

## Design and Implementation of Hypothetical Learning Trajectory on The Geometry of Cubes and Cuboids using Project-Based Learning

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*Abstract: The geometry of cubes and cuboids serves as a crucial foundation for learning other spatial shapes in mathematics education. Students often experience difficulties achieving objectives due to various learning obstacles. Mathematics teachers also struggle to design learning activities that address students' needs and effectively overcome learning obstacles. This research aimed to investigate the impact of Hypothetical Learning Trajectory (HLT) using project-based learning to overcome students learning obstacles on the topics of cubes and cuboids. Students collaborate in groups to create miniature building projects that are used to analyze the concepts of elements, nets, surface area, and volume. This study uses a design research method consisting of 2 cycles, each with 3 stages: preliminary design, teaching experiments, and retrospective analysis. 64 eighth-grade students from a secondary school in Indonesia were involved in the implementation of 8 HLT across 9 teaching experiments. Data were collected through classroom observations, interviews, tests, and problem-solving exercises. There are 21 types of characteristic learning obstacles that have been identified on the topic of cubes and cuboids. HLT combined with project-based learning successfully addressed 87% of students' learning obstacles related to this the topic, including epistemological, ontogenetic, and didactic learning obstacles.*

Keywords: design research, hypothetical learning trajectory, project-based learning, geometry, cubes and cuboids.

### INTRODUCTION

Accurate drawing and understanding of a drawing is of utmost importance to effectively support mathematical performances (Rellensmann et al., 2017), especially in geometry of cubes and cuboids (Gal & Linchevski, 2010; Hoffmann & Németh, 2021). As part of geometry, the topic of

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cubes and cuboids remains challenging in mathematics education (Dimla, 2018; Snyders, 1995). Many students still struggle to solve problems related to cubes and cuboids (Priskila et al., 2023). Students sometimes find it difficult to solve problems due to learning obstacles, including concept limitations, such as errors in using surface area and volume formulas, limitations in prerequisite material such as the Pythagorean Theorem and area unit conversion, and student limited experience in solving non-routine problems. This difficulty is also caused by low motivation and fear of mathematics (Saadati & Celis, 2023).

The results of previous research have shown that the identified learning obstacles cause students to have difficulty solving cubes and cuboids problems (Priskila et al., 2023; Safitri & Dasari, 2022). Students cannot achieve learning objectives optimally. This fact leads to the task of teachers to overcome problems in learning mathematics. However, current education systems are rarely equipped to tackle complex and thorny problems that emerge in schools and classrooms, which include a wide range of academic, social, mental, and emotional issues that students struggle with, along with distinct achievement gaps that have persisted and accelerated in recent years (Dorn et al., 2020). This has led to higher levels of stress, burnout, and depression among students and teachers (Baker et al., 2021; Henriksen et al., 2022). Teachers should understand the learning obstacles that students have and design a learning process that is able to overcome the various difficulties that students face in the classroom. Teachers need to study the phenomena that underlie the process of building hypothetical learning designs by, examining concept images and learning obstacles during the prospective analysis stage to serve as an analysis of didactic context before learning (Nurhikmayati et al., 2022).

HLT is a learning process design that considers students' responses based on their obstacles (Kuncoro et al., 2023a; Simon, 1995a). The conventional wisdom in the mathematics education community holds that HLTs are an important tool in improving mathematics education (Baroody et al., 2022). A HLT framework has the potential to help teachers develop knowledge about students, their conceptions and misconceptions, and their thinking and learning strategies (Bardsley & ellen, 2006). It is an ongoing challenge for teachers to integrate their teaching and learning goals with trajectories of students' thinking and learning; HLT may help in addressing these challenges (Simon, 1995a). This research focuses on developing HLT using Project-Based Learning (PBL) which is part of the 3 models recommended in the implementation of the 2013 curriculum in Indonesia (MoEC, 2016). These models allow teachers to select the most suitable learning method to achieve specific objectives (Lainufar et al., 2021). PBL provides more extensive problems than other models by offering more contextual and authentic experience (Capraro et al., 2013). It enhances the effectiveness of teaching and learning geometry (Chin, 2014), enabling students to construct knowledge through project experiences on a team. The criteria of one group was more successful in solving geometry problems in collaboration, such as through project experiences, while the other could have been more successful (Farida et al., 2023).

In geometry, PBL allows students to visualize and manipulate geometric shapes and structures, thereby deepening the understanding of abstract concepts. It is widely accepted that mathematics instruction should prioritize developing students' abilities to apply mathematical concepts to solve real-world problems (NCTM, 2000). Projects made by students about building space will provide a meaningful experience. Students can analyze each part of the project to generalize a new understanding. This research introduces the development of HLT through PBL as a novel method. Previous efforts to develop HLT for cubes and cuboids by Sari et al (2024a) and Surya et al (2021a) relied on problem-based learning. HLT developed in this research aims to enhance students' conceptual understanding of materials through projects contextualized with real-world applications.

The importance of cubes and cuboids in geometry as a basis for learning other spatial shapes alongside the identification of students' learning obstacles emphasizes a gap in this research. The development of HLT using PBL stages is expected to overcome students' learning obstacles on the topics of cube and cuboid geometry. This study is important as it aims to provide an overview to educators in an effort to improve student understanding based on the problems in the learning obstacles that students have.

This study aims to investigate the impact of HLT implementation using PBL in overcoming students' learning obstacles on the topics of cube and cuboid. To achieve this objective, three specific research questions have been established as follows:

1. What are the characteristics of students' learning obstacles in cubes and cuboids?
2. How is the HLT designed and implemented through PBL activities to address students' learning obstacles in understanding cubes and cuboids?
3. What is the impact of HLT using PBL in overcoming students' learning obstacles?

## LITERATURE REVIEW

### Student's Learning Obstacles

Learning obstacles is a condition where a person cannot follow the learning process properly, which is characterised by certain obstacles preventing individuals from achieving learning outcomes (Moru, 2009). Brousseau (2002) categorises learning obstacles into three types: ontogenic obstacles, epistemological obstacles, and didactical obstacles. Ontogenic obstacles are learning obstacles related to mental readiness. This mental readiness is related to the level of student development that is not in accordance with their age (Cesaria & Herman, 2019). If the obstacle arises only because of slow mental development and not because of an inherited disease, the obstacle will disappear by itself along with the growth of the student (Cesaria & Herman, 2019). Brousseau (2002b) noted that epistemological obstacles are related to the limited ability of students' concepts. Epistemological obstacles occur when students experience obstacles in concepts, procedures, and calculation techniques. Epistemological understanding is essentially a person's knowledge, which

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was only limited to certain contexts (Duval, 2006). If the person is faced with different contexts, the possessed knowledge becomes unusable, or she or he undergoes difficulty using it. In this case, students' views on other concepts are disjointed and not comprehensive. Finally, Brousseau (2002b) noted that didactical obstacles are obstacles related to teaching materials or learning presentations by teachers that are not appropriate to the conditions of students.

### **Hypothetical Learning Trajectory**

The term Hypothetical Learning Trajectory (HLT) in mathematics learning was first introduced by Simon in 1995. HLT is a learning process design that pays attention to learner responses consisting of three components, namely learning objectives, learning activities, and learning process conjectures predictions about how students' thinking and understanding will develop in the context of learning activities (Simon, 1995b). HLT is a hypothesis or prediction of how learners' thinking and understanding develop in learning activities. HLT also refers to a hypothetical learning path as a conjecture about the series of activities that children go through in solving a problem or understanding a concept. The design of HLT in the learning process is based on students' learning obstacles (Kuncoro et al., 2023b). Each activity and task in the HLT must aim to overcome learning obstacles that are in accordance with the suspected way of thinking for students when performing the given task. The presumption of students' way of thinking that has been designed by the teacher will make it easier for the teacher to provide didactical anticipation during the learning process. When the HLT designed is not in accordance with the students' expected learning process, the HLT is revised. This activity continues until the HLT is in accordance with the alleged learning process and, can overcome students' learning obstacles to achieve optimal learning objectives.

### **Project-Based Learning**

Project-Based Learning (PBL) has gained so much recognition mainly because it was the first method where teachers at the time could see students gaining new knowledge while working on activities. It allows students to construct their knowledge at their own pace (Gay, 2022a). PBL is one mode of learning that creates an active, collaborative atmosphere and can increase confidence in students (Hudson et al., 2015; Pablos et al., 2017). It is a learning model that focuses on the main concepts and principles of a discipline, involves students in problem-solving and reasoning activities, provides opportunities for students to work autonomously to construct their own learning, and produces products of student work (Fisher et al., 2020). In this study, we used PBL stages which include reflection, research, discovery, application, and communication (Laboy-Rush, 2011a). These five stages allow students to explore mathematical understanding openly, conduct scientific investigations in the learning classroom, and validate the process through application activities.

## Relevant Research on Geometry Learning

Various studies on geometry learning have been conducted with various methods and education levels. A literature review from 2017-2021 shows that most geometry learning interventions are effective, especially on van Hiele stage-based approaches, technological media, and concrete manipulative media (Waluya et al., 2022). The study recommended the development of more structured geometry scaffolding. Student-centered learning approaches were shown to be more effective than traditional methods (Juman et al., 2022), with activity-based learning also showing good results (Noreen & Rana, 2019). In addition, it is important for teachers to relate geometry concepts to the real world (Nazarovich & Kurudirek, 2024).

In addition to PBL, other approaches such as PBL are also effective in understanding geometry concepts. A literature review and meta-analysis showed that 86% of studies noted the positive impact of PBL supported by GeoGebra and Cabri 3D on students' geometry understanding (Ramadhanti & Juandi, 2022), as well as knowledge improvement and retention (Ahamad et al., 2017; Haji et al., 2023; Herawati & Sidik, 2023). The inquiry learning model can also improve students' ability to identify geometry concepts (Kedikli & Katranci, 2022; Ratmokohadi, 2022; Triwahyuningtyas & Suastika, 2021), through skills such as observation, measurement, and data presentation (Kedikli & Katranci, 2022). PBL and inquiry models are excellent at developing higher order thinking skills, but for highly visual geometry concepts, the real-world experience offered by PBL is needed. PBL excels in providing geometry understanding because it integrates contextual, collaborative learning and relevant real products. When combined with technology, this approach can provide a more immersive, engaging, and meaningful learning experience.

## METHOD

### Research Design

This research aimed to enhance the presentation of mathematics learning methods on the topic of cubes and cuboids by developing HLT based on students' identified learning obstacles. To achieve this objective, the design research method was adopted which focused on developing theories about learning process (Plomp, 2006). The design method was defined as the systematic analysis and evaluation of educational interventions with the dual purpose of producing research-based solutions to complex problems in educational practice. This design also advanced the knowledge of the characteristics of the interventions and the process of designing and developing the innovations (Plomp, 2006).

In implementing this research, the design method was applied which included 3 stages, namely preliminary design, teaching experiment, and retrospective analysis (Akker et al., 2006). The de-

sign method was a cyclical process of testing a series of learning activities. Part of the key characteristics of design research was the cyclic-iterative nature which could be divided into macro- and micro-cycles (Gravemeijer & Eerde, 2009a). In this research, 2 cycles were conducted as stages of HLT development on the topic of cubes and cuboids as presented in Figure 1.

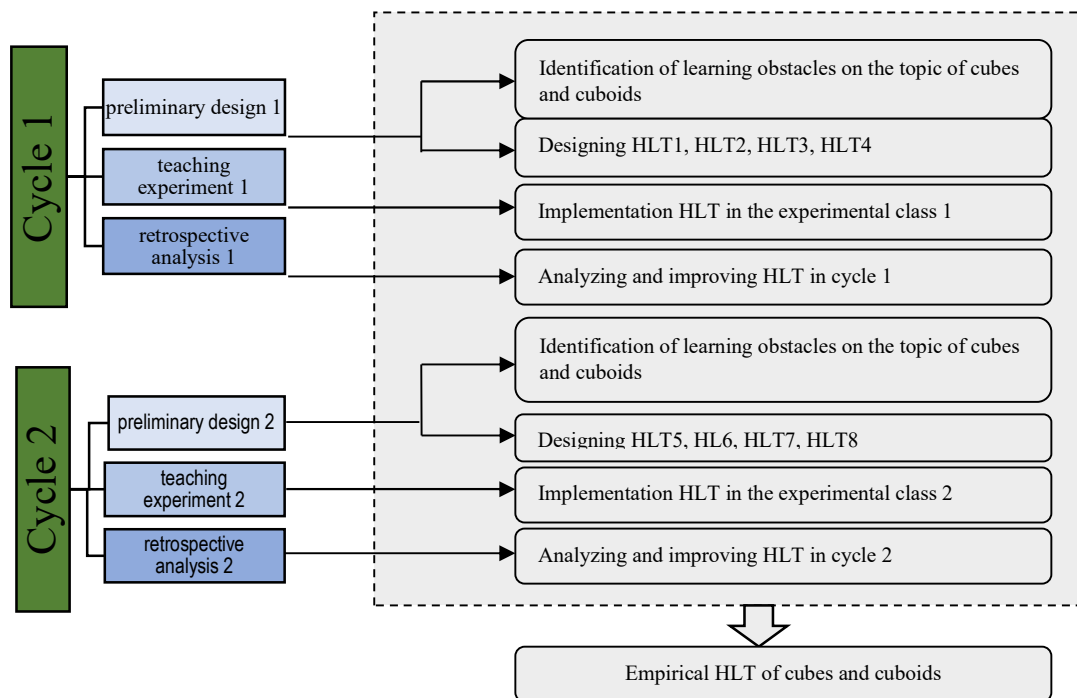


Figure 1. Flow of Research Design for Cubes and Cuboids

HLT in this research was divided into universal and partial. Universal HLT represented the final design derived from the partial HLT in each cycle. The HLT in each cycle was designed according to the stages of PBL, starting with project assigned to students during the reflection and research stages. HLT implementation scheme which incorporated the stages of PBL is presented in Figure 2.

The sub-topic of cubes and cuboids was studied in each HLT while ensuring adherence to the material hierarchy. Projects given to students were reviewed and developed at HLT1 and HLT5. Furthermore, the analysis of each sub-topic on cubes and cuboids was carried out based on projects completed by students which were reviewed during the discovery, application, and communication stages in HLT2, HLT3, HLT4, HLT6, HLT7, and HLT8.

### Sample and Data Collection

Participants of this research included 64 eighth-grade students from a public junior high school in Majalengka, Indonesia. In the first cycle of the experiment, 29 students participated while 31 took part in the second cycle. The first cycle served as the initial didactic design stage, and the second focused on improving the design from the initial cycle. Although all participants came from the same school, students were in different grades and participated at different stages of the cycle examining the same material. Participants engaged in a 3-month teaching experiment. During the first cycle, students used HLT1, HLT2, HLT3, and HLT4. In the second cycle, students used HLT5, HLT6, HLT7, and HLT8, which served as improvements on the first cycle.

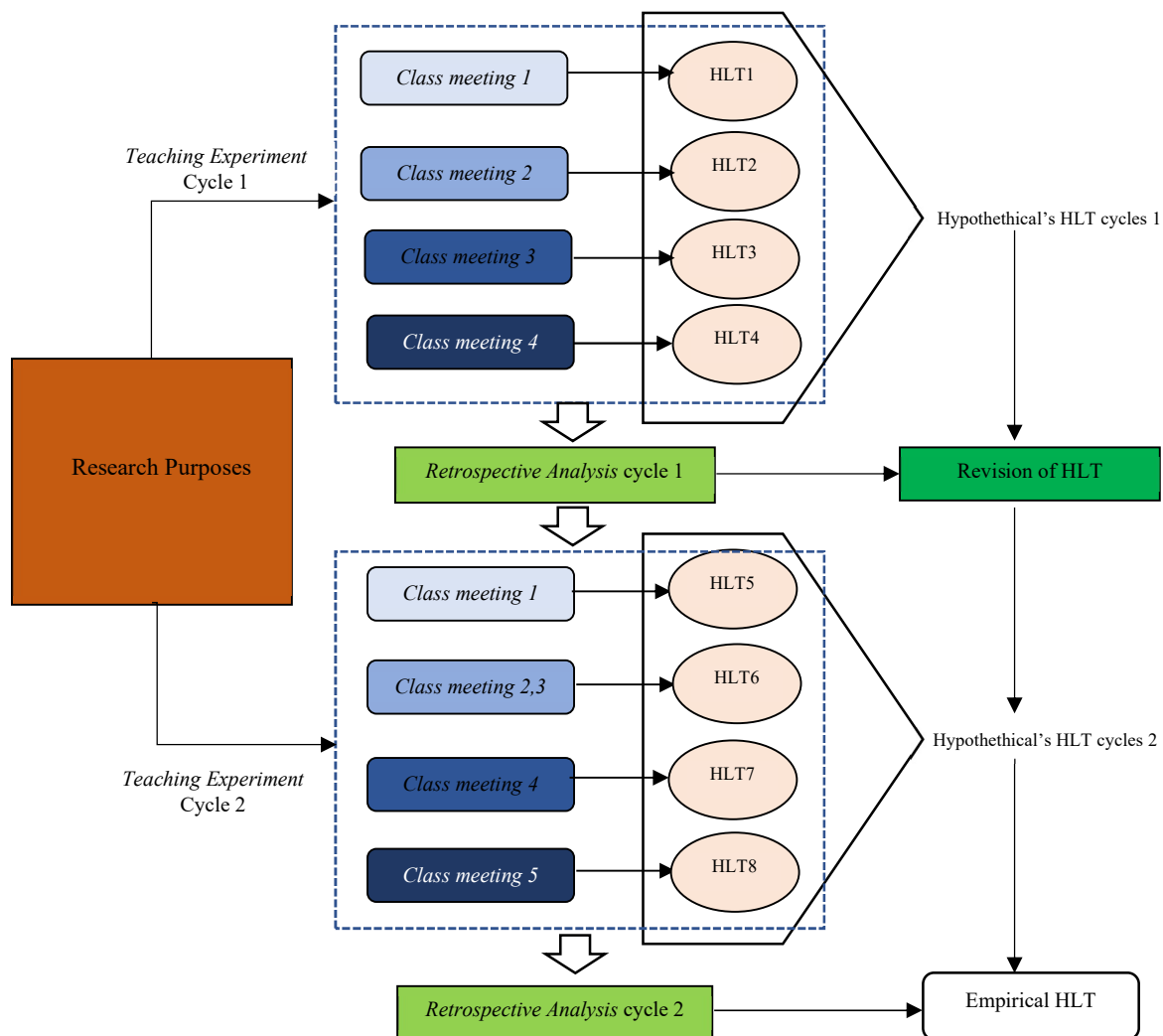


Figure 2. HLT Implementation Scheme for Cubes and Cuboids Topics with PBL

Learning obstacles data were collected through interviews with teachers and students, classroom observations and tests. This test used questions on the topic of cubes and cuboids that were descriptive in nature. At the end of the HLT implementation, We collected students' test data through practice questions to examine how the HLT helped reduce their learning obstacles and increased their understanding of the geometry concepts of cubes and cuboids.

### Analyzing of Data

HLT design on cubes and cuboids was developed based on the identified learning obstacles. The process of identifying students' learning obstacles on the topics of cubes and cuboids was analyzed using the qualitative analysis method from (Miles et al., 2014a). This analysis consists of three flows, namely data condensation, data display and conclusion. This research further used data triangulation which was a method for testing the validity of data by cross-checking the accuracy of information from different perspectives. Interview data, classroom observations, and tests were used to identify learning obstacles on the topic of cubes and cuboids. The interviews were semi-structured as it did not follow systematic and complete interview guidelines but focused on key issues to be addressed.

## RESULTS

### Characteristics of Learning Obstacles in Cubes and Cuboids

Learning obstacles in cubes and cuboids were identified through 3 methods namely learning observation notes (SO), interview questionnaire notes (SI), and student test results (ST). The analysis of learning obstacles was conducted using the data analysis model by (Miles et al., 2014a). The identified learning obstacles in cubes and cuboids were presented in Table 1.

| Didactic situation  | Source | Coding |
|---|--------|--------|
| <i>Context limitations in understanding the types of elements of cubes and cuboids</i>                      | SO, ST | EO-1   |
| <i>Context limitations in understanding the properties of cubes and cuboids</i>                             | SO, ST | EO-2   |
| <i>Limitations in depicting 3-dimensional space</i>   | SI, ST | EO-3   |
| <i>Context limitations in understanding the shape of the nets of cubes and cuboids</i>                      | ST     | EO-4   |
| <i>Context limitations in understanding cubes and cuboids skeletons</i>                                     | ST     | EO-5   |
| <i>Errors in identifying the value of elements in the formula for the surface area of cubes and cuboids</i> | ST     | EO-6   |
| <i>Errors in formula usage</i>  | ST     | EO-7   |
| <i>Limited understanding of the concept of the surface area of cubes and cuboids</i>                        | ST     | EO-8   |

|  |          |       |
|--|----------|-------|
| <i>Misconceptions on the meaning of volume of cubes and cuboids as the exact number of unit cubes stored on a cube or cuboid</i> | SI, ST   | EO-9  |
| <i>Context limitations on new problems in problem-solving</i>  | SO,SI,ST | EO-10 |

Table 1: Epistemological learning obstacles

Epistemological learning obstacles identified in this research referred to students' conceptual limitations in understanding cubes and cuboids. These conceptual limitations were closely connected to contextual limitations and difficulties in comprehending mathematical symbols. Students' limited understanding of the concepts related to cubes and cuboids contributed to the low performance in solving related problems. When students experienced a problem related to calculating the surface area of unit cuboids of different sizes, errors occurred in determining the correct formula (EO-7) and in assigning values in the surface area formula (EO-6). Epistemological learning obstacles in cubes and cuboids were often interrelated and connected to students' conceptual understanding. Several learning obstacles were identified from a single students' answer, each related to different concepts. Ontogenic learning obstacles identified in cubes and cuboids are further presented in Table 2.

| Didactic situation   | Source | Coding |
|--|--------|--------|
| <i>Errors in understanding the sentences in the question</i>                     | ST     | OO-1   |
| <i>Limited knowledge of the prerequisite material of the Pythagorean theorem</i> | SO, ST | OO-2   |
| <i>Operational learning obstacles</i>  | SO, ST | OO-3   |
| <i>Limited students' prior knowledge related to comparison material</i>          | SO, ST | OO-4   |
| <i>Students' low academic awareness and interest in learning mathematics</i>     | SO     | OO-5   |

Table 2. Ontogenic Learning Obstacles

The preliminary analysis further showed several ontogenic learning obstacles faced by students. Many of these learning obstacles originated from psychological and instrumental factors. Observations showed that students exhibited low interest in mathematics with some deliberately arriving late to class despite knowing it had already started. The test results also showed that many students had insufficient knowledge of prerequisite materials, such as the Pythagorean theorem and ratio concepts. Furthermore, a significant number of students struggled with basic calculation skills, which were essential for solving mathematical problems. Didactic learning obstacles identified in cubes and cuboids were further presented in Table 3.

| Didactic situation   | Source | Coding |
|--|--------|--------|
| <i>Misconception between the known volume formula and the application in the figure with the unit cube</i>       | ST     | DO-1   |
| <i>Context limitation on unfamiliar problems</i>   | SO, ST | DO-2   |
| <i>Limited use of teaching aids and technology in learning</i>   | SO, SI | DO-3   |
| <i>Limited discussion and question and answer between teachers and students as well as students and students</i> | SO     | DO-4   |

|   |        |      |
|---|--------|------|
| <i>Limited opportunities, experiences, and opportunities for students to think as a whole in constructing knowledge</i> | SO     | DO-5 |
| <i>Errors in conveying volume concepts</i>  | SO, ST | DO-6 |

Table 3. Didactical Learning Obstacles

Didactic learning obstacles identified in this research were related to the teaching methods used by teachers when instructing students on cubes and cuboids. Didactical obstacles arise from the didactic system, encompassing factors such as sequencing and curriculum stages, including their presentation in classroom instruction (Fauzi & Suryadi, 2020), such as the use of learning media. The observations outlined several didactic situations that contributed to the learning obstacles including the limited use of teaching aids such as models of cubes and cuboids as well as the skeletal structures. There was also a lack of integration of learning technologies such as GeoGebra, Maple, or Matlab mathematical applications. Teachers primarily relied on textbooks as visual aids for teaching cubes and cuboids. Additionally, the observations showed that students had limited opportunities and experiences in constructing individual knowledge, as students mainly focused on verifying the concepts presented by teachers.

### Design of HLT for Cubes and Cuboids

HLT was designed based on the identified learning obstacles to achieve the objectives on the topic of cubes and cuboids which was aimed to address and overcome the learning obstacles. Following the research design, HLT was developed during Cycles 1 and 2. The final HLT of each cycle became an empirical reference for teaching cubes and cuboids using PBL. It could be used by teachers and other scholars as a guide. In this research, there were 2 learning cycles namely Cycle 1 which included HLT1, HLT2, HLT3, and HLT4, as well as Cycle 2, comprising HLT5, HLT6, HLT7, and HLT8, as improvements from the initial HLT.

Each HLT was specifically designed for the 2 cycles. In Cycle 1, four HLT (HLT1, HLT2, HLT3, and HLT4) were created, following the stages of PBL model. According to HLT implementation scheme as shown in Figure 2, the design was not organized by material sub-topics but was centered around project initiated at the initial stage of learning. This project which was conducted during the reflection and research stages served as a basis for learning cubes and cuboids comprehensively in each HLT. The four HLT in Cycle 1 were refined based on the teaching experiment process, leading to the creation of HLT5, HLT6, HLT7, and HLT8 in Cycle 2.

### Implementation of HLT on Cubes and Cuboids

HLT for cubes and cuboids was designed in correlation with the stages of PBL model, which included reflection, research, discovery, application, and communication (Laboy-Rush, 2011b). The implementation of HLT included a teaching experiment based on the designed HLT.

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## Teaching Experiment Cycle 1

### *Implementation of HLT 1*

HLT 1 focused on designing learning activities on cubes and cuboids, addressing learning obstacles EO1–EO and DO4–DO5. The stages followed reflection and research. Students were introduced to related problems and encouraged to link prior knowledge to the project. They identified surrounding objects to recall concepts; some gave accurate examples, while others erred or didn't respond. In the research stage, students prepared a mini-proposal for a project involving building miniatures from cubes and cuboids using recycled materials. Several designs were illustrated through sketches and project outcomes (see Figure 1).

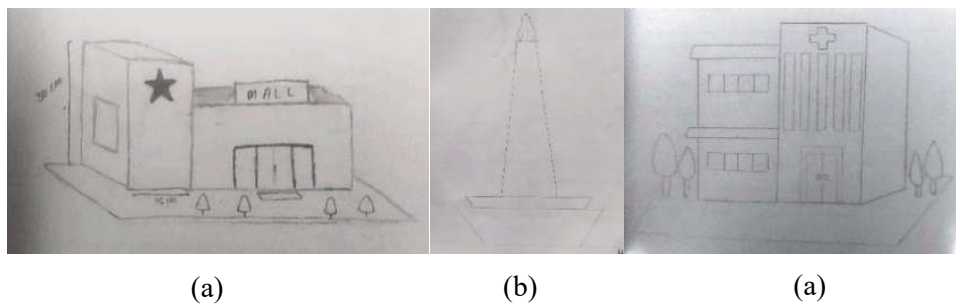


Figure 1: Students' Cubes and cuboids Building Sketches in Cycle 1

The sketches of the miniature building project designs develop by students mostly depicted structures encountered in daily living such as malls (Figure 1a), Monas (Figure 1b) as iconic buildings in Indonesia, and hospitals (Figure 1c). Students relied on imagination, drawing from group discussions and school textbooks. The results of these projects were shown in Figure 2.

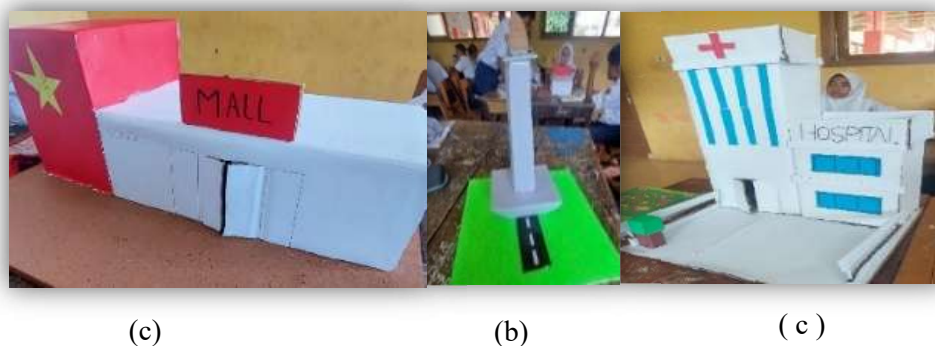


Figure 2: Students' cubes and cuboids building project in cycle 1

Students had gained an understanding of the shapes of cubes and cuboids as reflected in the buildings constructed. This process helped students develop a deeper understanding through observation and discussion with peers.

### *Implementation of HLT 2*

HLT 2 focused on the discovery stage of learning. Projects that students had previously created were then used as a foundation for exploring, analyzing, understanding, and generalizing concepts of the materials. Students analyzed projects to answer questions on the student worksheet which was designed to guide students toward a meaningful understanding of cubes and cuboids. The worksheet covered subtopics such as the elements, nets, surface area, and volume. Some student groups were able to correctly identify all the elements, while others could only identify some. Some student groups successfully described the nets with several different shapes. Some groups still made errors in the general formula for the surface area and volumes of cubes and cuboids. An example of knowledge construction about cubes and cuboids from a project that has been created is presented in Figure 3. Each group of students gave different results in constructing knowledge about cubes and cuboids. Figure 3 is one of the construction results from the student group. The process of finding your own understanding of cubes and cuboids is a very meaningful way for students.

### *Implementation of HLT 3*

The HLT focused on the application stage of learning. At this stage, students worked to understand all concepts related to cubes and cuboids using GeoGebra application and validated the understanding through practice problems. As students practiced the understanding through GeoGebra application, most were able to create accurate drawings of cubes and cuboids. Some students still struggled to completely identify all elements. Students also attempted to create nets of the shapes constructed to visualize how the nets formed complete spaces. In the surface area and volume subtopic, students were guided to calculate these values based on the dimensions of the spaces created and to match the spaces with the formulas previously learned.

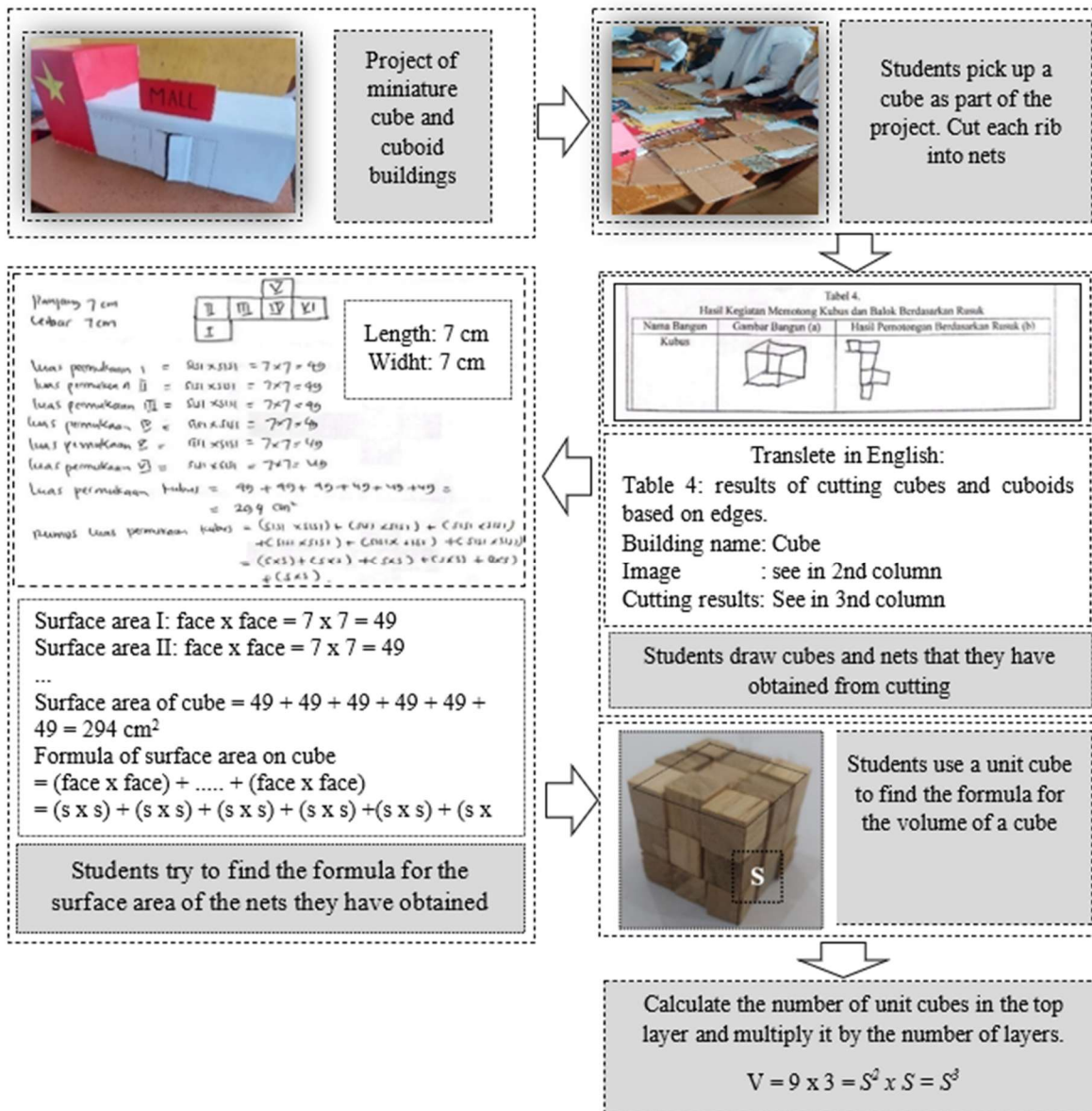


Figure 3: Students constructed knowledge on the topic of cubes through the discovery stage

### Implementation of HLT 4

HLT 4 focused on the communication stage of learning. Students were asked to present and explain the results of projects and to verbally communicate their comprehension. Due to time constraints, only group representatives participated in this activity. The scholar provided verbal validation when misunderstandings arose. Practice problems were given to assess students' application of the concepts which was learned about cubes and cuboids. The results showed that while some students

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still made errors in identifying the elements and properties of cubes and cuboids, all were able to draw the nets, though limited in variety. Some students knew the formulas for surface area and volume, but many still struggled when applying these to real-life problems.

## Teaching Experiment of Cycle 2

In teaching experiment cycle 2, every deficiency in HLT was corrected to obtain optimal results. HLT 5 was an improvement of the lesson plan from HLT 1 with a focus on incorporating the use of the internet to help students gather information for designing cubes and cuboids building projects, which was not included in the initial HLT. Based on the sketches, the result of the completed projects are depicted in Figure 4.

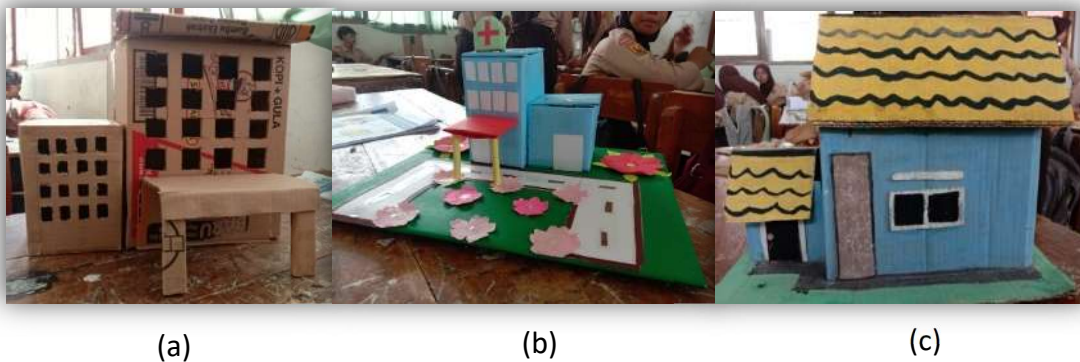


Figure 4: Students' cubes and cuboids building project in cycle 2

The results of projects in Cycle 2 showed a significant improvement in students' understanding of building with cubes and cuboids. Students further appeared to understand project purpose more effectively, and nearly all projects featured structures composed of cubes and cuboids. The improvement focused on extending the time allocated for students to explore, analyze, and build an understanding of cubes and cuboids sub-topic. In HLT 2, the time was set at 3 sessions of 40 minutes each for a meeting. Observations during the implementation of HLT 2 further showed that the duration was insufficient for students to fully grasp all concepts related to cubes and cuboids. HLT 6 increased the time to 5 sessions of 40 minutes each, spread over 2 meetings. HLT 7 there were no significant changes. All tasks are in accordance with the student's train of thought and no changes were made in HLT 8 from the implementation of HLT 4, as learning process met the research objectives. Students were allowed to present and explain the results of projects and showcase the understanding of the concept of cubes and cuboids. In HLT 8, students also practiced solving problems as an application of the understanding gained. The implementation results of

HLT 8 showed that 80% of students were able to explain projects relevant to the concept of cubes and cuboids, improving problem-solving skills.

### Improving Understanding of Students' Epistemological Learning Obstacles

Figure 5 shows an example of students' epistemological learning obstacles, namely limited context in understanding the properties of cubes and cuboids (EO-2) in Table 1.

Question: Mention what elements are in cubes and cuboids!

Answers (student 5):

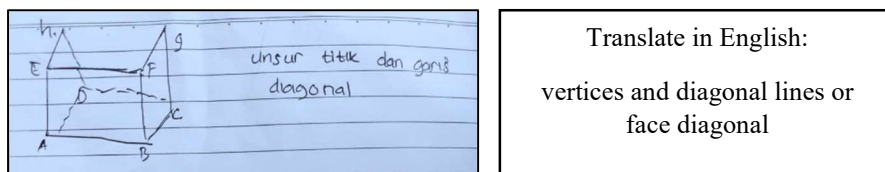


Figure 5: Student answers on epistemological learning obstacle

Figure 5 shows students' answers before learning by using project-based HLT. The researcher tried to ask questions related to the elements of cubes and cuboids. The students' answers showed that they did not know the elements. The correct answers were vertex, edge, face, face diagonal, space diagonal and diagonal plane, but student 5 only mentioned two of the supposed elements. This didactic situation shows the limited context of students in understanding each part of the cubes and cuboids.

To overcome the existing obstacles through project-based HLT, students make direct observations of the results of the miniature cube and cuboid building project. Students are asked to show each element in the cube and cuboid building. For example, students will take the cubes from the project product in Figure 2(a), then show and mention each element in the cube. This activity provides students with a direct learning experience, so that they easily remember and understand the concepts on the elements of cubes and cuboids. Through direct experience with projects, students are involved in problem solving and provide opportunities for students to construct their own understanding (Pablos et al., 2017). They can work together with a team for better understanding (Gay, 2022b). After the learning was conducted, students showed improvement on the limitations of previous understanding. Students can answer all elements in cubes and cuboids correctly. The didactical anticipation given is to give awards by saying 'great' to the correct group and provide scaffolding to students who are still mistaken by asking questions through pictures or project products directly.

## Improving Understanding of Students' Ontogenical Learning Obstacles

One of the most common ontogenical learning obstacles is low academic awareness and interest in learning (OO-5) and operational learning obstacles (OO-3) as shown in Table 2. This is illustrated by the results of the researcher's interview with the teacher of the school mathematics lesson.

- R (researcher) : *"In your opinion, what learning barriers have the most influence on students' success rate in learning math. Especially the material of cubes and cuboids?"*
- T (teacher) : *"Based on my years of teaching experience, I noticed that many students were not enthusiastic when the math lesson started. They arrive late, ignore the lesson time and do not focus during the lesson. Of course this affects any math material"*
- R (researcher) : *"Besides students' low interest in math, do you find other obstacles?"*
- T (teacher) : *"Yes, I do. Among the issues are students' low operational skills in arithmetic. When learning about the surface area or volume of cubes and cuboids, students are expected to understand multiplication and division, but many still make mistakes."*

The teacher's statement from the interview shows that students have low interest in mathematics, and many still make mistakes in basic mathematical calculations. This didactic situation indicates the presence of ontogenical learning obstacles. The teacher's statement was also proven when the researcher made observations at the beginning of learning. This illustrates the importance of applying learning methods that can attract students' interest and make the process of learning mathematics more enjoyable.

To overcome these obstacles, a project-based HLT was designed through a series of learning activities that involved making miniature building projects from cubes and cuboids. The learning started with a reflection activity, where students were asked to mention the types of buildings around them that contained cube or cuboid shapes. After that, they are asked to create miniature buildings that they like, according to the predetermined learning criteria. The products of this project are then used as learning media to understand various concepts in cube and block materials, such as the elements of a building, nets, surface area, and volume.

This series of activities gives students the space to be creative and imaginative. They can also work together in teams to produce the best product. This approach is a special attraction for students, so they are more motivated to learn the cube and block material with enthusiasm. The results showed significant positive changes in students' attitudes towards learning mathematics. They arrived on time when the lesson started and were more focused during the learning process. One student even said: *"I really like learning like this. This project makes us excited to learn maths. Can it be done all the time?"*. Another student expressed a similar opinion: *"Learning maths through projects like this is much more fun than just looking at the blackboard and books in class. I am always excited when the lesson starts"*. The students' statements reflect their increased academic awareness and interest in learning mathematics, especially in cubes and cuboids.

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## Improving Understanding of Students' Didactical Learning Obstacles

One of the didactical obstacles found in learning cubes and cuboids is the limited use of media and technology (DO-3, see Table 3). The results of observations and interviews show that teachers rarely use media or technology in explaining the material, and tend to teach traditionally. This obstacle disrupts the process of forming students' understanding, especially in imagining the shape of spatial shapes and developing their spatial imagination.

To overcome this, learning through project-based HLT is designed so that students gain a more concrete and visual learning experience. One of the activities is using unit cubes, which are arranged into larger cubes and cuboids. This activity helps students understand the concept of volume and surface area directly. In addition, at the application stage, students are given the opportunity to create digital models of cubes and blocks using GeoGebra software. The combination of using concrete media and technology is designed to improve students' spatial ability in understanding the concept of building space. The learning results showed a significant increase in student scores. This finding is in line with previous research which states that GeoGebra is able to improve students' spatial abilities and understanding of cube and cuboid materials (Gravemeijer & Eerde, 2009b; Miles et al., 2014b).

## Retrospective Analysis

The retrospective analysis stage was used to evaluate the overall HLT based on the results of the teaching experiments in cycles 1 and 2. These results illustrate the impact of HLT implementation in overcoming students' learning obstacles on the topics of cube and cuboid. Retrospective analysis involves a careful review of the data and a reflection on the process of the teaching experiment in order to develop an explanatory model of what induced the changes observed in the learning ecologies (Gravemeijer & Eerde, 2009b). According to the research objectives, the development of HLT aimed to address students' learning obstacles regarding the topic of cubes and cuboids. Learning obstacle was considered resolved when 80% of students no longer experienced it. The implementation of HLT in Cycles 1 and 2 showed that 85.7% of students' learning obstacles were effectively overcome. The improvement of students' learning obstacles on the topic of cubes and cuboids was presented in Table 4.

| Type of LO                                     | Pre-Class | Implementation of HLT |         | Pasca Class |
|--|-----------|-----------------------|---------|-------------|
|  |           | Cycle 1               | Cycle 2 |             |
| <i>Epistemological Learning Obstacles (EO)</i> |           |                       |         |             |
| EO-1   | √         | X                     | X       | X           |
| EO-2   | √         | √                     | X       | X           |

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| Type of LO                               | Pre-Class | Implementation of HLT |         | Pasca Class |
|--|-----------|-----------------------|---------|-------------|
|  |           | Cycle 1               | Cycle 2 |             |
| EO-3                                     | √         | √                     | X       | X           |
| EO-5                                     | √         | √                     | X       | X           |
| EO-6                                     | √         | √                     | X       | X           |
| EO-7                                     | √         | √                     | X       | X           |
| EO-8                                     | √         | √                     | X       | X           |
| EO-9                                     | √         | X                     | X       | X           |
| EO-10                                    | √         | X                     | X       | X           |
| <i>Ontogenic Learning Obstacles (OO)</i> |           |                       |         |             |
| OO-1                                     | √         | X                     | √       | √           |
| OO-2                                     | √         | X                     | X       | X           |
| OO-3                                     | √         | √                     | √       | √           |
| OO-4                                     | √         | √                     | X       | X           |
| OO-5                                     | √         | X                     | X       | X           |
| <i>Didactic Learning Obstacles (DO)</i>  |           |                       |         |             |
| DO-1                                     | √         | X                     | X       | X           |
| DO-2                                     | √         | √                     | √       | √           |
| DO-3                                     | √         | X                     | X       | X           |
| DO-4                                     | √         | X                     | X       | X           |
| DO-5                                     | √         | X                     | X       | X           |
| DO-6                                     | √         | X                     | X       | X           |

Table 4. Improvement of Learning Obstacles

Project-based HLT for learning cubes and cuboids prompted changes in students' cognitive, affective, and psychomotor skills. Cognitive improvements were particularly evident in overcoming EO. However, some issues such as OO-1, OO-3, and DO-2 were not fully resolved. Some students still misinterpreted problem statements, leading to incorrect problem-solving stages. Errors in basic calculations also persisted, as some students remained confused about unit conversions for both surface area and volume. Students were still not accustomed to solving non-routine problems, suggesting a need for changes in teaching methods by the classroom teachers. Significant changes in students' learning obstacles were observed from the pre-class stage to Cycle 1, and from Cycle 1 to 2. These changes showed that the designed HLT gradually helped students overcome learning obstacles.

Although this study quantitatively demonstrated a high level of success in overcoming learning barriers in understanding the geometry concepts of cubes and blocks, the qualitative data from the interviews revealed deeper insights into students' experiences in constructing their knowledge. One student gave his opinion after implementing the learning, "At first I didn't know how the surface area or volume formula came about, but after learning with this miniature building project, I understand better and can remember it better". This reflects the changes in students' reasoning generated through hands-on activities in acquiring new knowledge. Another student said, "I am very enthusiastic about learning with projects like this, I can exchange understanding with my friends

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and it helps me overcome what I don't understand". This statement shows the development of students' communication and problem-solving skills throughout the project.

During the learning process, the teacher gave feedback to each student's response to the task that was not in accordance with the expectations in the HLT by adjusting the didactical anticipation that had been prepared in cycle 1. This didactical anticipation was improved in cycle 2 to anticipate obstacles that might reappear. Improvements in feedback were made to encourage deeper exploration from students, by providing scaffolding such as: "What do you think?", "Let's try it again in another way", and "Try discussing it with your project team". These adjustments emphasize the dynamic relationship between plan and reflection in PBL that is continuously improved to overcome obstacles experienced by students.

## DISCUSSION

The reflection and research stages motivate students to recall previous knowledge about cubes and cuboids. Reflection helps students learn from past experiences, transforming surface learning into deep learning. This process is crucial in PBL, as reflection is considered an essential component (Duryan, 2023; Sense, 2007; Söderlund et al., 2008). When students are asked to develop projects including cubes and cuboids, students interpret their existing understanding and transform it into the knowledge required for project. Previously acquired knowledge and experience cannot be fully used in projects without reflection. This reflection is reinforced by research activities that help students gather information related to project. In PBL, students participate in authentic research, using and constructing knowledge similarly to scientists (Novak & Krajcik, 2019). In HLT1, students initially relied on school textbooks and group discussions for research, which limited the ability to imagine buildings shaped such as cubes and cuboids. An improvement over HLT1 for HLT5 allowed students to search for information online, helping to better design the building sketches.

Observations from the reflection and research stages show that students critically analyze cubes and cuboids concepts and identify relationships between the concepts to develop projects and integrate new knowledge. Through this process, students construct an understanding of cubes and cuboids independently. The implementation results showed that these stages significantly reduced students' learning obstacles. Project activities during the reflection and research stages motivate students to explore various concepts such as shapes, elements, properties, nets, and skeletons, addressing epistemological learning obstacles (EO1-EO5). PBL also stimulates curiosity, engagement, and motivation, as students take responsibility for learning while addressing real-world obstacles (Chiang & Lee, 2016; Hsu et al., 2024; Whelan et al., 2022).

In the discovery stage, students begin to explore the concepts of cubes and cuboids through guided questions leading to the conclusion of the concepts independently. As students create building projects such as houses, hotels, and hospitals, mathematical concepts are not only learned but also relate the relevance of the concepts to daily life. This stage allows students to identify the shapes, elements, nets, and skeletons of cubes and cuboids in projects. The questions on students' worksheets further deepen the understanding. The discovery stage had a significant impact on overcoming learning obstacles related to the materials, including epistemological (EO-1 to EO-10), ontogenic (OO-1, OO-4, OO-5), and didactic learning obstacles (DO-1, DO-4, DO-5). Observations from HLT2 and HLT6 show a gradual improvement in students' understanding of these concepts.

During the application stage, students successfully applied their understanding by developing and analyzing cubes and cuboids using GeoGebra application. Technology provides immediate feedback, allowing students to self-correct and adjust more effectively (Bright et al., 2024). GeoGebra offers a visually rich and interactive learning experience, making math more interesting and engaging (Cai et al., 2020). Students can visualize cubes and cuboids from various perspectives and observe how cube nets transform into solid shapes and vice versa. The retrospective analysis showed that students' understanding of materials improved significantly. Learning obstacles related to low spatial abilities (DO-3) and low motivation (OO-5) are effectively addressed using GeoGebra. This correlates with previous research showing that GeoGebra enhances students' spatial abilities and understanding of cubes and cuboids (Kunrade et al., 2023; Ramdaniah, 2020). Additionally, GeoGebra significantly increases student learning motivation in the classroom (Hidayat et al., 2023), as students eagerly collaborate to develop and analyze cubes and cuboids. Last, in the communication stage, students show significant improvement in cognitive, affective, and psychomotor skills. Each group effectively presents project results, clearly explaining the concepts related to the materials. Students strive to showcase the best outcomes of projects and actively engage in discussions during question-and-answer sessions, which helps validate the understanding. These activities build decision-making skills, communication skills, and evaluation skills (Munna & Kalam, 2021).

The retrospective analysis shows a positive impact of project-based HLT on students' understanding. This improvement is evident in students' performance on practice questions towards the end of learning and in the interviews with the students. These results correlate with previous research by (Astuti & Wijaya, 2021) showing that project-based HLT aids in understanding mathematical concepts. The research also shows that students successfully overcome the 3 types of learning obstacles including epistemological, ontogenic, and didactic. These learning obstacles correspond to the 3 domains of learning outcomes as outlined: cognitive, affective, and psychomotor (Bloom et al., 1956).

In the cognitive domain, students show improved understanding of concepts such as identifying elements and properties of cubes and cuboids, correctly applying formulas, overcoming the learning obstacles in visualizing 3-dimensional space, understanding the concept of volume in cubes and cuboids, and accurately using formulas. In the affective domain, students' academic awareness and interest in learning mathematics increase significantly. This enhancement in cognitive and affective domains parallels the improvement in students' psychomotor domain, particularly in spatial ability and learning independence, as showcased in projects. Generally, project-based HLT design supports meaningful geometry learning and effectively addresses learning obstacles related to cubes and cuboids.

During the implementation of project-based HLT, adjustments to the initial design were necessary, particularly concerning the time allocated for tasks. The original HLT design included 17 tasks with HLT2 containing 8 tasks completed in 1 meeting. However, during the retrospective analysis of Cycle 1, it became clear that more time was needed to ensure optimal construction of students' understanding. HLT5 is further extended to 2 meetings because students need more time to explore and develop the understanding of cubes and cuboids sub-topic. The freedom of time that teachers must give is an important concern in implementing PBL (Taylor et al., 2024). These results provide information to educational actors to anticipate more time when conducting PBL. An effort should be made to accommodate students' opportunities to construct knowledge more optimally.

## CONCLUSIONS

This research concludes that project-based HLT successfully overcame learning obstacles identified in the topic of cubes and cuboids. The analysis found that HLT had a positive impact on students' understanding of the topic. This conclusion was supported by the retrospective analysis of students' practice question results and interview data. Generally, the design of learning activities using project-based HLT proved effective in supporting meaningful geometry learning and overcoming obstacles related to cubes and cuboids.

Future research could explore the application of this project-based HLT methodology to other geometric topics such as prisms, pyramids, or coordinate geometry to assess its broader applicability. Additionally, testing its effectiveness across different educational levels or learning environments—such as rural schools, inclusive classrooms, or digital/blended learning settings—would provide valuable insights. Learning obstacles arose in implementing project-based HLT, particularly concerning time allocation. Project-based learning required more time compared to traditional methods. Therefore, teachers should carefully adjust the time allocated to match the number of tasks. Addressing the learning obstacle should be a priority for those planning to implement PBL activity designs.

These findings also can make a significant contribution to teacher training programs, particularly in developing teachers' skills in designing more structured, contextual, and student-centered learning experiences. By understanding how HLT can be used to anticipate and respond to students' thinking processes, teachers can design instructional strategies that are more adaptive and reflective of students' learning needs. In curriculum development, this approach can be utilized to design PBL sequences that are more exploratory and meaningful, while also promoting the integration of 21st-century skills such as collaboration, critical thinking, creative thinking, and problem-solving. A curriculum that accommodates HLT allows for flexibility in selecting activities that align with students' stages of thinking and the targeted learning outcomes.

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