Teachers' Conceptions of Critical Thinking in Mathematics Teaching

Duhwi Indartiningsih, Tatag Yuli Eko Siswono*, Wiryanto, Suryanti

State University of Surabaya, Indonesia

duhwi.22011@mhs.unesa.ac.id, *tatagsiswono@unesa.ac.id, wiryanto@unesa.ac.id, suryanti@unesa.ac.id

Abstract: The aim of this research is to describe teachers' conceptions of critical thinking in mathematics teaching. It also aims to evaluate critical thinking conceptions of teachers with high, moderate, and low conceptions. The research method used was Explanatory Sequential Mixed Methods with quantitative data collected sequentially through questionnaires distributed to 200 primary school mathematics teachers in in Sidoarjo District, East Java, Indonesia. Furthermore, qualitative data were collected through interviews and observations in the high, medium, and low categories. The results showed that teachers with high critical thinking conceptions had appropriate conceptions and were better able to design mathematics instruction. Teachers with medium and low critical thinking skills only had appropriate conceptions of the indicators, which makes them unable to design mathematics teaching activities.

Keywords: Teachers' Conceptions, Critical Thinking, Mathematical Critical Thinking, Mathematics Teaching

INTRODUCTION

Critical thinking is an indispensable skill in the 21st century (Tang et al., 2020; Toheri et al., 2020) because it improves the quality of thought (Prado-Arenas et al., 2022). This skill need to be incorporated into learning due to the relation with daily activities, which can affect the progress level (Nurdiana et al., 2023). According to Julianto et al. (2023), the skill also triggers self-actualization procedures in social, educational, and work life. An individual who possess this skill is confident that the decisions made were properly analyzed (Gass & Seiter, 2019). Therefore, students must utilize critical thinking at all levels of education, particularly for all elementary school. Some experts state that the skill is focused on resolving inferences on what to accept or execute (Ennis, 2015). The skill is a thought process aimed to solve problems, interpret meaning, and obtain proofs (Facione, 2015). It also refers to the capacity to assess and interpret a particularity by searching for relevant data or evidence before deciding to address the problems, assess circumstances, and take appropriate action (Hunaepi et al., 2020). This skill is adopted by developing or practicing ways to solve problems (Sutama et al., 2022), including exploring existing or relevant information,

as well as conducting analysis, synthesis, evaluations, and drawing inferences (Ulfiana et al., 2019). Critical thinking can also be adopted through mathematics teaching training. Previous research reported that mathematics was a science that evaluated critical reasoning, requiring careful thought and observation procedures due to its usefulness in daily activities (Munawaroh et al., 2018).

In Indonesia, the students' mathematics learning outcomes are currently unsatisfactory. Based on several research studies, students exhibit poor critical thinking skills in mathematics learning at all levels of education, including elementary school (Sarwanto et al., 2020). These students also experience many obstacles in solving related problems, resulting in unsatisfactory learning outcomes. This simply proves that critical thinking skill have not been properly adopted (Delfia et al., 2020; Maulidiya & Nurlaelah, 2019). The skill is lacking in elementary school students due to teachers' inability to understand the concept (Nurbaeti et al., 2019). It is one of the objectives achieved in learning mathematics, hence the need for teachers to consider the use of learning conceptions to obtain the desired objectives (Scristia et al., 2018).

Regarding this description, Thompson (1992) stated that teachers' perception of desired objectives play a role in the adoption of appropriate classroom activities, as well as preferred instructional methods, legitimate mathematical procedures, and acceptable outcomes (Rodriguez-Muniz et al., 2022). Critical thinking can be adopted in mathematics teaching by designing learning activities that direct students according to the stipulated objectives (Lusiana et al., 2024). In addition, teachers' conceptions of the skill in mathematics teaching are based on its understanding, including mathematical critical thinking relevant indicators, and teaching design alongside related learning activities.

The lack of conception by teachers, including associated learning activities, leads to difficulty in designing lessons that can develop students' critical thinking skills. Prior research reports the need to understand and practice teaching designs that develop critical thinking (Gunawan et al., 2020). Moreover, adopted teaching design requires specific learning strategies, basic questioning tactics, the ability to draw inferences, conduct analysis, synthesis, and evaluation (Acharya, 2018). Teachers' conceptions are relevant attributes that must be considered in teaching. In respect to critical thinking, it is essential to design mathematics teaching that fosters the skill through interactive and student-centered teaching methods (Moodley & Chetty, 2024).

Recent research focused on investigating secondary school teachers' conceptions of critical thinking. Data was collected by distributing questionnaires to 77 teachers, and the results showed the perception of an advanced reasoning competency, including reflective and careful thoughts alongside other mind related habits, processes, or strategies, such as curiosity, exploration, information gathering, and intellectual caution (Ma et al., 2023). The skill focused on the need for teachers to develop critical thinking with training programs used to improve teachers understanding and practice in the classroom (Oner & Aggul, 2022).

Based on the description above, several research studies are focused on elementary school teachers' conceptions of critical thinking in mathematics teaching, leading to the formulation of the following questions:

1. What is teacher's conception of critical thinking in mathematics teaching?
2. Why are teachers with high conceptions able to design mathematics teaching that fosters critical thinking compared to those with low or moderate conceptions?

The current research focuses on elementary school teachers of mathematics, investigating in depth three teachers who had high, medium, and low conceptions in designing related teaching activities that can develop critical thinking. The critical thinking conceptions are used to determine the methods used by teachers to design mathematics instruction and facilitate future practice.

LITERATURE REVIEW

Teacher Conception

Conception refers to personal understanding or interpretation of a concept, resulting in differing experience. This is outlined by Philips' perception of conception as a mental framework consisting of beliefs, meanings, concepts, propositions, rules, mental representations, and preferences (Rodriguez-Muniz et al., 2022). Conceptions are conscious and unconscious cognitive and affective beliefs that indicate the abilities and experiences that one has. (Amirali, 2010). According to Thompson (1992), conceptions include considering the desired objectives of mathematics teaching, roles played by teachers, and students, appropriate classroom activities, methods, and preferred instructional emphases, as well as valid procedures and acceptable results (Rodriguez-Muniz et al., 2022).

Based on previous research, teacher's conception refers to the understanding or interpretation of a concept. In respect to mathematics teaching, it is closely related to the understanding of critical thinking, mathematical critical thinking, indicators, and teaching design as shown in Figure 1.

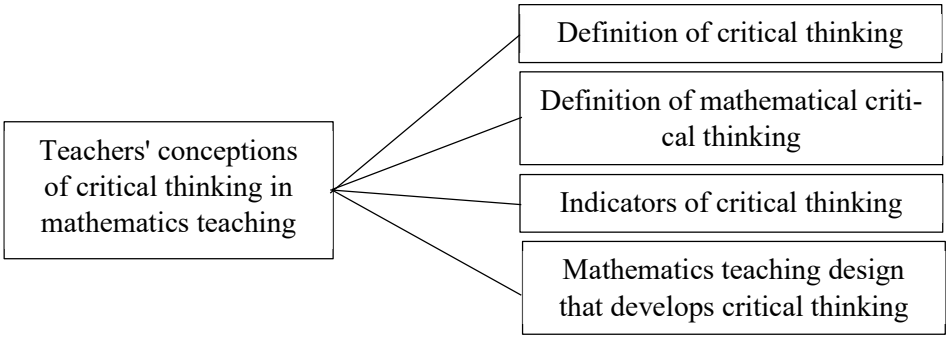


Figure 1: Teachers' conceptions of critical thinking in mathematics teaching

Critical Thinking and Critical Thinking Indicators

Critical thinking is a reflective thought that focuses on making decisions about what to believe or execute (Ennis, 2016). This thought process aims to solve problems, interpret the meaning of a circumstance, and prove drawn inferences (Facione, 2015). Previous research states that it also referred to as the capacity to accurately assess and explain phenomena by collecting related data or evidence before making choices, solving problems, assessing circumstances, and taking action (Hunaepi et al., 2020). Critical thinking is a clear and structured mental activity that can involve processes such as solving problems, analyzing existing assumptions, and making decisions based on evaluations (Fitriani et al., 2019; Munawaroh et al., 2018). It is a metacognitive process that includes the ability to analyze, synthesize, evaluate, and summarize ideas or solutions effectively to solve problems (Fitriana et al., 2023). Critical thinking is a skill that analyzes, evaluates, assesses, and justifies misinformation (Saputra et al., 2019). This includes the ability to examine assumptions, distinguish hidden values, assess, evaluate evidence, and draw inferences (Bayuningsih et al., 2018).

The possession of critical thinking skill can be viewed based on the following indicators: interpreting, analyzing, evaluating, concluding, explaining, and self-regulating (Facione, 2015). According to Ennis (2016), these indicators include basic clarification, giving reasons for a decision, drawing inferences, and further clarification. Critical thinking indicators consisted of clarification, judgment, conclusion, and strategy (Corich et al., 2007). Facione (2015) reported that these indicators were selected because individuals who think critically must be able to have self-regulation and re-examine the resolution of problem, ensuring appropriate decisions were taken. Self-regulation also enables the ability to think strongly and critically (Facione, 2015). An individual with great self-regulation is grouped in the high critical thinking category (Bayuningsih et al., 2018). Considering the definition by experts, it is inferred that critical thinking is a thought process in making reasoned decisions through analysis, evaluation, and conclusion based on existing and relevant evidence. The indicators include interpreting, analyzing, evaluating, concluding, explaining, and self-regulating.

Mathematical Critical Thinking

Mathematical critical thinking is a thought process in making reasoned decisions in solving related problems through analyzing, synthesizing and evaluating skills based on existing and relevant evidences (Saragih & Zuhri, 2019). Abilities and dispositions combined with knowledge, mathematical reasoning skills, and cognitive strategies to solve relevant problems (Nurbaeti et al., 2019). Mathematical critical thinking refers to the capability to solve related problems with the ability to understand, recall, distinguish, analyze, provide reasons, reflect, interpret, search for relationships, evaluate, and make temporary conjectures (Sahidun et al., 2023). This skill was used to analyze, evaluate or assess, and summarize opinions or solutions appropriately to solve a problem (Saputra

et al., 2019). Based on the diverse definitions from experts, it was inferred that mathematical critical thinking is a reasoned thought process in solving related problems by analyzing, evaluating, and drawing inferences on existing and relevant evidence.

Mathematics Teaching Design that Develops Critical Thinking

Critical thinking can be developed early, starting from elementary school (Siswono, 2016). In mathematics, effective teaching practices and student engagement play a significant role (Yanuari & Turmudi, 2023). This skill can also be absorbed by giving problems that develop one's ability to analyze and evaluate (O'Reilly et al., 2022). Teachers should give non-routine problems and allow students to argue or express respective opinions, analyze, or evaluate it, while drawing inferences (Willingham, 2019). The provided questions in discussions and role play in problem solving also aid in developing this skill (Toheri et al., 2020).

Teachers use learning methods or models in designing math lessons in accordance with critical thinking indicators. Previous research states that teaching activities using Realistic Mathematics Education (Cahyaningsih & Nahdi, 2021; Hikayat et al., 2020), scientific methods (Nurhikmayati & Jatisunda, 2018; Sukarna & Iman, 2019), problem-based (Aini et al., 2019; Maulidiya & Nurlaelah, 2019; Tanjung et al., 2020), Cooperative (Afrilianto et al., 2022; Siswono et al., 2019), and Discovery learning (Chusni et al., 2020, 2022; Putri et al., 2020) aid in the development process. Teaching procedure is designed to train certain abilities in line with critical thinking indicators. These include interpretation, analysis, evaluation, explanatory conclusions, and self-regulation (Seventika et al., 2018). The following steps based on the indicators are adopted in teaching mathematics.

A. Interpretation refers to the ability to understand and explain the meaning of information or an event. Teachers propose math problems at the beginning of learning activities, guiding students to identify and organize relevant information, as well as ensuring these are understood by correctly writing the known information and question asked.
B. Analysis is the ability to identify the relationship between questions, statements, concepts, descriptions, or other representations intended to express beliefs, judgments, experiences, reasons, information, or opinions. Teachers direct students with guided questions to find the relationship between statements and concepts presented, ensuring problem solving strategies are determined.
C. Evaluation is described as the ability to assess the credibility of questions or other representations in the form of reports or descriptions of perceptions, experiences, situations, judgments, beliefs, or opinions. This includes interpreting the logical strength of referential correlations or other intended representations. Teachers guide students to make judgments regarding the value of an idea, creation, or method used in solving problems.
D. Inference refers to the ability to draw logical conclusions, make assumptions, and generate hypotheses, while considering relevant information, consequences, situations, questions, and

other representations. The students are guided to make reasonable inferences by considering relevant information in problem solving.

E. Explanation is the capability to articulate results, as well as reasoning clearly and concisely. Students are facilitated to explain the results, procedures, and arguments related to the solution of the problem.

F. Self-regulation depends on the ability to monitor one's cognitive activities and elements used in problem solving, particularly analysis and evaluation. Teachers ask students to re-examine the working process and engage in self-reflection.

METHODS

This research adopted the Explanatory Sequential Mixed Method, a combination of quantitative and qualitative methods to provide a comprehensive understanding of complex phenomena. The methods consist of two phases namely the collection of quantitative, and qualitative data to describe quantitative results (Sofiyana et al., 2022). In this regard, quantitative data collection was conducted by distributing critical thinking conception questionnaires to elementary school teachers in Sidoarjo district, East Java, Indonesia. Meanwhile, Sidoarjo district was selected as the research sample because it is one of the most populous districts in East Java that is characterized by a rich and diverse culture. The region is closer to the city center of Surabaya, the capital of East Java province, a large area with a diverse population in terms of education and economic factors. The criteria for selecting the research sample were teachers with three years of experience teaching mathematics in grades III, IV, V, or VI. The number of elementary schools in Sidoarjo district amount to 585 schools. Of the 585 schools, 200 teachers met the sample selection criteria. Quantitative data were collected by distributing questionnaires consisting of 14 statements prepared in Bahasa, Indonesia. These were based on a grid related to conception of critical thinking, mathematical critical thinking, indicators, and teaching design. The questionnaire was assessed using a Likert scale, namely strongly agree, agree, disagree, and strongly disagree, as shown in Table 1.

Aspect	Description	Statement	
		Positive	Negative
Conception of critical thinking	Statements related to teachers' conceptions of critical thinking.	3	2
Conception of mathematical critical thinking	Statements related to teachers' conceptions of mathematical critical thinking.	2	1
Conception of critical thinking indicators	Statements related to teachers' conceptions of critical thinking indicators.	1	1
Conception of mathematics teaching design that develops critical thinking	Statements related to teachers' conceptions of teaching design that can develop critical thinking.	2	2

Table 1: Questionnaire grid for critical thinking conceptions

Each positive and negative statement show an appropriate and inappropriate conception, respectively. For positive statements, a larger scale shows a more appropriate conception of critical thinking in mathematics teaching. Meanwhile, for negative statements, a larger scale implies an inappropriate or incompatible conception. For example, a participant's answer agreeing with a negative statement implies a teacher has an incompatible conception of critical thinking in mathematics teaching. The critical thinking conception questionnaire was validated by two experts in mathematics education, a language expert, and two mathematics teachers.

The responses of the 200 teachers were converted to scores, for positive statements, the answers "strongly agree", "agree", "disagree", and "strongly disagree" were converted to the following scores 4, 3, 2, and 1. For negative statements, the answers "strongly agree", "agree", "disagree", and "strongly disagree" were converted to scores 1, 2, 3, and 4. The scores of each participant were then summed up, with a typical example of the calculated critical thinking questionnaire response score shown in Table 2.

Respondent 1	Response	Score
Statement 1 (Positive statement)	Strongly Agree	4
Statement 2 (Positive statement)	Agree	3
Statement 3 (Negative statement)	Disagree	3
etc	....	...
Total score		35

Table 2: Example of response score calculation

The formula for determining the percentage value of the questionnaire statements answered by respondents is shown in Figure 2 as follows:

$$\text{Percentage value} = \frac{the\ score\ obtained}{maximum\ score} \text{ x } 100\%$$

Figure 2: Percentage value formula

The percentage values obtained were then categorized for critical thinking conceptions as in Table 3.

Percentage value interval (%)	Category
$81{,}25 < x \leq 100$	Very high
$71{,}50 < x \leq 81{,}25$	High
$62{,}50 < x \leq 71{,}50$	Medium
$43{,}75 < x \leq 62{,}50$	Low
$0 < x \leq 43{,}75$	Very low

Table 3: Category of the percentage value of the results of critical thinking conception questionnaire

The subjects were selected according to the research objectives (Purposive sampling), namely teachers with very high or high critical thinking conceptions, another from the medium, and very low or low categories. Qualitative data were collected through semi-structured interviews and observation of lesson plans prepared by the three selected teachers. Additionally, to reduce the possibility of information sharing, each teacher was interviewed separately. The initial step was the confirmation of willingness to be interviewed. The interviews focused on exploring teachers' conceptions of critical thinking in mathematics teaching, alongside respective instructional designs. The results of the interview were transcribed and analyzed to show each teacher's conception of critical thinking and design in mathematics teaching.

RESULTS

Quantitative Data Analysis

The results of the questionnaire filled out by 200 teachers, showed 37 (18.5%), 159 (79.5%), and 4 (2%) had critical thinking conceptions in teaching mathematics with low categories, medium, and high categories, respectively. This implied that out of 200 teachers who filled out the questionnaire, 4 had appropriate critical thinking conceptions, while the remaining 196 had inappropriate conceptions in teaching mathematics. Based on the results obtained, one teacher from the high, medium, and low categories were selected to be interviewed regarding conception of critical thinking and design of mathematics teaching that enables the development of this skill.

Qualitative Data Analysis

Semi-structured interviews were conducted with three teachers selected as research subjects to collect qualitative data on conceptions of critical thinking, mathematical critical thinking, indicators, and mathematics teaching designs. More importantly, teaching design made by the three teachers to investigate learning activities that exhibited the development of critical thinking was observed. The results of the interviews were transcribed and coded R, S1, S2 and S3 for the research, teachers with low, medium, and high conceptions of critical thinking, respectively.

I. Conceptions of Critical Thinking

R: What do you mean by critical thinking?
S1: Critical thinking is a thought process associated with solving problems. An individual who can solve problems in several ways.
S2: Critical thinking refers to the ability to solve problems quickly using various methods.
S3: Critical thinking describes a meaningful thought process applied when solving problems based on accountable answers.

Building upon the description above, S1, S2, and S3 had different conceptions of critical thinking. S1 and S2 had similar conception related to critical thinking, namely the ability to solve problems in various ways. According to S3, critical thinking is a meaningful thought process applied when solving problems based on available evidence and the accountability of the answer.

II. Conceptions of Mathematical Critical Thinking

R: What is meant by mathematical critical thinking?
S1: The ability to answer questions or solve mathematical problems quickly.
S2: The ability to solve mathematical problems using different methods.
S3: Thinking reasonably when solving mathematical problems and using evidence to determine the appropriate answer.

Conceptions of S1, S2, and S3 differ in respect to mathematical critical thinking. According to S1, mathematical critical thinking refers to the ability to solve related problems quickly. Regarding this perspective, S2 described it as the ability to solve problems by using different methods. In this context, S3 stated that mathematical critical thinking is the process of applying meaningful reason and evidence to account for the right answer.

III. Conception of Critical Thinking Indicators

R: Describe the indicators of critical thinking?
S1: An individual who can give an explanation of the problem solution, understood and recognized by others.
S2: One who is able to convey reasons related to decision making from problem solving, clearly and appropriately.
S3: A person who is able to correctly identify answers based on facts, as well as evaluate which solutions were appropriate, an indication of critical thinking. In addition, critical people can also explain the reasons behind the selected answers.

Certain differences were detected in conceptions of S1, S2, and S3 regarding critical thinking indicators. According to S1 and S2, the indicators refer to the ability to explain the reasons for decision making related to problem solving. S3 defined it as being able to determine the correct answer based on evidence obtained from the analysis precisely conducted, including explaining the reasons behind the decisions taken.

R: Based on critical thinking indicators, are you a critical thinker? Explain the reason for your answer?
S1: I am critical thinker because I always prioritize quick thinking when solving problems.
S2: I am not critical thinker because I solve problems using the same method.
S3: I am critical thinker because I am able to determine which solution is appropriate for the problem based on existing evidence. Prior to solving the problem, I understand it, while deciding on the right solution.

In line with the description above, it was reported that S1 and S3 were believed to be critical thinkers. According to S1, being a critical thinker entails the prioritization of thinking quickly when solving problems. S3 stated that critical thinkers solve problems by determining the right solution based on existing evidence. In addition, they believe problems are solved by understanding and thinking of the right way to address the issue. S2 considers himself not a critical thinker because he uses the same method when solving problems.

R: What steps were adopted when working on math problems?
S1: I try to understand the problem, then find information on Google...including whether there any similar problems? ...this resulted in the modelling of the solution.
S2: I read and understood the problem then tried to solving it by adopting commonly used methods.
S3: I read and understood the problem, before writing the relevant information. I try to think of a way to answer it, and search for the solution.

The problem-solving steps adopted by S1, S2, and S3 were different. S1 solved the problem by searching for examples of existing or common solutions, then modifying the procedure. However, S2 solved the problem by understanding it and adopting commonly used methods. S3 solved the problem by understanding and writing relevant information, as well as trying to think of a solution or answer.

IV. Mathematics Teaching Design that Develops Critical Thinking

R: What are the learning activities associated with math teaching that can develop critical thinking?
S1: I give practical problems to students to solve and these were discussed together. The frequent practicing of math problems, helped develop students' critical thinking skills.
S2: Giving mathematical problems and providing guided triggering questions, that would enable students understand it and find solutions.
S3: Each group was given a problem and then asked to engage in discussions when solving the question. The students were guided by reading the problem together to understand it, followed by compiling relevant information regarding what is known and asked. This also included finding a way to solve the problem ... with what formula ... etc. The students were also asked to write the inferences drawn from the answer.

S1 stated that learning activities associated with developing critical thinking include the practice of mathematical problems and engaging in discussions. S2 provided mathematical problems and triggering questions that helped in understanding and finding a solution. S3 divided the students into groups and each were given a math problem, which was discussed together. The students were further guided by giving trigger questions that enabled an understanding of the problem. This also included writing relevant information, the formula used, and inferences drawn from the answers.

R: What teaching models or methods were applied to design mathematics teaching that developed critical thinking? Explain the reasons for your response?
S1: Inquiry method. The students were invited to find formulas or answers to math problems guided by teacher. This included asking questions that invited students to think about the stages of problem solving.
S2: A scientific method was adopted by giving students mathematical problems to practice.
S3: The problem-based model was used to design learning that develops critical thinking. Mathematical problems were given and students motivated to discuss the questions in groups.
R: Have you ever designed a lesson using this learning method or model?
S1: Never. I use existing teaching plans or copy those on Google.
S2: I have made one.
S3: I have also made teaching plans.

S1's answer was related to the learning model used to design lessons that develop critical thinking, namely the inquiry method. However, S1 never made a mathematics teaching design using an inquiry method. S2 stated that the use of a scientific method, as well as giving mathematical problems, develops critical thinking. Building upon the discussion, S2 prepared a mathematics teaching design using a scientific method. S3 stated that problem-based learning can develop critical thinking by starting with problems students needed to solve. Mathematics teaching design was made using a problem-based learning model.

R: Do you have any problems in designing mathematics teaching activities that develops critical thinking?
S1: I do not know how to make the designs, occasionally I search for it on Google. But I do not know if its right or wrong.
S2: I do not understand learning activities that develop critical thinking.
S3: I am still confused about the steps adopted, despite having made the designs, I do not know if it is right.

Similar obstacles were experienced by S1, S2, and S3 in designing mathematics teaching that enabled the development of critical thinking skills. S1 did not understand how to design lessons and tried to find examples on Google. However, S1 could not decipher the appropriateness of the examples on Google. S2 lacked an understanding of learning activities that developed critical thinking. Considering this perspective, S3 made a mathematics teaching design that developed critical thinking but doubts the accuracy of the steps adopted.

S1, S2, and S3 were assigned with the task of designing a mathematics teaching plan that could develop critical thinking.

R: Make a math lesson plan that develops critical thinking!

The following is a diagram of the math lesson plan made by S1 shown in Figure 3, using a project-based learning model with material on the characteristics of flat shapes.

Material: Characteristics of flat shapes with project-based learning model.
1. Understand the main problem
   a. The teacher divided students into three groups, with each consisting of three to four students.
   b. Students were shown regularly used objects with flat shapes such as books, triangular rulers and origami paper.
   c. The teacher asks, what shape is this origami paper? This included the shape of other objects.
2. Planning the project
   a. Students were shown a video by the teacher, regarding the characteristics of flat shapes.
   b. Discussion activities were carried out in each group, with students directed to complete the tasks on the worksheet.
   c. Instructions or signs was listened to in preparing products, namely making various flat shapes from folding paper.
3. Presenting project results
   Students present the results of the project, namely the products they have made.
4. Evaluating and reflecting on the process and results of the project
   Students from other groups give responses to the group that is presenting.

Figure 3: S1's mathematics teaching design

S1 designed the learning process into four phases. First, the teacher shows the students pictures of different objects, and asks them to identify the shapes, grouping those that are similar. The teacher asks questions related to the characteristics of the surface shapes. In the second phase of project planning, a video is played regarding the characteristics of flat surfaces, as well as instructing students to plan the task of making various flat shapes using origami paper and arranging it in a creative manner. The third phase focuses on the provision of opportunities for students to present the results of respective projects. The fourth phase focuses on evaluating and reflecting on the project results, with the teacher offering each group an opportunity to review the results of others.

The following is a diagram of the mathematics teaching design made by S2 (Figure 4). Based on this description, teaching design was realized using scientific method for numbers 1001 – 5000.

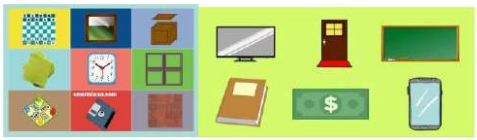
1. The teacher showed the number flashcards 1001 to 5000.
2. Learners mentioned the numbers according to the cards shown by the teacher.
3. The teacher distributed ice cream sticks and student worksheets to each group.
4. Learners were asked to count and write the number symbols.
5. While the learners were working on the LKPD, the teacher assessed the observation sheet of the students.
6. Learners presented the executed tasks.
7. This included working independently on numbers 1001 to 5000.
8. While the work was ongoing, the teacher assisted mostly those in the low groups.

Figure 4: S2's mathematics teaching design

The learning activities designed by S2 were in accordance with the scientific method of observing, questioning, reasoning, attempting, and communicating. The teacher shows the number cards and students read the written number symbols. Meanwhile, students are grouped and assigned with the task of counting the number of ice cream sticks and writing the symbols on the available worksheets.

The following is a diagram of the mathematics teaching design that develops critical thinking made by S3 (Figure 5). In addition, a problem-based learning model focused on the characteristics of shapes, particularly rectangles and squares, was adopted.

Learning activities with problem-based learning model
Phase 1: Orientation of learners concerning the problem
a. Students observe the pictures presented by the teacher.



b. The teacher inquires about the shapes of the object surfaces in the picture. The students were assigned to group the surfaces with similar shapes.
c. The teacher inquired about names of flat shapes similar to the object surfaces in the picture.

Phase 2: Organizing students to learn
a. The teacher divided students into two groups
b. Students joined respective groups
c. The guidance of the teacher led to the discussion of the differences between square and rectangular shapes.

Phase 3: Guiding individual and group investigations
a. Each groups started to investigate the characteristics of squares and rectangles, writing it on the worksheet.

No	Shape	Characteristics
1		
2		

Phase 4: Develop and present work
a. After the discussion on the properties of the square and rectangle, students continue to conclude and write on the worksheet.
b. Each group presents the conclusion of the properties of the square and rectangle.

Phase 5: Analyze and evaluate the problem solving process
a. Other groups evaluate the presentation results and the teacher provides confirmation regarding the results of all groups' presentations.
b. Students discuss the differences and similarities of the square and the rectangle.

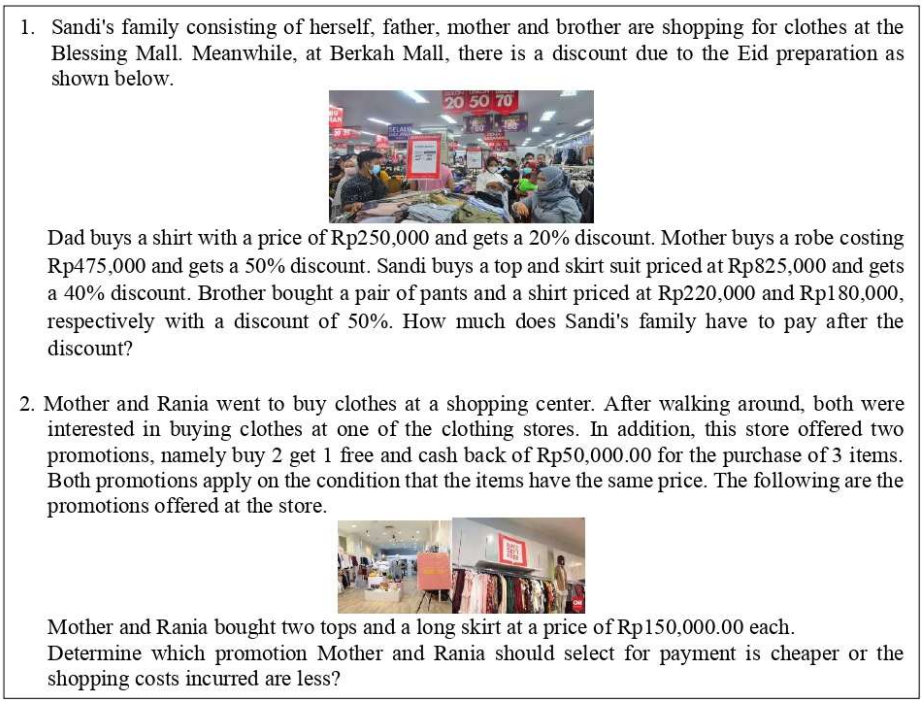
Figure 5: S3's mathematics teaching design

The math teaching design made by S3 is divided into five phases. In the first phase, the teacher provides pictures of objects with rectangular and square surfaces, assigning students to group it based on similar surface shapes. In the second phase, groups are formed to complete the task of observing the surfaces of rectangular and square objects. However, in the third phase, the teacher guides and provides the groups with worksheets to write relevant information related to the properties of rectangles and squares based on its sides and angles. For the fourth phase, students write respective results and draw inferences from the difference between a rectangle and a square. In the fifth phase, each group presents the results of analyzing and evaluating the properties of rectangles and squares from the surrounding objects and images.

R: Describe the characteristics of mathematical problems or issues that can develop critical thinking?
S1: Problems can be solved using diverse formulas, particularly those with a high level of difficulty.
S2: Problems that can train the ability to analyze and evaluate.
S3: Higher order thinking problems.

S1 stated mathematical problems that aid in the development of critical thinking are those with high level of difficulty and can be solved in various ways, using more than one formula. According to S2 and S3, high-level thinking mathematical problems enable the development of critical thinking.

R: Look carefully at the questions in Figure 6 and choose the one that can develop critical thinking! Give reasons why you selected a particular problem!


1. Sandi's family consisting of herself, father, mother and brother are shopping for clothes at the Blessing Mall. Meanwhile, at Berkah Mall, there is a discount due to the Eid preparation as shown below.

Dad buys a shirt with a price of Rp250,000 and gets a 20% discount. Mother buys a robe costing Rp475,000 and gets a 50% discount. Sandi buys a top and skirt suit priced at Rp825,000 and gets a 40% discount. Brother bought a pair of pants and a shirt priced at Rp220,000 and Rp180,000, respectively with a discount of 50%. How much does Sandi's family have to pay after the discount?

2. Mother and Rania went to buy clothes at a shopping center. After walking around, both were interested in buying clothes at one of the clothing stores. In addition, this store offered two promotions, namely buy 2 get 1 free and cash back of Rp50,000.00 for the purchase of 3 items. Both promotions apply on the condition that the items have the same price. The following are the promotions offered at the store.

Mother and Rania bought two tops and a long skirt at a price of Rp150,000.00 each.
Determine which promotion Mother and Rania should select for payment is cheaper or the shopping costs incurred are less?

Figure 6: Math problems

S1: Problem number 1 due to the discount. The students were expected to calculate each of the question to determine the shopping payment, and this was quite difficult. This led to the need for students to think harder to solve the problem.

S2: Problem number 2 because students must determine the shopping promotion that is cheaper. Therefore, students must assess which of the two promotions was the most profitable and cheaper.
S3: In my opinion, critical thinking question is number 2 because the problem required determining which promo should be selected for cheaper shopping payments. First, calculate the purchase of clothes with both promos individually. Second, compare which is cheaper and finally determine the promo to select.

S1 selected problem number 1 as a mathematical problem that can develop critical thinking. This was because problem number 1 included many steps and must be calculated individually, which is quite difficult. Meanwhile, S2 and S3 selected problem number 2 as a mathematical problem capable of developing critical thinking because students determine which promotion to take to enable cheaper shopping payments. This also implies evaluating and drawing inferences about the selected promotion.

R: What are the difficulties faced by teachers in terms of designing mathematics teaching that develops critical thinking? What lesson plan should be prepared to develops students' critical thinking?
S1: I do not understand critical thinking, and I am confused about the steps that should be adopted during teaching. Based on the result, teachers need to possess this skill, when preparing lesson ideas and challenges that foster critical thinking. I have never participated in training and do not read enough books thereby lacking the relevant knowledge.
S2: I do not know the steps associated with teaching math that develop critical thinking. Moreover, there is need for teachers to possess adequate understanding, of how to impact the learning activities related to the development of students' critical thinking. I have never attended training for teachers, particularly regarding critical thinking, so I search for information on YouTube, while reading several books.
S3: I still do not understand the steps related to teaching math that develops critical thinking. Teachers must have proper knowledge of the skill, because it cannot be taught if lacking. Moreover, teachers also need to be prepared, including possessing fundamental thinking skills. I had to attend teacher trainings and reading books to improve personal knowledge and teaching skills.

The answers provided by the three teachers were related to the readiness in teaching mathematics to develops students' critical thinking. S2 and S3 stated the relevance for teachers to possess a strong understanding of the skills and related steps. According to S1, teachers must also have the ability to think critically before impacting students. S3 improved personal knowledge and teaching skills by attending training sessions and reading books. S2 had never attended training but sought out information from YouTube and reading books. In line with this discussion, S1 read less books and had never attended training.

DISCUSSION

I. Conceptions of Critical Thinking

S1, S2, and S3 had different conceptions of critical thinking, based on their respective responses. According to S3, critical thinking is a thought process that enables solving problems and results in accountable answers. This conception was in accordance with the statements made by several experts. Facione (2015) reports that critical thinking focuses on proving a point, meaningful interpretation, and solving problems. It is also defined as a reflective thinking process that focuses on making decisions about beliefs or activities executed (Ennis, 2015). However, S1 and S2 stated that critical thinking is the ability to solve problems in various ways using different methods. This conception of critical thinking is inappropriate due to the direction towards the notion of creative thinking, namely a person's ability to solve problems in more than one way or solution (Nasution et al., 2021; Wijayanti et al., 2021).

II. Conceptions of Mathematical Critical Thinking

Differing conceptions of mathematical critical thinking was observed based on responses of S1, S2, and S3. According to S3, this factor refers to thinking reasonably when solving mathematical problems, including using evidence to determine the right answer. This conception is appropriate and in line with the result that mathematical critical thinking is a thought process in making reasoned decisions when solving related problems through analyzing, synthesizing, and evaluating skills based on existing and relevant evidence (Saragih & Zuhri, 2019). However, S1's conception is not appropriate because solving problems quickly does not necessarily mean that the solution is precise and accurate. The problem-solving process requires mathematical reasoning and solution strategies (Nurbaeti et al., 2019), as well as higher-order thinking procedures that use the capacity to investigate, assess, and make provisional predictions of the solution (Sahidun et al., 2023). S2's conception is also not appropriate due to the similarity with mathematical creative thinking, namely an individual's ability to create new ways of solving problems (Suherman & Vidákovich, 2022).

III. Conception of Critical Thinking Indicators

S1 and S2 reported that one of the indicators of critical thinking is the ability to explain or argue about certain solutions. However, S3 gave a detailed explanation of this factor, as well as the reasons for making decisions, analyzing, and evaluating relevant evidence by determining the appropriate solution or answer to the problem. Inference is drawn after evaluation to decide the final result of the decision made. Critical thinking indicators consist of interpretation, analysis, evaluation, inferences, explanation, and self-regulation (Cruz et al., 2020; Seventika et al., 2018; Tohir et al., 2021). Based on the analysis of S1, S2, and S3 interview answers, S1 and S2 were able to explain one of critical thinking indicators. S3 provided an in-depth explanation of critical thinking indicators.

S1 is believed to be a critical thinker because of the ability to solve problems quickly. This reasoning does not portray the characteristics of mathematical critical thinkers. Previous research stated that critical thinker can be viewed from the ability to solve problems based on critical thinking indicators (Facione, 2015). S2 is not a critical thinker due to the inability to use methods different from the ways adopted by others. This result is not in accordance with the characteristics of critical thinkers. S3 is believed to be a critical thinker because they solve problems based on understanding the problem and thinking of the right solution depending on existing evidence. S3 is able to conclude accurately and is able to convey strong reasons and evidence as a critical thinker, while the opposite occurs in S1.

Personal beliefs are thinking dispositions necessary for one to become critical thinker (O'Reilly et al., 2022). This is characterized by curiosity, accurate information, trust in reason, open- and fair-minded evaluation, flexibility, honesty in facing personal biases, thoughtfulness during judgments, willingness to reconsider, clear understanding about the problem, and organization in complex matters (Cruz et al., 2020). However, in deciding what to believe or do, a person is assisted by a set of dispositions and critical thinking abilities (Ennis, 2015). S3 is believed to be critical thinker because they showed a similar disposition of explaining the steps related to solving mathematical problems. S1's belief in the ability to think critically tends to affect personal views on teaching mathematics (Kusaeri & Aditomo, 2019), knowledge, as well as filtering all new information to form a particular mindset (Rodriguez-Muniz et al., 2022).

The responses of S1, S2, and S3 regarding the problem-solving steps differed. S1 solves problems by searching for examples of existing or common problem solutions, followed by its modification. However, S2 understands the problem, which is then solved using a common method. For S3, the problem is understood, then important information is written, linking the problem with the answer and drawing inferences. In addition, S3 provided a better explanation of the problem-solving process in respect to critical thinking indicators, namely interpretation, analysis, and inferences.

IV. Mathematics Teaching Design that Develops Critical Thinking

S1, S2, and S3 provided similar responses related to mathematics teaching activities that can develop critical thinking. S1 did not discuss in detail the abilities required by students to develop critical thinking. On the other hand, S2 and S3 provided a detailed explanation of teaching process adopted by teachers, namely the ability to interpret, analyze, and evaluate information derived from problems, which includes drawing inferences. These abilities can be acquired, improved, and refined through instructions with clear objectives (Lorencová et al., 2019). Students are trained to solve mathematical problems by finding relevant information and understanding the question. Teacher should also provide a triggering question to connect the problem and adopted strategy. The problem-solving process presented by S2 and S3 shows the diverse stages of training students to think critically. The ability to analyze, evaluate, and draw appropriate conclusions when solving problems is a component of the critical thinking process (Fitriana et al., 2023). This skill can also be developed through the support of appropriate ideas and learning exercises. An example is the

use of learning models and giving related questions (Julianto et al., 2023). Based on the description above, S2 and S3 had a better understanding of teaching activities that help students develop critical thinking. S1 was unable to provide a detailed explanation of teaching activities that develop this skill.

In line with the discussion, S1 reported that teaching activities associated with the inquiry method can develop students' critical thinking in learning mathematics. Students are guided to find answers to math problems with directed questions to prepare or foster critical thinking. This was in accordance with finding that that the adoption of an inquiry method can improve critical thinking skills (Chususiyah et al., 2020; Yasin et al., 2019). S2 stated that teaching activities requiring the use of a scientific method and given mathematical problems develop students' critical thinking. This was in line with the results of research that states the adoption of the steps could impact the skill (Nurhikmayati & Jatisunda, 2018; Sukarna & Iman, 2019). According to S3, the problem-based learning model, in which teachers provide mathematical problems to be solved both individually and in groups, helps students learn and develop critical thinking. The given problems were high-level thinking questions in the field of analysis and evaluation (Tanjung et al., 2020). S3's statement was in line with the results of research that show the effectiveness of problem-based learning in developing students' critical thinking in mathematics (Aini et al., 2019; Amin et al., 2020; Maulidiya & Nurlaelah, 2019; Saputra et al., 2019).

Responses from S1, S2, and S3 concerning the application of learning strategies or models that foster critical thinking differed. S1 had never made a mathematics learning design that developed critical thinking due to lack of knowledge about designs and lack of understanding about the diverse stages fostering critical thinking skills. S2 and S3 had designed mathematics teaching that developed critical thinking but were hesitant or confused about the right steps in designing related teaching activities. Both hoped that training sessions are organized for teachers.

The mathematics teaching design made by S1 used a project-based learning model in planning discoveries that could create critical thinking. The designed learning model was subjected to four phases, namely 1) understanding the main problem, 2) planning the project, 3) presenting the results, as well as 4) evaluating and reflecting on the project process and results. S1 lacked an understanding of teaching stages contained in each phase of model. Project-based learning consists of six phases, such as 1) determining the main question, 2) designing the project, 3) preparing the schedule, 4) observing, 5) testing the results, and 6) assessment. The design made by S1 was not explained in detail in each phase. Moreover, teaching activities that were in line with critical thinking indicators, include interpretation (common in the phase of determining the main problem), analyzation of the project planning, explanation of the presented work results, evaluation of the project, and reflection phases. The teaching design created by S1 is not able to develop critical thinking because it only develops two indicators of critical thinking, namely interpretation and self-regulation.

The mathematics teaching design made by S2 entailed the use of a scientific method. Teaching activities were in line with the learning practices in the scientific method, namely observation,

questioning, reasoning, attempting, and communicating. Learning activities and worksheets focus only on developing 2 indicators of critical thinking, namely interpretation and explanation, so that the teaching design created by S2 has not been able to develop critical thinking.

The mathematics teaching design made by S3 used a problem-based learning model. The teaching activities designed are based on the phases of the problem-based learning model, including 1) orienting students to the problem, 2) organizing learning, 3) guiding both individual and group investigations, 4) developing and presenting work, and 5) analyzing and evaluating the problem-solving processes. Learning activities using problem-based learning tend to impact critical thinking (Yu & Zin, 2023). Additionally, teaching activities and student worksheets designed by S3 are able to have an impact on these abilities based on critical thinking indicators. Discussions are held in groups to solve the problems given by teacher, including grouping objects with square and rectangular surfaces. The students must observe and discuss with friends to conclude the characteristics of square and rectangular shapes. Inferences are presented, with given assessment and input to improve the conclusions drawn.

S1 stated that mathematical problems associated with the development of critical thinking are those with answers realized using more than one interrelated formula and several methods. The response that critical thinking problems can be solved in many ways is not in line with the characteristics of mathematical problems responsible for developing this skill. S1 selected problem number 1 which was inappropriate in terms of developing critical thinking because it does not train the ability to evaluate. Meanwhile, S2 stated that mathematical problems that develop critical thinking are those able to impact the ability to analyze and evaluate. S3 reported that math problems responsible for developing this skill are higher order thinking problems. Building upon this result, problem number 2 was selected by S2 and S3 due to its capability. The problem can develop critical thinking because it is a high-level reasoning question based on Bloom's taxonomy in the analysis and evaluation fields (Tanjung et al., 2020). Higher Order Thinking Skills (HOTS) category questions made significant contributions to improving critical thinking skills including problem solving using various solutions, decision making, and inferences (Thornhill-Miller et al., 2023). Regarding non-routine questions with criteria requiring solution justification trains students to think critically according to the indicators (Khusna et al., 2024). Based on the description above, S2 and S3 were able to explain the characteristics of mathematics questions that developed critical thinking.

The difficulties experienced by the three teachers were consistent with the results of research that identified the difficulties faced by Indonesian elementary school mathematics teachers in integrating critical thinking into teaching. This includes deficiencies in pedagogical knowledge, self-confidence, and resources which hindered the design of effective mathematics teaching (Trisnani et al., 2024). The difficulties experienced are due to the lack of training in line with the needs of teachers in developing knowledge of critical thinking and skills in designing mathematics teaching. Therefore, there is a need to organize special training for teachers to understand critical thinking and teaching design (Badescu & Stan, 2020). Of the three teachers, S3 had attended teacher

training and read books related to mathematics teaching knowledge and skills, resulting in adequate understanding. Meanwhile, S1 and S2 had never attended training, with S2 obtaining information from watching YouTube and reading books.

Following the discussion above, teachers must have a strong conception of critical thinking, critical mathematical thinking, indicators, and teaching designs. Strong teacher conceptions provide a firm understanding of designing and implementing mathematics teaching that develop critical thinking. Teacher conceptions also play an important role in the implementation of strong educational experiences (Bezanilla et al., 2023). Additionally, these conceptions and skills in designing mathematics teaching can be impacted through training (Gunawan et al., 2020).

A comparison of teacher conceptions of critical thinking for the high (S3), medium (S2), and low (S1) categories is shown in Table 4 as follows:

The concept of critical thinking in mathematics teaching	Teachers with high critical thinking concepts	Teachers with medium critical thinking concepts	Teachers with low critical thinking concepts
Conception of critical thinking	Correct	Incorrect	Incorrect
Conception of mathematical critical thinking	Correct	Incorrect	Incorrect
Conception of critical thinking indicators	Correct	Correct	Correct
Mathematics teaching design	Teachers were able to explain mathematics teaching activities ensuring the design was able to develop critical thinking.	Teacher was able to explain mathematics teaching activities but the design was unable to develop critical thinking.	Teachers were unable to explain mathematics teaching activities, therefore the design was not able to develop critical thinking.

Table 4: Comparison of teachers' conceptions of critical thinking for high, low, and medium categories

This research obtained samples and subjects from a particular district in East Java, limiting its applicability to a broader context. The challenges faced by mathematics teachers in the future in terms of designing teaching activities that develop critical thinking outline the need for related knowledge and the integration of active methods into teaching practices (Shyshenko et al., 2022). Therefore, future research should be conducted on a broad scale to determine elementary school teachers' conceptions of critical thinking, alongside the ability to design and practice classroom teaching.

CONCLUSIONS

In conclusion, teachers with low critical thinking conceptions had inappropriate conceptions of critical thinking and mathematical critical thinking. Conceptions related to critical thinking indicators were appropriate but unable to explain mathematical problem-solving procedures and related teaching activities.

Teachers with medium critical thinking conceptions had inappropriate conceptions related to critical thinking and mathematical critical thinking. However, these teachers had appropriate conceptions related to critical thinking indicators and were able to explain mathematical problem-solving procedures and teaching activities.

Following the description above, teachers with high critical thinking conceptions had appropriate conceptions related to critical thinking, mathematical critical thinking, indicators, teaching steps, and characteristics of mathematics problems that develop critical thinking. These teachers were able to explain mathematical problem-solving procedures and teaching activities. One of the factors that aided teachers with high critical thinking conceptions was the ability to attend training, actively seeking information by reading books and practicing in the classroom.

Following the description, teachers must have high conception of critical thinking to be able to design and teach mathematics that developed critical thinking. The difficulty in designing mathematics teaching is due to the misconception of critical thinking and relevant steps. These teachers require regular training and guidance to be able to develop students' critical thinking in the future.

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