MATHEMATICS TEACHING RESEARCH JOURNAL
WINTER 2025
Vol 17 no 6

The Application of Problem-Based Learning in Improving Junior High School Students' Mathematical Communication Skills

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Abstract: Mathematical communication is a core competency in mathematics education, yet many students continue to experience difficulties in expressing mathematical ideas coherently. This study investigates the effectiveness of problem-based learning (PBL) in enhancing students' mathematical communication skills in junior high school mathematics. A quasi-experimental design with a nonequivalent control group was employed, involving 78 students from a public junior high school in Indonesia. Students' mathematical communication skills were measured using essay-based tests administered as pretests and posttests. The data were analyzed using nonparametric statistical techniques to examine differences in learning outcomes between groups. The results indicate that students who received instruction through problem-based learning demonstrated significantly higher gains in mathematical communication skills compared to those taught using conventional instructional methods. These findings highlight the pedagogical value of problem-based learning in fostering students' ability to articulate mathematical communication and suggest its potential as an effective instructional approach in secondary mathematics education.

Keywords: Problem-Based Learning, Mathematical Communication Skills

INTRODUCTION

One of the skills that students need to possess is mathematical communication. Through reasoning, deliberation, and decision-making, this skill enables pupils to comprehend mathematics (Wardono et al., 2020; Sercenia et al., 2023). Additionally, communication skills can help students illustrate mathematical concepts in a variety of ways. Thus, the main goal of learning mathematics should be developing mathematical communication skills in order to encourage students' passion and critical thinking as well as to communicate a variety of concepts (Kamid et al., 2020; Triana et al., 2019; Sa'adah et al., 2023).

Presenting mathematical concepts orally, in writing, and using graphs, pictures, and other visual aids is part of mathematical communication skills. Students' mathematical knowledge can be strengthened by mathematical communication, both written and verbal (Kusumah et al., 2020; Ikhsan et al., 2020). Students can exchange ideas through a mathematical communication process. Therefore, it is critical to effectively incorporate mathematical communication into the classroom and guide students in expressing and recording their thoughts, questions, and solutions. According to several accounts, mathematical communication is expressing mathematical concepts and symbols verbally, in writing, in pictures, or in diagrams (Çelik Demirci & Baki, 2023; Balyan et al., 2022; Smith et al., 2023).

In mathematical communication, one must ask questions, formulate arguments, and provide answers (Pourdavood et al., 2020). Given the enormous variation in intelligence across students, brain function plays a significant influence in determining how well individuals are able to think and communicate. In order to increase student engagement and yield more meaningful learning, teachers should create a fun learning environment and assess students' critical thinking skills according to van Dijk & Lane (2020) and Öztaş (2023).

In order for pupils to become proficient communicators in mathematics, teachers must encourage their curiosity. Teachers can give students the chance to communicate their thoughts by utilizing the learning opportunities that are offered (Fonseca, 2021; Chasanah et al., 2020). If a teacher consistently employs a learning model or strategy, learning can be successful. A learning platform that can assist teachers in teaching content such that pupils grasp it fast is called a learning model or approach. In order to facilitate students' understanding of the content being taught quickly, the problem-based learning approach is used. Problem-based learning is thought to improve students' ability to communicate mathematical ideas to others (Ahdhianto et al., 2020; Kurniasih et al., 2022; Greenstein & Fernández, 2023; Iwuanyanwu, 2021).

Problem-based learning is a learning approach that provides students with real-world situations to help them learn how to communicate mathematical ideas and solve problems, as well as help them retain the knowledge and concepts taught. When problem-based learning is used, students must be given problems (Darhim et al., 2020; Treepob et al., 2023). Problem-based learning involves the following steps: (1) the teacher poses a problem in front of the class; (2) students have conversations in small groups; (3) students carry out an independent study of problems that need to be solved; and (4) students review the problems that have been solved. Students work in home groups to solve problems, share knowledge, and learn from each other; (5) students present the solutions they found; and (6) teachers help students complete assessments related to all learning activities (Tadjer et al., 2022; Ukobizaba et al., 2021; Magaji, 2021). Specifically, the problem-based learning approach calls for the long-term development of students' mathematical communication skills, even while it theoretically facilitates the development of those skills based on applied designs. Through conversations and the presentation of issues that may be used with the problem-based learning paradigm, mathematical communication skills can be fostered (Dinç et al., 2022; Palupi et al., 2020; Cresswell-Yeager, 2021).

Creating new mathematical problems or questions by reformulating existing ones is known as posing mathematical problems (Lee, 2021; Putra et al., 2020). The process of creating a math problem consists of the following steps: (1) choosing a starting point; (2) quality recording (registering property); (3) What-if-not-ing (aka asking "what if not?"); (4) raising questions or concerns; and (5) evaluating problems and finalizing them after analysis. In addition to fostering students' mathematical thinking, mathematical problem-solving offers opportunities for critical interpretation and analysis of reality (Muhtarom et al., 2020; Passarella, 2021; Papadopoulos et al., 2022; Piñeiro et al., 2021).

Numerous pertinent studies, such as Domu et al. (2023), have provided evidence for the aforementioned claim. They found that problem-based learning in reverse online classes enhances students' statistical literacy skills in remote learning. Positive feedback was also observed from students working on entrepreneurship-related statistics projects. This is evident from the opinions of students in this study who expressed a high level of satisfaction with their entrepreneurship-related projects. Additionally, the study's findings support the notion that incorporating students into distance learning is crucial.

According to Susanti et al. (2023), their study developed a problem-based learning model implemented in a probability theory course, which consists of several instructional stages. These stages include introducing students to a given problem, organizing a structured learning schedule, facilitating individual and group investigations, guiding students in developing and presenting their solutions, and conducting analysis and evaluation of the problem-solving process. The findings indicate that both lecturers and students perceive problem-based learning as a practical and effective instructional approach. In the context of probability theory courses, the application of problem-based learning supports students in improving problem-solving abilities and fostering critical thinking skills. Therefore, the integration of problem-based learning is considered a viable strategy for optimizing learning outcomes in probability theory instruction.

According to Rohid et al. (2019), one in three pupils are able to communicate mathematical concepts through words, notation, and symbols as well as comprehend, analyze, evaluate, and respond to mathematical concepts. According to this research, students still need to improve their mathematics communication skills. Math teachers should take note of this research so they can use creative and innovative learning activities to help students develop their mathematical communication skills in addition to teaching mathematics.

Despite numerous studies emphasizing the significance of mathematical communication skills and the role of problem-based learning (PBL) in enhancing these skills (e.g., Wardono et al., 2020; Sercenia et al., 2023; Ahdhianto et al., 2020; Kurniasih et al., 2022), there remains a need for more comprehensive research investigating the specific impact of PBL on junior high school students' mathematical communication abilities. Existing studies have predominantly focused on theoretical frameworks and applied designs, emphasizing the long-term development of these skills (Dinç et al., 2022; Palupi et al., 2020; Cresswell-Yeager, 2021). However, there is limited empirical evidence demonstrating whether PBL directly enhances students' ability to communicate mathematical concepts effectively, particularly in the context of junior high school education.

Furthermore, although studies such as Rohid et al. (2019) have indicated that students continue to encounter challenges in mathematical communication, there is a lack of detailed exploration into how PBL can be tailored to address these challenges. Specifically, the effectiveness of PBL in fostering skills such as oral and written mathematical expression, the utilization of visual aids, and collaborative problem-solving among junior high school students remains underexplored. Additionally, while some research has examined PBL in higher education or specific subjects like probability theory (Susanti et al., 2023), its application and outcomes in junior high school mathematics classrooms require further investigation.

LITERATURE REVIEW

Mathematical Communication

Students' mathematical communication skills can be developed through the education process at school. This happens because one of the elements of mathematics is the science of logic, which develops students' thinking abilities. So, mathematics is vital in developing mathematical communication skills (Ya-Amphan et al., 2024; Rajagukguk et al., 2022).

Mathematical communication is a method of expressing and interpreting mathematical ideas orally or in writing, pictures, tables, diagrams, formulas, or demonstrations. Studying mathematics without being balanced with mathematical communication ensures the learning process will not run well. Mathematical communication can help students develop their personality and ability to solve problems in learning mathematics, convey ideas correctly, and make it easier for students to convey ideas to other people so that the information is easy to understand (Uyen et al., 2021; Kusumah et al., 2020).

Greenes and Schulman (in Ata Baran & Kabael, 2021) explain that mathematical communication is (1) the ability to express mathematical ideas through speech, writing, demonstration, and depict them visually in different types, (2) the ability to understand, interpret, and assess ideas presented in writing, orally, or in visual form, and (3) the ability to construct, interpret and connect various representations of ideas and their connections.

According to the National Council of Teachers of Mathematics, mathematical communication skills refer to the ability to organize mathematical thinking, communicate mathematical ideas logically and clearly to others, analyze and evaluate the mathematical thinking and strategies of others, and use mathematical language to express ideas. The ability to articulate mathematical ideas during problem solving and to communicate the processes and outcomes of problem solving also constitutes essential skills that foster higher-order mathematical thinking, including logical, analytical, systematic, critical, creative, and productive thinking (Trgalová & Tabach, 2023; Estabrooks & McArdle, 2022)

Mathematical communication skills can be classified into two parts, namely written and unwritten or verbal mathematical communication skills. Oral mathematical communication skills are mathematical communication built verbally in the form of speaking, listening, discussing, and exchanging opinions without being proven in writing. Meanwhile, written mathematical communication skills are students' mathematical communication skills using vocabulary, structures, and mathematical notation in the form of reasoning connections, as well as in problems presented in the form of graphs, pictures, tables, equations, or writing (Smith et al., 2023; Pourdavood et al., 2020).

The mathematical communication skills analyzed in this study are specifically focused on written communication. This is based on the objectives and indicators studied, which are to describe mathematical communication skills in writing. The indicators of written mathematical communication skills utilized in this study encompass Grammatical Ability, Sociolinguistic Ability, and Strategic Ability.

Grammatical Ability comprises two primary aspects. Firstly, the appropriate use of mathematical symbols or notations. This involves students' ability to present problems using tables, figures, mathematical models, or symbols. Students are expected to articulate statements or problems in mathematical language utilizing appropriate models, symbols, or visuals. In problem-based learning, students have been trained to solve problems using models and subsequently express their solutions in the form of mathematical representations or symbols. Secondly, the formulation of mathematical definitions. Students are expected to define terms in mathematics with clarity. This definition serves as a foundation for introducing concepts, elucidating mathematical topics, or discussing linear equations.

Sociolinguistic Ability also comprises two aspects. Firstly, the expression of problems in mathematical language. This entails students' ability to construct mathematical models and conclude problems presented. Secondly, the mathematical solution of problems. This includes students' proficiency in providing reasons for their solutions, explaining the methods employed to solve problems, and concluding appropriately.

Furthermore, Strategic Ability encompasses students' ability to extract information from given problems. Students must identify the available information in the question and determine whether it is sufficient to solve the problem. Additionally, this aspect includes students' ability to describe the strategies employed to solve contextual problems. Students must be able to articulate the systematic steps involved in solving problems with appropriate and correct strategies. In problem-based learning, students are trained to collaborate with peers and teachers, fostering their accustomedness to express their ideas or thoughts and optimizing the teaching and learning process.

Problem-Based Learning

The method of addressing problems through science is the main focus of a set of learning exercises known as problem-based learning. Problem-based learning is characterized by three key elements. First, problem-based learning is an activity-based learning approach, which means that students must complete multiple tasks in order to utilize it. Problem-based learning involves students actively thinking, communicating, searching and processing facts, and ultimately drawing conclusions. It does not assume that students will just listen, take notes, and then memorize the course information. Secondly, the goal of learning activities is to solve issues. Problem-based learning is a crucial term in education. This implies that in order for learning to occur, difficulties must exist. Third, a scientific thinking technique is used for solving problems. A scientific method of thinking involves both inductive and deductive reasoning. This methodical, empirical thought process is employed. Systematic refers to the steps that scientific thought takes, whereas empirical refers to the use of accurate data and facts as the foundation for problem-solving (Katsara, 2023; Liu et al., 2022; Rogers et al., 2021).

According to Duch (in Kelly et al., 2022), problem-based learning is an instructional approach that promotes students' ability to learn and collaborate in groups to find answers to issues in the real world. Before beginning to learn a subject, pupils' curiosity is piqued by problem simulations. Problem-based learning equips students with the critical and analytical thinking skills necessary to locate and make effective use of learning materials.

Problem-based learning is an educational approach that develops students' thinking, problem-solving, social, and autonomous learning skills as well as their ability to construct or acquire new knowledge by using authentic, real-world problems that are left open-ended for student solution (Dakabesi & Luoise, 2019; Tadjer et al., 2022). This type of learning is distinct from traditional forms of learning, which only use real-world situations as a means of applying the knowledge that have been acquired at the end of the learning process. Genuine issues were chosen according to how well they will help students develop key skills (Sancar-Tokmak & Dogusoy, 2023; Ernawati et al., 2022).

Problem-based learning is characterized by several characteristics, according to Barrows (in Buus & Pedersen, 2021): (1) student-centered learning; (2) learning occurs in small groups; (3) the teacher acts as a facilitator or guide; (4) the problems presented in the learning environment are arranged with a particular form and focus and function as a learning stimulus; (5) new information is obtained through independent learning (self-directed learning); and (6) problems serve as a means to develop clinical problem-solving abilities.

The following describes the characteristics of problem-based learning based on Barrows' explanation:

(1) Learning starts from problems.
(2) The problems should be relevant to students' daily lives.
(3) Learning must be organized based on problems rather than scientific disciplines.
(4) Students should be responsible for forming and implementing their learning process.
(5) Small groups should be used.
(6) Students must demonstrate their learning through performances and products.

This problem-based learning approach is based on students addressing real-world problems, but they also need to gain experience with unforeseen circumstances. Problem-based learning must so accomplish a number of particular goals (Noordzij & Wijnia, 2020). The following are the goals of putting this program into practice in order to raise the caliber of students:

(1) To enhance pupils' ability to choose and make decisions using critical thinking.
(2) Teach problem-solving techniques in a methodical, responsible manner, and make plans for successful outcomes.
(3) Problem-based learning aids kids in accurately comprehending the function of adults in society.
(4) Students are encouraged to develop into responsible, self-sufficient adults.

Problem-based learning must be implemented correctly and in accordance with the guidelines in order to meet its intended purpose. It is known that certain unique syntaxes or rules were purposefully developed in order to facilitate its ideal implementation in this regard. Figure 1 is the syntax for problem-based learning (Balac & Ozogul, 2024).

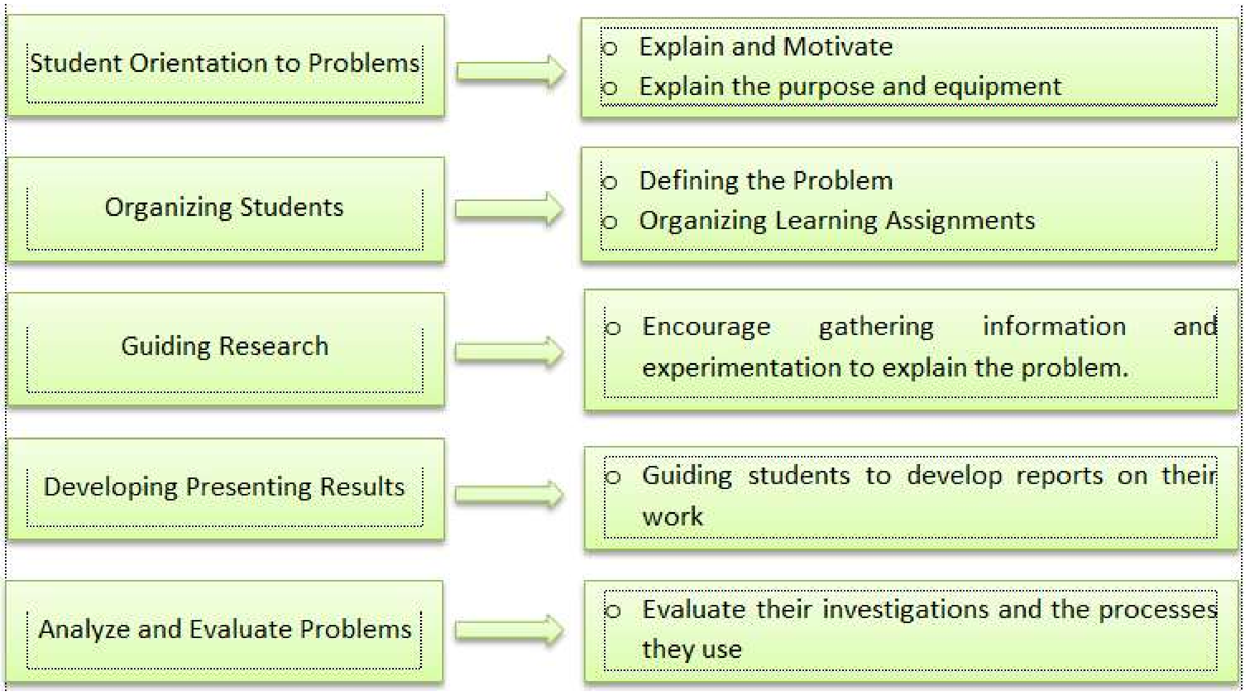


Figure 1: Problem-Based Learning Syntax

Based on the aforementioned explanation, the implementation of Problem-Based Learning (PBL) is designed to assist students in understanding the concept of linear equations. The design of the PBL model is structured not only to enable students to learn about formulas and graphs but also to effectively communicate their mathematical ideas.

Through the classroom arrangement using the PBL model, students not only learn about linear equations theoretically but also develop essential mathematical communication skills. They learn to articulate their ideas, listen to others' perspectives, and collaborate within groups. In this way, PBL creates a dynamic and interactive learning environment where students can refine their communication skills while comprehending complex mathematical concepts.

The primary objective of this study is to address these gaps by investigating the following:

(1) The extent to which problem-based learning influences junior high school students' mathematical communication skills.
(2) Does problem-based learning enhance the mathematics communication skills of junior high school students.

By conducting this research, this study seeks to provide actionable insights for educators to design innovative and effective learning strategies that enhance students' mathematical communication abilities.

METHODS

Design

This study uses a quantitative approach and is problem-based. The purpose of this study was to evaluate how problem-based learning and traditional learning environments are handled. As a result, this study employs quasi-experiments. Nonequivalent control group design was selected as the study design. The experimental and control groups in this design were not chosen at random. The researchers did not select the research subjects at random; instead, they accepted the students' circumstances exactly as they were, making no modifications to the other pupils in the class. Both the experimental and control groups are contrasted in this research design. While the control group did not receive treatment, the experimental class did.

To measure their mathematical communication skills, students in the experimental class were given treatment using problem-based learning. For students in the control class, they were given regular learning, as shown in Figure 2.

O_1	X	O_2
O_1		O_2

Figure 2: Nonequivalent Control-Group Design. Key: O1 = Pretest; O2 = Posttest; X = Problem-based learning.

Research Sample

The sample used in the research included 78 students at State Middle School 4, Ternate City, Indonesia, of VIII grade. The number of students was grouped based on study groups from the school without particular criteria. The school arranged the learning schedule and supporting administration. Therefore, researchers did not randomize individual students but accepted the subjects as they were in each class. Determination of the experimental and control classes was based on the researcher's direct appointment. The number of students in the experimental class was 39 people, and the control class was 39 people.

Research Instrument

The instruments utilized in this study encompass training materials, non-test instruments, and test instruments. Mathematical communication skills assessments are administered before and after the acquisition of knowledge for measuring the progression of students' abilities. The initial assessment (pre-test) is administered prior to the commencement of learning, while the final assessment (post-test) is administered upon the culmination of the learning process. This assessment comprises descriptive questions that encompass three primary domains: Grammatical Ability, Sociolinguistic Ability, and Strategic Ability. Grammatical Ability encompasses the accurate utilization of mathematical symbols or notations and the formulation of a lucid definition of a mathematical term. Sociolinguistic Ability involves students' proficiency in expressing mathematical concepts in language and subsequently solving them mathematically. Furthermore, Strategic Ability assesses students' aptitude for systematically recording information contained within a problem and describing the strategies employed in solving the problem.

One instance of a pre-test and post-test question utilized in this study is as follows: In the post-test, students were tasked with determining the equation of a straight line that passes through the point $(4,5)$ and is perpendicular to the line $y=-2x+4$. Concurrently, in the pre-test phase, students were tasked with determining the equation of a straight line that passes through two specified points: $(1,-2)$ and $(3,4)$. The comprehensive mathematical communication skills assessment instrument is available in the appendix.

Prior to its utilization in the study, the test instrument underwent a rigorous trial and validation process. The trial was conducted with classes that had studied the material in alignment with the content of the test instrument. Item analysis was conducted to assess the quality of the questions, ensuring their appropriateness based on difficulty, discriminatory power, validity, and reliability. The difficulty level was analyzed to strike a balance between easy, medium, and challenging questions. Discriminating power was employed to measure the extent to which the questions differentiated students with varying abilities. The validity of the instrument was evaluated to ascertain its alignment with the mathematical communication skills under investigation. Additionally, its reliability was assessed to ensure consistency in measurement outcomes across diverse situations. Based on the trial results, the test instrument employed in this study demonstrated high validity and reliability standards, rendering it an accurate and reliable data collection tool.

Data Collection Technique

Data collection was conducted through the administration of tests and documentation of the instructional treatment implemented in each group. Each group participated in a total of six meetings. The first meeting was allocated for the administration of the pre-test, while the final meeting was used to administer the post-test. The remaining four meetings were dedicated to the instructional treatment, with each treatment session conducted over a duration of 120 minutes per meeting. In the experimental class, treatment was provided through problem-based learning, as illustrated in the following steps:

Step 1: Student Orientation to the Problem

The researcher begins by introducing an engaging problem: “Creating a City Map Using Straight Lines.” Students are divided into small groups and tasked with designing a city map using linear equations. They must determine the locations of various places, such as schools, parks, and shopping centers, and illustrate the relationships between these locations using straight lines.

Step 2: Organizing Students

Each group begins exploring the concept of linear equations. They learn about the general form of a linear equation, $y = mx + b$, where m represents the slope and b is the y-intercept. Students investigate how straight lines are applied in everyday life, such as in architecture and design.

Step 3: Guiding Research

During this process, students engage in active discussions. They exchange ideas on how to determine the appropriate slope to connect various locations on their map. These discussions encourage them to use precise mathematical language and articulate their reasoning to their peers.

Step 4: Developing and Presenting Results

After completing their maps, each group presents their work to the class. They explain how they used linear equations to design the map, as well as the rationale behind their choice of locations and slopes. This presentation provides students with an opportunity to practice mathematical communication skills, such as explaining concepts, listening to feedback, and responding to questions.

Step 5: Analyzing and Evaluating Results

Following the presentations, the teacher guides students in reflecting on their experiences. Students are asked to discuss what they learned about linear equations and how they communicated with their peers during the project. This reflection helps students recognize the importance of communication in understanding and applying mathematical concepts.

The data presented in this study were collected through a mathematical communication skill test administered to students, consisting of four questions. This test was conducted in two stages: an initial test administered before the treatment and a final test administered after the treatment for each group. Each test session was scheduled for 60 minutes, during which students were required to work independently on the problems without discussing or interacting with each other to ensure the validity of the data obtained. Additionally, data was collected from the results of assignments given to students during the learning process. Documentation of these assignments serves as an additional source to evaluate students' mathematical communication skills in greater depth.

Data Analysis Technique

Based on the collected data, an analysis was conducted to evaluate students' mathematical communication skills in the context of implementing problem-based learning and conventional learning models. The data analysis process was carried out in several key stages. First, a comprehensive data collection was performed, followed by a data quality assessment to ensure that the data met the required standards. If any missing data were identified, data extraction procedures were conducted to complete the necessary information. Subsequently, prerequisite tests were carried out to examine the normality and homogeneity of the data, ensuring that it met the assumptions required for inferential statistical analysis.

Descriptive statistical analysis was applied by calculating pretest, posttest, and gain scores, including statistical measures such as mean, variance, and standard deviation. This analysis aimed to provide a general overview of the data distribution without making inferences about a broader population.

For inferential analysis, the Wilcoxon test and independent sample t-test were employed to assess the impact of the learning models. The Wilcoxon test was used to measure the improvement in mathematical communication skills within each class, while the independent sample t-test was utilized to compare the differences in improvement between the problem-based learning group and the conventional learning group.

Furthermore, the enhancement of students' mathematical communication abilities was evaluated employing the Normalized Gain metric (Hake, 1998), which is presented in Table 1.

Normalized Gain value Score	Interpretation
$g \geq 0{,}70$	High
$0{,}30 \leq g \leq 0{,}70$	Medium
$g < 0{,}30$	Low

Table 1: Normalized Gain Value Score Criteria (Hake, 1998)

The effectiveness of the learning process is also evaluated by determining the percentage of scores that fall within the categories outlined in Table 2.

Percent (%)	Interpretation
< 40	Ineffective
40 - 55	Less effective

56 - 75	Effective enough
> 76	Effective

Table 2: Category Interpretation of Gain value Effectiveness (Hake, 1998)

Thus, this analysis provides a comprehensive understanding of the effectiveness of problem-based learning in enhancing students' mathematical communication skills.

RESULTS

The discussion and results of this study are relevant to the goals of the study. The objectives of this study are based on current issues: (1) To determine the impact of problem-based learning on junior high school students' mathematical communication skills and (2) To determine the increase in junior high school students' mathematical communication skills after implementing problem-based learning. Data were assessed based on how students who participated in learning with the problem-based learning model and students who learned conventionally were influenced and improved in their mathematical communication skills. With the use of the SPSS 20 software program, descriptive and inferential statistics were used for data analysis. To describe the abilities of both student groups before and after treatment, as well as their improvement, a descriptive statistical analysis was conducted. Concurrently, inferential statistical analysis was carried out to determine the impact and variations on students' development of mathematical communication skills.

The quantitative data collected for the study were processed in accordance with the need to facilitate discussions about the issues brought up by the investigation. Table 3 displays data on students' mathematical communication skills based on their learning.

Statistics	n	Problem-Based Learning			Conventional Learning		
		Pretest	Posttest	Gain Value	Pretest	Postest	Gain Value
Minimum	39	8	60	.30	5	60	.42
Maximum		50	95	.95	35	85	.78
Mean		21,35	79,72	.7332	21,14	70.18	.6265
Standard Deviation		11,51	9,52	.14162	6,02	7.126	.08287
Ideal Maximum Score: 100							

Table 3: Description of Mathematical Communication Ability Test Data Based on Learning

Before the implementation of the Problem-Based Learning model (see Table 3), students exhibited varying levels of mathematical communication skills. The average pre-test score of 21.35 indicates that many students were at a low level of understanding. The wide range of scores (from 8 to 50) and the high standard deviation (11.51) suggest significant differences in students' initial abilities. Some students may have a stronger background in mathematics, while others may face difficulties.

After the application of the Problem-Based Learning model, there was a significant improvement in students' mathematical communication skills. The average post-test score reached 79.72, indicating that students, overall, made substantial progress. The narrower range of post-test scores (60 to 95) and the lower standard deviation (9.52) indicate that students' learning outcomes became more consistent. This suggests that the Problem-Based Learning model successfully enhanced students' understanding and reduced the gap in their abilities.

In the class that utilized conventional learning model, students had a slightly lower average pretest score (21.14) compared to the Problem-Based Learning model. The narrower range of scores (10 to 35) and the lower standard deviation (6.02) indicate that students in this class had more homogeneous abilities. Nevertheless, many students still remained at a low level of understanding. After the implementation of conventional teaching, students showed improvement in their mathematical communication skills, with an average post-test score of 70.18. Although there was an increase, the range of post-test scores (60 to 85) and the lower average compared to the Problem-Based Learning class indicate that the impact of conventional learning was not as strong as that of Problem-Based Learning. The lower standard deviation (7.126) suggests that students' learning outcomes remained consistent, but there was no significant leap in their understanding.

Based on the results of this descriptive analysis, it can be concluded that the Problem-Based Learning class demonstrated a greater improvement in students' mathematical communication skills compared to the conventional learning class. The Problem-Based Learning model not only increased the average post-test score but also reduced the variability in students' learning outcomes, indicating that more students achieved a better understanding. The conventional learning class showed a smaller increase, although students still demonstrated progress. However, the results were not as varied as those in the Problem-Based Learning class, suggesting that conventional learning model may not be sufficient to foster deeper understanding. To strengthen this hypothesis, inferential statistical tests were subsequently conducted.

1. The Effect of Problem-Based Learning on Students' Mathematical Communication Skills

In this section, we will test the influence of problem-based learning on students' mathematical communication skills. Before inferential statistical tests, prerequisite tests were carried out: data normality tests and equality of variance (homogeneous) tests. Data from the normality test results for the experimental and conventional classes are presented below, listed in Table 4.

Learning	Test	Shapiro-Wilk			Conclusion
		Statistics	df	*Sig.*	
Problem-Based	Pretest	0.863	39	0.001	Not Normal
	Posttest	0.996	39	0.279	Normal
	N-gain	0.943	39	0.076	Normal
Conventional	Pretest	0.971	39	0.404	Normal

	Posttest	0.943	39	0.048	Not Normal
	N-gain	0.965	39	0.261	Normal

Table 4: Summary of Normality Test for Mathematical Communication Ability Data

Pay attention to Table 4. The data obtained from the pretest and posttest results of both modes of learning, namely problem-based learning and conventional learning, shows that the data are not generally distributed because, from the results of the Shapiro-Wilk test, there are several significant values from the pretest and posttest in problem-based learning and conventional learning. Conventional is less than alpha (0.05). Based on the testing rules for statistical tests, testing can be carried out using nonparametric statistical tests; in this case, it can be done using the Wilcoxon test. The research hypotheses formulated are as follows:

H_0: Problem-based learning does not affect students' mathematical communication skills.

H_1: Problem-based learning influences students' mathematical communication skills.

The following is a summary of the results of nonparametric statistical tests (Wilcoxon), which can be used in decision-making when implementing problem-based learning. A summary of the test results are presented in Table 5.

	PostTets PBL - PreTest PBL	PostTest Conventional - PreTest Conventional
Z	-5.444^{b}	-5.481^{b}
Asymp. Sig. (2-tailed)	.000	.000

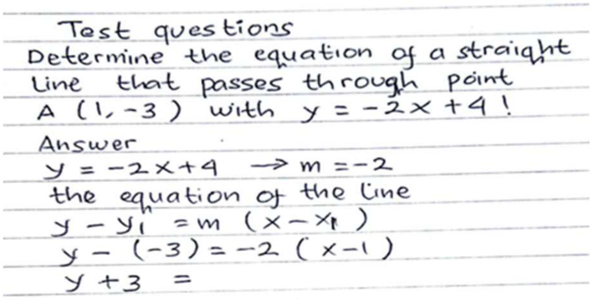
a. Wilcoxon Signed Ranks Test
b. Based on negative ranks.

Table 5: Summary of Wilcoxon Test of Students' Mathematical Communication Ability

The results of the Wilcoxon test for the problem-based learning class indicated a very small p-value (e.g., $p < 0.05$), suggesting that the difference between the pre-test and post-test scores is statistically significant. This implies that the implementation of the problem-based learning model effectively enhances students' mathematical communication skills. The Wilcoxon test results for the conventional class also revealed a significant p-value (e.g., $p < 0.05$), indicating that there is a significant difference between the pre-test and post-test scores. However, the smaller improvement compared to the problem-based learning class suggests that while the conventional method is effective, its impact is not as pronounced as that of the problem-based learning model.

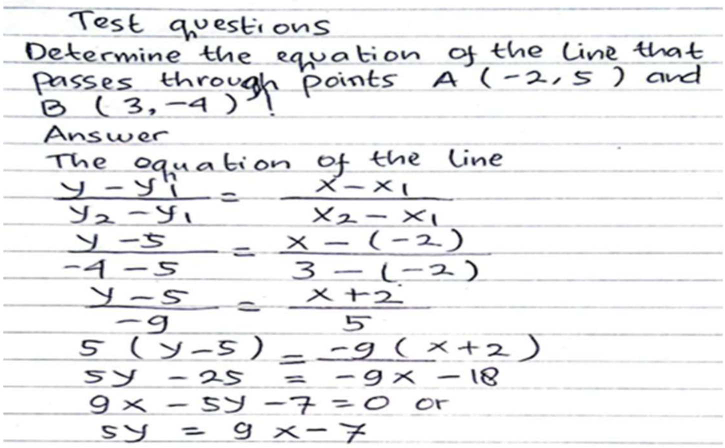
The analysis results demonstrate that the Problem-Based Learning model is more effective in enhancing students' mathematical communication skills compared to conventional learning model. The significant increase in post-test scores in the PBL class indicates that students are better able to understand and communicate mathematical concepts after participating in this learning approach. Although conventional learning also showed significant improvement, the results were not as robust as those observed in the PBL class. This suggests that the PBL method may be more capable of fostering student engagement and deeper understanding of the material.

The results of calculations using the Wilcoxon tests show that the application of problem-based learning has a significant influence on mathematical communication skills, as shown in the results of student work after the application of the problem-based learning model in the following Figure 2.



Test questions
Determine the equation of a straight line that passes through point A (1, -3) with $y = -2x + 4$!
Answer
$y = -2x + 4 \rightarrow m = -2$
the equation of the line
$y - y_1 = m\,(x - x_1)$
$y - (-3) = -2\,(x - 1)$
$y + 3 =$

Figure 2: Example of student pretest results



Test questions
Determine the equation of the line that passes through points A (-2, 5) and B (3, -4)!
Answer
The equation of the line

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y - 5}{-4 - 5} = \frac{x - (-2)}{3 - (-2)}$$

$$\frac{y - 5}{-9} = \frac{x + 2}{5}$$

$$5\,(y - 5) = -9\,(x + 2)$$

$$5y - 25 = -9x - 18$$

$$9x - 5y - 7 = 0 \text{ or}$$

$$5y = 9x - 7$$

Figure 3: Example of student posttest results

After implementing problem-based learning, (1) students can communicate the flow of thought or strategy used clearly and coherently so that other students can follow their thinking process; (2) students can discuss, assess, and provide additional explanations or clarifications to mathematical ideas in discussions; and (3) Students can write mathematical answers or solutions with logical, coherent steps that the reader can understand. This explanation is shown in Figure 3. Figure 2 shows the results of student work before the implementation of problem-based learning. Based on the results, students cannot write mathematical solutions using logical steps, so they have not reached a correct solution set. By looking at Figure 3, it can be said that students can communicate their understanding of determining the equation of a straight line.

2. Increasing students' mathematical communication skills by implementing problem-based learning

Based on the prerequisite test results for the normality test, the results show that the data were normally distributed. So, the test can be used with parametric statistics, namely the t-test. Testing the improvement in students' mathematical communication skills can be done using the Normalized Gain test. The results of the independent t-test are presented below in Table 6.

		Levene's Test for Equality of Variances		t-test for Equality of Means		
		F	Sig.	t	df	Sig. (2-tailed)
N-gain	Equal variances assumed	6.300	.014	4.211	76	.000
	Equal variances not assumed			4.211	60.861	.000

Table 6: Comparison of N-gain values in the two classes

The results of the independent t-test (assuming equal variances) revealed a significant difference ($p<0.05$) between the two groups of students taught with distinct learning models. This analysis primarily focused on the N-gain score of students' mathematical communication skills, which quantifies the enhancement in their abilities following the implementation of the learning model.

Students taught with the problem-based learning model exhibited a notable increase in their mathematical communication skills. The average N-gain for the problem-based learning model group (0.7332) was significantly higher than that of the conventional group (0.6229). This finding underscores the effectiveness of the problem-based learning model, which emphasizes active and collaborative learning, in fostering students' engagement and involvement in the learning process. Not only do students acquire mathematical concepts, but they also practice communicating and explaining their understanding to peers.

In contrast, students taught with the conventional learning model also demonstrated an improvement, albeit with a lower N-gain compared to the problem-based learning model group. While progress was made, the more passive conventional learning model may not provide sufficient opportunities for students to effectively practice mathematical communication. Students in this group tend to prioritize mastering the material without substantial interaction, which can hinder their ability to communicate effectively.

The gain value, which assesses the effectiveness of enhancing learning outcomes, indicates that the problem-based learning class has a value of 0.7332, placing it within the high improvement category. This result demonstrates a substantial increase of 73.32% of its potential, suggesting the efficacy of implementing the problem-based learning model. Conversely, the conventional class has a gain value of 0.6265 (categorized as moderate improvement), indicating an increase of approximately 62.65% of its maximum potential. This suggests the effectiveness of implementing the conventional model.

Overall, these findings demonstrate that the application of the Problem-Based Learning model exerts a greater impact on enhancing students' mathematical communication skills compared to the conventional learning model. The presence of problem-based challenges in the learning process encourages students in the problem-based learning class to engage actively in thinking, discussing, and developing more effective problem-solving strategies. While conventional learning also yields significant improvements, it falls short of the problem-based learning model in optimizing students' mathematical communication skills.

DISCUSSION

The findings of this study demonstrate that students who engaged in problem-based learning (PBL) exhibited significantly higher gains in their mathematical communication abilities compared to those who were taught using conventional methods. This aligns with the theoretical framework of PBL, which emphasizes active, collaborative, and contextual learning as a means to enhance both conceptual understanding and communication skills (Abate et al., 2022; Martin, 2022; Alenezi, 2023). The success of PBL in fostering mathematical communication can be attributed to its structured approach, which requires students to engage in group discussions, articulate their reasoning, and collaboratively solve real-world problems. This process not only deepens their understanding of mathematical concepts but also provides ample opportunities to practice and refine their communication skills.

The results of this study are consistent with prior research that highlights the effectiveness of PBL in improving students' ability to communicate mathematical ideas. For instance, Bailey (2022), Yong et al. (2020), and Ilhan and Akin (2022) argued that traditional, teacher-centered methods often fail to cultivate students' passion for communicating mathematical concepts, as these approaches typically prioritize rote memorization over active engagement. In contrast, PBL creates a dynamic learning environment where students are encouraged to explain their thought processes, justify their solutions, and respond to feedback from peers and teachers. This active participation is critical for developing the ability to communicate mathematical ideas in a clear, accurate, and logical manner.

Furthermore, the contextual nature of PBL, as highlighted by Nugraheni and Marsigit (2021) and Nurmasari et al. (2024), plays a pivotal role in enhancing students' motivation and engagement. By connecting mathematical concepts to real-world problems, PBL makes learning more relevant and meaningful for students. This relevance not only increases their enthusiasm for learning but also provides a practical framework for applying mathematical communication skills. For example, in this study, students were tasked with designing a city map using linear equations, a problem that required them to interpret, analyze, and communicate their solutions in a real-world context. This approach aligns with the findings of Das (2020) and Susanti et al. (2023), who emphasize that

PBL's five key components—orienting students, organizing learning, guiding research, developing and presenting results, and analyzing outcomes—collectively contribute to the development of higher-order thinking and communication skills.

The data presented in Table 3, 5 and 6 further underscore the superiority of PBL over traditional methods in enhancing mathematical communication skills. While both approaches showed some improvement, the difference in average scores between the two groups highlights the unique advantages of PBL. Specifically, students in the PBL group demonstrated a greater ability to interpret mathematical results, explain their reasoning, and use mathematical language effectively. These findings are consistent with the observations of Sakir and Kim (2020), who noted that PBL encourages students to articulate their ideas and collaborate on problem-solving tasks, thereby fostering both conceptual understanding and communication proficiency.

However, it is important to acknowledge that students in traditional learning environments still showed moderate improvement in their communication skills, albeit to a lesser extent. This suggests that while conventional methods may provide some foundation for mathematical communication, they are insufficient for developing the depth and fluency required for effective communication. As noted by Novikasari (2020), Sevinc and Lesh (2022), and Luritawaty et al. (2024), traditional approaches often fail to provide students with opportunities to engage in meaningful dialogue or present their work, which are critical components of communication skill development.

CONCLUSIONS

In conclusion, this study provides compelling evidence that problem-based learning is a highly effective approach for enhancing students' mathematical communication skills. By integrating real-world problems, collaborative learning, and structured reflection, PBL creates an environment where students can actively engage with mathematical concepts and practice communicating their ideas. These findings not only support the existing literature on PBL but also highlight the importance of adopting innovative teaching methods to address the challenges of mathematical communication in the classroom. Future research could explore the long-term impact of PBL on students' communication skills and its applicability across different cultural and educational contexts.

By applying learning models in the classroom, we can ignite a transformative process where students discover something new or make various updates that are formed in a product, idea, design, and so on. The problem-based learning model, in particular, harnesses the real world as a context that students use to learn, thereby enhancing their mathematical communication skills. This approach holds immense potential to revolutionize the way we teach and learn mathematics.

The implementation of mathematics learning should shift its focus from the teacher as the main source of knowledge to the students as active participants in their learning journey. Learning should be a process where students are empowered to create meaningful learning experiences. Fun learning, undoubtedly, is the first step to achieving quality learning outcomes. However, the true value of learning lies in its meaningfulness, which can be achieved when students are given the opportunity to experience and apply what they are learning (Ndiung & Menggo, 2024).

Before problem-based learning was implemented, most students had difficulty communicating mathematics. Problem-based learning helps students communicate abstract mathematics. The abstractness of mathematics and problem-based learning is a solution to recognize, understand, appreciate, and practice mathematics in communication (Ningsih et al., 2023; Aguilar & Telese, 2018).

Mathematical communication skills are reflected through several key indicators. Students should be able to: (1) express a situation or problem using language, symbols, ideas, or mathematical models, such as pictures, diagrams, graphs, or mathematical expressions; (2) explain mathematical ideas, situations, and relationships using everyday language; (3) listen to, discuss, and write about mathematical concepts; (4) interpret mathematical representations; and (5) re-express mathematical descriptions in their own words (Bron & Prudente, 2024).

Problem-based learning can improve students' mathematical communication skills in the high category. Meanwhile, conventional learning can also improve mathematical communication skills in the moderate category. Both lessons were quite effective in enhancing mathematical communication skills.

ACKNOWLEDGEMENTS

The author would like to thank all parties who have helped in this research. Especially the Head of State Junior High School 4, Ternate City, Indonesia, who has allowed researchers to collect data.

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APPENDIX

Pretest Questions

Please ensure that you answer these questions correctly:

1. Determine the equation of the line that passes through the point (3, 4) and has a gradient of 2.
2. Find the equation of the line that passes through the points (1, -2) and (3, 4).
3. Determine the gradients of the parallel and perpendicular lines to the given line $y = \frac{1}{2}x + 4$
4. Determine the equation of the line that is perpendicular to the given line $y = 4x + 1$. The line passes through the point (2, -3).

Posttest Questions

Please ensure that you answer these questions correctly:

1. Determine the equation of the line that passes through the point (5, -3) and has a gradient of $-\frac{3}{4}$.
2. Find the equation of the line that passes through the points (2, 3) and (6, -1).
3. Determine the gradients of the parallel and perpendicular lines to the given line $y = \frac{5}{3}x + 7$.
4. Determine the equation of the line that is perpendicular to the given line $y = -2x + 3$. The line passes through the point (4, 5).