

Characterizing the Reasoning of Moroccan Students in Problem-solving Tasks: the Case of Inductive Reasoning

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Abstract: This study provides a detailed characterization of inductive reasoning among Moroccan secondary school students, which remains underexplored in national and international contexts. Using an original analysis grid based on structural and processual reasoning steps, the research offers insight into how students engage with inductive thought when solving mathematical problems, revealing cognitive tendencies and curricular influences.

A test composed of five carefully designed problems was administered to elicit inductive reasoning through familiar concepts and representations. The results show that most students successfully engaged with the initial phases of reasoning, particularly observation and model identification. However, generalization was less frequently achieved. The study's originality lies in its fine-grained, step-by-step analysis of students' reasoning, highlighting the potential of inductive thinking in a context where it is often overshadowed by a dominant focus on deduction.

Keywords: Inductive reasoning, problem-solving, qualifying secondary cycle, analysis grid.

INTRODUCTION

In the Moroccan high school mathematics curriculum, several types of reasoning are introduced as foundational principles: reasoning by equivalence, contrapositive, disjunction of cases, contradiction, and mathematical induction (Pedagogical Orientations of Mathematics, Secondary Qualifying, 2014, p. 17). These reasoning modes are typically presented as formal tools for proof, often without sustained attention to how students develop them through problem-solving. Among these, deductive reasoning remains dominant, shaping the structure of both teaching and assessment (Nhiry et al., 2023).

Yet mathematical reasoning is not limited to deduction. Inductive reasoning plays a crucial role in helping learners identify patterns, make conjectures, and construct generalizations from specific

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cases. Despite its centrality in mathematical discovery and early learning, inductive reasoning often remains underrepresented in instructional practice, limiting students' capacity to explore and justify mathematical ideas flexibly (Brousseau, 2000; Duval, 1995; Lithner, 2008).

This study addresses a significant gap in literature. While existing research has explored mathematical reasoning broadly, very few studies have offered a detailed empirical analysis of how Moroccan students engage in inductive reasoning, especially through the lens of their written problem-solving processes. The originality of this study lies in its use of a custom-designed analysis grid—based on structural and processual dimensions of reasoning—to characterize how inductive thinking unfolds across multiple stages of problem-solving.

Although this work focuses primarily on student productions, it is situated within a broader reflection on the influence of curriculum and teaching practices. The didactic environment—heavily oriented toward formal proof—may shape how students approach or neglect inductive reasoning when required. Understanding how students reason inductively therefore offers key insights into cognitive processes and the curricular framework in which they operate.

This research contributes new empirical data to the field by analyzing the responses of first-year baccalaureate students to five carefully selected mathematical problems. It aims to reveal both the potential and the limits of inductive reasoning among secondary students and to open future pathways for research and teaching practices that value exploratory reasoning in mathematics.

THEORETICAL BACKGROUND

Inductive Reasoning: Historical, Epistemological, and Didactic Anchoring

David Hume (1748) challenged the validity of induction, suggesting it depends on habit rather than logical certainty. Observing events repeatedly — like the sun rising daily — doesn't logically ensure they will happen again, as the uniformity of nature cannot be proven. The assumption that the future will mirror the past is rooted purely in psychological habit.

Polya (1957) characterizes induction as “a way of reasoning that leads to combination.” Induction deals with the transition from the particular to the general, leading us to the knowledge of general truth (Arnaud & Nicole, 1965).

Osherson et al. (1990) and Sloman (1993) recognize the influence of perceptual similarity in their main theories of induction in adults, specifically in terms of overlapping structural characteristics between an inductive base and a target, which plays a major role in determining the strength of inductive inferences.

In mathematics education, inductive reasoning is defined as an *ampliative inference* leading to the construction of new knowledge from the observation of cases, generalized to a broader set (Pedemonte, 2002). Often applied locally, especially in experimental contexts, induction alone cannot account for all processes at play. A process-oriented perspective—considering actions such as

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generalizing, conjecturing, and classifying—helps identify two problem-solving approaches that are neither purely deductive nor purely inductive: the experimental approach and the successive test-adjustment approach. The experimental approach articulates phases of experimentation—conducting tests, observing results, drawing conclusions—together with the formulation and attempted proof of conjectures (Gardes, 2013).

Inductive reasoning follows the pattern-data-result-rule (Pedemonte, 2002; Cabassut, 2005; Jeannotte, 2015). The process begins with observing data, recognizing patterns, and drawing conclusions or creating a rule. Duquesne (2003) states, "Inductive inference is exercised on observed regularities from which more general conclusions can be drawn." Therefore, it involves deriving a rule from data and making assertions about that data. It moves from the specific to the general. According to Jeannotte (2015), generalization, conjunction, and the identification of regularities are essential for the induction process. It is also helpful to consider the dynamic aspect of reasoning in teaching and learning mathematical reasoning (MR). Jeannotte (2015) proposes a distinction between two dimensions to better understand the components of inductive reasoning in mathematics education.

- Structural aspect: This refers to the overall organization of the reasoning process, how a student moves from data to rule, and how different stages are ordered and connected.
- Processual aspect: This refers to the cognitive processes involved in constructing that reasoning, which Jeannotte categorizes as:
 - Processes of exploration, such as identifying regularities, comparing, classifying, conjecturing, and generalizing.
 - Processes of validation, where learners seek to confirm or justify the reasoning behind their conjectures.

In terms of problem-solving using inductive reasoning:

The study by Cañadas (2007) proposes a more precise categorization of inductive reasoning based on the steps outlined by Polya (1967) and Reid (2002), specifically: 1) observation of cases; 2) organization of cases; 3) search for and prediction of models; 4) formulation of conjectures; 5) validation of conjectures; 6) generalization of conjectures; and 7) justification of general conjectures. This framework was used to study the inductive reasoning of twelve Spanish secondary school students.

Cañadas et al. (2009) used a model for the characterization of inductive reasoning composed of seven steps: 1) working on particular cases; 2) organizing particular cases; 3) searching for and predicting models; 4) formulating conjectures; 5) justifying (validating the conjecture based on particular cases); 6) generalizing; and 7) justifying the generalization (formal proof) (Cañadas & Castro, 2007). This model was employed to analyze the responses of Spanish students in grades 9 and 10 when working on linear and quadratic sequence problems.

Papageorgiou (2009) proposes and evaluates a training program that integrates problem-solving through inductive reasoning with the development of mathematical concepts among sixth-grade students in elementary schools in an urban area of Cyprus.

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Sosa Moguel et al. (2019) proposed three processes to characterize secondary mathematics teachers' inductive reasoning when working with a quadratic model to obtain a general rule: 1) observing regularities; 2) establishing a model; and 3) formulating a generalization. The results showed that some teachers encountered difficulties in transitioning from observing regularities to formulating a generalization.

Inductive Reasoning: Characterization and Detection of Similarities and Differences With Deductive Reasoning

Inductive reasoning is a fundamental cognitive process in which we observe data (judged to be true) to deduce a conclusion that allows us to create generalizations.

This type of reasoning is often characterized as allowing one to move from the particular to the general (Polya, 1965). An inductive reasoning step can be characterized as data-rule-affirmation, as shown in the diagram (Figure 1). However, according to Jeannotte (2015), this characterization is insufficient to describe RM; the processual aspects—generalization, conjunction, and identification of a regularity—are also necessary for the induction process.

A disadvantage of inductive reasoning is that inferences are made from specific situations, which may not have any meaning in the real world.

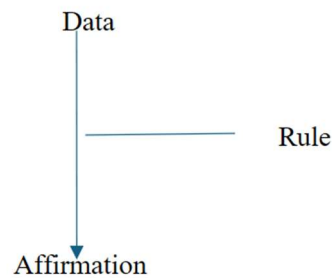


Figure 1: Model of an inductive reasoning step (Favier, Stéphane & Chanudet, Maud 2021, p.90)

On the other hand, deductive reasoning works in the opposite direction from inductive reasoning. It is a process that allows obtaining a statement from data and a rule. It involves the use of general hypotheses and logical premises to arrive at a logical conclusion.

Therefore, both reasoning types are considered alternatives; inductive reasoning produces generalizations, while deductive reasoning produces particularizations. Additionally, there is a difference in the production of proof, as the two reasons are at different and non-linear times.

In a proof approach, the deductive and inductive points of view are located at different and non-linear times. The inductive point of view is present in the development of a conjecture or the study of hypotheses when the conclusion is to be constructed. The deductive point of view is predominant in the writing of the proof. (Denise GRENIER, 2012, p.107)

On the other hand, there are also similarities. For example, the degree of certainty that allows deduction and induction to be placed on a common degree of argument strength (Aidan Feeney, Evan Heit, 2007).

Inductive Reasoning: Detection of the Place it Occupies in the Moroccan Program

Reasoning is the foundation of mathematics, as Ross wrote in 1998:

It is essential to emphasize that the foundation of mathematics is reasoning. While science verifies through observation, mathematics verifies through logical reasoning. Thus, the essence of mathematics lies in proofs, and the distinction among illustrations, conjectures, and proofs should be emphasized (Ross, 1998, p.254)

In Moroccan secondary school programs (Men, 2007), mathematical reasoning is central, appearing explicitly and implicitly in activities, problem-solving exercises, and problems. Explicit instruction on reasoning mainly occurs in the “Logic” course, specifically within the Mathematical Reasoning section, which covers direct reasoning (deductive), disjunction of cases, proof by absurdity, contrapositive, counterexample, and induction. This study follows the reference model of mathematical reasoning proposed by Jeannotte (2015), used in Nhiry et al. (2023), which examines the structural aspects of reasoning. Their findings highlight the dominance of the deductive step, with the inductive step also strongly represented in the curriculum—hence its inclusion here, especially in problem-solving contexts. Although inductive reasoning is rarely named in textbooks and appears only briefly in official teacher guidelines (Nhiry et al., 2023), it is more often encountered in preparatory activities than in exercises and problems designed to build understanding of mathematical concepts (see Appendix 2).

Structural Aspect in Inductive Reasoning

The structural aspect of inductive reasoning involves observing particular data, identifying regularities (also referred to as assertions), and ultimately inferring a rule that allows the passage of data to assertions (Jeannotte, 2015). In addition to its structural aspect, the grid presented (Nhiry and al. 2023) also described its procedural aspects: identifying regularities, conjecturing, and generalizing.

To describe inductive reasoning, four stages were considered, Pólya (1967):

- Observation of cases.
- Formulation of a conjecture based on previous particular cases.
- Generalization.
- Verification of the conjecture with new special cases.

For uncertain cases, Reid (2002) proposed a conjecture formulation with five detailed steps:

- Work on specific cases.
- Analysis model.

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- Information analysis.
- Analysis of information on specific cases (with doubts).
- Generalization, use of generalization to prove.

We utilized his writings in our study to develop an analysis grid (Table 1) that characterizes the inductive reasoning of secondary school students when they solve problems that can be solved using this type of reasoning.

Analysis criterion (C)	C1: Case observation	C2: Identification of a model	C3: Identification of regularities	C4: Formulation of a conjecture	C5: Generalization
Steps to follow in the generalization process	Examine several specific examples to identify patterns or regularities.	Observe subsequent cases and infer a pattern.	Infer a recursive relationship between mathematical objects by analyzing their similarities and differences.	Infer a statement about a regularity that has potential for mathematical theorization	Infer a property of a set of mathematical objects, or of the relationships within it, from a more restricted subset.

Table 1: Inductive Reasoning Analysis Grid.

METHODOLOGY

Participants

The participants were 130 Moroccan students from the first year of the baccalaureate: 1 bac (2nd year of high school) distributed as follows:

- 80 students in the first year of the experimental sciences baccalaureate (abbreviated as 1sc.exp) are divided into three classes, ages 16 to 17.
- 50 students in the first year of the baccalaureate in mathematical sciences (1SM), divided into two classes (ages 16-17).

The choice of school level was based on the idea that students at this level, with these two branches, often engage in mathematical reasoning during the "notion of logic" lesson, where they explore different types of mathematical reasoning.

Data Collection Tools

We developed a test consisting of five problems to document and characterize students' inductive reasoning. The test was designed not only to collect answers but also to elicit different stages of inductive reasoning, ranging from observation and model identification to conjecture and generalization. It thus served as both a research instrument and a didactic tool, enabling us to capture the ways in which students mobilize inductive reasoning when confronted with mathematical tasks.

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The full version of the test is provided in Appendix 2, while Appendix 3 summarizes the objectives, task types, and detailed reasoning steps associated with each problem. For clarity, Table 2 presents a summary of the five problems, indicating their registers and main objectives.

problem	Register	Objective of the problem
Prob1	Digital + Algebraic	Use inductive reasoning to identify the sequence and induce its explicit formula.
Prob2	Digital + Algebraic	Use inductive reasoning to determine the units digit of powers (e.g., 13^n) and generalize the pattern.
Prob3	Digital	Use inductive reasoning to observe the data of n^2-n+11 and predict its behavior for all natural numbers.
Prob4	Digital + Algebraic	Use inductive reasoning to examine the behavior of functions x^n as x reverses $-\infty$ and generalize the rule for their limits.
Prob5	Geometric + Algebraic	Use inductive reasoning to determine the number of sticks in a growing pattern and conjecture a general rule.

Table 2: Summary of Test Problems and Their Main Objectives

The Test Process

We administered a test of five problems to students individually in class at the end of the school year in May 2024, aiming to gather more results for analysis, given the particularity of the chosen school level (first year of the baccalaureate). The student has taken a reasonable basis of the acquired knowledge and new concepts at the level of the common core science and in the first year of the baccalaureate he is ready to analyze, think and use his knowledge and especially reason and also by the reason that in this level the student becomes more familiar with mathematical reasoning and logical thinking through the lessons of the notion of logic which is located at the beginning of the program. The estimated time to answer the test was 90 minutes. To collect more criteria on the students' inductive reasoning, we asked the students to write what they thought and answer in detail, explaining that it was a matter of providing reasoning rather than an immediate result of the questions. We also asked them to draft their answers on a copy.

Analysis Instrument

The analysis of student reasoning was guided by three main dimensions: (1) the structure of inductive steps, following Jeannotte's (2015) model of data, assertion, and rule; (2) the registers of semiotic representation mobilized (Duval, 1995); and (3) the curricular location of each task in the Moroccan secondary mathematics program. A complementary a priori analysis, presented in

Appendix 4, illustrates how the analytical grid was applied and ensures transparency in the coding process.

Method of Analysis

We used a study based on the grid to analyze students' inductive reasoning. It aims to characterize the inductive reasoning developed by the student (the structural step of the reasoning) and to document the different stages of inductive reasoning (criteria of the grid) which also describes the processual step of the reasoning and this through a descriptive analysis: numbers, percentages and averages for each problem of the test.

❖ Example of the analysis of the reasoning given by a student according to the grid:

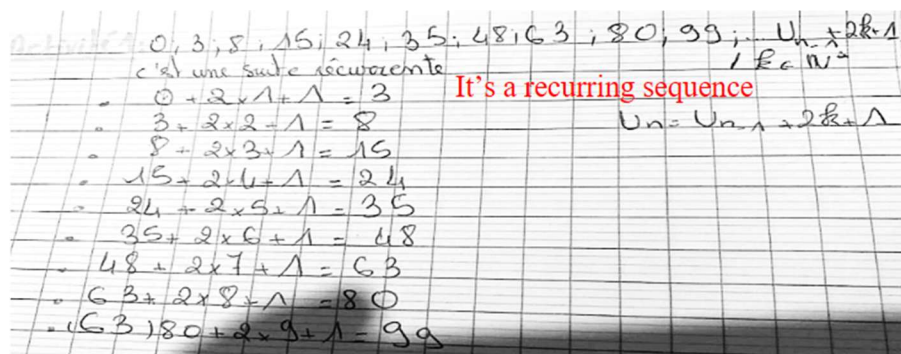


Figure 2: Example of inductive reasoning given by a student on problem

Analysis criterion (C)	C1: Case observation	C2: Identification of a model	C3: Identification of regularities	C4: Formulation of a conjecture	C5: Generalization
Approach to the reasoning process	F	F	NF	F	NF

Table 3: Criteria for analyzing students' reasoning; F: done; NF: not done; FE: done with error

This student employed inductive reasoning effectively to answer the question posed in Problem 1 of the test. We then analyzed the steps of this reasoning according to the grid, as shown in Table 4. The results indicated that the student followed inductive reasoning and completed most of the steps mentioned in the grid; however, he did not provide a generalization at the end. Inductive reasoning was considered successful if the student observed and formulated a conjecture, provided a generalization, or completed most of the steps and formulated a conjecture without generalizing. Conversely, inductive reasoning was considered unsuccessful if the student limited himself to simple observation or produced only a very brief conjecture. In this illustrative case, the student can therefore be qualified as having successfully employed inductive reasoning.

After presenting this example of student reasoning, it was important to clarify how the analytical grid was applied more broadly. For this reason, an a-priori analysis of the test tasks is provided in

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Appendix 4. This analysis presents one possible solution method for each problem and shows in advance how the grid was used to code reasoning steps. By including this complementary analysis, the coding procedure is made explicit and methodological transparency is ensured.

While both the student example and the a-priori analysis illustrate the application of the grid at the level of individual cases, the methodological framework extended well beyond these illustrations. The approach was not limited to a descriptive presentation of the sample, tasks, and procedures; rather, it was deliberately designed to investigate whether and how students were able to mobilize inductive reasoning. The test composed of five carefully selected problems, the explicit instructions emphasizing justification, and the analytical grid for coding reasoning steps, all constituted methodological interventions that directly shaped the outcomes. This integrated design allowed us to go beyond simple description and to characterize the presence and quality of inductive reasoning across the entire student sample.

RESULTS

Frequencies of the students' inductive structural step according to each problem.

Table 4 below shows the frequency with which students identified inductive reasoning in solving the 5 problems proposed in the test.

The first phase of our analysis involved reviewing the students' copies. We found students who did not answer all the problems and students who, despite answering the problems, provided a direct answer to certain questions, that is, an absence of a reasoning approach.

In Table 4, "used" refers to the number of students who employed the inductive step (observation, statement, or rule), while "not used" or "did not answer" refers to the number of students who either did not reason or did not answer the problem directly (resulting in an absence of response).

	<i>Frequency of inductive step presence</i>	
	<i>Used</i>	<i>Not used or not answered the problem</i>
<i>Prob1</i>	<i>50 (38.46%)</i>	<i>80 (61.53%)</i>
<i>Prob2</i>	<i>28 (21.53%)</i>	<i>102 (78.46%)</i>
<i>Prob3</i>	<i>102 (78.46%)</i>	<i>27 (19.23%)</i>
<i>Prob4</i>	<i>111 (85.38%)</i>	<i>19 (14.61%)</i>
<i>Prob5</i>	<i>100 (76.92%)</i>	<i>30 (23.07%)</i>
<i>Total</i>	<i>391 (60.15%)</i>	<i>259(39.84%)</i>

Table 4: Frequency of use of inductive step

For the five proposed problems and the 130 student scripts collected, a total of 650 effective resolutions were expected for analysis. However, 259 resolutions (39.84%) were excluded. Nearly half were removed because no solution appeared in the student's script (blank space or an explicit statement of inability to solve the problem), because only a brief answer was provided, or because the responses combined correct and incorrect elements without demonstrating any evidence of inductive reasoning.

Regarding the rate of inductive reasoning use, the academic track had a significant impact on the percentage observed. Table 5 presents a visualization of the difference between the two tracks in the first year of the Baccalaureate.

	<i>Frequency of the presence of inductive step according to the branches</i>			
	<i>Used</i>		<i>Not used or not answered the problem</i>	
	<i>ISM</i>	<i>1EXP</i>	<i>ISM</i>	<i>1EXP</i>
<i>Total reasoning according to the total copies analyzed</i>	230 (58.82%)	161 (41.11%)	20 (07.72%)	239 (92.27%)
<i>Total reasoning according to copies of the same branch</i>	230 (92%)	113 (28.25%)	20 (05.11%)	239 (59.75%)

Table 5: Frequency of use of the inductive step by branch at the 1st-grade school level.

Among the two branches studied, students in the 1SM branch showed a significant use of inductive reasoning (92%). This can be explained by the fact that students in the SM stream, being specialized in mathematics, develop more in-depth skills in this area. They also demonstrate a more attentive and methodical attitude, which can be interpreted as a better mobilization of methodological and cognitive skills, allowing them to approach tasks more efficiently than their counterparts in the 1EXP branch.

Subsequently, the analysis focused exclusively on the **391 responses (60.15%)** that exhibited evidence of inductive reasoning, in accordance with the previously developed analysis grid.

Frequency of Steps Taken by Students for Each Problem

To determine the frequencies of the steps performed by students on the test problems, we identified the steps taken by each student in the 391 reasonings related to each test problem by filling in the previously presented grid. We present these frequencies in Table 6 and Figure 3:

	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C4</i>	<i>C5</i>
<i>Prob1</i>	50 (100%)	31 (62%)	13 (26%)	20 (04%)	7 (14%)
<i>Prob2</i>	25 (89.28%)	19 (67.85%)	6 (21.42%)	10 (35.71%)	2 (07.14%)
<i>Prob3</i>	102 (100%)	93 (91.17%)	55 (53.92%)	100 (98.03%)	95 (93.13%)
<i>Prob4</i>	111 (100%)	106 (95.49%)	68 (61.26%)	63 (56.75%)	62 (55.85%)
<i>Prob5</i>	100 (100%)	90 (90%)	64 (64%)	34 (34%)	26 (26%)
TOTAL	388 (99.23%)	339 (86.70%)	206 (52.68%)	227 (58.05%)	192 (49.10%)

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Table 6: Frequency of inductive reasoning steps

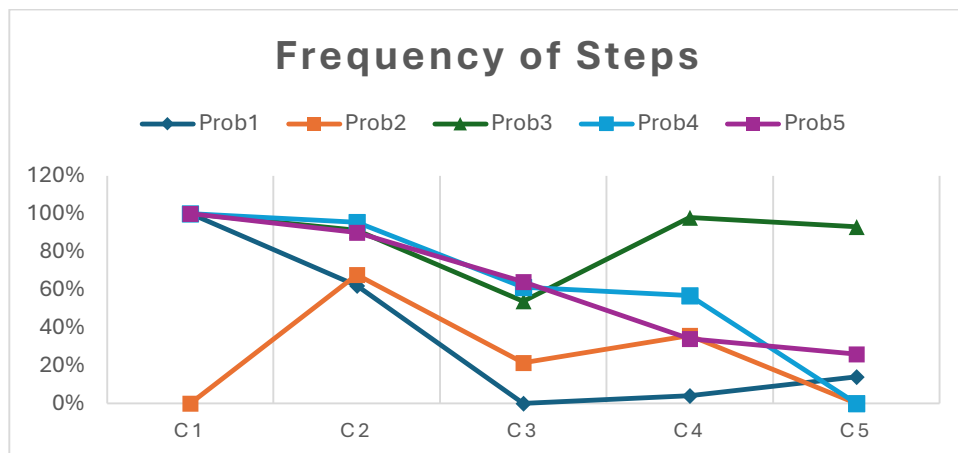
For criterion C1: Observation of cases, the results show that students who used the inductive step in their resolutions succeeded well in this initial step, with a success rate of **99.23%**. This shows their ability to make remarks and observations.

For criterion C2: Identification of a model, students were able to identify a model in 86.70% of the reasonings, with high percentages in problems 3, 4, and 5 and moderately high percentages in problems 1 and 2. In all the resolutions where the model identification step appears, there is already a case observation step, which allowed us to conclude that accurate data observation leads to the detection of the model.

For criterion C3: Identification of regularities, half of the solutions contain the identification of regularities, with most of these instances appearing in problems 3, 4, and 5, compared to lower percentages in problems 1 and 2. This is due, according to a brief interview conducted with students after the test, to the fact that problem 3 contains more cases, and problems 4 and 5 include, respectively, a table and a figure, which made the regularity more noticeable. So here, the identification is linked to the structure of the problem.

For criterion C4: Formulation of a Conjecture, the reasoning in which the formulation of conjectures was 58.05%, with slight moderation in problems 1, 2, and 5. According to the students' comments, these problems were slightly more challenging for them to identify compared to problems 3 and 4. Most students were able to find it, especially in the resolution of problem 3, with a large percentage achieving a score of **98.03%**.

For criterion C5: Generalization, we observed a low weighting, particularly in problems 1, 2, and 5, during the final step of inductive reasoning. However, in general, fewer than half of the reasons contain generalization, even in resolutions where students have already made a conjecture. So it is clear that the students have a confusion between making a conjecture and giving the generalization that is the expected rule. On the other hand, there are instances where students have reached generalization without going through the other steps, solely from observation (30.09%).



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Figure 3: Frequency of inductive reasoning steps per problem.

The frequency analysis of the steps across the five problems reveals a consistent trend, with similar increases and decreases from one problem to another. The most frequently employed steps (Figure 4) are: (C1) observation of given cases, (C2) model identification, and (C3) conjecture formulation.

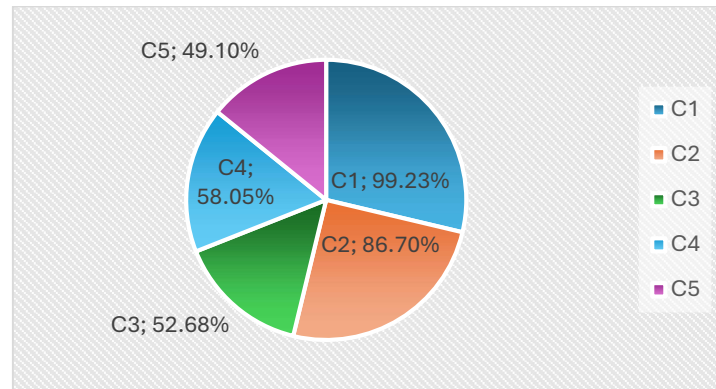


Figure 4. Frequency of inductive reasoning steps in the five problems.

The results previously obtained concern the students' copies where traces of inductive reasoning were identified. However, our study was conducted on a sample of 650 resolutions. To better visualize the results, Figure 5 presents the frequencies of the steps used by all students.

- **C1 (59%):** This category is the most dominant, accounting for more than half of the occurrences. This could indicate that step C1 is essential or very frequently used by students in their reasoning or resolutions.
- **C2 (52.15%):** This step is also very frequent, although less than C1. It can play a complementary or secondary role in resolutions.
- **C3 (31.69%):** This step is used by approximately a third of students, which could indicate that it is relevant but less central than the previous steps.
- **C4 (34.92%):** Its frequency is similar to that of C3, suggesting it is also used occasionally or in specific cases.
- **C5 (29.53%):** This category is the least frequent, accounting for approximately a quarter of the occurrences. It could correspond to a more specific or advanced stage, used in particular situations.

These data show an uneven distribution in steps or categories, with a marked preponderance of steps C1 and C2. This suggests that students focus more on key steps in their solutions, while others, such as C4 and C5, are less systematically used.

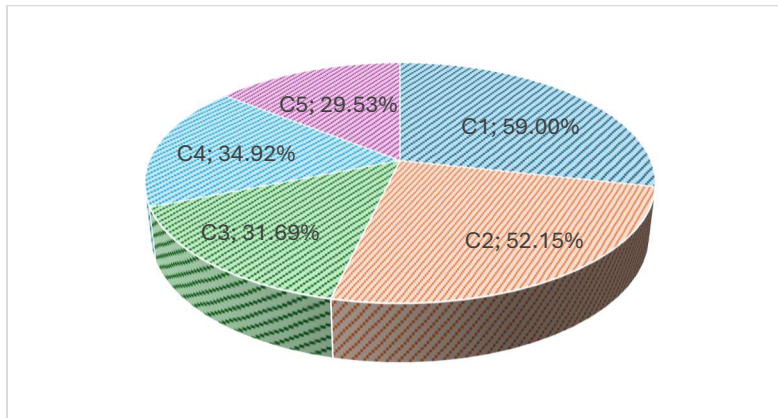


Figure 5: Frequency of steps in inductive reasoning.

These percentages reveal a clear hierarchy in the accessibility of the different stages of inductive reasoning. Nearly all students were able to observe cases and most could identify a model, but the proportion decreased significantly when moving to more advanced steps such as identifying regularities, formulating conjectures, and especially generalizing. This distribution suggests that while students are generally capable of initiating inductive reasoning, they struggle to bring it to completion. In particular, the transition from conjecture to generalization appears to represent a critical difficulty. These findings indicate that inductive reasoning is often only partially mobilized, with students demonstrating strengths in the initial phases but encountering obstacles in the later, more abstract stages.

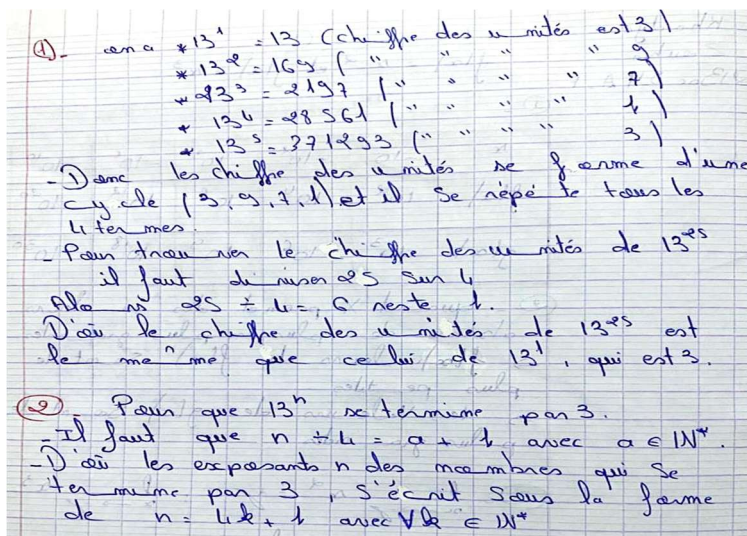
Examples of Illustrations of Students' Reasoning

We present some of the students' productions that relate to inductive reasoning, as illustrated by the given problems (10 examples). We illustrate successful and unsuccessful inductive reasoning:

Inductive Reasoning Succeeds

- ✚ Student production 1 of branch 1SM on problem 02 succeeds without doing all the steps presented in the grid:

The student here has respected the structural step of inductive reasoning: data - statement - rule. For the steps of reasoning, we find that the student has done the step of observing the data well (we have ... The units digit is ...), then the identification of the model has been produced well (the unit's digits are repeated in cycles of 4), then the student has reformulated the conjecture. To find the units digit, we must divide by 4 and observe the remainder. Since the remainder of 25 divided by 4 is 1, the units digit is 3. Towards the end, the student provided a generalization of the power formula to have a unit digit equal to 1 ($4K + 1 \forall K \in N$). In this production, we find that the student has not completed the step of identifying regularities; despite this, his reasoning is considered successful.



① - on a $*13^1 = 13$ (chiffre des unités est 3)
 $*13^2 = 169$ (" " " " " 9)
 $*13^3 = 2197$ (" " " " " 7)
 $*13^4 = 28561$ (" " " " " 1)
 $*13^5 = 371293$ (" " " " " 3)
 - Donc les chiffres des unités se forment d'une
 suite de (3, 9, 7, 1) et il se répète tous les
 4 termes.
 - Pour trouver le chiffre des unités de 13^{25}
 il faut de diviser 25 sur 4.
 Alors $n = 25 \div 4 = 6$ reste 1.
 Donc le chiffre des unités de 13^{25} est
 le même que celui de 13^1 , qui est 3.

② - Pour que 13^n se termine par 3.
 - Il faut que $n \div 4 = a + 1$ avec $a \in \mathbb{N}^*$.
 - Donc les exposants n des membres qui se
 terminent par 3, s'écrivent sous la forme
 de $n = 4k + 1$ avec $\forall k \in \mathbb{N}^*$

Translation:

so, the unit's digits are formed by a cycle (3,9,7,1) and they repeat every 4 terms.

to find the units digit of... you have to divide 25 by 4

hence the units digit of... is the same

so that... ends in 3

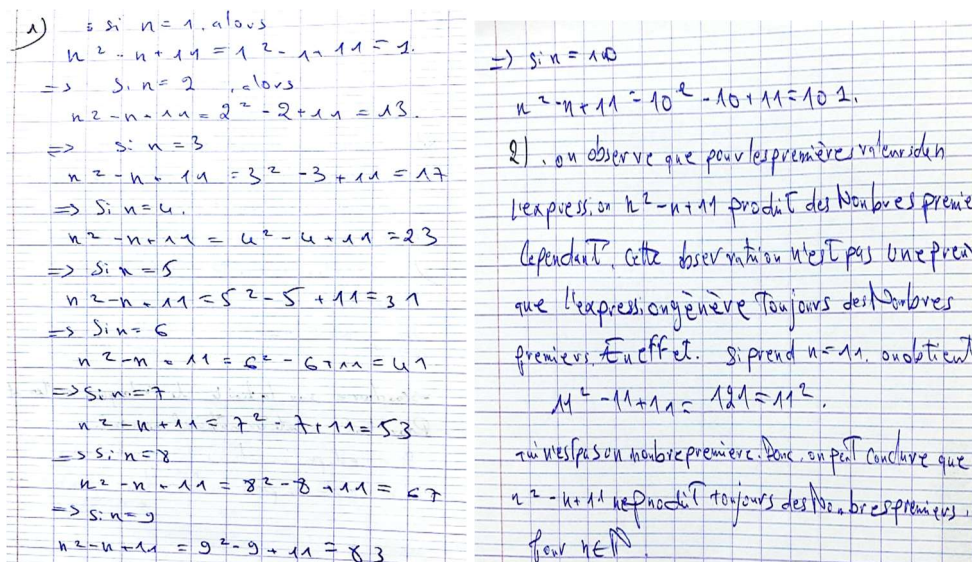
it is necessary that ... with..

hence the exponents n of numbers that end in 3 are written in the form of...

Figure 6: Student 1 SM's successful production on test problem 2.

Student production 2 of branch 1SM on problem 03, respecting all the steps and considering the epistemological value of the reasoning:

In problem 3, the student applied inductive reasoning in full accordance with the analysis grid—progressing through observation, model identification, and recognition of regularities—to formulate the generalization “product of prime numbers.” Demonstrating an understanding of the epistemological limitation of induction, he acknowledged that conclusions drawn from limited data are only probable and accordingly refined his generalization while supplying a counterexample. This production is therefore considered successful in inductive reasoning and of high quality in terms of mathematical logic.



1) si $n=1$, alors
 $n^2 - n + 11 = 1^2 - 1 + 11 = 11$
 \Rightarrow si $n=2$ alors
 $n^2 - n + 11 = 2^2 - 2 + 11 = 13$
 \Rightarrow si $n=3$
 $n^2 - n + 11 = 3^2 - 3 + 11 = 17$
 \Rightarrow si $n=4$,
 $n^2 - n + 11 = 4^2 - 4 + 11 = 23$
 \Rightarrow si $n=5$
 $n^2 - n + 11 = 5^2 - 5 + 11 = 31$
 \Rightarrow si $n=6$
 $n^2 - n + 11 = 6^2 - 6 + 11 = 41$
 \Rightarrow si $n=7$
 $n^2 - n + 11 = 7^2 - 7 + 11 = 53$
 \Rightarrow si $n=8$
 $n^2 - n + 11 = 8^2 - 8 + 11 = 67$
 \Rightarrow si $n=9$
 $n^2 - n + 11 = 9^2 - 9 + 11 = 83$

\Rightarrow si $n=10$
 $n^2 - n + 11 = 10^2 - 10 + 11 = 101$
 2) on observe que pour les premières valeurs de n
 l'expression $n^2 - n + 11$ produit des nombres premiers.
 Cependant, cette observation n'est pas une preuve
 que l'expression génère toujours des nombres
 premiers. En effet, si prend $n=11$, on obtient
 $11^2 - 11 + 11 = 121 = 11^2$,
 qui n'est pas un nombre premier. Donc, on peut conclure que
 $n^2 - n + 11$ ne produit pas toujours des nombres premiers,
 pour $n \in \mathbb{N}$.

Translation:

we observe that for the first values of n

the expression ... produces prime numbers. However, this observation is not a

proof that the expression

always generates prime

numbers. indeed, if we take

.... we obtain ...

which is not a prime

number. therefore, we can

conclude that ... does not

always produce prime

numbers.

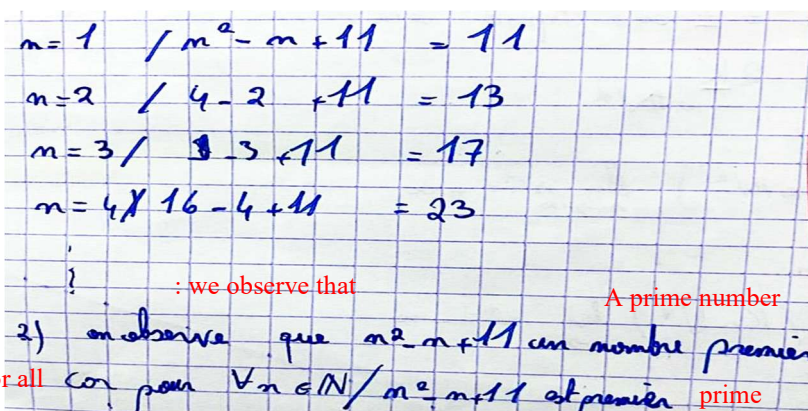
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Figure 7: Student 2 SM's successful solution to test problem 3.

✚ Student production 3 (1EXP branch, problem 3):

In this production, the student employed inductive reasoning by observing several cases of n , conjecturing that the expression $n^2 - n + 11$ yields prime numbers, and generalizing that it is always prime for any n . Although the steps of model identification and recognition of regularities were not explicitly stated, the reasoning is deemed successful. Unlike student 2's production for the same problem, this one does not address the epistemological value of induction; the generalization is thus considered valid for the observed cases, but the reasoning as a whole remains merely plausible.

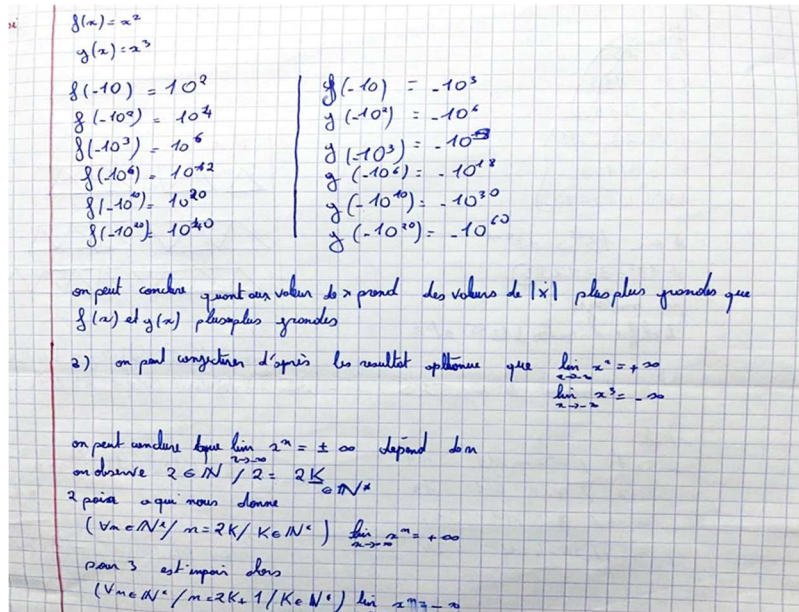


$n=1 / m^2 - m + 11 = 11$
 $n=2 / 4 - 2 + 11 = 13$
 $n=3 / 9 - 3 + 11 = 17$
 $n=4 / 16 - 4 + 11 = 23$
 ? : we observe that A prime number
 2) on observe que $m^2 - m + 11$ est un nombre premier
 Because for all $\forall n \in \mathbb{N} / m^2 - m + 11$ est premier prime

Figure 8: Student's successful production of EXP 3 on problem 3 of the test.

✚ Student production 4 of branch 1EXP on problem 04:

In identifying the usual limit, the student successfully applied inductive reasoning to generalize the rule. By calculating the values for each function, we observe and identify the model that emerges. In addition to the regularities between the two functions, the formulation of the conjectures related to each function was established as a result of the generalization (the expected rule).



Translation:

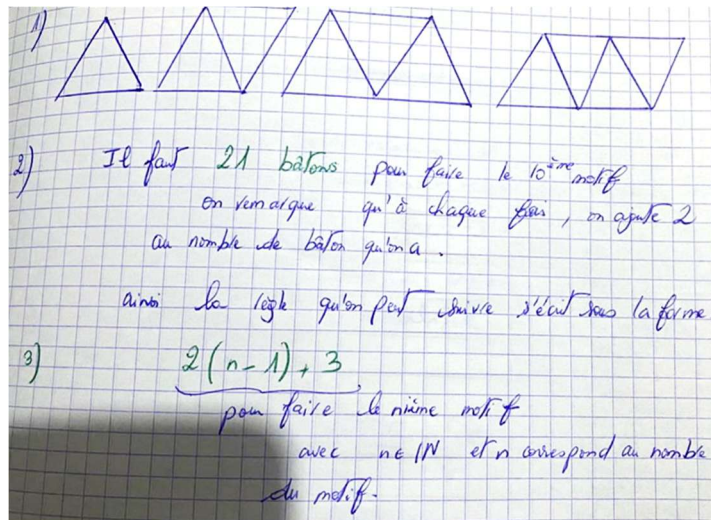
we can conclude that the value of x takes on values of and increasingly larger.

we can conjecture from the results that their....

Figure 9: Student's successful production of EXP 4 on problem 4 of the test.

- ✚ Student production 5 of branch 1SM on problem 05: successful inductive reasoning on a geometric problem.

The student illustrated the observation phase on the drawn patterns, then identified the pattern and regularities of adding 2 sticks each time. Then, he conjectured the number of sticks needed to create the 10th pattern and the nth pattern as part of a generalization process.



Translation:

2) it takes 21 sticks to make the 10th pattern

we notice that each time, we add 2 to the number of sticks we have

so the rule that we can follow is written in the form...

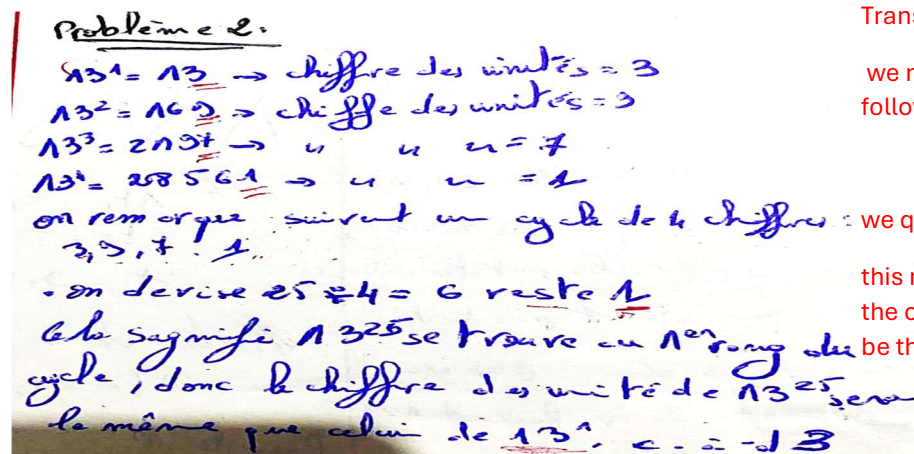
3) to make the nth pattern with and ... corresponds to the number of the pattern

Figure 10: Student 5 SM's successful solution to problem 5 of the test.

Inductive reasoning failed:

✚ Student production 6 of branch 1EXP on problem 02:

The student used inductive reasoning to answer question 1 of the problem. He completed all the steps, observing and analyzing the powers of 13. Then, he identified the patterns and regularities, revealing a conjecture about the units digit of 13 raised to the power of 25. Since division by 4 gives a remainder of 1, it follows that the units digit of 13 raised to the power of 1 is also 1. However, the student was unable to generalize or devise a sufficient rule for determining the exponent that ends in 3. This reasoning is considered unsuccessful.



Translation:

we notice that a cycle of 4 digits follows

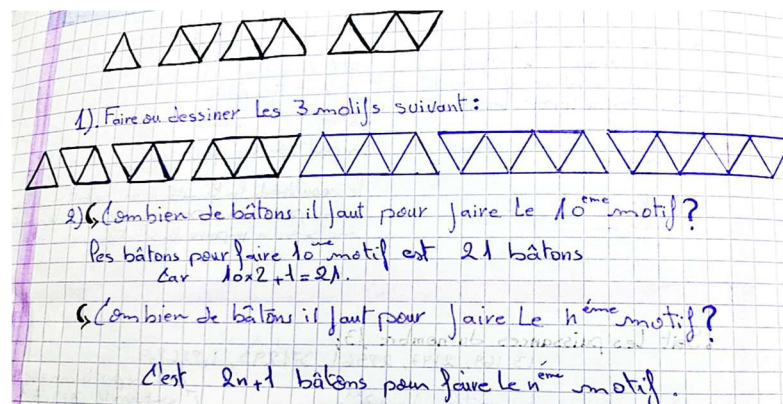
we quote ...

this means is at the 1st rank of the cycle, so the unit digit of will be the same as that of ...

Figure 11: Unsuccessful production of student 6's 1EXP on problem 2 of the test.

✚ Student production 7 of branch 1EXP on problem 05:

In this reasoning, the student was satisfied with two steps: drawing the patterns without detailing the observations and the conjecture itself, without specifying which model was followed or even the regularities detected that led to this conjecture. He lacks generality, and he did not give any information on the unknown for the conjectured expression. It is for these reasons that his reasoning is unsuccessful.



Translation:

1- make or draw the following 3 patterns:

2- how many sticks do you need to make the 10th pattern?

the sticks to make the 10th pattern are 21 sticks.

how many sticks do you need to make the nth pattern

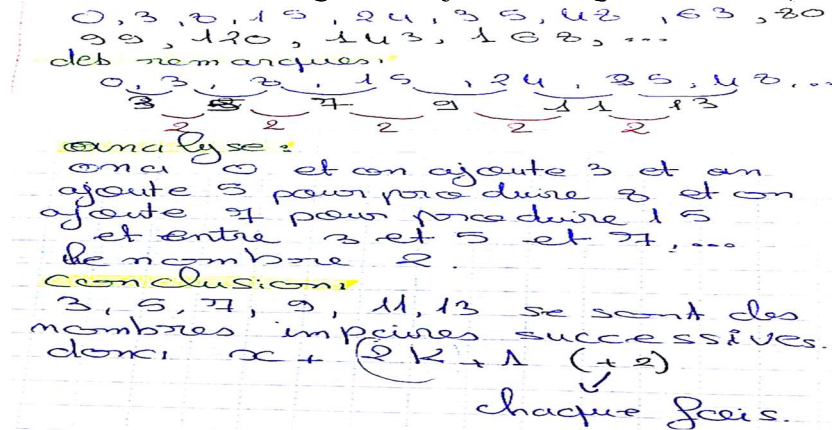
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Figure 12: Unsuccessful production of student 7's EXP on problem 5 of the test.

✚ Student production 8 of branch 1EXP on problem 01:

In this case, the student's use of inductive reasoning was deemed unsuccessful. While the reasoning began appropriately—with accurate observations, correct completion of the numerical sequence through model detection, and identification of a regularity explained in the “analysis” section—it failed at the stages of conjecture and generalization (FE: formulated with error).



0, 3, 8, 15, 24, 35, 48, 63, 80,
99, 120, 143, 168, ...

des remarques:

0, 3, 8, 15, 24, 35, 48, ...

analyse:

on a 0 et on ajoute 3 et on ajoute 5 pour produire 8 et on ajoute 7 pour produire 15 et entre 3 et 5 et 7, ... le nombre 2.

conclusion:

3, 5, 7, 9, 11, 13 se sont des nombres impaires successives. donc, $x + (2k + 1) (+2)$ chaque fois.

Translation:

we have 0 and we add 3 and we add 5 to produce 8 and we add 7 to produce 15 and between 3 and 5 and 7 ... the number 2 ...

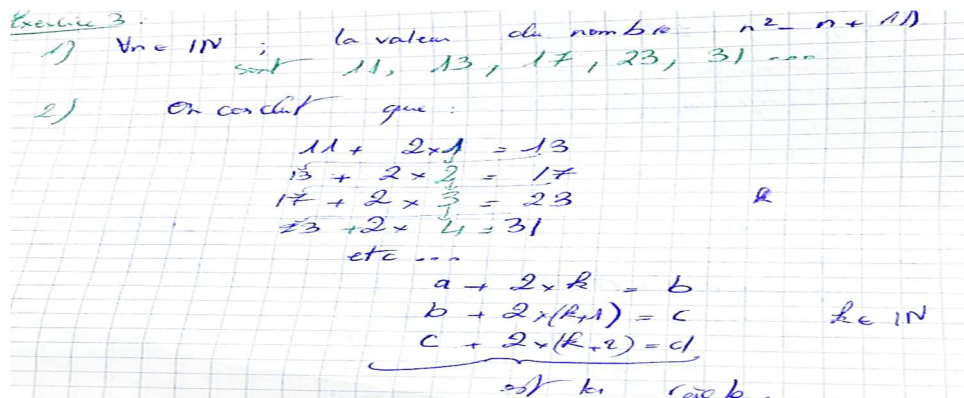
conclusion

3,5,7, 9, 11,13 are successive odd numbers. so...

Figure 13: Failed production of student 8 EXP on test problem 1.

✚ Student production 9 from branch 1SM on problem 03:

Conversely, although this student's inductive reasoning followed all steps of the analysis grid, it was deemed unsuccessful. After performing calculations and observations, the student formulated a conjecture and proposed a concluding rule; however, the conjecture was incorrect. The problem was treated as a recurrence sequence, but the student failed to generalize correctly (FE), particularly concerning the primality of the values of $n^2 - n + 11$.



Exercice 3:

1) Une IN ; la valeur du nombre $n^2 - n + 11$ sont 11, 13, 17, 23, 31 ...

2) On conjecture que :

$11 + 2 \times 1 = 13$
 $13 + 2 \times 2 = 17$
 $17 + 2 \times 3 = 23$
 $23 + 2 \times 4 = 31$
 etc ...

$a + 2 \times k = b$
 $b + 2 \times (k+1) = c$
 $c + 2 \times (k+2) = d$

est k_1 car k .

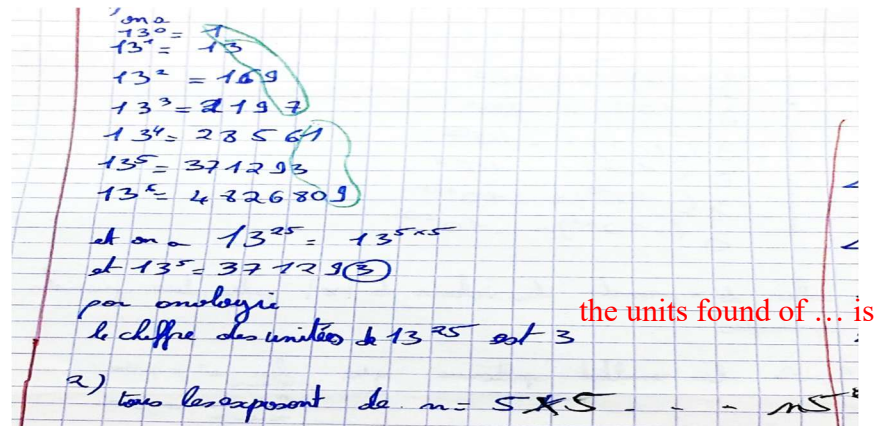
Figure 14: Unsuccessful production of student 9 SM on problem 3 of the test.

✚ Student production 10 of branch 1EXP on problem 02:

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Unsuccessful production: The student completed the observation step by circling the identified unit digits but failed to carry out model identification and recognition of regularities. Consequently, the generalization was flawed. Although the structural step of inductive reasoning was followed, the omission of the initial phases hindered the understanding and formulation of conjectures.



$13^0 = 1$
 $13^1 = 13$
 $13^2 = 169$
 $13^3 = 2197$
 $13^4 = 28561$
 $13^5 = 371293$
 $13^6 = 4826809$

et on a $13^{25} = 13^{5 \times 5}$
 et $13^5 = 371293$

par analogie
 le chiffre des unités de 13^{25} est 3

2) tous les exposant de $n = 5 \times 5 \dots n^2$

the units found of ... is

Figure 15: Student failed to produce 10 EXP on problem 2 of the test.

DISCUSSION

This study aimed to characterize the inductive reasoning of secondary school students, specifically those enrolled in the first year of the baccalaureate program. Drawing on the framework developed by Nhiry et al. (2023), which distinguishes between structural and processual dimensions of inductive reasoning, we applied a tailored analytical grid to examine students' responses to five mathematical problems. The test was administered to 130 students, and the results revealed both strengths and limitations in how inductive reasoning was mobilized in this context.

The analysis showed a clear hierarchy in the accessibility of inductive steps. Almost all students (99.23%) were able to make relevant observations, and the majority (86.70%) identified a model, demonstrating their capacity to recognize structure in specific cases. However, fewer students moved forward to more complex stages: 52.68% identified regularities, 58.05% formulated conjectures, and only 49.10% reached generalization. These results suggest that while students are generally comfortable initiating inductive reasoning, they encounter significant challenges in completing the reasoning cycle. The tendency to stop at conjecture without moving to generalization reflects both cognitive difficulties and didactic influences. In contexts where deduction dominates, inductive exploration often lacks explicit scaffolding, leaving students with limited opportunities to consolidate the later stages of inductive reasoning.

The partial nature of inductive reasoning observed here is consistent with international findings. Jeannotte (2015) highlighted that the transition from conjecture to generalization remains a persistent obstacle in secondary mathematics, with many students formulating conjectures that are never generalized. Similarly, Cañadas and Castro (2007) reported that students often succeed in producing conjectures but fail to extend them to broader generalizations. Our results thus corroborate these findings and extend them to the Moroccan context, showing that the challenge of completing the inductive process is not context-specific but resonates with international evidence. This alignment reinforces the idea that inductive reasoning, while fundamental to mathematical thinking, requires explicit support in order to develop fully.

Importantly, these results were not the product of descriptive observation alone but emerged from a methodological design intentionally structured to elicit inductive reasoning. The five problems were chosen to stimulate students' capacity to identify patterns and generalize. Instructions emphasized explanation and justification, encouraging students to articulate their reasoning rather than merely provide answers. The analytical grid offered a systematic framework for coding reasoning steps, ensuring consistency in interpretation. Together, these interventions constituted methodological choices that directly shaped the findings, allowing us to characterize inductive reasoning across the entire student sample. Thus, the study contributes not only empirical results but also a replicable methodological approach for analyzing reasoning.

From a pedagogical standpoint, the findings highlight the need to rebalance classroom practices. In the current Moroccan curriculum, as in many international contexts, deduction is strongly emphasized, often at the expense of inductive reasoning. To make more room for inductive reasoning, teachers could implement several strategies. For instance, beginning new topics with open-ended exploratory tasks rather than formal definitions would invite students to observe and conjecture before rules are introduced. Encouraging students to make and test conjectures before formal proofs can help them see the value of induction as a pathway to deduction. Classroom discussions structured around observed patterns, counterexamples, and student reasoning would promote collaborative exploration and critical reflection. In addition, scaffolding prompts—such as asking “How might this pattern continue?” or “Can you propose a rule that fits all these cases?”—can explicitly guide students toward generalization.

These pedagogical implications are directly connected to the empirical findings. The fact that nearly all students succeeded in observation (99.23%) and most in model identification (86.70%) suggests that exploratory tasks can serve as effective entry points for inductive reasoning. At the same time, the difficulty observed in moving from conjecture (58.05%) to generalization (49.10%) highlights the need for scaffolding prompts and structured guidance to help students complete the reasoning cycle. By explicitly aligning teaching strategies with these observed strengths and weaknesses, instruction can better support the development of inductive reasoning.

Beyond the immediate findings, this study makes a theoretical and methodological contribution to the literature on mathematical reasoning. It demonstrates how an analytical grid, based on structural and processual dimensions, can be used to systematically identify and evaluate inductive reasoning in student work. This tool has the potential to be applied or adapted in other instructional

contexts, enabling cross-national comparisons and fostering dialogue on how inductive reasoning is fostered in different educational systems. By connecting the Moroccan case to broader international research, this study contributes to a deeper understanding of the challenges and possibilities of inductive reasoning in mathematics education worldwide.

Overall, the study shows that inductive reasoning among secondary students is frequently partial: students readily engage in early stages such as observation and model identification but struggle to reach full generalization. This finding is consistent with international research and highlights a pressing didactic challenge. Addressing this challenge requires both methodological tools, such as the one proposed in this study, and pedagogical practices that deliberately scaffold inductive reasoning. In this sense, the study bridges theoretical analysis, empirical data, and practical implications, offering a contribution that is both contextually grounded and internationally relevant.

CONCLUSION

This study provides a detailed characterization of how Moroccan secondary school students engage in inductive reasoning when solving mathematical problems. Using a structured analysis grid grounded in recent theoretical work (Nhiry et al., 2023), we evaluated student responses across key reasoning stages: observation, model identification, regularity recognition, conjecture, and generalization. The findings showed that while many students demonstrated strong observational skills and could identify models, fewer advanced to formulating valid conjectures and generalizations.

One of this study's main contributions lies in its methodological approach. The use of a fine-grained, step-by-step analytical grid enabled a nuanced understanding of reasoning processes, moving beyond binary notions of success or failure. Rather than merely evaluating correctness, the study captured how students constructed generalizations from specific cases—an essential feature of inductive reasoning. This approach provides a replicable tool that can be adapted in other contexts to assess reasoning more systematically.

The results further revealed that although nearly 60% of students demonstrated inductive reasoning, the remaining 40% did not, particularly among the 1SEXP group. This finding raises concerns about the limited visibility of inductive reasoning in the Moroccan mathematics curriculum, which heavily emphasizes formal deduction. Similar challenges have been reported internationally: Jeanotte (2015) noted that the transition from conjecture to generalization remains a persistent obstacle, while Cañadas and Castro (2007) highlighted that students often formulate conjectures without extending them to valid generalizations. Our study therefore corroborates these findings and situates them within the Moroccan context, contributing to the international dialogue on mathematical reasoning.

While this research did not directly observe classroom teaching, the results point toward clear pedagogical implications. Inductive reasoning could be better supported by integrating open-ended exploratory tasks at the beginning of topics rather than starting with formal definitions,

encouraging students to make and test conjectures before introducing proofs, structuring classroom discussions around observed patterns and counterexamples, and providing scaffolding prompts that explicitly support generalization. Such strategies could help students progress beyond observation to higher levels of inductive reasoning, complementing deductive approaches and fostering a more balanced mathematical experience.

This research thus establishes a foundation for future studies on the role of teaching methods and task design in advancing inductive reasoning. It emphasizes the importance of pedagogical strategies that recognize induction as both a precursor to proof and a central aspect of mathematical thinking. By bridging theoretical insights with instructional considerations, the study contributes to academic discussions on reasoning and offers guidance for mathematics educators seeking to strengthen students' reasoning skills.

Nevertheless, the study has limitations. The sample size was restricted, which may limit generalizability to other student populations or educational settings. In addition, relying solely on written assessments, without triangulation through classroom observations or student interviews, constrained the depth of analysis regarding the dynamics of reasoning. Future research could adopt longitudinal approaches to monitor the development of inductive reasoning over time, investigate how specific teacher interventions foster generalization, or analyze classroom conversations to capture the evolving nature of reasoning during collaborative work.

CONFLICTS OF INTEREST STATEMENT

All authors declare that they have no conflicts of interest.

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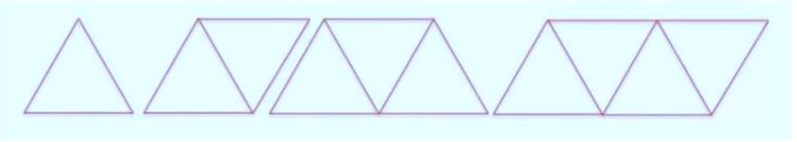
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APPENDIX

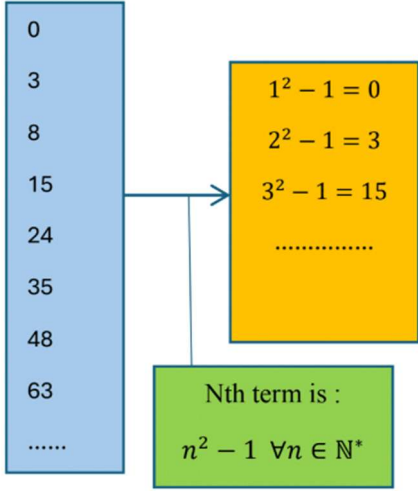
Appendix 01: Distribution of Task Types in the Moroccan Program and Manuals for the First Year of the Baccalaureate: 1SC.EXP and 1SM.

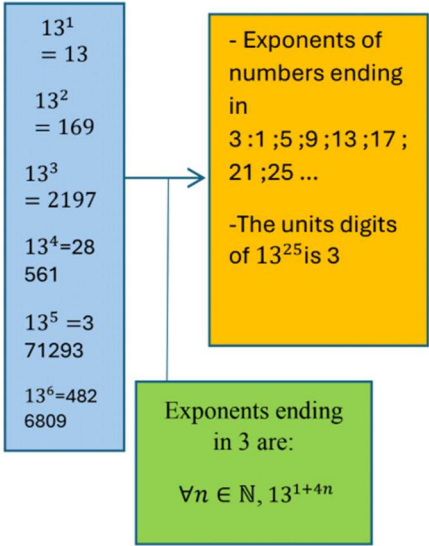
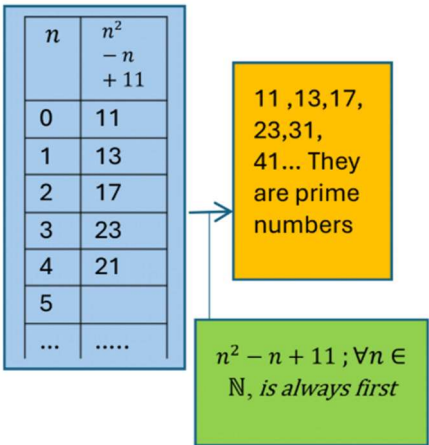
1sc.exp	1SM
<ul style="list-style-type: none"> - Preparatory activities (introductory, reminder, and constructive activities). - Exercises: Invest in my knowledge - Exercises: integrating my learning - Application exercises - Exercises to reinforce acquired skills. - Improvement exercises. 	<ul style="list-style-type: none"> -Preparatory activities (reminder activities; constructive activities). -Exercises: applications; reinforcement of acquired knowledge; improvement. -Problems.

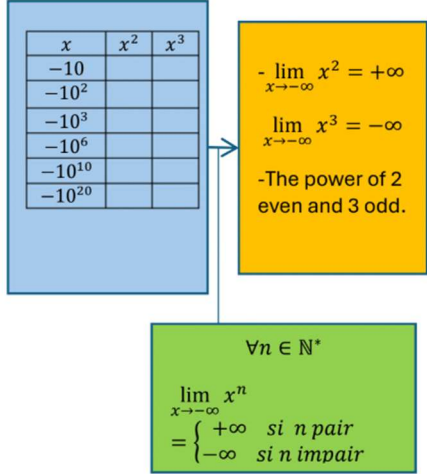
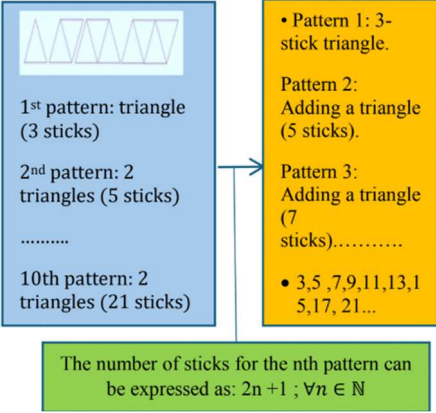
Appendix 02: Task Testing

Codification	Problems																					
Prob1	<p>Let the following sequence be: 0;3;8;15;24;35;48; 63.... Give the following 5 terms, then determine the nth term.</p>																					
Prob2	<p>Let the powers of the number 13 be: 13,169,2197, 28561, 371293, 4826809... 1) Determine the units digit of 13^{25} 2) Determine all the exponents of numbers ending in 3.</p>																					
Prob3	<p>1) Give $\forall n \in N$, the values of the number: n^2-n+11 2) By justifying, what can we conclude about: n^2-n+11</p>																					
Prob4	<p>Let two numerical functions be defined by: $f(x)=x^2$ And $g(x)=x^3$ 1) Complete the following table:</p> <table border="1" style="margin-left: 40px;"> <tr> <td>x</td> <td>-10</td> <td>-10^2</td> <td>-10^3</td> <td>-10^6</td> <td>-10^{10}</td> <td>-10^{20}</td> </tr> <tr> <td>$f(x)$</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>$g(x)$</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>2) What can we conclude about the values of $f(x)$ and $g(x)$ when x takes on increasingly larger absolute values? 3) Guess the limits: $\lim_{x \rightarrow -\infty} x^2$ and $\lim_{x \rightarrow -\infty} x^3$ 4) What can we conclude about $\lim_{x \rightarrow -\infty} x^n$; $\forall n \in N^*$</p>	x	-10	-10^2	-10^3	-10^6	-10^{10}	-10^{20}	$f(x)$							$g(x)$						
x	-10	-10^2	-10^3	-10^6	-10^{10}	-10^{20}																
$f(x)$																						
$g(x)$																						
Prob5	 <p>1) Make or draw the following 3 patterns 2) How many sticks does it take to make the 10^{ème} pattern? 3) How many sticks? if it is necessary to make the n^{ème} pattern?</p>																					

Appendix 03: Characteristics of Test Problems

	Inductive step structure	Register	Location in the 1EXP & 1SM program	Objective of the problem	Reason for choice
Prob1		Digital + Algebraic	Digital sequences (The 2 branches)	- Use inductive reasoning to determine the sequence of elements in the given list and induce its explicit form.	This problem is widely used for the introduction of numerical sequences in its explicit form. -an activity that allows the student to use inductive reasoning.

<p>Prob2</p>		<p>Digital + Algebraic</p>	<p>digital sequences (The 2 branches) Arithmetic for 1SM</p>	<p>- Determine the units digit of numbers raised to a power, here the example of 13 and the exponents which end with a precise digit using inductive reasoning.</p>	<p>-A task that uses inductive reasoning to determine the units digit of numbers with power. - Application exercise on the notion of divisibility and explicit sequence and also of congruence for 1SM.</p>
<p>Prob3</p>		<p>Digital</p>	<p>-Notion of logic (both branches) - Arithmetic for 1SM</p>	<p>-Make good observations of the data and arrive at a prediction about the nature of the expression $n^2 - n + 11$ for all natural integers n.</p>	<p>-An activity in which the use of inductive reasoning is necessary to determine the nature of the numbers in an expression, and which students encounter in the logic concept course.</p>

<p>Prob4</p>		<p>Digital + Algebraic</p>	<p>Limits of numerical functions (both branches)</p>	<p>-See the behavior of the two type functions x^n when x reverses $-\infty$. -Generalize the general rule for calculating the limit of these functions.</p>	<p>-An introductory activity on the usual limits in $-\infty$ which allows students to use inductive reasoning.</p>
<p>Prob5</p>		<p>Geometric + Algebraic</p>	<p>Digital sequences (The 2 branches)</p>	<p>-Determine the number of sticks to construct a pattern. -Conjecture a general rule to calculate this number.</p>	<p>This problem allows the use of inductive reasoning in the context of numerical sequences using a geometric figure.</p>

Appendix 04: A Priori Analysis of One Method for Solving Test Tasks According to the Grid.

✚ Problem 01

	C1:	C2:	C3:	C4:	C5: Generalization
Problem 01	Case observation	Identifying a model	Identification of regularities	Formulation of a conjecture	

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<p>Let the following sequence be: 0;3;8;15;24;35;48 ; 63.... Give the following 5 terms then determine the nth term.</p>	<p>- The student must observe the first given terms. -The student adds more terms (5 terms or more) According to the observation that makes it.</p>	<p>-The student should observe that these terms are related to perfect squares. $1^2=1$ $2^2=4$ $3^2=9$ $4^2=16$ $5^2=25$ $6^2=36$ -The student must compare this remark with the terms of the sequence and note that each term is expressed in the form of a square minus 1: $1^2-1=0$ $2^2-1=3$ $3^2-1=8$ $4^2-1=15$ $5^2-1=24$</p>	<p>-The student must ensure that there is a regularity that each term is formed by a square minus 1, by observing other terms: $6^2-1=35$ $7^2-1=48$ $8^2-1=63$ $9^2-1=80$</p>	<p>-The student must formulate a conjecture for the nth term of the sequence is n^2-1</p>	<p>-The student must infer a general rule, returning to the first term he must notice that: $0^2-1=-1$ and which is not the first term of the sequence (0), so the rule is true for all terms starting with 1 hence the rule is: $\forall n \in \mathbb{N}^*; n^2-1$</p>
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 **Problem 02**

Problem 02	C1: Case observation	C2: Identification of a model	C3: Identification of regularities	C4: Formulation of a conjecture	C5: Generalization
<p>Let the powers of the number 13 be: 13,169,2197,28561 ... 1) Determine the units digit of 13^{25} 2) Determine all the exponents of the numbers that end in 3.</p>	<p>-The student must observe the powers given and at the same time observe the units digit: $13^1=13$, the units digit is 3. $13^2=169$, the units digit is 9. $13^3=2197$, \leq <i>chiffre des unités es</i></p>	<p>-The student must notice that the unit digits of the powers follow a cycle of 4 numbers:</p>	<p>-The student must identify the repeating units-digit cycle in powers of 13 (3, 9, 7, 1) and uses the remainder</p>	<p>Formulates a conjecture about the units digit of 13^n by identifying the cyclic pattern (3, 9, 7, 1)</p>	<p>-The student must generalize that the units digit of 13^{25} is 3. - The student must generalize that the</p>

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	$13^4=28561$, the units digit is 7. $13^5=371293$, the units digit is 3. $13^6=4826809$, the units digit is 9.	3,9,7,1 and then it starts again.	when dividing the exponent by 4 to determine the units digit.	using modulo 4.	exponent ending in 3 is: $\forall n \in \mathbb{N}$, 13^{1+4n}
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✚ **Problem 03**

Problem 03	C1: Case observation	C2: Identification of a model	C3: Identification of regularities	C4: Formulation of a conjecture	C5: Generalization
<p>1) Give $\forall n \in N$, the values of the number: n^2-n+11</p> <p>2) In justifying, what can we conclude about: n^2-n+11</p>	<p>-The student must calculate and observe the particular cases of n^2-n+11, in answering question 1</p> <p>For $n=0$ $0^2-0+11=11$</p> <p>For $n=1$ $1^2-1+11=11$</p> <p>For $n=2$ $2^2-2+11=13$</p> <p>For $n=3$ $3^2-3+11=17$</p> <p>For $n=4$ $4^2-4+11=23$</p> <p>For $n=5$ $5^2-5+11=31$</p> <p>.....</p>	<p>-The student must observe the results and other cases:</p> <p>$n=0 \rightarrow 11$ $n=1 \rightarrow 11$ $n=2 \rightarrow 13$ $n=3 \rightarrow 17$ $n=4 \rightarrow 23$ $n=5 \rightarrow 31$ $n=6 \rightarrow 41$ $n=7 \rightarrow 53$...</p> <p>-The student must identify that The expression n^2-n+11 produces prime numbers.</p>	<p>- The student must see the regularities that for</p> <p>$n=0 \rightarrow 11$ $n=1 \rightarrow 11$ n^2-n+11 gives 11 which is a prime number and for $n \geq 2$ the values of are also prime numbers</p>	<p>-The student must guess that: 11, 13, 17, 23, 31, 41...these are prime numbers therefore n^2-n+11 product of prime numbers seems to be true for the observed cases</p>	<p>-The student must generalize that $\forall n \in N$; n^2-n+11, is always first</p>


✚ **Problem 04**

Problem 04

C1: Case observation	C2: Identificatio n of a model	C3: Identificat ion of regularitie s	C4: Formulati on of a conjecture	C5: Generalization
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<p>Let two numerical functions be defined by: $f(x)=x^2$ and $g(x)=x^3$</p> <p>1) Complete the following table:</p> <table border="1" data-bbox="191 527 548 688"> <tr> <td>x</td> <td>-10</td> <td>-10</td> <td>-10</td> <td>-10</td> <td>-10</td> </tr> <tr> <td>$f(x)$</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>$g(x)$</td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table>	x	-10	-10	-10	-10	-10	$f(x)$						$g(x)$						<p>- Identifies that for large x, the function grows rapidly and reaches large values, and recognizes that $g(x)$ exhibits similar behavior while preserving its sign.</p>	<p>- Identifies that both functions are powers of x; $f(x)$ remains positive and increases rapidly, whereas $g(x)$ retains the sign of x and grows faster in magnitude.</p>	<p>-The student must identify that the more x increases in absolute value, the more $f(x)$ and $g(x)$ reach very large values. However, $f(x)$ it always remains positive while $g(x)$ the sign of follows x.</p>	<p>- Observes that as x increases, both $f(x)$ and $g(x)$ grow significantly; however, $f(x)$ is always positive, whereas $g(x)$ retains the sign of x.</p>	<p>-The student must generalize that: $\forall n \in \mathbb{N}^*$</p> $\lim_{x \rightarrow -\infty} x^n = \begin{cases} +\infty & \text{if } n \text{ is even} \\ -\infty & \text{if } n \text{ is odd} \end{cases}$
x	-10	-10	-10	-10	-10																		
$f(x)$																							
$g(x)$																							
<p>2) What can we conclude about the values of $f(x)$ and $g(x)$ when x takes on increasingly larger absolute values?</p> <p>3) Conjecture the limits: $\lim_{x \rightarrow -\infty} x^2$ and $\lim_{x \rightarrow -\infty} x^3$</p> <p>4) What can we conclude about $\lim_{x \rightarrow -\infty} x^n ; \forall n \in \mathbb{N}^*$</p>																							

✚ **Problem 05**

Problem 05	C1: Case observatio n	C2: Identificatio n of a model	C3: Identificatio n of regularities	C4: Formulation of a conjecture	C5: Generalizatio n
 <p>1) Make or draw the following 3 patterns 2) How many sticks do you need to make the 10th pattern? 3) How many sticks does it take to make the same pattern?</p>	<p>- Observes that the triangles are equilateral and that each new pattern adds one triangle sharing a side with the previous figure; counts the number of sticks to identify the growing sequence (3, 5, ..., 21).</p>	<p>-The student must identify that for each additional pattern, 2 sticks are added for the reason that each triangle shares a side with the previous triangle.</p>	<p>-The student must observe the regularity of the number of sticks: for each additional pattern there is an addition of 2 sticks.</p>	<p>-The student must conjecture that: Pattern 1: 3-stick triangle. Pattern 2: Adding a triangle (5 sticks). Pattern 3: Adding a triangle (7 sticks). -The student must guess the formula $2n + 1$ which expresses the numbers of sticks for each pattern: 3,5,7,9,11,13,15,17, 21...</p>	<p>The student should generalize that the number of sticks for the $n^{\text{ème}}$ pattern can be expressed as: $2n + 1 ; \forall n \in \mathbb{N}$</p>