Improving Understanding of Trigonometric Functions Using GeoGebra

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Abstract: This study aimed to investigate the effect of the digital tool GeoGebra on vocational school students' understanding of trigonometric functions. The research used a quasi-experimental approach with a single group of students (N = 20), gathering both quantitative and qualitative data via pre-tests, post-tests, and structured interviews. Students initially engaged in regular instruction, followed by a number of interactive activities in GeoGebra. The findings indicated that the difference between pre-test and post-test outcomes was not statistically significant; nonetheless, the effect size (r = 0.46; g = 0.4742) suggests a moderate practical importance of the intervention. On the other hand, the interview findings indicated that students regard GeoGebra as an effective instrument for visualizing, accelerating problem-solving, and minimizing errors. They highlighted that the integration of regular lessons and interactive digital activities enhanced their understanding of the topic and increased motivation to learn. The findings suggest that incorporating GeoGebra into teaching trigonometry, even for short time as was the case in this study, can enhance the comprehension of abstract mathematical ideas and improve student interest and understanding.

Keywords: trigonometric functions, GeoGebra, quasi-experiment, vocational school students

INTRODUCTION

Due to its broad range of applications in science, engineering, and technology, trigonometry has become an important component of mathematics education. However, it also presents substantial challenge for both students and teachers. As Delice and Roper (2006) noted, the effectiveness of instruction in trigonometry often depends heavily on the teaching approach: some methods focus mainly on the mechanical application of trigonometric formulas, often at the expense of conceptual understanding and proofs, while others emphasize formal algebraic reasoning, which can reduce student engagement. In Croatia, where this research was conducted, trigonometry is introduced in the first year of upper secondary school (students age 15-16) and continues to be taught in the second and third grades, with varying degrees of emphasis (MZO, 2019). Despite its early inclusion in the curriculum, traditional teaching methods often fall short in addressing students' conceptual difficulties, especially in relation to functions and their graphical representations, which many students perceive as abstract and challenging.

In recent years, the integration of digital tools in mathematics instruction has gained an increasing attention, both as a means of modernizing teaching practices and as a way to enhance students' understanding and motivation. Among these tools, GeoGebra stands out as a widely used, free, interactive software that combines dynamic geometry with algebraic, graphical, and numerical representations (Hedi et al., 2023; Ziatdinov & Valles, 2022). Its usefulness in the teaching of functions has been particularly highlighted: GeoGebra enables students to manipulate parameters and instantly observe corresponding changes in the graph, thereby promoting exploration, conceptual insight, and intuitive understanding (Bedada & Machaba, 2022). This potential is particularly important in vocational education, where students frequently report low motivation to learn mathematics, often perceiving it as disconnected from their future professions (Schmid et al., 2021). This paper investigates the impact of using GeoGebra in teaching trigonometric functions to vocational school students enrolled in a program for electrical technicians whose digital skills and career orientation made them especially receptive to applied and technology-enhanced instruction. The study adopts a teaching-research approach, where a teacher-researcher designs, implements, and evaluates instructional interventions based on reflective practice. Specifically, the study aims to examine how the use of GeoGebra influences students' cognitive and affective characteristics. Therefore, we posed the following research question: *How does the use of GeoGebra affect vocational school students' understanding and engagement with learning trigonometric functions?*

LITERATURE REVIEW

Difficulties and Misconceptions Related to the Concept of Functions

Mathematics curricula around the world attempt to help students build competency in understanding and applying functions. This learning process begins in the early grades, when students investigate patterns; continues through middle school with activities involving covarying quantities; and concludes in secondary school with the formal study of functions as mappings between sets (Carlson & Oehrtman, 2005; Cooney et al., 2010; Hatisaru & Erbas, 2017). Difficulties in understanding the concept of functions emerge as early as students' first exposure to the topic (Panaoura et al., 2015; Walde, 2017; Widada et al., 2020). Numerous studies confirm that most secondary school students are unable to define a function clearly and precisely. Typically, they find it easier to provide concrete examples than to formulate coherent and accurate definitions (Dogan-Dunlap, 2007). However, a further issue is that the examples they offer are often mathematically incorrect, reflecting intuitive yet flawed conceptions of functions (Panaoura et al., 2015; Dogan-Dunlap, 2007). These challenges are exacerbated by students' limited use of precise mathematical terminology. Without mastering mathematical language, they find it difficult to express the formal meaning of a function (Panaoura et al., 2015). The abstract nature of the concept further hinders understanding, and textbooks often contribute to misconceptions by presenting different definitions of functions. Although these definitions generally convey the same mathematical meaning, students frequently perceive them as unrelated or even contradictory (Widada et al., 2020).

A particularly common obstacle is misunderstanding of mathematical notation, especially expressions such as $f(x)$=x, where students often struggle to interpret the role and meaning of the symbol "x" (O'Shea et al., 2016; Sajka, 2003; Walde, 2017). Related to this issue is the difficulty to transition between different representations of functions, such as moving from algebraic expressions to graphical forms and vice versa (Panaoura et al., 2015; Walde, 2017). Because the fundamental concept of the function often remains unclear, students frequently fail to connect algebraic notation with the representation of ordered pairs in a coordinate system (Eames et al., 2021). Furthermore, they often misinterpret function graphs as literal depictions of real-world situations rather than as visualizations of the relationships between input values (on the x-axis) and corresponding output values (on the y-axis) (Carlson & Oehrtman, 2005). They commonly overlook the fact that the values on the y-axis represent the codomain of the function, and that the notation $f(x)$=y expresses the mathematical relationship between two sets (Parhizgar et al., 2021).

These difficulties become especially pronounced when learning trigonometric functions. According Hamzah et al. (2021), students exhibit various misconceptions, ranging from the incorrect application of trigonometric formulas and computational errors when determining angles to a weak grasp of the function concept in the context of trigonometry. They are often confused by expressions such as $sinx$, as they struggle to distinguish between the symbolic representation of a function and its specific value at a given angle. The study in this paper focuses specifically on trigonometric functions.

Use of Technology in Mathematics Education

The integration of technology into mathematics education has transformed how students engage with mathematical concepts. Among the most influential tools are dynamic mathematics software (DMS) applications, formerly referred to as dynamic geometry software (DGS), such as GeoGebra, Cabri, Sketchpad etc. (He et al., 2025). A key strength of DMS lies in its ability to make abstract concepts more accessible. These programs allow students to interact with mathematical objects in real time by dragging points, modifying shapes, and dynamically measuring properties (Baccaglini-Frank, 2019). By enabling students to manipulate objects and test mathematical properties, for example, transforming a triangle and observing which characteristics remain unchanged, learners move beyond surface-level observation toward deeper theoretical insight. This hands-on approach supports intuitive and reflective mathematical thinking (Segal et al., 2018). Furthermore, DMS has proven especially valuable in promoting inquiry-based learning. Guan et al. (2024) found that, compared to physical manipulatives, digital tools like DMS more effectively support both immediate learning gains and long-term retention.

Research consistently shows that such tools can enhance a wide range of mathematical skills. A recent meta-analysis by He et al. (2025), which synthesized 68 effect sizes from as many studies, found a strong overall effect on K–12 students' mathematics learning (Hedges' $g = 0.820$, $p < .01$). While the overall gains were substantial, the effects were somewhat smaller in areas related to spatial reasoning. This finding suggests that the effectiveness of DMS may depend on the specific mathematical domain and underscores the importance of aligning digital tools with targeted instructional goals. When thoughtfully implemented, tools like DMS tools not only improve outcomes but also foster more engaging and meaningful learning experiences.

GeoGebra in Mathematics Instruction

GeoGebra is an open and free DMS tool for math education. Although it is most frequently linked with geometry instruction, it can be used for various mathematical areas. GeoGebra's interactive interface and visual representation of mathematical expressions allow students to investigate relationships between variables, modify function parameters, and immediately observe how these changes affect graphical representations. This type of visual, experiential learning promotes a greater comprehension of mathematical topics and connects abstract theory to actual applications (Breda & Santos, 2021; Zulnaidi & Zamri, 2017). Research indicates that utilizing GeoGebra can improve students' grasp of various functions, including quadratic, exponential, and trigonometric (Birgin & Acar, 2020; Bedada & Machaba, 2022; Hidayat et al., 2024; Pospos & Piñero, 2024; Tuda & Rexhepi, 2023). GeoGebra can be also used in the context of complex topics, such as multivariable functions, where it proves to be especially useful because students frequently struggle to visualize and grasp them (Hedi et al., 2023; Lepellere, 2021). Studies have demonstrated that GeoGebra not only enhances conceptual knowledge but also increases student motivation and engagement (Hidayat et al. 2024; Pospos & Piñero 2024; Zakaria et al., 2024). Lessons become more dynamic and captivating, and students are more engaged in the learning process. Students also report that mathematics feels more approachable when visuals and interactive tools are part of instruction.

Teachers also benefit from GeoGebra in their lessons. Its use improves explanations, saves time during education, and encourages classroom participation and collaboration (Bedada & Machaba, 2022; Triet et al., 2024; Zulnaidi et al., 2020). A recent meta-analysis by Zhang et al. (2025) synthesized 20 years of research on the use of GeoGebra in mathematics education. Their findings indicate a statistically significant medium-to-large effect (Hedges's $g = 0.653$) of GeoGebra-supported instruction on students' mathematics achievement. Notably, the study found that smaller sample sizes (fewer than 50 students) and shorter treatment durations (less than 4 weeks) were associated with stronger educational effects.

METHODOLOGY

This study used a quasi-experimental design, specifically a one-group pretest–posttest model. In this approach, no control group was used; instead, participants in a single experimental group were assessed both before and after the educational intervention. The sample consisted of 20 students aged 17 and 18, enrolled in the program for electrical technicians at the vocational secondary school Electrotechnics and Traffic School Osijek. These students possess digital skills, as their curriculum includes subjects focused on programming and application development. All participants completed both the pre-test and post-test assessments, and four of them also took part in follow-up interviews. In previous school years, they had studied trigonometric ratios involving the sides of right-angled triangles. However, during the current academic year, they encountered for the first time the formal definitions and graphical representations of trigonometric functions. Prior to the study, all participants were informed about the research objectives, content, and duration. They were explicitly informed that participation was voluntary, that they could withdraw at any time without negative consequences, and that all collected data would remain confidential and be used exclusively for scientific purposes. The study was conducted in accordance with ethical guidelines governing educational research.

Role of Teacher

Motivated by a desire to enhance her students' understanding of trigonometric functions and to explore the potential of digital tools in mathematics education, the second author, a mathematics teacher at the above-mentioned secondary school, created a set of GeoGebra applets. These applets were designed to support visual exploration of the relationships between parameters in trigonometric functions. In addition to developing the digital materials, the second author also designed the accompanying mathematical activities. The first author, a mathematics educator, reviewed these activities and provided advisory support throughout the implementation phase.

During the intervention, the second author assumed a dual role as both teacher and researcher. As a teacher, she guided students through the content and activities, offering explanations and support. As a researcher, she created and collected data on student learning using formal instruments, including a pre-test, a post-test, and interviews. This integration of teaching and research roles enabled her to gain a deeper understanding of students' learning experiences and knowledge acquisition.

Procedure

The instructional process began with regular classroom teaching, followed by the administration of the pre-test. Students then participated in the intervention involving GeoGebra applets, and finally completed the post-test. Each phase of instruction and assessment is described in detail below.

Classroom Instruction

The students spent three class periods, each 45 minutes long, studying trigonometric functions. The topic was introduced as follows. Students began by hand-drawing the basic graphs of $f(x)$=$sinx$, $f(x)$=$cosx$, $f(x)$=$tanx$ and $f(x)$=$cotx$ on paper. This initial activity familiarized them with the fundamental properties of these functions, such as periodicity, symmetry, and key points on the graphs. After mastering the basic graphs, the students, guided by the teacher explored how the parameters a, b, c, and d affect the graphs of the of functions such as $f(x)$=$asinx$, $f(x)$=$sin(bx)$, $f(x)$=$sin(x$+$c)$, and $f(x)$=$sinx$+d. For each type of transformation, they plotted three examples on the same coordinate plane to observe how changing the parameter values influenced the shape and position of the graphs. This approach enabled them to systematically analyze the effect of each parameter.

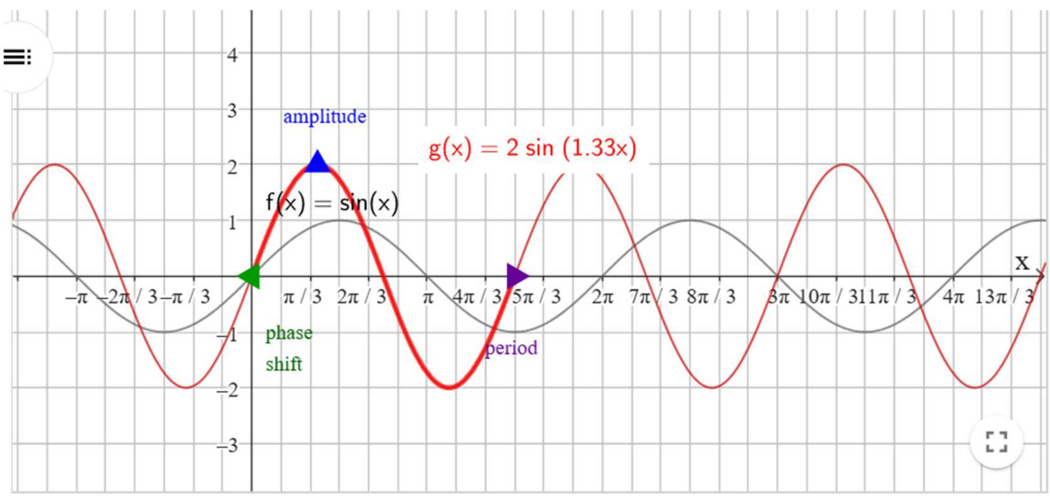
When investigating the effect of parameter a, students plotted the functions $f(x)$=$sinx$, $f(x)$=2$sinx$, and $f(x)$=-3$sinx$, and concluding that a affects the amplitude of the graph. They then examined the parameter b by plotting $f(x)$=$sin(x)$, $f(x)$=$sin(2x)$, and $f(x)$=$sin(0.5x)$, and observing that the period depends on b according to $P=\frac{2\pi}{|b|}$. To analyze the horizontal shift caused by parameter c, they graphed $f(x)$=$sinx$, $f(x)=sin\left(x+\frac{\pi}{6}\right)$, and $f(x) = sin\left(x - \frac{\pi}{3}\right)$, noting that the graph shifts left or right depending on the sign and magnitude of c. Finally, they studied the effect of parameter d by plotting $f(x)$=$sinx$+d, $f(x)$=$sinx$+2, and $f(x)$=$sinx$-3, and concluded that the graph moves up or down based on the value of d.

A similar procedure was applied to the functions $f(x)$=$acosx$, $f(x)$=$cos(bx)$, $f(x)$=$cos(x$+$c)$, and $f(x)$=$cosx$+d. Working with more complex functions such as $f(x)$=$asin(bx$+$c)$+d and $f(x)$=$acos(bx$+$c)$+d, students learned to identify key characteristics including amplitude, period, and phase shift. They also practiced sketching graphs from algebraic expressions and deriving expressions from given graphs.

Intervention

The intervention involved the use of specially designed GeoGebra applets, which were assigned as formative homework. A GeoGebra Classroom was set up by the teacher, and students were provided with access credentials, requiring them to log in using their real names. This setup enabled the teacher to monitor student engagement on a daily basis, specifically, whether students accessed the platform and how many points they earned while completing the assigned tasks. It should be noted that the system did not record the duration of student interactions, but only logged task access, completion status, and point accumulation. Students had access to the applets for one week. Throughout this time, the teacher used engagement data from the GeoGebra Classroom to identify students who had not yet begun the tasks and issued reminders during regular lessons to encourage their participation. The formative nature of the assignment allowed students to work at their own pace while still being held accountable through visible progress tracking. While students were working on the applets at home, they were encouraged to ask questions in class. The teacher did not provide general demonstrations but responded to student-initiated inquiries, offering clarifications only when students expressed difficulties. This ensured that teacher support was focused and directly responsive to students' individual learning needs.

During the intervention, students engaged in two types of activities within GeoGebra Classroom. The first type consisted of eight interactive applets, allowing students to manipulate the coefficients of trigonometric functions in the forms $f(x)=a\sin(bx+c)+d$ and $f(x)=a\cos(bx+c)+d$. After exploring these applets, students responded to questions addressing how changes in parameters affected amplitude, period, and horizontal and vertical shifts of the graphs. These questions were designed to deepen students' conceptual understanding of how each parameter influences the function's shape (see Figure 1). All questions were presented in a multiple-choice format, with a total of 72 questions associated with this set of applets. (Examples of given activities can be found here: https://www.geogebra.org/m/xa4hfbds)



1. Changing the coefficient b, 0<b<1 in the function f(x) = a sin(bx + c) will affect:

Uključite sve točne odgovore

A ☐ the x-coordinates of the extrema

B ☐ the fundamental period

C ☐ the amplitude

D ☐ the phase shift

Figure 1: Example of the first type of GeoGebra activities

The second type of activity, also consisting of eight applets, focused on assessing students' understanding of trigonometric function graphs through interactive problem-solving tasks. In these activities, students were asked to determine the parameters a, b, c and d based on graphical representations of $f(x)=asin(bx+c)+d$ and $f(x)=acos(bx+c)+d$. They were also required to plot graphs from algebraic expressions after identifying key features such as amplitude, period, and horizontal and vertical shifts. (This set of activities may be found here: https://www.geogebra.org/m/rzbjxpbr) Within these applets, points were awarded for correct answers and deducted for incorrect ones (see Figure 2 for an example). This scoring system encouraged careful reasoning, as each error affected the final score. When students made mistakes, the applets provided immediate feedback and contextual hints, guiding them toward the correct solution and highlighting key aspects to consider. This feature fostered self-regulated learning and allowed students to reflect on their reasoning in real time. The teacher informed students that earning ten points in each assigned activity indicated full mastery of the content.

At the end of the intervention period, the teacher reviewed students' interactions with the applets to identify common misunderstandings. She focused on activities in which students had earned the fewest points. Based on this analysis, a class discussion was conducted to address the most frequent errors and misconceptions revealed in students' digital work. This review session provided an opportunity for immediate feedback, clarification of misconceptions, and reinforcement of key concepts. Following the discussion, students completed a post-test.

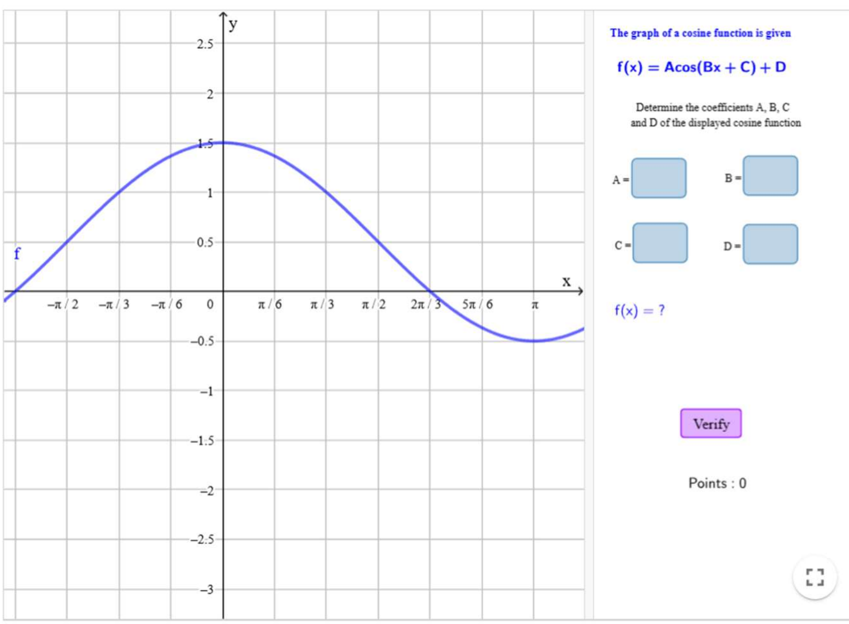


Figure 2: Example of the second type of GeoGebra activities

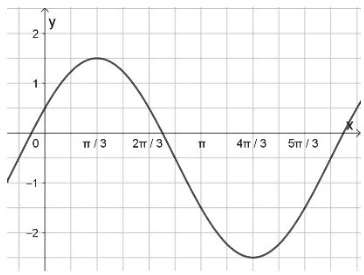
Pre-Test Assessment

Following the classroom instruction, students completed a pre-test aimed at assessing their initial knowledge. Each question/sub-question was assessed as either 1 (correct) or 0 (incorrect). Students could achieve 15 points in total. The pre-test tasks were presented below:

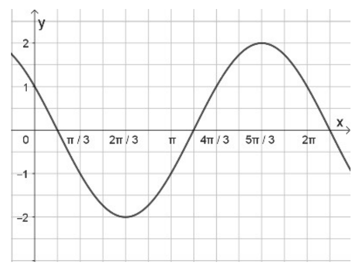
A. Tasks involving data interpretation from graphical representations

1. Read and identify the required coefficients from the graphs of trigonometric functions.

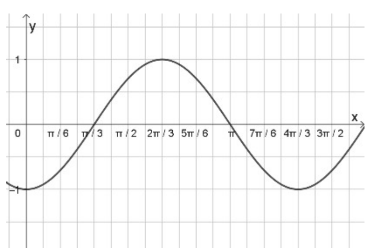
a) Determine the parameter a of the graph of the function $f(x)=a\cos(bx+c)+d$ shown in the figure.



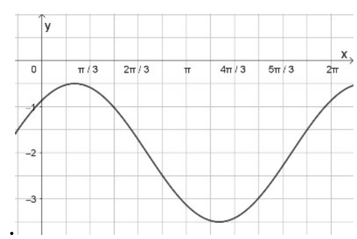
b) Determine the parameter b of the graph of the function $f(x)=a\cos(bx+c)+d$ shown in the figure.



c) Determine the parameter c of the graph of the function $f(x)=a\cos(bx+c)+d$ shown in the figure.



d) Determine the parameter d of the graph of the function $f(x)=a\cos(bx+c)+d$ shown in the figure.



B. Multiple choice

2. Determine the amplitude and the fundamental period of the function

$$f(x)=2\cos(3x\text{-}4)+5$$

Amplitude	a) 2	b) 3	c) 4	d) 5
Fundamental period	a) 2π	b) $\frac{\pi}{3}$	c) $\frac{\pi}{2}$	d) π

3. If the function is given as $f(x)=\text{-}3\sin x\text{-}2$, by how much is the original function $g(x)=\text{-}3\sin x$ translated along the y-axis?

a) 3 units up

b) 2 units down
c) 2 units up
d) 3 units down

4. If the function is given as $f(x)=2cos\left(x+\frac{\pi}{3}\right)$, by how much is the original function $g(x)=2cosx$ translated along the x-axis?

a) $\frac{\pi}{3}$ to the right
b) $\frac{\pi}{3}$ to the left
c) π to the right
d) $\frac{\pi}{2}$ to the left

C. Open tasks.

5. Write a cosine function with an amplitude of 4, a fundamental period of $\frac{\pi}{2}$, and a vertical shift of 2 units upwards.

6. Determine the amplitude, fundamental period, and vertical shift for the function

$$f(x)=-3sin2x-4.$$

7. Given the function $f(x)=4cos\left(3x-\frac{\pi}{2}\right)$ find its fundamental period and horizontal shift.

D. Drawing graphs

8. Draw the graph of $f(x)=sinx$ over the interval [0, 4π].

9. Draw the graph of $f(x)=cosx$ over the interval [0, 4π].

10. Draw the graph of $f(x)=-2cos0.5x-1$ over its fundamental period.

11. Draw the graph of $f(x)=3sin\left(2x-\frac{2\pi}{3}\right)$ over its fundamental period.

Post-Test Administration

The post-test was conducted to evaluate students' progress following the interactive activities in GeoGebra. The test targeted the same learning outcomes as the pre-test but featured modified numerical values in the tasks, enabling an objective assessment of students' understanding of trigonometric functions. The post-test results were used to analyze student progress, identify areas requiring further attention, and provide feedback to help students recognize their strengths and weaknesses. This approach also offered the teacher the opportunity to adjust instructional methods according to individual student needs and to foster active engagement in the learning process.

Interviews

The student interviews focused on their experiences and perceptions regarding the use of GeoGebra as a tool for learning about graphs of trigonometric functions. During the interviews, students shared their views on how working with GeoGebra enhanced their understanding of function graphs, influenced their motivation to learn mathematics, and how this method compared to classroom teaching. The following questions were posed:

- Do you believe that working in GeoGebra helped you better understand the graphs of trigonometric functions? Please provide an example.
- Did using GeoGebra increase your motivation to learn mathematics compared to traditional methods? Explain why.
- What advantages did you notice when changing the coefficients of trigonometric functions in GeoGebra compared to manually drawing graphs?
- Do you think that combining theoretical instruction with GeoGebra was more effective than traditional learning alone? Why?
- Can you explain how GeoGebra helped you avoid certain mistakes you made at the beginning?

Data Analysis

Descriptive statistics was used to analyze pre-test and post-test results as well as statistical methods appropriate for small sample sizes. Differences between the two sets of results were examined using the Wilcoxon signed-rank test for paired samples, which is suitable given the small sample size (N = 20) and the assumption of non-normal data distribution. Additionally, effect sizes were calculated using the *r* statistic and Hedges' *g*, both of which are recommended for small samples.

The qualitative part of the study was analyzed using thematic analysis (Saldaña, 2015). Although the interview questions were predefined, the analysis was not limited to direct responses. Instead, the answers were coded and interpreted inductively, and themes were organized to capture recurring patterns that went beyond literal responses. As a result, thematic categories were developed to reflect deeper aspects of students' perceptions rather than just the explicit content of the questions. This approach allowed for the identification of the meanings students' attributed to using GeoGebra, including their insights into understanding processes, motivation, effectiveness, feedback, and the integration of different teaching methods.

RESULTS

Assessment of Intervention Effectiveness

To evaluate the effectiveness of the implemented intervention, basic descriptive statistics were calculated for the pre-test and post-test results (Table 1). The mean score on the pretest was 11.45 (*SD* = 4.01), while the post-test showed a higher mean of 12.15 (*SD* = 3.79). This increase suggests a slight improvement in student performance following the intervention. Additionally, the median increased from 12 to 13, further indicating that most students achieved better results on the post-test. A decrease in standard deviation was also observed, suggesting reduced variability in scores after the intervention.

Measure	Mean (*SD*)	Median	Min	Max
Pre-test	11.45 (4.01)	12	3	16
Post-test	12.15 (3.79)	13	6	18

Table 1: Results of pre-test and post-test

The Wilcoxon test yielded a value of $W = 36.5$ with a corresponding p-value of 0.056. Since $p > 0.05$, the null hypothesis of no statistically significant difference between the two measurements was not rejected, meaning that the observed change was not significant at the conventional level of significance. Nevertheless, the calculated effect size using Wilcoxon's r measure was $r = 0.46$, which is interpreted as a moderate to large effect. In addition, the effect size calculated using Hedges' $g = 0.4742$, further supported the conclusion of moderate practical importance. These indicators suggest that, although the difference was not statistically significant, there was meaningful progress in student knowledge that merits attention from an educational practice perspective. It is important to note that the Wilcoxon test is based on ranks rather than raw scores, which enhances the reliability of the analysis when assessing changes.

Although the post-test results were generally higher, progress was not universal; some students showed little or no improvement. This indicates individual differences in response to the intervention. Analysis of students' engagement with the GeoGebra Classroom revealed those students did not log into GeoGebra Classroom. With regard to their prior performance, they had varying levels of knowledge on the pre-test, indicating that lack of progress was not strictly associated with low initial achievement. Other students' engagement with GeoGebra varied also in both participation and task success. While most students engaged with the platform, the depth and completeness of engagement varied. In the first type of activities (manipulation of parameters in functions having form $f(x)=asin(bx+c)+d$ and $f(x)=acos(bx+c)+d$), 8 students completed all 72 questions, 4 students completed approximately 62 questions, and the remaining completed fewer than 25. One student did not access this activity type at all. The second type of activities had lower engagement. Participation per applet ranged from 7 to 15 students, with the highest engagement in applets 1 and 2, and the lowest in applets 7 and 8. Only 8 students completed all eight applets. In terms of points earned, this instance showed great variability; for instance, 11 of 12 students achieved more than 10 points in applet 4, while only 4 of 13 students did so in applet 3. Overall, a decline in both participation and performance across later applets was observed, suggesting either increased difficulty or decreased motivation over time.

Students' Attitudes Toward GeoGebra Activities

To gain deeper insight into students' experiences and attitudes toward using GeoGebra for learning trigonometric functions, the interview results are presented below, organized into thematic categories.

a) Visualization and Understanding

Students emphasized that GeoGebra helped them better understand the concepts of amplitude, phase shift, and period due to visual and interactive features. One student noted that progress in understanding depended on the amount of effort invested in working with GeoGebra, highlighting the importance of consistent engagement with the tool.

- “It really helped me because I could visualize the information. I didn’t have to imagine it.”
- “When I started using GeoGebra, I could picture amplitude, shifts, and everything else.”
- “It helped me understand trigonometric functions when the coefficients changed. But I didn’t practice enough in GeoGebra. If I had used it more, it would have helped me even more.”

b) Increased Motivation and Engagement

Students' motivation to learn mathematics was enhanced by making the content more accessible, engaging, and less abstract. They expressed being more engaged, largely due to the dynamic nature of the digital tool, which supported autonomy and faster comprehension. They also noted that GeoGebra made mathematics feel "more approachable," which is particularly important given the subject’s inherent challenges.

- “I think learning with GeoGebra is better than just doing textbook exercises.”
- “Math is not many students’ favorite subject, so technology makes the material more appealing.”

c) Learning Efficiently and Faster Task Completion

Students almost unanimously agreed that GeoGebra enabled them to complete tasks faster and more accurate compared to manual graphing. One student specifically pointed out the advantage of being able to compare multiple graphs simultaneously, while another emphasized that hand-drawn graphs can introduce confusion due to inaccuracies.

- “Using GeoGebra, we can complete tasks much faster compared to drawing by hand.”
- “I don’t need to draw separate graphs to notice differences and understand the rules.”
- “When drawing graphs on paper, mistakes can happen—it’s not as precise as in GeoGebra.”

d) Feedback

GeoGebra provided immediate visual feedback, allowing students to quickly identify errors. This feature enhanced their ability to self-correct and facilitated a deeper understanding of their mistakes, which traditional instruction does not offer to the same extent or speed.

- “It’s easier when the program corrects you. You know you made a mistake, and it’s easier to find the problem.”
- “When working traditionally, we don’t even know when we make mistakes.”

e) Synergy Between Theory and Practice

Overall, students advocated for a combination of classroom teaching and using technology, though with varying emphasis. While some appreciated how GeoGebra made the content more concrete, others pointed out that the tool cannot replace the teacher’s role in developing mathematical reasoning.

- “In traditional lessons, you learn more theory, but this way, you apply what you’ve learned.”
- “GeoGebra helps, but it cannot provide as much as a teacher can to strengthen our thinking.”
- “I think both should be combined.”

DISCUSSION AND CONCLUSION

This study examined the impact of GeoGebra on vocational school students’ understanding and engagement in learning trigonometric functions. Although the Wilcoxon test did not reveal a statistically significant difference between pre- and post-intervention results ($p = 0.056$), the moderate effect size ($r = 0.46$; Hedges’ $g = 0.4742$) suggests that the observed change has practical relevance. In small-sample studies, effect size often provides a more informative measure of an intervention’s educational impact than statistical significance alone (Sullivan & Feinn, 2012). The results align with previous research confirming the positive influence of GeoGebra on students’ ability to connect algebraic expressions with their corresponding graphical representations (Bedada & Machaba, 2022). In the specific context of trigonometric functions, students demonstrated an improved understanding of the relationships between function parameters and their graphs and were better able to correct errors that occurred during manual graphing. Qualitative findings from student interviews enriched the quantitative results, revealing that most participants perceived GeoGebra as a helpful tool that enabled them to complete tasks more quickly and accurately. They emphasized that visualizing amplitude, period, and phase shift facilitated their conceptual understanding and that immediate digital feedback supported error detection and correction. Students particularly highlighted the value of combining traditional instruction with GeoGebra-based activities to achieve a balance between theoretical explanation and practical application.

Although overall results improved, progress was not uniform across students. Engagement within the GeoGebra Classroom platform varied considerably: while some students completed all assigned tasks, others did not access certain activities at all. The noticeable decline in participation in the later applets may indicate reduced motivation or increased task difficulty. These differences highlight the crucial role of the teacher in monitoring progress and encouraging participation. Previous research suggests that the effectiveness of digital tools such as GeoGebra depends heavily on clear teacher guidance (Bedada & Machaba, 2022). Merely providing access to digital technology is not sufficient; its full educational potential can be realized only when teachers are adequately prepared to integrate such tools into their instruction and when students actively engage with the material. In this study, the GeoGebra activities were assigned as homework, placing the responsibility for engagement on students and limiting the teacher's ability to provide real-time feedback or guide exploration. Ideally, GeoGebra activities should be embedded into regular classroom instruction, allowing the teacher to scaffold students' learning and ensure consistent participation. However, such integration is frequently constrained by limited instructional time and dense curricula, particularly in vocational schools, where priority is often given to technical and vocational subjects (Rodrigues & Pimenta, 2025; Rosvall et al., 2016).

Despite these limitations, the results suggest that even out-of-class engagement with GeoGebra can enhance students' understanding, provided that they actively and meaningfully participate in the activities. It is important to note that GeoGebra Classroom did not track the duration of student activity but only recorded task access, completion status, and points earned. Consequently, it was not possible to accurately estimate the amount of time students spent working on the tasks, which represents a limitation when interpreting student engagement. Although students who showed little or no progress were identified as those who had not accessed the digital activities, no clear relationship was observed between lack of engagement and prior knowledge. Some of these students had achieved relatively strong pre-test scores. This finding suggests that the lack of progress cannot necessarily be attributed to weaker prior knowledge but is more likely related to lower levels of engagement, motivation, or self-regulation. Future research should include measures of students' digital skills, learning strategies, and time-on-task data to gain a more comprehensive understanding of individual differences in performance.

Overall, the results of this study can be linked to the findings of Held and Mejeh (2024), who studied the effect of self-regulated learning in vocational schools. In their study, students were given greater autonomy in planning and organizing their own learning, with minimal but targeted support from teachers. Although the approach in our study was technically different (students worked on digital GeoGebra applets at home), the role of the teacher as a facilitator through progress monitoring and occasional interventions was similar. In both studies, students who actively participated showed increased understanding and engagement, while inactive students remained without visible progress, regardless of prior knowledge. This parallel confirms an important implication for practice: independent digital activities can support deeper understanding of mathematical concepts only if they are clearly structured and accompanied by systematic support and supervision. Autonomy, while valuable, does not guarantee success in itself. As Held and Mejeh

(2024) show, autonomy can lead to lower engagement, procrastination, or stress for some students. Therefore, we argue that when introducing digital tools and methods of independent learning, teachers should ensure a balance between freedom and structured support, in order to maximize their educational impact.

LIMITATIONS AND FUTURE RESEARCH

Several limitations of this study should be acknowledged, as they affect the generalizability of the findings. First, the research was conducted with a limited sample, consisting of a single class from one school, and no control group was included. Second, the small number of participants reduced the statistical power of the analysis, making it difficult to draw robust conclusions. Although a moderate effect size was observed, statistically significant differences in student performance before and after the intervention could not be established.

In light of these limitations, replicating this study with students from general secondary schools would be particularly valuable. In Croatia, these students are required to take a compulsory mathematics graduation exam that includes trigonometric functions, which may lead to higher engagement and potentially different learning outcomes. A comparative study across different school types could provide deeper insight into how GeoGebra supports conceptual understanding and motivation in diverse student populations. Future research should also include larger and more diverse samples, involve multiple schools, and incorporate control groups to enhance the external validity of the results.

RECOMMENDATIONS FOR USING GEOGEBRA

For teachers who are just starting to use GeoGebra, it is recommended to start with simple applets that allow manipulation of function parameters. Teachers can use existing resources available on the platform (www.geogebra.org). GeoGebra Classroom allows for tracking student progress, and Classroom integration can begin with short demonstrations, followed by independent exploration by students with teacher support. The key is to provide clear instructions, gradually introduce functionalities, and encourage students to reflect on the changes they observe.

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