Integrating STEM/STEM related tools like robotics in my elementary mathematics lessons would be easy for me.
Social Influence and Behavioral Intention	21. If my future colleagues integrate STEM/STEM related tools like robotics within their lesson, I will probably use it in my teaching and learning of elementary mathematics too.
	22. I would use STEM/STEM related tools like robotics in my teaching and learning once I become a teacher.

Table 2: Sample of Questions by Measured Characteristic

From table 3 below, it can be noticed that in every item the EPST's responses positively increased from the pre to the post implementation. It is important to mention that this does not means they are all statistically significant. To find evidence of this we have conducted a t-test and a covariance analysis. However, some inferences emerged from the raw data. For example, item 2, 6 & 17 shows that EPST's confidence to integrate some STEM ideas in their lessons, improved. This is relevant since, the ETPS's mindset toward STEM has positively shifted. In addition, in items 7, 10 & 22, the EPSTs depicted comprehension about the importance of engaging elementary students in STEM activities to support and promote conceptual understandings beyond simple lectures, same as the EPSTs were exposed in an experiential learning approach (Kolb & Kolb, 2009).

#	Pre		Post	
	Agree or Completely Agree	Disagree or completely disagree	Agree or Completely Agree	Disagree or completely disagree
2	60%	11%	74%	6%
6	21%	42%	72%	2%
7	81%	0%	91%	2%
10	53%	6%	79%	0%
13	53%	2%	85%	2%
15	40%	0%	70%	4%

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17	47%	17%	70%	0%
18	34%	15%	64%	2%
21	75%	0%	87%	0%
22	79%	2%	85%	0%

Table 3: Raw Data of Selected Survey Items

Also, items 17, 18 & 22 showed the EPSTs see the use and integration of STEM activities as something that would help them to better perform in their future teacher responsibilities and to achieve their teaching goals. This is relevant because the EPTS depicted to be inclined to accept the presented STEM ideas as new ways of approaching their own future pedagogies (Abbad, 2021).

Elementary Pre-service Teacher that participated in the study, were barely exposed and engaged to any structured STEM activity during their teacher preparation program. We decided to change this, because as researchers, we recognize that for young elementary scholars to develop a STEM mindset, they need to be exposed to it since a very young age, rather than later when they get to middle or high school. We understand that there are many more relevant aspects that need to be said and shared. These are shared in the next paragraph.

Continuing with our analysis of the quantitative data, we conducted a pair t-test analysis of the pre and post data by creating the mean scores for each construct and observing and comparing the differences between the pre and post responses. In table 4 below we depict the means of the EPST's responses and its standard deviation for each of the measured construct.

Measured Construct	**Pre (*SD*)**	**Post (*SD*)**	**p-value, effect size [95% CI]**
Self-confidence and ability	3.34 (0.39)	3.54 (0.51)	$p < 0.01$, $d = 0.33$ [0.20, 0.46]
Value and Enjoyment of STEM	3.70 (0.44)	4.00 (0.51)	$p < 0.01$, $d = 0.43$ [0.29, 0.57]
Performance expectancy	3.70 (0.59)	4.09 (0.58)	$p < 0.01$, $d = 0.53$ [0.38, 0.68]
Effort Expectancy	3.30 (0.75)	3.98 (0.74)	$p < 0.01$, $d = 0.64$ [0.48, 0.80]
Social influence	3.89 (0.69)	4.21 (0.66)	$p < 0.01$, $d = 0.33$ [0.19, 0.47]
Behavioral intention	3.60 (0.75)	4.18 (0.71)	$p < 0.01$, $d = 0.59$ [0.43, 0.75]

Table 4: Mean Responses and the Standard Deviation from the Survey

The paired sample t-test revealed that the EPSTs showed significantly higher responses in all the constructs after the implementation (see Table 4). All six measured constructs depict statistically significant improvements from pre to post measurements ($p < 0.01$). This consistent pattern indicates that the intervention had a meaningful impact across all measured areas. Effect sizes ranged from small to medium-large, indicating both statistical and practical significance. The largest effect sizes were observed for Effort Expectancy ($d = 0.64$) and Behavioral Intention ($d = 0.59$), suggesting that the experiential learning intervention was particularly effective in reducing

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perceived implementation difficulty and increasing intentions to use STEM approaches in future teaching. These findings align with technology acceptance theory predictions that effort expectancy and behavioral intention are primary determinants of adoption (Venkatesh et al., 2003).

Then, we subdivided participants (i.e., the EPSTs) based on their status (i.e., first generation), sex, program of study, and background knowledge in STEM, and compared the groups' post-survey responses with pre-survey as a covariate on each construct. In regard to the status of the EPSTs (i.e., first-generation versus non-first-generation), we found that there was no difference between the subgroups in all the constructs (see Table 5), where first-generation group showed higher responses than non-first generation after the implementation. This is relevant, since being a first-generation student, does not reflect to be a factor influencing the EPTSs mindset toward STEM.

Measured Construct	**First generation (n = 49)**	**Non-First generation (n = 26)**	**p-value**
Self-confidence and ability	3.54 (0.52)	3.54 (0.51)	p = 0.41
Value and Enjoyment of STEM	4.02 (0.51)	3.94 (0.52)	p = 0.53
Performance expectancy	4.13 (0.59)	4.03 (0.58)	p = 0.20
Effort Expectancy	4.07 (0.72)	3.80 (0.76)	p = 0.31
Social influence	4.27 (0.67)	4.12 (0.65)	p = 0.45
Behavioral intention	4.27 (0.65)	4.01 (0.80)	p = 0.49

Table 5: Mean Scores Comparison Between First-Generation and Non-First-Generation Groups in Post-Survey Responses with Pre-Survey as a Covariate

We also conducted a covariance analysis considering the EPST's previous experiences with STEM, for which most of them (*n*=71) mentioned not having any previous experience with STEM activities implementation or exposure. We did not find any evidence that the EPST's STEM mindset was different between the subgroups, which means that having or not experience with STEM has an impact on the EPST's minded toward it. In a similar pattern, we also analyzed the differences based on sex and the program of study of the EPSTs. (See tables 6 and 7 below).

	Male (n = 8)	**Female** (n = 67)	**p-value**
Self-confidence and ability	3.54 (0.24)	3.54 (0.54)	p = 0.86
Value and Enjoyment of STEM	3.98 (0.39)	4.00 (0.53)	p = 0.93
Performance expectancy	4.28 (0.49)	4.07 (0.59)	p = 0.36
Effort Expectancy	4.18 (0.96)	3.95 (0.72)	p = 0.43
Social influence	4.13 (0.99)	4.22 (0.62)	p = 0.68
Behavioral intention	3.88 (1.31)	4.22 (0.61)	p = 0.21

Table 6: Mean Score Comparison of Sex In Post-Survey with Pre-Survey as a Covariate

Based on the EPST's sex—i.e., male (*n*=8) and female (*n*=67) —the covariation analysis depicted above in Table 6 shows that there were no statistically significant differences between males and

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females on any of the measured constructs. However, this was one our biggest limitation, since the nature of the sample had this composition —i.e., the EPTSs that participated in the study. In terms of means and variability, we did notice that for the behavioral intention construct, it shows the largest mean difference between groups, with females scoring higher (4.22) than males (3.88). The high standard deviation for males (1.31) indicates considerable variation within this small group.

	Bilingual (40)	ESL (7)	Spe. Ed (6)	EC6 (13)	Early Ed (9)	p-value
Self-confidence and ability	3.56 (0.57)	3.32 (0.21)	3.57 (0.32)	3.44 (0.28)	3.62 (0.59)	p = 0.85
Value and Enjoyment of STEM	4.04 (0.53)	3.73 (0.72)	4.06 (0.38)	3.89 (0.35)	4.00 (0.59)	p = 0.85
Performance expec-tancy	4.17 (0.61)	3.95 (0.87)	3.97 (0.36)	3.89 (0.45)	4.25 (0.45)	p = 0.62
Effort Expectancy	4.06 (0.73)	3.96 (0.74)	4.02 (0.69)	3.69 (0.76)	3.87 (0.97)	p = 0.81
Social influence	4.31 (0.64)	4.00 (0.71)	4.22 (0.67)	4.00 (0.71)	4.17 (0.75)	p = 0.76
Behavioral intention	4.23 (0.72)	4.20 (0.69)	4.06 (1.02)	4.08 (0.48)	4.13 (0.72)	p = 0.97

Table 7: Mean Score Comparison Of Program of Study in Post-Survey with Pre-Survey as a Co-variate

The covariance analysis of the EPSTs based on their program of study depicted in table 7, shows that there were not no statistically significant differences, since all p-values are above 0.05 (ranging from 0.62 to 0.97), on all of the measured constructs. Similarly to the analysis conducted considering the EPST's sex, the distribution of the sample played an important role, however, once again this is an uncontrollable variable, since all EPSTs are required to take the mathematics methods course regardless of their program of study. Despite the above, our analysis shows that the bilingual education EPSTs scored higher on the constructs related to performance expectancy (4.17) and social influence (4.31), the ESL EPSTs scored the lowest across most constructs compared to other groups, and special education EPSTs show relatively higher scores on value and enjoyment of STEM construct (4.06). In terms of the behavioral intention construct, all EPST's groups depicted a consistency scored (p= .97), suggesting that it is likely the program of study was not relevant in terms of their behavioral intentions.

In general, throughout our analysis we found evidence that support our claim that the implementation of STEM activities with EPSTs effectively improved their perceptions, attitudes, and behavioral intentions related to STEM education and integration at the elementary level, with particularly strong impacts on effort expectations and intended future engagement. In addition, we found evidence that all the variables considered in the covariance analysis (i.e., sex, program of study, or ETPS's status) had an impact on their attitudes and self-efficacy toward the use, implementation, and integration of STEM relegated activities within their future lessons with elementary students.

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Qualitative Outcomes

As mentioned above, we decided to conduct interviews with a subset of EPSTs that volunteered for this purpose. In total, 8 EPSTs participated in semi-structured open-ended interviews (Zhang & Wildemuth, 2009) that were video recorded. Each interview lasted approximately 20-30 minutes. The EPSTs were asked 9 questions (See table 8) related to their experiential learning experience during the math methods, where they were exposed to several STEM-related activities as described above.

1. Do you consider that (STEM Teaching) should start earlier (e.g., elementary?)? Why?
2. Can you talk about any subject in elementary that can be taught using robotics/STEM?
3. Do you think robotics/STEM activities during the course were and will be helpful to you?
4. Please explain how using robotics/STEM can enhance mathematics learning and teaching.
5. How was your experience (or how would be your experience) creating a lesson plan that integrates the use of robotics/STEM to teach mathematics in elementary (if you did one)?
6. Did you (or your group) go above and beyond the class requirements about robotics/STEM?
7. Did you enjoy (or would enjoy) using robotics/STEM to create a lesson plan? Please explain your rationale.
8. Would you like to integrate robotics/STEM into your instruction once you become an elementary teacher?
9. What did you dislike about robotics/STEM activities during the course?

Table 8: Interview Questions

Once all the interviews were transcribed, we conducted an open-coding double-round (Saldaña, 2014) analysis to find the most common patterns, ideas, and opinions from the EPTSs responses. After discussing our findings, we obtained a final inter-rating agreement (Cole, 2024) of 88% for each of the pattern found in each question. In the following paragraphs we depict the thematic patterns we found, for which data revealed four major themes that shows the EPSTs' experiences and perspectives following the STEM activities integration:

Theme 1: Changes in STEM Perceptions

Participants consistently described a fundamental shift in how they viewed STEM education's role in elementary classrooms. Initially, many EPSTs perceived STEM as content too advanced or complex for young learners. Following the implementation of the activities, EPSTs reconceptualized STEM as a pedagogical approach that could enhance existing mathematics and science instruction rather than additional content to cover. EPSTs quotes include: "I used to think STEM was just for older kids, like middle school or high school, but now I see how it can make elementary math come alive and actually help kids understand better.", "Before this class, I thought STEM meant complicated engineering projects, but now I understand it's about connecting subjects in ways that make sense to kids." This theme aligns with research indicating that teacher beliefs about developmental appropriateness significantly influence instructional decisions (Stipek & Byler,

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1997). The shift from viewing STEM as "too advanced" to "developmentally appropriate and beneficial" represents a crucial shift for future elementary STEM implementation.

Theme 2: Enhanced Pedagogical Confidence

EPSTs reported substantial increases in confidence regarding their ability to implement hands-on, integrated learning experiences. This finding directly supports Bandura's (1997) self-efficacy theory, as participants gained mastery experiences through successful completion of STEM activities, leading to increased self-efficacy beliefs. Key expressions of enhanced confidence included: "I feel so much more prepared now to actually try these activities with my students. Before, I would have been terrified, but now I'm excited.", "The hands-on experience made me realize I can do this. It's not as scary as I thought it would be.", "I have actual examples now of what STEM looks like, so I feel ready to adapt and create my own activities." Participants frequently mentioned feeling "more prepared," "excited to try," and "confident" about implementing STEM approaches. This confidence growth is particularly significant given that 89% of participants had no prior STEM experience.

Theme 3: Recognition of Student Engagement Benefits

All interviewed EPSTs recognized STEM activities' potential to increase student engagement and conceptual understanding. They described anticipating higher levels of student motivation, deeper learning, and improved mathematical reasoning when STEM approaches are implemented effectively. Example of EPSTs responses include: "Students would be so much more engaged than just sitting and listening to me explaining fractions…when they're building and creating, they're really learning.", "I can see how this makes abstract math concepts concrete. Kids can actually see and touch what they're learning about.", "The hands-on approach helps different types of learners. Not everyone learns by just hearing about it." This theme demonstrates EPSTs' developing pedagogical content knowledge and understanding of how experiential learning can support diverse learning needs and styles.

Theme 4: Implementation Concerns and Realistic Planning

Despite increased enthusiasm, EPSTs expressed realistic concerns about implementation challenges, including time constraints, resource availability, classroom management with hands-on activities, and curricular pressures. Importantly, these concerns were accompanied by problem-solving mindsets and willingness to adapt activities to available resources. Common concerns included: "I worry about having enough materials for all students, but I think I could modify activities to work with what's available.", "Time is always an issue with all the testing requirements, but maybe I could integrate these into existing math lessons.", "Classroom management might be challenging with hands-on activities, but I think the engagement would actually help with behavior." This theme reflects realistic thinking about implementation challenges while

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maintaining commitment to STEM integration. The presence of both enthusiasm and realistic concern-solving suggests that EPSTs developed a practical perspectives on STEM implementation.

Integration of Qualitative and Quantitative Findings

The qualitative themes that emerged from the interviews strongly converge with the quantitative results obtained, providing triangulation support for the effectiveness of the implementation of the STEM activities with the EPSTs in the following ways. First, the transformation of STEM integration and implementation perceptions aligns with increased value and enjoyment scores. Second, we noticed that confidence was enhanced, which correspond with improved self-efficacy measures. Third, the EPSTs recognized the benefits for their future students. This relates to higher performance expectancy, and last but not less relevant, the realistic implementation planning, which connects with improved effort expectancy and behavioral intentions. These last aspects are relevant since depicts the intention of the EPSTs for adopting and integrating STEM activities in their classroom.

DISCUSSION AND CONCLUSIONS

Our study provides evidence that experiential learning approaches can effectively transform EPST's attitudes, self-efficacy, and behavioral intentions toward STEM integration. The consistent pattern of statistically significant improvements across all measured constructs ($d = 0.33$ to 0.64), combined with qualitative evidence of improved perspectives and enhanced confidence, suggests that structured hands-on experiential STEM activities address fundamental barriers to elementary STEM implementation. We aimed to contribute new knowledge to the field by depicting evidence that structured experiential learning integration can develop STEM acceptance and self-efficacy among EPSTs who typically have limited STEM backgrounds. Unlike previous research that has focused primarily on content knowledge development (e.g., Nadelson et al., 2013), our study reveals that acceptance-based approaches targeting performance expectations, effort expectations, and behavioral intentions may be effective for developing the confident EPSTs. In fact, the particularly strong effects we observed for effort expectancy ($d = 0.64$) and behavioral intention ($d = 0.59$) align with Venkatesh et al.'s (2003) findings that these constructs are primary determinants of technology adoption and use. In the context of STEM education, these results suggest that reducing perceived implementation difficulty while enhancing performance expectations can significantly increase the likelihood of future STEM integration.

Our findings extend previous research by Bursal and Paznokas (2006) and Tomas et al. (2019), who found that hands-on experiences improve pre-service teachers' STEM confidence. We advance the field by providing evidence through which experiential learning influences STEM adoption intentions, bridging self-efficacy theory with technology acceptance models in teacher education contexts.

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Implications for Mathematics Teacher Education Practice

The enhancement of EPSTs' perspectives on the developmental appropriateness of elementary STEM education represents a significant contribution to addressing gaps in early STEM exposure. EPSTs can develop both competence and confidence through structured experiences. Our study provided evidence for systematic approaches to improving elementary STEM education quality. Specifically, teacher preparation programs could integrate experiential STEM activities into required methods courses where natural connections exist between manipulative use and STEM integration, prioritizing self-efficacy development (Bandura, 1997) through successful hands-on STEM implementation, rather than theoretical coursework alone. Also incorporating culturally relevant activities to connect mathematical concepts to students' cultural backgrounds. Based on our findings, we recommend that mathematics teacher educators consider a structured lesson incorporating Kolb's (1984) experiential learning cycle, beginning with concrete experiences—such as constructing measurement tools or programming educational robots—followed by reflective observation, abstract conceptualization through explicit connections to elementary mathematics standards, and active experimentation as EPSTs adapt activities for their future classrooms.

Teacher educators could allocate about 90 min./activity, with 30 min. for hands-on engagement, 20 min. for guided reflection connecting mathematical concepts to STEM integration, 25 min. for collaborative discussion of implementation strategies, and 15 min. for individual planning of classroom adaptations, with timing adjusted according to course needs. The lesson design should emphasize culturally relevant contexts while addressing mathematics anxiety through successful mastery experiences (Bandura, 1997), as our results indicated these factors significantly contributed to increased confidence and value perceptions. We recommend that teacher educators model the pedagogical approaches they want EPSTs to adopt—including explicit verbalization of mathematical thinking, demonstration of how abstract concepts connect to real-world applications, and facilitation of collaborative problem-solving that mirrors effective elementary classroom practice. Pre- and post-lesson assessments using adapted versions of our self-efficacy and behavioral intention measures can help teacher educators document growth and adjust instruction accordingly. Our goal in recommending this empirically grounded lesson design is to provide a practical framework for transforming traditional mathematics methods courses into experiential learning environments that develop EPSTs who are both confident and competent in STEM integration at the elementary level.

Limitations and Future Research

We acknowledge several limitations in our study. The relatively small sample size (N = 75) evident sex imbalance (89% female), and single-institution design further limit generalizability, though the Hispanic-Serving Institution context provides valuable insights for similar settings. Additionally, our focus on immediate post-intervention effects precludes conclusions about long-term retention of attitudes and behavioral changes, self-report measures introduce potential social

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desirability bias, and EPSTs' actual future classroom implementation remains unobserved. The study's placement within a required mathematics methods course may also have influenced participants' responses and engagement. Future research should address these limitations through randomized controlled designs to establish causality, longitudinal follow-up tracking attitude and behavior persistence as EPSTs transition into classroom teaching, direct observation of STEM implementation, and expanded samples with greater sex diversity—directions we are actively pursuing as next steps. Notwithstanding these limitations, our study provides meaningful empirical evidence that experiential learning approaches within teacher preparation programs can effectively enhance elementary pre-service teachers' attitudes, self-efficacy, and intentions toward STEM integration, supporting broader efforts to ensure all students—particularly those in underserved communities—have access to high-quality, interdisciplinary STEM education from their earliest school years.

REFERENCES

[1] Abbad, M. M. (2021). Using the UTAUT model to understand students' usage of e-learning systems in developing countries. *Education and Information Technologies*, ***26***(6), 7205-7224.

[2] Aguilar, J. J., & Flores, Y. (2022). Analyzing the effectiveness of using mixed-reality simulations to develop elementary pre-service teacher's high-leverage practices in a mathematics methods course. *EURASIA Journal of Mathematics, Science and Technology Education*, ***18***(5). https://doi.org/10.29333/ejmste/12006

[3] Aguilar, J. J., & Telese, J. A. (2018, December). The implementation of mixed-reality simulations: An innovative approach to train first-year pre-service elementary mathematics teachers. *Association of Mathematics Teacher Educators: Connections*. https://amte.net/connections/2018/12/implementation-mixed-reality-simulations-innovative-approach-train-first-year

[4] Bandura, A. (1997). *Self-efficacy: The exercise of control*. Freeman.

[5] Bers, M. U., González-González, C., & Armas-Torres, M. B. (2021). Coding as a playground: Promoting positive learning experiences in childhood classrooms. *Computers & Education*, ***138***, 103-110.

[6] Brown, B. A. (2021). *Science in the city: Culturally relevant STEM education*. Harvard Education Press.

[7] Bursal, M., & Paznokas, L. (2006). Mathematics anxiety and preservice elementary teachers' confidence to teach mathematics and science. *School Science and Mathematics*, ***106***(4), 173-180.

[8] Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Heinemann.

[9] Cole, R. (2024). Inter-rater reliability methods in qualitative case study research. *Sociological Methods & Research*, ***53***(4), 1944-1975.

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[10] Creswell, J. W., & Guetterman, T. C. (2024). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Pearson.
[11] Creswell, J. W., & Plano Clark, V. L. (2017). *Designing and conducting mixed methods research* (3rd ed.). SAGE Publications.
[12] Emiru, E. K., & Gedifew, M. T. (2024). The effect of teacher self-efficacy on learning engagement of secondary school students. *Cogent Education*, *11*(1). https://doi.org/10.1080/2331186X.2024.2308432
[13] Ensign, T. I. (2017). *Elementary educators' attitudes about the utility of educational robotics and their ability and intent to use it with students* [Doctoral dissertation, West Virginia University]. WVU Research Repository. https://researchrepository.wvu.edu/etd/5546
[14] Fry, H., Ketteridge, S., & Marshall, S. (Eds.). (2009). *A handbook for teaching and learning in higher education: Enhancing academic practice* (3rd ed.). Routledge.
[15] Gado, I., Ferguson, R., & van't Hooft, M. (2006). Using handheld computers and probeware in a science methods course: Preservice teachers' attitudes and self-efficacy. *Journal of Technology and Teacher Education*, ***14***(3), 501-527.
[16] Grossman, P. (Ed.). (2021). *Teaching core practices in teacher education*. Harvard Education Press.
[17] Hebebci, M. T., & Usta, E. (2022). The effects of integrated STEM education practices on problem solving skills, scientific creativity, and critical thinking dispositions. *Participatory Educational Research*, ***9***(6), 358-379.
[18] Ingersoll, R., Merrill, E., Stuckey, D., & Collins, G. (2021). The demographic transformation of the teaching force in the United States. *Education Sciences*, ***11***(5), 234-249. https://doi.org/10.3390/educsci11050234
[19] Itter, D., & Meyers, N. (2017). Fear, loathing and ambivalence toward learning and teaching mathematics: Preservice teachers' perspectives. *Mathematics Teacher Education and Development*, ***19***(2), 123-141.
[20] Kelley, T. R., & Knowles, J. G. (2016). A conceptual framework for integrated STEM education. *International Journal of STEM Education*, *3*, Article 11. https://doi.org/10.1186/s40594-016-0046-z
[21] Kolb, A., & Kolb, D. (2009). The learning way: Meta-cognitive aspects of experiential learning. *Simulation & Gaming*, ***40***(3), 297-327.
[22] Kolb, D. A. (1984). *Experiential learning: Experience as the source of learning and development*. Prentice-Hall.
[23] Kolb, D. A. (2014). *Experiential learning: Experience as the source of learning and development*. FT Press.
[24] Margot, K. C., & Kettler, T. (2019). Teachers' perception of STEM integration and education: A systematic literature review. *International Journal of STEM Education*, ***6***(1), 1-16.
[25] Master, A., Cheryan, S., Moscatelli, A., & Meltzoff, A. N. (2017). Programming experience promotes higher STEM motivation among first-grade girls. *Journal of Experimental Child Psychology*, ***160***, 92-106.

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[26] McKinnon, M., & Lamberts, R. (2014). Influencing science teaching self-efficacy beliefs of primary school teachers: A longitudinal case study. *International Journal of Science Education, Part B*, ***4***(2), 172-194.
[27] Nadelson, L. S., Callahan, J., Pyke, P., Hay, A., Dance, M., & Pfiester, J. (2013). Teacher STEM perception and preparation: Inquiry-based STEM professional development for elementary teachers. *The Journal of Educational Research*, ***106***(2), 157-168.
[28] National Center for Education Statistics. (2022). *The condition of education 2022* (NCES Publication No. 2022-144). U.S. Department of Education. https://nces.ed.gov/pubs2022/2022144.pdf
[29] National Science Foundation, National Science Board. (2023). *Elementary and secondary STEM education* (Science and Engineering Indicators NSB-2023-31). https://ncses.nsf.gov/pubs/nsb202331
[30] Palmer, D., Dixon, J., & Archer, J. (2015). Changes in science teaching self-efficacy among primary teacher education students. *Australian Journal of Teacher Education*, ***40***(12), 27-40.
[31] Piaget, J. (1976). Piaget's theory. In B. Inhelder, H. H. Chipman, & C. Zwingmann (Eds.), *Piaget and his school* (pp. 11-23). Springer.
[32] Saldaña, J. (2014). *Thinking qualitatively: Methods of mind*. SAGE Publications.
[33] Saldaña, J., & Omasta, M. (2016). *Qualitative research: Analyzing life*. SAGE Publications.
[34] Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, ***15***(2), 4-14.
[35] Stipek, D. J., & Byler, P. (1997). Early childhood education teachers: Do they practice what they preach? *Early Childhood Research Quarterly*, ***12***(3), 305-325.
[36] Stohlmann, M., Moore, T. J., & Roehrig, G. H. (2012). Considerations for teaching integrated STEM education. *Journal of Pre-College Engineering Education Research*, ***2***(1), Article 4.
[37] Tomas, L., Jackson, C., & Carlisle, K. (2019). The transformative potential of engaging in science inquiry-based challenges: The STEM professionals in schools project. *Teaching Science*, ***65***(4), 4-13.
[38] Tschannen-Moran, M., & Hoy, A. W. (2001). Teacher efficacy: Capturing an elusive construct. *Teaching and Teacher Education*, ***17***(7), 783-805.
[39] Venkatesh, V., Morris, M. G., Davis, G. B., & Davis, F. D. (2003). User acceptance of information technology: Toward a unified view. *MIS Quarterly*, ***27***(3), 425-478. https://doi.org/10.2307/30036540
[40] Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
[41] Zhang, Y., & Wildemuth, B. M. (2009). Unstructured interviews. In *Applications of social research methods to questions in information and library science* (2nd ed., pp. 222-231). Libraries Unlimited.

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[42] Zhengtao, Z., & Hidayat, R. (2025). The Effects of Augmented Reality (AR) toward Achievement on the Graphs and Geometry Topic among Third-Grade Students. *Mathematics Teaching Research Journal*, *17*(1), 82-98.

APPENDIX A

Demographic component
Age
Sex
Are you first-generation College student?
Program Specialization - Selected Choice
Have you taken any STEM course (e.g., Computer Science, Biology, engineering) in College?
Do you have any STEM experience?

Self-confident and ability
Learning how to integrate STEM (e.g., robotics) in teaching elementary mathematics sound challenging.
I am confident I can integrate STEM (e.g., robotics) for teaching in elementary.
I feel at ease using STEM resources (e.g., like robots) to teach in elementary.
I would hesitate in integrating STEM (e.g., Robotics) in my teaching for fear of making mistakes I won't be able to correct.
I am unsure of my ability to integrate STEM (e.g., Robotics) in my teaching.
With my current experience in STEM, I feel prepared to integrate related concepts and tools (e.g., robotics) within my lesson plans and future teaching.

Value and enjoyment of STEAM
Children should learn STEM related concepts (e.g., Robotics) in Elementary.
Integrating STEM related concepts (e.g., Robotics) in elementary teaching would be enjoyable and stimulating for me.
Students would have hard time learning mathematics if STEM related concepts (e.g., Robotics) are integrated in a lesson.
Students would conceptually learn mathematics if STEM related concepts (e.g., Robotics) are used for teaching elementary mathematics.
I expect my future students to enjoy learning elementary mathematics through the use and implementation of STEM resources like robotics.

Performance Expectancy
The use of STEM (e.g., robotics) in elementary to teach and learn mathematics is useful.
Using STEM (e.g., robotics) in teaching and learning elementary mathematics would help me to accomplish my teaching goals quickly.

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Using STEM (e.g., robotics) in teaching and learning in elementary mathematics could increases my productivity.
If I use STEM (e.g., robotics) in teaching and learning elementary mathematics, I will increase my chances of getting a raise once I get a teaching position.

Effort Expectancy
My interaction with STEM related tools like robotics in teaching and learning would be clear and understandable for me.
My interaction with STEM related tools like robotics would be easy for me to become skillful at using it.
Integrating STEM related tools like robotics in my elementary mathematics lessons would be easy for me.
Learning how to use STEM related tools like robotics would be easy for me.
Learning how to operate and program STEM related tools like robotics would be easy for me.

Social Influence and Behavioral Intention
If my future colleagues integrate STEM related tools like robotics within their lesson, I will probably use it in my teaching and learning of elementary mathematics too.
I would use STEM related tools like robotics in my teaching and learning once I become a teacher.

Raking scale (1-10)
How likely is that you will seek ways to integrate STEM related ideas like the use of Robotics (or any other) in your elementary teaching once you start your teaching career? (1 not likely at all - 10 Very possible).

APPENDIX B

ITEM	Pre (%)		Post (%)	
	Agree or Completely Agree	Disagree or completely disagree	Agree or Completely Agree	Disagree or completely disagree
Learning how to integrate STEM (e.g., robotics) in teaching elementary mathematics sound challenging.	57%	15%	55%	28%
I am confident I can integrate STEM (e.g., robotics) for teaching in elementary.	60%	11%	74%	6%
I feel at ease using STEM resources (e.g., like robots) to teach in elementary.	32%	28%	64%	4%

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I would hesitate in integrating STEM (e.g., Robotics) in my teaching for fear of making mistakes I won't be able to correct.	57%	19%	26%	51%
I am unsure of my ability to integrate STEM (e.g., Robotics, engineering) in my teaching.	58%	15%	25%	47%
With my current experience in STEM, I feel prepared to integrate related concepts and tools (e.g., robotics) within my lesson plans and future teaching.	21%	42%	72%	2%
Children should learn STEM related concepts (e.g., Robotics) in Elementary.	81%	0%	91%	2%
Integrating STEM related concepts (e.g., Robotics) in elementary teaching would be enjoyable and stimulating for me.	64%	6%	85%	6%
Students would have hard time learning mathematics if STEM related concepts (e.g., Robotics) are integrated in a lesson.	19%	42%	15%	57%
Students would conceptually learn mathematics if STEM related concepts (e.g., Robotics) are used for teaching elementary mathematics.	53%	6%	79%	0%
I expect my future students to enjoy learning elementary mathematics through the use and implementation of STEM resources like robotics.	64%	0%	91%	0%
The use of STEM (e.g., robotics) in elementary to teach and learn mathematics is useful.	77%	0%	92%	0%
Using STEM (e.g., robotics) in teaching and learning elementary mathematics would help me to accomplish my teaching goals quickly.	53%	2%	85%	2%
Using STEM (e.g., robotics) in teaching and learning in elementary mathematics could increases my productivity.	66%	2%	83%	2%
If I use STEM (e.g., robotics) in teaching and learning elementary mathematics, I will increase my chances of getting a raise once I get a teaching position.	40%	0%	70%	4%
My interaction with STEM related tools like robotics in teaching and learning would be clear and understandable for me.	42%	23%	79%	0%
My interaction with STEM related tools like robotics would be easy for me to become skillful at using it.	47%	17%	70%	0%

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Integrating STEM related tools like robotics in my elementary mathematics lessons would be easy for me.	34%	15%	64%	2%
Learning how to use STEM related tools like robotics would be easy for me.	38%	17%	62%	4%
Learning how to operate and program STEM related tools like robotics would be easy for me.	34%	17%	70%	4%
If my future colleagues integrate STEM related tools like robotics within their lesson, I will probably use it in my teaching and learning of elementary mathematics too.	75%	0%	87%	0%
I would use STEM related tools like robotics in my teaching and learning once I become a teacher.	79%	2%	85%	0%
How likely is that you will seek ways to integrate STEM related ideas like the use of Robotics (or any other) in your elementary teaching once you start your teaching career? (1 not likely at all - 10 Very possible)	6.5		8.5	

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Secondary School Students' Critical Thinking Skills: A Comparison Between Impulsive and Reflective Students in Learning Geometry

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Abstract: This study aims to examine secondary school students' critical thinking skills and compare differences between reflective and impulsive cognitive styles in geometry learning. The participants were 69 eighth-grade students: 25 were classified as reflective, 18 as impulsive, and 26 were not categorized. Critical thinking skills were measured utilizing a geometry problem-solving test, while cognitive styles were identified using the Matching Familiar Figures Test (MFFT). Because the data were non-parametric, the Mann-Whitney U test was used for analysis. The results showed that, among the 43 students classified as reflective or impulsive, reflective students demonstrated significantly higher critical thinking skills than the impulsive ones. The former represented 36.2% of the sample, while the latter accounted for 26.0%. Moreover, the proportion of reflective and impulsive students (62.2%) was higher than that of students classified only by speed and accuracy (37.8%). Lastly, reflective students achieved higher average critical thinking scores because they tended to spend more time analyzing problems and answering carefully. In contrast, impulsive students responded more quickly but less thoroughly, which often resulted in incorrect answers.

***Keywords**: Critical thinking, cognitive style, reflective, impulsive*

INTRODUCTION

Learning mathematics at school plays a crucial role in developing students' critical-thinking skills. It helps students solve problems and prepares them for various daily challenges (Tong et al., 2020). The ability to think critically is also one of the essential competencies in contemporary education (Iskandar & Juandi, 2022), which is also relevant in learning mathematics and essential in various aspects of life (Elder & Paul, 2020). Furthermore, critical thinking involves analyzing information, evaluating arguments, and making rational decisions based on proof (Yang et al., 2024). In the context of learning, the ability to think critically enables students to understand concepts thoroughly, solve problems effectively, and adapt to complex situations.

Critical thinking is the foundation of effective and sustainable learning (Rachmawati & Siswono, 2020). Having critical thinking skills not only makes students passively accept information but

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also actively process, evaluate, and apply it (El Sadik & Al Abdulmonem, 2021). In today's ever-changing world filled with complex information, critical thinking enables students to identify relevant information, make appropriate decisions, and solve problems using innovative approaches (Warli & Nofitasari, 2021). Therefore, critical thinking should be emphasized in mathematical problem-solving activities (Tang et al., 2020). Another important factor in mathematics learning is individual differences in students' cognitive styles (Mann et al., 2021). The two cognitive styles most frequently discussed are reflective and impulsive, which influence how students process information and ultimately affect the effectiveness of mathematics learning (Xu et al., 2023).

Students with reflective characteristics tend to think carefully before making decisions (Shanta & Wells, 2022). They consider various alternatives, analyze information deeply, and give more accurate and detailed answers (Tang et al., 2020). However, their thought process is slower. This can make them look insufficiently responsive in situations requiring quick response (Bellaera et al., 2021). On the other hand, impulsive students tend to respond quickly without considering various alternatives (Alsaleh, 2020). They often give answers rapidly, but the decisions they take are usually shallow and are more prone to error. Thus, the speed at which they respond can become superior in certain situations (Fadilla et al., 2021).

Cognitive styles have been reported to significantly impact mathematical learning (Ahdhianto et al., 2020). It is suggested that students who are reflective are more likely to be successful in solving problems that require in-depth analysis and logical reasoning (Toker & Baturay, 2022). However, they might find difficulties in situations requiring a fast response (Dolapcioglu & Doğanay, 2022). On the other hand, students who are impulsive can finish a simple task faster, but might have difficulty with complex and demanding problems. Cases of students who are impulsive and reflective were also found in mathematical learning at the secondary level. At SMPN 1 Ambarawa, after identifying impulsive and reflective eighth-grade students with equal (high) mathematical skills, it turned out that the two were very different in solving problems. Impulsive students in mathematical problem-solving tended to respond spontaneously without profoundly considering the strategies used. Furthermore, they often rushed in solving mathematics problems and tended to make mistakes that could have been avoided if they were more patient. In addition, impulsive students also tended to ignore the understanding of fundamental mathematical concepts, so they focused more on the result without understanding the basis of problem-solving. This led to the possibility of frequent errors in mathematical problem-solving and a lack of deep understanding.

Impulsive and reflective characteristics significantly impact the quality of mathematical solutions produced by students. Impulsive students tend to create solutions more quickly but less thoroughly, making them prone to errors (Jamil et al., 2024). They also tend to use a trial-and-error approach without deep thinking, which results in solutions that may not be optimal. On the other hand, reflective students usually produce more thorough solutions and use a logical and systematic thinking approach, although it takes longer. This impacts the quality of mathematical solu-

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tions produced, where reflective students' solutions tend to be more accurate and correct (Mihail, 2022). Therefore, understanding the impact of these differing characteristics is essential in developing more effective mathematics teaching and learning strategies (Dolapcioglu & Doğanay, 2022).

This study is expected to contribute to improving educational quality by supporting the development of students' critical thinking and problem-solving abilities. Critical thinking goes beyond basic understanding as it involves applying, analyzing, evaluating, and creating knowledge, which are key components of Higher-Order Thinking Skills (HOTS). Therefore, developing critical thinking for all students, both with reflective and impulsive characteristics, is essential in mathematics learning. Teachers need to design instructional strategies that accommodate diverse student characteristics, for example, through structured problem-solving activities, guided reflection, and self-regulation strategies to support impulsive learners. Furthermore, research on differences between reflective and impulsive students and their impact on critical thinking can provide valuable insights for educators. A deeper understanding of students' cognitive styles enables teachers to implement more adaptive and effective teaching approaches, ultimately improving learning processes and student outcomes.

METHODS AND DESIGN

This study employed an exploratory-descriptive approach to examine the critical thinking skills of students with reflective and impulsive cognitive styles. It also utilized a comparative design to investigate the differences in critical thinking skills between these two cognitive style groups. The participants were 69 eighth-grade students from classes 8A to 8I at a junior high school. To identify their cognitive styles, all students were administered a cognitive style identification questionnaire, adapted from previous research, consisting of 15 items that assessed tendencies in decision-making speed and accuracy. Based on the questionnaire results, 25 students were identified to have the reflective cognitive style, while 18 students fell under the impulsive style. The remaining 26 students were uncategorized. The study was conducted in two stages. In the first stage, students' cognitive styles were identified using the Matching Familiar Figures Test (MFFT), originally developed by Kagan et al. (1964) and adapted by Warli (2010). The instrument was used to validate the initial categorization of students' cognitive styles. The MFFT consists of one standard picture accompanied by eight similar variations. Students were required to select the picture identical to the standard image. Two variables were recorded: the time taken to provide the first response and the number of attempts required to obtain the correct answer.
Based on the combination of response time and error frequency, students were categorized into four cognitive-style groups: fast–accurate, reflective (slow–accurate), impulsive (fast–inaccurate), and slow–inaccurate. The classification followed the median-split procedure proposed by Warli (2010), in which the median values of response time and error frequency were used to divide students into four quadrants.
The MFFT procedure included the following steps: (1) students individually completed the cognitive-style test by identifying the picture that matched the standard image; (2) the time required

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to produce the first response was recorded; (3) the number of attempts until the correct answer was obtained was documented; (4) the average response time and error frequency were calculated; and (5) median values of time and frequency were used to construct classification quadrants.

In the second stage, students identified as reflective and impulsive were selected as research subjects and were given geometry-based problem-solving tasks. Students' critical thinking skills were measured using five essay questions on circle geometry designed to assess four indicators: clarification, evaluation of evidence, drawing conclusions, and making judgments (see Figure 1).

The pipes in the form of a tube with a length of 2 m and a radius of 14 cm will be tied with a rope. Given the condition, determine

a. The length of rope needed to tie the 2 pipes. Explain how you found it.
b. The length of rope needed to tie 3 pipes. Explain how you found it
c. If the number of pipes is 4, determine the length of rope needed to tie the four pipes. Explain how you found it.
d. If the number of pipes is 5, determine the length of rope used to tie the five pipes, explain how you found it.
e. Draw a conclusion from points a, b, c, and d.

Figure 1: The problem-solution instrument

The questions were utilized to measure critical thinking skills that refer to subindicators (Ennis, 1996), which include: (1) identifying or establishing criteria to consider possible answers; (2) providing explanations; (3) identifying inaccuracies; (4) demonstrating the ability to give reasons; and (5) selecting criteria to consider possible solutions. Additionally, to determine the critical thinking skills score referring to standardization, the assessment criteria of Paul and Elder (2007) was consulted, which include: clarity, accuracy, precision, depth, breadth, and logic, adjusted to mathematical learning about geometry in particular circles.

The next stage involved conducting in-depth interviews with selected participants representing reflective and impulsive cognitive styles. One student from each cognitive style category was purposively selected based on their communication skills to ensure the richness and relevance of the data obtained. Furthermore, the data on students' critical thinking skills, categorized by reflective and impulsive cognitive styles, were analyzed using the SPSS software through the nonparametric Mann Whitney U test.

RESULTS

This section presents the results of the analysis of students' cognitive styles. The subjects in this study were eighth-grade students of SMPN 1 Ambarawa. To identify the cognitive styles of the

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subjects, cognitive style measurements were carried out on each student. In this study, students with reflective cognitive styles were identified as those who typically take more time to analyze a problem and carefully check their responses before answering. In contrast, students with impulsive cognitive styles tend to respond quickly, often without sufficient consideration, which can lead to more frequent errors. These tendencies were observed during the test sessions and further supported by the students' answer patterns. The aspect was observed and recorded in cognitive style measurements, which included the time (t) the student took for the first answer and the frequency (f) the student answered the response and got the correct answer. The mean time (t) and frequency (f) for each student were calculated, followed by the mean time (t) and frequency (f) of all students, which were tabulated in a table to determine the median time and frequency. The results of cognitive style measurements are presented in Table 1.

Amount student	Time (seconds)			Frequency			Total of reflective students	Total of impulsive students
	max	min	med	max	min	med		
69	62.01	13.29	39.12	2,923	1,231	2,000	25	18

Description: Max: maximum, Min: minimum, Med: median
Table 1: *Results of Cognitive Style Measurement*

The MFFT measurement results show four cognitive types thinkers, presented in Figure 2.

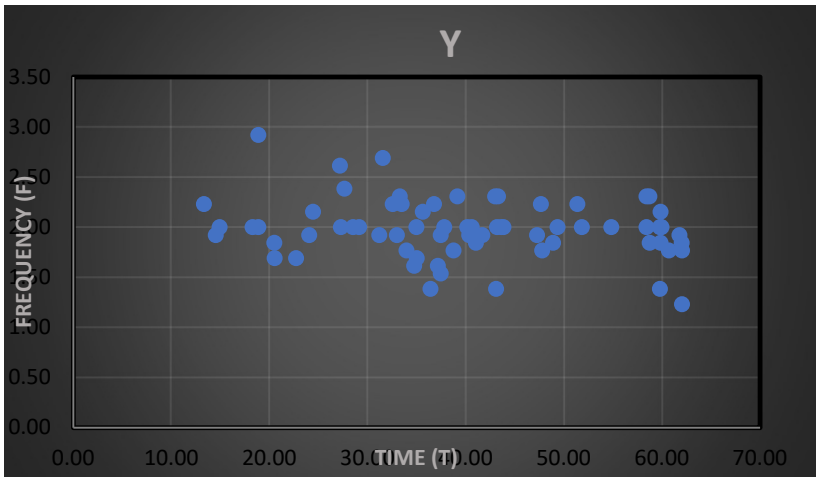


Figure 2: Distribution of MFFT scores representing four cognitive style categories (reflective, impulsive, fast-accurate, and slow-inaccurate)
Based on Table 1, 25 students (36.2%) were categorized as reflective, while 18 students (26.0%) were categorized as impulsive. In addition, 14 students (20.3%) were classified as fast-accurate, and 12 students (17.4%) as slow-inaccurate. Overall, the proportion of students categorized by cognitive style (reflective, impulsive, fast-accurate, and slow-inaccurate) provides a more com-

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prehensive representation (62.2%) compared to classification based solely on response speed and accuracy (37.8%). These findings indicate that cognitive style classification offers a broader analytical framework than relying only on observable response behavior during testing. Furthermore, this result is consistent with previous studies, such as Rozencwajg and Corroyer (2005), which reported that the proportion of students exhibiting reflective-impulsive cognitive styles reached 76.2%, and Warli (2010), who found similar tendencies in students' cognitive processing characteristics

Critical Thinking Skills Across Reflective and Impulsive Cognitive Styles

The measurement of students' critical thinking skills based on their cognitive styles (reflective and impulsive) was conducted outside of regular school hours following participants' consent, after completing a lesson on the topic of circles. The assessment employed a geometry problem-solving test comprising five open-ended questions. Each question was scored on a scale of 1 to 10, giving a maximum total score of 50 points. Table 2 displays the distribution of total scores for both cognitive style groups. The reflective group consisted of 25 students, while the impulsive group included 18 students. Due to the small sample size and limited score range, several students obtained identical scores, leading to repeated frequency values for specific score intervals.

Scores of critical thinking	Reflective (f)	Impulsive (f)
12-20	3	5
21-25	6	6
26-30	7	3
31-35	4	2
36-40	2	1
41-45	2	1
46-50	1	0
Total students	**25**	**18**
Total Score	**649.5**	**296.5**
Average score	**25.98**	**16.47**

Table 2: Distribution of students' critical thinking scores across reflective and impulsive cognitive styles

To facilitate interpretation, individual scores were grouped into score intervals. The distribution indicates that students with a reflective cognitive style tended to achieve higher critical thinking scores than those with an impulsive style. Because the data did not meet the assumption of normality, the non-parametric Mann–Whitney U test was employed. The results of this analysis are presented in Table 3.

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	Cognitive-Style	N	Mean Rank	Sum of Ranks
CT Scores	*Reflective*	25	25.98	649.50
	Impulsive	18	16.47	296.50
	Total	43		
Mann-Whitney U			125.50	
Wilcoxon W			296.50	
Z			-2.453	
Asymp . Sig. (2-tailed)			0.014	

a. Grouping Variable: Cognitive-Style

Table 3. Results of the Mann–Whitney U Test Comparing Critical Thinking Scores Between Reflective and Impulsive Students

Based on Table 3, the Asymp. Sig. (2-tailed) value for the Mann–Whitney U test was 0.014, which was below the significance threshold of 0.05. In this test, the null hypothesis (H_0) stated that there is no significant difference in students' critical thinking scores between reflective and impulsive cognitive styles. If the p-value was greater than 0.05, H_0 would be accepted; otherwise, it must be rejected. Since the p-value was 0.014, H_0 was rejected. This result indicates a statistically significant difference in critical thinking performance between the two cognitive style groups, especially in the context of geometry learning involving circle geometry problems. As shown in Table 2, students with reflective cognitive styles had higher average scores in critical thinking skills than students with impulsive cognitive styles. According to Rozencwajg and Corroyer (2005), students who have reflective cognitive processes prefer to take their time and consider questions carefully before answering, usually resulting in accurate answers. On the other hand, students with an impulsive cognitive style are known for answering questions in a hurry and tend to give incorrect answers within a given limited time. Figures 3 and 4 illustrate selected responses from students representing each cognitive style category.

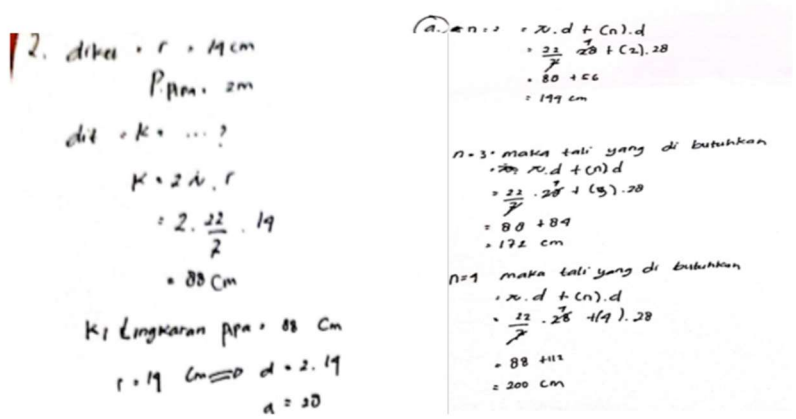


Figure 3: Sample answer of an impulsive student

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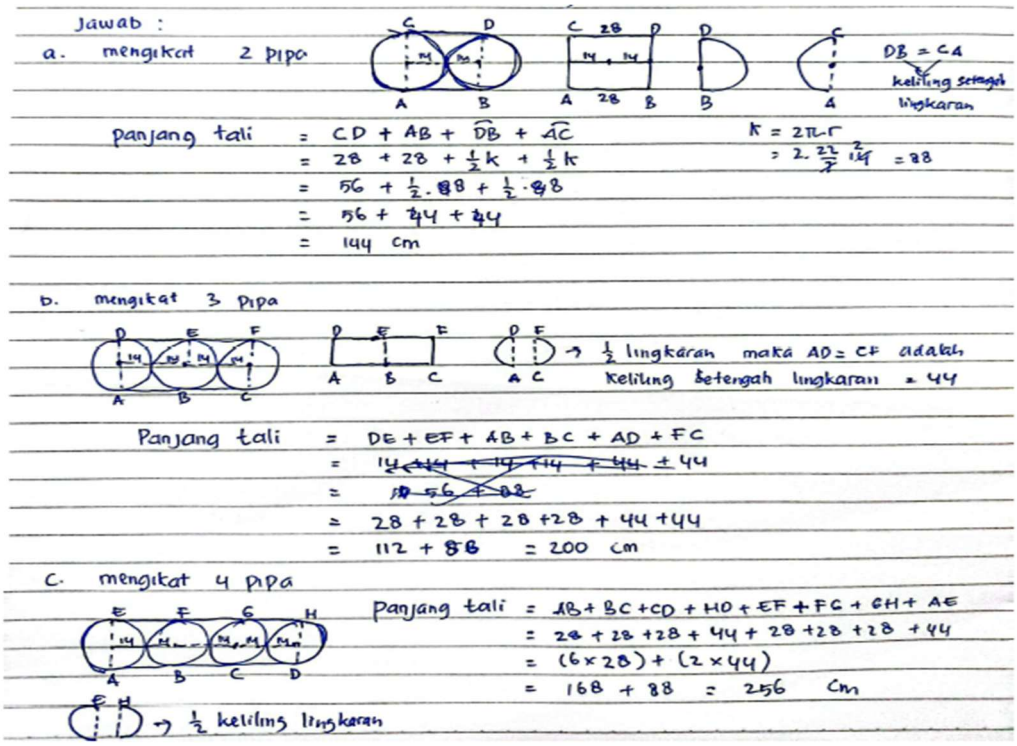


Figure 4: Sample answer of a reflective student

To gain deeper insights into students' cognitive processes, semi-structured interviews were conducted with two selected participants representing reflective and impulsive cognitive styles. The following excerpt illustrates an interview with the impulsive student (S1) after completing the geometry problem:

P: How did you begin solving the problem?

S1: First, I wrote down the known information, namely r =14cm. Then I identified that the problem involved a circle.

P: What did you do next?

S1: The question asked about the length of a rope around the pipe, so I interpreted it as finding the circumference of a circle. I used the formula $k = 2\pi r = 2\frac{22}{7}14 = 88\ cm$. *Substituting* $r = 14$, *I obtained* $d = 28$ *and calculated the result*.

P: How did you determine the method to use?

S1: (S1 writes n=2) *Then the length of the rope is* $\pi d{+}n.d{=}\frac{22}{7}.28{+}2.28{=}144\ cm$

P: Can you tell me why it is like that?

S1: Because the teacher had previously taught us to use this formula for similar problems.

As seen from the transcript above, the student responded quickly by immediately applying a memorized formula without first verifying whether the formula was appropriate for the specific

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context of the problem. This reflects a tendency to focus on procedural recall rather than conceptual understanding. Evidently, the student's justification, "this formula was already taught by the teacher," shows reliance on rote learning, which is typical of impulsive problem-solving behavior. This lack of deeper analysis can lead to mechanical answers that may not always be accurate or contextually correct.

Meanwhile, a snippet of the interview with the reflective student (S2) is as follows:

P: *How did you solve the problem?*

S2: *First, I identified the known information in the problem: the pipe has a length of* 2 *m and a radius of* 14 *cm. Then, I analyzed each sub-question: (a) determining the length of rope required to tie two pipes, (b) determining the length for three pipes, and (c) comparing different tying methods to identify which requires a longer rope, whether arranged linearly or combined.*

P: *What did you do next?*

S2: *I began by solving part (a). I illustrated the situation using two circles representing the pipes. The distance between points C and D corresponds to the diameter of the circle, which is 28 cm. The curved part represents half of the circle's circumference. Since there are two pipes, the total curved length equals one full circumference. With a radius of 14 cm, the circumference is 88 cm. Therefore, the total rope length required is: 28 cm + 28 cm + 88 cm =* 144 *cm.*

P: *What about part (c)?*

S2: *The approach is similar to part (b), but it involves three pipes, so the configuration differs. Based on the illustration, the total rope length is:*28 *cm* + 28 *cm* + 28 *cm* + 28 *cm* + 44 *cm* + 44 *cm* = 200 *cm.*

P: *Are there any alternative methods?*

S2: *Yes, the pipes can also be arranged using a combined configuration, as shown there.*

DISCUSSION

Based on the results of the data analysis, including student responses as shown in Figures 3 and 4, and supported by interview findings, several differences were identified between impulsive and reflective students. These differences are related to aspects such as speed and accuracy in problem-solving, strategy use, and responsiveness to instructions, as summarized in Table 3 below. The data were collected from 26 students: 13 identified as having an impulsive cognitive style and 13 with a reflective cognitive style. The classification was determined using the Matching Familiar Figures Test (MFFT), where time taken and response accuracy were the main indicators. The characteristics presented in Table 3 represent typical patterns observed from each group, supported by both written test results and qualitative interview data.

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Impulsive Student	Reflective Student
Speed and Accuracy - Tend to finish problems fast without thinking a lot - Missing important details or making error in calculating - Focus on speed - Not checking the answer - Ignore or not realizing errors due to a lack of reflection **Solving Strategy Problem** - Directly choosing the first strategy that comes to mind without considering whether there was another better approach - Relying often on instinct or answer intuitively rather than doing deep analysis - Tend to look for solutions fast, even when failing to look for the correct answer **Reception to Instructions** - Tend to ignore instructions or only skim through, directly trying to solve problems without understanding the instructions better.	**Speed and Accuracy** - Spend more time thinking and examining each step in problem solving - Be more thorough and tend to consider various possibilities before making a decision - Focus on rigor and accuracy even though it is slower to complete - Errors are often identified and fixed before resolving the problem **Solving strategy Problem** - Tend to consider some strategies before choosing a more approach effective - Use analysis deep and thoughtful critical for selecting and implementing strategies - Solving complex problems becomes more minor and more gradual. **Reception to Instructions** - Understand instructions and context problems before starting to do the task. - Tend to follow the steps given systematically.

Table 4: *Differences Student Impulsive and Reflective in Finish Problem*

From the differences shown in Table 4, it was observed that students with a reflective cognitive style had the tendency to demonstrate a greater variety of indicators of critical thinking compared to the impulsive students. These tendencies included more thorough reasoning, careful problem analysis, and more accurate responses, as observed in both their written work and interviews. Interestingly, these patterns are consistent with a number of other studies on reflective and impulsive cognitive styles and critical thinking, including those conducted by Fridiamanti et al. (2018), Amriwati and Cintamulya (2017), Muryani and Cintamulya (2018), Rofifah and Masriyah (2018), and Rahayu and Warsono (2018). Furthermore, Ennis (1985) describes the following criteria for critical thinking:

1. *Focus* – the ability to understand and clearly define the problem
2. *Reasoning* – the ability to provide logical arguments based on relevant facts or evidence throughout the problem-solving process.
3. *Inference* – the ability to draw appropriate conclusions based on valid reasoning
4. *Situation* – the ability to consider and utilize all relevant information related to the problem

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5. *Clarity* – he ability to clearly explain terms, provide relevant examples, and articulate conclusions.
6. *Overview* – the ability to review and evaluate the entire problem-solving process systematically.

Additionally, other criteria for critical thinking include clarity, accuracy, precision, depth, breadth, and logic (Paul & Elder, 2007). Understanding these criteria is essential for educators, particularly in recognizing differences in students' cognitive styles, such as reflective and impulsive tendencies, which influence how they engage in learning processes. Griggs (1991) emphasized that the development of critical thinking requires an awareness of students' learning styles. When instruction aligns with students' preferred learning styles, their attitudes toward learning can improve, leading to increased productivity, academic achievement, and creativity. Thus, cognitive style plays a significant role in shaping the effectiveness of the learning process (Dekker, 2020).

Teachers, therefore, need to consider students' cognitive styles to enhance their critical thinking abilities (Cortázar et al., 2021). By doing so, they can create a more inclusive learning environment that fosters critical thinking through appropriate instructional strategies (Bağ & Gürsoy, 2021). Previous studies have shown a positive relationship between a reflective cognitive style and critical thinking, and a negative relationship between an impulsive cognitive style and critical thinking (Afdareza et al., 2020; Shanta & Wells, 2022). These findings, alongside the ones generated in this study, highlight the importance of adapting instructional design to students' cognitive characteristics. As a response, this study has developed a lesson plan tailored to low-achieving students with impulsive cognitive styles in learning geometry, specifically on the topic of circles. The lesson is designed to enhance students' critical thinking skills through structured activities, scaffolded problem-solving, and cognitive regulation strategies. The detailed structure of the lesson plan is provided in the Appendix.

CONCLUSION

In conclusion, the critical thinking skills of students with reflective cognitive styles are better than the critical thinking skills of students with impulsive cognitive styles in learning geometry. Students with reflective cognitive styles have the characteristics of taking a long time to answer questions carefully so that the answers given tend to be correct, while students who have the characteristics of impulsive cognitive styles tend to respond quickly and answer questions more perfunctorily, so the answers given tend to be wrong. This cognitive style affects not only how students solve problems, but also the end result of solving those problems. The reflective cognitive style encourages careful, analytical, and evaluative thinking, which is the foundation of critical thinking. Therefore, students with reflective cognitive styles are better critical thinkers, resulting in a strong positive relationship between the two. Impulsive cognitive styles, on the other hand, often neglect the process of deep reflection and analysis, which can reduce the effectiveness of critical thinking. This creates a negative relationship between impulsive cognitive style

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and critical thinking ability. By understanding these relationships, teachers can tailor their teaching approach to help students with impulsive cognitive styles develop more reflective thinking habits, thereby improving their critical thinking skills. Based on this, teachers are advised to design lessons with explicit step-by-step instructions and clear learning objectives for impulsive students. Each task should be broken down into smaller components, with regular checkpoints for feedback and guided reflection. Interactive activities should be structured to include brief pauses, allowing students to self-monitor their work before moving on. This approach can help students slow down, stay focused, and improve the quality of their learning outcomes.

There were several limitations of the study that were acknowledged, including: 1) Individual Variability: Reflective and impulsive characteristics can vary greatly between individuals, making it difficult to generalize findings. Each student has different levels of reflectivity and impulsivity depending on the specific situation, which can affect the consistency of the research results. 2) Difficulty in Measurement: Accurately measuring reflective and impulsive levels can be challenging. The instruments used to measure reflective or impulsive behavior may have limitations in validity and reliability, thus affecting the findings' soundness. 3) Influence of Contextual Factors: The environment, time pressure, and difficulty level of the problems faced can affect student performance, both reflective and impulsive. These variables can be difficult to fully control in research, so the results can be biased. 4) Researcher Bias: The researcher's interpretation of reflective and impulsive behavior can be affected by subjective bias. This can occur during data observation or analysis, which can reduce the objectivity of the research results. 5) Difficulty in Distinguishing Behaviors: In practice, reflective and impulsive behaviors are not always easy to distinguish clearly. Some students may exhibit a combination of these two traits depending on the context, which can complicate data analysis and interpretation.
These limitations need to be considered when assessing the findings of the study. Some ideas for future research that can be carried out include: 1) Subsequent research can focus on developing or validating more accurate and reliable instruments to measure reflective and impulsive tendencies. This could involve using technologies such as eye-tracking or click pattern analysis to get more objective data on how students process geometry problems. 2) Longitudinal Studies: Conducting longitudinal research to observe the development of reflective and impulsive tendencies over time in the context of solving geometric problems can give a more elaborate outlook on the matter and its trends.

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REFERENCES

[1] Afdareza, M. Y., Yuanita, P., & Maimunah, M. (2020). Development of Learning Device Based on 21st Century Skill with Implementation of Problem Based Learning to Increase Critical Thinking Skill of Students on Polyhedron for Grade 8th Junior High School. Journal of Educational Sciences, 4(2), 273. https://doi.org/10.31258/jes.4.2.p.273-284

[2] Ahdhianto, E., Marsigit, M., Haryanto, H., & Nurfauzi, Y. (2020). Improving Fifth-Grade Students' Mathematical Problem-Solving and Critical Thinking Skills Using Problem-Based Learning. *Universal Journal of Educational Research*, *8*(5), 2012–2021. https://doi.org/10.13189/ujer.2020.080539

[3] Amiwati, A., & Cintamulya, I. (2017). Analisis kemampuan berpikir kritis siswa bergaya kognitif reflektif dan impulsif pada pembelajaran biologi melalui model SQ3R (Survey, Question, Read, Review, Recite) dengan media kartu bergambar. *Bio-Pedagogi*, 6(2), 13–18.

[4] Alsaleh, N. J. (2020). Teaching Critical Thinking Skills: Literature Review. Turkish Online Journal of Educational Technology-TOJET, 19(1), 21-39.

[5] Bağ, H. K., & Gürsoy, E. (2021). The Effect of Critical Thinking Embedded English Course Design to The Improvement of Critical Thinking Skills of Secondary School Learners☆. *Thinking Skills and Creativity*, *41*, 100910. https://doi.org/10.1016/j.tsc.2021.100910

[6] Bellaera, L., Weinstein-Jones, Y., Ilie, S., & Baker, S. T. (2021). Critical thinking in practice: The priorities and practices of instructors teaching in higher education. *Thinking Skills and Creativity*, *41*, 100856. https://doi.org/10.1016/j.tsc.2021.100856.

[7] Cintamulya, I., Warli, & Mawartiningsih, L. (2019). Critical Thinking Ability after Project-Based Learning: A Comparative Study on Students Who Have Different Cognitive Styles. *Journal of Physics: Conference Series*, *1422*(1), 012039. https://doi.org/10.1088/1742-6596/1422/1/012039

[8] Cortázar, C., Nussbaum, M., Harcha, J., Alvares, D., López, F., Goñi, J., & Cabezas, V. (2021). Promoting critical thinking in an online, project-based course. *Computers in Human Behavior*, *119*, 106705. https://doi.org/10.1016/j.chb.2021.106705

[9] Dekker, T. J. (2020). Teaching critical thinking through engagement with multiplicity. *Thinking Skills and Creativity*, *37*, 100701. https://doi.org/10.1016/j.tsc.2020.100701

[10] Dolapcioglu, S., & Doğanay, A. (2022). Development of critical thinking in mathematics classes via authentic learning: an action research. *International Journal of Mathematical Education in Science and Technology*, *53*(6), 1363–1386. https://doi.org/10.1080/0020739X.2020.1819573

[11] El Sadik, A., & Al Abdulmonem, W. (2021). Improvement in Student Performance and Perceptions through a Flipped Anatomy Classroom: Shifting from Passive Traditional to Active Blended Learning. *Anatomical Sciences Education*, *14*(4), 482–490. https://doi.org/10.1002/ase.2015.

[12] Elder, L., & Paul, R. (2020). *Critical thinking: Tools for taking charge of your learning and your life*. Foundation for Critical Thinking Press.

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[13] Ennis, R.H. (1985). A Logical Basis for Measuring Critical Thinking Skills. *Educational Leadership, 43*, 44-48.
[14] Fridanianti, A., Purwati, H., dan Murtiyanto, Y,H. (2018). Analisis Kemampuan Berpikir Kritis Dalam Menyelesaikan Soal Aljabar Kelas VII SMP Negeri 2 Pangkah Ditinjau dari Gaya Kognitif Reflektif dan Kognitif Impulsif. *Aksioma.*, 9 (1), 11-20.
[15] Griggs, S. A. (1991). *Learning styles counseling.* Ann Arbor: ERIC Counseling and Personnel Services Clearinghoause.
[16] Fadilla, N., Nurlaela, L., Rijanto, T., Ariyanto, S. R., Rahmah, L., & Huda, S. (2021). Effect of problem-based learning on critical thinking skills. *Journal of Physics: Conference Series*, *1810*(1), 012060. https://doi.org/10.1088/1742-6596/1810/1/012060
[17] Griggs, S. A. (1991). *Learning styles counseling*. Ann Arbor, MI: ERIC Clearinghouse on Counseling and Personnel Services.
[18] Iskandar, R. S. F., & Juandi, D. (2022). Study Literature Review: Realistic Mathematics Education Learning on Students' Mathematical Creative Thinking Ability. *SJME (Supremum Journal of Mathematics Education)*, *6*(1), 35–42. https://doi.org/10.35706/sjme.v6i1.5739.
[19] Jamil, M., Hafeez, F. A., & Muhammad, N. (2024). Critical Thinking Development for 21st Century: Analysis of Physics Curriculum. *Journal of Social & Organizational Matters*, *3*(1), 01–10. https://doi.org/10.56976/jsom.v3i1.45.
[20] Mann, L., Chang, R., Chandrasekaran, S., Coddington, A., Daniel, S., Cook, E., Crossin, E., Cosson, B., Turner, J., Mazzurco, A., Dohaney, J., O'Hanlon, T., Pickering, J., Walker, S., Maclean, F., & Smith, T. D. (2021). From problem-based learning to practice-based education: a framework for shaping future engineers. *European Journal of Engineering Education*, *46*(1), 27–47. https://doi.org/10.1080/03043797.2019.1708867
[21] Mihail, R. (2022). The Relevance of Critical Thinking from the Perspective of Professional Training. *Postmodern Openings*, *13*(2), 499–513. https://doi.org/10.18662/po/13.2/468
[22] Paul, R., & Elder, L. (2007). *The miniature guide to critical thinking concepts and tools.* Berkeley: The Foundation for Critical Thinking.
[23] Rachmawati, D. A., & Siswono, T. Y. E. (2020). Impulsive And Reflective Students' Understanding To Linear Equations System: An Analysis Through Apos Theory. *MATHEdunesa*, *9*(1), 128–135. https://doi.org/10.26740/mathedunesa.v9n1.p128-135
[24] Shanta, S., & Wells, J. G. (2022). T/E design based learning: assessing student critical thinking and problem solving abilities. *International Journal of Technology and Design Education*, *32*(1), 267–285. https://doi.org/10.1007/s10798-020-09608-8
[25] Sternberg. (2016). Critical thinking: Its nature, measurement, and improvement. *National Inst. Of Education*, *11*(1), 217–232. https://pdfs.semanticscholar.org/80a7/c7d4a98987590751df4b1bd9adf747fd7aaa.pdf
[26] Tang, T., Vezzani, V., & Eriksson, V. (2020). Developing critical thinking, collective creativity skills and problem solving through playful design jams. *Thinking Skills and Creativity*, *37*, 100696. https://doi.org/10.1016/j.tsc.2020.100696
[27] Toker, S., & Baturay, M. H. (2022). Developing disposition to critical thinking and

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problem-solving perception in instructional design projects for producing digital materials. *International Journal of Technology and Design Education*, 32(2), 1267–1292. https://doi.org/10.1007/s10798-020-09646-2

[28] Tong, D. H., Loc, N. P., Uyen, B. P., & Son, T. H. (2020). Enhancing Creative and Critical Thinking Skills of Students in Mathematics Classrooms: An Experimental Study of Teaching the Inequality in High Schools. *Universal Journal of Educational Research*, *8*(2), 477–489. https://doi.org/10.13189/ujer.2020.080219

[29] Warli, & Nofitasari, Y. (2021). Junior high school students' mathematical connection: a comparative study of children who have reflective and impulsive cognitive styles. *Journal of Physics: Conference Series*, *1776*(1), 012036. https://doi.org/10.1088/1742-6596/1776/1/012036

[30] Xu, E., Wang, W., & Wang, Q. (2023). The effectiveness of collaborative problem solving in promoting students' critical thinking: A meta-analysis based on empirical literature. *Humanities and Social Sciences Communications*, *10*(1), 16. https://doi.org/10.1057/s41599-023-01508-1

[31] Yang, L., Jiang, J., Ji, F., Li, Y., Yung, K.-L., Ferreira, A., & Zhang, L. (2024). Machine learning for micro- and nanorobots. *Nature Machine Intelligence*, *6*(6), 605–618. https://doi.org/10.1038/s42256-024-00859-x

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APPENDIX

1. Lesson Identity and Objectives

Component	Description
School Level	Junior High School (Grade VIII)
Semester	Second Semester
Subject	Mathematics
Topic	Geometry - Circle
Duration	2 x 40 minutes
Student Characteristics	Low-achieving students with impulsive cognitive style
Learning Objectives	1. Identify and illustrate the elements of a circle accurately. 2. Apply formulas to calculate circumference and area of a circle. 3. Solve circle-related geometry problems in a structured and careful manner. 4. Practice self-regulation before answering.

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2. Learning Activities

Stage	Learning Activities	Time Allocation
Introduction	Teacher greets students and builds motivation. Explain the learning objectives and introduce "STOP – THINK – ANSWER" strategy.	10 minutes
Main Activities	**a. Concept Introduction (15 min)** Explain elements of a circle with visual aids. Students redraw and annotate diagrams with pause time. **b. Formula Practice (20 min)** Guided explanation and practice with structured problem steps. **c. Simulation and Evaluation (15 min)** Solve contextual problems with "think aloud". Immediate feedback from teacher. **d. Mini Test (10 min)** - Structured response format: Steps – Answer – Reason.	60 minutes
Closing	Joint summary of lesson. Student self-reflection: how to avoid rushing. Assign scaffolded homework.	10 minutes

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3. Assessment and Follow-Up

Component	Description
Assessment Techniques	Written tests. Observation of problem-solving behavior Student reflections Think-aloud responses
Instruments	Worksheets with structured thinking columns Reflection rubric
Remedial Actions	✓ Step-by-step problem exercises ✓ Paused response simulation ✓ STOP – THINK – CHECK strategy
Enrichment	Advanced contextual problems Mini project on circular measuring tools

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The Strategies Behind the PhD in Mathematics Education Graduates' Professional Growth: Insights from a Graduate Tracer Study (2016–2023)

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Abstract: This study explored the professional growth of PhD in Mathematics Education graduates from Don Mariano Marcos Memorial State University (2016–2023), focusing on the teaching strategies that contributed to their advancement. Motivated by the limited research linking doctoral training to practical classroom application, the study aims to identify which instructional approaches were most relevant to graduates' teaching practices and career outcomes. Using a descriptive tracer study design, data were gathered from all 16 program alumni through a structured online survey and analyzed using both quantitative and qualitative methods. Findings revealed that research-based instruction, problem-based learning, and digital tool integration were the most impactful strategies, contributing to improved teaching performance, increased confidence, job promotions, and professional recognition. Graduates reported high alignment between their doctoral training and real-world educational demands, particularly during the pandemic's transition to remote learning. Framed by Human Capital and Social Systems Theories, the results highlight how doctoral education functions as both a personal investment and an institutional lever for change. The study concludes that embedding flexible, research-informed pedagogies in doctoral programs strengthens instructional leadership and positions graduates as key agents of innovation in mathematics education.

Keywords: graduate tracer study, mathematics teacher education, professional growth benefits, teaching strategies

INTRODUCTION

What happens after the doctorate? In the high-stakes world of mathematics education, earning a PhD is often seen as the pinnacle of academic preparation, but what comes next is less frequently studied. While most tracer research ends at employment status, this study opens a new line of inquiry: How do doctoral graduates actually develop after graduation, and how do the teaching strategies they experienced shape their career trajectories? By tracing the professional paths and classroom practices of PhD Mathematics Education graduates over seven years, this study reveals not just where graduates work but also *how* they were taught, *why* certain strategies matter, and

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what makes their expertise relevant in today's educational landscape. This intersection of pedagogy and professional growth offers a fresh perspective on the true impact of doctoral education, one that moves beyond diplomas and dives into the heart of teaching.

In the evolving landscape of mathematics education, the role of doctoral programs has grown increasingly vital, not only in shaping scholarly research but also in transforming classroom instruction and educational leadership (Regan & King-Sears, 2023). Doctoral graduates are expected to serve as instructional innovators, research mentors, and curriculum leaders across diverse learning environments (Holewik et al., 2024). However, little empirical data demonstrates how doctorate-level training, especially in mathematics education, translates into real teaching methods and career advancement in the field, in spite of these expectations (Reys et al., 2019).

The Doctor of Philosophy in Mathematics Education program, which is in line with Level 8 of the Philippine Quality Framework (PQF), aims to produce graduates who exhibit advanced proficiency in curriculum design, research, teaching, and leadership (Atweh et al., 2023). Individuals are expected to lead professional practice, produce new knowledge, and use educational innovation to advance national development at this level. Yet, the real-world application and impact of these competencies, particularly in mathematics teaching, remain insufficiently documented (Chekulaeva et al., 2021). This gap presents an opportunity to examine how doctoral graduates apply PQF-aligned outcomes in their professional roles.

Several studies in higher education tracer research have historically focused on employment outcomes, income increases, or job satisfaction (Ferreras, 2023). While useful, such studies often overlook the pedagogical impact of doctoral programs, how teaching strategies developed during graduate studies are applied in practice, and how these strategies contribute to educators' professional growth, recognition, and leadership. Furthermore, in the wake of significant shifts in instructional delivery, especially during the COVID-19 pandemic, understanding which teaching strategies have remained relevant and impactful is more critical than ever (Bautista & Valtoribio, 2024; Koh & Daniel, 2022; Siani, 2024).

This study is grounded in the local context of the PhD in Mathematics Education program at Don Mariano Marcos Memorial State University (DMMMSU-SLUC). Between 2016 and 2023, the program produced graduates who entered or returned to roles in teaching, supervision, and administration. Yet questions remain: What strategies did these graduates take with them into their classrooms? And which strategies helped them advance professionally?

This research addresses those questions by tracing the teaching strategies employed with PhD Mathematics Education graduates and linking them to the benefits they reported after graduation, including improved job performance, increased confidence, promotion, and professional recognition. In doing so, the study moves beyond general tracer objectives and applies a pedagogical lens to graduate outcomes. It aims to contribute to the broader conversation on how doctoral programs can align teaching and research in meaningful, measurable ways.

Specifically, the study aims to answer the following questions:

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1. What are the common teaching strategies employed with PhD in Mathematics Education graduates from 2016 to 2023 that are perceived as most relevant to their professional practice?
2. Which among these teaching strategies are associated with the greatest career-related benefits for graduates of the PhD in Mathematics Education program?
3. What specific career-related benefits have PhD in Mathematics Education graduates reported after completing the program?

This study draws upon empirical data to explore the relationship between instructional practices and professional growth among PhD Mathematics Education graduates. By linking applied teaching strategies to measurable professional outcomes, the research highlights the reciprocal nature of teaching and research in advanced mathematics education. The study's findings have implications for doctoral curriculum design, institutional development, and the cultivation of research-informed practitioners.

Rather than concluding with the attainment of a doctoral degree, this study begins at that pivotal moment, unfolding in four key parts. The Methodology section provides insight into how the study was designed, introducing the participants, data collection tools, and the analytical approach used to trace teaching strategies and professional growth. The Results section shares what the graduates revealed: the teaching practices they carried forward and the benefits they experienced in their careers. In the Discussion, these findings are brought to life through the lens of educational theory, including Human Capital and Social Systems Theory, and are situated within current national and global shifts in mathematics education. The paper concludes by synthesizing the implications in the Conclusion section, where recommendations are offered for improving doctoral programs, elevating instructional leadership, and informing future directions in research and practice. Each section builds toward a clearer understanding of how doctoral preparation can leave a lasting mark, not just on academic transcripts, but in real classrooms and careers.

Teaching Strategies in Graduate Studies

Effective teaching strategies are central to graduate education, particularly in fields like mathematics education, where instructional leadership and research competence are equally critical (Connolly et al., 2018). In doctoral programs, the emphasis often shifts from traditional content delivery to more complex forms of instruction, such as research-based teaching, problem-based learning (PBL), and inquiry-driven pedagogy (Öztürk et al., 2022; Wijnia et al., 2024). These strategies support the PQF Level 8 competencies, which include generating new knowledge, leading transformative practice, and addressing learning problems using innovative approaches (Erdem et al., 2025; Liu & Pásztor, 2022).

Moeini (2020) emphasized that research-based instruction, when embedded in coursework and dissertation processes, empowers doctoral students to design lessons and learning environments

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that are both evidence-informed and adaptable to diverse contexts. Similarly, problem-based learning encourages graduates to approach instruction as inquiry, enabling the development of learners' critical thinking, collaboration, and problem-solving, key targets of PQF-aligned outcomes in mathematics education (Kozhoshov & Sakybekova, 2024; Kasemsap, 2021; Atmaja et al., 2024).

With the advent of distance education, technology-enhanced instruction has emerged as another key strategy at the graduate level (Mondragon-Estrada et al., 2023). The COVID-19 pandemic accelerated the integration of digital tools such as GeoGebra and LMS platforms, providing doctoral students not only with technological fluency but also with the ability to model flexible and resilient teaching (Engelbrecht & Borba, 2023; Assadi & Cretu, 2023). These competencies reflect the PQF's emphasis on lifelong learning, adaptability, and innovation in educational leadership.

Professional Growth Benefits Gained After Graduation

Graduate education, particularly at the doctoral level, is expected to result in a range of personal, academic, and institutional benefits. The PQF Level 8 descriptors emphasize that graduates should demonstrate advanced knowledge, independent research skills, and the ability to lead in their field. Studies have affirmed that PhD graduates frequently experience professional gains such as promotion, enhanced confidence, and expanded responsibilities, all of which align with these national qualifications benchmarks (Spronken-Smith et al., 2024; Cadilhe et al., 2023).

Painter and Clark (2015) noted that PhD holders often assume key roles in academic governance, faculty development, and curriculum innovation. These roles represent not just vertical advancement but evidence of applied leadership, a PQF-aligned outcome that doctoral programs aim to develop. Moreover, Spies et al. (2021) pointed out that teaching strategies gained through doctoral-level coursework often translate into leadership in classroom reform and research mentorship, further illustrating the intersection of academic preparation and practical influence.

The experience of navigating instruction during the pandemic also led to the development of digital literacy and pedagogical adaptability, reinforcing PQF objectives related to responding to national and global developments (Arissaputra et al., 2023). Graduates who completed their degrees during this period demonstrated not only instructional resilience but also the capacity to apply emerging tools and frameworks in ways that enhanced teaching effectiveness and program continuity (Schell, 2023; Marotta & Van De Laar, 2024).

Further, this study is anchored in two complementary theoretical frameworks: Human Capital Theory and Social Systems Theory.

Human Capital Theory (HCT), as articulated by economists such as Becker (1964), posits that investment in education enhances an individual's skills, productivity, and value in the labor market (Leoni, 2023). In the context of mathematics education, doctoral study represents a high-level educational investment that is expected to result in professional benefits, such as career advancement, teaching effectiveness, and institutional leadership (Rivas, 2024). The findings of this study, particularly the link between teaching strategy use and promotions, confidence, and recognition,

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demonstrate how human capital is not only accumulated during doctoral study but actively deployed through instructional practices that meet institutional goals and learner needs (Pozas et al., 2022).

Complementing HCT, *Social Systems Theory* views educational institutions as systems where individual components (students, faculty, programs) interact to achieve common goals (Vanderstraeten, 2023). In this framework, doctoral programs serve as developmental subsystems that shape graduates who, in turn, influence broader systems, such as schools, universities, and educational policy structures (Estrada et al., 2019). Graduates from the PhD in Mathematics Education program do not operate in isolation; their teaching strategies, curriculum development efforts, and leadership roles contribute to the functioning and transformation of the educational institutions they serve (Reys et al., 2008). As shown in this study, strategies like research-based instruction and curriculum alignment are mechanisms through which doctoral graduates exert systemic influence.

Together, these frameworks support the core proposition of this study: that doctoral training in mathematics education produces not only knowledge producers but also knowledge applicators, whose teaching practices carry both personal and institutional value.

METHODS

This section outlined the research design, participants, instrumentation, data collection, ethics, and analytical procedures employed in the study. Grounded in a tracer study framework, the methodology was designed to capture both the instructional strategies and professional growth benefits of PhD Mathematics Education graduates. By examining the teaching strategies employed during the program and the perceived benefits that followed, the study aims to provide a meaningful account of how doctoral-level preparation translates into professional impact in the field of mathematics education.

Research Design

This study employed a *descriptive* research design within a *tracer study* framework. As a tracer study, it aimed to monitor the professional trajectories and instructional practices of PhD in Mathematics Education graduates from 2016 to 2023. The descriptive approach was used to identify and summarize the teaching strategies most commonly employed by the graduates, as well as the career-related benefits they reported after completing the program.

This design was considered appropriate, as the study did not seek to manipulate variables or establish causal relationships. Instead, it focused on observing naturally occurring patterns, drawing on graduate feedback to document the perceived relevance of doctoral training, the alignment of program components with real-world teaching practices, and the reported impact on their professional growth and educational contributions. By using a descriptive tracer model, the study offers

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insights into how doctoral-level preparation in mathematics education translates into practical and career-enhancing outcomes.

Participants

The participants of the study were all 16 graduates of the PhD in Mathematics Education program at Don Mariano Marcos Memorial State University – South La Union Campus (DMMMSU-SLUC) from 2016 to 2023. A census approach was used due to the manageable population size, allowing for a complete set of feedback across all cohorts. The participants were employed in various educational institutions, including state universities, public schools, and private colleges, with most holding academic, supervisory, or leadership positions.

Instrumentation

The primary instrument used in this tracer study was a structured survey questionnaire, which was made accessible online via Google Forms to ensure ease of participation for respondents. The questionnaire was adapted from a standardized survey framework utilized by the university for all its tracer studies, thereby supporting the validity, consistency, and comparability of data across different academic programs at the graduate level. Consisting of 57 items, the survey was designed to gather information on several key domains: personal and demographic background, teaching strategies employed, current employment status, enrolment, the perceived relevance of the doctoral program to career development, and elements that contribute to employability and professional advancement.

The questionnaire incorporated a variety of item formats, including open-ended questions to elicit in-depth qualitative responses, a five-point Likert scale to measure levels of agreement or satisfaction, and multiple-choice items to capture specific categorical data. This design offers a comprehensive portrayal of alumni experiences following their completion of the PhD in Mathematics Education program.

Data Collection Procedure and Ethics

Tracing the participants for this study posed notable logistical challenges, as the graduates were no longer concentrated in a single institution or geographic location – some are already outside the country. Having embarked on diverse professional trajectories across various provinces and regions, spanning state universities, public school systems, and private educational institutions, coordinating communication and encouraging participation required deliberate effort and flexibility. The dispersed nature of their employment rendered traditional face-to-face follow-ups impractical. As a result, the researcher employed multiple digital platforms, including email, mobile messaging, Facebook, and Messenger, to trace, reach out to, and engage the participants.

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Each graduate received a formal invitation outlining the study's purpose, voluntary nature, and assurances of confidentiality. Informed consent was obtained electronically prior to participation. Despite the geographical dispersion, a complete census was successfully achieved through persistent communication, utilization of institutional networks, and the accessibility of online data collection tools such as Google Forms. Data collection was conducted over a four-month period in 2024. All responses were anonymized and securely stored.

Data Analysis

The collected data were analyzed using descriptive statistical techniques. Frequency counts and percentages were used to summarize the most commonly applied teaching strategies, their perceived usefulness, and the professional benefits reported by graduates. These metrics provided a comprehensive snapshot of patterns in instructional practice and career development across cohorts.

In addition to quantitative summaries, qualitative responses from open-ended survey items were subjected to thematic coding. Responses were grouped by recurring patterns and categorized under themes that reflected key areas of growth, strategy application, and program impact. Coding was done manually by the researcher, and categories were cross-validated through peer checking to enhance the trustworthiness of interpretation. This qualitative layer provided context and explanatory depth to support the quantitative findings.

Trustworthiness and Rigor

To ensure the credibility and integrity of the qualitative findings, the study observed strategies aligned with the criteria of trustworthiness: credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985; Ahmed, 2024). Credibility was supported by the use of a validated institutional tracer questionnaire and by triangulating data from responses. Participants' narratives were examined systematically through thematic coding to capture recurring patterns and meaningful insights. To enhance dependability, coding and interpretation were checked through peer debriefing with a senior faculty member, ensuring consistency in theme development. Confirmability was addressed by maintaining a clear audit trail of responses, coding decisions, and interpretive summaries. Although the study was context-specific, efforts were made to present detailed descriptions of participants and settings to support the transferability of findings to similar doctoral programs.

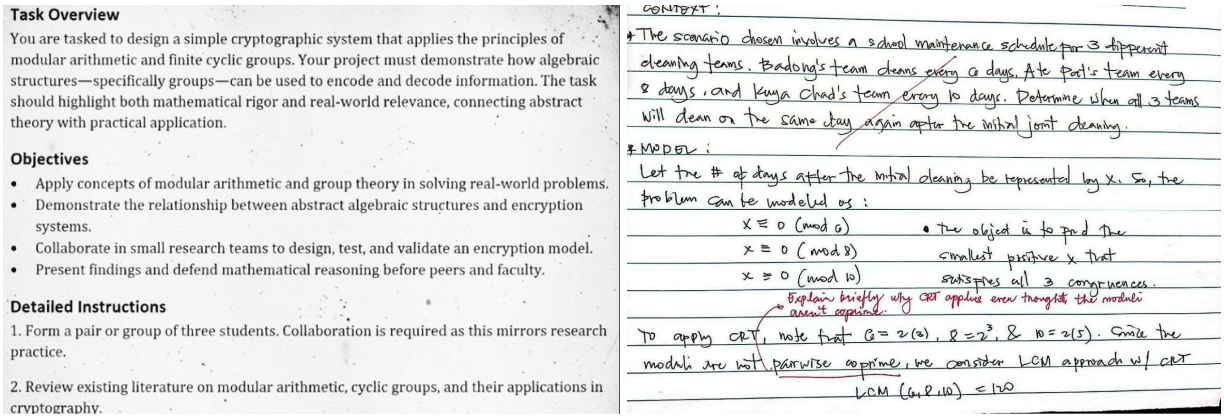
RESULTS

Inferred from Table 1, the teaching strategies applied to PhD Mathematics Education graduates evolved across time periods in response to both pedagogical developments and external factors such as the COVID-19 pandemic.

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Table 1 shows that from 2016 to 2018, during traditional face-to-face instruction, strategies were largely grounded in constructivist approaches. Problem-based learning (PBL) and lecture-discussion with real-world modeling were dominant. Use of manipulatives, peer teaching, and oral defense formats supported communication, reasoning, and collaborative learning, core competencies emphasized during this period.

Course records show that one of their major tasks in Abstract Algebra involved developing a simple encryption model using modular arithmetic and cyclic groups (Figure 1a). Conducted as a problem-based project, the task required collaborative planning, literature review, and data validation, culminating in a class presentation and critique session.



Task Overview

You are tasked to design a simple cryptographic system that applies the principles of modular arithmetic and finite cyclic groups. Your project must demonstrate how algebraic structures—specifically groups—can be used to encode and decode information. The task should highlight both mathematical rigor and real-world relevance, connecting abstract theory with practical application.

Objectives

- Apply concepts of modular arithmetic and group theory in solving real-world problems.
- Demonstrate the relationship between abstract algebraic structures and encryption systems.
- Collaborate in small research teams to design, test, and validate an encryption model.
- Present findings and defend mathematical reasoning before peers and faculty.

Detailed Instructions

1. Form a pair or group of three students. Collaboration is required as this mirrors research practice.

2. Review existing literature on modular arithmetic, cyclic groups, and their applications in cryptography.

(a) (b)

Figure 1. (a) Excerpt of the Abstract Algebra Task and (b) Student's output in Number Theory

Similarly, in the Number Theory course, lecture-discussion was complemented with modeling activities, such as applying the Chinese Remainder Theorem to real scheduling or synchronization problems in their workplaces (Figure 1b). Such experiences of the students became templates for how they later designed classroom investigations in their own teaching practice.

Further, in 2019, a shift toward research-integrated instruction became more evident. Graduates began to gain insights from their dissertations and coursework, particularly through action research, the use of formative assessment with rubrics, and early trials of the flipped classroom model. These emerging strategies reflect deeper engagement with curriculum design and student-centered learning. Faculty interviews indicated that dissertation findings were frequently used as case materials in research-related courses, reflecting the program's commitment to flexible and research-driven pedagogy. For example, one faculty of Qualitative Research required students to analyze data sets from completed dissertations to practice coding and thematic analysis, an approach that allowed learners to engage directly with authentic research processes rather than relying solely on textbook exercises. The faculty explained: *"I realized that many of our students find research abstract until they see how findings are derived. That's why I bring data from previous dissertations into class. We analyze them together, students practice coding, identifying themes, and connecting them to theoretical frameworks"*. In addition, Course syllabi and sample materials

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from 2018–2020 confirmed that many readings and examples were drawn from completed dissertation research within the program. This practice illustrates how flexible pedagogical approaches enabled faculty to adapt instruction to real research contexts, strengthening the integration of inquiry, reflection, and evidence-based learning in doctoral coursework and contributing to the institution's evolving culture of scholarly inquiry.

Period	Teaching Strategies	Description
2016–2018 (Classroom-Based Instruction)	Problem-Based Learning (PBL)	Used in teaching topics like number theory and abstract algebra.
	Lecture-Discussion with Mathematical Modeling	Integrating real-world applications in calculus or statistics.
	Use of Manipulatives and Visual Aids	Employed especially in geometry and combinatorics.
	Peer Teaching and Collaborative Group Work	Often used during group studies or as extension strategies.
	Oral Defense as an Instructional Tool	Frequent use of presentation and oral critique formats for fostering communication and logical reasoning.
2019 (Pre-pandemic Period)	Action Research-Driven Teaching	With more graduates conducting dissertation research, many PhD faculty members began integrating research findings into their classrooms.
	Use of Formative Assessment and Rubrics	Developed from coursework in curriculum and instruction.
	Flipped Classroom (Experimental Pilots)	Some started experimenting with this to increase student engagement and critical thinking.
2020–2022 (Pandemic and Distance Learning Period)	Synchronous and Asynchronous Online Teaching	Conducted via Google Meet or Zoom
	Video Lectures with Guided Practice	Pre-recorded math lectures followed by Google Classroom-based quizzes.
	Online Discussion Forums	For topics in educational philosophy, graduate seminars, and research methods.
	Digital Assessment Tools	Such as Google Forms and GeoGebra assessments.
	Modularized Instruction	Self-learning modules prepared in PDF/ print formats to serve learners with poor connectivity.
	ICT Integration in Mathematics	Applying ICT in teaching real analysis, statistics, and operations research.
2023 (Post-Pandemic/ Hybrid)	Blended Learning Strategies	A mix of online and face-to-face methods.
	Differentiated Instruction Using Data	Grounded in dissertation work focused on student profiling and performance tracking.
	Capstone- or Research-Based Instruction	Students work on mini-projects or papers as assessment tools.
	Technology-Supported Demonstrations	Incorporating math software like GeoGebra, or MATLAB in calculus and linear algebra.

Table 1: Common Teaching Strategies

The 2020–2022 pandemic period marked a significant transformation in instructional delivery. Teaching rapidly adapted to remote modalities, with widespread use of synchronous and asynchro-

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nous platforms like Zoom and Google Meet. Graduates implemented video lectures, online discussion forums, and Google Classrooms. The development of modularized instruction and ICT integration in mathematics also demonstrated a commitment to equity and continuity despite technological barriers.

By 2023, as schools transitioned to hybrid modes, a new blend of traditional and digital strategies emerged. Graduates reported learning through blended learning, data-driven differentiated instruction, and capstone or research-based projects to assess deeper learning. The increasing use of technology-supported demonstrations, such as GeoGebra and MATLAB, also illustrates how digital tools have become normalized in post-pandemic pedagogy.

Moreover, Table 2 revealed a clear hierarchy among the teaching strategies employed. The top-ranked strategy, Research-Based Instruction, consistently stood out as the most impactful. With 87.5% of graduates rating research activities as the most useful aspect of the program, this strategy was not only central to dissertation work but also translated into classroom practices such as action research, literature reviews, curriculum design, and research colloquium. Its consistent application in both face-to-face and distance learning settings highlights its versatility and alignment with doctoral-level competencies. To illustrate how research-based instruction and literature reviews were implemented in doctoral coursework, the following examples of instructional designs were drawn from the 2023 implementation of the Curriculum Development, Assessment, and Instructional Methodologies in Education and the 2019 midyear offering of the Special Topics in Abstract Algebra courses, as shown in Appendices A and B, respectively.

Next, Problem-Based Learning (PBL) was particularly effective in traditional classroom instruction, especially in subjects such as algebra, calculus, and number theory. This strategy was considered key to developing students' problem-solving and critical thinking skills, attributes they found most useful in their teaching and workplace settings. Its strong alignment with core mathematical thinking makes it a cornerstone of effective pedagogy in mathematics education. Third, the Use of Digital Tools such as GeoGebra, Google Forms, and learning management systems gained prominence during the 2020–2022 pandemic period. Graduates demonstrated a strong capacity to adapt these tools for assessment, content delivery, and engagement in both synchronous and asynchronous modes. While this strategy emerged in response to remote learning needs, it has persisted into the post-pandemic context, particularly in hybrid environments, signaling its lasting pedagogical value.

In contrast, the ninth- and tenth-ranked strategies were less widely adopted or had limited classroom application. The Flipped Classroom, while innovative, was only used by a small number of

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Rank	Teaching Strategies	Context	Justifications From the Analysis of the Tracer Data
1	Research-Based Instruction	Face-to-face and Distance	87.5% rated research activities as most useful. Widely applied in dissertations and later translated to classroom strategies and professional development sessions.
2	Problem-Based Learning (PBL)	Face-to-Face	Frequently used in algebra, calculus, and number theory instruction. Promotes problem-solving and critical thinking skills, rated most useful in the workplace.
3	Use of Digital Tools (GeoGebra, Google Forms, LMS)	Distance Learning & Blended	Strong adaptation noted in 2020–2022. Widely applied in assessment and instruction during pandemic-era learning.
4	Collaborative/Group-Based Learning	Face-to-Face and Distance	Used across multiple years for promoting peer interaction, especially in teaching higher math and conducting research seminars.
5	Modular and Self-Paced Learning	Distance Learning (‘20–‘22)	Practical and widely adopted due to connectivity issues. Reinforced by institutional guidance during the pandemic.
6	Lecture-Discussion with Application Tasks	Face-to-Face	Especially effective in mathematics in finance, statistics, and linear programming. Combines concept delivery with real-world modeling.
7	Formative Assessment with Rubrics	Face-to-face and Distance	Tied to program coursework in curriculum and assessment. Rated highly relevant in linking instruction to performance-based evaluation.
8	Blended Learning (Post-pandemic Hybrid)	Hybrid	As classes returned to physical campuses, blended strategies offered continuity and flexibility. Still in the early reintegration phase among the graduates.
9	Flipped Classroom	Select Cases *(Experimental)*	Some 2019 and 2023 graduates reported using this in higher ed teaching, but adoption was not widespread.
10	Oral Presentation/ Defense as an Instructional Tool	Face-to-Face	Common in seminars and research subjects, but limited to academic assessments rather than direct classroom teaching.

Table 2: Ranked Teaching Strategies by Relevance and Usability in the Program

graduates (notably in 2019 and 2023) and remained in the experimental stage. It holds potential for increasing student autonomy and engagement, but lacks broad implementation during the study period. Meanwhile, Oral Presentation or Defense as an Instructional Tool, although common in research seminars and academic evaluations like dissertation proposals and final defenses, was rarely applied as a core teaching strategy in mathematics classes. Its use was largely limited to formal academic settings rather than everyday instruction.

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The data presented in Table 3 reflect the wide-ranging professional growth benefits experienced by graduates of the PhD Mathematics Education program. The most consistently reported gains occurred in the domain of Instructional Competence, where 87.5% of graduates indicated improved job performance and increased teaching confidence. In the domain of Career Advancement, 81.25% of graduates experienced promotion or received new professional designations, with half of them (50%) attaining such advancement within a year of completing the program.

Graduates also reported high levels of Research and Academic Engagement, with 87.5% finding research activities to be the most useful component of the program. This reinforces the centrality of research-based instruction in both academic practice and career progression, further supported by the high percentage (56.25%) of graduates recognized as resource persons or speakers.

Domains	**Premier Professional Growth Benefits per Domain**	**%**
Instructional Competence	Improved job performance	87.5
	Increased teaching confidence	87.5
Career Advancement	Promotion or new designation	81.25
	Promotion within 1 year post-graduation	50
Research and Academic Engagement	Found research activities most useful	87.5
Professional Recognition	Recognized as a resource person or speaker	56.25
Program Relevance	Reported program courses and strategies as highly relevant	100
Adaptability and Retention	Retained employment during the pandemic	100
	Successfully adapted to online/remote teaching	93.75
Leadership and Responsibility	Assigned greater roles/ responsibilities post-graduation	50

Table 3: Summary of professional growth benefits

Further, the domain of Program Relevance achieved the highest rating, with 100% of graduates affirming that the program's courses and strategies were highly applicable to their current roles.

This affirms strong curricular alignment with the real-world demands of mathematics education. Equally notable are the outcomes under Adaptability and Retention, where 100% of graduates reported retaining employment during the COVID-19 pandemic, and 93.75% successfully adapted

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to remote teaching. These findings highlight the program's role in fostering digital resilience and instructional flexibility.

Finally, Leadership and Responsibility were reported by 50% of graduates who assumed greater institutional roles post-graduation. These responsibilities likely stem from the research, mentoring, and curriculum development competencies cultivated throughout the program.

Table 4 shows a relationship between specific teaching strategies and the professional benefits gained by PhD Mathematics Education graduates. In the Table, three emerged as the most strongly associated with meaningful career advancement and performance outcomes: Research-Based Instruction, Problem-Based Learning (PBL), and the Use of Digital Tools. Research-Based Instruction was the most frequently cited strategy linked to career gains. Graduates credited it with enhancing their *confidence*, *improving work performance*, and enabling their *promotion to leadership roles* or *recognition as resource persons*. This aligns with tracer data showing that 87.5% of graduates rated research-related activities as the most useful component of their doctoral training. Its high relevance to program competencies, such as *critical thinking*, *instructional design*, and *leadership*, reinforces its transformative impact on practice. Further, Problem-Based Learning (PBL) also stood out, with graduates reporting improvements in *teaching performance, promotion* (due to the effect of the teaching effectiveness rating), and *overall job satisfaction*. This strategy's alignment with core doctoral courses such as Curriculum Development and Advanced Mathematics subjects underscores its direct application to content delivery and student-centered instruction. The focus on inquiry and critical thinking reflects a shift toward modern pedagogical approaches emphasized in graduate education.

Additionally, during the 2020–2022 pandemic period, the Use of Digital Tools played a crucial role in helping graduates retain their teaching roles and adapt to new instructional environments. Tracer data show a strong link between this strategy and enhanced *teaching confidence*, increased *innovation*, and sustained *employability*. Notably, there were no reports of job loss among graduates during this period, indicating the practical value of digital fluency in times of disruption.

Moreover, other strategies such as Collaborative Learning and Formative Assessment with Rubrics were linked to growth in areas like *leadership, mentoring*, and *instructional responsibility*. These approaches helped graduates build strong interpersonal and evaluation skills, key elements of educational leadership and reflective practice. Meanwhile, Lecture-Discussion with Application, Modular Instruction, and Blended Learning offered context-specific benefits, particularly in delivering complex mathematical content or maintaining instructional continuity during remote learning phases. However, their influence on career advancement was more limited compared to research-driven strategies.

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Teaching Strategies	Graduate Benefits from Tracer Data	Support from Tracer Data
Research-Based Instruction	Promotions, increased confidence, expert/resource person roles, improved work performance	87.5% found research activities most useful; high alignment with program competencies like critical thinking and leadership.
Problem-Based Learning (PBL)	Improved performance, promotion, and job satisfaction	Linked to high-rated subjects like Curriculum Development and Advanced Math topics; aligns with critical thinking and problem-solving.
Use of Digital Tools (GeoGebra, Google Forms)	Improved confidence, innovation in teaching, and job retention	Strategies tied to pandemic-era instruction (2020–2022); sustained employability with no reported job loss; increased adaptability noted.
Collaborative/Group-Based Learning	Confidence in job, leadership roles, commitment to teaching, and peer mentorship	Aligned with reported gains in human relations and collaborative spirit, used in seminar-based and extension instruction.
Formative Assessment with Rubrics	Better performance feedback, greater responsibility in academic functions	Reflects skills in assessment and curriculum relevance; supports research and instructional effectiveness.
Lecture-Discussion with Application Tasks	Improved clarity in content delivery, effective math communication	Supports job performance in the core content area; aligns with improved communication and content mastery
Modular and Self-Paced Learning	Job retention during distance learning, flexibility in delivery	Particularly relevant in 2020–2022 for continued teaching and program completion; practical but less emphasized post-pandemic.
Blended Learning	Smooth transition post-pandemic, continuity of teaching	Applied mainly in 2023; supports sustainability and integration, but has limited long-term data on benefits.
Flipped Classroom	Encouraged learner independence and critical thinking, used in higher education settings	Used experimentally; relevant but not widely adopted (mentioned qualitatively, not strongly reflected in benefit frequency).
Oral Presentation and Defense as an Instructional Tool	Supports communication and confidence, but is limited to academic contexts	Mostly applicable in seminars and graduate-level presentations; less impact on broader teaching strategies or promotion.

Table 4: Teaching strategies by reported graduate benefits

Lastly, at the lower end of the benefit spectrum, Flipped Classroom and Oral Presentation/Defense as Instructional tools were acknowledged for their potential in developing student autonomy and communication skills, but were not widely adopted or reported as significantly impactful across graduates. Their use was mostly limited to experimental or academic settings, and did not strongly correlate with broader professional benefits like promotion or institutional leadership.

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DISCUSSION

The evolution of teaching strategies among PhD Mathematics Education graduates from 2016 to 2023 reflects a dynamic response to pedagogical shifts, institutional expectations, and external pressures such as the COVID-19 pandemic (Biehler et al., 2024). As shown across the tables, the most relevant and widely adopted strategies, Research-Based Instruction, Problem-Based Learning (PBL), and the Use of Digital Tools, correlate with the most impactful professional benefits reported by the graduates, including improved job performance, increased confidence, promotion, and recognition as resource persons. The examples above show how doctoral coursework has embedded flexible and research-driven pedagogies into instruction. The students engaged in collaborative, problem-based, and modeling activities that linked theory to authentic inquiry. Similarly, the use of dissertation data in research courses exemplified flexible pedagogy, enabling students to experience the actual research process. Collectively, these practices demonstrate how the program bridges theoretical learning with applied research, fostering scholarly engagement and reflective practice among doctoral students. This progression supports the notion that doctoral training not only imparts knowledge but fosters adaptable, forward-thinking educators equipped to lead in varied instructional contexts (Patterson et al., 2019; McAlpine & Inouye, 2021).

This alignment is best understood through the lens of Human Capital Theory (HCT). Becker (1964) posits that educational attainment and skill development serve as investments in one's productivity and future professional value. The tracer data affirm this, showing that a significant majority of the graduates rated research activities, the cornerstone of doctoral preparation, as the most useful part of the program. Their application of research-based instruction in their teaching, curriculum development, and professional development sessions is a direct manifestation of capitalized learning (Gerber et al., 2011). Furthermore, strategies like PBL and digital integration reflect the program's role in enhancing instructional capacity and increasing graduates' market value and employability, particularly during times of crisis (D'Elia et al., 2025).

The results also reveal substantial evidence of professional growth across multiple domains. Under Instructional Competence, a high percentage of graduates reported improved job performance and increased teaching confidence, indicating that the pedagogical training they received had an immediate and lasting impact on their classroom effectiveness (Li et al., 2025). In terms of Career Advancement, a very high number of graduates were promoted or received new designations, with half of them advancing within just one year of graduation. These outcomes underscore the practical return on educational investment and further validate HCT's core assertion that advanced qualifications enhance workforce potential (Mustafa & Lleshi, 2024; Thwin, 2023).

In addition, beyond individual advancement, these outcomes also reflect how doctoral graduates function within larger educational systems, as articulated in Social Systems Theory. Graduates are not isolated agents; they are subsystems functioning within broader educational ecosystems (Khuzwayo, 2020). Their adoption of collaborative learning, formative assessment, and modular instruction demonstrates how doctoral-level training enables them to contribute meaningfully to institutional goals (Schmidt et al., 2023). The integration of new strategies during pandemic-disrupted learning phases shows systemic adaptability; graduates acted as mediators of change within

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their departments and communities, ensuring continuity of education and modeling resilience (Harmey & Moss, 2021).

The tracer study further highlights the role of the program in supporting research and academic engagement, with the majority of graduates citing research activities as most beneficial. This not only aligns with core expectations of doctoral education but also reflects how graduates translated their research competencies into practice, leadership, and policy influence (Cunningham et al., 2023). Supporting this, more than half of their number were recognized as resource persons or speakers, reinforcing the program's contribution to their professional recognition and visibility within academic circles (Van Winkel et al., 2025).

Equally relevant is the alignment of the observed outcomes with the Philippine Qualifications Framework (PQF) Level 8, which outlines the expected competencies of doctoral degree holders. Graduates demonstrated outcomes such as critical thinking, innovation, leadership, and reflective practice. The use of research-based and differentiated instruction exemplifies the ability to generate new knowledge and lead professional practice, both of which are embedded in PQF descriptors. Notably, all of the graduates confirmed the relevance of program courses and strategies to their work, demonstrating curricular coherence with national standards (Cuoco & McCallum, 2017).

The program's responsiveness to external disruptions is further validated in the domain of Adaptability and Retention, where all the graduates retained their employment during the pandemic, and almost all successfully adapted to remote teaching modalities. These results highlight the role of the doctoral program in fostering digital resilience and innovation, particularly through the use of ICT and modular delivery. Such capabilities are essential in navigating uncertainty and reflect a future-ready approach to teacher preparation (Bakker et al., 2021).

Moreover, half of the graduates assumed greater institutional responsibilities, including roles in supervision, curriculum leadership, and research mentorship. This is consistent with PQF expectations and illustrates that graduates not only improve their personal capacities but also strengthen the systems in which they operate, serving as agents of institutional transformation (Ahl et al., 2025).

The prominence of Problem-Based Learning in traditional classroom instruction further supports literature highlighting its value in developing mathematical reasoning and higher-order thinking skills (Alreshidi & Lally, 2024; Purba et al., 2023; Aba-Oli et al., 2024). PBL's relevance to graduate-reported benefits: promotion, teaching confidence, and job satisfaction, emphasizes its practical and professional value, especially in core subjects such as algebra and number theory (Ahl et al., 2025; Mohamad et al., 2024).

The Use of Digital Tools during 2020–2022 was not merely a response to pandemic conditions; it signaled a pedagogical shift that has persisted into post-pandemic practice (Driskell et al., 2025). These tools, such as GeoGebra, Google Forms, and LMS platforms, supported not only content delivery but also graduate adaptability, innovation, and job retention. Notably, no job loss was reported during this period, which highlights the role of technological fluency as a protective factor for employment continuity and a marker of educational relevance (Razalli et al., 2025). However,

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some graduates experienced challenges during remote teaching, noting that limited internet access occasionally hindered full use of digital tools, especially in less connected regions.

Meanwhile, strategies like Collaborative Learning and Formative Assessment with Rubrics, though ranked slightly lower, were significant in fostering leadership, mentoring, and institutional engagement (De Paula et al., 2024). These reflect the human relations and evaluative skills expected of doctoral graduates who must lead curriculum implementation and support instructional improvement in their workplaces (Kaur et al., 2023). Such findings echo the work of Fadzil and Osman (2025) and Johnson and Williams (2023), who highlight the importance of research-informed, collaborative teaching in shaping effective educational leaders.

On the other hand, less widely adopted strategies, such as the Flipped Classroom and Oral Presentation as an Instructional Tool, were acknowledged for fostering student autonomy and academic discourse but were limited to experimental or academic settings. Their lower ranking and limited impact on career advancement suggest the need for more systematic support and training for these strategies to gain traction in mainstream instructional practice (Cevikbas & Kaiser, 2020; Baybayon & Lapinid, 2024).

The teaching strategies employed for the graduates were not only pedagogically sound but also career-shaping tools, reflecting the convergence of theory, policy, and lived experience. These findings affirm the doctoral program's success in producing mathematics educators who are research-informed, context-responsive, and professionally agile, educators who embody the outcomes prescribed by national qualification standards, and who are well-positioned to lead instructional innovation and institutional improvement across diverse educational contexts (Biehler et al., 2024).

CONCLUSIONS

This tracer study confirms that doctoral training in Mathematics Education serves as a transformative experience that significantly enhances both instructional competence and professional identity. Graduates reported learning from and applying a range of impactful teaching strategies, particularly research-based instruction, problem-based learning, and digital integration, which influenced how they taught, led, and adapted in their professional contexts.

A key finding is the strong alignment between the program's pedagogical design and its real-world application. This integration of theory and practice translated into a range of professional benefits, including enhanced teaching competence, increased confidence, career advancement, and institutional leadership. Graduates also demonstrated resilience in times of disruption, highlighting the program's role in fostering digital fluency and instructional adaptability, skills now essential in the post-pandemic educational landscape.

These findings offer important implications for teaching practice within and beyond the doctoral level. Doctoral programs should more intentionally embed research-driven, collaborative, and flexible pedagogies into coursework and instructional training. Doing so ensures that graduates are

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not only content experts but also reflective practitioners and transformational leaders. Post-graduation, institutions must continue to support professional development and recognize doctoral alumni as key contributors to innovation in education.

Ultimately, the study reinforces that doctoral education is not only a scholarly pursuit but a systemic investment, one that prepares educators to drive meaningful change in mathematics education, both in the classroom and at the institutional level.

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REFERENCES

[1] Aba-Oli, Z., Koyas, K., & Husen, A. (2024). Higher-order thinking skills-oriented problem-based learning interventions in mathematics: A systematic literature review. *School Science and Mathematics*. https://doi.org/10.1111/ssm.12676

[2] Ahl, L. M., Helenius, O., Færch, J. V., Koichu, B., Misfeldt, M., Aguilar, M. S., & Jankvist, U. T. (2025). The current state of implementation research in mathematics education. *Implementation and Replication Studies in Mathematics Education*, 1–17. https://doi.org/10.1163/26670127-00501001

[3] Ahmed, S. K. (2024). The pillars of trustworthiness in qualitative research. *Journal of Medicine Surgery and Public Health*, *2*, 100051. https://doi.org/10.1016/j.glmedi.2024.100051

[4] Alreshidi, N. a. K., & Lally, V. (2024). The effectiveness of training teachers in problem-based learning implementation on students' outcomes: a mixed-method study. *Humanities and Social Sciences Communications*, *11*(1). https://doi.org/10.1057/s41599-024-03638-6

[5] Arissaputra, R., Sobandi, A., Sentika, S., Adib Sultan, M., & Putu Nurwita Pratami Wijaya, N. (2023). Trend Analysis Using Bibliometric Study on Digital Literacy in Education. *International Journal Of Humanities Education and Social Sciences (IJHESS)*.

[6] Assadi, N., & Cretu, C. (2023). The Influence of Integrating GeoGebra Software into the Educational Setting on the Affective, Behavioral, and Cognitive Aspects of Pre-Service Mathematics Teachers. *Creative Education*, *14*(12), 2503–2519. https://doi.org/10.4236/ce.2023.1412161

[7] Atmaja, T.A., Yustitia, V., Naim, S., Fantoni, Santosa, T.A., Meriyati, & Pasaribu, D. (2024). Meta-analysis Problem Based Learning on Students' Problem-Solving Skills in Higher Education. *Indonesia Journal of Engineering and Education Technology (IJEET)*.

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[8] Atweh, B., Lapinid, M. R. C., Limjap, A. A., Elipane, L. E., Basister, M., & Conde, R. L. (2023). Critical Analysis of Mathematics Education Doctoral dissertations in the Philippines: 2009–2021. In *Mathematics education - an asian perspective* (pp. 69–95). https://doi.org/10.1007/978-981-99-0643-7_4

[9] Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: an international survey before and during the pandemic. *Educational Studies in Mathematics*, *107*(1), 1–24. https://doi.org/10.1007/s10649-021-10049-w

[10] Bautista, R.M., & Valtoribio, D. C. (2024). Flexible Teaching-Learning Modality in Mathematics Education of a State University in West Philippines. *Mathematics Teaching-Research Journal. 16*(3), 5 – 24.

[11] Baybayon, G., & Lapinid, M.R. C. (2024). Effects of Differentiated Instruction in Flipped Classrooms on Students' Mastery Level and Performance in Quadratic Equations. *Mathematics Teaching-Research Journal. 16*(1), 213 – 237.

[12] Becker, G. S. 1964. *Human Capital Theory*. New York: Columbia University Press.

[13] Biehler, R., Durand-Guerrier, V., & Trigueros, M. (2024). New trends in didactic research in university mathematics education. *ZDM*, *56*(7), 1345–1360. https://doi.org/10.1007/s11858-024-01643-2

[14] Cadilhe, M., Almeida, B., Rodrigues, A. I., & Santos, M. (2023). Professional experience before a PhD. Does it pay off? *Frontiers in Education*, *8*. https://doi.org/10.3389/feduc.2023.1129309

[15] Cevikbas, M., & Kaiser, G. (2020). Flipped classroom as a reform-oriented approach to teaching mathematics. *ZDM*, *52*(7), 1291–1305. https://doi.org/10.1007/s11858-020-01191-5

[16] Chekulaeva, M. E., Sidorova, N. V., Kuzina, N. G., & Veselovskaya, J. A. (2021). Application mathematical problems as means of implementing competency-based approach. *Revista on Line De Política E Gestão Educacional*, 713–725. https://doi.org/10.22633/rpge.v25iesp.1.15008

[17] Connolly, M. R., Lee, Y., & Savoy, J. N. (2018). The Effects of Doctoral Teaching Development on Early-Career STEM Scholars' College Teaching Self-efficacy. *CBE,Life Sciences Education*, *17*(1), ar14. https://doi.org/10.1187/cbe.17-02-0039

[18] Cunningham, K. M. W., Rangel, V. S., Lochmiller, C. R., & Farmer, M. (2023). Developing Educational leadership in mathematics and Science: Insights from teaching and learning faculty. *Journal of Research on Leadership Education*, *19*(2), 241–265. https://doi.org/10.1177/19427751231170243

[19] Cuoco, A., & McCallum, W. (2017). Curricular coherence in mathematics. In *Advances in STEM education* (pp. 245–256). https://doi.org/10.1007/978-3-319-61434-2_13

[20] D'Elia, P., Stalmach, A., Di Sano, S., & Casale, G. (2025). Strategies for inclusive digital education: problem/project-based learning, cooperative learning, and service learning for students with special educational needs. *Frontiers in Education*, *9*. https://doi.org/10.3389/feduc.2024.1447489

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[21] De Paula, V. B. M., De Moraes Fonseca, A., Ferreira, W. J., & Da Silva Richetto, K. C. (2024). Math Education: Collaborative Learning for Educational Equity. *EIKI Journal of Effective Teaching Methods*, *2*(3). https://doi.org/10.59652/jetm.v2i3.233

[22] Driskell, S., Wheeler, A., & Rhine, S. (2025). Technologies that persist in mathematics education instruction after emergency remote teaching. *Contemporary Issues in Technology and Teacher Education, 25*(2).

[23] Engelbrecht, J., & Borba, M. C. (2023). Recent developments in using digital technology in mathematics education. *ZDM*, *56*(2), 281–292. https://doi.org/10.1007/s11858-023-01530-2

[24] Erdem, C., Kaya, M., Toptaş, H. T., & Altunbaşak, İ. (2025). Problem-based learning and student outcomes in higher education: a second-order meta-analysis. *Studies in Higher Education*, 1–22. https://doi.org/10.1080/03075079.2025.2498084

[25] Estrada, M., Zhi, Q., Nwankwo, E., & Gershon, R. (2019). The influence of social supports on graduate student persistence in biomedical fields. *CBE,Life Sciences Education*, *18*(3), ar39. https://doi.org/10.1187/cbe.19-01-0029

[26] Fadzil, N. M., & Osman, S. (2025). Scoping the landscape: Comparative review of collaborative learning methods in mathematical problem-solving pedagogy. *International Electronic Journal of Mathematics Education*, *20*(2), em0820. https://doi.org/10.29333/iejme/15935

[27] Ferreras, U.L. (2023). Assessing the contemporary employment Prospects of DEBESMSCAT Graduate Programs: A Comprehensive Tracer study. *www.academia.edu*. https://doi.org/10.5281/ZENODO.10211794

[28] Gerber, B. L., Marek, E. A., & Martin, E. P. (2011). Designing Research-Based Professional Development for Elementary school science and mathematics. *Education Research International*, *2011*, 1–8. https://doi.org/10.1155/2011/908014

[29] Harmey, S., & Moss, G. (2021). Learning disruption or learning loss: using evidence from unplanned closures to inform returning to school after COVID-19. *Educational Review*, *75*(4), 637–656. https://doi.org/10.1080/00131911.2021.1966389

[30] Holewik, K., Nowak, B., & Przybyła-Kasperek, M. (2024). Teaching doctoral students: Best practices for modern education. *Beyond Philology an International Journal of Linguistics Literary Studies and English Language Teaching*, *21/2*, 79–109. https://doi.org/10.26881/bp.2024.2.04

[31] Johnson, K. G., & Williams, T. (2023). Mathematics Instructional leadership. *Journal of School Administration Research and Development*, *8*(1), 1–12. https://doi.org/10.32674/jsard.v8i1.3692

[32] Kasemsap, K. (2021). Advocating Problem-Based Learning and Creative Problem-Solving Skills in Global Education. *Research Anthology on Developing Critical Thinking Skills in Students*.

[33] Kaur, A. T., Narayana, K. A., & Aslam, S. (2023). The Relationship of Headmasters' Distributive Leadership towards Professional Learning Community Practices. *International Journal of Academic Research in Progressive Education and Development*, *12*(2). https://doi.org/10.6007/ijarped/v12-i2/16878

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[34] Khuzwayo, Q. O. (2020). Systems theory conceptualised and pasted to teaching and learning. *Deleted Journal*, *8*(10), 01–16. https://doi.org/10.31686/ijier.vol8.iss10.2593

[35] Koh, J. H. L., & Daniel, B. K. (2022). Shifting online during COVID-19: A systematic review of teaching and learning strategies and their outcomes. *International Journal of Educational Technology in Higher Education*, *19*(1). https://doi.org/10.1186/s41239-022-00361-7

[36] Kozhoshov, T., & Sakybekova, N. (2024). Problem-based learning issues and interdisciplinary relations in mathematics classes. *Математиканы Физиканы Және Информатиканы Оқытудың Өзекті Мәселелері*, *4*, 20–32. https://doi.org/10.52081/mpimet.2024.v08.i4.044

[37] Leoni, S. (2023). A historical review of the role of education: From human capital to Human Capabilities. *Review of Political Economy*, 1–18. https://doi.org/10.1080/09538259.2023.2245233

[38] Li, L., Kanchanapoom, K., Deeprasert, J., Duan, N., & Qi, Z. (2025). Unveiling the factors shaping teacher job performance: exploring the interplay of personality traits, perceived organizational support, self-efficacy, and job satisfaction. *BMC Psychology*, *13*(1). https://doi.org/10.1186/s40359-024-02324-1

[39] Lincoln, Y. S. (1985). *Naturalistic inquiry* (Vol. 75). Sage.

[40] Liu, Y., & Pásztor, A. (2022). Effects of problem-based learning instructional intervention on critical thinking in higher education: A meta-analysis. *Thinking Skills and Creativity*, *45*, 101069. https://doi.org/10.1016/j.tsc.2022.101069

[41] Marotta, J., & Van De Laar, M. (2024). Education as an e-resilient system: Empirical insights from stakeholder perspectives in public affairs education. *Journal of Public Affairs Education*, 1–28. https://doi.org/10.1080/15236803.2024.2388919

[42] McAlpine, L., & Inouye, K. (2021). What value do PhD graduates offer? An organizational case study. *Higher Education Research & Development*, *41*(5), 1648–1663. https://doi.org/10.1080/07294360.2021.1945546

[43] Moeini, A. (2020). Theorising Evidence-Informed Learning Technology Enterprises: A Participatory Design-Based Research Approach. *Education Computer Science*.

[44] Mohamad, I. Z., Nik, A. H. N. N., Adul, H. A., Najua, S. A. A., Rasidi, P., & Wanda, N. Y. (2024). Unlocking the Future: Mathematics Teachers' Insight into Combination of M-learning with Problem-Based Learning Teaching Activities. *Mathematics Teaching-Research Journal. 16*(3), 196 – 216.

[45] Mondragon-Estrada, E., Kirschning, I., Nolazco-Flores, J. A., & Camacho-Zuñiga, C. (2023). Fostering digital transformation in education: technology enhanced learning from professors' experiences in emergency remote teaching. *Frontiers in Education*, *8*. https://doi.org/10.3389/feduc.2023.1250461

[46] Mustafa, B., & Lleshi, S. (2024). The impact of lifelong learning and investments in employee development on employee productivity and performance. *Multidisciplinary Reviews*, *7*(8), https://doi.org/10.31893/multirev.2024175

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[47] Öztürk, B., Kaya, M., & Demir, M. (2022). Does inquiry-based learning model improve learning outcomes A second-order meta-analysis. *Journal of Pedagogical Research*. https://doi.org/10.33902/jpr.202217481

[48] Painter, S., & Clark, C. M. (2015). Leading Change: Faculty Development through Structured Collaboration. *International Journal of Doctoral Studies*, *10*, 187–198. https://doi.org/10.28945/2267

[49] Patterson, C.A., Chang, C., Lavadia, C., Pardo, M.L., Fowler, D., & Butler-Purry, K.L. (2019). Transforming doctoral education: preparing multidimensional and adaptive scholars. *Studies in Graduate and Postdoctoral Education*.

[50] Pozas, M., Letzel, V., Bost, N., & Reichertz, J. (2022). Confident, positive, but interested? Exploring the role of teachers' interest in their practice of differentiated instruction. *Frontiers in Education*, *7*. https://doi.org/10.3389/feduc.2022.964341

[51] Purba, E. R., Siregar, N., & Lubis, R. D. (2023). Implementation of LKPD-Assisted Problem-Based Learning Models as an effort to improve students' mathematical reasoning ability at Percut Sei Tuan State Junior High School. *Paradikma Jurnal Pendidikan Matematika*, *16*(1), 40–45. https://doi.org/10.24114/paradikma.v16i1.42288

[52] Razalli, A. R., Idris, N., Jamil, M. R. M., Ismail, R., Janudin, S. E., & Azmi, S. H. (2025). A Quantitative Study on Graduate Employability: Insights from Sultan Idris Education University, Malaysia. *International Journal of Research and Innovation in Social Science*, *VIII*(XII), 2127–2142. https://doi.org/10.47772/ijriss.2024.8120179

[53] Regan, K., & King-Sears, M. (2023). A scaffolded model for preparing doctoral students to teach in higher education. *Journal of Special Education Preparation*, *3*(3), 38–47. https://doi.org/10.33043/josep.3.3.38-47

[54] Reys, R., Glasgow, R., Teuscher, D., & Nevels, N. (2008). Doctoral programs in mathematics education in the United States: 2007 status report. *Issues in Mathematics Education*, 19–37. https://doi.org/10.1090/cbmath/015/02

[55] Reys, R., Reys, B., & Shih, J. C. (2019). Doctoral Preparation in Mathematics Education: Time for Research and a Widespread Professional Conversation. *Journal of Mathematics Education at Teachers College*, *10*(2), 21–27. https://doi.org/10.7916/jmetc.v10i2.4192

[56] Rivas, C. (2024). Supporting the professional and career development of doctoral students. *Encyclopedia*, *4*(1), 337–351. https://doi.org/10.3390/encyclopedia4010024

[57] Schell, J. A. (2023). Designing for Academic Resilience in Hands-On Courses in Times of Crisis: Two models for supporting Hands-On Online Learning drawn from the COVID-19 Pandemic. *American Behavioral Scientist*, *67*(13), 1632–1654. https://doi.org/10.1177/00027642221118292

[58] Schmidt, J. M., Porter, J. T., Rivera-Amill, V., & Appleyard, C. B. (2023). Promoting crucial team building, collaboration, and communication skills in graduate students through interactive retreats. *AJP Advances in Physiology Education*, *47*(4), 919–929. https://doi.org/10.1152/advan.00125.2023

[59] Siani, A. (2024). Impact of the COVID-19 pandemic on teaching, learning, assessment, and wellbeing in higher education. *Advances in Experimental Medicine and Biology*, 263–275. https://doi.org/10.1007/978-3-031-61943-4_17

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[60] Spies, T., Pollard-Durodola, S., Bengochea, A., Falomir, G. C., & Xu, Y. (2021). Teacher Leadership in Systemic Reform: Opportunities for graduate education programs. *The CATESOL Journal.*, *32*(1). https://doi.org/10.5070/b5.35923

[61] Spronken-Smith, R., Brown, K., & Cameron, C. (2024). Work happiness of PhD graduates across different employment sectors. *New Zealand Journal of Educational Studies*. https://doi.org/10.1007/s40841-024-00339-1

[62] Thwin ,N.P.Z., Janarthanan, M., Bhaumik, M. (2023). Impact of Career Advancement on Employee Retention. *International Journal of Latest Engineering and Management Research. 8*(1).

[63] Van Winkel, M., Van Der Rijst, R. M., Kuijer-Siebelink, W., Basten, F., Sools, A. M., Poell, R. F., & Van Driel, J. H. (2025). Identifying story types of PhD candidates lecturing in higher professional education: "ups and downs", "turnaround", "continuous growth", and "scholarly recognition." *Studies in Graduate and Postdoctoral Education*. https://doi.org/10.1108/sgpe-02-2023-0015

[64] Vanderstraeten, R. (2023). Systems Theory approaches to researching educational organizations. *Oxford Research Encyclopedia of Education*. https://doi.org/10.1093/acrefore/9780190264093.013.1885

[65] Wijnia, L., Noordzij, G., Arends, L. R., Rikers, R. M. J. P., & Loyens, S. M. M. (2024). The Effects of Problem-Based, Project-Based, and Case-Based Learning on Students' Motivation: a Meta-Analysis. *Educational Psychology Review*, *36*(1). https://doi.org/10.1007/s10648-024-09864-3

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Appendix A

Sample Classroom Learning Design in the Course Curriculum Development, Assessment, and Instructional Methodologies in Education

Philo 306: Curriculum Development, Assessment, and Instructional Methodologies in Education (Mathematics)

Project Title: Designing and Implementing Data-Driven Lessons in Mathematics
Semester: 2nd Semester, Academic Year 2022–2023
Instructor: [redacted]
Learning Modality: Blended (online and face-to-face)
Duration: 6 weeks

I. Rationale

The course emphasizes the integration of curriculum theory, assessment strategies, and innovative instructional methodologies to enhance mathematics teaching. Consistent with these objectives, the activity provides students with authentic opportunities to connect educational theory with practice by investigating real classroom problems, developing data-informed interventions, and analyzing their instructional outcomes. Through this process, doctoral students engage in reflective inquiry, collaborative problem solving, and evidence-based decision-making—core competencies aligned with the course's aim of developing scholarly and adaptive mathematics educators capable of designing responsive and research-grounded learning experiences.

II. Learning Outcomes

By the end of the activity, students were expected to:

1. Apply research methodologies in analyzing actual mathematics classroom problems;
2. Design differentiated and data-driven instructional strategies to address identified learner needs;
3. Implement lessons flexibly across various modalities (face-to-face, online, or hybrid);
4. Analyze collected classroom data to draw conclusions on instructional effectiveness; and
5. Present research findings and reflections through scholarly discussion and written report.

III. Learning Design

Stage	Description of Activity	Pedagogical Feature
Stage 1: Problem Identification (Week 1)	*Students identify a learning issue in their mathematics classrooms, supported by classroom data or learner performance records.*	Research-driven (rooted in authentic problems)
Stage 2: Literature Review and Lesson Design (Weeks 2–3)	*Each student conducts a brief literature review on strategies addressing the issue, then designs a short lesson sequence or intervention (e.g., differentiated algebra tasks, problem-based geometry).*	Flexible pedagogy (students choose focus and context)
Stage 3: Implementation and Data Gathering (Week 4)	*Students implement the designed lesson in their schools, using formative assessments, observation checklists, or interviews to collect data on student responses and outcomes.*	Research-driven, context-based learning
Stage 4: Analysis and Reflection (Week 5)	*Students analyze the collected data using basic descriptive or qualitative techniques to evaluate lesson effectiveness.*	Inquiry-based reflection
Stage 5: Presentation and Peer Critique (Week 6)	*Results are presented in class through PowerPoint and a brief report. Peers provide feedback and suggest methodalogical improvements.*	Collaborative learning

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Stage	Description of Activity	Pedagogical Feature
Stage 5: Presentation and Peer Critique (Week 6)	*Results are presented in class through PowerPoint and a brief report. Peers provide feedback and suggest methodological improvements.*	Collaborative learning

IV. Assessment Scheme

Criteria	4	3	2	1
Research Application	Demonstrates outstanding ability to apply appropriate research methods and analytical tools; collects valid, well-organized data directly linked to research questions; integrates literature effectively to support findings.	Applies suitable research methods with generally clear data collection and analysis; minor gaps in consistency or interpretation.	Attempts to apply research processes but shows limited methodological rigor; data collection or analysis is partially appropriate.	Shows minimal understanding of research processes; data are incomplete or unrelated to identified problem.
Instructional Design Quality	Designs a coherent, innovative, and context-responsive lesson or intervention; instructional activities are differentiated and well-aligned with identified learner needs.	Lesson design is clear and mostly aligned with learner needs; some differentiation and contextualization are evident.	Lesson design shows partial alignment with learner needs; limited evidence of differentiation or innovation.	Lesson design is unclear or disconnected from learner needs; lacks differentiation and coherence.
Data Analysis and Interpretation	Presents comprehensive data analysis with accurate interpretation; demonstrates clear linkage between results, theory, and implications for teaching practice.	Provides sound data analysis and interpretation with some connection to theory and practice.	Data analysis is basic or descriptive; interpretations are limited and only loosely connected to theory.	Minimal or inaccurate analysis; conclusions are unsupported or missing.
Reflection and Synthesis	Produces deep, evidence-based reflection showing critical insight into teaching practice, learning outcomes, and professional growth; connects results to broader educational theories.	Reflection shows awareness of insights gained and links findings to teaching improvement.	Reflection is superficial or descriptive with limited evidence of synthesis.	Reflection is absent or purely narrative, lacking analytical or theoretical connection.
Collaboration and Communication	Actively contributes to group discussions and peer critiques; communicates ideas clearly and scholarly both in oral and written form; demonstrates respect and responsiveness to feedback.	Participates constructively in group work and presentations; communicates ideas clearly with minor gaps in coherence or engagement.	Participates inconsistently in collaboration; contributions or communication need clarity and substance.	Shows minimal engagement in group collaboration; communication is unclear or incomplete.

4 – Excellent (3.50–4.00): Exceeds expectations; demonstrates mastery of research-driven pedagogy and reflective practice.
3 – Proficient (2.50–3.49): Meets expectations; shows strong understanding with minor areas for refinement.
2 – Developing (1.50–2.49): Partially meets expectations; requires further development of research and instructional integration.
1 – Beginning (1.00–1.49): Does not meet expectations; limited evidence of understanding or application.

V. Instructions for Conducting the Research Activity

A. Preparation and Permission

1. **Select a Focus:** Identify a teaching or learning issue in mathematics relevant to curriculum, assessment, or methodology.
2. **Prepare a 1–2 Page Proposal:** Include title, objectives, description of intervention, data plan, and ethical considerations.

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3. **Secure Permission:** Submit the proposal for instructor approval, then request authorization from your school head or supervisor using a signed letter of permission.
4. **Ensure Ethical Compliance:** Maintain confidentiality, voluntary participation, and data anonymity at all times.

B. Conduct of the Research

1. **Design the Intervention:** Develop a short lesson or strategy (e.g., differentiated task, blended module, modeling activity).
2. **Collect Baseline Data:** Administer a pre-assessment or diagnostic tool to identify learner needs.
3. **Implement the Lesson:** Apply your design in your class or simulated setting; document engagement and observations.
4. **Gather Post-Data:** Conduct a post-test or reflection interview to evaluate results and learning gains.
5. **Analyze and Reflect:** Use simple statistics or thematic analysis, then interpret results in relation to theory and practice.

C. For Non-Teaching Students

If not currently teaching, choose one:

- **Collaborate** with a teaching colleague as co-researcher.
- **Simulate** a lesson with volunteer learners or peers.
- **Analyze Documents** such as anonymized lesson plans or assessment data.

D. Report Format

1. **Title and Objectives**
2. **Description of Intervention**
3. **Methodology (Participants, Data, Tools)**
4. **Results and Reflection**
5. **References and Appendices**

Formatting: Times New Roman 12, 1.5 spacing, 1-inch margins, 5–7 pages.
Filename: *LastnameFirstname_RA_MathEd2023*

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Appendix B

Sample Literature Review Task in the Special Topics in Abstract Algebra

Med 317 – Special Topics in Abstract Algebra
Midyear Term, School Year 2018-2019
FINAL Requirement
Literature Review

I. Objectives

By the end of the term, the students should be able to:

1. Conduct a systematic and scholarly literature review on a self-selected topic under *Abstract Algebra*;
2. Demonstrate research-driven inquiry by locating, analyzing, and synthesizing peer-reviewed sources;
3. Collaborate in scholarly discussions to critique, refine, and present findings; and
4. Identify research gaps and propose possible directions for future study in mathematics education or abstract algebra research.

II. Activity Overview

This activity was developed to integrate the literature review process into the PhD in Mathematics Education coursework as a means of cultivating authentic scholarly practice. It engages students in independent inquiry, collaborative critique, and reflective synthesis—core elements of advanced research training. Flexibility is embedded in the design by allowing students to select their specific subtopic within Abstract Algebra and determine their preferred synthesis framework (thematic, chronological, or theoretical). Through structured consultation sessions, students engage in critical dialogue and peer evaluation, replicating the iterative knowledge-building process characteristic of the research community.

III. Scope and Considerations

1. **Topic and Focus:**
   Students select any research area under *Abstract Algebra* based on their academic or professional interests.
2. **Time Frame:**
   Sources should primarily cover studies from the last **10 years (80% recent literature)** to ensure relevance.
3. **Sources:**
   Use at least **10 scholarly references** (peer-reviewed journals, conference proceedings, books, or theses).
4. **Inclusion and Exclusion Criteria:**
   Students set criteria based on methodology, relevance, or scope. Criteria must be justified in the Introduction.
5. **Synthesis Approach:**
   Choose a synthesis type — **thematic**, **chronological**, or **theoretical** — as appropriate to your topic.
6. **Gap Identification:**
   Conclude with the identification of **research gaps** and implications for further study.

III. Learning Flow

Time Frame	Stages	Student Tasks
Week 2	**Topic Proposal**	Submit a one-page proposal identifying the chosen subtopic in Abstract Algebra, its rationale, and initial keywords for literature search. This establishes the foundation for a research-driven inquiry.
Week 3	**Guided Search Workshop**	Attend a guided database search workshop (onsite or online) on accessing research databases such as Scopus, ERIC, and JSTOR. Students practice constructing search strings and evaluating scholarly sources, demonstrating flexible, technology-enhanced learning.

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Time Frame	Stages	Student Tasks
Week 5	**Peer Consultation**	Participate in a peer consultation session to present preliminary findings, discuss challenges, and refine inclusion and exclusion criteria. This stage fosters collaborative learning and critical academic dialogue.
Week 7	**Draft Submission**	Submit a partial draft of the literature review (Introduction and initial synthesis). Receive formative feedback from both instructor and peers to refine analysis and organization. This promotes iterative, research-based improvement.
Week 10	**Final Submission and Presentation**	Submit the final version of the literature review and deliver a short oral presentation summarizing findings, identified gaps, and future research directions. This stage emphasizes reflective synthesis and scholarly communication.

V. Output Format

Length: 5-7 pages (excluding references)
Format: Trebuchet, 12pt, 1.5 spacing, 1-inch margins
File name: *Lastname_LitReview_AbsAlgebra.docx*
Reference Style: APA 7th edition
Submission Deadline: Week 10, Saturday
Presentation: Week 11 (10-minute summary face-to-face)

VI. Assessment Criteria

Criteria	4 - Excellent	3 - Proficient	2 - Developing	1 - Beginning
Research Rigor	Comprehensive review using diverse, credible sources; clear inclusion/exclusion justification.	Adequate review with some methodological clarity.	Limited justification or uneven source selection.	Incomplete or poorly justified review.
Synthesis and Organization	Insightful and coherent synthesis showing clear conceptual patterns or theoretical linkages.	Logical synthesis with minor organizational lapses.	Partially organized or descriptive synthesis.	Disconnected or unstructured review.
Gap Identification	Clearly identifies gaps and proposes meaningful future research directions.	Gaps identified but with limited elaboration.	Minimal attempt to discuss gaps.	No discussion of gaps.
Collaboration and Feedback Use	Actively engaged in peer discussions and used feedback effectively in revision.	Participated in discussions; feedback somewhat integrated.	Limited collaboration or reflection.	No collaboration or feedback evidence.
Writing and Scholarly Style	Academic tone, accurate citation, and logical argumentation throughout.	Minor lapses in style or citation accuracy.	Several style or citation inconsistencies.	Poor academic tone and missing citations.

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The Computational Thinking Profile of Vocational Students in Solving Mathematical Problems in Terms of Logical-Mathematical Intelligence

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Abstract: Computational thinking has been widely recognized as an effective problem-solving approach and plays a crucial role in the digital era, making it essential for students to master. These skills can be developed through engagement with mathematical problems such as Bebras tasks, which require logical reasoning, problem-solving abilities, and logical-mathematical intelligence. This study aims to describe students' computational thinking skills in solving mathematical problems containing Bebras tasks based on their levels of logical-mathematical intelligence and to determine the extent of its contribution. The study involved 55 tenth-grade students from a Center of Excellence Vocational School in Bali, an institution designed to enhance students' competencies, particularly in preparation for the digital world. A mixed-method approach was employed, combining qualitative methods to analyze students' computational thinking skills and quantitative methods to measure the contribution of logical-mathematical intelligence. The results indicate that students with high, moderate, and low logical-mathematical intelligence mastered five, three, and one indicators of computational thinking, respectively. Additionally, logical-mathematical intelligence contributed 81.1% to students' computational thinking skills. These findings suggest that higher levels of logical-mathematical intelligence are associated with greater mastery of computational thinking. In conclusion, logical-mathematical intelligence has a significantly strong contribution to students' computational thinking skills.

Keywords: Computational Thinking, Mathematical Problems Containing Bebras Task, Logical Mathematical Intelligence, Center of Excellence Vocational School

INTRODUCTION

The digitalization in the 21st century has led to an extraordinary increase in the flow of knowledge, making the use of computers in various fields inevitable (Aminah *et al*., 2022; Threekunprapa & Yasri, 2020; Puente & Perez, 2023). The learning process in the world of education is also required to adapt and create new innovations (Ayub *et al.*, 2021 & Pambudi *et al.*, 2023). In order to meet

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these challenges, students need to be equipped with 21st-century skills that focus on analytical thinking, technological programming, critical analysis, complex problem-solving, as well as system analysis and evaluation (Angeli & Giannakos, 2020; Helsa *et al.*, 2023; Miftah *et al.*, 2024). These skills are part of computational thinking (Munawarah *et al*., 2021; Izu *et al.*, 2017; Farib *et al.,* 2019). Computational thinking in this study is defined as a thinking ability that involves the application of computational concepts and logic to understand, formulate, and solve problems effectively. Computational thinking can enhance analytical abilities, facilitate the use of technology, and train complex problem-solving skills (Rahaju *et al.*, 2019 & Yunianto *et al.*, 2024). Moreover, Wing as cited in Sondakh *et al.* (2020) states that Computational Thinking (CT) is a computational concept used to approach and solve problems. In line with this statement, research by Persson (2022) has also recognized computational thinking as an efficient approach to problem-solving, making it highly demanded in the digital work environment.

Recognizing the importance of CT in supporting digitalization in the 21st century, the Indonesian government has realized the significance of implementing CT-based learning in schools (Rafiepour & Farsani, 2021; Saputra & Zaulmaulida, 2021; Setiarini *et al.*, 2023). This is because the computational thinking as an approach can help students structure complex problem-solving into simpler steps (Nuvitalia *et al.*, 2022 & Sulistya, 2021). CT-based learning is highly suitable for students, especially the students at Center of Excellence Vocational School, because they have been prepared to enter the professional world, which includes the digital work environment (Zamzami *et al.,* 2020 & Ersozlu *et al.,* 2023). CT-based learning also refers to the initiatives taken by the Indonesian government to integrate computational thinking into the national education system, particularly through programs that match the demands of industry 4.0 and the digital economy (Ersozlu *et al.,* 2023). Based on this information, CT is an interesting topic that has been widely studied in educational research. However, studies that specifically focus on the integration of CT into mathematics learning at the vocational high school (SMK) level are still relatively new and worth further investigation, especially considering the characteristics of vocational school students who tend to have a strong orientation toward the workforce and a high demand for advanced thinking skills (Aminah *et al*., 2022).

However, the learning process at the Center of Excellence Vocational School has not extensively trained students' computational thinking skills, particularly their problem-solving abilities (Nuvitalia *et al.,* 2022). This is evident from the field observations, where vocational high school students in Indonesia still weak in solving mathematical problems (Tresnawati *et al.,* 2020). Indahsari & Fitriana (2019) found a similar fact on their research of 33 10th-grade students from a vocational high school in Cimahi. They found that only 2.57% of the students were able to understand the problem, 8.63% were able to develop a plan, and only 15% were able to create and revise their plans. The implementation of computational thinking should be promoted mathematics learning activities in the classroom to prepare students for the workforce.

Efforts to train CT skills in students can be carried out through mathematical problems containing Bebras tasks (Dagiene *et al.,* 2022). Mathematical problems containing Bebras tasks are defined as problem-solving activities embedded with computational concepts and challenges that encourage logical reasoning, pattern recognition, algorithmic thinking, and decomposition. Bebras is an

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activity specifically designed to train students' computational thinking abilities (Ayub *et al.,* 2021; Dagiene *et al.,* 2022; Izu *et al.,* 2017; Zamzami *et al.,* 2020). The use of mathematical problems containing Bebras task to train students' computational thinking skills is an appropriate solution. This is because each mathematical problems containing Bebras task contains all five aspects of computational thinking (Mufidah, 2018). Computational thinking requires proficiency in calculation, problem formulation, and solving complex mathematical calculations. This proficiency is known as logical mathematical intelligence (Faizah, 2017). Logical mathematical intelligence is defined as the ability to analyze and solve mathematical problems using logic and structured reasoning, which directly supports students' mastery of computational thinking skills. Additionally, logical thinking ability is one of the competency standards for vocational high school graduates (Permendiknas, 2006). Therefore, if you want to train students' computational thinking skills, it is important to consider their logical mathematical intelligence (Yuliana, 2015).

Based on the description above, in this era of digitalization, teachers should take advantage of technological and information developments through CT-based mathematics learning using mathematical problems containing Bebras tasks. However, prior to that, teachers need to consider students' logical mathematical intelligence (Khotijah, 2016). The use of mathematical problems containing Bebras tasks is not a new concept. The research result by Nuvitalia *et al.* (2020) describe that the use of mathematical problems containing Bebras tasks can enhance students' computational thinking abilities. However, there is no empirical and definitive information stating the contribution of logical mathematical intelligence to students' computational thinking abilities in solving mathematical problems containing Bebras tasks. Additionally, Mufidah (2018) mentions that there are differences in the profiles of students' computational thinking based on their levels of logical mathematical intelligence. The research does not clearly reveal the extent of that contribution. Therefore, the author raises the issue of the profile of computational thinking abilities of vocational high school students in terms of students' logical mathematical intelligence.

This study aims to describe students' computational thinking skills in solving mathematical problems involving Bebras tasks based on their levels of logical-mathematical intelligence and to determine the extent of its contribution. Accordingly, the research questions are: (1) What is the profile of vocational high school students' computational thinking skills in terms of their logical-mathematical intelligence? (2) To what extent does logical-mathematical intelligence contribute to these skills in solving mathematical problems involving Bebras tasks?

Computational Thinking

Computational thinking (CT) is a thinking process that involves computational steps or algorithms to obtain problem-solving (Angeli & Giannkos, 2020 and Threekunprapa & Yasri, 2020). Computational thinking has the characteristic of breaking down problems into simpler components. Another perspective, presented by Ian Horswill, states that computational thinking is an algorithmic approach to finding solutions of problems based on given input (Budiyanto *et al.*, 2022 & Dagiene *et al.,* 2017). This viewpoint suggests that computational thinking is a problem-solving skill that emphasizes logical reasoning. Angeli & Giannkos (2020) and Rahmawati *et al.* (2024)

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describe the five components of computational thinking, which include decomposition, pattern recognition, abstraction/generalization, algorithms, and debugging. Decomposition is a method of breaking down a problem into simpler parts, making it easier to understand (Purwasih *et al.,* 2020 & Izu *et al.,* 2017). Pattern recognition involves identifying patterns, equations, relationships, and utilizing known information. Algorithmic thinking is a method of obtaining a solution using sequential steps (Purwasih *et al.,* 2020). Abstraction involves selecting relevant information to use and disregarding unnecessary information (Prahmana *et al.,* 2024). Debugging is the process of reviewing and checking the problem-solving process to ensure correctness and accuracy (Sondakh *et al.,* 2020). To avoid overlapping interpretations of CT components, the integration of CT in this study, through mathematical problems involving Bebras tasks, does not necessarily have to include all five CT components simultaneously. For example, decomposition the process of breaking down complex problems into smaller, manageable parts can be emphasized when students are required to analyze multi-step Bebras problems. The integration can involve only one or several components, depending on the mathematical topic and the context of the Bebras tasks used.

Mathematical Problems Containing Bebras Tasks

In mathematics learning, mathematical problems containing Bebras task are used to improve students; computational thinking skills. Bebras refers to an activity specifically designed to train students' computational thinking skills (Ayub *et* al., 2021; Bavera *et al.,* 2020; Izu *et al.,* 2017; Yuliana, 2015). The term "Bebras" is derived from the Lithuanian word for "beaver" (Rahaju *et al.,* 2019). The beaver is used as a symbol of challenge because in their daily activities, beavers strive hard to achieve their goals perfectly (Mufidah, 2018). The purpose of organizing the Bebras competition in international level is to promote computational thinking and problem-solving skills Kalelloglu *et al*., 2022 & Khotijah, 2016). Each question in Bebras encompasses the five aspects of computational thinking (Mufidah, 2018) and is presented in the form of descriptions accompanied by images to facilitate students' understanding of the problems. Since each question incorporates the five components of computational thinking, students are required to think algorithmically in solving them (Hubwieser & Mühling, 2014).

Logical Mathematics Intelligence

In this study, logical-mathematical intelligence is defined as the ability to manipulate algebraic expressions, recognize patterns, and apply logical reasoning in solving mathematical problems (Kasus, 2014). This scope was chosen because these aspects form the core of logical-mathematical intelligence as described by experts and are most relevant to the context of the Bebras tasks and the mathematics content used in this study (Suarca *et al*., 2016). Logical-mathematical intelligence itself refers to the ability to conduct scientific investigations and analyze problems logically in order to discover or create mathematical formulas based on logical rules, numerical pattern analysis, and logical thinking in problem-solving (Dwita *et al*., 2022; Khotijah, 2016). By focusing on

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these aspects, this research aims to more precisely explore the relationship between logical-mathematical intelligence and students' computational thinking abilities. Another perspective is presented by Walters, the important aspects of logical-mathematical intelligence include the ability to: perform mathematical calculations, think logically, solve problems, use deductive and inductive reasoning, and recognize patterns or relationships (Dwita *et al*., 2022). From these explanations, it can be concluded that logical-mathematical intelligence is the ability to understand numerical concepts, perform mathematical calculations, solve problems, and think logically (Wulandari & Fatmahanik, 2020).

Center of Excellence Vocational School

The Center of Excellence Vocational School is an expansion of a Vocational School that includes certain expertise programs through collaboration with the World of Work Industry. Vocational School with this program is also used as a foothold and center for improving the quality of performance for other vocational schools (Suherman *et al.*, 2022). In the learning process, the curriculum used at the Center of Excellence Vocational School involves the professional world and is competency-oriented according to the interests of the students' vocational interest. Pudyastuti *et al.* (2022) explained that the existence of a Center of Excellence Vocational School is aimed at producing graduates who are ready to enter the workforce or become entrepreneurs through a comprehensive alignment of vocational education.

METHODS

Research Design

The research employs a mixed-methods approach, defined as a procedure for collecting and analyzing data using both qualitative and quantitative methods (Masrizal, 2021; Am *et al*., 2015). In this study, qualitative analysis is used to understand the underlying reasons for the relationship between logical-mathematical intelligence and computational thinking, while quantitative analysis is used to investigate whether logical-mathematical intelligence correlates with computational thinking. The qualitative approach is used to describe the profile of students' computational thinking skills in solving mathematical problems containing Bebras tasks in terms of their level of logical-mathematical intelligence. A quantitative approach, specifically simple linear regression, is used to determine the extent to which logical-mathematical intelligence contributes to students' computational thinking skills in solving mathematical problems involving Bebras tasks. This research employs two variables, namely the independent (predictor) variable and the dependent (criterion) variable. In this research, the independent variable is the level of students' logical-mathematical intelligence, and the dependent variable is students' computational thinking skills.

Subject

The study population consisted of 137 tenth-grade students specializing in automotive studies at

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the Center of Excellence Vocational School in Bali Province, enrolled in the 2022/2023 academic year. This population was chosen because it is based on lessons in the automotive vocational field which are dominated by mathematics subjects (Mirati, 2015). Then the sample selection was carried out using Non-Probability Sampling technique, namely Purposive Sampling. Purposive sampling is known as a sampling technique based on the considerations of the researcher or evaluator by looking at which sample is the most useful and representative (Telaumbanua, 2021). This sampling process resulted in a sample of 55 tenth-grade students from the Center of Excellence Vocational School, who were selected as participants in this study.

Data Collection

There are two methods of data collection in this research, namely the test method which includes a written test of students' logical mathematical intelligence and a written test of computational thinking skills (Alanda *et al.,* 2019) and the interview method by giving questions orally (Sumartini, 2016).

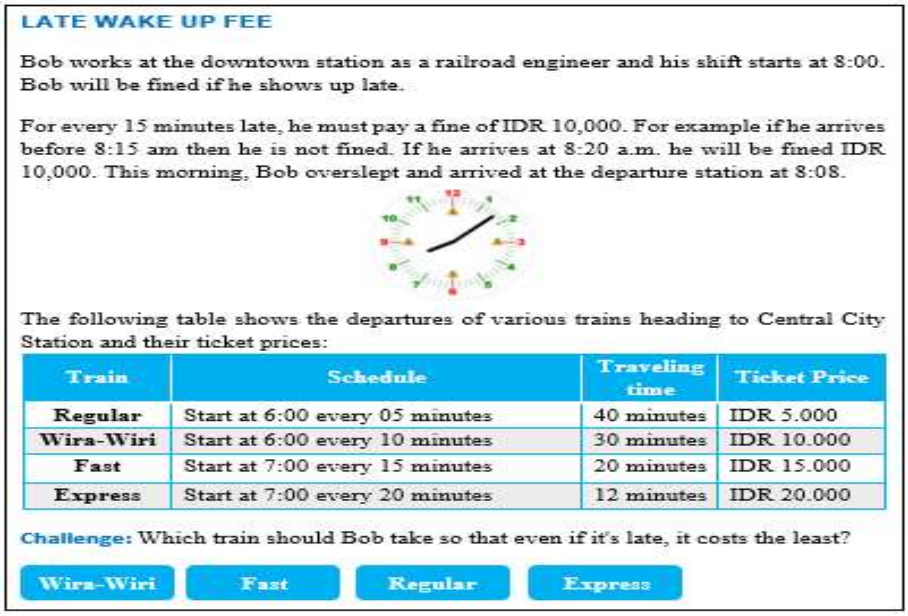
Logical mathematical intelligence in this study is defined as students' ability to understand numerical concepts, patterns, algebra, and logical reasoning. To determine whether students possess a high, moderate, or low level of logical-mathematical intelligence, a test instrument adapted from Rejeki (2021) is used. The logical-mathematical intelligence test consists of 25 objective questions to be completed within 50 minutes. Students receive 1 point for each correct answer and 0 points for incorrect answers. The test sheet includes four indicators of logical-mathematical intelligence: numerical ability, algebraic concepts, number sequences/patterns, and logic (reasoning) (Rejeki, 2021). Numerical capability is one of the indicators of logical-mathematical intelligence, which relates to the ability to process, understand, and interpret numerical data. Algebra concept ability is one of the indicators of logical-mathematical intelligence, which relates to the ability to understand, manipulate, and solve problems involving algebraic concepts. This ability includes skills in identifying variables, formulating equations or inequalities, and solving them logically and systematically. Series/pattern of number ability is one of the indicators of logical-mathematical intelligence, which relates to the ability to recognize, understand, and predict numerical patterns or sequences. This ability involves skills in analyzing regularities within a number series, determining the rule that forms the pattern, and applying that rule to identify the next or missing element. Logic (reasoning) ability is one of the indicators of logical-mathematical intelligence, which relates to the capacity to think rationally, draw conclusions both deductively and inductively, and solve problems based on logical relationships between pieces of information. This ability involves skills in analyzing premises, identifying implications, evaluating the truth of statements, and determining valid conclusions. Therefore, students are considered to possess logical-mathematical intelligence if they are able to complete or answer a set of tests encompassing the four indicators.

At first, the logical mathematical intelligence test consisted of 35 objective items. After being validated, the test was tested on 30 students. Thus, 29 valid questions, 6 invalid questions were

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obtained. At a significance level of 0.05, the critical value obtained was 0.361. Out of the 29 questions, 29 had higher r-value than r-table, indicating their validity. After analysis, the Kuder Richardson Formula 20 (KR-20) reliability coefficient was found to be 0.892. Subsequently, a differential power test was conducted on the 29 questions that were both valid and reliable. The results of the differential power test showed that 4 questions had poor discriminatory power, leaving 25 questions suitable for use as a post-test.

The instrument for the computational thinking skills test was adapted from the challenges of Bebras and modified to suit the vocational school students' domain. This instrument consists of 2 essay-type items with a time allocation of 60 minutes. The mathematical problems containing Bebras task instrument for the computational thinking skills test is presented in Figure 1.



LATE WAKE UP FEE

Bob works at the downtown station as a railroad engineer and his shift starts at 8:00. Bob will be fined if he shows up late.

For every 15 minutes late, he must pay a fine of IDR 10,000. For example if he arrives before 8:15 am then he is not fined. If he arrives at 8:20 a.m. he will be fined IDR 10,000. This morning, Bob overslept and arrived at the departure station at 8:08.

The following table shows the departures of various trains heading to Central City Station and their ticket prices:

Train	Schedule	Traveling time	Ticket Price
Regular	Start at 6:00 every 05 minutes	40 minutes	IDR 5.000
Wira-Wiri	Start at 6:00 every 10 minutes	30 minutes	IDR 10.000
Fast	Start at 7:00 every 15 minutes	20 minutes	IDR 15.000
Express	Start at 7:00 every 20 minutes	12 minutes	IDR 20.000

Challenge: Which train should Bob take so that even if it's late, it costs the least?

Wira-Wiri | Fast | Regular | Express

Figure 1. Mathematical Problems Containing Bebras Task Instrument

The scoring for each question is divided into 5 components of computational thinking, namely: decomposition, pattern recognize, algorithms, abstraction/generalization, and debugging. Each student's answer will be given a score in the range 0-3. The interview instrument that researchers used in this study was based on Sulistya (2021). The interview guide contains several questions to ask to students related to students' computational thinking skills. The interview instrument contains questions related to the five indicators of computational thinking.

Procedure

Before conducting the research, the researcher prepared instruments which included logical-mathematical intelligence instruments, computational thinking instruments, and interview instruments. The instruments that have been compiled, then validated by experts and tested first on students outside the research sample to ensure the effectiveness of the instruments.

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When researchers began to conduct research, students were first given a logical-mathematical intelligence test to group students into three categories of logical-mathematical intelligence, namely high (S1), medium (S2), and low (S3). After being grouped, students were given a computational thinking skills test in the form of Bebras questions. From the results of the answers to the computational thinking skills test, the researchers conducted interviews to students' answers.

The researcher tested the validity of the data by making comparisons between the results of the computational thinking skills test and the results of the interviews. If the two data that have been collected have the same tendency, then the data can be said to be valid. Based on the answers to the computational thinking test and interview results, 3 subjects were purposively selected to represent students' abilities at each level of logical-mathematical intelligence. Then an analysis of the answers of the three subjects was carried out and outlined in the form of a description. Finally, to see the contribution, a simple linear regression test was carried out to describe the relationship of one independent variable (level of logical-mathematical intelligence) with one dependent variable (computational thinking skills).

Data Analysis

The researcher conducted a qualitative analysis to describe the profile of students' computational thinking skills. Prior to qualitative data analysis, the data on students' logical intelligence test results were categorized into three levels with reference to the opinion of Sudjana (2013), which classifies them into three categories: high, medium, and low.

Quantitative analysis was carried out using simple linear regression to see the contribution of logical-mathematical intelligence to students' computational thinking abilities. Data on the results of logical mathematical intelligence tests and computational thinking skills tests were analyzed using descriptive statistical tests. Then a classical assumption test was carried out with the help of SPSS version 25. This was intended so that parametric statistical tests could be applied. This test includes: normality test, linearity and significance of the regression direction, multicollinearity, heteroscedasticity, and autocorrelation. After the classical assumption test is fulfilled, a hypothesis test is carried out with simple linear regression. Guidelines for providing an interpretation of the correlation coefficient R that has been obtained refers to the opinion (Sugiyono, 2016) in Table 1.

Correlation Coefficient	Interpretation
0.00	Nothing
0.00 – 0.19	Very Low
0.20 – 0.39	Low
0.40 – 0.59	Medium
0.60 – 0.79	High
0.80 – 0.99	Very High
1.00	Perfect

Table 1. Guidelines for Interpreting the Correlation Coefficient

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To determine the contribution of logical mathematical intelligence to CT skills, the coefficient of determination R^2 is calculated by squaring the correlation coefficient. The research methodology described in this study can be utilized by teachers and other researchers to examine similar topics or conduct further studies to explore the profile of computational thinking and logical-mathematical intelligence, either at different educational levels or involving other relevant variables.

RESULTS

Qualitative Research Result

The qualitative findings describe students' computational thinking profiles based on their levels of logical-mathematical intelligence in solving Bebras-based mathematical problems. Computational thinking is analyzed through five indicators: decomposition, pattern recognition, algorithmic thinking, abstraction, and debugging, while logical-mathematical intelligence is classified into three levels: high, medium, and low, in order to obtain more in-depth and diverse results.

The following are the answers of students with high logical-mathematical intelligence (S1).

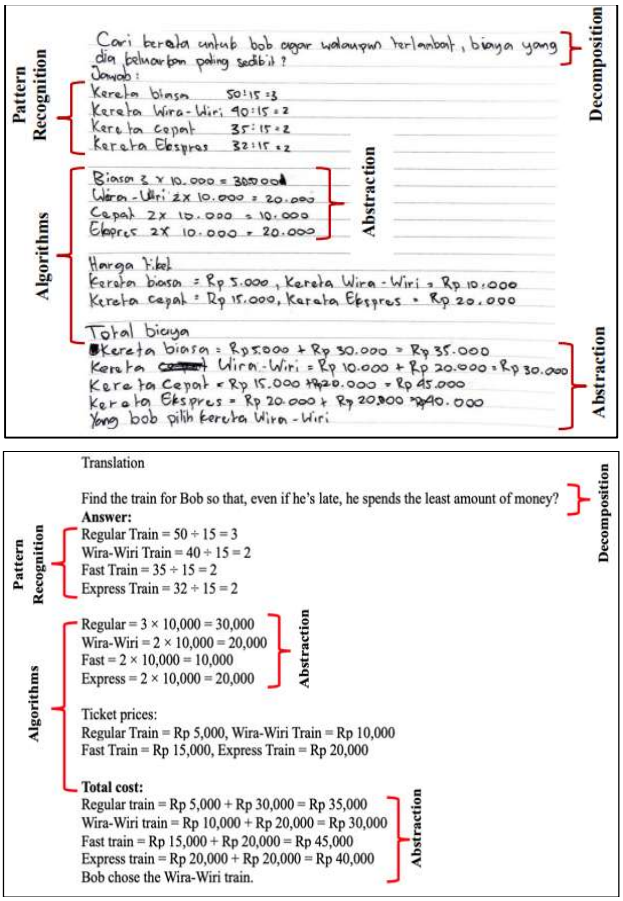


Figure 2. Answers of Students with High Logical Mathematical Intelligence (S1)

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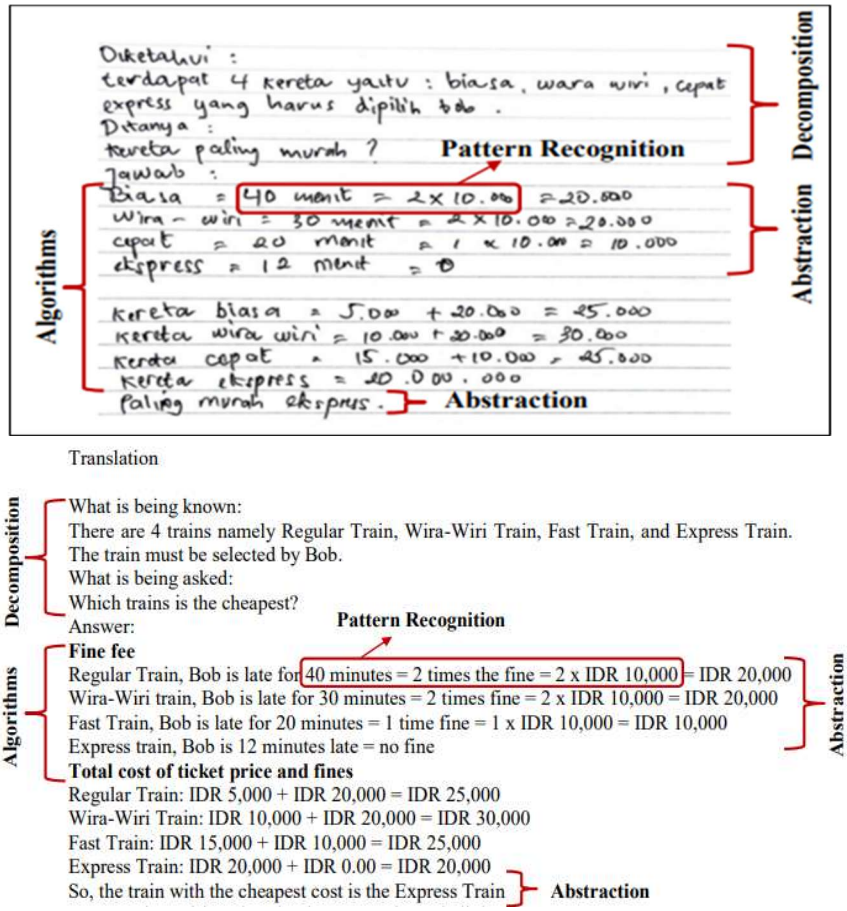
Based on the answer above, an interview is conducted to clarify the steps of solving Bebras problems according to the indicators of computational thinking. The excerpts from the interview are provided in Appendix 3.

The interview findings indicate that even though the student did not write down the known information on the answer sheet, subject S1 was able to clearly explain both the given and the required information in the Bebras questions. This indicates that subject S1 demonstrated strong decomposition skills. Based on the student's answers and the interview results, it can be concluded that the decomposition ability of students with high logical-mathematical intelligence is classified as very good, which supports their ability to solve problems involving Bebras tasks. The interview results also show that subject S1 applied the same pattern to determine Bob's fine for each train. In addition, subject S1 was able to clearly identify patterns in late fines and demonstrated a strong understanding of the rules for imposing fines. This indicates that subject S1 has mastered pattern recognition skills. Based on the student's answers and the interview results, it can be concluded that subject S1 applied appropriate solution steps. This indicates that subject S1 has a strong mastery of algorithmic thinking skills. The interview findings further indicate that subject S1 identified the train with the lowest cost, *Wira-Wiri*. Subject S1 was able to describe the identified pattern and explain how to determine the fines for each train in a consistent manner. This indicates that subject S1 has mastered the skills of abstraction and pattern generalization. Additionally, the interview results indicate that after being asked to review the answer, subject S1 was able to verify that the obtained solution was correct. This demonstrates that subject S1 has strong debugging skills.

Based on the examination and analysis of students' answers, as well as in-depth interview results, it was found that students with high logical-mathematical intelligence are able to master all five indicators of computational thinking, but tend not to write down the given and required information in the problems. Therefore, students with high logical-mathematical intelligence also demonstrate a high level of computational thinking skills in solving mathematics problems.

The following are the answers of students with moderate logical-mathematical intelligence (S2).

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Translation

Decomposition
What is being known:
There are 4 trains namely Regular Train, Wira-Wiri Train, Fast Train, and Express Train.
The train must be selected by Bob.
What is being asked:
Which trains is the cheapest?
Answer:

Pattern Recognition

Algorithms
Fine fee
Regular Train, Bob is late for 40 minutes = 2 times the fine = 2 x IDR 10,000 = IDR 20,000
Wira-Wiri train, Bob is late for 30 minutes = 2 times fine = 2 x IDR 10,000 = IDR 20,000
Fast Train, Bob is late for 20 minutes = 1 time fine = 1 x IDR 10,000 = IDR 10,000
Express train, Bob is 12 minutes late = no fine
Total cost of ticket price and fines
Regular Train: IDR 5,000 + IDR 20,000 = IDR 25,000
Wira-Wiri Train: IDR 10,000 + IDR 20,000 = IDR 30,000
Fast Train: IDR 15,000 + IDR 10,000 = IDR 25,000
Express Train: IDR 20,000 + IDR 0.00 = IDR 20,000
So, the train with the cheapest cost is the Express Train

Abstraction

Abstraction

Figure 3. Answers of Students with Moderate Logical Mathematical Intelligence (S2)

Based on the answers above, interviews were conducted in order to clarify the stages of solving Bebras questions in accordance with the indicators of computational thinking. The excerpts from the interview are provided in Appendix 4.

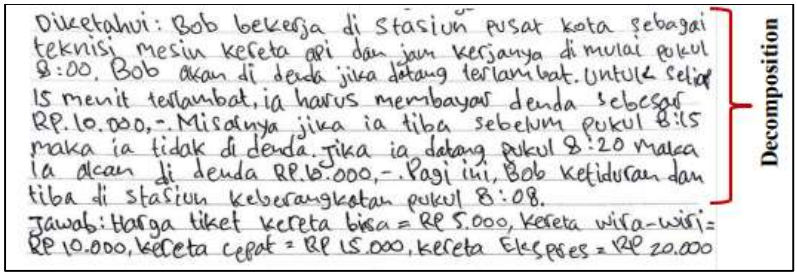
Based on the interview excerpts, it was concluded that even though students wrote information that was known and asked incompletely on the answer sheet, the S2 subject was able to explain precisely the information that was known and asked in the Bebras questions. This shows that subject S2 has medium decomposition skills. The interview excerpt also shows that subject S2 applies the same patterns in finding fines on each train. However, subject S2 has not been able to correctly mention the pattern of late fines, where subject S2 only calculates travel time without calculating train departure schedules. This shows that subject S2 has medium pattern recognition skills. In relation to the results of the answers and interviews, it was concluded that the S2 subject had used the appropriate resolution steps, although on the answer sheet they were not written clearly. This shows that subject S2 has medium algorithmic thinking skills. From the interview findings, it can be seen that subject S2 gets the train with the cheapest cost, namely the express train. Subject S2 was able to describe the patterns found; however, because the pattern identified was not quite correct, the conclusion of the answer obtained by subject S2 was also incorrect. This indicates that

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the S2 subject has not yet optimally mastered the skills of abstraction and pattern generalization. Considering the overall interview results, it can be concluded that the S2 subject did not check their answers before they were collected. In addition, after being asked to review their answer, subject S2 was not able to find mistakes and correct errors on the answer sheet. This shows that subject S2 has not mastered debugging skills well.

Based on the examination and analysis of students' answers, as well as in-depth interview results, it was found that students with moderate logical-mathematical intelligence are able to master three indicators of computational thinking, namely decomposition, pattern recognition, and algorithmic thinking, but tend to work hastily and without sufficient thoroughness when solving problems. Therefore, students with moderate logical-mathematical intelligence also demonstrate a moderate level of computational thinking skills in solving mathematical problems involving Bebras tasks.

The following are the answers of students with low logical-mathematical intelligence (S3).



Translation

What is being known:
Bob Works at the Downtown station as a railroad engineer and his shift starts at 8:00. Bob will be fined if he shows up late. For every 15 minutes late, he must pay a fine of IDR 10,000. For example, if he arrives before 8:15 am then he is not fined. If he arrives at 8:20 a.m. he will be fined IDR 10,000. This morning, Bob overslept and arrived at the departure station at 8:08.
Dekomposition

Answer:
Ticket Price
Regular Train : Rp5.000
Wira-Wiri Train : Rp10.000
Fast Train : Rp15.000
Express Train : Rp20.000

Figure 4. Answers of Students with Low Logical Mathematical Intelligence (S3)

Based on the answers above, interviews were conducted in order to clarify the stages of solving Bebras questions in accordance with the indicators of computational thinking. The excerpts from the interview are provided in Appendix 5.

Based on the interview excerpts, it was concluded that even though he only wrote down the information he knew on the answer sheet, subject S3 was able to explain exactly the information that was known and asked in the Bebras questions. This shows that subject S3 has good mastery of decomposition skills. The interview excerpt also shows that subject S3 was confused in determining the fines for each train. Then, subject S3 only used the ticket prices of each train to calculate the cheapest cost. It was concluded that subject S3 had not been able to correctly identify the pattern of late fees. This shows that subject S3 has not mastered pattern recognition skills. Drawing

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on students' responses and interview findings, in working on these questions, there was only one step taken by subject S3. To find the cheapest price, subject S3 looked at the ticket prices of each train without calculating the fines. According to subject S3, the fines for each train were the same, while the ticket prices were different. It was concluded that subject S3 had not used systematic solution steps. This shows that subject S3 has not mastered algorithmic thinking skills. Referring to the interview findings, it can be seen that subject S3 did not explain the pattern of late fees, which led to an inaccurate solution. This occurred because subject S3 was unable to recognize the existing patterns. By considering only the ticket prices, subject S3 immediately concluded that the train with the lowest cost was the Regular Train. Subject S3 assumed that late fees had no effect on the total cost, as the fine was the same for each train, IDR 10,000. This indicates that subject S3 has not mastered abstraction and pattern generalization skills. According to insights gained from the interviews, it can be concluded that subject S3 did not check the answers before submitting them. In addition, when asked to review the answers, subject S3 did not know where the error was and was unable to correct it. This shows that subject S3 has not yet mastered debugging skills.

Based on the examination and analysis of students' answers, as well as in-depth interview results, it was found that students with low logical-mathematical intelligence are only able to master one indicator of computational thinking, namely decomposition, and tend to have difficulty understanding the meaning of the questions, although their numeracy skills are fairly good. Therefore, students with low logical-mathematical intelligence also demonstrate a low level of computational thinking skills in solving mathematical problems involving Bebras tasks.

Based on the above description, the qualitative findings of this study indicate that logical-mathematical intelligence serves as the primary foundation for the development of computational thinking in mathematics learning. The higher the students' logical-mathematical intelligence, the greater their ability to break down complex problems into simpler parts, analyze patterns, develop systematic strategies, and apply mathematical concepts in a digital context. Therefore, logical-mathematical intelligence and computational thinking should be developed simultaneously in mathematics learning, so that students are not only proficient in calculations but also well-prepared to face a data-driven and automated world.

Quantitative Research Result

Previously, qualitative results indicated a relationship between logical-mathematical intelligence and computational thinking skills in mathematics learning. The following section presents the quantitative research findings, which examine the contribution of students' logical-mathematical intelligence to the improvement of their computational thinking skills in mathematics learning.

Results of Descriptive Statistics Logical Mathematical Intelligence Test Data

The central tendency of logical mathematical intelligence data obtained from the test results for 55 subjects is presented in Table 2.

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Mean	Median	Mode	Standard Deviation	Maximum Value	Minimum Value
52.11	56	32	18.88	88	20

Table 2. Central Tendency of Logical Mathematical Intelligence Test Data

Based on the mean value and standard deviation, logical mathematical intelligence data is classified into 3 categories as shown in Table 3.

Value Range	Absolute Frequency	Relative Frequency	Criteria
$X \geq 70.99$	10	18%	High
$33.23<X<70.99$	29	53%	Medium
$X \leq 33.23$	16	29%	Low

Table 3. Frequency of Data Classification Criteria for Logical mathematical Intelligence

Results of Descriptive Statistics Computational Thinking Skills Test

The central tendency of data on computational thinking skills obtained from the test results for 55 subjects are presented in Table 4.

Central Tendency					
Mean	Median	Mode	Standard Deviation	Maximum Value	Minimum Value
58.24	60	23.34	23.88	96.67	23.34

Table 4. Central Tendency of Computational Thinking Skills Test

Based on the mean value and standard deviation, computational thinking skills data is classified into 3 categories as shown in Table 5.

Value Range	Absolute Frequency	Relative Frequency	Criteria
$X \geq 82.12$	10	25.5%	High
$34.36<X<82.12$	31	56.4%	Medium
$X \leq 34.36$	14	18.2%	Low

Table 5. Frequency of Data Classification Criteria For Computational Thinking Skills

Normality Test Results

The normality test uses the Kolmogorov Smirnov test. The data is normally distributed if the

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value. The normality test results are presented in Table 6.

Testing	*Sig (2 – tailed)* K-S Value	Condition	Information
RES _ Y	0.200	*Sig (2 – tailed)* > 5%	Normally distributed

Table 6. Normality Test Result

Based on the results in the table it is known that the significance value of 0.2 is greater than 0.05. So it can be concluded that the data is normally distributed.

The Result of the Linearity and The Significance of The Regression Direction

The relationship between variables is linear if the value. The significance value is obtained from the SPSS output results in the Deviation from Linearity line ANOVA table. The following presents the results of the linearity test in Table 7.

Independent Variable	Dependent Variable	Sig. Deviation from Linearity	Information
X	*Y*	0.149	Linear

Table 7. Linearity Test Results

Based on the results in the table it is known that the Deviation from Linearity Significance value is 0.149 greater than 0.05. So, it can be concluded that there is a linier relationship between logical mathematical intelligence and computational thinking skills

Multicollinearity Test Results

In the regression model, it is preferable to avoid multicollinearity. The criteria for variables that do not exhibit multicollinearity are if the variables have values *VIF<10* and tolerance>*0.1*. The results of the multicollinearity test are presented in Table 8.

Independent Variable	Dependent Variable	Tolerance	VIF	Information
X	*Y*	1.00	1.00	Does not contain multicollinearity

Table 8. Multicollinearity Test Results

Based on the results in the table it is known that the VIF value of 1.00 is less than 10 and the Tolerance value of 1.00 is greater than 0.1. So, it is concluded that the variable does not contain multicollinearity.

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Heteroscedasticity Test Results

In the heteroscedasticity test, the Glejser test was used. The criteria for data that does not have heteroscedasticity if the value of *Sig>5%*. The following presents the results of the heteroscedasticity test in Table 9.

Independent Variable	Dependent Variable	Significance Value	Information
X	$\|_\|RESY$	0.204	There is no evidence of heteroscedasticity.

Table 9. Heteroscedasticity Test Results

Based on the results, it is known that the significance value of 0.204 is greater than 0.05. So, it can be concluded that the data does not evidence of heteroscedasticity.

Autocorrelation Test Results

The autocorrelation test was carried out using the Durbin Watson test. The criteria for data that satisfy the autocorrelation test if the DW value is greater than 1.526 and less than 2.3986. The following shows the results of the autocorrelation test in Table 10.

Independent Variable	Dependent Variable	*DW*	Information
X	Y	1.846	There is no autocorrelation

Table 10. Autocorrelation Test Results

Based on the results, it is known that the Durbin Watson value is 1.846 which is greater than 1.526 and less than 2.3986. So, it can be concluded that the data does not evidence of autocorrelation.

Hypothesis Test Results

Based on the results of the one-way correlation test using SPSS, logical-mathematical intelligence (X) showed a statistically significant relationship with computational thinking skills (Y), with a significance value of less than 0.01. Because *Sig = 0.000<0.01,* so H_o was rejected. So, it can be concluded that there is a contribution of logical-mathematical intelligence to computational thinking skills.

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Simple Linear Regression Test Results

Regression Function	*R*	R^2	*Relatively Contribution*	*Effective Contribution*
$\hat{Y}$=1.139X-1.114	0.901	0.811	87.8%	71.2%

Table 11. Simple Linear Regression Test Results

The further explanation regarding the results of the simple linear regression test in Table 11 are as follows:

1. The R value obtained is 0.901, it means that the relationship between logical-mathematical intelligence and computational thinking skills is in a very strong category.
2. The value R^2 obtained is 0.811, it means that the contribution of logical-mathematical intelligence level simultaneously to computational thinking skills is 81.1%. The remaining 18.9% % is the contribution of other variables that were not examined in this research.
3. The SR value obtained is 87.8% indicates that the contribution of the logical mathematical intelligence variable to the computational thinking skillls variable, without considering other variables outside the research variables, is 87.8%.
4. The SE value obtained is 71.2% indicates that the contribution of logical mathematical intelligence variable to the computational thinking skills variable is 71.2%, while the remaining 28.8% is the contribution of other variables that were not examined in this research.
5. The obtained simple linear regression equation is $\hat{Y}$=1.139X-1.114. The constant value of -1.114 indicates that if students don't have logical mathematical intelligence (X), their computational thinking ability (Y) is -1.114. The regression coefficient of logical mathematical intelligence 1.139, indicates that for every unit increase in logical mathematical intelligence (X), the computational thinking skill (Y) of students increases by 1.139. This simple linear regression equation interprets that as the logical mathematical intelligence (X) increases, the computational thinking ability (Y) of students also increases, and vice versa.

DISCUSSION

The results of this qualitative study are important in the early stages, as differences in mathematical intelligence influence students' computational thinking (CT) skills. Therefore, in mathematics learning, it is essential to incorporate CT by providing differentiated instruction based on each student's mathematical intelligence characteristics. The findings from this qualitative research can serve as a foundation or a preliminary step for conducting a quantitative study to examine the relationship between logical-mathematical intelligence and students' computational thinking skills in solving mathematical problems involving Bebras tasks.

Based on the description of the research results and the analysis presented, the following conclusions can be drawn: (1) Students with high logical-mathematical intelligence are able to master all

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five indicators of computational thinking but tend not to write down the information known and the questions asked; students with moderate logical-mathematical intelligence are able to master three indicators of computational thinking, namely decomposition, pattern recognition, and algorithmic thinking, but often work in a hurry and are not thorough in solving problems; students with low logical-mathematical intelligence are only able to master one indicator of computational thinking, namely decomposition, and tend to have difficulty understanding the meaning of the questions, although their numeracy skills are quite good. (2) The level of logical-mathematical intelligence has a very high contribution to the computational thinking abilities of 10th-grade students at the Center of Excellence Vocational School in Bali. (3) The practical implications for teachers and mathematics curriculum developers based on these findings are that mathematics learning should actively foster computational thinking skills integrated into the mathematics curriculum. Logical-mathematical intelligence serves as the primary foundation for the development of computational thinking in mathematics learning.

Computational thinking is a thinking process to solve problems that leads to solutions using computational steps or algorithms (Threekunprapa & Yasri, 2020; Lockwood & Mooney, 2018). Students who possess computational thinking skills can solve problems using logical and systematic steps (Yunianto *et al.,* 2024; Zamzami *et al.,* 2020). In this study, to obtain data on students' computational thinking skills, a test containing mathematical problems embedded with Bebras tasks was administered to the research subjects (Rahmawati *et al.,* 2024; Munawarah *et al.,* 2021). The results of our analysis revealed a clear distinction in computational thinking performance among students with high, moderate, and low logical-mathematical intelligence (Aminah *et al.,* 2022; Faizah *et al.,* 2017; Mufidah, 2018).

This study show that students with high logical-mathematical intelligence successfully mastered all five computational thinking indicators. While this aligns with previous studies (Mufidah, 2018; Purwasih *et al.*, 2024), we found a unique behavioral tendency: these students often did not write down the given information or the questions, even though they demonstrated strong understanding. This observation adds a behavioral dimension that is not emphasized in earlier studies. Although Dwita *et al.* (2022) and Ersozlu *et al.* (2023) also mentioned this behavior, our research further connects it to efficiency in processing and internalizing information, which may stem from cognitive confidence. This contrasts with Wulandari & Fatmahanik (2020), who argued that such students tend to document both knowns and unknowns explicitly. Hence, our findings emphasize the variability in strategy even among high performers, pointing to possible instructional adjustments.

Students with moderate logical-mathematical intelligence demonstrated mastery of three computational thinking indicators, decomposition, pattern recognition, and algorithmic thinking (Purwasih *et al.*, 2024; Setiarini *et al.*, 2023; Threekunprapa & Yasri, 2020). This corroborates existing literature (Nuvitalia *et al.*, 2022; Mufidah, 2018), yet our study offers a new insight: these students frequently rushed through the tasks and lacked thoroughness, leading to incomplete answers. While Faizah *et al.* (2017) acknowledged limitations in logical analysis among moderate students, our research brings attention to process-related behaviors, such as time mismanagement and inadequate attention to task structure. This behavioral tendency weakens their computational thinking implementation despite moderate intelligence levels. Such a pattern was also noted in Aminah *et*

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al. (2022), yet our study emphasizes its instructional implications more directly.

As for students with low logical-mathematical intelligence, our research confirms they generally master only the decomposition aspect of computational thinking (Angeli & Giannakos, 2020 & Tresnawati *et al.*, 2020). Consistent with Faizah *et al.* (2017), these students exhibited low interest in numerical and analytical tasks. However, a distinct contribution from our findings is that while some of these students possessed reasonable numeracy skills (Khotijah, 2016), they struggled to interpret verbal information in the problems, leading to significant comprehension gaps (Asmal, 2020 & Izu *et al.,* 2017). This highlights a disconnection between procedural skills and semantic understanding. Our study, therefore, not only confirms prior conclusions but also adds depth by highlighting this semantic barrier as a critical challenge for low-ability students.

These findings are generally supported by previous research. For instance, studies by Faizah *et al.* (2017), Pudyastuti *et al.* (2022), and Saputra & Zulmaulida (2021) confirm the influence of logical-mathematical intelligence on computational thinking skills. Similarly, Lockwood & Mooney (2018) and Setiarini *et al.* (2023) support the use of Bebras tasks in developing these skills. However, some research (Dwita *et al.,* 2022; Mufidah, 2018; Purwasih *et al.*, 2024) suggests that other cognitive factors may also play significant roles. In light of these perspectives, our study provides a distinctive contribution by linking cognitive levels with observable problem solving behavior such as skipping written elements or working hastily thereby offering a more behaviorally grounded instructional lens. Thus, while our findings align with the dominant discourse, they also enrich it by emphasizing the behavioral nuances associated with each intelligence level and their pedagogical implications.

CONCLUSIONS

Based on the description of the research results and the analysis presented, it can be concluded that students with high logical-mathematical intelligence are able to master all five indicators of computational thinking, although they tend not to explicitly write down the known and unknown information in the questions. Students with moderate logical-mathematical intelligence are able to master three indicators (decomposition, pattern recognition, and algorithmic thinking), but often work hastily and lack thoroughness in solving problems. Meanwhile, students with low logical-mathematical intelligence are able to master only one indicator, namely decomposition, and tend to have difficulty understanding the meaning of the questions, although their numeracy skills are relatively good. Logical-mathematical intelligence serves as the primary foundation for the development of computational thinking in mathematics learning. The higher the level of students' logical-mathematical intelligence, the greater their ability to break down complex problems into simpler components, analyze patterns, design systematic strategies, and apply mathematical concepts in a digital context. Therefore, logical-mathematical intelligence and computational thinking should be developed simultaneously in mathematics learning so that students are not only proficient in calculation but also prepared to face a data-driven and automated world.

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Teachers should first diagnose students' logical-mathematical intelligence before integrating computational thinking into mathematics learning. Furthermore, teachers should adopt different approaches for students with high, moderate, and low levels of logical mathematical intelligence to effectively develop computational thinking skills in mathematics lessons. For example, students with high logical-mathematical intelligence may benefit from more complex and exploratory computational tasks, while those with moderate or low intelligence levels may require structured guidance and scaffolded support. Based on this study, future researchers are encouraged to examine other factors that influence computational thinking skills in mathematics learning, such as 21st-century skills. Specifically, they could investigate the profile of computational thinking in terms of critical thinking, creativity, collaboration, and communication, either partially or holistically, in the context of mathematics learning. Such research could provide a broader understanding of how these skills interact and contribute to students' ability to solve mathematical problems using computational thinking approaches. Thus, this study has successfully described students' computational thinking skills in solving mathematical problems involving Bebras tasks based on their level of logical-mathematical intelligence, as well as determined the extent to which logical-mathematical intelligence contributes to students' computational thinking skills. The results of this study are recommended as a guide for mathematics teachers to apply its findings by integrating computational thinking into mathematics learning through problem-solving using Bebras-based tasks.

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REFERENCES

[1] Alanda, Y., Mustangin & Hasana, S. N. (2019). Kemampuan pemecahan masalah dan berpikir kritis matematis melalui model flipped classroom dengan media edmodo pada materi bangun ruang sisi datar. *Jp3*, *14*(6), 24–32.

[2] Am, A. P., Faan, F., White, H., Hons, F. B., Bsn, S. S., Apostolo, J. & Ma, P. K. (2015). A mixed-methods approach to systematic reviews. https://doi.org/10.1097/XEB.0000000000000052.

[3] Aminah, N., Sukestiyarno, Y. L., Wardono, W. & Cahyono, A. N. (2022). Computational thinking process of prospective mathematics teacher in solving diophantine linear equation problems. *European Journal of Educational Research, 11*(3), 1495–1507. https://doi.org/10.12973/eu-jer.11.3.1495.

[4] Angeli, C. & Giannakos, M. (2020). Computational thinking education: Issues and challenges. In Computers in Human Behavior. https://doi.org/10.1016/j.chb.2019.106185.

[5] Asmal, M. (2020). Pengaruh kecerdasan logis matematis terhadap kemampuan pemecahan masalah siswa kelas vii smpn 30 makassar. *ELIPS: Jurnal Pendidikan Matematika, 1*(1), 30–36. https://doi.org/10.47650/elips.v1i1.122.

[6] Ayub, M., Wijanto, M. C., Adelia, A., Panca, B. S., Edi, D., Kasih, J., Toba, H., Risal, R., Christianti, M., Tan, R. & Surjawan, D. J. (2021). Service learning in teachers and students mentoring for 2020 bebras challenge in pandemic era at maranatha bebras bureau christian university. *Journal of Innovation and Community Engagement, 2*(2), 75–88. https://doi.org/10.28932/jice.v2i2.3802.

[7] Bavera, F., Quintero, T., Daniele, M. & Buffarini, F. (2020). Computational thinking skills in primary teachers: evaluation using bebras. *Communications in Computer and Information Science, 1184 CCIS(May),* 405–415. https://doi.org/10.1007/978-3-030-48325-8_26.

[8] Budiyanto, C. W., Fenyvesi, K., Lathifah, A. & Yuana, R. A. (2022). Computational thinking development: benefiting from educational robotics in stem teaching. *European Journal of Educational Research, 11*(4), 1997–2012. https://www.researchgate.net/profile/Suntonrapot-Damrongpanit/publication/356662582_Effects_of_Mindset_Democratic_Parenting_Teaching_and_School_Environment_on_Global_Citizenship_of_Ninth-grade_Students/links/61a6dda685c5ea51abc0f7b6/Effects-of-Mindset-Dem.

[9] Dagiene, V., Sentance, S. & Stupuriene, G. (2017). Developing a two-dimensional categorization system for educational tasks in informatics. *Informatica (Netherlands), 28*(1), 23–44. https://doi.org/10.15388/Informatica.2017.119.

[10] Dwita, R., Muchtadi, M. & Risalah, D. (2022). Kecerdasan logis-matematis terhadap

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kemampuan pemecahan masalah pada penyelesaian soal cerita materi spldv siswa kelas x multimedia smkn hulu gurung. *JagoMIPA: Jurnal Pendidikan Matematika Dan IPA, 2*(2), 1–14. https://doi.org/10.53299/jagomipa.v2i2.176.

[11] Ersozlu, Z., Swar, M. & Skourdoumbis, A. (2023). Developing computational thinking through mathematics: an evaluative scientific mapping. April, 0–16. https://doi.org/10.3390/educsci13040422.

[12] Faizah, F., Sujadi, I., Setiawan, R. (2017). Proses berpikir siswa kelas vii e dalam memecahkan masalah matematika pada materi pecahan ditinjau dari kecerdasan logis-matematis. *Jurnal Pendidikan Matematika, 1*(4), 15–25.

[13] Farib, P. M., Ikhsan, M. & Subianto, M. (2019). Proses berpikir kritis matematis siswa sekolah menengah pertama melalui discovery learning. *Jurnal Riset Pendidikan Matematika, 6*(1), 99–117. https://doi.org/10.21831/jrpm.v6i1.21396

[14] Helsa, Y., Turmudi., & Juandi, D. (2023). TPACK-based hybrid learning model design for computational thinking skills achievement in mathematics. *Journal on Mathematics Education, 14*(1), 225-252. http://doi.org/10.22342/jme.v14i2.pp225-252.

[15] Hubwieser, P. & Mühling, A. (2014). Playing pisa with bebras. *ACM International Conference Proceeding Series*, November, 128–129. https://doi.org/10.1145/2670757.2670759.

[16] Indahsari, A. T. & Fitrianna, A. Y. (2019). analisis kemampuan pemecahan masalah siswa kelas x dalam menyelesaikan spldv. *JPMI (Jurnal Pembelajaran Matematika Inovatif), 2*(2), 77. https://doi.org/10.22460/jpmi.v2i2.p77-86

[17] Izu, C., Mirolo, C., Settle, A., Mannila, L. & Stupuriene, G. (2017). Exploring mathematical problems containing bebras task content and performance: a multinational study. *Informatics in Education, 16*(1), 39–59. https://doi.org/10.15388/infedu.2017.03.

[18] Kalelİoğlu, F., Doğan, D. & Gülbahar, Y. (2022). Snapshot of computational thinking in turkey: a critique of 2019 bebras challenge. *Informatics in Education, 21*(3), 501–522. https://doi.org/10.15388/infedu.2022.19.

[19] Kasus, S., Siswa, P., Xi, K., Ma, I. P. A., Huda, M., Irvaniyah, I. & Akbar, R. O. (2014). Analisis kecerdasan logis matematis dan kecerdasan linguistik siswa berdasarkan jenis kelamin. *3*(1).

[20] Khotijah, S. (2016). Pengaruh kecerdasan logis matematis dan kecerdasan analitik terhadap kemampuan peserta didik dalam menggambar grafik fungsi eksponensial kelas x sma negeri 13 semarang tahun pelajaran 2015/2016. 10–47.

[21] Lockwood, J. & Mooney, A. (2018). Developing a computational thinking test using bebras problems. *CEUR Workshop Proceedings*, 2190(September).

[22] Masrizal. (2021). Mixed method research. *Jurnal Kesehatan Masyaraka, 6*(1), 53–56.

[23] Miftah, R., Kurniawati, L., & Musa, K. F. (2024). Development of prospective teacher

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student worksheets through interactive case-based learning model assisted by cublend app to improve mathematical literacy skills. *Mathematics Teaching Research Journal Early Spring 2024, 16*(4), 76-93.

[24] Mirati, L. (2015). Analisis kesulitan belajar matematika pada topik logika pada siswa smk muhammadiyah 3 klaten utara. *Jurnal Pendidikan Matematika, 2*(1), 25–40. https://123dok.com/document/qopo7lmz-analisis-kesulitan-belajar-matematika-topik-logika-muhammadiyah-klaten.html.

[25] Mufidah, I. (2018). Profil berpikir komputasi dalam menyelesaiakan bebras task ditinjau dari kecerdasan logis matematis siswa. *Skripsi*, November, 1–110. http://digilib.uinsby.ac.id/id/eprint/28697.

[26] Munawarah, Thalhah, S. Z., Angriani, A. D., Nur, F., & Kusumayanti, A. (2021). Development of instrument test computational thinking skills ijhs/jhs based rme approach. *Mathematics Teaching Research Journal, 13*(4), 202-220

[27] Nuvitalia, D., Saptaningrum, E., Ristanto, S. & Putri, M. R. (2022). Profil kemampuan berpikir komputasional (computational thinking) siswa smp negeri se-kota semarang tahun 2022. *13*(2), 211–218. https://doi.org/10.26877/jp2f.v13i2.12794

[28] Pambudi, D. S., Sunardi, Kurniati, D., & Lestari, N. D. S. (2023). The mathematics prospective teachers activities when solving outdoor learning mathematics projects in the campus garden. *Mathematics Teaching Research Journal Early Spring 2023, 15*(3), 19-33.

[29] Persson, H. I. (2022). Has computational thinking a place in the curriculum? September.

[30] Prahmana, R. C. I., Kusaka, S., Peni, N. R. N., Endo, H., Azhari, A., & Tanikawa, K. (2024). Cross-cultural insights on computational thinking in geometry: indonesian and japanese students' perspectives. *Journal on Mathematics Education, 15*(2), 613-638. http://doi.org/10.22342/jme.v15i2.pp613-638.

[31] Pudyastuti, E., Ginting, R. S. & Ginting, M. (2022). Sosialisasi program smk pusat keunggulan pada smk immanuel. *PUBARAMA: Jurnal Publikasi Pengabdian Kepada Masyarakat, 2*(1), 35–38.

[32] Puente, M. D. L., & Perez, H. (2023). Assessing the impact of brilliant.org on enhancing mathematics academic performance among high school students in colombia: a quasi-experimental study. *Mathematics Teaching Research Journal Early Spring 2023, 15*(2), 82-103.

[33] Purwasih, R., Turmudi, & Dahlan, J. A. (2024). How do you solve number pattern problems through mathematical semiotics analysis and computational thinking?. *Journal on Mathematics Education, 15*(2), 403-430. http://doi.org/10.22342/jme.v15i2.pp403-430.

[34] Rafiepour, A., & Farsani, D. (2021). Cultural historical analysis of iranian school mathematics curriculum: the role of computational thinking. *Journal on Mathematics Education, 12*(3), 411-426. http://doi.org/10.22342/jme.12.3.14296.411-426.

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[35] Rahaju, R., Purwanto, P., Parta, I. N. & Rahardjo, S. (2019). Students' critical thinking skills in making mathematical problems. *Journal of Physics: Conference Series, 1318*(1). https://doi.org/10.1088/1742-6596/1318/1/012094.

[36] Rahani, F. F. & Jones, A. H. S. (2020). Pelatihan computational thinking dan lomba bebras untuk guru dan siswa Sekolah Dasar se- Bantul. *Seminar Nasional Pengabdian Kepada Masyarakat*, 0(0), 851–858. https://ojs.unm.ac.id/semnaslpm/article/view/18284.

[37] Rahmawati, R. D., Sugiman., Wangid, M. N., & Purnomo, Y. W. (2024). Designing model of mathematics instruction based on computational thinking and mathematical thinking for elementary school student. *Mathematics Teaching Research Journal Early Spring 2024, 16*(1), 143-166.

[38] Rejeki, W. S. (2021). Analisis keterampilan metakognitif siswa kelas XI SMAN 2 Sukoharjo dalam memecahkan masalah matematika tipe hots pada materi turunan fungsi aljabar ditinjau dari kecerdasan logis matematis. 00(00), XX–XX. https://doi.org/10.1007/XXXXXX-XX-0000-00

[39] Saputra, E. & Zulmaulida, R. (2021). Implementation of reflective thinking process approach to students' mathematical critical thinking. *Journal of Physics: Conference Series, 2123*(1). https://doi.org/10.1088/1742-6596/2123/1/012033.

[40] Setiarini, T., Lisnawati, I., Prastyo, T. D., Studi, P., Informatika, P. & Komputer, P. (2023). Analisis berpikir komputasional mata pelajaran informatika siswa kelas X DPIB SMK Negeri 1 Pacitan pada kurikulum merdeka. *Edumatic: Jurnal Pendidikan Matematika, 4*, 39–46.

[41] Sondakh, D. E., Osman, K. & Zainudin, S. (2020). A proposal for holistic assessment of computational thinking for undergraduate: content validity. *European Journal of Educational Research, 9*(1), 33–50. https://doi.org/10.12973/eu-jer.9.1.33.

[42] Suarca, K., Soetjiningsih, S. & Ardjana, I. E. (2016). Kecerdasan majemuk pada anak. Sari Pediatri, 7(2), 85. https://doi.org/10.14238/sp7.2.2005.85-92.

[43] Suherman, A. I., Suharyanto, S. & Sauri, S. (2022). Manajemen program penyelarasan kurikulum SMK 2013 dengan Industri, dunia usaha dan dunia kerja (IDUKA) dalam meningkatkan keterserapan tenaga kerja lulusan SMK Kota Bandung. *JIIP - Jurnal Ilmiah Ilmu Pendidikan, 5*(2), 460–465. https://doi.org/10.54371/jiip.v5i2.430.

[44] Sulistya, H. (2021). Analisis kemampuan berpikir komputasi siswa kelas VII A SMP Pangudi Luhur ST. Vincentius Sedayu Tahun ajaran 2020/2021 dalam menyelesaikan soal bebras task pada materi perbandingan.

[45] Sumartini, T. S. (2016). Kemampuan pemecahan masalah matematika dalam menyelesaikan soal cerita berdasarkan langkah polya. *Jurnal Pendidikan Matematika STKIP Garut, 5*(2), 1–7.

[46] Telaumbanua, Y. N. (2021). Peranan matematika terhadap kewirausahaan. *Intelektium,*

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2(1), 89–98. https://doi.org/10.37010/int.v2i1.383.

[47] Threekunprapa, A. & Yasri*, P. (2020). Patterns of computational thinking development while solving unplugged coding activities coupled with the 3s approach for self-directed learning. *European Journal of Educational Research, 9*(3), 1025–1045. https://doi.org/10.12973/eu-jer.9.3.1025.

[48] Tresnawati, D., Latifah, A., Nashrulloh, M. R., Fitriani, L., Rahayu, S., Mulyani, A., Cahyana, R., Satria, E., Setiawan, R., Septiana, Y. & Kurniadi, D. (2020). Edukasi cara berpikir komputasi melalui tantangan bebras 2020 di Garut. *Jurnal PkM MIFTEK, 1*(2), 181–186. https://doi.org/10.33364/miftek/v.1-2.181\

[49] Wang, P., Zhu, F., Song, H., & Hou, J. (2017). A Bibliometric Retrospective of the journal Eurasia journal of mathematics, science and technology education between 2012 and 2017. EURASIA Journal of Mathematics, Science and Technology Education, 14(3), 765-775. https://doi.org/10.12973/ejmste/80911.

[50] Wulandari, L. & Fatmahanik, U. (2020). Kemampuan berpikir logis matematis materi pecahan pada siswa berkemampuan awal tinggi. *Laplace: Jurnal Pendidikan Matematika, 3*(1), 43–57.

[51] Yuliana, E. (2015). Pengembangan Soal Open-Ended pada Pembelajaran Matematika untuk Mengidentifikasi Kemampuan Berfikir Kreatif Siswa. *In Prosiding Seminar Nasional Pendidikan Matematika (SNAPTIKA)..*

[52] Yunianto, W., Lavicza, Z., Kastner-Hauler, O., & Houghton, T. (2024). Investigating the use of ChatGPT to solve a GeoGebra based mathematics+computational thinking task in a geometry topic. *Journal on Mathematics Education*, *15*(3), 1027–1052. https://doi.org/10.22342/jme.v15i3.pp1027-1052.

[53] Zamzami, E. M., Tarigan, J. T., Zendrato, N., Muis, A., Yoga, A. P. & Faisal, M. (2020). Exercising the students computational thinking ability using bebras challenge. *Journal of Physics: Conference Series, 1566*(1). https://doi.org/10.1088/1742-6596/1566/1/012113.

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APPENDIX 1

Sample Questions Based on the Four Indicators of Logical-Mathematical Intelligence

The theories and sample questions related to the four indicators of logical-mathematical intelligence used in this study are presented as follows.

Part I: Numerical Capability is one of the indicators of logical-mathematical intelligence, which relates to the ability to process, understand, and interpret numerical data. Students with strong numerical capability demonstrate a high level of logical-mathematical intelligence, whereas those with low numerical capability exhibit a lower level of logical-mathematical intelligence. An example of a logical-mathematical intelligence test item that falls under the numerical capability indicator is presented as follows.

Part I: Numerical Capability

1. If the calculation result $504 : a = 36$, then $a = \ldots\ldots$
   A. 12
   B. 14
   C. 16
   D. 19
   E. 24

The example above illustrates a condition designed to assess students' numerical capability, as indicated by their ability to calculate or determine the value of aaa as the divisor of 504 to obtain the result of 36. If students are able to solve this problem correctly, it can be concluded that they possess a high level of logical-mathematical intelligence.

Part II: Algebra Concept Ability is one of the indicators of logical-mathematical intelligence, which relates to the ability to understand, manipulate, and solve problems involving algebraic concepts. This ability includes skills in identifying variables, formulating equations or inequalities, and solving them logically and systematically. Students with strong Algebra Concept Ability demonstrate a high level of logical-mathematical intelligence in the domain of symbol- and relation-based problem-solving, whereas those with low ability in this area exhibit a lower level of logical-mathematical intelligence. An example of a logical-mathematical intelligence test item that falls under the Algebra Concept Ability indicator is presented as follows.

Part II: Algebra Concept Ability

2. If $x < 0, y < 0$ and $|x| > |y|$, the the relationship between x and y is ....
   A. $x = y$
   B. $x \neq y$
   C. $x > y$
   D. $x < y$
   E. x dan y can not be determined

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The example above presents a condition designed to assess students' Algebra Concept Ability, as indicated by their capacity to understand the relationship between two variables, x and y, by considering their signs and absolute values. This problem requires students to analyze the given information, apply the concept of absolute value, and draw the correct conclusion regarding the relationship between x and y. Successfully solving this problem demonstrates a high level of logical-mathematical intelligence in the domain of algebraic concept understanding.

Part III: Series/Pattern of Number Ability is one of the indicators of logical-mathematical intelligence, which relates to the ability to recognize, understand, and predict numerical patterns or sequences. This ability involves skills in analyzing regularities within a number series, determining the rule that forms the pattern, and applying that rule to identify the next or missing element. Students with strong Series/Pattern of Number Ability demonstrate a high level of logical-mathematical intelligence in the domain of pattern-based reasoning, whereas those with low ability in this area often struggle to identify numerical regularities. An example of a logical-mathematical intelligence test item that falls under the Series/Pattern of Number Ability indicator is presented as follows.

Part III: Series/Pattern of Numbers Ability

9. 1, 3, 7, 13, 21, 31, 43, ...
   A. 55
   B. 57
   C. 59
   D. 60
   E. 62

The example above presents a condition designed to assess students' Series/Pattern of Number Ability, as indicated by their capacity to recognize and determine numerical patterns within a sequence. This problem requires students to analyze the regularity of the given number series, identify the applicable rule of progression, and predict the next number in the sequence. Successfully solving this problem demonstrates a high level of logical-mathematical intelligence in the domain of pattern-based reasoning.

Part IV: Logic (Reasoning) Ability is one of the indicators of logical-mathematical intelligence, which relates to the capacity to think rationally, draw conclusions both deductively and inductively, and solve problems based on logical relationships between pieces of information. This ability involves skills in analyzing premises, identifying implications, evaluating the truth of statements, and determining valid conclusions. Students with strong Logic (Reasoning) Ability demonstrate a high level of logical-mathematical intelligence in the domain of general reasoning, whereas those with low ability in this area tend to struggle in connecting facts and drawing accurate conclusions. An example of a logical-mathematical intelligence test item that falls under the Logic (Reasoning) Ability indicator is presented as follows.Through the mathematical intelligence test, which includes the four components mentioned above as illustrated in the example, a comprehensive and objective mapping of students' logical-mathematical intelligence can be obtained. This

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ensures that the research results are accountable and free from bias.

Part IV: Logic (Reasoning) Ability

19. Premise 1 : Randu wood is a type of wood that is useful.
    Premise 2 : Useful wood is expensive.
    The conclusion from these two premises is: .....
    A. All very useful wood is not cheap
    B. Randu wood is expensive
    C. Randu wood is not for sale
    D. All wood that is cheap is randu wood.
    E. All wrong

The example above presents a condition designed to assess students' Logic (Reasoning) Ability, as indicated by their capacity to draw a valid conclusion based on two given premises. This problem requires students to analyze the logical relationship between the first and second premises and then determine a conclusion that is accurate and consistent with the provided information. Successfully solving this problem demonstrates a high level of logical-mathematical intelligence in the domain of logical reasoning.

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APPENDIX 2

Guidelines for Evaluating Computational Thinking Tests

Guidelines for evaluating computational thinking tests according to Sulistya, (2021) are presented in the following table.

Aspects of Computational Thinking	Assessment Criteria	Score
Decomposition	Students are able to write down or mention known information and questions asked in the questions correctly and completely.	3
	Students are able to write down or mention the information that is known and the questions asked in the questions correctly but incompletely.	2
	Students are able to write down or mention information that is known or information that is asked correctly.	1
	Students are able to write down or mention information that is known and questions that are asked in questions incorrectly.	0
Pattern Recog-nize	Students are able to recognize the patterns contained in the questions and use them to carry out appropriate solutions	3
	Students are able to recognize the pattern contained in the prob-lem and use it to carry out an almost precise solution	2
	Students are able to recognize the patterns contained in the questions and use them to carry out solutions with most of the calculation steps being inaccurate	1
	Students are not able to recognize the patterns contained in the questions and are unable to use them to make solutions that are not quite right	0
Algorithm Thinking	Students are able to write down or mention the steps of comple-tion correctly	3
	Students are able to write down or mention the completion steps almost precisely	2
	Students are able to write down or mention the completion steps with most of the calculation steps being inaccurate	1
	Students are not able to write down the steps of completion	0

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Abstraction/ Generalization	Students are able to describe the pattern found into an appropriate solution and draw conclusions from the pattern found correctly	3
	Students are able to describe the patterns found into an appropriate solution and draw conclusions from the patterns found inaccurately	2
	Students only write or mention conclusions without being accompanied by calculation steps	1
	Students do not write down or mention conclusions and do not write down the final results of calculations	0
Debugging	Students are able to ensure the correctness of the answer and correct the answer appropriately if there is an error.	3
	Students are able to ensure the correctness of the answer and correct the answer inaccurately	2
	Students are able to ensure the correctness of their answers and have not been able to correct errors in their answers	1
	Students are not able to ensure the correctness of their answers and correct them	0
	Total Score	15

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APPENDIX 3

Interview Excerpts of Students with High Logical-Mathematical Intelligence (S1)

Based on the answers of students with high logical-mathematical intelligence (S1) in Figure 2, an interview was conducted to clarify the steps involved in solving Bebras problems according to the indicators of computational thinking.

Decomposition

In Figure 2, it can be seen that subject S1 did not write down the known information but wrote down the problems asked in the questions. Then conducted interviews related to decomposition skills. The excerpts from the interview results are presented as follows.

R : What information is known in the problem?
S : Known information about fines and ticket prices, Ma'am. If Bob is late, he will be fined IDR 10,000 for every 15 minutes late. Then if Bob arrives before 8.15 he will not be fined, but if he arrives past 8.15 he will be fined. One day Bob overslept and arrived at the departure station at 8:08. To get to work, there are four trains that Bob can choose from with different ticket prices and departure times.
R : Then, state what problems was asked?
S : The question asked is which train should Bob take so that even if he is late, the expenses for fines and tickets are the least?

Based on the interview, it can be concluded that even though the student did not write down the known information on the answer sheet, the S1 subject was able to explain exactly the information that was known and asked in the Bebras questions. This shows that subject S1 mastered decomposition skills well. Based on students' answers and interview results, it can be concluded that the decomposition ability of students with high logical-mathematical intelligence is classified as very good, which helps them solve problems involving Bebras tasks.

Pattern Recognition

Figure 2 shows that S1 subject was also able to recognize the pattern contained in the question. When looking for a solution to the problem, subject S1 first looked for how many fines that Bob would get on each train. It can be seen that the method used by S1 subjects is to divide Bob's arrival time at work by 15 minutes. Then conducted interviews related to pattern recognition skills. The excerpts from the interview results are presented as follows.

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R : What was the first thing you did when answering the Bebras question?
S : First, I looked for fines on Regular Train, by finding the time of arrival at Bob's work. The Regular Train starts at 6:00 and departs every 5 minutes, while Bob arrives at the departure station at 8:08. He will be able to catch the Regular Train at 8:10, then it will take 40 minutes so he will be at work by 8:50. So Bob was 50 minutes late, so by choosing the Ordinary Train Bob was fined 3 times. Then look for other train fines using the same method ma'am.
R : Why is the Regular Train, Bob was fined 3 times?
S : Since the fine is calculated every 15 minutes late, calculate a multiple of 15 which is closer to 50.

The interview above shows that subject S1 applied the same pattern to determine Bob's fine for each train. In addition, subject S1 was able to clearly identify patterns of late fines and demonstrated a strong understanding of the rules for imposing fines. This indicates that subject S1 has mastered pattern recognition skills.

Algorithms

Figure 2 shows that S1 subject has taken systematic steps to find a solution to the problem. It can be seen that in working on the questions there are three steps carried out by S1 subject. Then conducted interviews related to algorithms skills. The excerpts from the interview results are presented as follows.

R : Describe the steps you took to solve the problem?
S : First, I searched for fines in Regular Train. The method is the same as before Ma'am. Then I look for fines for other trains using the same method. Second, I multiplied the number of fines by the cost of each fine, which is IDR 10,000. Third, add up the ticket prices with the fines for each train, ma'am and choose the train with the smallest total cost.

Based on the results of the answers and interviews, it can be concluded that subject S1 used the correct solution steps. This indicates that subject S1 has a strong mastery of algorithmic thinking skills.

Abstraction

In Figure 2, it can be seen that subject S1 can describe the patterns that have been obtained. In addition, subject S1 can also conclude the answer of the question correctly. Then conducted interviews related to abstraction skills. The excerpts from interview results are presented as follows.

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R : After compiling the steps for solving the problem, how do you conclude the answer you got?
S : So, the train chosen by Bob with the lowest total ticket cost and fine is the Wira-Wiri Train, Ma'am.
R : How do you describe the pattern you found?
S : After I got the pattern of late fines for Regular Trains, I also looked for fines on other trains using the same method. Then the number of fines on each train is multiplied by IDR 10.000

Based on the interview, it can be seen that subject S1 identified the train with the lowest cost, *Wira-Wiri*. Subject S1 was able to describe the identified pattern and explain how to determine the fines for each train in the same way. This indicates that subject S1 has mastered the skills of abstraction and pattern generalization.

Debugging

The researcher then continued the interview to see debugging skills. The excerpts from interview results are presented as follows

R: Have you checked your answers before submitting them?
S: Yes, Ma'am
R: Now, try to check again, is there a mistake in the answer you made?
S: I'll see first Ma'am (while reading the answer sheet). I think my answer is correct Ma'am and the calculations I made were also not wrong. So, I believe my answer is correct ma'am.

Based on the results of the interview, it can be concluded that after S1 subject was asked to reviewed their answer, it was seen that the S1 subject was able to ensure that the answers obtained were correct. This shows that the S1 subject has mastered debugging skills well.

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APPENDIX 4

Interview Excerpts of Students with Moderate Logical-Mathematical Intelligence (S2)

Based on the answers of students with moderate logical-mathematical intelligence (S2) in Figure 3, an interview was conducted to clarify the steps involved in solving Bebras problems according to the indicators of computational thinking.

Decomposition

In Figure 3, it can be seen that subject S2 wrote down information that was known incompletely and wrote down the problems asked in an unclear manner. Then conducted interviews related to decomposition skills. The excerpts from interview results are presented as follows.

R : What information is known in the problem?
S : Known information is about Bob who works as a mechanic and arrives late at work. Bob's working hours start at 8.00 and every 15 minutes late, Bob will be fined IDR. 10,000. If Bob comes before 8.15, he will not be fined, but if he is later than 8.15 he will be fined. In addition, the table contains information on the departure schedules and travel times of the four trains that Bob can choose to arrive at his workplace.
R : Then, try to state what problems were asked?
S : Which train requires the cheapest cost?

Based on the interview excerpts, it was concluded that even though students wrote information that was known and asked incompletely on the answer sheet, the S2 subject was able to explain precisely the information that was known and asked in the Bebras questions. This shows that subject S2 has medium decomposition skills.

Pattern Recognition

Figure 7 shows that subject S2 applies the same rules to find the number of fines in each train. The method used by S2 is to divide the travel time by 15 minutes. This shows that the S_2 subject already recognizes the pattern of giving fines on the questions, but the patterns found are not correct. Then conducted interviews related to pattern recognition skills. The excerpts from interview results are presented as follows.

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R : What was the first thing you did when answering the Bebras question?
S : At first, I was looking for fines for all four trains, Ma'am. For example, the Regular Train takes 40 minutes, then I divide by 15 minutes. Because it will be fined every 15 minutes late. So, the Regular Train will be fined 2 times, because once a fine of IDR 10,000 means 2 times a fine of IDR 20,000. For Wira-Wiri, Fast, and Express trains, the method is the same with Regular train, Ma.am.
R : Why do you divide the travel time by 15 minutes?
S : Because the travel time affects the time Bob arrives at the office

The interview excerpt above shows that subject S2 applies the same patterns in finding fines on each train. However, subject S2 has not been able to correctly mention the pattern of late fines. Where subject S2 only calculates travel time without calculating train departure schedules. This shows that subject S2 has medium pattern recognition skills.

Algorithms

Figure 3 shows subject S2 has taken systematic steps to find a solution to the problem. In working on the questions, there are two steps taken by subject S2. Then conducted interviews related to algorithms skills. The excerpts from interview results are presented as follows.

R : Describe the steps you took to solve the problem?
S : At first, I looked up how many times Bob would be charged a late fee if he departed on each train and I multiplied the number of fines by IDR 10,000. Then I added up the cost of the fine with the ticket price of each train. From the total cost of fines and ticket prices, I chose the train with the cheapest cost, Ma'am.

Based on the results of the answers and interviews, it was concluded that the S2 subject had used the appropriate resolution steps. Although in answer sheet is not written clearly. This shows that subject S2 has medium algorithmic thinking skills.

Abstraction

In Figure 3, it can be seen that subject S_2 can describe the patterns that have been obtained. However, subject S2 has not been able to conclude the answer of the question correctly. Then conducted interviews related to abstraction skills. The excerpts from interview results are presented as follows.

R : After compiling the steps for solving the problem, how do you conclude the answer you got?
S : The train with the cheapest total ticket costs and fines is the Express Train, Ma'am
R : How would you describe the patterns you found?
S : After I got how much fine Bob will get on each train, then I multiply the number of fines on each train by IDR 10,000

Based on the results of the interview above, it can be seen that subject S_2 gets the train with the

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cheapest cost is Express train. Subject S2 was able to describe the patterns found. However, because the pattern found was not quite right, the conclusion of the answer obtained by subject S2 was not correct. This indicates that the S2 subject has not yet optimally mastered the skills of abstraction and pattern generalization.

Debugging

The researcher then continued the interview to see debugging skills. The excerpts from interview results are presented as follows.

R: Have you checked your answers before submitting them?
S: Not yet, Ma'am, because I was short on time so I didn't have time to check again
R: Now, try to check again, is there a mistake in the answer you made?
S: I'll see first, Ma'am (while reading the answer sheet). I don't see the mistake, Ma'am, because I think this answer is correct.

Based on the results of the interviews, it can be concluded that the S2 subjects did not check their answers before it was collected. In addition, after being asked to reviewed their answer, subject S2 has not been able to find mistakes and correct errors on the answer sheet. This shows that subject S2 has not mastered debugging skills well.

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APPENDIX 5

Interview Excerpts of Students with Low Logical-Mathematical Intelligence (S3)

Based on the answers of students with low logical-mathematical intelligence (S3) in Figure 4, an interview was conducted to clarify the steps involved in solving Bebras problems according to the indicators of computational thinking.

Decomposition

In Figure 4, it can be seen that subject S3 only wrote down the known information but did not write down the problems asked in the questions. Then conducted interviews related to decomposition skills. The excerpts from interview results are presented as follows.

R : What information is known in the problem?
S : Bob Works at the Downtown station as a railroad engineer and his shift starts at 8:00. Bob will be fined if he shows up late. For every 15 minutes late, he must pay a fine of IDR 10,000. For example, if he arrives before 8:15 am then he is not fined. If he arrives at 8:20 a.m. he will be fined IDR 10,000. This morning, Bob overslept and arrived at the departure station at 8:08.
R : Then, try to state what problems were asked?
S : Looking for a train with the lowest cost

Based on the interview excerpts, it was concluded that even though he only wrote down the information he knew on the answer sheet, S_3 subject was able to explain exactly the information that was known and asked in the Bebras questions. This shows that subject S3 has good mastery of decomposition skills.

Pattern Recognition

Figure 4 shows that subject S3 did not write a pattern of late fines for each train. The subject of S3 is directly based on the ticket price of each train. Then conducted interviews related to pattern recognition skills. The excerpts from interview results are presented as follows.

R : What was the first thing you did when answering the Bebras question?
S : I use the ticket price for each train Ma'am.
R : Why are you using ticket prices? Then what about the fine that Bob got?
S : I think the choice of the train depends on the ticket price, ma'am. While fines are not counted

The interview excerpt above shows that subject S3 was confused in finding fines on each train. Then subject S3 only uses the ticket price of each train to calculate the cheapest cost. It was concluded that the S3 subject had not been able to correctly mention the pattern of late fines. This shows that subject S3 has not mastered pattern recognition skills.

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Algorithms

Figure 4 shows that when looking for a solution to the Bebras problem, subject S3 only looks at the ticket prices for each train in determining the train with the cheapest cost. Then conducted interviews related to algorithms skills. The excerpts from interview results are presented as follows.

R : Describe the steps you took to solve the problem?
S : I only saw the ticket price for each train Ma'am. Then I get the train with the cheapest cost.
R : Don't you add the fine fee to the ticket price?
S : I think the fine for each train is IDR 10,000. The difference is the ticket price. So I only use the ticket price.

Based on the results of the answers and interviews, in working on these questions there is one step taken by the S3 subject. To find the cheapest price, subject S3 look at ticket prices on each train, without calculating the fine. According to subject S3, the fines for each train are the same, while the ticket prices are different. It was concluded that the S3 subject had not used systematic completion steps. This shows that the S3 subject has not mastered algorithmic thinking skills.

Abstraction

In Figure 4, it can be seen that subject S3 has not been able to describe the pattern of late fines. Then conducted interviews related to abstraction skills. The excerpts from interview results are presented as follows.

R : After compiling the steps for solving the problem, how do you conclude the answer you got?
S : The train that costs the least for Bob is Regular Train, Ma'am
R : How would you describe the patterns you found?
S : Look at the ticket price, Ma'am
Regular Train = IDR 5,000, Wira-Wiri Train = IDR 10,000, Fast Train = IDR 15,000, and Express Train = IDR 20,000
So, the cheapest is the Regular Train. So, the train that Bob chose is Regular Train

Based on the interview results above, it can be seen that subject S3 did not explain the pattern of late fines, which led to an inaccurate solution to the questions. This is because subject S3 was unable to recognize the existing patterns. By looking only at the ticket prices, subject S3 immediately concluded that the train with the lowest cost was the Regular Train. Subject S3 assumed that late fees had no effect on the total train cost, as the fine was the same for each train, IDR 10,000. This indicates that subject S3 has not mastered the skills of abstraction and pattern generalization.

Debugging

The researcher then continued the interview to see debugging skills. The excerpts from interview results are presented as follows.

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R: Have you checked your answers before submitting them?
S: Not yet ma'am, because my time is up
R: Now, try to check again, is there a mistake in the answer you made?
S: I'll see first Ma'am (while reading the answer sheet). Sorry Ma'am, I don't know where is the mistake from my answer

Based on the results of the interviews, it can be concluded that the S3 subject did not check their answers before submitting them. In addition, after being asked to review the answers, the S3 subject did not know where the error was and was unable to correct it. This shows that the S3 subject has not yet mastered debugging skills.

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