Cognitive Disconnect in Middle School Algebra: A Phenomenological Study of Students' Understanding of Variables

Analisa Fitria, Subanji*, Susiswo, Hery Susanto

Universitas Negeri Malang, Indonesia

analisa.fitria.2103119@students.um.ac.id, subanji.fmipa@um.ac.id*, susiswo.fmipa@um.ac.id, hery.susanto.fmipa@um.ac.id

Abstract: *This study aims to explore the cognitive obstacles experienced by middle school students in understanding the concept of variables in algebraic expression learning. Using a descriptive phenomenological approach, this study introduces the term cognitive disconnect to describe a disruption of meaning caused by incompatibility between students' prior knowledge (Met-Before the Lesson/MBL) and the new concepts acquired during the lesson (Met-During the Lesson/MDL). Data were collected through classroom observations, analysis of student worksheets (LKPD), error-based interviews, and examination of textbooks and worksheets used by the teacher. The results show that students tend to interpret variables as concrete objects rather than quantitative symbols. Although algebraic expressions can be written correctly in terms of syntax, the meaning of letter symbols remains tied to prior arithmetic experiences, resulting in inaccurate interpretations of variables. The incompatibility between MBL and MDL is not explicitly bridged in the learning process and is reinforced by didactical situations, such as inconsistencies in learning resources and the dominance of procedural approaches. These findings highlight the importance of learning that connects arithmetic experiences with variables through representations and scaffolding.*

Keywords: cognitive disconnect, met-before, variable, didactical situation

INTRODUCTION

Algebra is a major branch of mathematics that forms the foundation for mastering concepts in geometry, statistics, and calculus (Bush, 2013; Edogawatte, 2011; Joanna & Jacquelynn, 2019; Nataraj & Thomas, 2017; Wettergren, 2022; Yew et al., 2020). At the middle school level, one of the essential topics is the introduction to algebraic expressions, particularly the concept of variables, which functions as a starting point for the cognitive transition from concrete arithmetic to abstract symbolization (Kieran, 2011; Öztürk, 2021; Pratiwi et al., 2019). Algebraic expressions include elements such as variables, coefficients, constants, and terms (Girit & Akyuz, 2017; Tekin-Sitrava, 2017).

However, various studies have shown that many students experience difficulties in understanding the conceptual meaning of variables (Blanton et al., 2017; Edogawatte, 2011; Ely & Adams, 2012;

Radford, 2006; Wettergren, 2022). This phenomenon is also found in Indonesia, where students often hold misconceptions regarding the function and meaning of symbols in algebraic expressions (Das, 2020; Jupri et al., 2014; Natalia et al., 2016; Wahyuni & Herman, 2019). Fitria et al. (2023) found that although students can identify algebraic expressions symbolically, they often interpret variables as concrete objects rather than abstract quantities (Fitria et al., 2023).

The transition from arithmetic to algebra demands a cognitive restructuring in the way students think. Their prior experiences with arithmetic, which are concrete and object-oriented, often conflict with the abstract nature of algebra, leading to conceptual gaps and increasing the likelihood of misconceptions (Akkan & Baki, 2016; Humberstone & Reeve, 2017; Malisani & Spagnolo, 2009; Sudianto & Kisno, 2021). These difficulties become one of the key barriers to effective algebra learning at subsequent stages. A large proportion of students who fail to understand variables and subsequently algebraic expressions tend to struggle with mathematics at higher levels (Blanton et al., 2018; Sugiman, 2023). One of the main contributing factors to this difficulty is the activation of pre-existing thinking schemas that are misaligned with the new symbolic context.

According to Tall (2004), before receiving formal learning in the classroom, students have already been exposed to various mathematical ideas and knowledge through everyday experiences. This knowledge is informally constructed by students before they encounter the formal definition of a concept in school learning (Beyene, 2023; Levin & Walkoe, 2022; Tall, 2008). Tall refers to this prior knowledge or experience as *met-before*, which is a mental structure formed from previous interactions with mathematical concepts (De Lima & Tall, 2008; Mallet, 2013; McGowen & Tall, 2010; Tall, 2004). In learning, *met-before* can support understanding if it aligns with the concept being taught (Liaw et al., 2021; Özdemir & Yıldız, 2021). However, *met-before* can also become an obstacle when applied in a different context, namely when previous rules or ways of thinking are no longer relevant or suitable to be used in a new learning situation (De Lima & Tall, 2008). For example, experiences in arithmetic can become a hindrance when students are accustomed to the assumption that "*addition always results in a larger number*," whereas this assumption does not always apply in the context of algebra (Kieran, 2011). An obstacle can hinder the learning process and cause students to experience difficulties in understanding the concepts being taught (Beyene, 2023; Brousseau, 2006; Herscovics, 1978; Mallet, 2013; Tall, 2004).

The idea of this obstacle was first introduced by Bachelard (1938) as an epistemological obstacle, namely a disruption in thinking that hinders the understanding of new concepts (Trindade et al., 2019). This concept was later adapted by Herscovics (1978, 1997) into a cognitive obstacle in the context of learning, which arises when students' knowledge schemas conflict with new concepts, particularly during the transition from concrete to abstract understanding, such as in the shift from arithmetic to algebra (Herscovics, 1978).

Tall (2004) stated that prior experiences (*met-before*) often become a source of obstacles in learning mathematics. Students have a tendency to apply existing schemas to new contexts, so when the concept being learned conflicts with previous experiences, cognitive conflict arises and hinders the learning process. In such conditions, students prefer to rely on familiar procedures rather than engage in the reorganization of more complex concepts, in line with theory of equilibration by Piaget, which postulates that individuals are more likely to assimilate than to accommodate when facing knowledge discrepancies (De Lima & Tall, 2008; McGowen & Tall, 2010). Expanding on this idea, Beyene (2023) introduced the Met Before the Lesson (MBL) and Met During the Lesson (MDL) framework to explain in greater detail how cognitive obstacles occur. Cognitive obstacles arise when there is a incompatibility between MBL, which refers to students' prior experiences and knowledge schemas, and MDL, which represents the new concepts introduced during the learning process. This incompatibility may take the form of misconceptions or inaccurate interpretations of new concepts and is strongly influenced by cultural contexts and teaching approaches in each country (Artigue et al., 2014; Beyene, 2023; Jupri & Drijvers, 2016). Therefore, understanding the nature of obstacles stemming from the MBL–MDL incompatibility is a crucial step in designing more effective and conceptually grounded mathematics learning.

Since cognitive obstacles are related to the incompatibility between MBL and MDL, it is important to trace how MDL is formed during the classroom learning process, particularly in the context of introducing algebraic expressions. The Theory of Didactical Situations (TDS) is used to analyze the formation of MDL through didactical situations, which include the *milieu* and the didactical contract (Artigue et al., 2014; Brousseau, 2002; Maknun et al., 2022). In this study, the *milieu* refers to the learning environment, which includes learning resources such as textbooks, teaching modules, and student worksheets (LKPD) that are used and designed by the teacher. If the provided *milieu* is procedural and does not present challenging problematic situations, then students' understanding tends to be shallow (Artigue et al., 2014; Brousseau, 2002). Meanwhile, the didactical contract refers to the teacher's instructional strategies, including the use of analogies and the ways in which student responses are addressed during the lesson (Artigue et al., 2014; Radford, 2014). Exploration-based instructional strategies can provide opportunities for students to connect their MBL with the new concepts being learned in a more meaningful way.

Furthermore, indications of cognitive obstacles can be identified through student errors that are repeated and consistent. Such misconceptions are not random occurrences but rather reflect an underlying structure of understanding that has been formed (Beyene, 2023; Brousseau, 2002; Mallet, 2013). Persistent misconceptions indicate the presence of obstacles in the thinking process that cannot be resolved merely through additional instruction but require a deeper analysis of how students construct understanding during the lesson process.

Various studies have examined students' lesson difficulties in algebra through the perspectives of misconceptions, epistemological obstacles, and didactical design (Fitriani et al., 2023; Joanna & Jacquelynn, 2019; Jupri et al., 2014; Lenz, 2022; Noto et al., 2020; Öztürk, 2021; Rakes & Ronau, 2019; Riastuti et al., 2023; Stacey & MacGregor, 1999; Weinberg et al., 2016). However, studies that explicitly highlight cognitive obstacles in understanding the concept of variables based on the framework of incompatibility between MBL and MDL are still limited. In fact, at the early stage of introducing algebra in middle school, this incompatibility can lead to students' failure to connect symbolic meaning with their prior arithmetic experiences. Based on this background, this study is formulated to answer two main questions:

1. How do middle school students experience cognitive obstacles in understanding the concept of variables in algebraic expression lessons as a result of the incompatibility between MBL and MDL?
2. How do classroom instructional practices (through teacher strategies and lesson resources) contribute to the formation of these cognitive obstacles?

By using a phenomenological approach and didactical situation analysis, this article is expected to provide both theoretical and practical contributions to the design of algebra instruction that is more responsive to students' prior knowledge structures.

LITERATURE REVIEW

This section discusses the main conceptual framework used in this study, namely, cognitive obstacles, the incompatibility between MBL and MDL, and the contribution of didactical situations in shaping or obstructing students' conceptual understanding of variables in algebra.

Cognitive Obstacle in Mathematics Learning

The concept of obstacles in the lesson was first introduced by Bachelard (1938) through the idea of epistemological obstacles, namely that the development of scientific knowledge does not proceed linearly, but rather through discontinuous leaps that occur due to obstacles in thinking (Trindade et al., 2019). This idea was later adapted into the context of mathematics education by Herscovics (1978), who emphasized that difficulties in understanding mathematical concepts are often not caused by a lack of information, but by a conflict between old cognitive structures and the newly introduced concepts (Eraslan, 2005; Herscovics, 1978). In his subsequent work, Herscovics (1989) stated that cognitive obstacles occur when students fail to assimilate new concepts into the knowledge framework they have previously constructed, particularly in higher-level algebraic concepts.

This concept has become the foundation for various subsequent studies. Cornu (1991), in his study on the concept of limits, and Mallet (2013), in a lesson about line integrals, showed that students' initial mental models often remain unchanged despite the introduction of formal concepts (Cornu, 1991; Kartinah et al., 2021; Mallet, 2013). These findings emphasize that cognitive restructuring does not happen automatically following the delivery of a formal lesson, but rather requires an active process of reconstruction by the students. Other studies have also contributed to expanding the understanding of cognitive obstacles. Bishop et al. (2015) explained that cognitive obstacles may initially support the formation of understanding, but later become barriers when that knowledge is applied in a different context (Bishop et al., 2014). Therefore, gradual instructional strategies are important to facilitate students' conceptual transitions. In a fraction lesson, Yoshida and Sawano (2017) showed that students' initial understanding of parts and division often leads to cognitive conflict, which can be alleviated through the use of concrete materials and appropriate conceptual approaches (Yoshida & Sawano, 2002). Antonijević (2016) added that obstacles may also arise when mathematical tasks exceed students' cognitive capacity; however, this can be overcome through cooperative learning that encourages collaboration in solving complex problems.

In a more recent study, Beyene (2023) developed the conceptual framework of cognitive obstacles by referring to the theoretical construct of met-before proposed by Tall (2004; 2008), which is the prior knowledge structure formed from previous lesson experiences. This *met-before* can serve as a supportive foundation, but also has the potential to become a barrier to understanding new concepts. Beyene adopted this concept by distinguishing *Met-Before the Lesso*n (MBL) as students' prior knowledge and *Met-During the Lesson* (MDL) as the new concepts introduced during the lesson. According to Beyene, cognitive obstacles happen when there is a incompatibility between MBL and MDL. This incompatibility can hinder the process of constructing and applying concepts, especially when instructional strategies or didactical transposition do not explicitly connect students' prior experiences with the new ideas being taught during the lesson.

This cognitive tension becomes increasingly complex when the students' concept image, formed from MBL, contradicts the formal concept definition provided in MDL (Tall & Vinner, 1981). This condition often causes dissonance in thinking, particularly when students are required to shift from concrete arithmetic understanding to symbolic representation in algebra. Vinner and Dreyfus (2010) refer to this phenomenon as compartmentalization, which is a condition in which students possess two systems of thinking that are not interconnected. For example, students may solve the expression $2x + 3x = 5x$ procedurally, yet still believe that x holds a specific value as in arithmetic, rather than recognizing it as a free variable in the algebraic system.

In the context of learning algebraic expressions at the middle school level, an incompatibility between MBL and MDL is highly likely to happen. This is primarily due to the transition from the concrete world of arithmetic to the more abstract forms of algebraic symbolization and manipulation. Therefore, this study directly adopts Beyene's (2023) conceptual framework to analyze how the incompatibility between MBL and MDL contributes to the emergence of cognitive obstacles in students' understanding of variables and algebraic expressions.

***Met Before the Lesson* (MBL), *Met During the Lesson* (MDL), and Didactical Situations as Components Shaping Cognitive Obstacles**

Within the framework of this study, *Met Before the Lesson* (MBL) refers to the cognitive structures or prior knowledge that students have before receiving a formal lesson on algebraic expressions. This structure is formed through previous learning experiences, including arithmetic lessons in elementary school, textbooks previously used, and students' interactions with various mathematical situations before the algebra lesson begins (Tall, 2008; McGowen & Tall, 2010; Beyene, 2023). Meanwhile, *Met During the Lesson* (MDL) refers to the new concepts introduced during the formal classroom lesson process. MDL reflects the conceptual structures expected to be developed through lessons, such as the concept of variables, algebraic expressions, as well as the operations and modeling of algebraic expressions. Cognitive obstacles in this study are defined as the incompatibility between MBL and MDL. When students' prior knowledge structures (MBL) do not align with or even contradict the new concepts being taught (MDL), students experience difficulties in constructing meaningful understanding. Beyene (2023) identifies three forms of such incompatibility: (1) prior knowledge cannot be applied directly; (2) difficulty in integrating prior knowledge with new concepts; or (3) the presence of deceptive compatibility, namely the impression that prior knowledge remains relevant, while it is misleading in the new context.

Furthermore, Prediger (2008) classifies the knowledge structures within MBL into three main dimensions: (1) algorithmic knowledge, which refers to procedures or steps that students are accustomed to performing; (2) intuitive knowledge and mental models, which are informal representations and beliefs formed through previous lesson experiences; and (3) formal knowledge, which refers to symbolic and conceptual understanding. An incompatibility in one or a combination of these three dimensions with the conceptual structure of the new content in MDL may lead to cognitive obstacles in learning algebraic expressions.

To understand how the incompatibility between MBL and MDL is formed and sustained throughout the learning process, this study analyzes the accompanying didactical situations using the framework of the *Theory of Didactical Situations* (TDS) (Brousseau, 2002; Artigue et al., 2014). This analysis is not intended to conclude that didactical situations are the direct cause of cognitive obstacles, but rather to examine whether and how the elements within the didactical situation, namely the *milieu* and the didactical contract, contribute to the emergence or reinforcement of the incompatibility between students' prior knowledge (MBL) and the new concepts being taught (MDL). The *milieu* includes all learning resources used during the lesson, such as textbooks, student worksheets (LKPD), and teaching modules. LKPD (Indonesian: *Lembar Kerja Peserta Didik)* refers to teacher-designed student worksheets used as instructional materials during the lesson. If the materials in these resources are presented without considering students' prior knowledge structures, the likelihood of an incompatibility between MBL and MDL will increase. The didactical contract, on the other hand, refers to the teacher's instructional strategies and the expectations established during the learning process. When learning places greater emphasis on procedures rather than on the understanding of meaning, students are likely to memorize rules without comprehending the underlying concepts. Effects such as the Topaze effect and the Jourdain effect (Brousseau, 2002; Artigue et al., 2014) serve as indicators that students may appear to succeed procedurally but experience conceptual obstacles in developing a deeper understanding of algebraic expressions.

Thus, cognitive obstacles in learning algebraic expressions cannot be separated from the complex interaction between students' initial cognitive structures (MBL), the conceptual structures built during instruction (MDL), and how the didactical situation, including both the *milieu* and the didactical contract, either supports or weakens the connection between the two. Therefore, understanding the relationship among these three components is essential for designing an algebra lesson that is more conceptual and meaningful. Figure 1 presents the conceptual framework of the study, illustrating the interaction between MBL, MDL, and the didactical situation as a potential source of cognitive obstacles in learning the concept of variables.

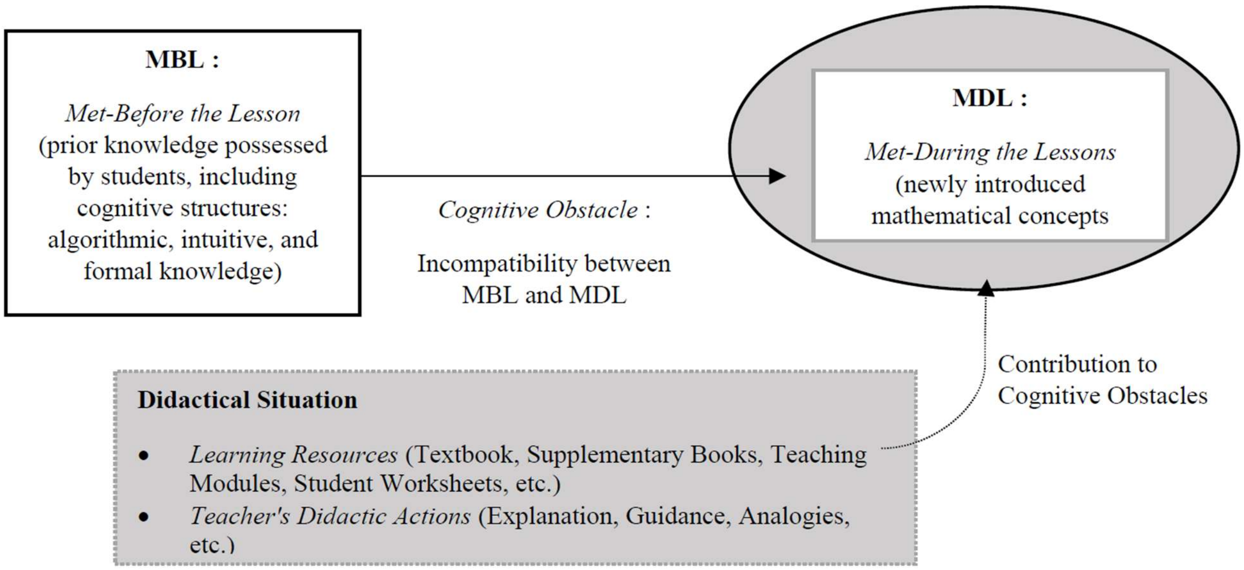


Figure 1: Conceptual Framework of the Study

METHOD

This study aims to describe the cognitive obstacles experienced by students in understanding the concept of variables in algebraic expression material, as well as to analyze the contribution of the didactical situation to the incompatibility between students' prior knowledge (Met Before the Lesson/MBL) and the new concepts acquired during lesson (Met During the Lesson/MDL). To achieve these objectives, a descriptive phenomenological approach was used, focusing on students' experiences as directly lived, without researcher intervention (Cohen et al., 2018; Polkinghorne, 1989; Qutoshi, 2018; Stolz, 2023; Zahavi, 2019). In this context, cognitive obstacles are understood not merely as procedural errors but as manifestations of the incompatibility between the MBL and MDL structures (Beyene, 2023), as observed during the classroom lesson process of algebraic expressions. To systematically implement this approach, the study adopted a descriptive phenomenological method by Giorgi (2009), which consists of three main stages: (1) Phenomenological reduction, (2) Description of experience, and (3) Essence analysis.

Research Context and Subjects

This study was conducted in one of the seventh-grade classes at a public junior high school in Banjarmasin City, which showed an increase in numeracy scores based on the Computer-Based National Assessment (Indonesian: *Asesmen Nasional Berbasis Komputer*; ANBK) results, although students' achievement in the algebra domain was still relatively low. According to the curriculum, the topic of algebraic expressions was taught over 25 instructional periods in 10 sessions, spanning approximately 1.5 months (6–7 weeks), covering algebraic expressions and their components, operations on algebraic expressions, and modeling. The concept of variables was introduced during the first two sessions (five instructional periods), which focused specifically on the formal introduction of variable notation. In the subsequent sessions, variables continued to be applied in activities involving algebraic operations and problem modeling. The observation was conducted in class VII A, which consisted of 34 students and was taught by one mathematics teacher. The observation was carried out without intervention, with the researcher acting as a passive observer, as recommended in the phenomenological approach (Cohen et al., 2018). All learning resources, including the textbook and student worksheets (LKPD), were entirely designed by the teacher, allowing the learning context to proceed authentically. As seventh-grade students, the participants had completed six years of elementary education, during which they systematically studied arithmetic and its properties. This arithmetic background constituted their met-before knowledge, which likely influenced how they perceived and interpreted the new concept of variables introduced in algebra lessons.

Participant selection in phenomenological research is conducted purposively and based on empirical data. By the principles of phenomenology, participants are selected because they have direct experience with the phenomenon being studied (Qutoshi, 2018; Polkinghorne, 1989; Williams, 2021). Therefore, the subjects in this study were students identified as experiencing cognitive obstacles based on conceptual errors in assignments (student worksheets) and responses during lessons. These errors are understood as indications of cognitive obstacles, namely the incompatibility between students' prior knowledge (Met-Before the Lesson/MBL) and the new concepts introduced during the lesson (Met-During the Lesson/MDL) (Brousseau, 2006; Beyene, 2023). During the lesson, the teacher divided the students into six groups, each consisting of 5 to 6 members. Based on the results of observation and analysis of the student worksheets, two groups showed indications of typical conceptual errors that represent cognitive obstacles, namely Group A (5 members) and Group D (5 members). This study focused its analysis on Group A because their seating position was the most feasible for conducting observation and interviews without disrupting the flow of the lesson. Although the in-depth analysis focused on Group A, similar patterns of conceptual errors and cognitive obstacles were also observed in Group D, based on their worksheet responses and classroom discussions. While not all groups exhibited these difficulties, the recurring patterns found in Group A and Group D suggest that the focal group represents common cognitive challenges experienced by a subset of students in this classroom context. This follows the principles of the phenomenological approach, which emphasizes a natural learning context free from intervention (Cohen et al., 2018; Polkinghorne, 1989).

Data Collection

This study has two main focuses: (1) to explore students' cognitive obstacles in understanding the concept of variables, particularly those that arise from the incompatibility between prior knowledge (MBL) and new concepts introduced during lesson (MDL); and (2) to analyze the contribution of the didactical situation to this incompatibility, both in terms of learning resources and the teacher's instructional strategies. To address these two focuses, data were collected from various sources:

1. Classroom observation, used to examine the learning situation, the teacher's instructional strategies, students' responses to tasks, and indications of cognitive obstacles.
2. Student worksheets, which were analyzed to identify errors as initial indicators of cognitive obstacles.
3. Error-based interviews, conducted in groups or individually (based on the worksheets provided by the teacher) to explore students' thinking processes and trace the forms of prior knowledge (MBL).
4. Learning resource documents, such as textbooks, student worksheets, and teaching modules, were analyzed to evaluate how the presentation of material may reinforce or reduce the incompatibility between MBL and MDL.

It is important to emphasize that all student worksheets and tasks were analyzed as they were, within the context of the regular lesson, fully designed by the teacher. The researcher did not intervene or modify the materials or tasks. This is following the phenomenological approach, which emphasizes the exploration of students' experiences in a natural context (as lived), without external manipulation.

Data Analysis

Data analysis in this study followed a descriptive phenomenological approach by Giorgi (2009) which is oriented toward the essential description of students' experiences in understanding the concept of variables in the algebraic expression lesson. The main focus is to reveal how cognitive obstacles emerge as a result of the incompatibility between students' prior knowledge (MBL) and the new concepts acquired during the lesson (MDL). The analysis process was carried out in three main stages:

1. *Phenomenological Reduction (Epoché)*: The researcher suspended theoretical assumptions or initial interpretations and focused solely on students' experiences as they are. The data were analyzed without being directly connected to a specific theoretical framework, so that the meaning that emerged truly originated from the experiences as observed in classroom activities, student work, and interviews.
2. *Description of Experience*: Data from classroom observations, responses in the student worksheets, and error-based interviews were organized into a coherent descriptive narrative. The principle of horizontalization was applied, in which each data element (students' actions,

statements, and responses) was treated equally and then categorized based on experiential themes related to the emergence of cognitive obstacles.

3. *Essence Analysis*: this stage aims to identify the essential structure of the emerging cognitive obstacles. Using imaginative variation, the researcher explored how the incompatibility between MBL and MDL shaped recurring patterns of error. The analysis also highlighted the contribution of the didactical situation, both in terms of learning resources and teacher strategies, to the emergence of cognitive obstacles.

The final results were compiled in the form of a thematic narrative that represents the interrelation between students' thinking processes, prior knowledge structures (MBL), new concepts (MDL), and the conditions of the accompanying didactical situation. This approach enables a deep and contextual understanding of the phenomenon of cognitive obstacles, in accordance with the main principles of descriptive phenomenology.

RESULT

Initial Responses to the Teacher's Stimulating Questions

The classroom lesson on the topic of algebra began with stimulating questions delivered by the teacher to all students. The purpose was to explore students' prior knowledge (MBL) related to the concept of algebra, particularly concerning variables. Two open-ended questions were posed:

What do you know about algebra?
Is there any algebra in daily life?

However, the teacher did not directly call on students, resulting in only a few students responding spontaneously. The students' assumptions indicated limited and non-conceptual understanding. The majority stated that they did not know what algebra is, while a small number associated it with letter symbols, such as x, or with mathematical activities in daily life.

These observational results indicate that most students did not yet possess a strong initial conceptual structure related to algebra, even at the introductory level. The majority of students did not mention concepts such as variables, algebraic expressions, or relationships between quantities. The responses that emerged reflected recognition-based knowledge, for example, the presence of the letter x, without an understanding of the function of the symbol (Booth and McGinn, 2016). This phenomenon indicates that students' knowledge of algebra tends to be superficially symbolic and not yet rooted semantically. In the literature, however, variables are considered fundamental to the understanding of algebra and serve as an essential starting point in the cognitive transition from arithmetic to more abstract forms of symbolic representation (Knuth et al., 2005; Tekin Sitrava, 2017; Ventura et al., 2021).

The absence of a semantic foundation for mathematical symbols has the potential to create obstacles when students begin to formally study algebraic expressions. Furthermore, the teacher did not provide follow-up questions to explore more deeply the possible forms of MBL possessed by the students. No apperception activity connected previous lesson experiences, particularly in arithmetic, with the concept of variables. As a result, the lesson immediately shifted to the completion of student worksheets without providing the necessary conceptual transition. This situation closed the opportunity for students to realize that symbols in algebra, such as letters, have a representational function that can be linked to their previous arithmetic experiences. This condition became the starting point for the emergence of the cognitive disconnect identified in the subsequent findings.

Literal Interpretation and Pseudo-Construction in Variable Representation

After the session with stimulating questions, the teacher distributed student worksheets designed to introduce the concept of algebraic expressions through the visualization of concrete objects such as apples, mangoes, boxes, and balls. In this activity, students were asked to write algebraic expressions based on the given images. The findings in this section are focused on the analysis of Group A, which was identified as experiencing cognitive obstacles during the lesson on algebraic expressions. The selection of this group was conducted purposively based on classroom observation results and analysis of the student worksheets, as well as the consideration of their seating position, which allowed for observation and interviews without disrupting the lesson process. Data were obtained through classroom observation, student worksheet results, and error-based interviews, and were organized into thematic narratives following the principles of descriptive phenomenology. Although this section presents a qualitative analysis of Group A, a brief review of worksheets from all six groups (N = 34) showed that students in Groups A and D tended to interpret variable notation through concrete-object references. These 10 students interpreted variable symbols as physical items rather than as abstract representations of unknown or varying quantities.

The responses from students in Group A indicated that they tended to interpret the images literally as the number of concrete objects that could be counted, for example, writing 3 for a picture of three apples, or 2 + 1 for a picture of two apples and one mango. The results of Group A's work can be seen in Figure 2 below.

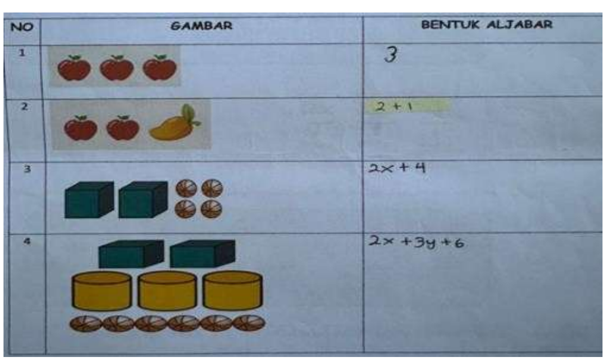


Figure 2. Student Group Responses on Worksheet (Group A)

The worksheet results from Group A indicate a tendency among students to interpret algebraic symbols literally and concretely. In the first question, students wrote the algebraic expression as 3 for the three apple images, reasoning that the number 3 directly represented the number of apples. Meanwhile, in the second question, which displayed two apples and one mango, students wrote 2+1 as the algebraic expression, because they observed two types of fruit and recorded them according to their physical quantities. Arithmetically, the representation 2+1 reflects the total number of concrete objects. However, when simplified to 3, the expression is identical to the first problem, even though the visual context and types of objects are different. This indicates that students have not yet internalized the idea that symbolic expressions in algebra represent not only quantities, but also the identity of objects or quantities whose values may change.

The interview with Group A reinforced this finding. When asked why they did not write 3x for three apples, the students rejected the idea because, according to them, there was nothing unknown in the image, and everything could be counted. They also explained that letter symbols are only used when the objects are not directly visible or are placed inside a container, such as in the case of a box or a tube filled with balls. The following interview excerpt clarifies how the students interpreted letters in that context:

Question: *In this picture, why did you answer that the algebraic expression is* 3*?*
Group A: *Because there are three apples in the picture.*
Question: *Is it acceptable to write it as 3x?*
Group A: *No, Miss. We also discussed it earlier, and we think there is no need to use x because all the apples are clearly visible.*

In another question involving two boxes and four balls, the students wrote $2x + 4$. When asked about the meaning of x, the students explained that x represented the box because the balls inside it were not known with certainty. In contrast, the number 4 was considered not to require a letter because those balls were clearly visible. This indicates that students tended to use letters as physical labels for unseen objects rather than as mathematical symbols representing quantities. This is illustrated in the following interview quotation

Question: *For this* $2x+4$*, how did you come up with it?*
Group A: *From the picture, there are two boxes, so that makes* $2x$*, and there are* 4 *visible balls, so it remains* 4.
Question: *What do you think x represents?*
Group A: *The box, Miss. Because there are two boxes, so it's* $2x$*. The balls do not need letters.*
Question: *Why did you not use letters for the apples and mangoes earlier?*
Group A: *Because there were no boxes. All the fruits were visible.*

These findings indicate that students understood letter symbols in a formalistic manner, but their semantic understanding remained tied to concrete objects. This phenomenon is referred to as pseudo-construction (Subanji, 2015), which occurs when students write symbolic forms that appear syntactically correct but are conceptually inaccurate. In addition, there were indications of deceptive compatibility, where symbolic structures such as $2x$ appeared to align with the visual context, but in fact did not represent the actual meaning of algebra. The connection with previous learning experiences in arithmetic was also strongly obvious. Students were accustomed to interpreting numbers as direct representations of concrete objects (for example, 2 rabbits means two actual rabbits).

The content of textbooks at the earlier grade levels further reinforced this association, as seen in examples that link images with quantities of objects (Figure 3). In the context of algebra, the lesson provided was insufficient to support students' transition toward understanding variables as abstract and flexible quantitative symbols. Without an explicit conceptual bridge in a lesson, the transition from arithmetic to algebra becomes hindered. When students assign object-based meanings to letter symbols rather than viewing them as variables, they are unable to comprehend the core concept of variables as quantities that are subject to change or whose values are unknown (Radford, 2011).

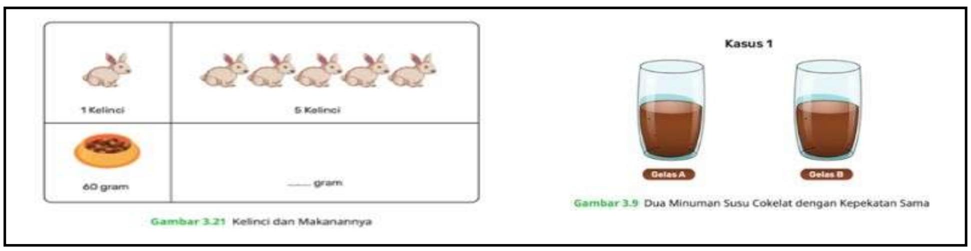


Figure 3: Textbook Material Connecting Images and Object Quantities

Figure 4 provides a schematic representation of the cognitive obstacle observed, illustrating how students' met-before experiences (MBL) in arithmetic—particularly associating numbers with visible objects—interfere with their understanding of variables as abstract symbols (MDL). This visual captures the deceptive compatibility between students' prior symbolic habits and the new algebraic representations expected in the lesson.

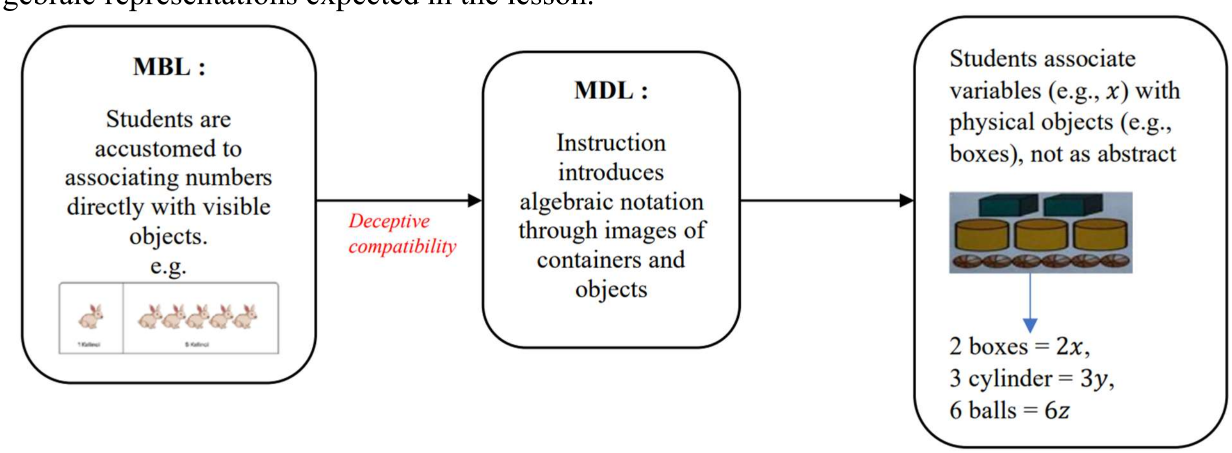
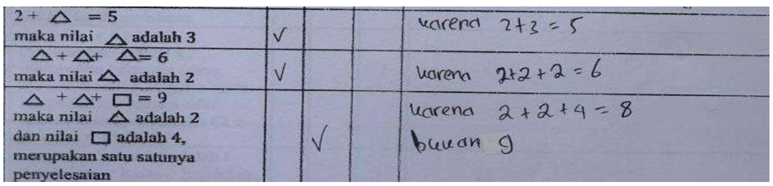


Figure 4: Deceptive Compatibility Between Students' MBL and MDL on the Concept of Variables

Cognitive Disconnect in the Transition from Placeholder to Variable

To follow up on the previous findings of pseudo-construction, the researcher administered a reflective task to all members of Group A as part of the interview preparation (Wijaya et al., 2022). The purpose was to further explore students' symbolic experiences related to the structure of MBL. The task consisted of four arithmetic statements using the symbol ⊡ (*placeholder*), such as 2+ ⊡ = 5. Students were asked to choose "Agree," "Disagree," or "Don't Know" and to provide a justification. All students in Group A responded correctly and consistently, indicating that they had prior experience using arithmetic forms involving explicitly unknown numbers. Students understood ⊡ as a *placeholder*, namely an empty space that must be filled with a specific number.



2 + △ = 5 maka nilai △ adalah 3	√			karena 2+3 = 5
△ + △ + △ = 6 maka nilai △ adalah 2	√			karena 2+2+2 = 6
△ + △ + □ = 9 maka nilai △ adalah 2 dan nilai □ adalah 4, merupakan satu satunya penyelesaian		√		karena 2+2+4 = 8 bukan 9

Figure 5: Student S1's Response Sheet on the Placeholder Task

To further explore the meaning structure that students constructed regarding the symbol, a follow-up interview was conducted with two group members, namely S1 and S2. The interview was carried out after the instructional activity had concluded, to ensure the process did not interfere with the learning situation, following the principles of the phenomenological approach (Cohen et al., 2018; Qutoshi, 2018). Although conducted separately, both students provided similar responses in terms of content and the reasoning behind their answers. This interview indicates that although students understood the symbol ⊡ as an unknown number, they did not automatically transfer that understanding to letter symbols. The following are excerpts from the interviews with S1 and S2 (member of Group A).

Question	***Student 1 (S1)***
Suppose it is given 2 + . . . = 5, *can you solve it?*	3
How did you get that?	*From* 5 - 2
What about this form, 2 + ⊡ *= 5, can you solve it?*	*Same as before,* 5 - 2 *is* 3
In your opinion, what does the form 2 + ⊡ = 5 *mean?*	*Two plus this (points to the box) equals five, so what is this (box)?*
What about the previous form, 2 + . . . = *5?*	*Same, Miss, only the dots are different (box versus dots)*
What about the lesson just given earlier, do you think it is related to these forms?	*Hmm… I don't think so, earlier we were learning algebra with letters*

Question	Student 1 (S1)
So that means you were learning variables?	*Hmmm… (nods)*
Can you explain what a variable is?	*A variable is a letter*
Do you know the function of the letter?	*For example, earlier it was for marbles inside a jar, also x for apples, and y for mangoes*
Earlier, you were able to fill in the box with the correct number to make $2 + \boxdot = 5$*, meaning the box contains* 3*. If we replace the box with a random letter, what do you think?*	*For example, like* $2 + a = 5$*, is that what you mean, Miss?*
Does it have the same meaning?	*If it's written like that, then it becomes algebra, Miss.*
What do you mean? So is it different from the earlier one?	*Yes, if it already uses a letter (a variable), then it's considered algebra.*
Can a be replaced with 3*?*	*In the previous lesson, we had to assume what the letter stands for, so I didn't really understand.*

Table 1: Excerpt Interview with S1

Interview data from S2 (a student from Group A) illustrates this interpretation

Question	Student 2 (S2)
For a question like this, $2 + \boxdot = 5$*, can you fill it in?*	*Yes, Miss. The answer is* 3*.*
How do you know it's 3*?*	*Because* 5 *minus* 2*, Miss.*
If we replace the box with a letter, like $2 + a = 5$*, do you think it still works?*	*It's different, Miss. If you use a letter, it becomes algebra.*
What's the difference?	*If it's a letter, we need to know first what a means. For example, a might mean apples or marbles.*
So does the letter mean an object in that case?	*Yes, Miss. Usually, a letter is used to replace the name of an object. If it's just a blank box, we just calculate it directly. But if we use a letter, we need to be told what the letter stands for.*
So, in your opinion, letters in algebra are not used for unknown numbers?	*No, Miss. If it hasn't been explained, I don't know what it's supposed to mean. Letters are like labels for objects.*

Table 2: Excerpt Interview with S2

The statements of both students indicate that they did not interpret letters in algebraic expressions as a continuation of the placeholder concept. Letters were understood as symbols that only carry meaning when assigned to specific objects, rather than as flexible quantitative representations that can stand independently. They were unaware that letter symbols such as a or x in algebra can function in the same way as the placeholder symbol ⊡, namely as a substitute for an unknown number (Knuth et al., 2005; Schoenfeld and Arcavi, 1988). Based on this analysis, the author refers to this phenomenon as *cognitive disconnect*, which is the students' failure to integrate meaning between their prior symbolic experiences (MBL) and the new concepts introduced during the lesson (MDL). This incompatibility happened due to the absence of an explicit bridge linking the function of placeholders and variables throughout the learning process.

This cognitive obstacle is not merely a procedural error, but a failure to construct a meaning structure that is conceptually connected. Although students can write symbolic forms formally, the foundational mathematical meaning is not understood. This situation indicates the presence of pseudo-construction (Subanji, 2015), which occurs when students produce semantically incorrect and syntactically correct answers because they interpret variables as object labels rather than as abstract quantities that are unknown or subject to variation. This finding highlights the importance of lessons that explicitly connect symbolic experiences in arithmetic with the concept of variables in algebra, thus, students can develop a conceptual understanding that is coherent and meaningful.

This finding is consistent with the report by Ely and Adams (2012), which shows that the transition from placeholder symbols to formal variables is often confusing for students, especially when teachers do not provide an explicit bridge connecting the two. Within the MBL and MDL framework (Beyene, 2023), this phenomenon reflects a cognitive disconnect, namely the failure of students to integrate their prior symbolic experiences (MBL) with the new concepts being taught (MDL). In summary:

- MBL: Students had prior experience solving expressions such as 2+□=5 with correct interpretation, where the placeholder represents an unknown number.
- MDL: The lesson introduced letters such as x or a as variables in algebraic expressions, but did not establish a connection between their meaning and the placeholder.
- *Cognitive Disconnect*: Students viewed *the placeholder* and the variable as two distinct entities. Letters were interpreted merely as labels for objects rather than as standalone quantitative symbols.

This finding directly addresses the research question, namely, how the incompatibility between students' MBL and the MDL, specifically the introduction of the variable concept in the classroom, contributes to the emergence of cognitive obstacles.

Contribution of Didactic Situations to the Emergence of *Cognitive Disconnect*

The *cognitive disconnect* experienced by students is closely related to the didactical situation that shapes the learning process. This situation involves various learning resources, including how materials are presented in textbooks, teaching modules, and student worksheets (LKPD), as well as the instructional strategies employed by the teacher. In the observed classroom, the teacher primarily referred to the Mathematics Textbook for Junior High School Grade VII (Kemendikbud, 2022), supplemented by a companion book published by PT Global Offset (2022) and custom-designed student worksheets.

1. Presentation of the Concept in the Textbook

The textbook introduces the concept of variables through the context of pattern generalization, such as constructing squares using matchsticks, and then expressing it in the form of an expression like $1+3n$. In this approach, the symbol n is positioned as an arbitrary number representing the number of pattern repetitions. However, students find it difficult to understand the meaning of this symbol because they do not see its connection to their prior experiences in arithmetic. The textbook does not connect the symbol n with the placeholder that is already familiar to students, resulting in n not being understood as a varying quantity, but rather being perceived merely as part of a form or an ordinary number (Ely and Adams, 2012). In fact, the transition from arithmetic to algebra requires students to recognize pattern structures as the foundation for constructing symbolic expressions (Apsari et al., 2020; Linchevski and Livneh, 1999; Taban and Cadorna, 2018).

Radford (2011) emphasizes that an effective algebra lesson must involve a transition from concrete representations to abstract symbols. When symbols such as n are introduced without sufficient explanation of their symbolic meaning, students will perceive them as labels for physical objects rather than as representations of quantitative relationships (Radford, 2014). This reinforces students' tendency to interpret variables in a concrete and inflexible manner.

2. Supplementary Book and Worksheet Approach

Unlike the textbook, the supplementary book and student worksheet (LKPD) frequently present variables through contextual problems, such as the unknown number of marbles in a bag. In this approach, the variable is understood as an unknown number, which is structurally closer to students' experiences in arithmetic. The following interview supports this tendency:

Question: Which book is easier to understand, the textbook or the worksheet (LKPD)?
Student 1: The worksheet, Miss. Because you used the worksheet in class. I don't understand this one (the textbook), it's about arranging matchsticks.
Question: So the worksheet is easier?
Student 1: Yes. Because it already uses letters (variables) directly.

The worksheet (LKPD) is connected to students' prior knowledge (MBL), such as the association between numbers and quantities of objects, making it cognitively more comfortable. However, this approach does not always support an abstract understanding of the variable concept.

3. Visual Representations in the Worksheet That Reinforce Misconceptions

In the worksheet used by the teacher, students were provided with images of concrete objects such as apples, mangoes, cubes, cylinders, and marbles. Students were asked to write algebraic expressions based on these images. The question boxes contained only pictures without labels or semantic cues, while the answer boxes were left blank and filled in by students based on their own interpretations. However, due to the absence of explicit guidance, students tended to associate algebraic symbols directly with types of objects rather than with varying quantities. For example, an image such as two cubes, three cylinders, and six cakes was converted into $2x+3y+6$, indicating that students perceived each physical shape as a distinct symbol. This condition reinforces the understanding of variables as labels for objects rather than as flexible quantitative symbols. Algebraic symbols are not interpreted as representations of relationships between quantities, but merely as labels for different objects. This phenomenon illustrates a failure in the transition from concrete to symbolic representation, as described by Radford (2014). Without teacher intervention that explains that variables represent quantities that vary, students will continue to form incorrect associations based solely on visual forms.

4. Incompatibility of Approaches and Cognitive Load

The discrepancy between the approaches used in the textbook and the worksheet (LKPD) creates two inconsistent representations of variables. The textbook leads to an understanding of variables as varying quantities, while the worksheet emphasizes variables as unknowns. Without a bridge or explanation that explicitly addresses this distinction, students become confused and are unable to understand that the same symbol can carry different meanings depending on the context. According to Malisani and Spagnolo (2009), variables as unknowns are easier to understand because they are more concrete, whereas variables as varying quantities require a higher level of abstraction. When students are presented with two approaches without integration, they experience representational conflict and an increase in cognitive load (Kirschner, 2002; Prendergast and Treacy, 2018; Takir, 2011). The following interview supports this finding:

Question: Did you read the section about variables in the textbook?
Group A: Yes, Miss. But it was difficult. For example, arranging matchsticks. We did not understand what it was for.
Group A: We found the worksheet easier to understand.

5. Limited Role of the Teacher in Bridging Concepts

Observations showed that the teacher did not provide a stimulating question to connect students' prior experiences in arithmetic, such as the use of placeholders, with the concept of variables in algebra. Although the teacher designed the lesson using a Problem-Based Learning (PBL) approach, in practice, the teacher directly introduced symbolic forms such as $2x+3y+6$ through the problem, without first developing students' understanding of the meaning of each symbol. In general, the lesson activities were dominated by question-answering tasks, both in the form of visual manipulation and the construction of algebraic expressions. The main focus of the lesson appeared to be on procedural problem-solving rather than on the exploration of the underlying concepts. The teacher guided students to complete the tasks in the student worksheet (LKPD) and evaluated their work based on the correctness of the algebraic forms, but did not provide space for in-depth discussion about the meaning of the algebraic symbols used by the students.

When students experienced confusion, the teacher assisted in the form of technical guidance, but it remained procedural. Correct answers in terms of form were immediately accepted, without encouraging students to reflect on the symbolic meaning behind them. For example, students were given an example of writing $2x+3y+6$ as a representation of images of concrete objects, without any explanation of why the variables x and y were used or what quantitative relationships were embedded in the algebraic expression. When students gave answers that were formally correct, the teacher approved them directly without uncovering the symbolic meaning behind them. As a result of this approach, students did not undergo the abstraction process necessary to understand variables as flexible symbols and representations of mathematical relationships. They only understood variables as labels for fixed objects depicted in the images, rather than as representations of quantities that can vary. Consequently, students' understanding remained at the pre-variable stage, a stage in which letter symbols are interpreted concretely and have not yet been understood functionally within algebraic structures (Ventura et al., 2021).

A brief interview with the teacher revealed her awareness of students' difficulties in learning algebraic expressions. In response, she designed worksheets featuring contextual visual representations such as apples, boxes, and marbles. She assumed these would help students grasp the concept of algebraic expressions by linking symbols to familiar concrete objects. However, the teacher did not fully recognize that the main difficulty lay in students' limited understanding of variables—not just algebraic notation. While these visual aids supported symbol recognition, they did not facilitate students' transition toward interpreting variables as unknown or varying quantities. Instead, the representations reinforced the misconception that algebraic symbols always correspond to specific concrete objects. This suggests that the instructional strategy, although well-intentioned, did not adequately support students in developing a structural understanding of variables. Moreover, the teacher's conceptual understanding of variables may still be developing, which limits her ability to effectively support students in constructing this understanding.

Overall, the didactical situation in the learning process, including the approach used in the textbook and student worksheet (LKPD), the visual representations in the tasks, and the teacher's lesson strategies, did not succeed in bridging students' prior experiences with the symbolic concepts in algebra. The lack of integration between approaches, the absence of a conceptual bridge, and the dominance of procedural learning reinforced the cognitive disconnect. This finding addresses the second research question, indicating that an instructional design that does not explicitly connect MBL and MDL is a major factor contributing to the emergence of conceptual obstacles in understanding variables. To summarize the dynamics of the incompatibility between students' prior knowledge (MBL), the new concepts introduced during lesson (MDL), and the contribution of the didactical situation, Figure 6 presents the interaction among these three elements in shaping the *cognitive disconnect*.

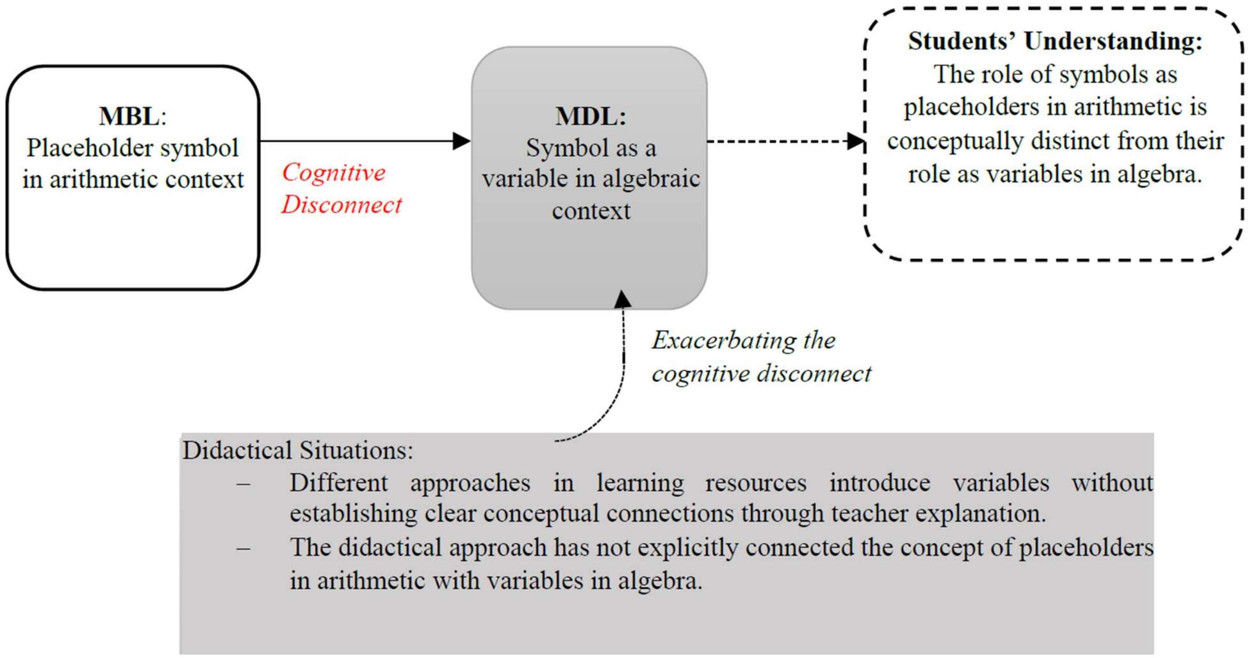


Figure 6: Cognitive Disconnect in the Concept of Variables

DISCUSSION

The findings of this study indicate that a primary conceptual obstacle in students' understanding of algebraic expressions is *cognitive disconnect*. This term, introduced in the current study, refers to students' failure to construct continuity of meaning between arithmetic placeholders (e.g., □ in 2+□=5) and letter symbols in algebraic expressions (e.g., $2+x$). Rather than viewing variables as a semantic continuation of prior symbolic experiences, students perceive them as entirely new and unrelated forms. This disconnect results in a rupture between students' prior knowledge (MBL) and the mathematical concepts introduced during instruction (MDL), as framed by Beyene (2023).

Although students are able to produce syntactically correct expressions such as $2x+4$, their interpretation remains literal. Variables are viewed as concrete labels—such as "apples" or "boxes"—rather than as abstract representations of varying quantities. This suggests that the semantic structure of variables has not yet been fully developed. Ely and Adams (2012) emphasize the importance of instructional approaches that explicitly guide students from using symbols as placeholders toward understanding variables as relational constructs. The inability to associate $2 + \square$ with $2 + x$ is similarly highlighted by Nathan and Koellner (2007) and Wang (2015), who stress the importance of symbolic continuity. Furthermore, Schoenfeld and Arcavi (1988), along with Nataraj and Thomas (2017), note that placeholders can serve as a foundation for understanding variables—if supported by deliberate instructional mediation. Without such scaffolding, this potential often remains unrealized (Glogger-Frey et al., 2018).

Two cognitive phenomena further reinforce this disconnect: pseudo-construction and deceptive compatibility. Pseudo-construction occurs when students write syntactically correct expressions based on incorrect conceptual reasoning (Subanji, 2015). Deceptive compatibility refers to students' confidence in misinterpretations that appear to align with prior knowledge (Beyene, 2023). For example, interpreting $2a$ as "2 apples" may feel accurate, but this superficial alignment conceals a failure to develop the abstract concept of a variable.

From the perspective of the *Theory of Didactical Situations* (Artigue et al., 2014; Brousseau, 2002), the learning milieu often fails to provide sufficient conceptual support. The textbook introduces variables through generalized patterns (e.g., $1+3n$) without explicitly connecting them to prior symbolic experiences. Conversely, the student worksheet (LKPD) relies on concrete visuals without symbolic explanation. This inconsistency is not bridged by the teacher through reflective discussion or conceptual questioning. As a result, instruction emphasizes procedural form over semantic development. Although the textbook presents opportunities for conceptual exploration through pattern generalization, this potential is left untapped without teacher mediation. As Radford (2014) explains, the development of symbolic reasoning involves *sensuous cognition*, in which concrete and perceptual experiences gradually evolve into abstract forms. Without guided support, students interpret variables as arbitrary visual symbols rather than tools for expressing generalized relationships.

This disconnect also explains students' preference for worksheets over textbooks. The issue lies not only in the procedural nature of the worksheets, but also in the perception that textbooks are too abstract and disconnected from existing knowledge structures. As Maknun et al. (2022) argue, when conceptual and procedural aspects of instructional resources are not integrated—and when teachers fail to mediate this gap—misconceptions are likely to deepen. A similar pattern has been identified in trigonometry instruction, where procedural emphasis without conceptual clarity creates epistemological barriers (Prendergast & Treacy, 2018). In algebra, such barriers emerge when instructional resources are not aligned through a coherent representational trajectory. Finally, within Tall's (2004) framework of symbolic representation, *cognitive disconnect* reflects students' failure to transition from the concrete operations of arithmetic to the abstract reasoning of algebra. Letter symbols such as x or a are not yet perceived as generalized mathematical objects, but remain linked to concrete associations. Without reflective mediation to support this conceptual progression, students remain confined to literal interpretations. At its core, *cognitive disconnect* emerges when students fail to recognize variables as a semantic extension of placeholders—thus separating the two in meaning rather than understanding them as part of a coherent conceptual progression.

While these findings offer valuable insights into students' cognitive disconnect in understanding variables, they should be interpreted in light of the study's methodological limitations. This study focused on a single group of students, purposively selected to maintain the authenticity of the learning process given the researcher's passive role. This approach allowed for in-depth analysis of students' collective cognitive dynamics, in line with the phenomenological emphasis on lived experience. However, the findings may not be broadly generalizable, as students' meaning-making can vary across different groups and contexts. Future studies are recommended to involve more diverse student populations, to design instructional interventions that explicitly connect placeholders and variables through visual, symbolic, and conceptual representational transitions, and to examine how teachers mediate students' conceptual development in authentic classroom contexts.

CONCLUSION

This study concludes that one of the most fundamental cognitive obstacles in learning algebraic expressions among junior high school students is *cognitive disconnect*, defined as a break in meaning between students' prior knowledge (Met Before the Lesson or MBL)—such as placeholder symbols in arithmetic—and the new concepts introduced during instruction (Met During the Lesson or MDL), namely variables expressed through letters such as x or a. Although students are able to construct algebraic expressions that are syntactically correct, they tend to interpret the letters as labels for concrete objects (such as apples, boxes, or cylinders) rather than as abstract symbols representing unknown or varying quantities. This phenomenon indicates the presence of pseudo-construction, in which students manipulate symbols formally without understanding the mathematical meaning they represent.

These findings suggest that *cognitive disconnect* is not merely a cognitive obstacle arising internally from the student, but also a consequence of a didactical situation that fails to provide adequate conceptual support. When textbooks and worksheets (LKPD) present inconsistent instructional approaches, and teachers do not offer explicit mediation of meaning, students are unable to connect prior symbolic experiences with newly introduced concepts. This study proposes the concept of *cognitive disconnect* as a specific form of conceptual cognitive obstacle that results from the incompatibility between MBL and MDL, compounded by insufficient didactical support that fails to promote meaning integration.

Practically, the study recommends the importance of:

1. Initiating instruction from students' arithmetic experiences as a conceptual foundation for constructing the meaning of variables,
2. Designing representations progressively, beginning with concrete contexts and evolving into symbolic forms, to support gradual abstraction, and
3. Applying scaffolding strategies systematically, to assist students in transitioning from arithmetic-based thinking to symbolic understanding in algebra.

A meaningful understanding of variables can only be achieved when instruction is deliberately designed to build upon students' existing knowledge structures and anticipates the potential cognitive obstacles that may emerge during the shift toward abstract symbolic reasoning. Therefore, teachers need to develop approaches that explicitly and gradually connect arithmetic experiences with the conceptual meaning of algebraic symbols.

Future research may focus on developing and evaluating didactical interventions that explicitly bridge students' prior experiences (MBL) with new concepts (MDL), including the use of representational strategies and scaffolding techniques that support conceptual transition. Experimental studies in various algebra learning contexts will further enrich our understanding of how to address and overcome cognitive disconnect.

.

REFERENCES

[1] Akkan, Y., & Baki, A. (2016). Investigation of the Secondary School Students' Transition Processes from Arithmetic to Algebra: Usage of Symbols and Meaning of the Letters. *Journal of Bayburt Education Faculty*, *11*(2), 270–305.

[2] Antonijević, R. (2016). Cognitive activities in solving mathematical tasks: The role of a cognitive obstacle. *Eurasia Journal of Mathematics, Science and …*. https://www.ejmste.com/article/cognitive-activities-in-solving-mathematical-tasks-the-role-of-a-cognitive-obstacle-4614

[3] Apsari, R. A., Putri, R. I. I., Abels, M., & Prayitno, S. (2020). Geometry Representation to Develop Algebraic Thinking: A Recommendation for a Pattern Investigation in Pre-Algebra Class. *Journal on Mathematics …*. https://eric.ed.gov/?id=EJ1237692

[4] Apsari, R. A., Sariyasa, S., Putri, R. I. P., Gunawan, G., & Prayitno, S. (2020). Understanding Students' Transition from Arithmetic to Algebraic Thinking in the Pre-Algebraic Lesson. *Journal of Physics: Conference Series*, *1471*(1). https://doi.org/10.1088/1742-6596/1471/1/012056

[5] Ardiansari, L., Suryadi, D., & Dasari, D. (2023). Desain Didaktis Pembelajaran Matematika untuk Mengatasi Learning Obstacles Siswa SMP dalam Mempelajari Materi Aljabar. *JNPM (Jurnal Nasional Pendidikan Matematika)*, *7*(1), 119–128.

[6] Artigue, M., Haspekian, M., & Corblin-Lenfant, A. (2014). Introduction to the Theory of Didactical Situations (TDS). In *Networking of Theories as a Research Practice in Mathematics Education* (pp. 249–265). Springer International Publishing. https://doi.org/10.1007/978-3-319-05389-9

[7] Beyene, A. K. (2023). *Obstacles to students ' learning of the Limit Concept. A comparative study*. Stockholm University.

[8] Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., & ... (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. *Journal for Research …*. https://pubs.nctm.org/view/journals/jrme/45/1/article-p19.xml

[9] Blanton, Brizuela, B., Stephens, A., Knuth, E., Isler, I., Gardiner, A., Stroud, R., Fonger, N., & Stylianou, D. (2018). Implementing a Framework for Early Algebra. In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5- to 12-year-olds: The global evolution of an emerging field of research and practice.* (pp. 27–49). Springer International Publishing. https://doi.org/10.1080/14794802.2020.1725613

[10] Blanton, M., Brizuela, B. M., Gardiner, A. M., Sawrey, K., & Newman-Owens, A. (2017).

A progression in first-grade children's thinking about variable and variable notation in functional relationships. *Educational Studies in Mathematics*, *95*(2), 181–202. https://doi.org/10.1007/s10649-016-9745-0
[11] Booth, J., & Mc Ginn, K. (2016). Misconceptions and Learning Algebra. In *And the Rest is Just Algebra* (pp. 1–238). Springer International Publishing. https://doi.org/10.1007/978-3-319-45053-7
[12] Breiteig, T., & Grevholm, B. (2006). The Transition From Arithmetic To Algebra: To Reason, Explain, Argue, Generalize and Justify. In I. N. J & H. Moraová (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 225–232). Prague : PME.
[13] Brousseau, G. (2002). Theory of Didactical Situations in Mathematics. In N. Balacheff & M. Cooper (Eds.), *Theory of Didactical Situations in Mathematics* (Ba). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47211-2
[14] Brousseau, G. (2006). Epistemological Obstacles, Problems, and Didactical Engineering. In A. . Bishop (Ed.), *Theory of Didactical Situations in Mathematics* (19th ed., Issue 1 983). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47211-2_6
[15] Bush, S. B. (2013). Analyzing common algebra-related misconceptions and errors of middle school students. In *Dissertation Abstracts International Section A: Humanities and Social Sciences* (Vol. 73, Issues 7-A(E)).
[16] Cohen, L., Manion, L., & Morrison, K. (2018). *Research Methods in Education* (Eight). Routledge Taylor and Franchis.
[17] Cornu, B. (1991). Limits. In David Tall (Ed.), *In: Tall, D. (eds) Advanced Mathematical Thinking. Mathematics Education* (pp. 153–166). Springer Netherlands. https://doi.org/10.1007/0-306-47203-1_10
[18] De Lima, R. N., & Tall, D. (2008). Procedural Embodiment and Magic in Linear Equations. *Educational Studies in Mathematics*, *67*(1), 3–18. https://doi.org/10.1007/s10649-007-9086-0
[19] Demonty, I., Vlassis, J., & Fagnant, A. (2018). Algebraic thinking, pattern activities and knowledge for teaching at the transition between primary and secondary school. *Educational Studies in Mathematics*, *99*(1), 1–19. https://doi.org/10.1007/s10649-018-9820-9
[20] Edogawatte, G. (2011). *Secondary school students misconceptions in algebra* (Issue 1).
[21] Ely, R., & Adams, A. E. (2012). Unknown, placeholder, or variable: What is x? *Mathematics Education Research Journal*, *24*(1), 19–38. https://doi.org/10.1007/s13394-011-0029-9
[22] Eraslan, A. (2005). *A qualitative study: Algebra honor students' cognitive obstacles as they explore concepts of quadratic functions* [search.proquest.com]. https://search.proquest.com/openview/09f36b40ebc70cccc5f2e8708ea1481a/1?pq-origsite=gscholar&cbl=18750&diss=y
[23] Fitria, A., Subanji, Susiswo, & Susanto, H. (2023). Cognitive Map: Diagnosing and Exploring Students' Misconceptions in Algebra. *Mathematics Teaching-Research Journal*, *15*(5), 49–75.
[24] Fitriani, N., Sugiman, S., & Arfah, A. (2023). Didactical design of algebraic expression

from a linear pattern with a realistic mathematics education approach. *Jurnal Riset Pendidikan Matematika*, *10*(1), 1–16. https://doi.org/10.21831/jrpm.v10i1.55212

[25] Giorgi, A. (2009). The Descriptive Phenomenological Method in Psychology: A Modified Husserlian Approach. Pittsburgh: Duquesne University Press.

[26] Girit, D., & Akyuz, D. (2017). Investigating Teaching Practices for Algebraic Expressions Within a Multiple Case Study. *The Eurasia Proceedings of Educational & Social Sciences (EPESS)*, *6*, 31–37.

[27] Glogger-Frey, I., Deutscher, M., & Renkl, A. (2018). Student teachers' prior knowledge as prerequisite to learn how to assess pupils' learning strategies. *Teaching and Teacher Education*, *76*, 227–241. https://doi.org/10.1016/j.tate.2018.01.012

[28] Greening, N. (2019). Phenomenological Research Methodology. *Scientific Research Journal*, *VII*(V). https://doi.org/10.31364/scirj/v7.i5.2019.p0519656

[29] Gueudet, G., Bosch, M., DiSessa, A. A., Kwon, O. N., & Verschaffel, L. (2016). Transitions in Mathematics Education: The Panel Debate. In G. Kaiser (Ed.), *AProceedings of the 13th International Congress on Mathematical Education ICME-13* (Vol. 1, p. 101). Springer Nature.

[30] Herscovics, N. (1978). Cognitive obstacles encountered in the learning of algebra. *Research Issues in the Learning and Teaching of Algebra. In S. Wagner & C. Kieran (Eds.)*, *4*, 60–86.

[31] Humberstone, J., & Reeve, R. A. (2017). The conceptual overlap between arithmetic and algebraic referential mapping. *Learning and Instruction*, *54*, 138–146. https://doi.org/10.1016/j.learninstruc.2017.09.001

[32] Joanna, S., & Jacquelynn, malloy. (2019). Addressing Misconceptions in Algebra 1. *Mathematics Teacher*, *112*(6), 450–454. https://doi.org/10.5951/mathteacher.112.6.0450

[33] Jupri, A., & Drijvers, P. (2016). Student difficulties in mathematizing word problems in Algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(9), 2481–2502. https://doi.org/10.12973/eurasia.2016.1299a

[34] Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683–710. https://doi.org/10.1007/s13394-013-0097-0

[35] Kartinah, Nusantara, T., Sudirman, & Daniel Chandra, T. (2021). Identifying Cognitive Obstacle on the Topic of Definite Integral among Prospective Teacher. *İlköğretim Online*, *20*(3), 186–192. https://doi.org/10.17051/ilkonline.2021.03.18

[36] Kieran, C. (1992). The learning and teaching of school algebra. In *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (In D. A. G). Macmillan Publishing Co, Inc.

[37] Kieran, C. (2011). Cognitive Processes Involved in Learning School Algebra. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and Cognition : A Research Synthesis by the International Group for the Psychology of Mathematics Education* (pp. 96–112). Cambridge University Press. https://doi.org/10.1017/cbo9781139013499.007

[38] Kirschner, P. A. (2002). Cognitive load theory: Implications of cognitive load theory on the design of learning. *Learning and Instruction*, *12*(1), 1–10.

https://doi.org/10.1016/S0959-4752(01)00014-7

[39] Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. C. (2005). Middle School Students' Understanding of Core Algebraic Concepts: Equivalence & Variable. *ZDM - International Journal on Mathematics Education*, *37*(1), 68–76. https://doi.org/10.1007/bf02655899

[40] Lenz, D. (2022). The role of variables in relational thinking: an interview study with kindergarten and primary school children. *ZDM - Mathematics Education*, *54*(6), 1181–1197. https://doi.org/10.1007/s11858-022-01419-6

[41] Levin, M., & Walkoe, J. (2022). Seeds of algebraic thinking: a Knowledge in Pieces perspective on the development of algebraic thinking. *ZDM - Mathematics Education*, *54*(6), 1303–1314. https://doi.org/10.1007/s11858-022-01374-2

[42] Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics*. https://doi.org/10.1023/A:1003606308064

[43] Maharani, I. P., & Subanji, S. (2018). Scaffolding Based on Cognitive Conflict in Correcting the Students' Algebra Errors. *International Electronic Journal of Mathematics Education*, *13*(2), 67–74. https://doi.org/10.12973/iejme/2697

[44] Maknun, C. L. L. Il, Rosjanuardi, R., & Jupri, A. (2022). Epistemological Obstacle in Learning Trigonometry. *Mathematics Teaching Research Journal*, *14*(2), 5–25.

[45] Malisani, E., & Spagnolo, F. (2009). From arithmetical thought to algebraic thought: The role of the "variable." *Educational Studies in Mathematics*, *71*(1), 19–41. https://doi.org/10.1007/s10649-008-9157-x

[46] Mallet, D. G. (2013). An example of cognitive obstacles in advanced integration: the case of scalar line integrals. *International Journal of Mathematical Education in Science and Technology*, *44*(1), 152–157. https://doi.org/10.1080/0020739X.2012.678897

[47] McGowen, M A, & Tall, D. O. (2010). Metaphor or Met-Before? The effects of previouos experience on practice and theory of learning mathematics. *The Journal of Mathematical Behavior*. https://www.sciencedirect.com/science/article/pii/S0732312310000404

[48] McGowen, Mercedes A., & Tall, D. O. (2010). Metaphor or Met-Before? The effects of previouos experience on practice and theory of learning mathematics. *Journal of Mathematical Behavior*, *29*(3), 169–179.https://doi.org/10.1016/j.jmathb.2010.08.002

[49] Nataraj, M. S., & Thomas, M. (2017). Teaching and Learning Middle School Algebra: Valuable Lessons from the History of Mathematics. In *And the Rest is Just Algebra* (pp. 1–238). Springer International Publishing. https://doi.org/10.1007/978-3-319-45053-7

[50] Nathan, M. J., & Koellner, K. (2007). A Framework for Understanding and Cultivating the Transition from Arithmetic to Algebraic Reasoning. *Mathematical Thinking and Learning*, *9*(3), 179–192. https://doi.org/10.1080/10986060701360852

[51] Norton, A., Arnold, R., Kokushkin, V., & Tiraphatna, M. (2022). Addressing the Cognitive Gap in Mathematical Induction. *International Journal of Research in Undergraduate Mathematics Education*, *0123456789*. https://doi.org/10.1007/s40753-022-00163-2

[52] Noto, M. S., Pramuditya, S. A., & Handayani, V. D. (2020). Exploration of Learning Obstacle Based on Mathematical Understanding of Algebra in Junior High School. *Eduma :*

Mathematics Education Learning and Teaching, *9*(1), 14. https://doi.org/10.24235/eduma.v9i1.5946

[53] Öztürk, M. (2021). An embedded mixed method study on teaching algebraic expressions using metacognition-based training. *Thinking Skills and Creativity*, *39*. https://doi.org/10.1016/j.tsc.2021.100787

[54] Öztürk, Mesut. (2021). An embedded mixed method study on teaching algebraic expressions using metacognition-based training. *Thinking Skills and Creativity*, *39*(September 2020). https://doi.org/10.1016/j.tsc.2021.100787

[55] Polkinghorne, D. E. (1989). Phenomenological research methods. In *In Existential-phenomenological perspectives in psychology: Exploring the breadth of human experience* (pp. 41–60). Springer US. https://doi.org/10.4135/9781412995658

[56] Pratiwi, E., Nusantara, T., Susiswo, & Muksar, M. (2019). Students' thinking process when experiencing cognitive conflict. *International Journal of Innovation, Creativity and Change*, *9*(2), 6–16.

[57] Pratiwi, W. D., Kurniadi, E., & Astuti, P. (2019). Learning design for transition from arithmetic thinking to algebraic thinking. *Journal of Physics: Conference Series*, *1166*(1). https://doi.org/10.1088/1742-6596/1166/1/012031

[58] Prediger, S. (2008). The Relevance of Didactic Categories for Analysing Obstacles in Conceptual Change: Revisiting the case of multiplication of fractions. *Learning and Instruction*, *18*(1), 3–17. https://doi.org/10.1016/j.learninstruc.2006.08.001

[59] Prendergast, M., & Treacy, P. (2018). Curriculum reform in Irish secondary schools–a focus on algebra. *Journal of Curriculum Studies*, *50*(1), 126–143. https://doi.org/10.1080/00220272.2017.1313315

[60] Qutoshi, S. B. (2018). Phenomenology: A Philosophy and Method of Inquiry. *Journal of Education and Educational Development*, *5*(1), 215. https://doi.org/10.22555/joeed.v5i1.2154

[61] Radford, L. (2011). Grade 2 students' non-symbolic algebraic thinking. *Early Algebraization: A Global Dialogue from Multiple …*. https://doi.org/10.1007/978-3-642-17735-4_17

[62] Radford, Luis. (2006). Algebraic Thinking and the Generalization of Patterns: A Semiotic Perspective. *28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *1–21*(March 1987), 2–21.

[63] Radford, Luis. (2014). The Progressive Development of Early Embodied Algebraic Thinking. *Mathematics Education Research Journal*, *26*(2), 257–277. https://doi.org/10.1007/s13394-013-0087-2

[64] Rakes, C. R., & Ronau, R. N. (2019). Rethinking mathematics misconceptions: Using knowledge structures to explain systematic errors within and across content domains. *International Journal of Research in Education and Science*, *5*(1), 1–21.

[65] Riastuti, A., Juandi, D., & Suryadi, D. (2023). Kecenderungan Hasil Tentang Penelitian Learning Obstacle Pada Materi Aljabar Dalam Sepuluh Tahun Terakhir. *Jurnal MATH-UMB.EDU*, *10*(3), 2023.

[66] Schoenfeld, A. H., & Arcavi, A. (1988). On the Meaning of Variable. *The Mathematics Teacher*, *81*(6), 420–427. https://doi.org/10.5951/MT.81.6.0420

[67] Stacey, K., & MacGregor, M. (1999). Learning the algebraic method of solving problems. *The Journal of Mathematical Behavior*. https://www.sciencedirect.com/science/article/pii/S0732312399000267

[68] Stolz, S. A. (2023). The practice of phenomenology in educational research. *Educational Philosophy and Theory*, *55*(7), 822–834. https://doi.org/10.1080/00131857.2022.2138745

[69] Subanji, S. (2015). *Teori Kesalahan Konstruksi Konsep dan Pemecahan Masalah Matematika* (T. Nusantara (ed.)). UM PRESS.

[70] Sudianto, S., & Kisno, K. (2021). Potret kesiapan guru sekolah dasar dan manajemen sekolah dalam menghadapi asesmen nasional. *Jurnal Akuntabilitas Manajemen Pendidikan*, *9*(1), 85–97. https://doi.org/10.21831/jamp.v9i1.39260

[71] Sugiman, S. (2023). Didactical design of algebraic expression from a linear pattern with a realistic mathematics education approach. *Jurnal Riset Pendidikan Matematika*. https://journal.uny.ac.id/index.php/jrpm/article/view/55212

[72] Taban, J. G., & Cadorna, E. A. (2018). Structure Sense in Algebraic Expressions and Equations of Groups of Students . *Journal of Educational and Human Resource Development*, *6*(January), 140–154.

[73] Takir, A. (2011). *The Effects of an Instruction Designed by Cognitive Load Theory Principles on 7th Grade Students' Achievement in Algebra Topics and Cognitive Load* (Issue June). Middle East technical University.

[74] Tall, D. O. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, *20*(2), 5–24. https://doi.org/10.1007/BF03217474

[75] Tall, D. (2004). Introducing three worlds of mathematics. In *For the learning of mathematics*. Citeseer. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=1a91ed4cb77347755cc37344d6eb3ecf94b69f41

[76] Tall, David. (2004). Thinking through three worlds of mathematics. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education.*, *4*, 281–288.

[77] Tall, David, & Vinner, S. (1981). Concept image and concep definition in mathematics with particular reference tokimit and continuity. *Educational Studies in Mathematics*, *3*(September), 151–169. https://doi.org/10.1007/BF00305619

[78] Tekin-Sitrava, R. (2017). Middle Grade Students' Concept Images of Algebraic Concepts. *Journal of Education and Learning*, *6*(3), 299. https://doi.org/10.5539/jel.v6n3p299

[79] Trindade, D. J., Nagashima, L. A., & Andrade, C. C. (2019). Epistemological obstacles from the perspective of Bachelard. *Brazilian Journal of Development*, *5*(10), 17829–17843. https://doi.org/10.34117/bjdv5n10-050

[80] Ventura, A. C., Brizuela, B. M., Blanton, M., Sawrey, K., Gardiner, A. M., & Newman-Owens, A. (2021). A learning trajectory in Kindergarten and first grade students' thinking of variable and use of variable notation to represent indeterminate quantities. *Journal of Mathematical Behavior*, *62*(December 2019), 1–17. https://doi.org/10.1016/j.jmathb.2021.100866

[81] Vinner, S., & Dreyfus, T. (2010). Images and Definitions for the Concept of Function. *Journal for Research in Mathematics Education*, *20*(4), 356–366.

[82] Wang, X. (2015). The literature review of algebra learning: Focusing on the contributions to students' difficulties. In *Creative education*. scirp.org. https://www.scirp.org/journal/paperinformation?paperid=53882

[83] Wang, Xiong. (2015). The Literature Review of Algebra Learning: Focusing on the Contributions to Students' Difficulties. *Creative Education*, *06*(02), 144–153. https://doi.org/10.4236/ce.2015.62013

[84] Weinberg, A., Dresen, J., & Slater, T. (2016). Students' understanding of algebraic notation: A semiotic systems perspective. *Journal of Mathematical Behavior*, *43*, 70–88. https://doi.org/10.1016/j.jmathb.2016.06.001

[85] Wettergren, S. (2022). Identifying and promoting young students' early algebraic thinking. *LUMAT: International Journal on Math, Science and Technology Education*, *10*(2), 190–214. https://doi.org/10.31129/LUMAT.10.2.1617

[86] Wijaya, A. P., Nusantara, T., Sudirman, & Hidayanto, E. (2022). Students' Analytical Questions and Interaction Patterns in Group Discussion Facilitated with a Scientific Approach Learning. *Mathematics Teaching-Research Journal*, *14*(2), 61–71.

[87] Williams, H. (2021). The meaning of "Phenomenology": Qualitative and philosophical phenomenological research methods. *Qualitative Report*, *26*(2), 366–385. https://doi.org/10.46743/2160-3715/2021.4587

[88] Yew, W. T., AL-Rababaha, Y., & Meng, C. C. (2020). Misconceptions in School Algebra. *International Journal of Academic Research in Business and Social Sciences*, *10*(5), 803–812. https://doi.org/10.6007/ijarbss/v10-i5/7250

[89] Yoshida, H., & Sawano, K. (2002). Overcoming Cognitive Obstacle in Learning Fractions : Equal-partitioning and Equal-whole. *Japanese Psychological Research*, *44*(4), 183–195.

[90] Zahavi, D. (2019). Applied phenomenology: why it is safe to ignore the epoché. *Continental Philosophy Review*, *54*(2), 259–273. https://doi.org/10.1007/s11007-019-09463-y