Empowering Primary School Student's Numeracy Skills Through Augmented Reality: The Didactical Technology Design

Sugiman[1], Mohammad Archi Maulyda[2*], Rina Dyah Rahmawati[3]

[1]Universitas Negeri Yogyakarta, Indonesia,

[2]Universitas Mataram, Indonesia,

[3]Universitas PGRI Yogyakarta, Indonesia

sugiman@uny.ac.id, archimaulyda@unram.ac.id* (Corresponding Author), rinadyah_r@upy.ac.id

Abstract: This study aims to enhance primary school students' numeracy skills through the Didactical Technology Design (DTD) framework integrated with Augmented Reality (AR) technology. It focuses on identifying learning obstacles, developing and validating AR-based instructional media, and evaluating their effectiveness through teaching experiments. A total of 301 participants, including teachers and students, were involved across three phases: Preliminary, Development, and Teaching Experiment. Data collection utilized a numeracy test, analyzed through descriptive and parametric tests. Results showed significant improvement in numeracy skills, with a mean score of 69.68, highlighting the potential of DTD with AR in fostering interactive, effective learning environments.

Keywords: early years; learning trajectory; mathematics; visualization

INTRODUCTION

The rapid development of technology in education has transformed the way knowledge is delivered, particularly in mathematics education (McKnight, 2020). One promising innovation is Augmented Reality (AR), which enhances learning by overlaying digital information onto the real world (Barteit, 2021; Elmqaddem, 2019). AR offers an interactive and immersive experience, bridging the gap between abstract mathematical concepts and their real-world applications, ultimately improving educational outcomes in primary mathematics (Zhengtao & Hidayat, 2025)(Zhengtao & Hidayat, 2025). For primary school students, who often struggle with numeracy, AR can provide visual and hands-on experiences that facilitate understanding (Kobayashi, 2018). In Indonesia, however, the integration of AR in primary mathematics education remains limited, despite its potential to address persistent learning challenges (Maulyda et al., 2024). This context underscores the need for a didactic approach that leverages AR to enhance numeracy skills.

Despite the growing recognition of AR's potential, many primary school students in Indonesia face significant barriers to mastering numeracy (Perso, 2006; Tout, 2020). These include limited exposure to engaging and contextual learning media, as well as instructional methods that fail to cater to diverse learning needs. Traditional teaching methods, often focused on rote memorization, struggle to foster deep conceptual understanding (Purpura et al., 2011). As a result, students frequently encounter difficulties in applying mathematical knowledge to real-life problems. This issue is compounded by learning obstacles such as low motivation and limited spatial reasoning abilities (Lusardi, 2012; Tariq, 2014). Addressing these challenges requires innovative approaches that make learning mathematics more engaging and effective, particularly for younger learners.

A review of the existing literature reveals a gap in the integration of AR technology within a structured didactical framework (Kobayashi, 2018; Lai, 2019; Rau, 2018). Most studies on AR in education focus on its technical implementation or its general impact on student engagement and motivation (Kiv, 2020; Molnár, 2018; Rossano, 2020). However, few studies have examined its potential within a systematic model that combines technology with didactic principles tailored to numeracy development (Klimova, 2018; Sorko, 2019). Additionally, while some research explores AR's role in secondary education, its application in primary school settings remains underexplored. This gap highlights the need for a model that not only incorporates AR but also aligns with the specific pedagogical and cognitive needs of primary school students.

This study introduces the Didactical Technology Design (DTD), a novel model that integrates AR into mathematics education through a structured, three-phase approach: Preliminary Phase, Development Phase, and Teaching Experiment Phase. The DTD model combines elements of Design-Based Research (DBR) (Nieveen et al., 2013), Technology Integration Planning (TIP) (Roblyer & Doering, 2014), and ASSURE (Smaldino et al., 2013) to create an iterative, evidence-based framework. This combination ensures that AR is not only effectively integrated but also adapted to address specific learning obstacles and enhance students' numeracy skills. The novelty of this research lies in its didactical focus, offering a new lens for leveraging AR to foster mathematical understanding among primary school students.

The primary aim of this study is to empower primary school students' numeracy skills through the implementation of the DTD model with AR technology. Specifically, this research seeks to: (1) identify learning obstacles and product needs in numeracy education, (2) develop and validate AR-based instructional media within the DTD framework, and (3) evaluate the effectiveness of these media in improving numeracy skills through teaching experiments. By addressing the identified gaps, this study aims to contribute to the growing body of knowledge on the didactical use of AR in education, providing practical insights for educators and policymakers.

LITERATURE REVIEW

Integer Operation Knowledge

Understanding integer operations (Integer Operation Knowledge) is an essential aspect of mathematics education in primary schools. According to de Walle et al. (2016), learning integers requires an approach that simultaneously involves both conceptual and procedural understanding. Students must be able to comprehend the meaning of addition, subtraction, multiplication, and division of integers in real-world contexts before mastering formal algorithms. This approach aims to build mathematical reasoning skills, which involves linking real-world experiences with formal mathematical concepts.

Creating mathematical models actually originates from real-life situations and contexts (Mulbar et al., 2023). In the context of Realistic Mathematics Education (RME), the process of mathematical modeling plays a crucial role in helping students understand integer operations. According to (Heuvel-Panhuizen, 1996; Oonk et al., 2019), RME emphasizes the use of contextual problems that can be bridged through two stages of mathematization: horizontal and vertical mathematization. Horizontal mathematization involves transforming real-life situations into mathematical models, while vertical mathematization focuses on refining these models into formal procedures (Hidayati et al., 2020). For example, to understand the concept of adding negative integers, teachers may use contexts such as temperature or depth below sea level.

This process can be applied, for instance, by presenting a problem like: "The initial temperature in city A is -3°C. Afterward, the temperature increases by 5°C. What is the current temperature?" Students first translate the situation into the mathematical equation (-3) + 5 through horizontal mathematization. Then, they solve the equation using informal strategies before eventually mastering the formal algorithm, as part of vertical mathematization. Students' understanding of negative integers is very important as a basis for understanding broader mathematical concepts (Zuhriawan et al., 2024). Through this approach, students not only gain an intuitive understanding of integers but also develop the ability to apply these concepts in various contexts, strengthening their mathematical thinking skills. Thus it is expected that students are able to solve problems because, problem solving is one of the important activities in learning mathematics (Kamariah et al., 2023).

Augmented Reality Facilitation

Augmented Reality (AR) has emerged as a promising technological innovation in education, particularly at the primary school level. AR allows for the integration of digital elements into the real-world environment, creating an interactive and immersive learning experience. According McKnight (2020), AR combines the real and virtual worlds in real-time, enabling students to interact with abstract concepts through three-dimensional visualizations. The use of augmented reality-based interactive media is one of the answers to the implementation of a non-discriminatory information and communication technology-based learning system (Buliali et al., 2022). In the context of primary education, AR provides opportunities to develop experiential learning, where students can gain a deeper understanding of mathematical concepts through concrete and interactive visual representations (Jesionkowska, 2020; Molnár, 2018; Nechypurenko, 2018).

One of the key contributions of AR in mathematics education at the primary level is its role in the process of mathematization. According to Rossano (2020), mathematization consists of two types: horizontal and vertical mathematization. AR can facilitate both of these through interactive learning scenarios. For instance, in the context of measuring area and volume, students can use AR applications to visualize how various three-dimensional geometric shapes can be filled with specific measurement units (Rashevska, 2020). In horizontal mathematization, students solve real-world problems, such as calculating the volume of water in a virtual tank. Then, through vertical mathematization, they are guided to formulate mathematical equations based on their observations, such as calculating volume using the formula $V=l \times w \times h$.

The implementation of AR also aligns with constructivist theory, which emphasizes active learning. Piaget stated that students build knowledge through direct experience, and AR provides a platform that supports this process (Slavin, 2018). By actively engaging students in exploring and manipulating virtual objects, AR not only enhances student engagement but also helps reinforce their conceptual understanding. Thus, AR becomes an effective facilitator in supporting meaningful mathematical learning at the primary school level.

Didactical Technology Design

Didactical Technology Design (DTD) is a development model designed to integrate technology into didactic learning. This model aims to create an interactive and effective learning environment (milieu) through the use of technology, particularly in mathematics education. DTD is based on a combination of three key frameworks: Design-Based Research (DBR) from Nieveen et al. (2013), Technology Integration Planning (TIP) from Roblyer & Doering (2014), and ASSURE from Smaldino et al. (2013). By combining the principles from these three models, DTD provides a structured approach to designing and implementing learning solutions that are oriented toward the didactic needs of students.

Several recent studies have begun to adopt and refine the DTD model to address various educational contexts. For instance, Kerr (2020) applied DTD to develop augmented reality (AR) media for elementary mathematics, showing improvements in students' conceptual understanding and engagement. Similarly, Ahsani et al. (2022) implemented DTD in developing digital flipbooks for numeracy learning, which enhanced both teachers' instructional quality and students' learning motivation. In another study, Firdaus et al. (2024) utilized DTD to design culturally responsive digital modules for multicultural classrooms, highlighting its adaptability in diverse learning environments. These findings support the practicality and effectiveness of DTD, reinforcing its potential to meet varied didactic needs.

DTD consists of three main phases. The Preliminary Phase is the initial stage where learning obstacles and product development needs are identified. This process refers to DBR, which emphasizes in-depth exploration of learning issues in real-world contexts (Nieveen et al., 2013). Additionally, TIP is used to plan the integration of technology that aligns with learning needs (Roblyer & Doering, 2013). The results of this analysis become the foundation for developing products that are pedagogically and didactically relevant. Past research

(Bunari et al., 2024; Firdaus et al., 2024) demonstrates how thorough needs analysis and contextual understanding at this stage contribute to creating targeted learning solutions.

The next stage, the Development Phase, involves the development of learning media using an iterative approach based on the ASSURE framework (Smaldino et al., 2013). Studies by Fahmi et al. (2019) and Kerr (2020) highlight the importance of continuous feedback loops and iterative improvements, which lead to more refined and user-centered educational products.

The final stage, the Teaching Experiment Phase, aims to evaluate the effectiveness of the developed product in real learning contexts. This phase also serves as an empirical validation process, ensuring that the learning media is not only effective but also adaptable to various learning situations. Previous studies utilizing DTD (Abdullah et al., 2022; Bunari et al., 2024) have shown positive outcomes in both student achievement and teacher satisfaction, underscoring the model's value in practical educational settings.

By integrating DBR, TIP, and ASSURE, DTD offers a comprehensive and didactic solution for developing technology-based learning media. Its use in recent studies highlights its growing relevance and ability to address contemporary educational challenges. This model is expected to contribute to improving the quality of education through an evidence-based approach.

Conjecture of Study

This study hypothesizes that integrating Augmented Reality (AR) into mathematics learning through Didactical Technology Design (DTD) can significantly enhance primary school students' numeracy skills. AR, with its interactive and visual elements, enriches learning experiences by facilitating students’ understanding of abstract mathematical concepts and boosting active engagement. AR enables direct virtual interaction with mathematical objects such as 3D shapes and arithmetic operations, thus supporting the development of numeracy skills through horizontal and vertical mathematization.

The DTD framework ensures a systematic design and evaluation process, incorporating Preliminary, Development, and Teaching Experiment Phases. This approach guarantees that AR-based learning tools are both pedagogically sound and effective. The hypothesis suggests that AR, when designed within the DTD model, enhances numeracy by promoting interaction, visualization, and conceptual understanding, ultimately improving students’ mathematical competencies in a structured and impactful manner.

METHODS

Research Design

The research employs a mixed-method approach using the Didactical Technology Design (DTD) framework to enhance primary students' numeracy skills through Augmented Reality (AR). As explained earlier, this research took place in three stages: Introduction, Development, and Teaching Experiment. This design ensures AR media aligns with didactic goals, fostering interactive and effective learning environments while empirically validating its impact on student numeracy. The details of DTD framework are in figure 1.

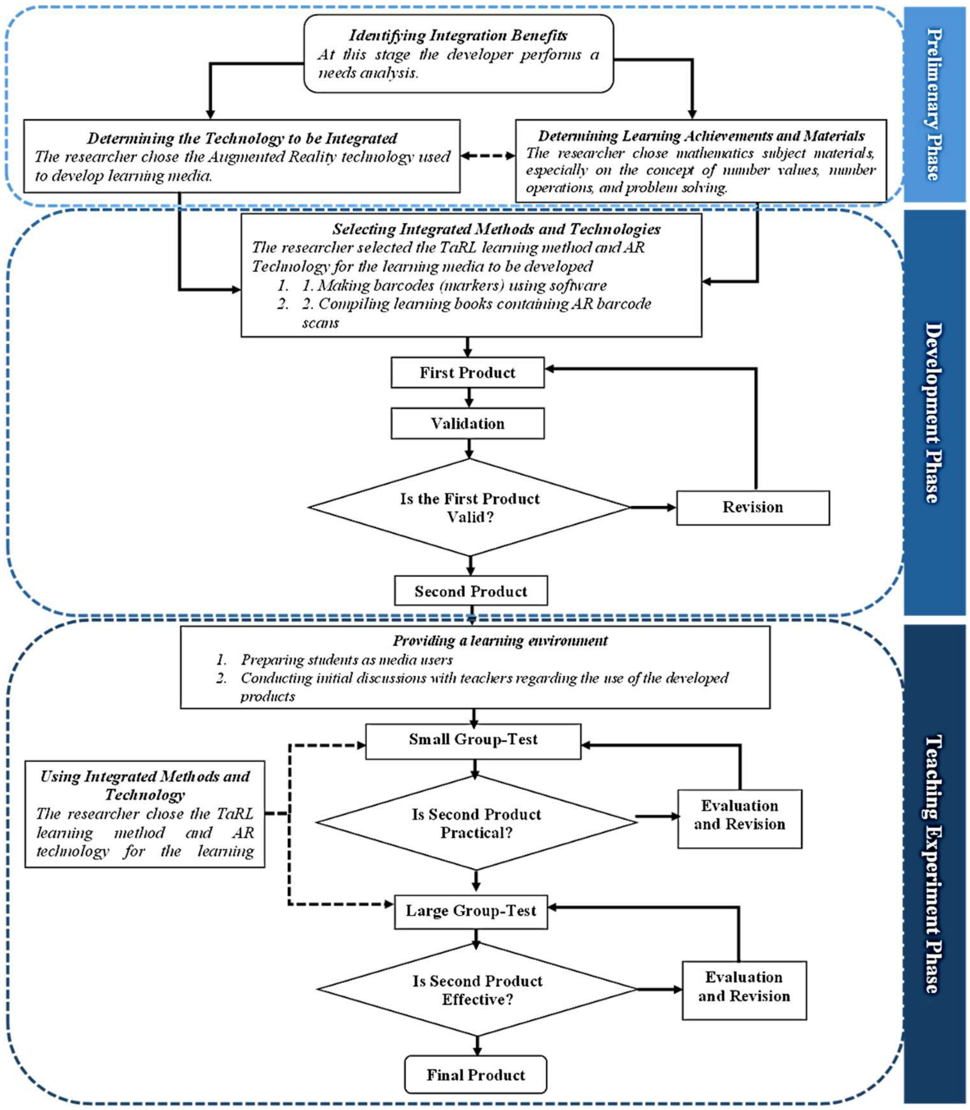

Figure 1: The Dicactical Technology Design Frameworks

Participant

This study involved a total of 301 participants, comprising both teachers and students, across different phases of the research. During the Preliminary Phase, 26 students from SDN Kraton were interviewed to analyze the learning obstacles they encountered. Concurrently, 213 teachers were surveyed using questionnaires to gather data on the perceived needs for educational products that could address these learning challenges. Following the analysis and development stages in the Development Phase, the study proceeded to the Teaching Experiment Phase, where the developed product was tested on 62 students. These participants were drawn from two different schools: 28 students from Kleco Public Elementary School (SDN) and 34 students from Baluwarti Public Elementary School (SDN). The demographic data of the students involved in the Teaching Experiment Phase is detailed in Table 1. This structured participant recruitment ensured comprehensive data collection, aligning with the study's aim of addressing learning obstacles through the Didactical Technology Design framework.

Aspect	Count (%)
Gender	
Male	25 (40.3)
Female	37 (59.7)
Age	
11 years old	42 (67.7)
12 years old	20 (32.3)
Geographical Location	
Rural	34 (54.8)
Urban	28 (45.2)

Table 1: Demographic data of participant in teaching experiment

The demographic distribution of the 62 students involved in the Teaching Experiment Phase is summarized as follows. In terms of gender, the sample comprised 25 male students (40.3%) and 37 female students (59.7%). Regarding age, the majority of participants were 11 years old (42 students, 67.7%), while the remaining 20 students (32.3%) were 12 years old. Geographically, 34 students (54.8%) resided in rural areas, whereas 28 students (45.2%) were from urban areas. This diverse demographic composition provided a balanced representation of students across different backgrounds, ensuring the generalizability of the study's findings.

Measurement and Data Collection

Learning Obstacle Interviews

The data collection process began with sequential face-to-face interviews conducted separately with teachers and students. These interviews aimed to gather in-depth and accurate information directly from the research subjects. The primary focus of the student interviews was to identify specific learning obstacles they encountered, particularly in understanding mathematical concepts. The interview instrument was adapted from Carvalho et al. (2004). It consists of 12 in-depth questions designed to explore the learning obstacles faced by students. This instrument has a reliability of 0.78, indicating its effectiveness in gathering data. The details of the instrument are presented in Table 2 below.

No	Dimensions	Items	Example item wording
1	Student	1-7	Explain if there are any topics you haven't understood so far.
2	Teacher	8-10	What teaching style do you like from your teacher?
3	Milieu	11-12	Have you used a gadget? If so, have you used it for learning mathematics?

Table 2: Learning Obstacle Interviews

Need Assesment Survey

This study utilized a survey questionnaire to assess elementary school teachers' digital competencies, covering the 27 item in three dimensions: Pedagogical Knowledge, Didactic Skills, and Teacher Attitudes. The instrument, based on indicators by Torres-Hernández & Gallego-Arrufat (2022), was validated by education and technology experts. Reliability tests were conducted using Pearson Product Moment and Cronbach's Alpha via IBM SPSS 25. The reliability test yielded a Cronbach's Alpha of 0.893, indicating high reliability. The reliable items were subsequently used with the primary study respondents, ensuring robust data collection to evaluate teachers' digital competencies. The details of the instrument can be found in Table 3 below.

No	Dimensions	Items	Example item wording
1	Pedagogical Knowledge	1-9	Can you display learning videos using an LCD?
2	Didactic Skills	10-17	Have you ever accessed the Quizizz website or other learning evaluation platforms?
3	Teacher's Attitude	18-27	Have you ever heard the term digital security?

Table 3: Need Assessment Survey

Numeracy Test

The numeracy test developed for this study consists of several key dimensions, aligned with the standards from PISA and AKM, as well as mathematical knowledge for teaching (Ball et al., 2008). The test includes a total of 20 items, distributed across different content and context areas. The content areas include Geometry, Measurement, Numbers and Algebra, and Data and Uncertainty. The context areas are Socio-Cultural, Personal, and Scientific, along with a Teaching Work dimension. The instrument has been tested for reliability, yielding a result of 0.83, indicating very high reliability. The details of the instrument can be found in Table 4 below.

No	Dimensions	Items	Example item wording
1	Socio-Cultural	1, 2, 4, 10, 13, 15	The Sekatenan celebration is a traditional festival regularly held at the Yogyakarta town square. This year, the head of Baleharjo Village intends to participate in the festival. One of the required items is 500 traditional cakes. The price of one box containing 72 traditional cakes is Rp. 199,500. How many boxes of traditional cakes need to be purchased, and what is the total cost?
2	Personal	3, 5, 8, 9, 11, 12, 14	Andi lives on the coast of Depok Beach, Yogyakarta. During his work holiday, he plans to travel to Mount Merapi. He departs from Depok Village to Pakem Village, located at the foot of Mount Merapi, using a private car. The following map shows the distance, route, and time required to reach Pakem Village. Andi departs from Depok Village at 8:30 AM. After driving for 3 hours and 10 minutes, he stops to rest at Ambarukmo Mall. After resting for 45 minutes, Andi continues his journey to Pakem Village. What time does Andi arrive at Pakem Village?
3	Scientific	6, 7, 20	The World Health Organization (WHO) recommends limiting sodium intake to 2,400 mg, or about one teaspoon of salt per day. The likelihood of sodium deficiency for those living in Indonesia is very low because most foods in the country are high in sodium content. Even diet menus are high in sodium, such as the South Beach Diet, which includes 2,300 to 6,700 mg of sodium per day. Foods high in sodium, such as chips, canned soups, processed meats, salted fish, soy sauce, ketchup, chicken bouillon cubes, and cheese, should be avoided or minimized as they can cause high blood pressure (hypertension), which can trigger heart disease, stroke, and kidney problems.
4	Teaching Work	16, 17, 18, 19	An elementary school teacher is teaching multiplication. The students provide various answers. As a teacher, it is essential to under-

stand the students' thinking. Below are some of their work results.

Siswa A	Siswa B	Siswa C
45 x 15 75 +60 675	45 x 15 225 +450 675	45 x 15 25 200 50 +400 675

Based on the students' answers above, which student correctly applied the method for multiplying two numbers? Provide an explanation!
Mark an (x) on the correct answers for each method.

Table 4: Numeracy Test

Development Phase

Workshop on Didactic Design Development

The development of Augmented Reality (AR) media to address students' learning obstacles began with a workshop on refining the didactic design for numeracy. This workshop aimed to improve learning tools that focus on reducing gaps in students' numeracy understanding. The activity involved elementary school teachers as active partners in discussions and Q&A sessions. The development team, consisting of the researchers (authors) themselves and the teachers as practitioners, facilitated the workshop. The researchers played a key role in presenting the initial AR-based didactic design, moderating discussions, and documenting feedback. Teachers, as participants, provided constructive feedback and raised relevant questions, contributing valuable insights to the refinement of the instructional materials. This collaborative approach ensured that both theoretical and practical perspectives were integrated into the development process. The outcomes of these discussions were summarized to refine the didactic design and to assign teachers the task of studying the Student Worksheet (LKPD, which stands for Lembar Kerja Peserta Didik in Indonesian) and AR media in preparation for the next session. This participatory workshop ensured that the resulting learning tools were practical and aligned with classroom needs.

In the second phase, a simulation workshop was conducted to test the implementation of the AR-assisted didactic design in a classroom-like setting. Teachers were given the opportunity to practice the design, while the development team conducted simulations and provided guidance. This simulation aimed not only to validate the learning tools but also to identify potential improvements through hands-on experience. This phase reinforced the synergy between didactic theory and classroom practice, ensuring the tools' applicability and effectiveness in real teaching scenarios.

FGD with Experts and Practitioners

Following the workshops, the development process continued with a Focus Group Discussion (FGD) involving education experts and practitioners, specifically elementary school teachers. This stage aimed to obtain a comprehensive validation of the AR-assisted numeracy didactic design. The FGD began with a presentation of the workshop and simulation results, followed by an in-depth discussion on the effectiveness of AR media in addressing learning obstacles. Experts provided feedback based on didactic and pedagogical theories, while teachers shared their empirical experiences and potential challenges in classroom implementation.

The FGD yielded a multidimensional evaluation of the AR media, covering aspects such as design, numeracy content, and interactivity. Input from experts ensured the tools aligned with constructivist learning principles, while feedback from teachers enhanced the tools' applicability in real classroom contexts. Discussions also included developing adaptive learning scenarios to meet the needs of students with varying numeracy skills. The development team recorded all feedback to refine the LKPD and AR media, integrating it into the final product.

The combination of workshops and FGD ensured that the development process was collaborative, and evidence based. By involving diverse stakeholders, the resulting AR media effectively addressed learning obstacles, enhanced student engagement, and supported more meaningful numeracy learning.

Data Analysis

The data analysis process begins with the use of descriptive analysis techniques to identify and summarize data related to learning obstacles and needs assessment. Descriptive analysis aims to describe the general conditions of the observed variables, such as frequency, percentage, mean, and data distribution. In this context, data on learning obstacles and students' needs are analyzed to identify common patterns or issues faced by students during the learning process. The results of this descriptive analysis provide an initial overview of the challenges faced by students and areas that require further attention in designing more effective learning interventions.

Following the descriptive analysis, normality testing is conducted to assess whether the data follows a normal distribution, which is a prerequisite for conducting parametric tests. This step ensures that the data meets the assumptions necessary for valid statistical analysis. If the data is found to be normally distributed, parametric tests such as the One-Sample T-Test can be applied. This test compares the meaning of the teaching experiment data with the theoretical or expected average, allowing the identification of whether there is a significant difference between the experimental condition and the standard or expected condition. In addition, covariate analysis is performed to control for potential confounding variables that may affect the results, ensuring that the effects observed are attributed to the intervention itself.

These steps, normality testing and covariate analysis are crucial in ensuring the robustness and validity of the statistical findings, and they strengthen the interpretation of the results. By integrating these analyses, researchers can confidently draw conclusions about whether the teaching method applied in the experiment has had a significant impact on improving learning outcomes and whether the changes observed can be considered statistically strong enough.

RESULTS

Product Validation Phase

To validate the content of the AR-based instructional materials, the product was assessed by three experts in education and numeracy. The experts reviewed the relevance, accuracy, and clarity of the AR media and accompanying materials. The validation process was quantified using Aiken's V coefficient, a statistical measure to assess content validity based on expert ratings. Aiken's V ranges from 0 to 1, where a value closer to 1 indicates higher validity. The experts provided their ratings for each item on a scale from 1 (not valid) to 4 (highly valid), and Aiken's V was calculated for each item of the AR media and worksheets. The final content validity for the entire product was determined by calculating the average Aiken's V score across all items, which yielded a score of 0.87, indicating that the content was highly valid for educational purposes.

For construct validity, the AR-based instructional materials were analyzed to ensure they effectively measure the intended construct numeracy skills and engagement with AR tools. A factor analysis was conducted to assess the underlying structure of the materials and to determine if the items loaded onto the expected factors. The results of the factor analysis indicated that all items of the AR media and worksheets were strongly associated with the construction of numeracy learning, with factor loadings ranging from 0.65 to 0.85. This shows that the items contributed significantly to the construction being measured. Additionally, model fit indices were assessed to evaluate the overall construct validity. The indices indicated a good fit for the model, with a Comparative Fit Index (CFI) of 0.91, a Tucker-Lewis Index (TLI) of 0.90, and a Root Mean Square Error of Approximation (RMSEA) of 0.048, all of which suggest that the materials effectively measure the intended constructs and are suitable for use in classroom settings.

During the development of the AR-based instructional materials, several iterations were conducted to refine the content, design, and functionality. Feedback from expert panels and FGD sessions played a critical role in shaping the final materials. Based on expert input, the language and examples used in the materials were refined for clarity and relevance, resulting in a higher Aiken's V score. Additionally, based on feedback regarding the AR design, interactivity was enhanced, and visual elements were added to ensure better engagement and usability for students. The final materials, therefore, reflect both the content and design improvements made in response to the feedback, ensuring that the AR-based materials are pedagogically sound, engaging, and aligned with the intended educational goals.

Prelimenary Phase

In this study, the interviews conducted with 26 students were part of a broader data collection process aimed at understanding learning obstacles. However, the responses from only three students (S1, S2, and S3) are presented in this section due to their representative nature in highlighting the key challenges encountered by students in mathematics. These students were selected because their individual struggles with multiplication exemplified distinct levels of difficulty, ranging from memorization issues to fundamental misconceptions, thus providing a comprehensive view of the variety of obstacles faced. The focus on these three students allows for a detailed exploration of their specific challenges and the types of interventions needed, which directly informs the design of educational products to address similar issues among other students in the sample.

The interviews with three students, identified as S1, S2, and S3, revealed various learning obstacles in mathematics, particularly in multiplication. S1, despite understanding the concept of multiplication, struggled with memorizing multiplication tables from 1 to 9 and often made errors due to inaccuracy in calculations. She noted that practice exercises were insufficient, although she felt comfortable with story problems and preferred when teachers provided initial explanations before problem-solving. Conversely, S2, who had memorized multiplication tables up to 10, encountered difficulties in multi-digit multiplication, especially involving hundreds and thousands. S2 often made errors in carrying numbers due to limited practice in this type of problem. Although familiar with story problems, the limited exposure to them affected her ability to master this skill.

Meanwhile, S3 exhibited more fundamental learning obstacles, particularly in understanding basic mathematical operations. She often confused addition and multiplication and experienced misconceptions in basic operations. Her scores were generally low, except in topics that were repeatedly practiced. Moreover, S3 was unfamiliar with the concept of story problems and struggled to understand and solve them unless the teacher read them aloud. This highlights her limited exposure to and understanding of different types of mathematical problems.

The findings indicate that these students require more intensive and varied instructional approaches. S1 and S2 need additional practice, particularly in story problems and multi-digit multiplication. S3, on the other hand, requires foundational support in understanding basic mathematical operations and a gradual introduction to story problems. These insights emphasize the crucial role of teachers in providing consistent practice, leveraging engaging instructional media, and fostering a supportive learning environment, particularly for topics perceived as difficult or uninteresting.

Teaching Experiment Phases

Descriptive statistics aim to provide an initial overview of the numeracy data. Based on the analysis, from a total of 62 samples, the minimum value is 60, and the maximum value is 85, with a mean of 69.68. The standard deviation of 8.818 indicates a degree of variation in the numeracy scores. This interpretation suggests that, on average, numeracy scores fall within the medium range, with a relatively varied data distribution. The results of the descriptive statistics analysis are presented in Table 5.

Descriptive Statistics					
	N	**Minimum**	**Maximum**	**Mean**	**Std. Deviation**
Numeracy	62	60	85	69.68	8.818
Valid N (listwise)	62				

Table 5: Descriptive Test

Subsequently, the researcher conducted a normality test to examine the distribution of the data. This step is crucial to determine the appropriate hypothesis test to be employed in addressing the research hypothesis. The results of the normality test are presented in Table 6 below.

Tests of Normality						
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Numeracy	.251	62	.248	.848	62	.098
a. Lilliefors Significance Correction						

Table 6: Normality Test

The normality test using Kolmogorov-Smirnov and Shapiro-Wilk aimed to determine whether the numeracy data followed a normal distribution. Based on the Kolmogorov-Smirnov test, the significance value was 0.248, while the Shapiro-Wilk test showed a significance value of 0.098. Since the significance values of both tests exceed 0.05, it can be concluded that the numeracy data is normally distributed. Given the normal distribution, the researcher proceeded with parametric testing. Accordingly, a One-Sample Test was conducted. The results of this test are presented in Table 7 below.

One-Sample Test						
	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper

Numeracy	62.216	61	.000	69.677	67.44	71.92

Table 7: One-Sample Test

The hypothesis test was conducted to determine whether the mean numeracy score differs significantly from zero. The analysis results show a t-value of 62.216 with 61 degrees of freedom (df) and a significance level of 0.000. Since the significance value is less than 0.05, the null hypothesis is rejected, indicating that the mean numeracy score (69.68) is statistically significantly different from zero. The 95% confidence interval for the mean numeracy score ranges from 67.44 to 71.92. To further explore these findings, the researcher examined other variables that might influence students' numeracy performance. The results of the covariate analysis are presented in Table 8 below.

Tests of Between-Subjects Effects					
Dependent Variable: Numeracy					
Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3400.701[a]	3	1133.567	48.961	.000
Intercept	10407.440	1	10407.440	449.516	.000
GND	2251.307	1	2251.307	97.238	.000
AGE	65.089	1	65.089	2.811	.099
GEO	242.667	1	242.667	10.481	.002
Error	1342.847	58	23.153		
Total	305750.000	62			
Corrected Total	4743.548	61			
a. R Squared = .717 (Adjusted R Squared = .702)					

Table 8: Covariate Test

The covariate analysis aimed to assess the influence of the independent variables (GND, AGE, and GEO) on numeracy, while accounting for the covariate effect. The analysis results indicate a significant corrected model (F = 48.961, Sig. = 0.000) with an Adjusted R Squared of 0.702, suggesting that 70.2% of the variance in numeracy can be explained by this model. Both GND (F = 97.238, Sig. = 0.000) and GEO (F = 10.481, Sig. = 0.002) have a significant effect on numeracy. However, AGE (F = 2.811, Sig. = 0.099) does not show a significant effect. This interpretation indicates that gender and geographical location significantly influence numeracy, whereas age does not have a significant impact.

DISCUSSION

Product Validation

The results of the content and construct validity assessments for the AR-based instructional. The results of the content and construct validity assessments for the AR-based instructional materials demonstrate that the developed tools are highly valid and effectively aligned with their intended educational purpose. The content validity, as indicated by an Aiken's V score of 0.87, suggests that the materials were considered highly relevant, accurate, and appropriate by the expert panel. The strong content and construct validity of the AR-based instructional materials developed in this study are consistent with findings from previous research that underscores the importance of expert validation and rigorous assessment in educational material development. According to Ariffin et al. (2010), expert validation plays a crucial role in ensuring that instructional materials are not only accurate but also contextually appropriate for the intended educational settings. The high Aiken's V score of 0.87, which improved to 0.90 after refinements based on feedback, aligns with this notion, emphasizing that expert feedback is pivotal in enhancing the relevance and clarity of instructional content. This expert validation process ensures that the materials align with pedagogical goals and can meet the specific needs of students. Furthermore, the expert panel's positive assessment of the AR design further supports the findings of Nieveen et al. (2013), who argued that the integration of engaging technology, such as AR, can significantly enhance numeracy instruction by fostering interactive and hands-on learning experiences.

Moreover, to content validity, the construct validity results reinforce the effectiveness of the AR-based materials in measuring and improving numeracy skills, as well as fostering student engagement. The high factor loadings and strong model fit indices (CFI = 0.91, TLI = 0.90, RMSEA = 0.048) support the notion presented by Hair et al. (2019) that technology-enhanced learning tools can play a significant role in supporting constructivist approaches to education. The refined AR materials, which emphasize gamification and interactive elements, align with Slavin's argument that such tools can promote deeper engagement and improve cognitive outcomes. The details of validation and FGD are shown in table 9.

Validated Aspect	Validity Score	Improvements Made	FGD Feedback	Final Evaluation Score
Content	Aiken's V = 0.87	Refinement of language and example problems	The materials were found to be highly relevant and easy to understand	Aiken's V = 0.90
AR Design	-	Added interactivity and visualizations	FGD appreciated the use of AR in numeracy materials but suggested adjustments to the interface controls	Score 4.5/5

Student Engagement	-	Strengthened gamification elements to increase motivation	FGD showed that students were more engaged with more interactive elements	Score 4.8/5

Table 9: Validation and FGD Summary

These findings suggest that the AR-based instructional materials developed in this study show considerable potential in addressing common numeracy learning challenges faced by students. Previous research has demonstrated that technology-enhanced learning tools, such as augmented reality (AR), can offer innovative solutions to improving numeracy skills by engaging students in interactive learning experiences (Nieveen et al., 2013). The materials developed in this study are designed with pedagogical best practices in mind, ensuring that they not only enhance numeracy understanding but also foster deeper cognitive engagement. This approach aligns with the constructivist learning theory, which emphasizes active learning through interaction with technology to build a more meaningful understanding of complex concepts (Purpura et al., 2011). By incorporating interactive features and adaptive learning elements, the AR materials provide an immersive environment that supports students in mastering numeracy concepts at their own pace.

Moreover, the alignment of both content and construct validity with established educational frameworks further strengthens the potential impact of the AR materials on student learning. The expert validation process confirmed that the content is relevant and accurate, while the construct validity analysis demonstrated that the AR-based materials effectively measure numeracy skills and student engagement. These results are consistent with studies by Ariffin et al. (2010) and Sari et al. (2024), which emphasize the importance of expert feedback and validation in ensuring that instructional materials are not only pedagogically sound but also effective in achieving desired educational outcomes. As the use of AR technology in education continues to grow, these findings suggest that the developed materials could offer valuable contributions to improving numeracy education and student engagement in the classroom.

Thinking Trajectories

Thinking trajectories describe the cognitive steps students take as they develop their numeracy skills. In the context of Augmented Reality (AR), these trajectories are dynamic and interactive, providing students with opportunities to explore, manipulate, and reflect on mathematical concepts through direct engagement with virtual environments (Szilágyi et al., 2013). This approach aligns with Sztajn et al. (2016) theory of teaching and learning trajectories, which helps educators understand the typical developmental stages of students' mathematical reasoning.

For instance, in our study, when students engaged with Augmented Reality (AR) tools to learn number operations such as addition and subtraction, they first interacted with visual representations that allowed them to manipulate objects directly on their devices. The results indicated that students in the AR group demonstrated a significant improvement in their ability to understand

and perform these operations compared to those in the traditional learning group. The concrete level of interaction provided by AR allowed students to visualize the mathematical concepts in a dynamic way, enhancing their comprehension and retention of the material. Initially, students might use virtual manipulatives, such as blocks or counters, to represent numbers and perform basic operations. This hands-on approach mirrors the concrete operational stage described by Piaget, where children understand mathematical concepts through tangible experiences (Slavin, 2018).

Moreover, the thinking trajectory has several stages (Panhuizen, 2019). At the first stage, students use AR tools to visualize and manipulate objects. For example, in an addition task, the AR app might display 5 virtual cubes and ask students to add 3 more. The student interacts with the AR by virtually adding 3 cubes to the existing 5, seeing the total number increase to 8. This operation is grounded in the real-world experience of counting physical objects, which aids students in forming the concept of addition.

In mathematical notation, this stage can be represented as:

$$5+3=8$$

This stage is foundational as students connect the abstract symbol "+" with the physical act of combining quantities. Once students are comfortable with the concrete representation, the AR environment may prompt them to focus on the abstract aspects of number operations. The AR system might hide the virtual cubes and present the equation $5+3=?$ on the screen. Here, students must rely on their previous experiences with physical manipulatives to understand and solve the problem. In this stage, students begin to make connections between the physical action of counting and the symbolic representation of numbers and operations. This stage represents the transition from concrete manipulatives to abstract mathematical reasoning. The equation becomes a symbol-based problem rather than a physical one, leading to a deeper understanding of numerical relationships.

Next, students begin to fully understand and apply mathematical symbols and operations independently of the physical objects. In AR, students may interact with virtual number lines or calculators that display abstract problems like 8-3=?. The use of symbols such as "+" and "=" is now internalized, and students can perform operations without needing the visual aid of objects. For example, in a subtraction task, students may see the equation:

$$8-3=5$$

Here, the student no longer needs to manipulate virtual cubes but instead relies on their mental model of the operation. This shift represents the formalization of mathematical thinking, where the student can now use symbolic operations fluidly and with confidence.

At the final stage, students begin to reflect on their own thinking and justify their solutions. Using AR, students might be asked to explain why 8-3=5 or to explore different methods of solving the problem, such as counting backward on a number line or using the inverse relationship between addition and subtraction. This stage promotes higher-order thinking, where students not only perform operations but also understand the reasoning behind them. For example, students might explain: "To find 8-3, I know that 3 is the same as taking away 3 from 8, which leaves 5.

This is the opposite of adding 3 to 5 to make 8." The ability to articulate mathematical reasoning indicates a mature understanding of both the symbols and the underlying concepts.

Through AR, students' thinking trajectories evolve from concrete manipulation of objects to abstract mathematical reasoning as they internalize the symbols and operations that define numeracy. These stages demonstrate how AR not only supports cognitive development but also aligns with Piaget's stages of cognitive development and Vygotsky's socio-constructivist theory, where social interactions and tools such as AR mediate learning and help students move from concrete experiences to abstract concepts (Glassman, 2001; Liu & Matthews, 2005). These findings have significant implications for real-world educational practices, particularly in designing numeracy instruction that is contextual and responsive to students' needs.

For educators seeking to implement AR-based numeracy interventions, it is recommended to integrate features such as adaptive problem-solving prompts tailored to students' cognitive development levels. Teachers can also leverage AR to create learning environments that foster collaboration and discussion, thereby strengthening the internalization of mathematical concepts. Future research could further explore how different AR features influence students' progression through these thinking stages and how their implementation can be adapted to diverse classroom needs.

AR for Numeracy

Augmented Reality (AR) has emerged as a transformative tool in mathematics education, particularly in enhancing numeracy skills. AR integrates virtual content with the real world, offering an immersive learning environment that promotes engagement and motivation (Elmqaddem, 2019; McKnight, 2020). In this study, AR was utilized to teach number operations, providing interactive simulations where students could explore and practice addition, subtraction, multiplication, and division.

The hypothesis test confirmed that students' numeracy scores were significantly above the baseline, with a 95% confidence interval between 67.44 and 71.92. These results are consistent with prior research (Bacca, 2019; Hwang et al., 2023), which found that AR fosters active learning by enabling students to visualize abstract concepts in tangible ways. These findings have practical implications for real-world educational settings, particularly in classrooms where engaging students in numeracy learning can be challenging. By incorporating AR, teachers can create a more interactive and student-centered learning environment, which may lead to improved long-term retention and understanding of mathematical concepts.

AR environments encourage active exploration, aligning with the experiential learning model proposed by Kolb (2014). For example, when solving arithmetic problems, students can manipulate virtual counters or number lines, supporting their procedural fluency and conceptual understanding simultaneously. The immediate feedback and gamified elements in AR also cater to intrinsic motivation, as highlighted by self-determination theory (Abramovich et al., 2019; Kobus et al., 2008). Integrating AR within a Desain Teknologi Didaktik (DTD) framework ensures that technology use is aligned with instructional goals. The DTD approach, which combines the TIP and ASSURE models, emphasizes planning, implementation, and evaluation stages that help teachers guide students to focus on learning objectives rather than merely interacting with technology.

In classroom practice, educators can apply the DTD framework by first identifying learning goals related to numeracy, selecting AR tools that align with these objectives, and designing activities that encourage meaningful student interaction (Rahmatih et al., 2021). For instance, during lessons on number operations, teachers can incorporate AR-based number lines to help students visualize numerical relationships, while structured guidance ensures that technological engagement remains purposeful (Meeks et al., 2014). Future research could explore the longitudinal impacts of AR on numeracy, particularly in varying instructional settings, to provide further insights into its sustained educational benefits.

Age and Geographical Location

The covariate analysis revealed significant effects of gender (GND) and geographical location (GEO) on numeracy scores, while age (AGE) did not emerge as a significant predictor. Specifically, the model's Adjusted R Squared value of 0.702 indicates that 70.2% of the variance in numeracy outcomes can be attributed to the model, underscoring the critical importance of gender and geographical location in explaining differences in numeracy achievement. The significance of geographical location aligns with findings from Schleicher (2019), which highlight disparities in educational resources and opportunities that exist across different regions.

These disparities are often linked to access to high-quality teaching, technological resources, and infrastructure. Students from urban areas, for instance, typically benefit from more advanced educational technologies, such as digital tools and online learning platforms, which can facilitate enhanced numeracy learning experiences. In contrast, students from rural or remote areas may face challenges related to limited access to such resources, which can hinder their learning progress, particularly in the context of AR-based education. The importance of infrastructure and technological literacy in the successful implementation of AR learning is well-documented by Ennis & Chen (1995), Rakhmawati et al. (2024), and Sumardi (2019), who emphasize the need for robust technological infrastructure to ensure that students can fully engage with and benefit from AR-based learning environments.

Conversely, the lack of a significant effect of age on numeracy scores suggests that developmental differences among the sampled students were relatively minimal within the age range represented in this study. This finding is particularly interesting because it contrasts with Piaget’s theory of cognitive development, which suggests that cognitive growth and mathematical reasoning are strongly influenced by age-related stages (Slavin, 2018). According to Piaget, students at different ages may exhibit distinct cognitive abilities, which in turn affect their capacity to understand and apply mathematical concepts. However, the current study’s results align more closely with research by Allcoat (2021), who found that numeracy skills are more closely linked to the quality of the learning environment and instructional methods than to age itself. This suggests that factors such as teaching strategies, classroom support, and access to educational technologies may play a more significant role in shaping numeracy outcomes than the biological age or developmental stage of students.

The findings further underscore the need for an inclusive and effective learning environment that transcends age-related assumptions about students' capabilities. More importantly, these results point to the crucial role of equitable educational policies that address geographical disparities in access to resources. For instance, the implementation of AR-based learning programs in rural schools could help bridge the gap in numeracy achievement between urban and rural students. By providing innovative and interactive learning experiences, AR technology has the potential to offer rural students the same high-quality educational opportunities that their urban peers enjoy, overcoming traditional limitations such as lack of access to specialized teaching resources. Moving forward, future studies should focus on examining how regional policies and improvements in infrastructure can positively influence the efficacy and sustainability of AR-based numeracy programs. Investigating the relationship between these factors would provide valuable insights into how technology can be leveraged to create more equitable educational systems that benefit all students, regardless of their geographical location.

CONCLUSIONS

Based on the results of this study, it can be concluded that the use of Augmented Reality (AR) in numeracy learning has a significant impact on improving students' numeracy skills. Descriptive analysis shows that the average numeracy score of students is 69.68 with a standard deviation of 8.818, indicating a reasonable variation in the data distribution, but generally falling within the medium range. The normality test results demonstrate that the numeracy data follows a normal distribution, which allows for the use of parametric testing. The hypothesis test shows that the mean numeracy score is significantly different from zero, with a t-value of 62.216 and a significance level of 0.000, indicating that AR-based numeracy learning has a positive effect. Additionally, the covariate analysis reveals that gender and geographical location significantly influence students' numeracy abilities, while age does not have a significant effect.

The impact of using AR in numeracy learning is highly positive, as this technology offers a more interactive and visual approach for students to understand abstract concepts like arithmetic operations. Through AR, students can see and manipulate virtual objects that help them conceptualize mathematical operations in a way that is easier to understand. This is consistent with previous findings that suggest technology-based learning can enhance students' comprehension of mathematical content and strengthen their numeracy skills. AR also supports the development of students' mathematical thinking through more structured thinking stages, from manipulating concrete objects to understanding more abstract mathematical symbols.

However, while AR has proven effective in improving numeracy skills, this study also indicates that factors such as gender and geographical location can affect student learning outcomes. Therefore, it is recommended that educators consider these factors when designing AR-based learning experiences to ensure inclusivity and accommodate the diverse needs of students. Furthermore, further research is needed to explore the influence of age and how AR implementation can be tailored to students' age groups to maximize their learning outcomes.

Overall, the use of AR in numeracy education holds great potential to enrich students' learning experiences and improve their learning outcomes. Therefore, it is recommended that education be further integrated with AR technology, particularly in mathematics, to create a more engaging and effective learning environment.

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