Exploring Writing in Mathematics: A Case Study analyzing Grade 10 Classroom Practice through Four Types of Writing

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Abstract: Writing is an integral component of classroom instruction. This qualitative ***case study*** *examined the significance of teachers' writing as a critical component of mathematics instruction, focusing on how different types of writing contribute to effective classroom practices. Three video-recorded function lessons from one Grade 10 class were purposively sampled and analyzed to investigate their integration of the four writing forms during teaching and learning using thematic analysis. The teacher's writing was categorized into four distinct types, namely, exploratory, explanatory, argumentative, and mathematically creative writing. Utilizing classroom observations, we explored how these writing practices facilitate the communication of mathematical concepts, foster learner engagement, and support the development of critical thinking skills. Our findings indicate that varied writing approaches, enhancing teachers' ability to convey complex ideas and promoting a more interactive and dynamic learning environment were underutilized. This research underscores the necessity of incorporating diverse and structured writing strategies into mathematics teaching to elevate instructional quality and enhance students' understanding. The study concludes with recommendations for teachers and curriculum designers to prioritize development in writing as an essential element in mathematics education, ultimately aiming to enrich the overall learning experience for students.*

Keywords: Exploratory, Informative, Argumentative, Mathematically creative writing, Critical thinking.

INTRODUCTION

Writing is an important aspect of communication, especially in formal environments like schools and workplaces. In schools, teachers and students interact with one another verbally, but also in the form of writing. Currently, people prefer to use social media to communicate in writing form. This demonstrates that writing does not only take place formally but also informally at home and other places.

The concept of writing is not new in research; in the United States in particular, many authors published works on students' writing in mathematics and its influence on learning. However, its incorporation in teaching is still problematic in many countries. In mathematics most teachers rely more on the writing of symbols and diagrams (images), with limited use of words. It is, therefore, essential that such concepts resurface in research to get a better understanding of how they are incorporated into mathematics teaching and their contribution to learning. Generally, writing is a powerful tool for learning, enabling students to articulate ideas, reason through problems, and explore concepts. However, there is still a lack of empirical research in writing (Herbert & Powell, 2016). Through writing, teachers can see the level of students' understanding, knowledge gaps, and misconceptions, thus providing opportunities for remedial work or intervention (Graham et al., 2020). Therefore, writing provides evidence of students' thinking and understanding of learning (NCTM, 2014).

Researchers support writing in classrooms. For instance, Silva and Limongi (2019) contend that writing about the subject matter aids the learning process, whereas Galbraith and Baaijen (2018) created a framework indicating that writing prompts learning and stimulates cognitive processes. School students still depend on their teachers' guidance to be proficient in mathematics. To do this, teachers need to shift from conventional ways of teaching and embrace teaching strategies that enhance learner engagement and subsequently, understanding (Zeeshan, 2022). By modeling good and effective writing, teachers can influence students to engage in writing activities to enhance their understanding of mathematical concepts. However, some teachers are still reluctant to move away from traditional teaching methods and embrace new or different ways of teaching (Engelbrecht & Borba, 2024). Writing provides affordances of multimodal representation of mathematical ideas and allows students to think beyond talk (Casa et al., 2016).

Graham et al. (2020) argue that,

> "*A variety of different types of writing should support academic learning, including using writing to summarize information, compare and contrast ideas, connect new and old information, describe one or more processes, explain how something works, create a story or poem to illustrate or extend ideas, construct analogies, and build an argument*" (p. 181).

The action words used (summarize, compare and contrast, explain, and create or extend), link directly with Casa et al.'s (2016) writing types namely, exploratory, informative/explanatory, argumentative, and mathematically creative writing. These writing types offer distinct yet complementary approaches to engaging with mathematics in the sense that using one unconsciously leads to another. This study analysed the extent to which these writing types were evident in observed lessons on functions and explored their potential to deepen mathematical understanding and expression. Although they may not all be used at once, however, the integration of these types of writing in mathematics classrooms has a powerful influence on students' ability to express ideas to enhance understanding.

Casa et al. (2016) describe the four writing types as follows:

1. Exploratory Writing**:** Involves brainstorming, questioning, and reflecting to make sense of mathematical concepts.
2. Informative/Explanatory Writing**:** Focuses on clearly explaining mathematical processes, definitions, or solutions.
3. Argumentative Writing**:** Encourages students to justify solutions, critique reasoning, and engage in mathematical debate.
4. Mathematically Creative Writing: Involves generating original problems, exploring patterns, and creating unique solutions.

Teachers' instructional strategies have a big impact on how mathematical topics, primarily functions, are taught and learnt (Yang & Kaiser, 2022). With an emphasis on functions, this study intends to investigate the different forms of writing that mathematics teachers in grade 10 employ in their instruction. Knowing the kinds of writing that are frequently incorporated into mathematics classes helps teachers explain difficult ideas and involve students in worthwhile learning activities. Additionally, we explore the ways in which Grade 10 mathematics teachers encourage their students to write. Writing in mathematics is more than just an extracurricular activity; it is an essential tool for students to express their comprehension, make sense of their ideas, and evaluate their learning (Shen & Chong, 2023). We also emphasize the value of writing as a crucial part of mathematics instruction by identifying teachers' strategies used to encourage writing. Finally, the study looks at how Grade 10 maths teachers' writing techniques improve students' understanding of functions. Teachers can model mathematical reasoning, dispel misconceptions, and promote student interest in the material by using good writing. The goal of this research is to better understand how writing and mathematics instruction interact by examining how teachers' writing affects students' learning outcomes related to functions. This will help teachers improve their pedagogical strategies and promote better content comprehension and mathematical achievement.

To achieve these objectives, the study seeks to answer the following research questions:

- What types of writing do Grade 10 mathematics teachers commonly use in their teaching of functions?
- What opportunities do Grade 10 mathematics teachers create to promote writing for their students?
- How do Grade 10 mathematics teachers' writing enhance the learning of functions?

LITERATURE REVIEW

Writing in mathematics has garnered increasing attention as a pedagogical tool for deepening understanding and fostering communication. Unlike other writing genres, mathematical writing necessitates a combination of general writing skills, mathematical content knowledge, and discipline-specific abilities like using mathematical representations, using precise mathematical language, and combining equations and expressions with words (Colonnese, 2023). Research highlights that incorporating writing in mathematics classrooms helps students clarify their thinking, make connections between concepts, and express reasoning (Burns, 2004). Writing-to-learn frameworks emphasize that writing enables students to move beyond rote memorization and engage with ideas on a conceptual and analytical level (Minasyan & Supriatna, 2024). Incorporating writing in mathematics can be a panacea for poor achievement which is a continuing problem in schools because when writing, students become engaged with content, and subsequently learning is supported (Graham et al., 2020).

Exploratory Writing

Exploratory writing allows students to reflect on their understanding, pose questions, and brainstorm ideas. It enables students to use their own words when trying to understand the problem (Casa et al., 2016). Students can also use their mother tongue to help them explore the problem (Colonnese, 2023). This form of writing aligns with constructivist theories of learning, which emphasize active engagement and the co-construction of knowledge (Vygotsky, 1978). Therefore, in exploratory writing, students can make drafts and represent a problem in different ways as they try to answer the "what" questions in a mathematical problem. In this way, exploratory writing assists in learning and promotes understanding. Studies suggest that reflective journals and informal prompts encourage students to engage in metacognition, helping them identify gaps in their understanding and articulate new insights (Lavi et al., 2019).

Informative/Explanatory Writing

Informative writing involves describing mathematical processes, definitions, and solutions clearly (Arsenault et al., 2024), and it is crucial for building procedural fluency and ensuring students can communicate their understanding. Explanatory (informative) writing deals with the "how" questions that require students to describe procedures. Because these questions require students to regurgitate procedures already learned from their teachers, if used excessively it can limit their ability to use writing to reason mathematically (Colonnese, 2023). Explanatory tasks such as summarizing steps to solve an equation or defining a mathematical concept help reinforce learning by requiring students to organize their thoughts logically.

Argumentative Writing

A mathematical argument is a description of how one arrived at a certain conjecture (Sriraman & Umland, 2020). Argumentative writing encourages students to justify their solutions and critique the reasoning of others in a mathematics discourse where the audience is fellow students and their teacher (Colonnese, 2023). Incorporating arguments in teaching aligns with the Standards for Mathematical Practice outlined by the National Council of Teachers of Mathematics (NCTM), which emphasizes constructing viable arguments and critiquing the reasoning of others (NCTM, 2000). Research shows that engaging in mathematical argumentation fosters critical thinking and helps students develop deeper conceptual understanding (Knipping, 2008). Colonnese (2019) proposed that teachers should ask students "why" questions to encourage them to explain mathematical ideas to their peers. Toker (2021) supports this view by stating that teachers can benefit from students' explanations of their thinking because they can recognize the sources of students' misconceptions. Therefore, prompting students to reason develops their cognitive thinking skills and fluency in communication. It also builds a foundation for mathematical proofs required in higher classes (Colonnese, 2023).

Mathematically Creative Writing

Creativity in mathematics often involves problem generation, pattern exploration, and the formulation of novel solutions. Mathematically creative writing encourages students to approach problems from multiple perspectives and express their unique ideas, extending them beyond what they have learned (Colonnese, 2023). Research by Nilimaa (2023) underscores the importance of fostering creativity in mathematics to cultivate innovation and adaptability. Furthermore, students who understand the learning content can create mental representations of a concept, which leads to good problem-solving skills (Toker, 2021). Therefore, giving students tasks that require the creation of their problems and solving them would not only incorporate mathematically creative writing but also assist teachers in seeing if learning has occurred.

CONCEPTUAL FRAMEWORK

This study is grounded in a framework that integrates Casa et al. (2016) four writing types namely, exploratory, informative/explanatory, argumentative, and mathematically creative writing as essential components of mathematical pedagogy. The conceptual framework is guided by the following principles:

Writing as a Tool for Learning

Researchers (Santos et al., 2019; Van Schalkwyk et al., 2019) encourage active learning and assert that students learn better when actively involved in the learning process. However, active learning seems to be a problem and is insufficiently incorporated in most classrooms (Reinholz et al., 2022). One way of actively engaging students in learning is by writing. Writing allows students to process and internalize mathematical concepts. Exploratory and informative writing helps students build a foundational understanding of mathematical concepts (Kooli, 2023). With exploratory writing, students are allowed to explore on their own or in groups to make sense of the problem before teaching commences, while informative writing assists in describing the problem (Casa et al., 2016). Therefore, informative writing activities require students to use words to describe mathematical concepts or situations. In argumentative writing, students should provide justifications for their reasoning (Colonnese, 2019).

Communication and Reasoning

Communicating effectively in mathematics is significant to learning (Sari & Fauzi, 2025). Mathematics is a language of logic and structure (Coregliano & Razborov, 2020), consisting of semiotic and grammatical aspects. Semiotics involves the use of signs (words, images, and objects), and grammatical aspects which entail words, phrases, clauses, and sentences (Cholili & Lapele, 2020). These (signs and words/sentences) are then used by teachers and students to communicate mathematical ideas. Mathematics contains technical vocabulary that may not carry the same meaning as ordinary words (Lin, 2021). This is where symbols are integrated to distinguish technical vocabulary from words used in everyday language. It is impossible to teach or study mathematics without carefully utilizing mathematical tools. These tools include words, symbols, and images.

Writing provides students with a medium to communicate their reasoning, critique arguments, and justify solutions, fostering critical thinking and collaborative learning. Teachers are more likely to spark lively discussions involving cognitive thinking when they ask questions that challenge students' knowledge retention (Gentaz & Richards, 2022; Slate & Charlesworth, 1988).

Creativity and Exploration

Mathematics is inherently creative, involving pattern recognition, problem-solving, and innovation. These characteristics are at play in individuals who have reached a level of conceptual understanding. Mathematically creative writing allows students to explore these aspects, encouraging flexibility and originality in their thinking. Mathematically creative writing involves students' creations to demonstrate a deeper understanding of mathematical content. Therefore, argumentative and creative writing both encourage deeper engagement and application, which are traits of cognitive engagement.

Holistic Engagement by Integrating the Four Writing Types

Holistic engagement holds a multi-dimensional perspective in learning (Shen & Chong, 2023). Among others, it constitutes learner engagement in perspectives such as cognitive, affective, and social (Wong & Liem, 2022). The cognitive aspect requires students to engage with the subject content through being actively involved in learning, that is, by reading, writing activities, and responding to questions. Wong and Liem further state that in the affective aspect, students learn to appreciate knowledge when it is contextual and relatable, while the social aspect develops students into creating relationships when learning, and this can be developed through cooperative learning. The cognitive dimension can be developed by integrating the four writing types in mathematics teaching and learning. This holistic approach ensures that students develop technical skills, confidence, and curiosity when communicating mathematical ideas.

Effective mathematics instruction integrates multiple writing types to address diverse learning goals. Studies by Pugalee (2004) demonstrate that combining explanatory, argumentative, and creative writing tasks promotes both procedural fluency and conceptual understanding. Additionally, providing students with varied writing opportunities ensures that they can engage with mathematics as both a logical and creative discipline (Bicer, 2021). Figure 1 is a framework showing how teachers can interact with students, giving them tasks utilizing the four writing types.

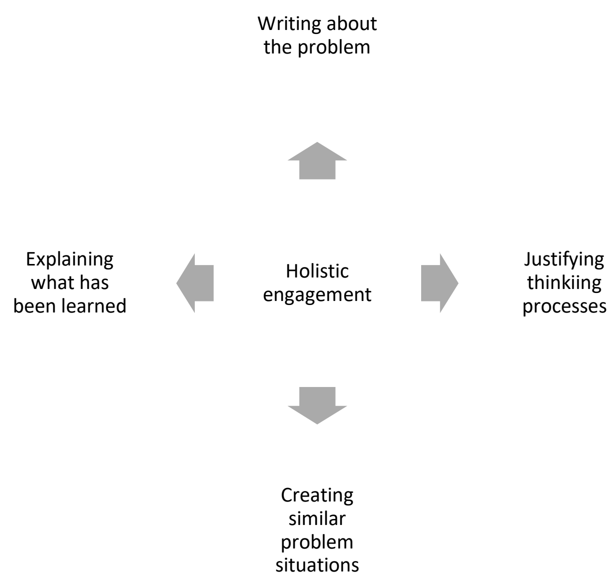

Figure 1: Holistic engagement of students utilizing the four writing types

Figure 1 shows that utilizing all four writing types assists in the holistic engagement of students. The four aspects in the holistic engagement framework are unpacked as follows:

> ***Writing about the problem***: Teachers can allow students to work in groups as they write about how they see and understand the problem, and the strategies they can apply in solving it. For example, they can be asked to explain how a linear function graph will be affected if its slope changes. When asking such questions, teachers can provide visuals/diagrams to assist students in exploring the problem. This allows students to put their thoughts into

words when exploring different facets of a problem (Sa'adah, 2020). They combine exploratory and explanatory writing to express ideas that involve organization of information and reflection, which leads to critical thinking and greater insights about the problem (Zulyusri, 2023). Therefore, students are not only cognitively engaged, but they also develop a personal connection with the study material.

Justifying thinking processes: This can be prompted by asking open-ended questions (Kim et al., 2021) to allow students the opportunity to critically evaluate their thinking and consider alternative perspectives to a problem using argumentative writing. Teachers should encourage them to "think aloud" so that they can be helped to phrase their arguments in collaborative environments. Prompting students to justify their reasoning assists in developing open-mindedness in problem-solving (Huang & Sang, 2023), including an understanding that problems may have different solutions and various factors might influence these solutions.

Creating similar problem situations: This is a higher level of thinking in mathematically creative writing that involves the application of learned content, showing mathematical understanding. It also shows relevance and applicability of the learned content beyond the original problem, thereby solidifying the learning process. Prompting students to generate problems similar to those given in teaching activities and solve them (Nilimaa, 2023) can be a clear demonstration of their understanding. This can be done in informal assessments where teachers can ask students to generate problems related to those given.

Explaining what has been learned: This is a reflective practice that utilizes explanatory writing and requires students to share their personal experiences with the learning material (Marshall et al., 2021). This can be done by asking students to write journals or mathematics essays where they reflect on their learning. When teachers prompt students to write about what they have learned, they reinforce their engagement with the study material by synthesizing information and connecting various parts of the content (Graham et al., 2020). They are compelled to think as they write about strategies that worked or did not work, and how they can approach such problems in their future learning.

When these four aspects are applied in teaching and learning, students engage holistically and naturally, allowing them to engage with the study material in a deep and interconnected manner, enhancing understanding and knowledge retention.

It shows that each form of writing does not necessarily lead to another, but can be utilized on its own. However, if all are utilized, students can be mathematically proficient.

The framework emphasizes the interplay between writing types and their collective potential to enhance mathematical proficiency. In analyzing classroom practice, this study seeks to identify how these writing types are employed in classrooms and how they can be better integrated to support diverse students.

METHOD

Research Approach, Design and Sampling

The study utilized qualitative data collection methods to get deeper insights into the teacher's writing activities and how his writing enhanced the learning of functions in general. A qualitative case study was conducted in one of the South African secondary schools in 2024 to get deeper insights about teachers' writing in Grade 10. There was only one Grade 10 mathematics teacher at the selected school, therefore, the teacher was selected as the only participant in the case studied. The participant was selected based on the researchers' judgment that he would generate the required data for the study. The teacher offered five linear function lessons to **42 learners**, and from the five, the first three were sampled to get a continuation of how the teacher utilized writing in his teaching. In addition, there was only one participant, therefore, we needed sufficient data to corroborate the discussion. Moreover, the researchers felt that the first lesson would serve as the introduction to the linear functions topic and would not provide clear and sufficient evidence of the teacher's writing.

Data Collection and Analysis

Video-recorded lesson observations were conducted because videos capture real-time context of the classroom environment (Cheung et al., 2021). Additionally, they provide visual and auditory evidence that complements qualitative data. Data were transcribed verbatim from the three lessons observed. To attain the study's credibility, researchers had a prolonged engagement with data by observing three lessons from the same class, transcribing the recorded lessons, and reading the data several times during transcription and analysis. The data from the transcribed lesson observations included stammering, pauses, and repetition of words as spoken by the teacher (Moser & Korstjens, 2018). Thematic analysis was used to analyze data in the study because it is flexible and can be adapted to a wide range of research questions and data types (Helou, 2020). Furthermore, it allows for the inclusion of diverse voices and perspectives, hence it is suitable for studies aiming at understanding complex social phenomena (Yamamoto & Keogh, 2018). The analysis outlined the importance of including the different types of writing in teaching functions, and how they connect to enhance learning (Jossey et al., 2021). The lessons were analyzed to identify instances where the four writing types were either explicitly incorporated or could have been utilized to enhance learning. Data analysis was specifically focused on emerging patterns that were associated with pre-determined themes, namely:

- The teacher's instructional strategies.
- Opportunities for student writing and verbal expression.
- The alignment of discourse with each writing type.

The pre-determined themes guided the researchers to focus only on the teacher's writing activities and their alignment with Casa et al.'s (2016) four types of writing. Steps followed in analyzing data were: 1) Reading and transcribing data for familiarization and immersion, 2) Data coding, 3)

Comparing codes with predetermined themes, 4) Naming and articulating what each theme represented, and 5) Reporting the findings

To enhance the trustworthiness of the findings, raw data transcripts from lesson observations were included. **The researchers gave the teacher participant the pseudonym of "Mike" to ensure confidentiality.**

RESULTS

In answering the research questions, data were coded according to the intended objective of the words spoken and a sample of these are presented in Table 1.

Codes Identified

Code	Strategy/technique used	Supporting Text
Explanation/ definition:	Informative writing	*"So, when it comes to functions, the functions can be, we can have many, er, many to one, where more than one x-value will have more than one y-values."* *For, for as long as it will be able to touch the graph ga one fela (only once), it means the graph satisfy the condition of being a function.*
Student engagement	Question and answer	*At how many points am I touching the graph?* *"What do you think will happen to the mother graph, compared to the, the graph that we are going to draw next? Will it go up when we add 2, or will it go down?"*
Critical thinking prompts	Argumentative/ Exploratory writing	*So you think this graph passes the vertical line test? Does it pass?*

Table 1: Codes identified from data transcripts

Coding used did not focus on the frequency of words, but on meaning (Hennink & Kaiser, 2020); therefore, they were later synthesized into findings using predetermined themes listed in the data collection and analysis of the methodology section.

Exploratory Writing

Exploratory writing was minimally evident in the observed lessons. The teacher occasionally posed reflective questions, such as, "*So, I want us to check, what do you think will happen to the mother graph, compared to the, the graph that we are going to draw next? Will it go up when we add 2, or will it go down?"* Students rarely had opportunities to respond to such questions and explore these ideas through written reflection. The teacher prompted students to think, however, he guided them toward the correct answer by saying, "*Will it go up when we add 2, or will it go down?"* In this way, students were allowed limited opportunities to explore. Although exploratory writing opportunities were limited during teaching, some activities did include some form of exploration, see Figure 2.

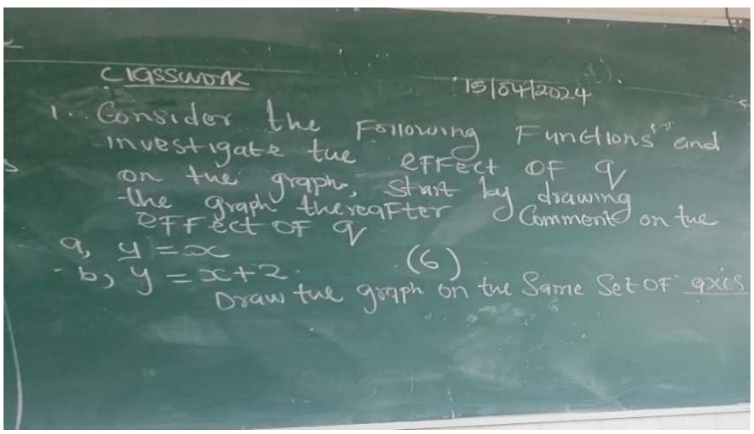

Figure 2: Mike's first activity on linear functions

In the activity, students were asked to explore or investigate the effect of q in linear function graphs. The activity required students to draw two graphs on the same axis before comparing them. It combines exploration with graph-drawing procedures (informative writing) that students should have in their activity books, showing that types of writing complement one another to develop students holistically. However, the exploratory writing form incorporated in the activity is minimal and at a foundational stage since students' answers could be in the form of a word or short phrase. For example, after sketching the two graphs, students could respond by saying, q increased or the graph shifted up. Moreover, exploration could have been part of the teacher's activities during lessons to encourage critical thinking before assessing the students. Therefore, integrating exploratory journals or quick writing before teaching a concept could encourage students to brainstorm and connect ideas more deeply.

Informative/Explanatory Writing

Informative writing was a dominant feature of the lessons. The teacher often explained mathematical concepts, such as defining a function: "A function is a mathematical relationship where each input has exactly one output" and "*So, when it comes to functions, the functions can be, we can have many, er, many to one, where more than one x-value will have more than one y-values.*" Students were prompted to record these definitions and explanations in their notes, reinforcing procedural fluency. Furthermore, the teacher gave more explanations instead of allowing for exploration, hence limiting students' opportunities to engage critically or explore. For instance, Mike explained:

> *Mike: So what do we do to check the vertical line test, you can take a ruler and check at one point or at one x-value how many times will the vertical line touch the graph. The line that you have drawn in broken lines, how many times is it going to touch the graph?*
>
> *Students: Two*
>
> *Mike: Two? How many times? You can take a ruler, and someone won't give you a ruler. This is my graph let's say this is f(x), how many times am I touching it here at this point? Am I touching this graph?*
>
> *Students: No*
>
> *Mike: no neh? And then here? Am I touching?*
>
> *Students: Yes (chorus).*
>
> *Mike: How many times? At how many points am I touching the graph?*
>
> *Students: Two. Am I touching it here? If the ruler is like this, am I touching it here? Here, here?*
>
> *Students: Mhm (chorus).*
>
> *Mike: Only here neh?*
>
> *Students: Yes.*
>
> *Mike: At one point. I am shifting now to a different x-value of a negative one. How many times am I touching the graph at this point?*
>
> *Students: One*

In the lesson observation transcript above, students were asked yes or no questions, while the teacher explained the procedure for conducting the vertical line test. Furthermore, the activities given also reinforced procedural fluency which is linked to informative writing, see Figure 3.

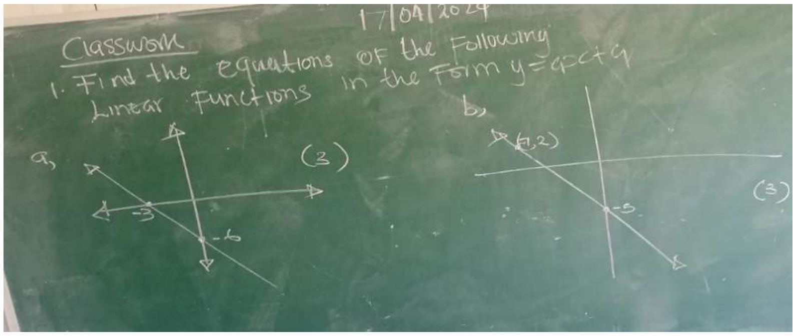

Figure 3: Mike's second activity on linear functions

Both the transcript and the activity show that the teacher relied more on explanatory (informative) writing and students had limited opportunities to create their explanations, which could have strengthened their conceptual understanding and assisted in identifying gaps and misconceptions.

Argumentative Writing

Argumentative writing, where students justify their reasoning, was occasionally encouraged but not systematically integrated. For example, during discussions on the vertical line test, the teacher provided a reason why the graph failed the vertical line test, saying:

> *Mike: Ga one (once). I am only touching it here, you see? For, for as long as it will be able to touch the graph ga one fela (only once), it means the graph satisfy the condition of being a function.*

Mike used another example and after using a ruler for the vertical line test, he said:

> *Mike: So you think this graph passes the vertical line test? Does it pass?*
>
> *Students: No.*
>
> *Mike: The vertical line test you only pass it when the ruler touches the graph once.*

Instead of asking students to explain why the graphs failed or passed the vertical line tests in both instances, Mike provided the reasoning behind his questions, which in a way, encouraged arguments. However, this demonstrates that the teacher provided limited opportunities for students to be exposed to argumentative writing because students were not allowed to respond to the "why" questions. Instead of asking why the graph does not pass the vertical line test, the teacher immediately substantiates it by providing the reason. Moreover, these instances lacked written justifications as they were only verbal utterances. Therefore, structured activities like writing mathematical proofs or defending solutions could enhance students' argumentative skills.

Mathematically Creative Writing

Mathematically creative writing was largely absent in all the lessons. The latter focused more on procedural tasks, such as plotting graphs and solving equations, with limited opportunities for students to pose original problems or explore patterns. Students were given activities and not encouraged to create their functions or problems. The activities given also tested students' skills in drawing graphs and procedures, see Figures 2 and 3 under exploratory and informative writing respectively. Interactive engagements during lessons were also about procedures when doing calculations.

Encouraging mathematically creative writing, by incorporating activities like asking the students to create a function that meets certain criteria or write a story involving a linear function was absent during lessons and in the activities. Such questions could foster students' creativity and effective engagement with the study material.

DISCUSSION OF THE FINDINGS

The Alignment of Discourse With Each Writing Type

In answering the first research question, *What types of writing do Grade 10 mathematics teachers commonly use in their teaching of functions?,* the results of this study revealed that students were exposed to explanatory (informative) writing more than any other type of writing when they described and defined functional concepts (Casa & Choppin, 2020; Colonnese, 2023). They demonstrated the sketching of graphs, calculations involving substituting in formulas, and testing whether graphs were functions or not. This type of writing focuses on talking more about facts and details and is not necessarily about understanding. It encourages regurgitation of procedures, which in turn promotes rote learning. The teacher's strategy of talking and explaining more during lessons reflects the traditional way of teaching where they transmit knowledge and students become passive recipients (Spiteri & Chang Rundgren, 2020). The attempt to incorporate exploratory writing was limited to certain sections of the content whereas informative writing was incorporated throughout the lessons, covering every section of the functions topic. Similarly, argumentative and mathematically creative writing was absent. Therefore, students were allowed limited opportunities for writing in function lessons because they were exposed to explanatory writing more than other types of writing. The National Council of Teachers of Mathematics (NCTM) (2000) recommends that students be encouraged to communicate in writing form during teaching and learning to allow them to express ideas. Learning environments incorporating extensive writing provide opportunities for practical engagement where students express ideas and reasoning behind their thinking verbally and in writing. This crucial learning feature is barely present in the observed lessons.

Opportunities for Student Writing and Verbal Expression

For the second research question *"What opportunities do Grade 10 mathematics teachers create to promote writing for their students"?* the results revealed that the students' responses to questions were mostly yes or no answers, confirming the teacher's actions and procedures. Additionally, questions were about algorithms and procedures, also encouraging informative writing. However, some sections of work as prescribed in the CAPS required students to compare graphs and investigate the effect of a and q on linear and quadratic functions. Questions asked in these sections enabled students to respond in words or phrases to describe the behavior of the graphs. In these sections, the type of questions used encouraged students to utilize foundational explanatory writing where they described mathematical situations in a few words, such as decrease or increase. Therefore, limited opportunities were provided to promote writing for students. The results support Powell et al. (2020) when they said many educators regard writing as important but still struggle to incorporate it into mathematics teaching.

The Teacher's Instructional Strategies

In addressing the third research question, *"How do Grade 10 mathematics teachers' writing practices enhance the learning of functions?"* Our findings revealed that the emphasis on explanatory writing in teachers' instructional materials led to an informative approach to teaching functions. This type of writing primarily involves the reproduction of factual knowledge, which, in turn, promotes rote learning and memorization of procedures. Although students remained engaged throughout the lessons, the questions posed focused mainly on procedures and algorithms. Unfortunately, a holistic approach incorporating all four writing types was not observed. The results highlight a lack of proficiency among teachers in integrating mathematical writing into their instructional practices, thereby supporting Powell et al.'s (2020) argument that teachers continue to struggle with incorporating mathematical writing into their teaching.

On the one hand, exploratory writing sparks curiosity, enhances engagement with content, and develops an individual's investigative and explorative skills. On the other hand, students involved in argumentative writing can interact with peers to advance arguments and justify their reasoning.

These kinds of environments support learning and conceptual understanding because teachers can use peer assessment to critique students' work and provide feedback. Therefore, instead of providing detailed conceptual explanations, teachers could lead inquiry-based teaching to allow students to explore, argue, and justify their thinking (Evans & Dietrich, 2022). Peer assessment is another form of student engagement that enhances self-regulated learning (Segaran & Hasim, 2021). Depriving students of the opportunity to engage in arguments limits their communication and critical thinking skills. It also inhibits their ability to interact effectively with the learning material and their peers, which is an aspect of socio-cognitive theory. Lastly, mathematically creative writing displays a high level of conceptual understanding because students create problems and solve them using previous knowledge. However, in this study, the teacher provided limited opportunities for students to create their own problems. Leikin and Sriraman (2022) advocate for the advancement of creativity in mathematics learning. Without creativity, mathematics teaching and learning would be a mere reproduction of facts and procedures that encourage rote learning. Therefore, the creative aspect of mathematics learning was under-developed. Teachers utilized informative writing rather than exploratory, argumentative, and mathematically creative writing, placing holistic engagement in a compromised state. When one writing type is dominant over others, as is the case with the observed lessons, students' communication and critical thinking skills, and creativity become underdeveloped.

CONCLUSION AND RECOMMENDATIONS

This paper examined the use of four writing types—exploratory, informative/explanatory, argumentative, and mathematically creative writing within a Grade 10 mathematics classroom. Sure, here's a refined version of your paragraph in clearer and more polished English:

The findings revealed a strong emphasis on explanatory writing, often at the expense of other forms of writing. This focus was evident in the teacher's explanations and definitions of concepts. Other types of writing such as exploratory, argumentative, and mathematically creative writing, were largely underutilized. As a result, learners were not given adequate opportunities to explore or engage in critical discussions about mathematical concepts.

Drawing on transcription data from lessons on functions, the study identified how each writing type could enhance mathematical understanding, communication, and creativity. The paper emphasizes the importance of integrating diverse writing practices to foster holistic engagement in mathematics learning.

The four types of writing offer a rich framework for enhancing mathematical learning. While the observed lessons provided a strong foundation in explanatory (informative) practices, integrating exploratory, argumentative, and creative writing could promote more profound conceptual understanding and student engagement. By embracing a multi-faceted approach to writing, mathematics teachers can empower students to develop mathematical thinking and problem-solving skills, and communicate effectively.

To integrate the four writing types more effectively, the study recommends:

1. Introduction of reflective journals or prompts encouraging students to speculate and connect ideas in exploratory writing.
2. Provision of opportunities for informative writing by asking students to explain concepts in their own words, both verbally and in writing
3. Implementing tasks requiring students to provide written justifications, proofs, or debates about solutions.
4. Encouraging students to generate original problems, explore patterns, and create narratives involving mathematical concepts.

Although we provide specific recommendations for each writing type, it is important to recognise that all four types of writing complement one another and can be integrated into a single task. However, to do this, teachers need to invest more time in planning and designing challenging tasks incorporating the four writing types. Additionally, students should be guided through similar tasks prior to the implementation of the designed tasks to achieve optimal results. Lastly, policy-makers should include mathematical writing in mathematics educational policies to promote its integration into teaching.

LIMITATIONS

The findings were derived from three function lessons taught by a single Grade 10 teacher; therefore, they cannot be generalized to a greater population. Future research should consider a broader pool of participants that includes different grades and topics to either affirm the findings or yield different results.

ACKNOWLEDGMENTS

We acknowledge the use of AI, namely, Chatbot, in writing some paragraphs in improved English and Grammarly, in editing the document to reduce grammar and spelling errors.

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