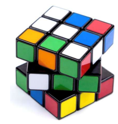
The Problem Corner



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The Purpose of ***The Problem Corner*** is to give Students and Instructors working independently or together a chance to step out of their "comfort zone" and solve challenging problems. Rather than the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to the posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word using math type or equation editor, and email your proposed problem and its solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Greetings, problem solvers!

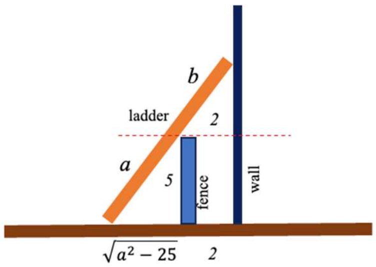
We are pleased to share that Problems 42 and 43 in *The Problem Corner* have received exceptional solutions, marked by both accuracy and originality. Highlighting these remarkable approaches aims to inspire others and foster a deeper appreciation for mathematical thinking worldwide.

Solutions to **problems** from the previous issue.

Problem 42
Proposed by Ivan Retamoso, BMCC, USA.

A 5-foot-tall fence stands parallel to a tall building, 2 feet away from it. Determine the length of the shortest ladder that can extend from the ground, over the fence, to touch the wall of the building.

First solution to problem 42
By JUDEL V. PROTACIO, Capiz State University, Capiz, Philippines.



Solution. Let $c = a + b$ be the length of the ladder where a is the part of the ladder from its foot to the point that meets the fence while b is the remaining part as shown. By Pythagorean theorem, the distance of the foot of the ladder from the base of the fence is $\sqrt{a^2-25}$. By similarity of two right triangles,

$$\frac{a}{\sqrt{a^2-25}} = \frac{b}{2}$$

so that $b = \frac{2a}{\sqrt{a^2-2}}$. Since $c = a + b,$ then

$$c = a + \frac{2a}{\sqrt{a^2-25}}.$$

We calculate $\frac{dc}{da}$.

$$\frac{dc}{da} = 1 + \frac{2\sqrt{a^2 25} - 2a(\frac{1}{2})(a^2-25)^{-\frac{1}{2}}(2a)}{a^2-25}$$

so that

$$\frac{dc}{da} = \frac{(a^2-2\ \ )^{3/2}-50}{(a^2-2\ \ )^{3/2}}$$

We set $\frac{dc}{da} = 0$ and solve for a to get $a \approx 6.21\ ft$. We then solve for b,

$$b = \frac{2a}{\sqrt{a^2-25}} = \frac{2(6.21)}{\sqrt{6.21^2-2}} \approx 3.37ft$$

Therefore, $c = a + b = 9.58\ ft.$

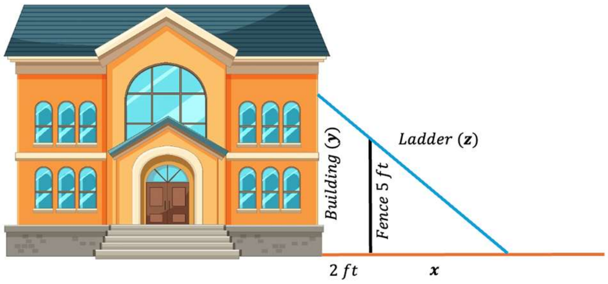
To verify that c is minimum, we compute $\frac{d^2c}{da^2}$. Since

$$\frac{d^2c}{da^2} = \frac{50}{(a^2-25)^3} > 0\ for\ a > 5, c\ is\ minimum.$$

Second solution to problem 42

Dr. Abdullah Kurudirek, Al-Naji University, Baghdad.

Solution: First, let’s begin our work by representing the given information on a diagram



First, using the Pythagorean theorem, we can write

$$z^2 = y^2 + (x + 2)^2 \quad (1)$$

Next, using the similar triangles formed by the building, the ground, and the fence, we can write

$$\frac{5}{x} = \frac{y}{2+x} \quad (2)$$

From equation (2), we obtain $y = \frac{5(2+x)}{x}$, and after substituting this into equation (1), we obtain

$$z^2 = (\frac{5(2+x)}{x})^2 + (x + 2)^2 \quad (3)$$

To find the minimum length, we differentiate z with respect to x and set the derivative to zero.

$$\frac{dz}{dx} = \frac{(x+2)(x^3-50)}{x^2\sqrt{x^4+4x^3+29x^2+100x+100}} = 0 \quad (4)$$

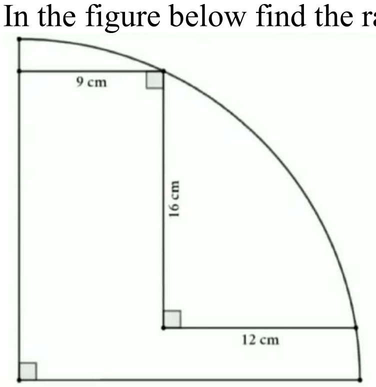
$x = -2$ (impossible) so $x = \sqrt[3]{50}$

The shortest ladder length (z) is approximately 9.58 ft.

Problem 43

Proposed by Ivan Retamoso, BMCC, USA.

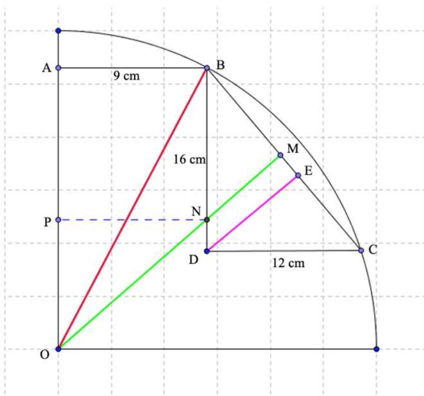
In the figure below find the radius of the circle.



First Solution to problem 43

By JUDEL V. PROTACIO, Capiz State University, Capiz, Philippines.

In the figure, find the radius of the circle.



Solution.

1. Initially label the intersection points as A, B, C, D and O as the center of the quadrant as shown.
2. Draw $\overline{BC}$, the hypotenuse to ΔBCD so that by Pythagorean theorem, $BC = 20\ cm$. Draw the altitude $\overline{DE}$ to the hypotenuse $\overline{BC}$.
3. Locate the midpoint M of $\overline{BC}$. Draw $\overline{OM}$. By chord-radius relationship, $\overline{OM}$ is the perpendicular bisector of $\overline{BC}$ so that $BM = CM = 10\ cm$.
4. Label the intersection of $\overline{BD}$ and $\overline{OM}$ as point N.
5. With $\overline{DE}$ as altitude to the hypotenuse,

   $EC = \frac{36}{5}$ cm, so that $EC = \frac{64}{5}$ cm and $DE = \frac{48}{5}$ cm.
6. Likewise, $\Delta DEB \sim \Delta NMB$ so that $NB = \frac{25}{2}$ cm and $NM = \frac{15}{2}$ cm.
7. Draw $\overline{PN}$ parallel to $\overline{AB}$ so that $PN = 9\ cm$ and $AP = 12.5\ cm$. It follows that $\Delta NMB \sim \Delta OPN$ so that by similarity,

   $ON = \frac{45}{4}$ cm and $OP = \frac{27}{4}$. This gives $AO = \frac{77}{4}$ cm.
8. Finally, draw $\overline{OB}$, the radius of the circle, so that by Pythagorean theorem, $OB = \frac{85}{4}\ cm.$

Second Solution to problem 43

Dr. Abdullah Kurudirek, Al-Naji University, Baghdad.

Solution: In solving this problem, we can make use of a coordinate system. For this purpose, I placed our quarter circle as shown below with its center at $(0,0)$, and added auxiliary lines with dashed lines alongside the x- and y-axes to support our solution. From here, we can easily identify two points on the quarter circle, $A(9, h)$ and $B(21, h-16)$, and these points will satisfy the circle equation $x^2 + y^2 = r^2$.

Now, when we write these points, we obtain the following two expressions:

$$9^2 + h^2 = r^2 \quad (1)$$

$$21^2 + (h-16)^2 = r^2 \quad (2)$$

From this, $9^2 + h^2 = 21^2 + (h-16)^2$ and we obtain $h = \frac{77}{4}$

If you substitute this value h into either of the above equations (1) or (2), you get radius $r = \frac{85}{4}$.