Learning Obstacles in Fraction Concept: A Hermeneutic Phenomenology Study

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Abstract: The concept of fractions is a fundamental mathematics, particularly for junior high school students in Indonesia. Despite its importance, learning obstacles are frequently encountered in the educational process. This study investigates the learning obstacles experienced by students in the concept of fractions. This research employs a hermeneutic phenomenological approach to identify the types of learning obstacles—ontogenic, didactic, or epistemological—based on the analysis of students' test answers, which are further explored through interviews. The study involved 30 junior high school students, with eight students selected for in-depth interviews. Data analysis includes stages of explanation, naïve understanding, in-depth understanding, and appropriation. The results indicate learning obstacles primarily in the ontogenic and epistemological types. Ontogenic obstacles are categorized into three types: psychological, instrumental, and conceptual. These are attributed to students' lack of interest in mathematics, failure to apply fundamental fraction concepts, and gaps in prerequisite knowledge of fractions. Epistemological obstacles are characterized by students' limited understanding of fractions exclusively in the $\frac{a}{b}$ form without grasping alternative interpretations of fractions. Based on these results, this study also recommends instructional solutions that can be used as alternatives in anticipating the occurrence of learning obstacles in students.

Keywords: fraction concept, hermeneutic phenomenology, learning obstacle

INTRODUCTION

The concept of fractions is a crucial element in mathematics that students must master. Lamon (2020) identifies five interpretations of the fraction concept, which are further elaborated by Isnawan et al. (2022a): (1) part of a whole, tending to the proper fraction; (2) quotients; (3) ratios, namely the comparison of two numbers; (4) measure, that is, the measure of an object; and (5) operator refers to the process of scaling a quantity. In line with Lamon, Van de Walle et al. (2010) posits that the introduction to the fraction concept also includes: (1) fraction parts indicating the quantity needed to compose a whole; (2) the more fraction parts required, the smaller the fraction part; (3) the denominator indicating how many parts the whole is divided into; and (4) two equivalent fractions representing the same quantity using parts of different sizes.

Fractions are crucial for understanding mathematical concepts, especially at the primary and secondary education levels (Hariyani et al., 2022; Getenet & Callingham, 2021; Kullberg & Runesson, 2013; Roni et al., 2017; Unaenah et al., 2023, 2024). The concept of fractions has numerous benefits, not only related to mathematics itself but also impacting other disciplines (Ballard & Johnson, 2004; Coetzee & Mammen, 2017; Gagani & Diano JR, 2019; Lortie-Forgues et al., 2015; Tian & Siegler, 2017). It is important to note that the concept of fractions is also employed in the solving of everyday problems as an alternative representation approach when integer solutions are not adequate (Purnomo et al., 2021). Therefore, the concept of fractions plays an indispensable role in both mathematics and real-world applications.

In Indonesia, based on the Minister of Education and Culture's Decree No. 37 of 2018, the concept of fractions is studied at the primary and secondary education levels. In elementary school, the concept of fractions is introduced from grade 2 to grade 6 (Unaenah et al., 2024). At this level, students are introduced to fraction symbols, albeit not in depth, while in junior high school, fractions are taught alongside the concept of integers. According to the curriculum documents, the competencies for fractions taught in junior high school are as follows: (1) explaining and determining the order of fractions (proper, mixed, decimal, and percent); (2) solving problems related to the order of fractions; (3) explaining and performing arithmetic operations with fractions using various properties of operations; and (4) solving problems related to arithmetic operations with fractions. In addition to primary and secondary education, fractions are also studied in some university courses. This indicates that fractions are one of the fundamental concepts in mathematics.

Given the significance of fractions, it is important for students to master this concept. However, Unaenah et al. (2024) state that there are still issues in the learning process related to fractions. Several studies show that students still find it difficult to understand the concept of fractions (Isnawan et al., 2022a, 2022b; Unaenah et al., 2023, 2024). These findings indicate that students' mastery of fractions remains limited, reflecting the existence of learning obstacles as one of the urgent and unavoidable problems in the mathematics learning process (Fauzi & Suryadi, 2020; Fritz et al., 2019). Douroux states that learning obstacles are pieces of knowledge not based on students' ignorance (Brousseau, 2002). Therefore, learning obstacles do not occur because students are unaware of the mathematical concept but due to certain factors that hinder students from fully mastering the concept.

Brousseau (2002) identifies three types of learning obstacles: ontogenic, epistemological, and didactical. Hendriyanto et al. (2024) states that ontogenic obstacles arise from a mismatch between the learning process and students' mental readiness. Consequently, learning can become problematic if the demands exceed or fall short of students' mental readiness. Nurhayati et al. (2023) classifies ontogenic obstacles into three types: (1) psychological, related to a lack of interest or motivation.; (2) instrumental, which include technical aspects that are important in the learning and problem-solving process; and (3) conceptual, which arise from demands on students to think at a level that is higher or lower than their abilities.

Didactical obstacles are linked to the didactic system, emphasizing obstacles based on the sequence of material presentation that hinders students' continuity of thought (Fauzi & Suryadi, 2020). Therefore, the order of material presentation should consider both structural sequences, showing the interrelation of concepts, and functional sequence, ensuring the continuity of students' thinking processes. Regarding epistemological obstacles, Brousseau (2002) states that these obstacles occur when knowledge that was previously meaningful and effective in a certain context continues to be used, but at a certain stage produces incorrect or inadequate answers, thereby creating contradictions. This indicates that epistemological obstacles occur due to the limitations of the applicability of knowledge or only limited to specific contexts. These epistemological obstacles arise when students encounter new and different problem situations compared to those usually presented during instruction.

Research on learning obstacles in the concept of fractions has been conducted by several researchers in Indonesia (Fauzi & Suryadi, 2020; Hariyani et al., 2022; Isnawan et al., 2022b, 2022a; Unaenah et al., 2023, 2024). However, most of this research focuses on the elementary school level, with limited studies on junior high school students. As previously mentioned, the concept of fractions is taught from elementary school through university, including junior high school. According to Unaenah et al. (2024), the concept of fractions still poses difficulties for students in learning mathematics. Therefore, it is likely that learning obstacles also exist for junior high school students regarding fractions. This study aims to identify potential learning obstacles related to the concept of fractions among junior high school students. The findings are expected to serve as a reference for developing didactic designs to minimize the potential occurrence of learning obstacles in teaching fractions.

METHODS

Research Design

This study employed a hermeneutic phenomenological approach. Hermeneutic phenomenology is a research approach based on the philosophies of phenomenology and hermeneutics. According to Langdridge (2007), phenomenology focuses on revealing the meaning or understanding of an individual's experience of phenomena that occurr around them. Hermeneutics refers to the interpretation of meaning. Through hermeneutic phenomenology, one can describe and interpret an individual's experience and the meaning and interpretation related to that experience (Lindseth & Norberg, 2004; Regan, 2012). Therefore, this study focused on examining the learning obstacles students encountered in the concept of fractions based on their experiences after receiving instruction (phenomenology) and then investigating the causes of these learning obstacles (hermeneutics) into ontogenic, didactical, and epistemological obstacles.

Participants and Data Collection

The participants in this study were 30 seventh-grade students from a junior high school in Surakarta, Indonesia. All thirty students were given a mathematics test on the topic of fractions. The selection of grade 7 is based on the consideration of the curriculum in Indonesia, in which the fraction material is taught. This finding suggests that the subjects have gained direct learning experience. Based on the test responses, eight students (labeled as S1 to S8) were selected for further investigation through in-depth interviews. These eight students were chosen based on their test answers, considering their communication abilities as recommended by their mathematics teacher. The eight selected students represented the errors that occurred in each of the given problems. Thus, purposive sampling was used in this study.

Data collection was conducted through both test and non-test methods. The test method aimed to gather initial data on potential learning obstacles in the fractions. The instrument employed is a mathematics question concerning fractions that has been validated by experts in mathematics education from the content aspect related to suitability for measurement purposes. Additionally, one of the questions used, specifically number 1 in Table 1, was taken from the test instrument used by Isnawan et al. (2022a). Table 1 shows the test instrument blueprint used in this study. The blueprint in Table 1 refers to the Basic Competencies in the Number element, namely: (1) explaining and determining the order of integers and fractions; (2) explaining and performing arithmetic operations on integers and fractions using various operational properties; and (3) solving problems related to arithmetic operations on integers and fractions. The test was completed individually by all participants. The test was administered for a duration of 45 minutes, during which the subjects were presented with written essay as answers.

In addition to using the test instrument, data collection in this study was also conducted through in-depth interviews with students who had answered the test questions. The interviews were guided by an open-ended question sheet while referring to the students' test answers. Activities were conducted face-to-face with each selected participant. Responses from each participant were audio-recorded with the participant's verbal consent. The interviews were conducted in-depth to gather data and confirm students' responses to identify the causes of learning obstacles. This allowed the researchers to classify the learning obstacles into categories of ontogenic, didactical, or epistemological obstacles.

No	Indicators	Item Test
1	Representing fractions using images/models/diagrams	Using the given illustrations/models, visualize/draw/shade the values of $\frac{1}{6}$ and $\frac{2}{3}$ on models 1 and 2 below! Model 1

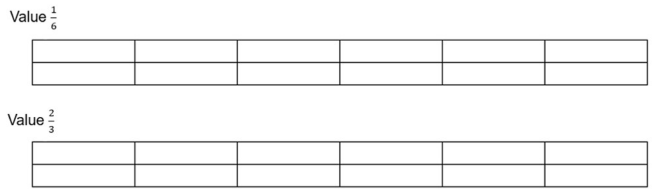
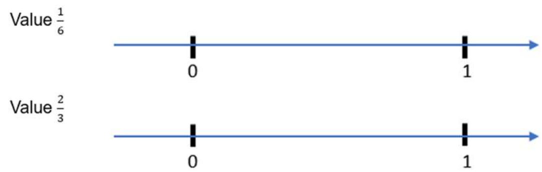
		 Model 2 
2	Calculating the value of fraction operations	Calculate the value of $1\frac{2}{3}:\frac{2}{5}$-$2\frac{3}{5}+\frac{1}{2}$.
3	Ordering the values of fractions	Order the following fractions from smallest to largest! Provide explanations if necessary. $\frac{2}{3}$; $\frac{1}{2}$; 0,6; 20%
4	Applying fraction operations to solve contextual problems	A farmer has $1\frac{2}{3}$ hectares of land to plant crops. The farmer feels that the land is insufficient, so he buys another $2\frac{1}{4}$ hectares. If the land is to be planted with maize, peanuts, and cassava, calculate the area of land for each crop.

Table 1: The Mathematics Test Instrument for Identifying Learning Obstacles

Data Analysis

The data analysis in this study referred to qualitative data analysis. Specifically, the analysis was conducted using a synthesis of the phenomenological data analysis steps according to Creswell (2007) and the hermeneutic data analysis steps based on Ricouer's Interpretation Theory (Ghasemi et al., 2011; Tan et al., 2009). The steps for hermeneutic phenomenological analysis are as follows.

1. *Explanation*: In this stage, the researchers compiled the research data, including students' answers, and recorded interviews with the students. The audio recordings of the interviews were transcribed, and the test answer sheets were reviewed and analyzed.
2. *Naïve Understanding*: The researchers developed research notes to obtain significant and specific statements based on the analysis of the interview transcripts and students' answer sheets. The developed statements were then grouped into information units (meaning units) related to learning obstacles. Textural and structural descriptions were then created. The textural description involved detailing what the students experienced regarding the meaning of the concept of

fractions, while the structural description referred to how the students acquired the concept of fractions.

3. *In-depth Understanding*: Further analysis was conducted on the relationship between the textural and structural descriptions for each respondent to derive the essence and meaning of the concept of fractions, including the learning obstacles faced by the students. Additionally, a composite description was created by combining the textural and structural descriptions based on the previous analyses.
4. *Appropriation*: In the appropriation stage, the researchers analyzed the relationship between the composite description and the significant statements from the students, including relevant theories. This process aimed to identify the learning obstacles encountered by the students.

Validity of Data

The validity of the data in this study was ensured through tests of credibility, transferability, dependability, and confirmability. Credibility was achieved through technique triangulation, comparing students' test answers with the results of the interviews conducted. Transferability was evaluated based on the research setting, participant selection, and data processing (Puspita et al., 2023), ensuring that the research findings provided sufficient information to the readers. Dependability was ensured by examining the entire research process, starting from problem identification, instrument development, data accuracy, and data analysis. Meanwhile, confirmability was achieved by examining the objectivity and transparency of the research results and discussions, ensuring the findings could be independently verified and re-examined.

RESULTS AND DISCUSSION

Based on these test responses, in-depth interviews with eight students were conducted, identifying learning obstacles related to the concept of fractions. Errors found in the test answers showed notable similarities among students. This section presents the research results and discussion, divided into three sub-chapters: (1) Learning obstacles in representation the meaning of fractions; (2) Learning obstacles in fraction operations and their application in solving contextual problems; and (3) Learning obstacles related to ordering fractions.

Learning Obstacles in Conceptual Representation of Fractions

The representation of the meaning of fractions in this section is based on question number 1 presented in Table 1. In Table 1, students were asked to represent the values of $\frac{1}{6}$ and $\frac{2}{3}$ on two models, each illustrating a specific meaning of fractions. Errors were still found in students' representations of fractions in both Model 1 and Model 2. Below is an example of a student's response to the first problem.

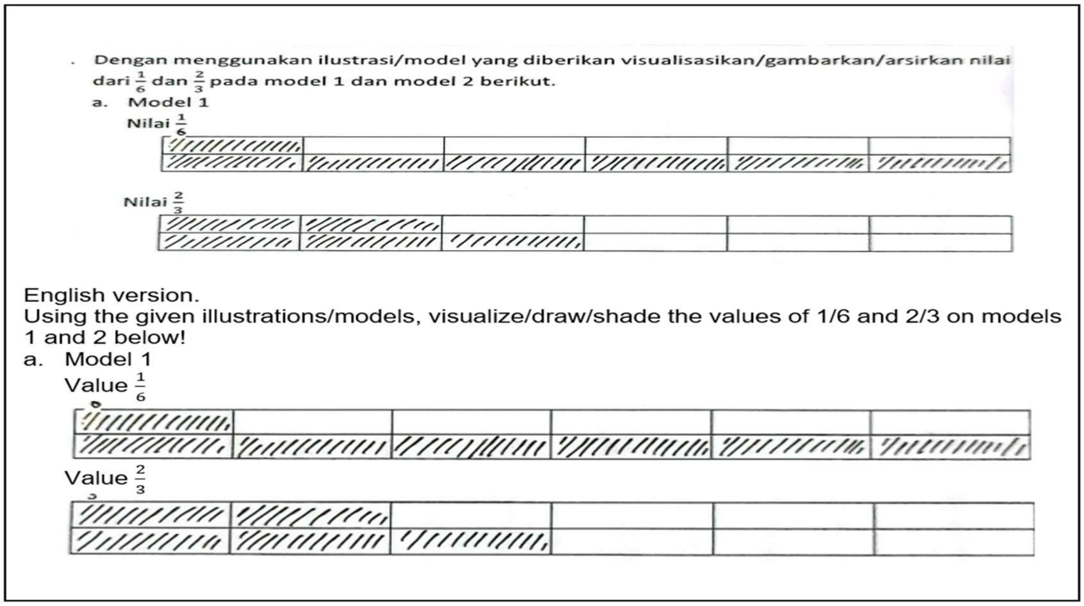


Figure 1. S1's Answer on the Representation of Fraction Meaning in Model 1

Figure 1 shows that the student represented $\frac{1}{6}$ and $\frac{2}{3}$ in Model 1 as two separate parts. The value $\frac{1}{6}$ in Figure 1 was represented with the numerator 1 on the top and the denominator 6 on the bottom. The student (S1) interpreted Model 1, consisting of 12 boxes, as representing equal parts corresponding to the numerator and denominator. A similar approach was taken by the student for the value $\frac{2}{3}$, as shown in Figure 1. This condition was explicitly stated based on an interview excerpt with S1 regarding the first problem.

Researcher	:	Okay. From your answer, in Model 1 you were asked to shade $\frac{1}{6}$ and $\frac{2}{3}$. For $\frac{1}{6}$, you shaded the part with 1 on top and 6 on the bottom. Why did you do that?
S1	:	Because the number 1 is on top, so I shaded 1, and the number 6 is on the bottom, so I shaded 6.
Researcher	:	Okay, did you understand what the boxes represent?
S1	:	Each box represents one unit.
Researcher	:	So, you shaded the numerator on top and the denominator on the bottom?
S1	:	Yes.
Researcher	:	And for $\frac{2}{3}$?
S1	:	The same.
Researcher	:	So what does it mean when you shade and don't shade parts?

S1	:	The shaded parts are the numbers being asked for, and the unshaded parts are the rest.
Researcher	:	Has the teacher ever given a problem like this before?
S1	:	No, never.
Researcher	:	Okay, so this is the first time. Were you confused when doing this?
S1	:	Yes, I was confused.
Researcher	:	Do you think you can imagine $\frac{1}{6}$ without illustrating it?
S1	:	No, I can't.

Based on the interview excerpt, S1 understood that the values of the numerator and denominator in a fraction indicate that they can be visualized as separate parts. This condition demonstrates a misunderstanding of the meaning of fractions by the student. Specifically, the student perceived Model 1 as a representation of fractions but displayed a partial understanding of the fraction's value. The student's errors in understanding the meaning of fractions are partly attributed to their lack of familiarity with such problems. This situation reflects a potential epistemological learning obstacle, which Brousseau (2002) describes as a barrier related to limited understanding of a concept based solely on specific contexts related to one's learning experiences. The student still perceived the basic concept of a fraction as a number denoted in the $\frac{a}{b}$ form. Therefore, when expressed in a different format, such as the representation of fraction meaning, the student made errors.

According to Lamon (2020), the meaning of fractions can be understood in several contexts, one of which is that a fraction represents a part of a whole. In the context of the first problem, $\frac{1}{6}$ should be visualized as one part of six equal parts. Thus, if given 12 equally sized boxes, only two of the 12 boxes should be shaded as shown in Figure 1. The epistemological obstacle encountered by the student is further reinforced by a contradiction in their answer when asked about the meaning of fractions. When asked what a fraction is, the student responded, "a fraction is one whole divided into several equal parts." In this context, the student understood what a fraction means, but when presented with a different situation, the student became confused.

This condition indicates that the student has not yet fully applied their understanding to solve problems in different contexts. Carpenter et al. (2015) asserts that students who have a clear misunderstanding of a concept will struggle to solve or apply that concept in varied situations. This condition may arise due to reliance on rote memorization rather than understanding the underlying principles (Catrambone, 1994). Nokes et al. (2007) suggest that to address this issue, teachers should provide assistance by explaining the relationships between principles and examples, as well as offering appropriate explanations and analogies. Additionally, Quintero (1983) emphasizes that the diversity of problem types provided should be considered to clearly identify students' difficulties.

In Model 2, Figure 2, students were asked to visualize the values $\frac{1}{6}$ and $\frac{2}{3}$ on a number line. Most students first converted $\frac{1}{6}$ and $\frac{2}{3}$ into decimal form and then placed these decimal values on the number line, with $\frac{2}{3}$ positioned to the right of $\frac{1}{6}$. This indicates that students understood how to place $\frac{1}{6}$ and $\frac{2}{3}$ on the number line. However, there are still students who are not correct in solving this problem. One of them is the answer from S2. The following is an example of a student's response and interview to the research problem.

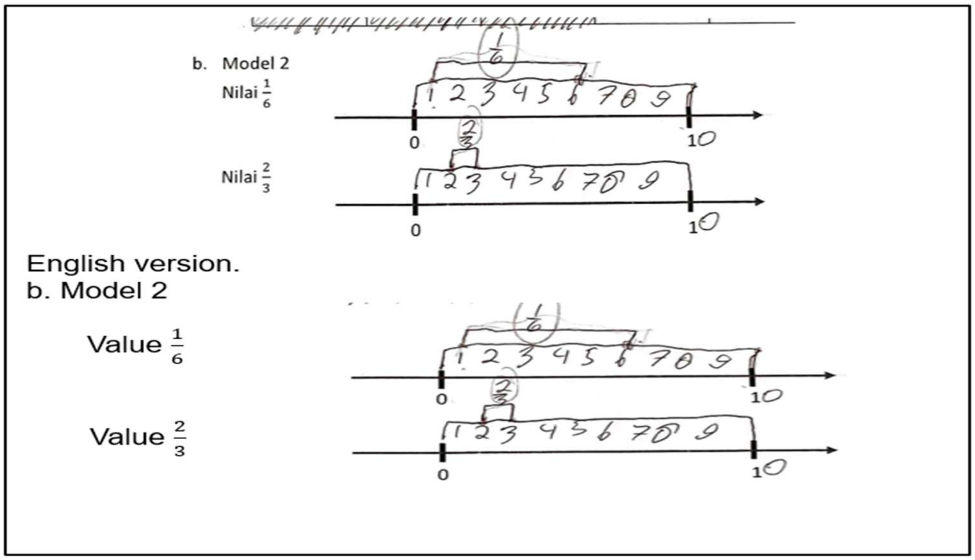


Figure 2. S2's Answer on the Representation of Fraction Meaning in Model 2

Researcher	:	Okay, let's move to Model 2. The question is the same. Can you explain where $\frac{1}{6}$ is located?
S2	:	Here it says 1-9 between 0 and 10. So, $\frac{1}{6}$ is from 1 to 6.
Researcher	:	So, is $\frac{1}{6}$ located at the far right, far left, or somewhere along 1 to 6?
S2	:	Somewhere along 1 to 6.
Researcher	:	Do you know what this line represents?
S2	:	I have seen it before but I have forgotten.
Researcher	:	What are the numbers before 0 and after 0?
S2	:	Before 0 is negative, after 0 is positive.
Researcher	:	Okay, and what about $\frac{2}{3}$?
S2	:	It's the same. Along 2 and 3.

Based on Figure 2 and the interview excerpt, S2 incorrectly answered the problem by visualizing $\frac{1}{6}$ or $\frac{2}{3}$ as distances between the numerator and denominator values. In this case, the student converted the number line interval into integers ranging from 0 to 10. This mistake aligns with findings from Isnawan et al. (2022a), where similar errors were found, with students marking integers on the number line. A notable finding from the interview is that the student struggled to understand the real number line, leading to errors in addressing the problem. This situation indicates that the student encountered an ontogenic obstacle.

Referring to the answers in Figure 2 and the interview results, the student faced a conceptual ontogenic obstacle related to previous learning experiences, specifically the number line. The number line concept in Model 2 is crucial as a previous learning experience for understanding one meaning of fractions, such as measurement, as explained by Lamon (2020). Furthermore, Isnawan et al. (2022a) illustrate that a fraction can represent a measurement, such as a length of 50 centimeters, which can also be expressed as a fraction value $\frac{1}{2}$ meters. This suggests that fraction values can be represented on a number line. The incorrect prior experience with the number line caused an obstacle when students attempted to visualize values like $\frac{1}{6}$ or $\frac{2}{3}$, despite the number line concept having been taught at the elementary school level.

Lutfi et al. (2021) states that a conceptual ontogenic obstacle refer to the mismatch between the conceptual level targeted in teaching and the way students think, which is shaped by their previous learning experiences. In this case, the students' previous learning experiences impact the obstacles they face, indicating that students have not fully grasped their knowledge related to the number line. According to Loewenberg Ball et al. (2008), this situation occurs when the knowledge acquired has not been fully integrated with the basic concepts being studied. Ideally, at the junior high school level, the concept of fractions should build upon a broader range of concepts, requiring higher-level thinking. However, the observed situation reveals a gap between the expected cognitive demands and the actual design of the instruction.

Learning Obstacles in Fraction Operations and Application in Contextual Problem Solving

The analysis of fraction operations and their application in contextual problem solving is based on students' answers to test questions 2 and 4 from Table 1. Question 2 required students to calculate the value of arithmetic operations involving fractions, including both proper and mixed fractions, covering operations such as division, subtraction, and addition. Based on students' answers, several errors were identified in question 2: (1) Errors in converting mixed fractions to proper fractions; (2) Errors in performing operations on fractions, particularly in addition and subtraction; and (3) Errors in arithmetic operations involving integers. Examples of these errors are illustrated in Figure 3 and Figure 4.

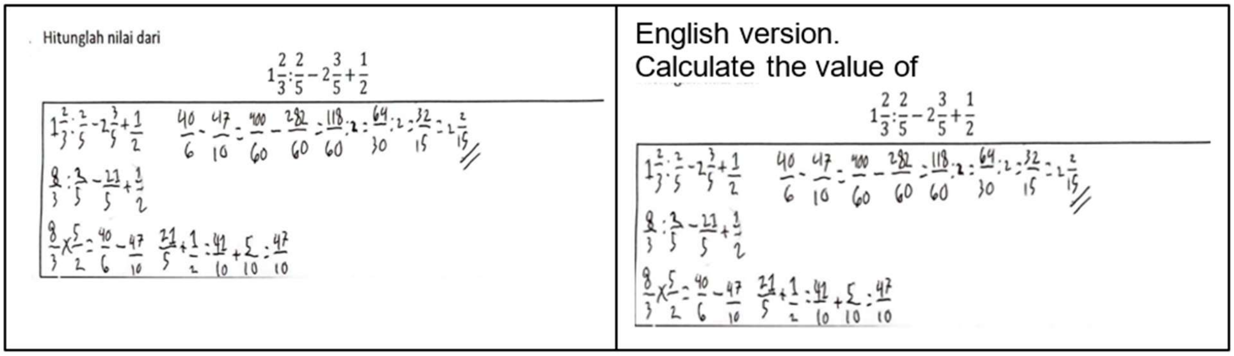


Figure 3. Errors in Converting Mixed Fractions to Proper Fractions and Performing Fraction Operations from S3

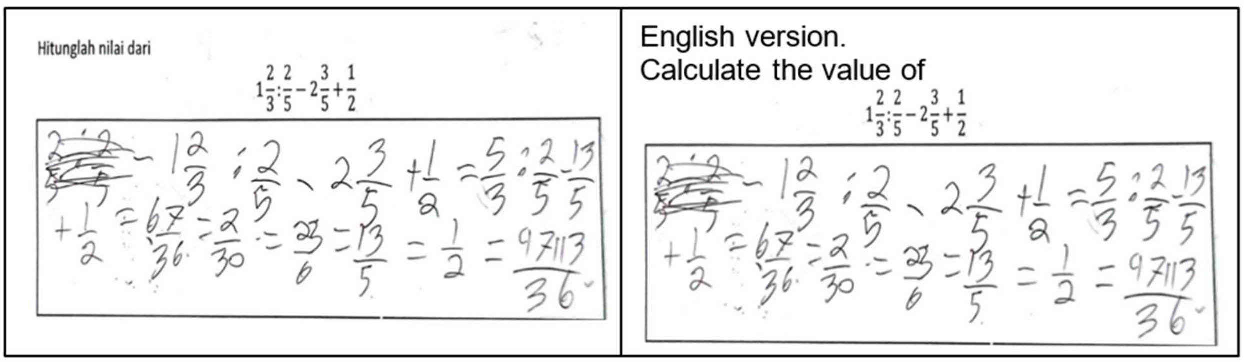


Figure 4. Errors in Fraction Operations and Integer Calculations from S4

Fraction operations differ from integer operations, particularly in the conversion of mixed fractions to proper fractions. Figure 3 shows S3 making errors in converting mixed fractions to proper fractions, such as incorrectly converting $1\frac{2}{3}$ into $\frac{8}{3}$. Similar conversion errors were noted with other mixed fractions, such as changing $2\frac{3}{5}$ into $\frac{21}{5}$, as confirmed in subsequent interviews with the researcher.

Researcher	:	Okay, let's move on to the calculations. You have just rewritten the problem. Can you explain why this conversion is done?
S3	:	So it could be easier to calculate.
Researcher	:	Okay, how is this conversion done? For instance, how does one and two third become eight-thirds.
S3	:	First, add and then multiply, add and then multiply.
Researcher	:	What is added? Can you point it out?
S3	:	Add 3 and 1.
Researcher	:	Okay, so 3 added to 1 is 4. What happens next?

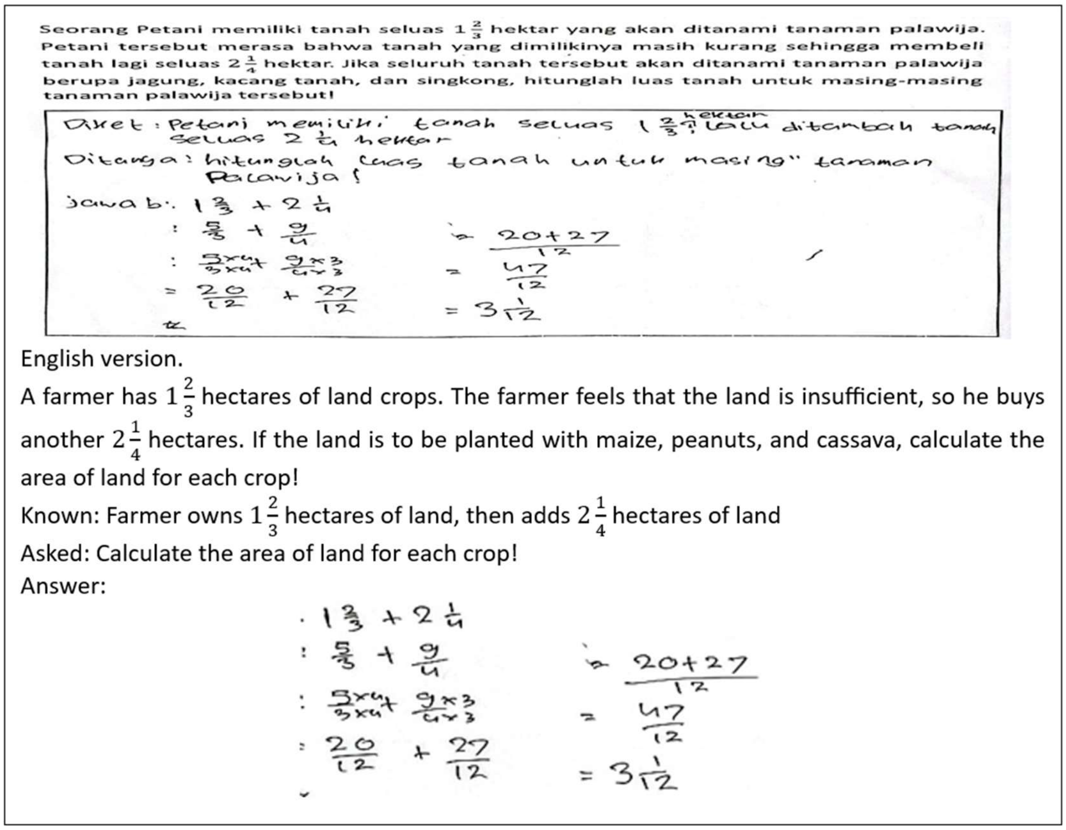
S3	:	Four times two.
Researcher	:	So, you obtained 8. If the 3 remains the same?
S3	:	Yes.

Based on Figure 3 and the interview results, S3 incorrectly converted mixed fractions $1\frac{2}{3}$ into proper fractions by first adding 1 to 3 and then multiplying by 2. The correct method should involve multiplying 1 by 3 and then adding 2, resulting in $\frac{5}{3}$. In addition to errors in converting mixed fractions to proper fractions, students also made mistakes in calculating the values for fraction operations, including integer arithmetic, as shown in Figure 3 and Figure 4. For fraction operations, specifically addition and subtraction, students need to equalize the denominators by finding the Least Common Multiple (LCM) of the denominators involved. However, S4 in Figure 4 incorrectly determined the LCM for the denominators 2, 3, and 5.

Based on the students' answers and interviews, the errors indicate the presence of ontogenic obstacles, specifically both instrumental and conceptual ontogenic obstacles. Ontogenic instrumental obstacles refer to technical errors that hinder students from understanding the given conditions. In this case, students made mistakes in fraction operations, where a crucial technical aspect is determining the LCM of the denominators in fraction addition and subtraction. Ontogenic conceptual obstacles, as defined by Nurhayati et al. (2023), refer to obstacles that arise due to a misalignment between the conceptual level of the questions designed and the students' understanding. Puspita et al. (2023) clarify that conceptual ontogenic obstacles pertain to problem conceptualization that is not aligned with the students' cognitive experiences. Hendriyanto et al. (2024) note that conceptual ontogenic obstacles may arise from a lack of foundational mathematical knowledge that students should have acquired earlier. This notion aligns with Septyawan et al. (2019), who emphasize that prior knowledge of a concept is necessary for understanding advanced topics. In this case, the conceptual obstacle is evident in the students' difficulty in converting mixed fractions to proper fractions. It is important to note that fraction operations are applicable when dealing with $\frac{a}{b}$ fractions rather than mixed fractions. Thus, mastering the conversion from mixed fractions to proper fractions is essential prior knowledge that students need before performing fraction operations.

The concept of fractional numbers is closely related to the numerical development of individuals. Siegler et al. (2011) state that one fundamental reason for students learning fractions is related to the reorganization of children's numerical knowledge, where children who have not yet learned fractions assume that the properties of integers apply to all numbers. In the concept of fractions, Lazić et al. (2017) emphasize the importance of understanding the differences and individual development in both conceptual knowledge and procedural knowledge related to fractions. Lazić et al. (2017) further explain that conceptual knowledge pertains to understanding the properties of fractions, including their magnitude, principles, and notation, whereas procedural knowledge involves understanding and proficiency in using the four arithmetic operations.

Subsequently, in question number 4 in Table 1, students were asked to solve a simple contextual problem by applying fraction operations. In this case, two broad categories of answers were identified, as shown in Figures 5 and 6.

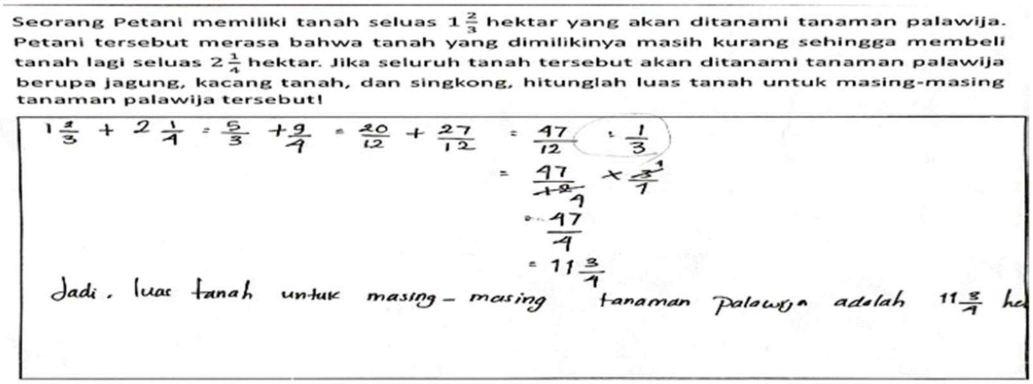

Seorang Petani memiliki tanah seluas $1\frac{2}{3}$ hektar yang akan ditanami tanaman palawija. Petani tersebut merasa bahwa tanah yang dimilikinya masih kurang sehingga membeli tanah lagi seluas $2\frac{1}{4}$ hektar. Jika seluruh tanah tersebut akan ditanami tanaman palawija berupa jagung, kacang tanah, dan singkong, hitunglah luas tanah untuk masing-masing tanaman palawija tersebut!

Diket: Petani memiliki tanah seluas $1\frac{2}{3}$ hektar lalu ditambah tanah seluas $2\frac{1}{4}$ hektar
Ditanya: hitunglah luas tanah untuk masing" tanaman palawija!
Jawab: $1\frac{2}{3} + 2\frac{1}{4}$
$= \frac{5}{3} + \frac{9}{4}$
$= \frac{5\times4}{3\times4} + \frac{9\times3}{4\times3}$
$= \frac{20}{12} + \frac{27}{12}$
$= \frac{20+27}{12}$
$= \frac{47}{12}$
$= 3\frac{1}{12}$

English version.
A farmer has $1\frac{2}{3}$ hectares of land crops. The farmer feels that the land is insufficient, so he buys another $2\frac{1}{4}$ hectares. If the land is to be planted with maize, peanuts, and cassava, calculate the area of land for each crop!
Known: Farmer owns $1\frac{2}{3}$ hectares of land, then adds $2\frac{1}{4}$ hectares of land
Asked: Calculate the area of land for each crop!
Answer:
$1\frac{2}{3} + 2\frac{1}{4}$
$= \frac{5}{3} + \frac{9}{4}$
$= \frac{5\times4}{3\times4} + \frac{9\times3}{4\times3}$
$= \frac{20}{12} + \frac{27}{12}$
$= \frac{20+27}{12}$
$= \frac{47}{12}$
$= 3\frac{1}{12}$

Figure 5. Example of Incorrect Answers from S5

In Figure 5, S5 answer only includes the addition of the areas of the two plots of land based on the operation of two fractions, but does not proceed to the division operation to show the area of each part of the land for intercropping. The interview with S5 revealed that the student considered the final answer to question number 4 to be the sum of the first and second land areas, which is $\frac{47}{12}$. This was confirmed by the interview when the S5 was asked, "Is this answer sufficient to address the question?" and responded, "Yes, it is sufficient." In Figure 6, S6 correctly calculated the total area of the two plots of land but mistakenly divided the area of each of the three intercropping sections by $\frac{1}{3}$ instead of dividing by 3.



Seorang Petani memiliki tanah seluas $1\frac{2}{3}$ hektar yang akan ditanami tanaman palawija. Petani tersebut merasa bahwa tanah yang dimilikinya masih kurang sehingga membeli tanah lagi seluas $2\frac{1}{4}$ hektar. Jika seluruh tanah tersebut akan ditanami tanaman palawija berupa jagung, kacang tanah, dan singkong, hitunglah luas tanah untuk masing-masing tanaman palawija tersebut!

$$1\frac{2}{3} + 2\frac{1}{4} = \frac{5}{3} + \frac{9}{4} = \frac{20}{12} + \frac{27}{12} = \frac{47}{12} : \frac{1}{3} = \frac{47}{12} \times \frac{3}{1} = \frac{47}{4} = 11\frac{3}{4}$$

Jadi, luas tanah untuk masing-masing tanaman palawija adalah $11\frac{3}{4}$ ha

English version.

A farmer has $1\frac{2}{3}$ hectares of land crops. The farmer feels that the land is insufficient, so he buys another $2\frac{1}{4}$ hectares. If the land is to be planted with maize, peanuts, and cassava, calculate the area of land for each crop!

$$1\frac{2}{3} + 2\frac{1}{4} = \frac{5}{3} + \frac{9}{4} = \frac{20}{12} + \frac{27}{12} = \frac{47}{12} : \frac{1}{3}$$
$$= \frac{47}{12} \times \frac{3}{1}$$
$$= \frac{47}{4}$$
$$= 11\frac{3}{4}$$

So, the land area for each crop is $11\frac{3}{4}$ ha.

Figure 6. Example of Incorrect Answers from S6 in Divided the Area

The errors observed in both Figures 5 and 6 are not attributed to the nature of the problem, as the interviews indicated that similar problems to question number 4 had been presented to the students during fraction concept lessons. Didactically, the teacher had provided an understanding of how to solve contextual problems related to fractions, specifically fraction operations. However, the errors were due to the students' lack of attention in understanding the problem, including a tendency to forget. An interesting finding was that students appeared to perform tasks by imitation without understanding the reasoning behind the tasks. Thus, relating to the types of learning obstacles, this indicates an ontogenic obstacle in the psychological form. Hendriyanto et al. (2024) state that psychological ontogenic refers to the students' lack of readiness in aspects related to motivation or interest, which impacts errors in task completion or the learning process.

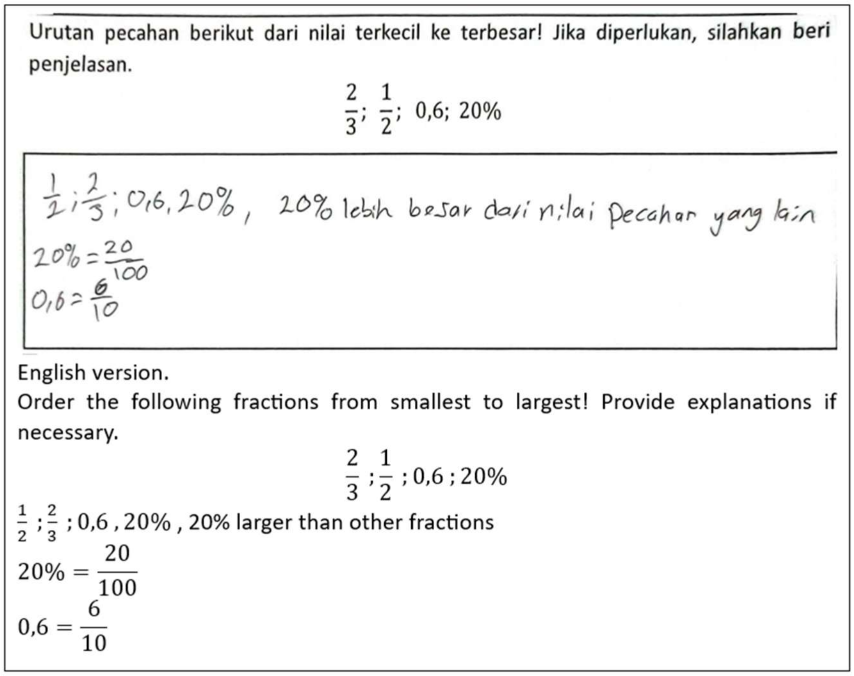
Researcher	:	Regarding the addition of fractions, do you remember the conditions?
S5	:	No, I don't understand these types of problems.
Researcher	:	Did you ask your friend about this?
S5	:	Yes, I asked a friend.

Researcher	:	Did you ask why it is done this way?
S5	:	No.

The interview results revealed a misunderstanding of the concept of fraction operations, coupled with the students' reluctance to ask questions. This lack of curiosity is likely due to insufficient motivation in the learning process. Skilling et al. (2021) suggested that low motivation to learn can lead to the emergence of learning obstacles. Furthermore, Hendriyanto et al. (2024) state that a lack of motivation and interest in mathematics can result in reduced attention in class, failure to solve problems, and a lack of enthusiasm in asking questions to understand the material.

Learning Obstacle in Ordering Fractions

The concept of ordering fractions is inherently tied to comparing two fractions. Thus, the fundamental concept of ordering fractions involves comparing two fractions to determine whether one is greater than, less than, or equal to the other. Using a number line visualization, if given two fractions A and B where $A>B$, then A is positioned to the right of B on the number line. In this study, two methods were identified for ordering fractions in question three of Table 1: one involves equalizing the denominators and performing division between the numerator and denominator, while the other method interprets fractions as quotients, as described by Lamon (2020). However, errors were found in the students' answers regarding the ordering of fractions from smallest to largest, specifically for $\frac{2}{3}, \frac{1}{2}$, 0.6, and 20%. Some students correctly ordered the fractions as 20%, $\frac{1}{2}$, 0.6, and $\frac{2}{3}$. Nonetheless, there were also incorrect answers, both in total and partially. Below are examples of incorrect student answers from S7 and S8.



Urutan pecahan berikut dari nilai terkecil ke terbesar! Jika diperlukan, silahkan beri penjelasan.

$$\frac{2}{3};\ \frac{1}{2};\ 0{,}6;\ 20\%$$

$\frac{1}{2};\frac{2}{3}$; 0,6, 20%, 20% lebih besar dari nilai pecahan yang lain

$20\% = \frac{20}{100}$

$0{,}6 = \frac{6}{10}$

English version.

Order the following fractions from smallest to largest! Provide explanations if necessary.

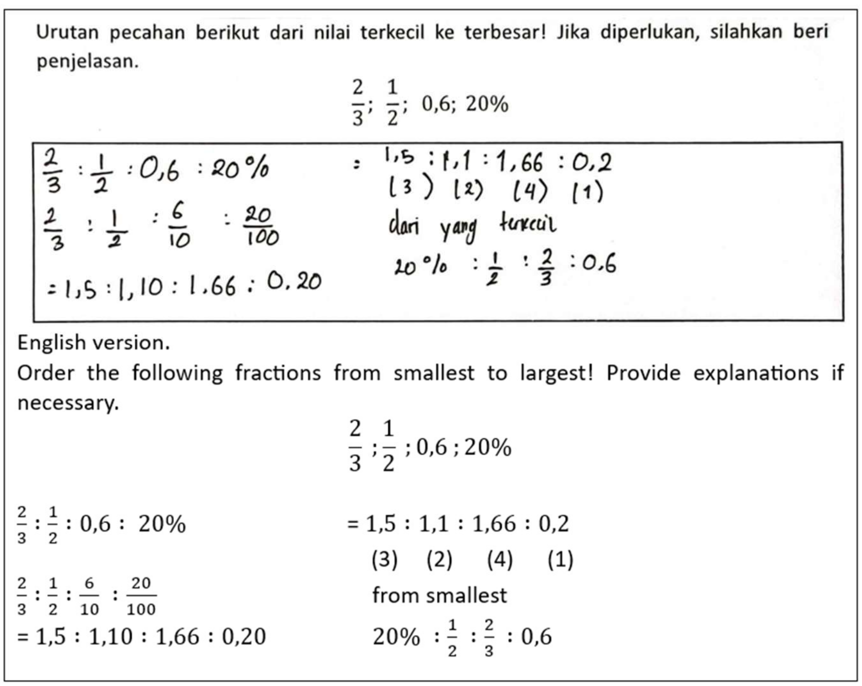
$$\frac{2}{3};\frac{1}{2};0{,}6;20\%$$

$\frac{1}{2};\frac{2}{3}$; 0,6 , 20% , 20% larger than other fractions

$20\% = \frac{20}{100}$

$0{,}6 = \frac{6}{10}$

Figure 7. Example of an Incorrect Student Response from S7 in Ordering Fractions



Urutan pecahan berikut dari nilai terkecil ke terbesar! Jika diperlukan, silahkan beri penjelasan.

$$\frac{2}{3};\ \frac{1}{2};\ 0{,}6;\ 20\%$$

$\frac{2}{3} : \frac{1}{2} : 0{,}6 : 20\%$

$\frac{2}{3} : \frac{1}{2} : \frac{6}{10} : \frac{20}{100}$

= 1,5 : 1,10 : 1.66 : 0.20

= 1,5 : 1,1 : 1,66 : 0,2

(3) (2) (4) (1)

dari yang terkecil

$20\% : \frac{1}{2} : \frac{2}{3} : 0{,}6$

English version.

Order the following fractions from smallest to largest! Provide explanations if necessary.

$$\frac{2}{3};\frac{1}{2};0{,}6;20\%$$

$\frac{2}{3} : \frac{1}{2} : 0{,}6 :\ 20\%$

$\frac{2}{3} : \frac{1}{2} : \frac{6}{10} : \frac{20}{100}$

= 1,5 : 1,10 : 1,66 : 0,20

= 1,5 : 1,1 : 1,66 : 0,2

(3) (2) (4) (1)

from smallest

$20\% : \frac{1}{2} : \frac{2}{3} : 0{,}6$

Figure 8. Partial Error in Ordering Fractions from S8

In Figure 7, S7 incorrectly ordered the fractions by stating that 20% is the largest fraction. The following is an excerpt from the interview with the student.

Researcher	:	Okay, let's continue with question 4. Could you read the question again? What does it ask?
S7	:	(Reads the question) It asks to order the fractions.
Researcher	:	Alright, how did you come to order them this way?
S7	:	Because as far as I know, $\frac{1}{2}$ is smaller than $\frac{2}{3}$.
Researcher	:	Why is that? What did you observe?
S7	:	The numbers.
Researcher	:	So 1<2 and 2<3?
S7	:	Yes. Also, I thought 0.6 is 0.60, and the largest is 20%.
Researcher	:	Why do you think 20% is the largest?
S7	:	Because 20% is $\frac{20}{100}$.
Researcher	:	So it's the largest because it's under 100?
S7	:	Yes. 100 is the largest.

S7 stated that the ordering was done by comparing the numerator of the first fraction with the numerator of the second fraction, and similarly for the denominators, resulting in the order $\frac{1}{2}$ and $\frac{2}{3}$. However, an interesting finding is that 20% was identified as the largest fraction based on the assumption that the denominator value is the largest compared to the denominators of the other fractions. This condition indicates a misunderstanding in determining the order of fractions. According to (Moyo & Machaba, 2021), this error suggests that students still perceive fractions similarly to integers. In this case, the student views the numerator and denominator in a fraction as separate integers, leading to a failure in understanding the concept of fractions (Deringöl, 2019; Moyo & Machaba, 2021). When the student identified 20% as the largest fraction based on the assumption that the denominator value of 100 is the largest, it shows that the student ordered the fractions based on the value of the denominator. This aligns with the view of Aliustaoğlu et al. (2018), who stated that in ordering fractions, students rely on comparing numerators individually and then apply this to the fractions as a whole.

The errors observed in students' ordering of fractions, as presented above, stem from their failure to understand the concept of fractions themselves. Students perceive fractions in terms of separate numerators and denominators, treating them similarly to integers. In this context, the difficulty in ordering fractions reflects an instrumental obstacle as it is one of the key technicalities in fraction concepts. In fraction ordering, a fraction is expressed as an indivisible whole rather than as separate numerators and denominators. Pavlovičová and Vargová (2020) state that comparing fractions relies on a compensatory relationship between the size and the number of equal parts within a partitioned unit. Hence, a fraction with more partitioned parts has a smaller value. This understanding aligns with the interpretation of fractions as quotients.

The Design of Alternative Instructional Strategies for Fraction Learning

The results of the learning obstacle analysis indicate the presence of ontogenic obstacles, as well as obstacles stemming from psychological, instrumental, and conceptual obstacles. Furthermore, students encounter epistemological challenges manifesting as constrained fractional context. The existence of learning obstacles can serve as a foundation for the development of learning designs, with the objective of reducing these obstacles in the future. In addressing the challenges posed by psychological barriers, Lin and Singh (2015) asserted the necessity of guided scaffolding during the learning process. This pedagogical approach has been shown to facilitate learning and enhance student motivation. Teachers should avoid delivering concepts that advance very quickly to complex concepts, but should be in accordance with the ontogenic development of students (Unaenah et al., 2023). Furthermore, the delivery of concepts through concrete manipulatives of fraction concepts has the potential to reduce the emergence of instrumental and conceptual ontogenic obstacles (Mohamed et al., 2021; Powell, 2023). The use of visual representations helps students make connections between the concept of fractions and real situations so as to encourage stable mental development, thereby reducing the occurrence of misconceptions stemming from the use of procedural that students do not understand conceptually (Pantziara & Philippou, 2012; Siegler et al., 2011). Thus, the use of concrete manipulatives or visual representations such as fraction blocks, number lines, or fraction circles, including digital learning media builds deep student understanding.

In the epistemological obstacle aspect, learning can be designed by involving multi-representation of concepts. Teachers provide a diversity of representations so that students are not limited to one meaning of fractions. The use of bar representations, pictures, number lines, or other visual models for the concept of fractions can help students in understanding the concept. Robotti et al. (2015) stated that the use of representation models such as colored number lines plays an important role in supporting the construction of fraction concepts. In addition, during the learning process teachers can develop the belief that fractions are not only limited to $\frac{a}{b}$ notation but emphasize meaning and context (Gupta & Elby, 2011). Teachers do not focus on only one meaning, but rather explore other meanings, including at the initial stage of learning (Purnomo et al., 2021). Fraction concept learning does not focus on the "how" problem but on the "why" problem. Involve a variety of real contexts to be explored such as through open-ended problems, discussions, or reflections including critical questions related to the interpretation of fractions. Thus, teachers can develop a learning design in the form of a learning trajectory based on the above considerations with the main basis of learning obstacles that occur to students. Table 2 provides a detailed description of the lesson plan design with instructional strategies as alternative, which aim to correct the identified student obstacles.

Phase	Instructional Activities	Strategy	Purpose
Engage	Presenting concrete-real life situations or problems related to fraction concepts, such as involving sharing food equally pizza. *Example Situations-One has 4 circular pizza pans that will each be cut into 8 pieces.* *1. If the person is going to share the pizza with friends, how many pieces of pizza will each friend get?* *2. If there are 10 pieces of pizza left, how many pieces have been eaten?*	Contextual Learning	Connecting fraction concepts to real life, activating prior knowledge, and overcoming psychological obstacle
Explore	Using concrete manipulations (i. e fraction strips, circles, or use virtual laboratory such as PhET) to explore fraction concepts such as part-whole use area of various shapes in paper *Example Situations-Students fold a circle piece of paper into four equal parts, then shade two of them, and then direct students to express the shaded areas as part values in the form of fractions to show how many parts are shaded.*	Concrete-Representational-Abstract	Address conceptual obstacles due to lack of intuitive understanding
Explain	Discuss in groups and present findings (can use representational forms such as number lines or symbols) *Example Situations- In groups, students are given different pieces of paper cut into equal parts (e.g. one-half, one-third, one-quarter, one-sixth) then ask students to represent each fraction on a number line and explain how $\frac{1}{2}$ is equal to $\frac{2}{4}$ and $\frac{3}{6}$, using visual models and symbols during group presentations. Furthermore, if it leads to arithmetic operations, namely addition, for example, students are asked to use two fraction strips*	Sociocultural Approach and Scaffolding	Encourage collaborative development of knowledge, address instrumental obstacles

	(for example $\frac{1}{2}$ and $\frac{1}{3}$), then cut and rearrange the strips so that they have equal parts, then represent $\frac{1}{2}+\frac{1}{3}$ visually and conclude for themselves why it is necessary to equalize the denominator before adding (LCM concept).		
Elaborate	Elaborate various concepts of fractions as part-whole, measure, ratio, and operator through various problems, including contextual problems *Example Situations- Students work in groups to solve four different contextual problems, each of which reflects one meaning of fraction (part-whole with cases such as cake cutting, measure with cases such as on distance traveled, ratio with cases such as mixture or composition of food ingredients, operator with cases such as taking part of an item), then asked to classify the problem based on the meaning of the fraction used and present the explanation visually and symbolically.*	Multiple Representation	Counter epistemological obstacles by broadening conceptual understanding
Evaluate	Reflect on initial problems and present interpretations of fraction concepts in different contexts	Formative Assessment	Assess conceptual shift, metacognitive awareness

Table 2: Lesson Design with Instructional Strategies for Fraction Learning.

The results of this study showed that some of the obstacles that occurred were mainly related to the fundamentals of the concept of fractions. The series of activities designed in Table 2 are designed to construct students' understanding gradually, especially on the fundamental understanding of the concept of fractions. In the engage phase, students are introduced to problems that may occur in life. This phase emphasizes on activating prior knowledge and generating students' interest in learning so that it has an impact on anticipating students' psychological obstacles. The explore phase provides opportunities for students to gain a deeper understanding of the concept of fractions, especially as part-whole through concrete or visual manipulation. This phase can be designed as a continuation of the engage phase through problems or situations. The involvement of group discussion activities in the design is raised through the explain phase where students discuss in representing fractions. In this phase, students discuss and give each other reinforcement or correction so that the key understanding in the concept of fractions becomes correct. The next phase is elaborate which is designed to introduce students to various fundamental understandings of fractions, not only as part-whole but also to other meanings of fractions. Through the variety of contexts presented, students do not only recognize one meaning, but several meanings so that students' understanding of the concept of fractions is not limited to only one concept.

CONCLUSION

The research results reveal that junior high school students encounter learning obstacles in the concept of fractions, specifically in the representation of fraction meaning, fraction operations and their applications, and the ordering of fractions. The types of learning obstacles identified include ontogenic and epistemological obstacles. Ontogenic obstacles observed in students encompass psychological, instrumental, and conceptual dimensions. Psychological ontogenic obstacles refer to a lack of motivation and interest in mathematics. Instrumental ontogenic obstacles pertain to students' failure to solve fraction-related problems, particularly in determining key aspects such as the Least Common Multiple (LCM) of denominators. Conceptual ontogenic obstacles involve students' insufficient mastery of prerequisite concepts related to fractions. Epistemological obstacles in the concept of fractions are related to students' limited context, where students are confined to understanding fractions only in the form of $\frac{a}{b}$. This restriction does not extend to other contexts or interpretations of fractions, such as part of a whole and measurement. However, this study is limited to identifying learning obstacles, specifically ontogenic and epistemological, while didactical ontogenic aspects remain unexplored. Future research should delve into didactical obstacles and include the development of didactic designs based on identified learning obstacles, as well as the creation of learning trajectories for teaching fraction concepts in junior high school.

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