

Discovering The Power of Realistic Mathematics Education: A Hypothetical Learning Trajectory for Teaching Social Arithmetic

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Abstract: The limited use of contextualized mathematical problem design in social arithmetic, particularly with concepts of profit and loss, restricts student opportunities to develop the mathematical skills needed to understand and solve real-world financial problems. This study aimed to analyze students' thought processes using a hypothetical learning trajectory (HLT). To learn social arithmetic concepts, the HLT consisted of three components: learning objectives, mathematical tasks, and learning process hypotheses. Employing the design research methodology, this study was conducted in three phases: initial design, experimental teaching, and retrospective analysis. Three seventh-grade students from a junior high school in Indonesia were selected according to their mathematical levels, determined by low, medium, and high scores. Various methods were used to collect data, including in-depth interviews, observations, video recordings, and analysis of student work during the implementation of the intervention. To ensure students' understanding of social arithmetic concepts, the teacher provided guidance during the mathematical learning process; the students were encouraged to act and express their opinions. The results of this study indicated that the learning process, which was sequentially arranged based on the activities included in the HLT, encouraged students to think mathematically. They could construct knowledge and improve their understanding of social arithmetic concepts.

Keywords: hypothetical learning trajectory, realistic mathematics education, social arithmetic

INTRODUCTION

Social arithmetic is a branch of mathematics that directly connects with students' daily lives. Through social arithmetic, students learn practical skills, such as setting buying and selling prices, and calculating profits and losses, which are applicable in real-world scenarios (Laurens et al., 2017). Through studying mathematics, students not only acquire mathematical skills but also develop cognitive abilities that support strategic thinking and problem-solving, which are essential in analyzing and making decisions in everyday situations. Their daily interactions with storekeepers provide relevant experiences which make the concepts being taught familiar (Lerman, 2020). However, when social arithmetic appears as word problems in a mathematics class, students may find these problems challenging to interpret and solve. This gap between daily experiences and formal mathematical tasks highlights the need for teaching methods that connect practical knowledge with academic mathematical problem-solving (Dila & Zanthy, 2020).

Reports of the Programme for International Student Assessment show that in Indonesia, student abilities to solve and interpret contextual problems are still relatively low (OECD, 2018). Several studies have indicated that students experience conceptual and verbal difficulties in solving contextual problems (Haji et al., 2018), especially within the social arithmetic concept (Lestari et al., 2019). Students have conceptual difficulties understanding questions and transforming them into formulas (Khasanah & Utama, 2015). Polya (1957) posited that students need to be able to use the formulas they have learned in solving problems. Difficulties in understanding social arithmetic concepts are additionally reflected in the students' struggle with prerequisite aspects. For example, students fail to write down what is known from the question and what is asked, have difficulty converting informal information into mathematical models, and lack mastery of relevant concepts. Hence, students find it difficult to determine which formula to use (Khasanah & Utama, 2015; Fitri et al., 2019). Verbal challenges often manifest in students' misinterpretation and misunderstanding of questions, difficulty identifying question intent, and inability to rephrase the questions in their own words. Students frequently experience boredom with lengthy questions, struggle with interpreting their meaning, and tend to focus on memorization rather than conceptual understanding (Fitri et al., 2019). Additionally, students rarely complete problem-solving exercises, which contributes to frequent calculation errors. This is consistent with White's (2010) opinion that "the child identified an appropriate operation or sequence of operations but did not know the procedures necessary to perform these operations accurately." Errors in solving word problems in arithmetic are common, often resulting from student inability to correctly follow the procedures needed to execute operations (Dila & Zanthy, 2020; Zakiah, 2017).

To address the difficulties in solving social arithmetic problems, the teacher may provide learning content and context related to students' daily activities (Lestari et al., 2009; Khasanah & Utama, 2015; Fitri et al., 2019). This will introduce a greater degree of efficiency and fun into teaching the material. Mathematics learning should be designed so that students are provided with continued

enjoyment and do not feel uncomfortable or forced. Students need to have a willingness to learn. Therefore, the employed learning approach must encourage the desire to learn (Laurens et al., 2017). A learning approach that can address the problems posed by traditional and abstract mathematics learning is realistic mathematics education (RME) (Bray & Tangney, 2016). Freudenthal's idea (1971) that mathematics is part of human life constitutes the roots of RME. This approach to learning allows students to reinvent mathematics by processing real-life situations into mathematical knowledge (Freudenthal, 2002).

Learning trajectory is fundamental in implementing RME across various subjects. It comprises a sequence of activities and tasks designed to foster students' understanding and help them achieve specific instructional goals (Simon, 1995). Gravemeijer (2004) argued that neither teachers nor researchers can rely on a fixed teaching sequence, as teachers must continually adapt to students' progress and thought processes. As such, a learning trajectory begins with a hypothesis and is therefore termed a hypothetical learning trajectory (HLT) (Simon, 1995; Gravemeijer (2004); Gravemeijer & Cobb, 2013).

This study aims to investigate students' thinking within an HLT framework applied to social arithmetic. The research focuses on examining students' mathematical reasoning, as well as the steps and decisions made by the teacher during the implementation of student-centered activities. The study addresses the following research questions:

1. How can a learning trajectory enhance students' strategies and thought processes in learning social arithmetic?
2. How does the learning trajectory affect students' independence from teacher assistance, their adaptability in strategy use, and their ability to communicate mathematically while learning social arithmetic?

LITERATURE REVIEW

RME is a learning theory derived from the ideas of Freudenthal and his colleagues at the Freudenthal Institute in the Netherlands (Sumirattana et al., 2017; Gravemeijer & Cobb, 2006; Van den Heuvel-Panhuizen, 2000). The RME theory emerged with the reform of the mechanistic approach in mathematics learning that had previously been used in the Netherlands (Freudenthal, 2000). Freudenthal (2002) emphasized that mathematics should be connected to reality, reflect human activities, and be closely tied to students' experiences and societal needs. In this context, social arithmetic is vital as it bridges the gap between abstract mathematical concepts and real-world situations, such as calculating profit, loss, and percentages. The emphasis on making things imaginable by the mind means that RME does not always use real-life problems but can employ contextual problems related to social arithmetic that students can relate to and visualize. Well-chosen

contextual problems in social arithmetic offer opportunities for students to develop informal and contextual solution strategies that can support the construction of mathematical concepts. Mathematics education is organized as a guided rediscovery process, allowing students to experience mathematics similarly to how it was invented (Sumirattana et al., 2017; Gravemeijer, 2004; Gravemeijer & Cobb, 2006).

According to the theory of RME, the real world is a source or starting point for the development of mathematical concepts (Sumirattana et al., 2017). The term realistic in RME comes from the Dutch term “zich realiseren,” which means “to imagine.” This term emphasizes the importance of problems that can be imagined by students, particularly in the context of social arithmetic. The relevance of RME in teaching social arithmetic is supported by studies indicating that contextual problems, such as those involving household finances or budgeting, facilitate a deeper understanding of mathematical processes (Cárcamo Bahamonde et al., 2017). The RME approach fosters an environment where students can engage with problems that reflect their daily lives, enhancing both their motivation and their ability to apply mathematical reasoning in practical contexts. This underlies the naming of RME (Freudenthal, 2002).

In designing instruction under RME principles, Gravemeijer (2004) proposed three core ideas: guided reinvention, didactical phenomenology, and emergent models. In the guided reinvention principle, students should have the opportunity to experience a process similar to how mathematics was invented. To accomplish this, a developer or instructional designer must design a route to allow students to discover mathematics on their own, particularly in social arithmetic scenarios, such as calculating discounts or interest rates. To implement the didactical phenomenology principle, the developer must provide students with contextual problems derived from phenomena that are real and meaningful to them (Gravemeijer & Cobb, 2013). As a result of the real phenomena, students will have the opportunity to observe the process of horizontal and vertical mathematization, especially in the context of social arithmetic. The emergent model’s principle allows students to develop their own models when solving these contextual problems, beginning with informal models and gradually moving toward formal mathematical representations. The following characteristics must be present when the RME approach is used for teaching and learning: contextual problems (use context) and models (use models, bridged by vertical instruments); student contributions; interactivity; and intertwinement (Gravemeijer & Cobb, 2006).

HLTs provide a structured pathway for students to build mathematical understanding, aligning with the principles of RME. Simon (1995) defined HLTs as a set of assumptions about students’ mathematical thinking, which guide the design of learning tasks to foster progressively sophisticated reasoning (Cárcamo Bahamonde et al., 2017; Akdeniz & Argün, 2021; Callejo et al., 2022). HLTs are particularly relevant in the context of social arithmetic, in which teachers hypothesize about the sequence of students’ understanding as they engage with financial concepts and develop instructional experiences that support this progression (Callejo et al., 2022; Ivars et al., 2017; Wilson et al., 2014). An HLT includes three components: learning objectives, mathematical tasks (cognitive processes), and a hypothetical learning process that encompasses assumptions about

student understanding and reasoning that will develop during the learning process. For example, in social arithmetic, the learning trajectory may involve students starting with basic financial literacy concepts and gradually progressing to more complex calculations involving expenditure, income, profit, loss, and the method for calculating percentages (Daro et al., 2011). Teacher feedback is crucial in helping students achieve their learning goals and in ensuring that the learning trajectory remains aligned with their evolving understanding. In mathematics learning, two or more interrelated and interacting trajectories are aimed at achieving goals: (1) the teacher uses a hypothesized set of instructional experiences and tasks to “generate mental processes or actions” that develop or progress in the desired direction; and (2) students “think and learn... in a particular mathematical domain” through “developmental level progression,” which must lead to the desired goal if the choice of instructional experience is to be successful (Clements & Sarama, 2004; Simon & Tzur, 2004; Dickinson et al., 2020).

An HLT focuses on the process by which students rediscover mathematical concepts. This process is divided into several levels of models that appear during learning (Gravemeijer, 2020). The models are the manners in which students perform every activity observed in learning. In the context of social arithmetic, four levels of activity are involved in the teacher’s design of an HLT, namely situational, referential, general, and formal levels (Gravemeijer & van Eerde, 2009). The situational level refers to situational knowledge and strategies related to financial contexts, such as budgeting. The referential level demonstrates the financial situations described in the problems students encounter. The general level focuses on predominant strategies for solving mathematical problems related to finance, and the formal level requires reasoning with conventional symbolization to support mathematical activities in social arithmetic. Students go through these four levels of activity to solve financial problems with an informal approach and construct formal knowledge in stages (Andrews-Larson et al., 2017). Hans Freudenthal (2002) emphasized that in the RME approach, every HLT series must be connected to realistic mathematics. Realistic problems in social arithmetic can be integrated into contextual questions that can be divided into several different learning activities within HLTs (Dickinson et al., 2020; Van den Heuvel-Panhuizen & Drijvers, 2014).

Treffers (1987) distinguished two types of mathematization: horizontal and vertical. In the context of social arithmetic, horizontal mathematization involves students using mathematics to transform realistic financial problem situations into mathematical situations in the form of mathematical models, and vertical mathematization entails students working within the realm of symbolic mathematics through a process of model reorganization until problem solutions are found (Treffers, 2012; Deniz & Uygur-Kabael, 2017; Aniswita, 2024). Horizontal mathematization is related to the generalizing process: students identify regularities and relationships through visualization and schematization to develop mathematical concepts in the context of financial literacy (De Lange, 1987). By contrast, vertical mathematization formalizes these concepts, building upon the mathematical models obtained from horizontal mathematization to develop a more advanced mathematical understanding. There is no strict separation between the two processes, and they often occur

in tandem, allowing students to gradually develop their understanding of both mathematical concepts and their applications in social arithmetic (Deniz & Uygur-Kabael, 2017; Fauzan et al., 2018; Yilmaz, 2020).

METHODS

This study used a design research model introduced by Gravemeijer and Cobb (2013). The model has three stages: preparation for the experiment, experimenting in the classroom, and retrospective analysis. This activity began with a thought experiment (thinking about the learning trajectory that students would go through), followed by reflecting on the results of the trial experiment and further by the next thought experiment, as illustrated in Figure 1.

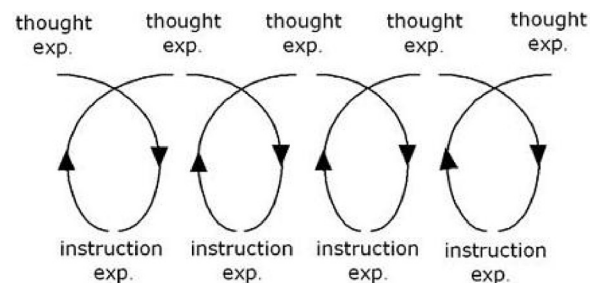


Figure 1: Gravemeijer & Cobb learning experiment (2013)

To prepare for the experiment, we established the end points of the instructions. The objectives of our social arithmetic classes were to help students rediscover concepts such as profit, loss, and the method for calculating percentages. Given that the students were already familiar with activities such as calculating sums of money and buying or selling items, we used these activities as a starting point for the lesson. Based on these starting and end points, we designed an HLT that consisted of three main activities involving solving contextual problems. This would facilitate students' horizontal and vertical mathematization while enhancing their conceptual thinking. A prototype HLT was developed (see Table 1).

Learning Activities	Social Arithmetic Concepts	Informal to Formal Mathematics Process
In the first activity, students solved a problem relating to buying goods and then selling them.	Capital (expenditures) and results (income)	Contextual understanding: Students familiarize themselves with real-world contexts by role-playing as sellers and buyers, which allows them to recognize how goods are bought and sold.

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Learning Activities	Social Arithmetic Concepts	Informal to Formal Mathematics Process
In the second activity, students solved a different problem relating to buying and selling goods.	Profit and loss	<p>Initial calculations: Students begin by calculating and organizing the costs of the items purchased. They estimate prices and consider which items might sell well, building a foundation for expenses and budgeting.</p> <p>Defining income and expenditures: They record the costs and revenue from sales, differentiating between money spent (capital) and money earned (income).</p> <p>Calculating profit and loss: They compare total income with initial expenditure to determine if they gained (profit) or lost (loss) money.</p> <p>Introduction to profit and loss terms: The terms “profit,” “loss,” “selling price,” and “cost price” are introduced. Students learn to define these terms in relation to their activities.</p>
In the third activity, the students created financial reports.	Selling price, purchase price, and the percentages of profit and loss	<p>Formulating financial statements: Students consolidate their earnings and expenses in the form of a simple financial report, transitioning from informal calculations to structured reporting.</p> <p>Using symbols and equations: Students start using mathematical symbols and equations to represent the selling price (HJ), purchase price (HB), and percentage profit ($U\% = \frac{U}{HB} \times 100$) or percentage loss ($R\% = \frac{R}{HB} \times 100$)</p> <p>Formal calculation of profit and loss: They use formulas to determine exact values for profit and loss percentages, applying these concepts to real-life data from their role-play.</p> <p>Reflection and analysis: Students analyze the cooperative’s financial health based on their reports, which reinforces the importance of accurate calculations and leads to a formal understanding of profit, loss, and financial management.</p>

Table 1: Learning trajectory of social arithmetic concepts

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In addition, we designed the instructor's role in supporting students' mathematical thinking development through a sequence of tasks. Three seventh graders from junior high school in Indonesia participated in the experimental phase in small groups. We selected students based on their ability levels by grouping them into three categories: high, medium, and low. This grouping was based on the exam scores obtained from the school, as well as recommendations from their teachers. Additionally, the selection also considered gender differences to observe cognitive variations among the students. Therefore, two female students and one male student were chosen as participants in the study. This experiment aimed to observe and gain an understanding of how learning trajectories worked in shaping students' thinking from presumptive responses. In this experiment, one researcher acted as a teacher and another as an observer of the learning. Research data were collected through the triangulation technique. In the implementation process, all classroom activities, such as actions and conversations between students and teachers, were voice recorded, and students' answer sheets were collected and documented. The researchers' findings and impressions of the learning were discussed and documented immediately after the learning as additional supporting data to clarify the learning. Besides developing an HLT, the researchers conducted observations and analyses of the impact of the HLT on students' confidence when using their own strategies to solve context-related problems. Following this line of activities, student reasoning skills were examined in a pilot program.

A retrospective analysis was conducted to collect and identify student-generated data that could shape student understanding of social arithmetic. This analysis was formulated based on several questions: what learning activities were performed to generate conjectures from student responses in the HLT? How did this work? Why did it work or not work? To obtain answers to these questions, all the collected experimental data were comprehensively and critically analyzed. We reviewed the students' answer sheets according to the answer key. Next, we transcribed the interviews conducted during the learning process. This aimed to garner an understanding and explanation of the "what," "how," and "why" of the role of HLT in shaping student cognitive development. To answer the first question ("what"), we compared students' answers and responses generated during learning with the predicted student responses contained in each set of HLTs. After that, the information was examined to answer the second question ("how"). Finally, to answer the third question ("why"), we presented or discussed the findings and related them to relevant theories to gain a better understanding of the phenomenon.

RESULTS

The results of this study are presented here with a focus on the four components of an HLT, namely learning objectives regarding conceptual understanding, sequence of tasks (contextual questions),

evolution/development of student's conceptual understanding, and the instructor's role in supporting the development of students' mathematical thinking throughout the task sequence. The developed HLT includes three learning objectives: (1) discovering the concept of capital (expenditure) and results (income); (2) finding the concept of profit, loss, and break-even; and (3) conceptualizing the percentage of profit and loss from informal knowledge to formal knowledge.

Discovering the Concepts of Capital, Income, Profit, Loss, and Break-Even

The purpose of the first activity was to have the students discover the concepts of profit, loss, and break-even. This activity provided opportunities for students to analyze what a school cooperative officer experiences in their sales activities. A school cooperative officer is typically a student or staff member responsible for managing and overseeing the activities of a school cooperative, which is a student-run business or organization that provides goods or services to the school community. The contextual problem posed through this activity was as follows:

Bu Ani is a cooperative officer at school. Every day, buying and selling activities occur with school residents. Bu Ani sells various types of fried foods, including bakwan, stuffed tofu, and rissoles. To cook this food, Bu Ani needs to buy some ingredients. The ingredients needed by Bu Ani include 2 liters of oil, which is priced at Rp 30,000, vegetables, priced at Rp 40,000, 3 kg of flour, priced at Rp 36,000, and 15 eggs, priced at Rp 25,000. Bu Ani also needs to buy plastic packaging for Rp 10,000 and pay a cook's wage at Rp 50,000. Bu Ani will spend some money to buy these items.

In this activity, students were asked four questions: The first task was (a) *Help Bu Ani calculate the capital needed to buy all these items!* To this, students provided various answers. In the following section, a student's answer is described.

Student with Medium Ability (MA)

Only one student successfully provided the correct answer immediately. The other two students could only answer after the teacher asked several questions to prompt them to think about the correct answers. The researcher directed students to discover the concept of capital by asking contextual questions and directed them back to the expected flow. In this guided reinvention process, students had the opportunity to construct their own mathematical knowledge. The following is an excerpt of the teacher's questions and the student's answers:

Teacher : Why is it called capital? What are expenses?

Student MA : All items purchased, totaling Rp 241,000.

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- Teacher : Look again, have these items been calculated correctly?
 Student MA : Hmmm, the salary wage of a cook should still be Rp 50,000.
 Teacher : Why did you get Rp 300,000?
 Student MA : Because I thought all capital was for one week. It turns out that all capital was for one day.
 Teacher : How much capital should Bu Ani have?
 Students MA: Rp 191,000, sir.

In this dialogue, the teacher's probing questions revealed the student's thought process and understanding of social arithmetic concepts. Student MA initially showed a basic understanding of calculations by stating total expenses but struggled to distinguish between daily and weekly capital. The teacher's prompts encouraged Student MA to reevaluate their calculations, demonstrating a developing independence from teacher assistance. This interaction highlights the student's flexibility in strategy use, as they adjusted their reasoning upon realizing their misunderstanding of capital. Overall, the dialogue reflects Student MA's ability to communicate mathematically, although their errors indicate areas for further development in understanding and applying mathematical concepts related to capital and expenses. The subsequent answer can be observed in Figure 2.

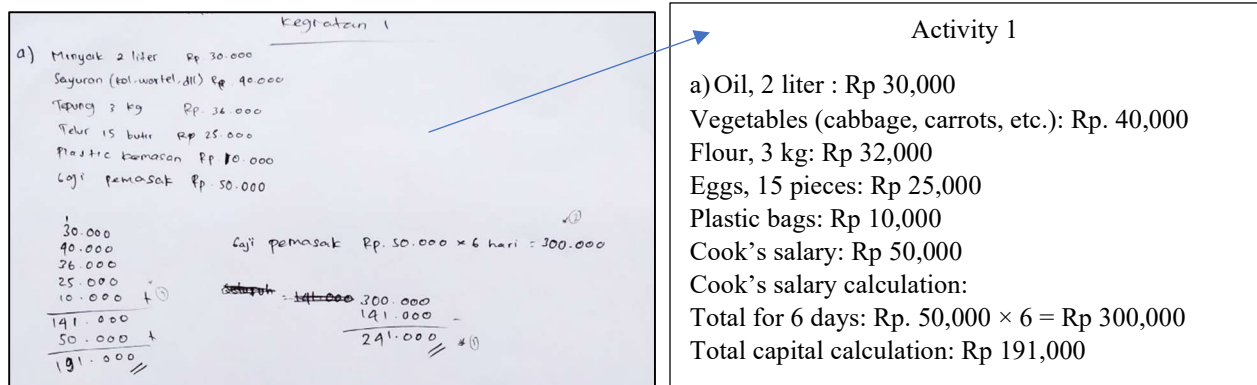


Figure 2: Student MA's answer to Question (a) in Activity 1

Based on the answers of the three students (see appendix), it is concluded that Activity 1 part (a) could lead students to discover the concept of capital (expenses). This laid a foundation for solving the following problem, which was Question (b) *If Bu Ani succeeds in selling out her fried foods and earns Rp 250,000 on Monday, what will she experience? State your opinion!* Figure 3 shows the answer from one of the students.

<p>b) Penjualan hari Senin Rp. 250.000. maka bu ani akan mendapat untung : 59.000 //</p> $\begin{array}{r} 250.000 \\ - 191.000 \\ \hline 59.000 \end{array}$	<p>Activity 1</p> <p>b) Monday's sales: Rp 250,000. Therefore, Mrs. Ani will get a profit of Rp 59,000.</p>
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Figure 3: Student MA's answer to Question (b) in Activity 1

The students' answers indicated that only one student had difficulty answering Question (b). After clarification and re-inquiring regarding Bu Ani's capital and income, the student MA explained the concepts of income and profit. The student MA was asked the following questions to explore their understanding:

Teacher : How did Bu Ani make a profit?

Student MA: Because the principal capital is Rp 191,000. Then, for example, she has fried foods, they sell well, and she earns a profit.

Teacher : Then how much should Bu Ani earn to experience a profit?

Student MA: Because the initial capital is Rp 191,000 and the income is more than that at Rp 250,000, then the profit is Rp 59,000.

In this dialogue, Student MA showed an emerging thought process in calculating profit. Student MA identified the initial capital of Rp 191,000 and recognized the successful sale of fried foods. When prompted by the teacher, the student accurately calculated the profit (Rp 59,000) by subtracting the initial capital from the income (Rp 250,000). This demonstrated Student MA's growing independence from teacher assistance and ability to apply mathematical concepts in real-world contexts. Additionally, Student MA effectively communicated their reasoning, reflecting flexibility in their strategy use, which aligned with the research question on student thought processes.

Next, students worked on Activity 1 part (c), with the following question: *If it rains heavily on Wednesday, and Bu Ani earns a total income of Rp 150,000 from selling her fried foods, what will Bu Ani experience on that day? Explain your opinion!* Figure 4 shows the answer from Student MA.

c.) maka ~~buani~~ buani mendapat untung 59.000, kerugian: 100.000

$\begin{array}{r} 191.000 \\ - 132.000 \\ \hline 59.000 \end{array}$	$\begin{array}{r} 250.000 \\ - 150.000 \\ \hline 100.000 \end{array}$	$\begin{array}{r} 191.000 \\ - 150.000 \\ \hline 41.000 \end{array}$
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c) Therefore, Mrs. Ani earns a profit of Rp. 59,000, or a loss of Rp 100,000.
Loss calculation:

- Income: Rp 191,000
- Total expenses: Rp 150,000
- Loss: Rp 41,000

Figure 4: Student MA's answer to Question (c) in Activity 1

- Teacher : What happens to Bu Ani on Wednesday?
- Student (MA): Bu Ani does not sell as much as on Monday.
- Teacher : (An income of) Rp 240,000 (means she) does not sell as much as on Monday, how about that?
- Student (MA): A loss of Rp 100,000.
- Teacher : Look at it again. How can it be called a loss? What is the benchmark we use to say that we incur a loss or earn a gain today?
- Student (MA): Previous sales.
- Teacher : Look again at the previous answer. Is the previous sales the correct answer?
- Student (MA): Hmmm, capital, sir.
- Teacher : Our benchmark is always capital, right? Not previous sales. So, what will happen to Bu Ani then?
- Student (MA): It is a loss about Rp 41,000, sir.
- Teacher : Why does Bu Ani suffer a loss?
- Student (MA): Because the sales are less than the capital.

There was little doubt in Student MA's answer when the researcher attempted to clarify how the student obtained the answer. Student MA answered correctly, but they were unsure of what loss means. Upon direction from the researcher, Student MA gained an accurate understanding of the concept of loss. This exchange demonstrated Student MA's flexibility in adapting strategies when provided guidance as they shift from an initial, incorrect assumption to a correct understanding. The dialogue highlights Student MA's emerging independence and ability to communicate mathematically, as they corrected their reasoning and articulated that a loss occurs when sales are lower than the capital, aligning with the research focus on independence, strategy flexibility, and mathematical communication. As stated by Fauzan et al. (2018) and Möhring et al. (2021), a teacher's anticipation can help students process their knowledge well. Based on the answers of the three students (see appendix), it is concluded that Activity 1 part (c) could direct students to find the concept of loss.

Next, students worked on Activity 1 part (d), with the following question: *If on Saturday Bu Ani manages to sell fried foods with an income of Rp 191,000, what will Bu Ani experience on that day? State your opinion!* Figure 5 is the answer from the student MA.

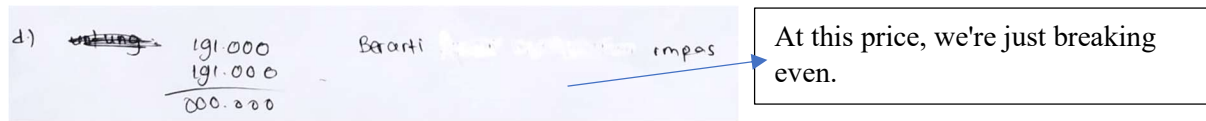


Figure 5: Student MA's answer to Question (d) in Activity 1

Teacher : What does Bu Ani experience?

Student (MA): Hmm, break-even, sir.

Teacher : Why does Bu Ani experience a return on investment or break-even?

Student (MA): Because the sales that day are valued at Rp 191,000, and the capital is also Rp 191,000.

The excerpt above shows that the student could answer correctly that Bu Ani experiences a break-even (return on investment). In this dialogue, the teacher clarified Student MA's answer regarding why they stated break-even. Then, the student explained that break-even occurs when the sales on that day match the capital spent. This shows the student's independence in thinking, as they were able to explain their reasoning without further prompting from the teacher. This exchange illustrates how the learning trajectory supports the development of students' thought processes, enhancing their ability to communicate mathematical concepts accurately.

A second activity with a more complex problem (consisting of contextual questions) was performed to retest students' understanding. This activity allowed students to determine what Bu Ani experiences as a school cooperative officer when she sells various gifts on National Teacher's Day. The contextual problem posed in this activity was as follows:

Besides food and stationery, Bu Ani intends to also sell gifts to commemorate National Teacher's Day, including flowers, chocolates, and brownies. As such, Bu Ani goes shopping for these items at once; the details are as follows:

15 stalks of red roses, bought for Rp 120,000;

10 stalks of white roses, bought for Rp 90,000;

25 pieces of chocolate, bought for Rp 100,000;

25 pieces of brownies, bought for Rp 50,000;

Plastic wrapping and gift tape, bought for Rp 30,000.

With these items, it turns out that 25 ready-to-sell gift parcels can be produced, which are sold for Rp 18,000 per unit.

In this activity, students were presented with the first question (Question (a)), which asked, *Help Bu Ani determine what she will experience if 15 parcels are sold. Explain your answer!* The students exhibited unique characteristics in their responses, which were often unsystematic and appeared as rough scribbles. The researcher analyzed the students' thought processes by examining their answers and inquiring about their reasoning. Studies suggest that a teacher's readiness to ask probing questions can significantly enhance students' cognitive development (Sari et al., 2024; Gravemeijer & Cobb, 2006; Polya, 1957). The following questions and examples offer guidance for teachers to support students in processing information effectively (Paredes et al., 2020; Andrews-Larson et al., 2017).

Student with Low Ability (LA)

Student LA had misconceptions about the concept of loss. They mistakenly believed that subtracting the number of parcels sold from the total number of parcels available for sale would result in a loss (as shown in Figure 6). The teacher posed the following anticipatory questions to guide the students through a process of mathematization:

Teacher : Why does Bu Ani suffer a loss?

Student LA : Because she produces 25 parcels, but only 15 are sold.

Teacher : Look again! Is it right to consider that Bu suffers a loss by counting the number of parcels sold?

Student LA : Yes, sir.

Teacher : Let me give you an example, if you buy apples for Rp 2,000 per apple and you have three apples at hand, how much money did you spend on buying the apples?

Student LA : Rp 2,000 multiplied by 3, Rp 6,000, sir.

Teacher : Then you sold (the apples) for Rp 4,000 per apple. It turned out that you only sold two of apples and you ate the rest. What did you experience?

Student LA : It is a loss, sir.

Teacher : Look again! How much money did you spend earlier?

Student LA : Rp 6,000, sir.

Teacher : How much did you earn from your sales?

Student LA : Rp 8,000, sir.

Teacher : What did you experience?

Student LA : Earning a profit, sir.

Teacher : You made a profit, even though you didn't sell all of the apples. So, what should we pay attention to in answering the questions?

Student LA : Sales and capital.

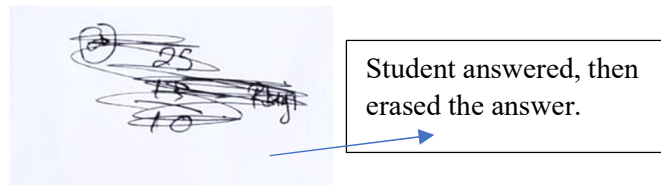


Figure 6: Initial Student LA's answers to Question (a) in Activity 2

In this dialogue, the teacher helped Student LA understand the concepts of profit and loss through a practical example. Initially, Student LA thought loss was determined by the number of items sold. However, guided by the teacher's questions, they realized that profit was based on total costs and revenues, reflecting a deeper understanding of financial concepts. Based on Figure 7, Student LA miscalculated the capital. Therefore, several questions were asked to direct Student LA to the correct answer, as shown in the excerpt below.

Teacher : Look again. Was the capital correctly calculated?

Student LA : Rp 290,000 (is) correct, sir.

Teacher : Be more careful. What is the result again?

Student LA : Rp 390,000, sir.

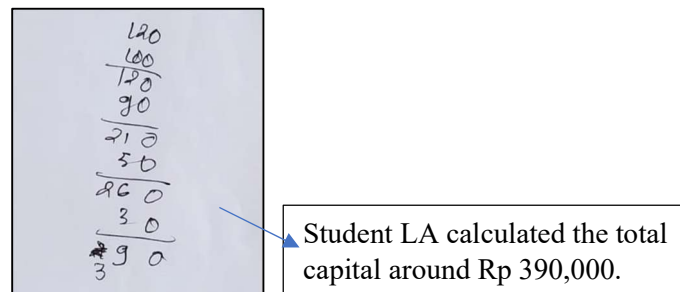


Figure 7: Student LA's answer to Question (a) in Activity 2

Although Student LA correctly identified the necessary capital, their answer lacked systematic organization, and they miscalculated the capital (as shown in Figure 7). Consequently, several questions were posed to guide Student LA to the correct answer. Student LA was then asked to contemplate the answer to the question: *What she (Bu Ani) would experience if 15 parcels were sold? Explain your answer!* Figure 8 shows a continuation of the student's answer.

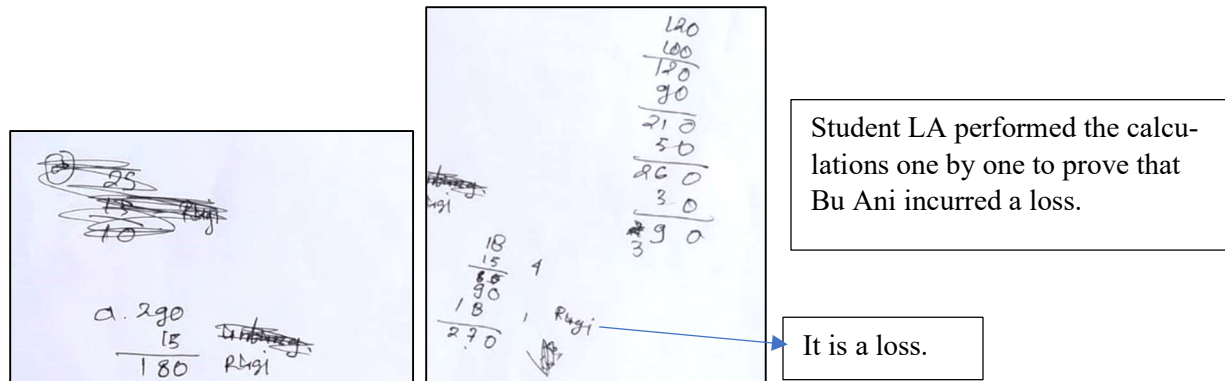


Figure 8: Student LA's final answers to Question (a) in Activity 2

Based on their answer shown in Figure 8, Student LA concluded that Bu Ani would experience a loss. However, in arriving at this conclusion, Student LA made a mistake: they did not compare the capital and income, but the capital and number of parcels sold. Additionally, Student LA still used the incorrect amount of capital, namely Rp 290,000. Therefore, to direct the student to identify the concept of loss, the following anticipatory questions were asked:

Teacher : Bu Ani only sells 15 parcels. How much will Bu Ani earn?

Student LA : Hmm.

Teacher : Pay attention to the problem. How much does one parcel cost?

Student LA : Rp 18,000, sir.

Teacher : Then, how about 15 parcels?

Student LA : Hmm, Rp 60,000, sir.

Teacher : Look again. Is Rp 60,000 correct?

Student LA : Yes, sir. Rp 18,000 multiplied by 15. Oh my God, it is Rp 270,000, sir.

Teacher : That's right. What happens to Bu Ani then?

Student LA : It is a loss, sir.

Teacher : Why does Bu Ani suffer a loss?

Student LA : The capital is more significant, sir. Rp 390,000, sir.

Despite some mistakes in the calculation, with the help of anticipatory questions, the teacher managed to direct Student LA to understand the concept of loss. After being asked anticipatory questions, the student could understand the concept of loss by comparing capital and income. Although students successfully discovered the concept of loss with the teacher's guidance through anticipatory questions, they also significantly contributed to their learning process, allowing them to

achieve the expected goals. This means that all student thinking processes (construction and production) could function adequately (Araújo & Lima, 2020; Amala & Ekawati, 2020; Jupri et al., 2021).

Next, students worked on Activity 2 part (b) with the question: *What is the minimum number of parcels that must be sold so that Bu Ani does not lose money?* This question was asked so that students could better understand the concepts of profit and loss, which are necessary for solving more complicated problems. Figure 9 depicts some answers from Student LA.

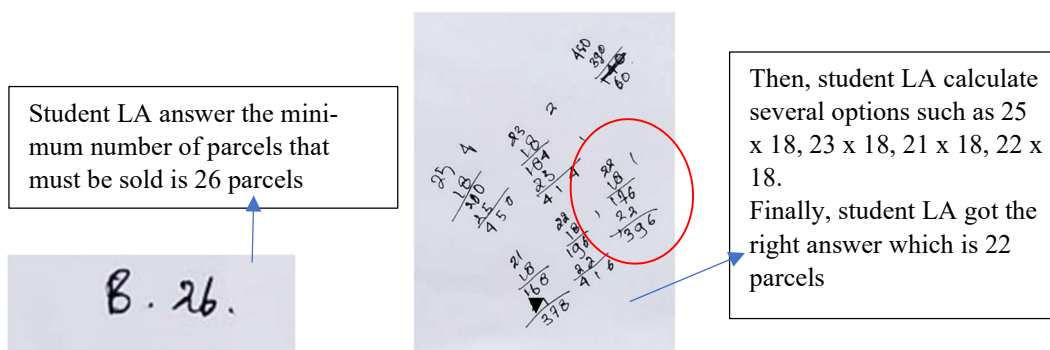


Figure 9: Student LA's answers to Question (b) in Activity 2

Based on these answers shown in Figure 9, the student LA initially answered that Bu Ani would not lose money if she sold 26 parcels. Since the student needed to understand the concepts of profit and loss through Question (b), the following anticipatory guiding questions were posed:

Teacher : Take a look. How many parcels does Bu Ani have?

Student LA : There are 25 parcels.

Teacher : What is the minimum number (that must be sold) so that Bu Ani doesn't suffer a loss?

Student LA : 26, sir.

Teacher : Why 26?

Student LA : The capital amounts to Rp 390,000. If (the parcels) are sold (in that number, the income) will be Rp 468,000. This way Bu Ani will earn a profit.

Teacher : Take a look. What is asked in the question?

Student LA : The minimum number of parcels (that must be sold) so that Bu Ani will not suffer a loss.

Teacher : What is meant by minimum?

Student LA : The least number.

Teacher : If Bu Ani has 23 parcels, what will she experience?

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Student LA : It is a loss, sir.

Teacher : Why does Bu Ani suffer a loss? Take a look. If 25 parcels are worth Rp 450,000.
What will 23 parcels be worth?

Student LA : Rp 414,000, sir.

Teacher : Then, what will Bu Ani experience?

Student LA : Earning a profit, sir.

Teacher : How can we know if Bu Ani suffers a loss? For example, if there are 24
packages or 21 gifts, can we know the profit or loss?

Student LA : I don't know, sir. You have to try one at a time, sir.

Teacher : After doing so, how many parcels should Bu Ani sell in order not to
experience a loss did you find?

Student LA : 22 parcels, sir.

Teacher : Why does it have to be 22 parcels?

Student LA : Rp 396,000 (is the worth of) 22 parcels than the capital (it is higher than the capital), which is Rp 390,000.

In this dialogue, Student LA showed a developing understanding of profit and loss concepts in social arithmetic. Initially, they confidently stated that 26 parcels must be sold to avoid a loss, indicating a basic grasp of the problem. However, they struggled to explain the outcome when asked about selling 23 parcels, revealing gaps in their understanding. This exchange highlights the importance of teacher guidance in promoting student independence and flexibility in strategy use. Although Student LA ultimately correctly concluded the number of parcels needed to avoid a loss, the dialogue underscores the need for ongoing support to enhance their reasoning skills.

With directions and anticipatory questions, the student could finally discover the concepts of profit, loss, and break-even. However, the student could only do so by multiplying a certain number of parcels by the unit price of the parcels, and if the answer was wrong, they would repeat the process with a different number of parcels until they found the correct answer. They did not have a faster method to find an answer to Question (b). An alternative way to answer this is to divide the capital by the unit price of the parcel to determine the number of parcels that Bu Ani must sell to not lose money. Nevertheless, students were able to answer the questions by constructing their own steps and thoughts. In a realistic approach to mathematics, students may take a variety of pathways to solve a problem according to their own thinking abilities. Therefore, there is no wrong answer because the most important thing in this approach is the ability of students to construct nonformal knowledge into formal knowledge, which will allow them to solve contextual problems (Treffers,

2012; Aniswita et al., 2024; Zubainur et al., 2020). Based on the answers of the three students (see appendix), it is concluded that Activity 2 part (b) could direct students to find the concept of profit.

Then, students worked on Activity 2 part (c) with the question: *What is the maximum profit that can be earned?* In answering this question, students encountered some difficulties. First, a student answered that the maximum profit was Rp 400,000, as shown in Figure 10.

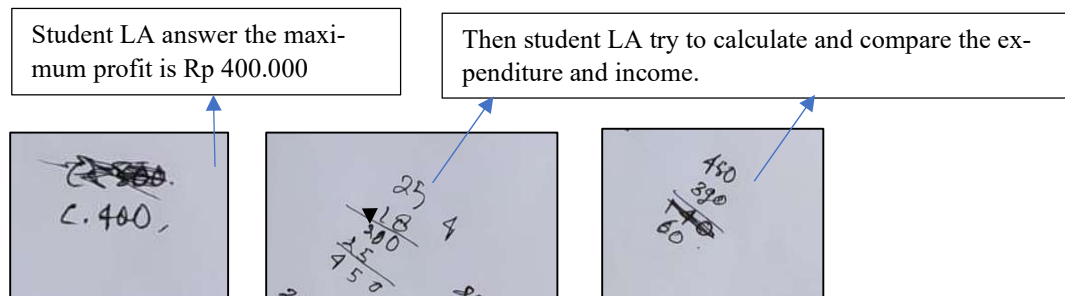


Figure 10: Student LA's answers to Question (c) in Activity 2

The teacher guided the student to the correct answer to Question (c) by presenting the following questions:

Teacher : How can Bu Ani get the maximum profit? What is meant by maximum?

Student LA : (It is) the most. All (items should be) sold.

Teacher : If all items should be sold, how many of them are there? What's the profit?

Student LA : There are 25 of them, sir. So, (the profit is) Rp 450,000.

Teacher : Is it correct to call Rp 450,000 a profit?

Student LA : Hmmm, (It is the value of the) sales, sir.

Teacher : If you want to make a profit, how do you do it? Try to remember activity (b) earlier. What was it? Profit or loss?

Student LA : Profit.

Teacher : Why was it called profit?

Student LA : Because the sales were more significant than the capital.

Teacher : How much profit does Bu Ani earn in Question (b)?

Student LA : The profit is Rp 6,000, sir.

Teacher : What is the maximum profit for sales? Take another look. How many parcels does Bu Ani sell?

Student LA : There are 25 of them, sir. So, it is Rp 450,000.

Teacher : Then, how much is Bu Ani's capital?

Student LA : Rp 390,000, sir.

Teacher : What can you conclude from this?

Student LA : Bu Ani earns a profit of Rp 60,000, sir.

The dialogue illustrates Student LA's cognitive process as they engaged with the concept of profit in a practical context. Initially, the student confused sales with profit, indicating a need for deeper understanding and independence from teacher assistance. However, as the conversation progressed, Student LA demonstrated improved flexibility in strategy use and mathematical communication, ultimately arriving at the correct conclusion about Bu Ani's profit; this reflects their developing cognitive skills in social arithmetic. However, Student LA did not write answers systematically using symbols and visual interpretations. This implies that although Student LA understood the concepts, their ability to achieve formal mathematical communication remained inadequate. The teacher should emphasize from the outset the importance of using symbols and clear interpretations when writing answers, especially for students with lower skill levels. Nevertheless, we confirmed that Student LA achieved a good understanding of the concepts of profit and loss, as demonstrated through the interview.

After being directed through anticipatory questions, students could find the concepts of profit, loss, and break-even. Students could also discover the concepts of maximum and minimum profit. Activities 1 and 2 involved RME characteristics. The use of context in problems made it easy for the students to imagine the problems. According to Freudenthal (2002), *realistic* in the RME approach means "easy to imagine," not only in direct relation to the real world but instead to the use of problems that students can imagine in the learning process.

Conceptualizing Percentages of Profit and Loss

The purpose of the third activity was for the students to discover the concept of the percentages of profit and loss and transform it from informal to formal knowledge. In this activity, students determined the percentage of profit earned by Bu Ani as a school cooperative officer. The contextual problem posed in this activity is as follows:

Bu Ani makes a school cooperative sales report every week. The report consists of the number of items sold, the percentage of profit, and the percentage of loss. Every week, Bu Ani buys goods with a total purchase of Rp 2,000,000 on average. Bu Ani wants to find the percentages of profit and loss that she experienced in the first week of June.

(a) If this week's total sales were worth Rp 2,600,000, what percentage of profit did the school cooperative earn?

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(b) Based on the answer to Question (a), if Bu Ani made a profit with a purchase price HB, what is the profit percentage (P%)?

(c) Based on the answer to Question (b), if Bu Ani incurred a loss with a purchase price HB, what is the loss percentage (L%)?

In this activity, students were asked the first question in part (a): *If this week's total sales were worth Rp 2,600,000, what percentage of profit did the school cooperative earn?* Students were expected to find it easier to solve this problem because they already gained an understanding of the concepts of profit and loss through the previous activities. They answered correctly right away, as shown in appendix.

Student with High Ability (HA)

As shown in Figure 11, Student HA could easily find the profit percentage. Therefore, the teacher attempted to clarify the student's reasoning and learn about the student's thought processes. This was useful so that the process of mathematization performed by the student could be explored more deeply. Several questions below were asked to elicit the student's reasons for the answer.

Teacher : How much profit did Bu Ani get?

Student HA : Rp 600,000, sir.

Teacher : How could Bu Ani get this profit?

Student HA : Bu Ani's sales proceeds were Rp 2,600,000, and her capital was Rp 2,000,000.

If it (the capital) is deducted (from the proceeds), she earned (a profit of)

Rp 600,000.

Teacher : How to find the profit percentage?

Student HA : The profit is divided by the capital, sir, and then multiplied by 100%.

In this dialogue, Student HA demonstrated a clear understanding of the concepts related to profit calculation and percentage, indicating a strong mathematical thought process. Student HA accurately calculated the profit by deducting the capital from the sales proceeds, showcasing their ability to apply relevant mathematical operations independently and reflecting their level of independence from teacher assistance. Additionally, Student HA showed flexibility in strategy use by moving from a straightforward profit calculation to the concept of profit percentage, which requires a more advanced understanding of ratios and percentages. This ability to communicate mathematically is evident in the student's clear explanations, which not only provided the answers but also articulated the underlying processes. Overall, this dialogue highlights how the learning trajectory

has positively influenced the student's independence, strategy flexibility, and communication skills in social arithmetic.

Kegiatan 3

a). 600.000 karena bukt Ani mendapatkan rata² penjualan 2.000.000

b). $\frac{600.000}{2.000.000} \times 100\%$ $\frac{600.000}{2.000.000} \times 100\%$

$\frac{60}{2} \% = 30\%$

Activity 3

a) The profit is Rp 600,000 because Mrs. Ani get her capital Rp 2,000,000

$$U\% = \frac{600000}{2000000} \times 100\% = 30\%$$

Figure 11: Student HA's answers to Question (a) in Activity 3

It also can be seen from the answers to Question (a) that Student HA could find the concept of profit percentage. Therefore, it is concluded that Activity 3 part (a) could lead students to discover the concept of profit percentages. Furthermore, Student HA was directed to establish formal knowledge in Questions (b) and (c). They were given anticipatory questions such as, "Is dividing the purchase price by the profit the correct way? What should we compare?" and the question, "What is known in the problem? Try to relate it back to the previous answer!" Through these questions, student HA could establish formal knowledge from the informal concepts found in Question (a), as shown in Figure 12.

b). $\frac{U}{HB} \times 100\%$

c). $\frac{R}{HB} \times 100\%$

Activity 3

b) $U\% = \frac{U}{HB} \times 100\%$ c) $R\% = \frac{R}{HB} \times 100\%$

U = profit, R = Loss, HB = Purchase price

Figure 12: Student HA's answer to Questions (b) and (c) in Activity 3

DISCUSSION

Fostering Students' Thought Processes, Independence Strategies, and Mathematical Skills Through a Learning Trajectory

The obtained results suggest that students with high, medium, and low abilities had different strategies and thought processes (ways of thinking) in solving problems in each activity. The way of

thinking had already been established when the students started reading the assigned social arithmetic contextual problems; gradually, the students identified the problems. They began to develop models, formulas, or patterns for solving the problems, which led them to think formally. The answers provided in this article to questions in Activity 1 through Activity 3 show that Student HA could apply their mathematical abilities and explain the reasons for their answers well. Student MA applied a strategy that was not much different from that applied by the student with HA in Activity 1. However, Student MA took more time to answer the questions than Student HA. Student MA needed guidance from the teacher in completing procedures and applying concepts, facts, and arguments. By contrast, Student LA applied a strategy that was different from those applied by the other two students. The LA student tended to answer without communicating the answer mathematically. Student LA could understand the concepts of profit and loss well, but they needed guidance from the teacher to communicate these concepts. Overall, the activities provided here allowed students to use their own strategies in solving the various contextual problems given. This aligns with Aniswita's research (2024) which indicates that students with low abilities tend to require longer to solve problems and often respond with words and scribbles. Meanwhile, students with medium and high abilities tend to draw conclusions more quickly and use symbols.

De Lange (1987) referred to this as a “student’s free production” or “self-developed or emergent model” (Gravemeijer, 2020). Students were provided with the freedom to solve contextual problems so that they could build their confidence in learning mathematics (Fauzan et al., 2018). The results also illustrate that RME can be a reference for teachers to construct students’ problem-solving skills through the context of everyday problems. As explained by Yilmaz (2020), teachers must be trained in designing contextual problems based on RME. During the learning process, teachers are required to develop their creativity and critical thinking. The teacher’s creativity will encourage them to develop HLTs with the RME approach, which encompasses cognitive domain, authenticity, and openness (Paredes et al., 2020). In testing an HLT, teachers must avoid directly supplying the answers to students. They must provide questions for each student’s answer that diverges from the learning objectives. This was also the case in the implementation of the HLT developed in this study, as can be seen from the various questions asked to clarify some student answers (Möhring et al., 2021; Zubainur et al., 2020).

This learning trajectory helped students explore their ability to think mathematically by solving contextual problems about profit, loss, break-even, and percentages (Amala & Ekawati, 2020; Fauzan et al., 2018). The mathematization process occurred in every learning activity performed by the students (Widada et al., 2020; Fauzan et al., 2022). The first activity involved students acting as cooperative officers, who bought various items to be sold. Next, the students took turns acting as sellers and engaged in buying and selling transactions with other students in the school cooperative cafeteria. In the third activity, students played the role of the head of the school coop-

erative, preparing financial reports to evaluate the profit and loss experienced by the school cooperative. From the implementation of the social arithmetic learning trajectory, it was found that the LA student had limitations in developing their mathematical communication skills. This can be seen from all figures depicting answers to questions in Activity 2. The student was unique in their way of thinking, and they needed more intensive guidance from the teacher (in the form of probing questions) to solve problems. This indicates that the learning trajectory helped the student with low ability identify how to solve problems and discover social arithmetic concepts through in-depth interviews. However, the learning trajectory was not perfect for improving the communication and critical thinking skills of the student with low ability. Observations revealed that the teacher did not remind the students to write answers systematically using symbols and visual interpretation and did not tell them how to draw conclusions. The teacher only posed questions to encourage the students to find the concept of social arithmetic on their own.

Nirawati (2021) introduced a way of thinking about HLT testing with students using the RME approach. The way of thinking can take the form of various visual and symbolic interpretations, how students explain the approaches or strategies that they use in problem-solving, and how they draw conclusions about a phenomenon. The more varied the students' thinking patterns or ways of thinking, the more developed the students' mathematical abilities. This is evident from the answers of the students with high and medium abilities who were able to develop their problem-solving skills and mathematical communication skills during the learning process. Some errors were still found when the students were solving contextual problems. These errors included counting errors and errors in applying procedures. Nevertheless, these students managed to construct their own ways of thinking to achieve the learning objectives of the HLT.

The implementation of a learning trajectory, especially through HLT within the RME framework, plays a significant role in promoting student independence from teacher assistance. By guiding students to explore and interpret contexts autonomously, they develop a more self-reliant approach to problem-solving. This independence fosters adaptability in strategy use, as students are encouraged to experiment with and select the strategies that best suit their understanding of a problem (Sahinkaya et al., 2024; Jupri et al., 2021; Fauzan et al., 2022). Consequently, this exploration of diverse strategies enhances their ability to communicate mathematical ideas clearly and effectively. When students are encouraged to articulate their way of thinking, both in explaining their strategies and in drawing conclusions, they reinforce their understanding of mathematical concepts and improve their mathematical communication skills (Nirawati et al., 2021). In social arithmetic learning, such autonomy and adaptability allow students to connect mathematical ideas to real-world contexts; this deepens their conceptual understanding and equips them with essential skills for communicating their solutions and reasoning comprehensively. Moreover, it is essential for both teachers and students to engage more thoroughly in checking activities to solidify student understanding of the mathematics learning process, ensuring that misconceptions are addressed

and that knowledge is effectively consolidated (Nusantara et al., 2023; Samporno & Meiliasari, 2019).

The activities in this research were designed to involve horizontal and vertical mathematization. As stated by Van den Heuvel-Panhuizen and Drijvers (2014), Jupri et al. (2021), and Widada et al. (2020), the mathematical process forms the basis of the HLT design that implements four levels of activity: situational, referential, general, and formal. The HLT implementation applied several RME key principles, namely guided rediscovery, didactic phenomenology, and self-developed models. In addition, the learning trajectory met the characteristics of RME, namely (1) use of context, (2) use of models for progressive mathematization, (3) use of students' construction results, (4) interactivity, and (5) linkage (Astuti, 2019; Gravemeijer, 2020). In the learning process, interactions between student and teacher and between student and student, such as negotiations, explanations, justifications, agreements, questions, and reflections, are used to achieve formal mathematical knowledge derived from informal mathematical knowledge based on the students' discoveries. In a realistic approach to mathematics, this is called interactivity. By designing an HLT, the teacher can develop a teaching and learning process that promotes students' creative thinking skills and ways of thinking and facilitates students' ability to construct new knowledge as a means of improving their understanding of mathematical concepts (Wewe & Juliawan, 2019; Fessakis et al., 2019; Laurens et al., 2017).

Practical Implications for The Teaching and Learning of Social Arithmetic

Teachers can use this learning trajectory as a reference for teaching social arithmetic and other mathematical topics in a more creative way by integrating cultural contexts and technology. The simplest initial step that teachers can take is to design a learning route that begins with clearly defined learning objectives. Subsequently, they can develop learning tasks based on contextual problems. In this regard, the use of context in RME has great potential, especially when the implementation is directly related to students' daily lives or to the cultural practices found in society. Sari et al. (2024) discussed the application of ethnomathematics in the batik-making tradition, specifically through the use of South Sumatera's Songket motif designs, to teach mathematical concepts such as symmetry, translation, and geometric transformation. Therefore, teachers can adopt a contextualizing problem to local culture approach by replacing generic examples with culturally relevant scenarios, such as traditional market transactions, local festivals, or community-based saving practices. In relation to ethnomathematics research, Nurjanah et al. (2021) suggested that the traditional Marosok livestock trading system could be used to teach social arithmetic by applying the ethnomathematical practices involved in buffalo trading within Minangkabau culture.

The use of ethnomathematical contexts presents a new experience for students in learning and teaching arithmetic through the RME approach. This implementation can enhance students' critical

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thinking and problem-solving abilities by facilitating the transfer of knowledge from school to real-life situations (Laurens et al., 2017; Nirawati et al., 2021). In teaching RME, teachers are also expected to be creative in preparing anticipatory questions based on various possible student responses. This aims to guide and deepen students' thinking during the problem-solving process. This approach encourages a process of reinvention and strengthens connections to broader mathematical concepts, aligning with the core principles of RME, which include: presenting problems through realistic contexts, identifying key mathematical elements, facilitating meaningful social interactions and guided teacher interventions to refine students' models, promoting reinvention throughout the problem-solving process, and emphasizing the interconnectedness of mathematical ideas (Van den Heuvel-Panhuizen, 2000).

Teachers can also combine the use of technology with the RME approach in teaching social arithmetic. Various studies have shown that technology has the potential to enhance student motivation, increase student engagement in learning, and create culturally relevant learning experiences (Zaranis, 2013; Zaranis & Exarchakos, 2018; Zhengtao & Hidayat, 2025; Bray & Tangney, 2016). Adaptive learning that utilizes emerging technologies has become an alternative solution for teachers in delivering social arithmetic lessons. For example, Zaranis and Exarchakos (2018) reported positive outcomes in teaching mathematical concepts to low-achieving students using a combination of RME and information and communication technologies (ICTs). Through this integration, students are able to investigate, solve problems, and understand concepts using various steps and tools provided by technology, particularly in the context of geometry (Zaranis, 2013).

Similarly, Zhengtao and Hidayat (2025) demonstrated that augmented reality (AR) technology can increase student interest and self-efficacy, thereby enhancing their mathematical abilities. Social arithmetic learning can empower students by incorporating cultural contexts, such as the traditional Marosok buying and selling technique or culturally contextualized word problems, with the aim of improving mathematical communication and problem-solving skills (Sahinkaya et al., 2024). Social arithmetic can also be taught through technological integration—such as AR or other ICTs-based software—to bridge cultural gaps by making abstract mathematical concepts more relatable. For instance, teachers can use real-world math applications that are connected to students' cultural backgrounds. For future research, scholars may explore the combination of Marosok techniques with AR for students with disabilities to support their numeracy development or investigate economic patterns and architectural designs from various cultures through educational software, including educational games. When thoughtfully integrated, mathematics teaching and learning can become more meaningful and engaging, offering students valuable and relevant experiences for everyday life.

CONCLUSION

In general, the designed learning trajectory has helped students discover the concepts of profit, loss, break-even, and percentage formulas. The activities in the HLT provided students with experiences that built their conceptual understanding and mathematical thinking skills through solving problems. For instance, in activities simulating buying and selling scenarios, students independently developed strategies such as comparing selling prices to purchase prices to determine profit or loss. The teacher's role in the learning process constituted one of the main factors guiding the rediscovery of social arithmetic concepts, especially for students who faced difficulties and required more time to understand mathematical ideas. Through strategic questioning and scaffolding, teachers guided students toward rediscovering social arithmetic concepts rather than directly providing solutions, fostering deeper cognitive engagement.

Furthermore, the HLT effectively enriched students' strategies and thought processes in learning social arithmetic by creating an environment where they could engage more deeply with the material. As students navigated through the learning trajectory, those with intermediate and advanced abilities demonstrated good independence, strategies, and mathematical communication skills in solving problems. However, students with lower abilities required more intensive guidance from the teacher to achieve formal knowledge. In this study, students were intentionally provided with the flexibility to answer, but it was found that additional guidance was still necessary to enhance the mathematical communication skills of students with lower abilities. Overall, the HLT with a sequential RME approach gradually encouraged students to think mathematically, construct knowledge, and improve their understanding of social arithmetic material. It illustrated the relationship between students' cognitive processes and their independence, strategic flexibility, and mathematical communication skills within the framework of a learning trajectory in their educational experiences.

LIMITATIONS

The limitation of this study is its use of a small research sample. Nevertheless, the in-depth interviews conducted in this study were able to identify students' thought processes while they were discovering social arithmetic concepts through buying and selling activities. We suggest that future researchers conduct more in-depth research with a larger sample and expand the learning trajectory related to problem-solving and mathematical communication abilities within the context of technology and local culture.

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APPENDIX

Appendix 1. Student HA's answer to question in HLT

a). $30.000 + 40.000 + 36.000 + 25.000 + 10.000 + 50.000$
 $= 191.000$
 Jadi jumlah modal yang diperlukan adalah $\text{Rp } 191.000$

b). mendapatkan untung sebanyak $\text{Rp } 59.000$ rupiah.

c). mendapatkan rugi / penurunan dari hasil hari senin
 yaitu sebanyak $\text{Rp } 10.000$ rupiah.

d). mendapatkan balik modal karena jumlah modal yang diperlukan bukk Ani adalah 191.000 dan jumlah ~~pe~~ penghasilan hari sabbu juga 191.000

a). $30,000 + 40,000 + 36,000 + 25,000 + 10,000 + 450,000 = 191,000$
 So, the total capital needed is 191,000.

b). Obtained a profit of 59,000 rupiah.

c). Incurred a loss/decline from the result on Monday, which was 11,000 rupiah.

d). Break-even because the total capital needed by Mrs. Ani is 191,000, and the total income on Saturday is also 191,000.

Figure 1. Student HA's answer to question in activity 1

Activity 1 part A & B

- Teacher : How did you get a profit in part B? Why is there a profit?
 Student HA : Because the main capital is Rp 191,000. Then, if, for example, the fried snacks sell out of them, we get a profit.
- Teacher : How much is the profit? From where?
 Student HA : Because the initial capital is Rp 191,000, then the profit is Rp 59,000 from the total calculation. The total sales are Rp 250,000.

Activity 1 part C

- Teacher : What about part C?
 Student HA : To determine a loss because sales went down.
- Teacher : Why was there a loss?
 Student HA : Because Mrs. Ani's sales weren't as high as on Monday.
- Teacher : If sales were Rp 240,000, which is less than Monday, what happens?
 Student HA : There's a loss of Rp 10,000.
- Teacher : Why is there a loss? What's the benchmark for determining profit or loss when selling? Do we compare it to previous sales or something else?
 Student HA : We compare it to previous sales.
- Teacher : What if Mrs. Ani's sales today were Rp 200,000?
 Student HA : It's a loss, because it's less than previous sales.
- Teacher : How much preparation cost does Mrs. Ani need?

- Student HA : Rp 191,000, Teacher.
Teacher : If sales are Rp 200,000, is it profit or loss?
Student HA : Profit.
Teacher : So what should be our benchmark: previous sales or something else?
Student HA : The capital.
Teacher : in part C, why is there a loss?
Student HA : Because sales were lower than the previous day.
Teacher : What's the comparison? The previous day, right? So the profit is not Rp 41,000?
Student HA : Compared to the capital, Teacher.

Activity 1 part D

- Teacher : Then for part d, breaking even means reaching the point where there's no profit or loss, right? Why is that?
Student HA : Because the sales that day were Rp 191,000, and our capital was also Rp 191,000.
Teacher : Okay, that's correct.

Appendix 2. Student LA's answer to question in Activity 1

Activity 1 part A

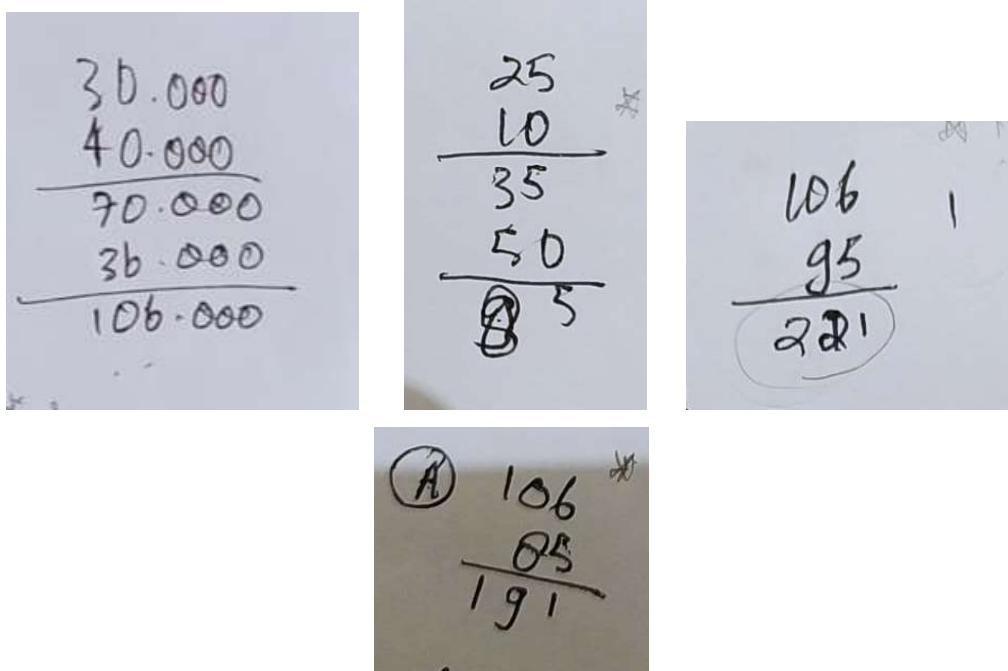


Figure 2. Student LA's answer (a) to question in activity 1

- Student LA : What does this mean, sir?
 Teacher : Has the number (a) been completed? What is the result?
 Student LA : Rp 221,000, sir.
 Teacher : Hmm, Rp 221,000, okay. Now, where does the Rp 95,000 come from?
 Student LA : This is Rp 25,000 plus Rp 10,000, so Rp 35,000, then add Rp 50,000, making it Rp 95,000.
 Teacher : Hmm, what's 35 plus 50? Hmm, what is 3 plus 5?
 Student LA : Hmm, 3 plus 5 is 9, ehh, 8.
 Teacher : Yes, 8, okay. Try correcting it and just cross it out there. Try to fix your answer at the beginning.
 Student LA : So, what's the answer?
 Student LA : Six plus five, hmm, eleven; one plus eight, hmm, nine, so the answer is 191,000

Activity 1 part B

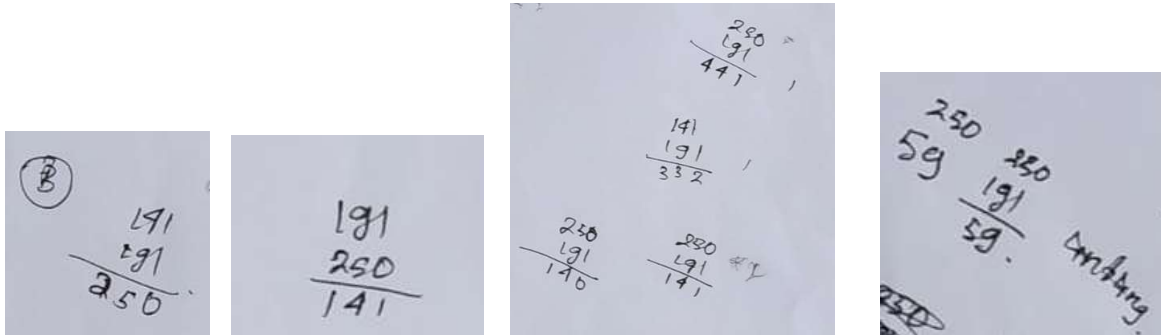
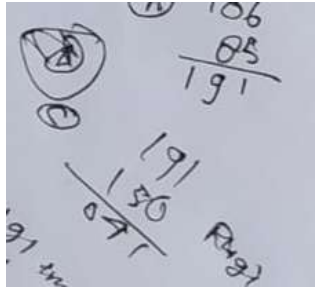


Figure 3. Student LA's answer to question (b) in activity 1

- Teacher : How about the answer for (b)?
- Student LA : It's done, sir. Here, Rp 191,000 plus Rp 141,000 equals Rp 250,000.
- Teacher : Rp 141,000, is it correct?
- Student LA : Yes, sir.
- Teacher : It seems like there's still an error. Check if it was added correctly.
- Student LA : Eh, it should be subtracted, sir. So it's 140,000, sir.
- Teacher : Look carefully, why does 5 minus 9 give you 4?
- Student LA : Ehhh. It's because I borrowed using the borrowing system, so 250,000 minus 191,000. The difference is 59,000, sir.
- Teacher : How did you know that's a profit? What did you understand?
- Student LA : Hmm, sales.
- Teacher : Yes, so what does it mean?
- Student LA: If the sales are higher than expenditure, it's a profit, sir.

Activity 1 part C



Activity 1 part D

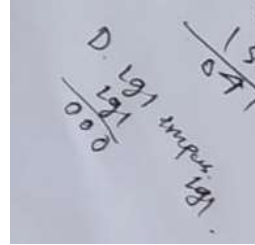
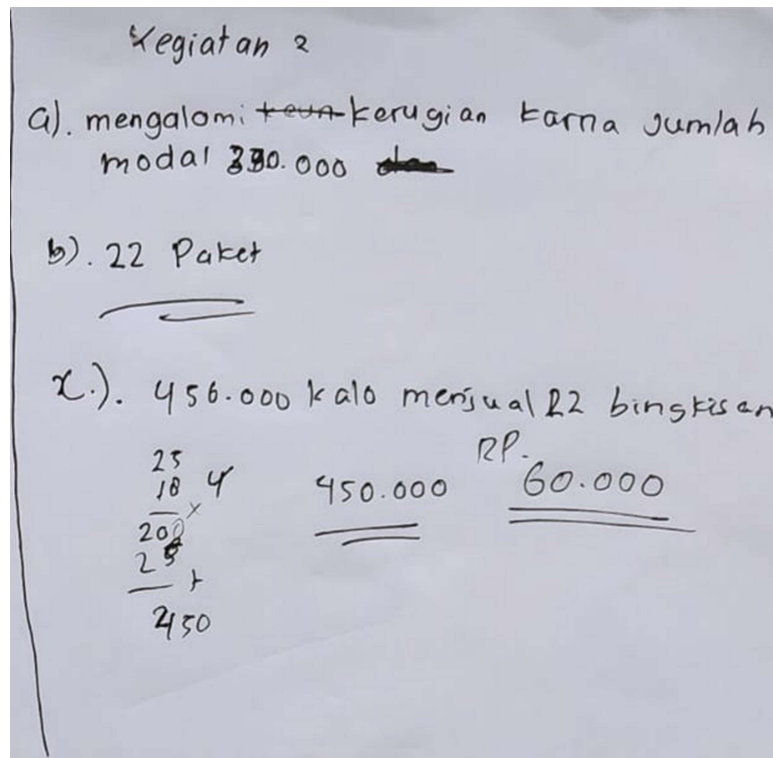


Figure 4. Student LA's answer to question (c) and (d) in activity 1

- Teacher : Now, for (c). How much did Bu Ani earn from sales?
 Student LA : Rp 150,000, sir.
 Teacher : What was the cost price earlier?
 Student LA : Rp 191,000, sir.
 Teacher : So, what does that mean for Bu Ani now?
 Student LA : A loss, sir.
 Teacher : Why is it a loss?
 Student LA : Hmmm, because the cost is higher than the sales, eh, the revenue.
 Teacher : So, it's a loss. That's what you need to understand, Dimas.
 How much is the loss?
 Student LA : Hmmm.
 Teacher : So, what's the difference?
 Student LA : Rp 41,000, sir.
 Teacher : How much is the sales for (d)?
 Student LA : Rp 191,000, sir.
 Teacher : What was the cost price?
 Student LA : Rp 191,000, sir.
 Teacher : So, which is greater?
 Student LA : They're the same, sir.
 Teacher : If they're the same, what does that mean?
 Student LA : Hmmm.
 Teacher : Is it a loss, a profit, or what?
 Student LA : Break-even, sir.

Appendix 3. Student HA's answer to question in Activity 2



Activity 2

a) Experiencing a loss because the total capital is Rp 250,000.

b) 22 Packages

c) Rp 456,000 if selling 22 packages.

Calculation:

- Total = Rp 450,000

- Profit = Rp 60,000

Figure 5. Student HA's answer to question in activity 2

Activity 2 part A

Teacher : Why did Mrs. Ani experience a loss?

Student HA : Because the income earned is less than the capital.

Activity 2 part B

Teacher : What's the minimum number of packages needed so that Mrs. Ani does not incur a loss?

Student HA : 22 packages.

Teacher : Why is it 22?

Student HA : Because 22 times 18 equals 456,000.

Teacher : How did you figure that out?

Student HA : multiplying by 18.

Teacher : What did you multiply by 18?

Student HA : 22 times 18, Sir.

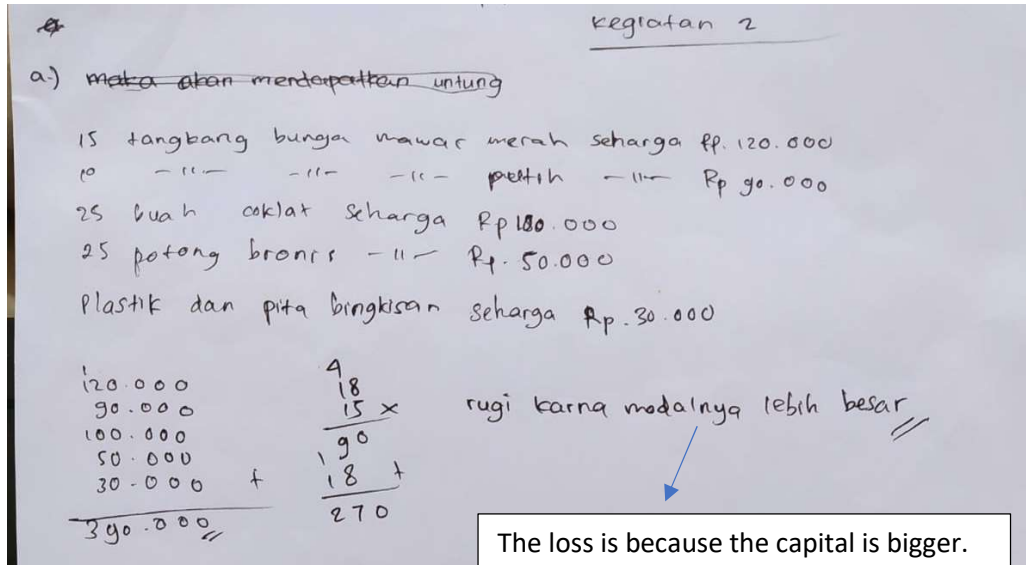
Teacher : What if it's 21? Would it be a profit or a loss?
Student HA : A loss.
Teacher : If she sells 21, how much money would that be?
Student HA : 378,000, still below 390,000.
Teacher : And if 22 is multiplied by 18, what do you get?
Student HA : 456,000, Sir.
Teacher : Hmm, is that correct? It seems there's an error in the multiplication.
Student HA : 396,000 for 22 packages.

Activity 2 part C

Teacher : What's the maximum profit Mrs. Ani can make?
Student HA : Rp 450,000, Sir.
Teacher : When is the maximum profit achieved? What does "maximum" mean?
Student HA : It means the most, Sir.
Teacher : So, how many packages need to be sold for the maximum profit?
Student HA : More than 22, around 29, Sir.
Teacher : Why is it 29?
Student HA : Because it's above the capital, Sir.
Teacher : What is Mrs. Ani selling to make a profit?
Student HA : Gift packages, Sir.
Teacher : How much is each package?
Student HA : Rp 18,000 each.
Teacher : How many packages are prepared?
Student HA : 25, Sir.
Teacher : To achieve maximum profit, does everything have to be sold?
Student HA : Yes, everything has to be sold, Sir.
Teacher : What's the total if everything is sold?
Student HA : Rp 18,000 times 25, Sir.
Teacher : That's the sales revenue, not the profit. So, how do you calculate the profit?
Student HA : By subtracting the capital from the sales revenue, Sir.

Appendix 4. Student MA's answer to question in Activity 2

Activity 2 part A



Kegiatan 2

a) ~~maka akan mendapatkan untung~~

15 tangkang bunga nawar merah seharga Rp. 120.000
 10 " " " " putih " Rp 90.000
 25 buah coklat seharga Rp 180.000
 25 potong bronis " Rp. 50.000
 Plastik dan pita bingkisan seharga Rp. 30.000

$\begin{array}{r} 120.000 \\ 90.000 \\ 180.000 \\ 50.000 \\ 30.000 \\ \hline 390.000 \end{array}$	+	$\begin{array}{r} 18 \\ 15 \times \\ \hline 90 \\ 18 \\ \hline 270 \end{array}$	<p>rugi karna modalnya lebih besar //</p>
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The loss is because the capital is bigger.

Figure 6. Student MA's answer to question (a) in activity 2

- Teacher : If there are 15 packages, how much money do they get?
- Student MA : If it's 25 packages, it's Rp18,000.
- Teacher : Now only 15 packages are sold. How much money does Bu Ani get?
- Student MA : Hmm...
- Teacher : How much is one package?
- Student MA : Rp 18,000 multiplied by 15 packages... hmm... Rp 270,000.
- Teacher : Hmm, what's the cost price?
- Student MA : the correct answer is Rp 390,000.
- Teacher : What about the sales?
- Student MA : Hmm, Rp 270,000 pak.
- Teacher : What does it mean?
- Student MA : It is a loss, sir.

Teacher : Why is it a loss?

Student MA : Because the cost is higher than capital.

Activity 2 part B

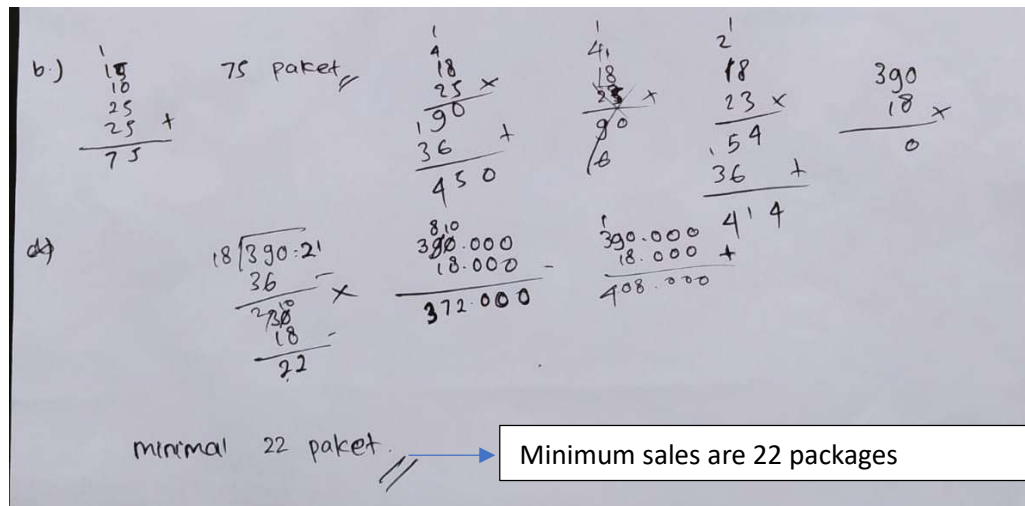


Figure 6. Student MA's answer to question (b) in activity 2

Teacher : To know the total, how do we calculate it?

Student MA : Make it larger than this (total of package).

Teacher : For example, how many?

Student MA : it is 25, sir.

Teacher : If it's 25, is there a loss or not?

Student MA : (Hesitant)

Teacher : How about 23?

Student MA : (Hesitant)

Teacher : Is it a loss or a profit?

Student MA : Profit, Pak.

Teacher : What if it's break-even, is it a loss or not?

Student MA : Not a loss, sir.

Teacher : How do we know for sure?

Student MA : Multiply, sir.

Teacher : What do we multiply?

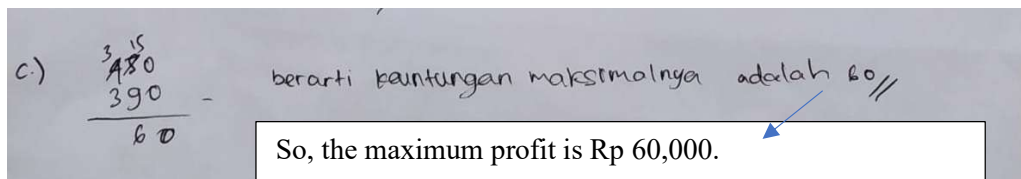
Student MA : The cost, sir, Rp 390,000.

Teacher : Multiply by what?

Student MA : Rp 18,000, sir.

- Teacher : If we multiply, it will be very large. How do we handle that?
 Student MA : Divide, Pak.
 Teacher : The result?
 Student MA : 21.1, Pak.
 Teacher : So, what's the minimum to avoid a loss?
 Student MA : Round it up to 22, Pak.
 Teacher : What if it's 22?
 Student MA : Profit, Pak. So, the answer to avoid a loss is 22 packages.

Activity 2 part C



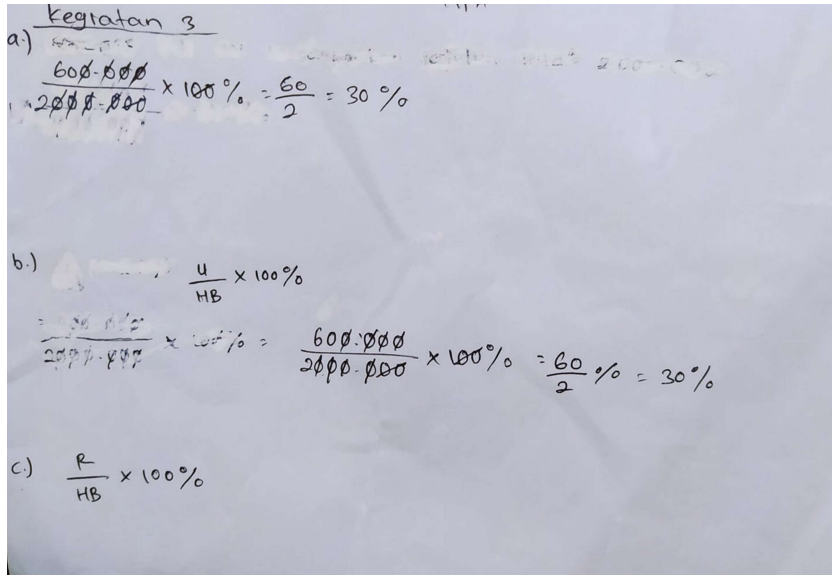
c.)
$$\begin{array}{r} 380 \\ - 390 \\ \hline 60 \end{array}$$
 berarti keuntungan maksimalnya adalah 60 //

So, the maximum profit is Rp 60,000.

Figure 7. Student MA's answer to question (c) in activity 2

- Teacher : If everything is sold out, what is it called?
 Student MA : The profit is maximized, Pak.
 Teacher : How many packages did Bu Ani make in total?
 Student MA : 25, Pak.
 Teacher : Price per package?
 Student MA : Rp 18,000.
 Teacher : So, if 25 packages are sold, what's the total sales?
 Student MA : Rp 450,000, Pak.
 Teacher : Is that profit or sales?
 Student MA : Sales, Pak.
 Teacher : What's the cost price?
 Student MA : Rp 390,000.
 Teacher : So, what's the profit?
 Student MA : Rp 60,000, Pak.
 Teacher : Correct. Please write the explanation.

Appendix 5. Student MA's question in activity 3



kegiatan 3

a) $\frac{600.000}{2.000.000} \times 100\% = \frac{60}{2} = 30\%$

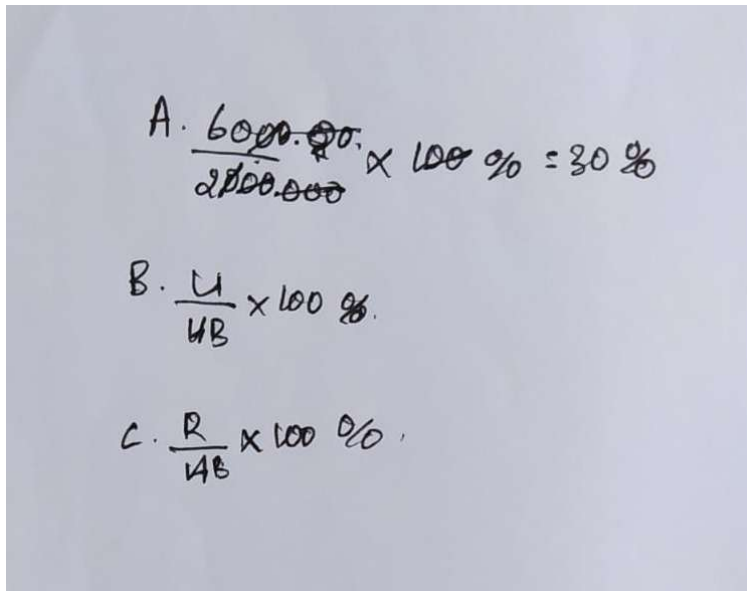
b) $\frac{600.000}{2.000.000} \times 100\% = \frac{60}{2} \% = 30\%$

c) $\frac{R}{HB} \times 100\%$

Figure 7. Student MA's answer to question in activity 3

- Teacher : Where did this come from? Why is the answer Rp 600,000 divided by 2,000,000?
- Student MA : Yes, because the average sales are Rp 2,000,000.
- Teacher : Is that really the sales?
- Student MA : Hmm...
- Teacher : Then, what's this Rp 600,000?
- Student MA : Hmm... that's the profit.
- Teacher : How do you know that's the profit?
- Student MA : The sales are Rp 2,600,000.
- Teacher : So, what is this Rp 600,000?
- Student MA : Hmm... just remove the 2's, Pak. You get Rp 600,000. It can't be bigger than Rp 2,000,000.
- Teacher : Why can't it be?
- Student MA : Hmm... this Rp 2,000,000 is the capital.
- Teacher : What is the sales?
- Student MA : Rp 2,600,000, Pak.
- Teacher : So, how much is the profit?
- Student MA : Hmm... 600,000, Pak.

Appendix 6. Student LA's question in activity 3



A. $\frac{6000.00}{2000.00} \times 100\% = 30\%$

B. $\frac{U}{UB} \times 100\%$

C. $\frac{R}{UB} \times 100\%$

Figure 7. Student LA's answer to question in activity 3

- Teacher : Now, if we want to know the profit percentage, what should we compare?
- Student LA : Hmm...
- Teacher : What terms have we discussed so far?
- Student LA : Sales, capital, loss, profit, break-even.
- Teacher: So, how do we know if an item has a profit or loss?
- Student LA : Sales.
- Teacher : What else?
- Student LA : Compared to the capital.
- Teacher : How can there be a loss?
- Student LA : If the capital is higher.
- Teacher : How can there be a profit?
- Student LA : If the capital is lower.
- Teacher : That's the basic idea behind finding the profit percentage. Try to find it, okay?