Bridging the Gap: Understanding Students' Struggles with Algebraic and Graphical Representations of Functions

Santosh Pathak

Department of Mathematics, University of Utah Asia Campus, Incheon, South Korea
spathak2039@gmail.com

Abstract: Students often find it challenging to move between algebraic and graphical representations of functions, which can lead to a fragmented understanding and make problem-solving more difficult. Research and classroom observations suggest that putting too much emphasis on algebraic procedures may limit students' ability to think graphically and connect different representations. In this study, we looked at how well students could use both algebraic and graphical approaches to solve function-related problems. We also explored their preferences for different representations and how aware they were of alternative strategies. To do this, we gave a five-question survey to 300 Grade 12 STEM students in Nepal. The survey included a variety of function types and required students to reason both symbolically and visually. The results showed a strong preference for algebraic methods, while performance on graph-based tasks was significantly lower. Many students were not even aware that graphical strategies were an option and had trouble transferring their knowledge between the two forms. These findings point to a clear need for changes in teaching and curriculum design. A better balance between symbolic and visual reasoning—including more graph-based tasks, the use of visualization tools, and training to build metacognitive skills—could help students develop a more flexible and deeper understanding of mathematics.

Keywords: Functions and Graphs, Algebraic and Graphical Approach, Problem-Solving, Mathematics in Nepal

INTRODUCTION

Understanding functions through both algebraic and graphical representations is essential in high school mathematics. Functions are considered central to mathematics education and foundational for more advanced topics such as calculus and statistics (Dubinsky & Harel, 1992; Romberg et al., 2012). Algebraic forms support symbolic manipulation, while graphs offer intuitive insights into behavior such as *trends*, *intercepts,* and *slopes* (NCTM, 2000; Schoenfeld et al., 1993)

Despite their importance, many students enter higher education with a weak grasp of functions, often due to insufficient focus on graphical reasoning in earlier education (Durant & Garofalo, 1994). Even those who encounter functions in high school struggle to understand them deeply

(Leinhardt et al., 1990; Markovits et al., 1988). Students typically prefer algebraic manipulation over graphical interpretation (Hitt, 1998; Knuth, 2000a), a tendency linked to curricular and instructional practices that emphasize procedures over conceptual understanding (Booth & Newton, 2012).

Graphical literacy is essential across disciplines. Shuard and Neill (1977) highlight that students unable to interpret graphs are disadvantaged, especially since graphs serve roles ranging from data recording to predicting behavior. However, differing views between mathematics and science educators on the role of graphical understanding can lead to inconsistencies in instruction (Booth, 1981).

The ability to move between algebraic and graphical representations—known as representational fluency—is a critical skill for mathematical reasoning and problem-solving (Lesh et al., 1987). Yet research shows students often fail to connect these forms, resulting in fragmented understanding (Even, 1990; Leinhardt et al., 1990). This gap highlights the importance of *metacognitive awareness* and instructional approaches that emphasize multiple representations (Dubinsky & Harel, 1992; Shuard & Neill, 1977). Moreover, developing this fluency supports cognitive growth as students transition to abstract thinking (Piaget, 1972) and builds communication skills necessary for mathematical literacy (CCSSM, 2010). By helping students interpret functions across symbolic systems, educators can promote deeper, more adaptable mathematical understanding.

Theoretical Framework

This study is grounded in a representational perspective on mathematical understanding, which emphasizes the central role of external representations—such as algebraic symbols, graphs, diagrams, and verbal descriptions—in the learning and use of mathematics. Rather than perceiving mathematical *objects* as directly accessible or intuitive, this perspective suggests that such *objects* are constructed and understood through representational systems (Duval, 1999, 2017). Each *system*, or *register*, provides a distinct way of expressing mathematical ideas and contributes uniquely to conceptual development.

For instance, the function $f(x) = 2x + 3$ can be expressed algebraically or visualized as a straight line on a Cartesian plane. Effective learning involves not only the ability to perform procedures within a single *register* (e.g., solving $f(1) = 5$) but also the ability to translate between *registers*—such as moving from a symbolic expression to a graphical representation, or vice versa. This distinction between *treatment* (within-register operations) and *conversion* (*cross-register translations*) is crucial in understanding students' reasoning (Duval, 1999; Pavlopoulou, 1993).

Empirical studies show that students frequently face challenges in coordinating algebraic and graphical representations. Many students can carry out symbolic manipulations but struggle to interpret or generate corresponding graphs, often treating representations as disconnected entities rather than as linked expressions of the same concept (Lobato et al., 2012; Wilkie, 2024). Particularly difficult is the reverse *conversion*—from graph to equation—which demands a conceptual grasp that goes beyond procedural skill (Arnal-Palacián et al., 2022; Lesh et al., 1987; Schliemann et al., 2021). This asymmetry in representational fluency suggests that some *conversions* are cognitively more demanding or less intuitive than others.

Students often rely heavily on procedural strategies like table-making or formula substitution, which may allow them to answer specific questions but hinder their ability to recognize underlying structures or make generalizations (Lobato et al., 2012; Wilkie, 2024). These difficulties become especially visible in applied contexts—such as motion problems—where students must relate changes over intervals (e.g., speed or acceleration) to features of a graph. However, rather than attending to rates of change, many focus on accumulated values (such as total distance), which can lead to a superficial understanding of function behavior. Similarly, students may fail to grasp how parameters in a quadratic function like $f(x) = ax^2 + bx + c$ influence the graph's shape and position, suggesting weak connections between symbolic manipulation and visual interpretation (Arnal-Palacián et al., 2022; Wilkie, 2024). Graphs are often viewed at a surface level, reproduced by memory or habit rather than interpreted meaningfully (Duval, 1999).

Relevance to this Study

The representational perspective described above is central to this study, as it helps explain the specific challenges students faced when working with algebraic and graphical representations of functions. Many students relied mostly on algebraic methods and either overlooked graphs or weren't sure how to use them effectively. Their difficulties in connecting equations to graphs—and in interpreting key features like *domain, intercepts*, and *turning points*—reflect well-known struggles with moving between different forms of representation (Lobato et al., 2012; Wilkie, 2024).

This framework also shaped the study's analysis. It guided the examination of how well students could work within a single *register* (e.g., solving an equation or reading a graph) and how effectively they could move between *registers*. Particular attention was given to the direction of these translations, such as whether students found it easier to graph an equation or to write an equation based on a graph. Finally, the study used this lens to assess students' development of representational fluency, a key aspect of flexible and meaningful mathematical thinking.

By highlighting where students struggle and why, this perspective provides useful insights for teaching—especially in helping students make stronger connections between algebra and graphs.

Objective

The objective of this study is to investigate students' ability to navigate between algebraic and graphical representations of functions, and to identify the *cognitive* and *instructional* barriers that hinder this transition. Specifically, the study aims to assess students' problem-solving strategies, determine their representational preferences, and evaluate their metacognitive awareness of alternative solution methods. By analyzing responses from 300 grade 12 STEM students in Nepal, the study seeks to uncover the extent to which students can flexibly apply both symbolic and visual reasoning when working with functions. The findings are intended to inform instructional practices and curricular reforms that promote a more balanced and conceptually grounded understanding of functions.

METHOD

This section outlines the participants, the design of the assessment instrument, and the approach taken to analyze student responses, with the aim of understanding students' use of algebraic and graphical representations in problem-solving.

Sample and Context

The study was conducted with 300 Grade 12 STEM students from various high schools in Kathmandu, Nepal. These students had completed instruction in graphing functions as part of their Grade 11 curriculum (*linear* and *quadratic* functions are also part of Grade 10 curriculum) and had some exposure to applying these concepts in calculus. The selection was made using a random sampling method to ensure a diverse and representative group across schools. In the *Nepali education system*, there is a strong emphasis on algebraic manipulation, and graphical reasoning often receives less instructional time. As a result, students were expected to be more comfortable with symbolic procedures than with visual or graphical approaches, making this group suitable for exploring the contrast in representational fluency.

Instrument

To assess students' ability to work with and move between algebraic and graphical representations, a paper-based questionnaire was developed. It contained five problems with a total of fifteen tasks, each involving different function types: *linear, quadratic, square root,* and *rational*. The tasks were designed to evaluate both procedural fluency and conceptual understanding through algebraic and graphical reasoning. Some questions offered multiple ways to solve the problem, inviting students to reflect on different approaches. The questionnaire was inspired by earlier studies on students' function understanding (Hitt, 1998; Knuth, 2000a; Van Dyke & White, 2004), and students were encouraged to show their reasoning and reflect on alternative strategies when appropriate.

Data Analysis

Student responses were analyzed using a *cognitive framework* that integrates both *constructivist* and *metacognitive* perspectives. From a *constructivist* viewpoint (Gale & Steffe,1995), we examined how students built upon their prior knowledge of algebra and graphing to approach unfamiliar problems. At the same time, a *metacognitive* lens (Hacker et al., 2009) allowed us to assess how students monitored and adapted their strategies—particularly their awareness of different solution methods and their flexibility in choosing between them. Each task was evaluated in terms of *metacognitive knowledge* (what students knew and how accurately they applied it) and *metacognitive regulation* (how they planned, monitored, or revised their approach). For example, in Question 1, *metacognitive knowledge* was reflected in students' understanding of the equation $2x + 3y - 6 = 0$ and their ability to apply algebraic and graphical methods. *Metacognitive regulation* was evident in how students chose their strategy, checked their work, and responded to part (c), which explicitly asked them to reflect on alternative approaches. Responses were coded as *correct*, *partially correct*, or *incorrect*, and attention was given to whether students demonstrated an awareness of multiple representations and switched between them appropriately. This combined framework supports a deeper understanding of students' problem-solving processes and provides insights into how instruction can better balance symbolic and visual reasoning for more effective mathematical learning.

RESULTS

Question 1

The graph of the equation $2x + 3y - 6 = 0$ is given below in Figure 1.

(a) What value of x gives $y = 2$?
(b) What value of y gives $x = 3$?
(c) Can you answer the questions in part (a) and (b) by a method different than the one you used? Explain your answer.

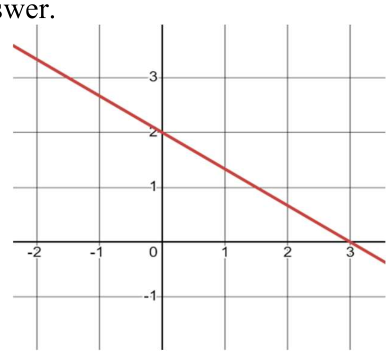

Figure 1: Graph of a straight line

The problem assesses whether students can interpret a linear equation both symbolically (through algebraic computation) and visually (by analyzing the graph in Figure 1). Students must realize that the same problem can be solved using either an algebraic approach or a graphical method. The third part of the question explicitly asks students to reflect on alternative methods.

Out of 300 students, 290 participated students responded at least one part of the question. Of these, 195 students (about 67% of the respondents) chose to answer parts (a) and (b) algebraically, showing a clear preference for algebraic methods, even though the graphical method is often quicker and easier. Only 95 students, which is nearly 33% of the 290 students chose to answer it graphically. This preference for algebra over graphical solutions was consistent across both parts. In part (c), only 178 students responded to the question. Of the 195 who answered part (a) and (b) algebraically, only 83 responded to the question in part (c), and of these 83, only 60 students were aware of the alternative approach for part (a) and (b). Hence 155 (about 52%) students were aware of graphical and algebraic approach of solving the problem whereas 145 (about 48%) students either had no idea at all or no graphical approach of solving the problem. Some students expressed this lack of awareness of the graphical approach with statements like the following:

- "No, I cannot answer the question by a different method."
- "There is no other method to answer the question in part (a) and (b)."
- "Sorry, I have no clue."
- "It can't be explained, sorry."

This indicates that while students overwhelmingly began with algebraic methods, many of them only realized the possibility of a graphical solution when confronted with part (c). As one student noted, "The easiest way to do it is by putting $x = 0$ $y = 0$,"highlighting the strong reliance on algebraic techniques. The responses in part (c), particularly the statements like "I have no clue" and "There is no other method to answer the question," suggest a lack of *meta-cognitive regulation*. Many students were not consciously aware that graphical solutions were an option, indicating that they were unaware of alternative methods beyond the algebraic ones they were comfortable with. The bar graph below (Figure 2) summarizes the outcome of the students' responses to question 1.

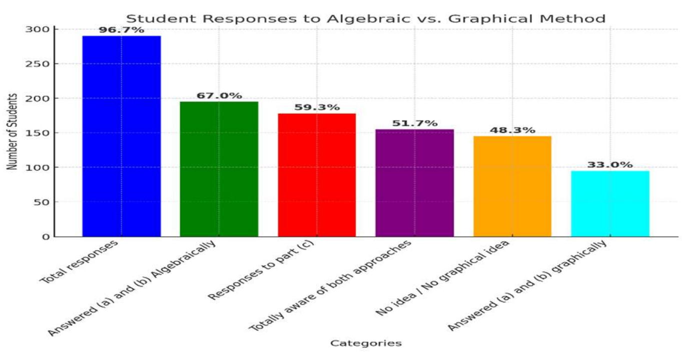

Figure 2. Students' Responses to Question 1.

Figure 3 below (from left to right) consists responses from three students in which first student demonstrates a clear understanding of the question and provides a correct answer. The second student initially applies the algebraic method but recognizes the alternative approach upon reaching part (c). In contrast, the third student's response reveals a lack of awareness or knowledge of the alternative approach for solving the problem.

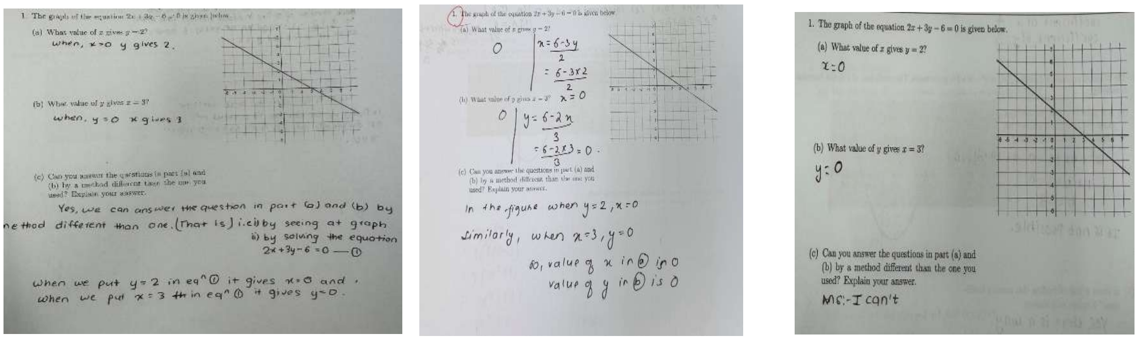

Figure 3. Sample Responses to Question 1.

Question 2

The graph of a linear function $?x + ?y = 6$ is given below (Figure 4).

(a) Can you find some obvious solution of the given equation without the missing coefficient? Explain your answer.
(b) If possible, find the missing coefficient of the equation. Explain your answer.

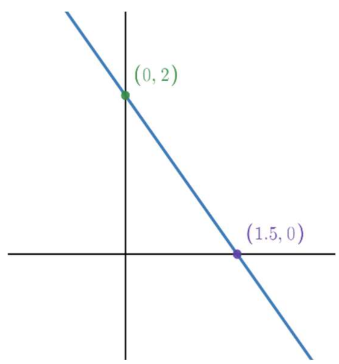

Figure 4. Graph of a straight line.

The purpose of this problem was to assess whether students can relate the graph to the equation (e.g., recognizing intercepts as direct solutions), whether they express a preference for graphical or algebraic methods and justify their choice, and if they demonstrate *metacognitive regulation* by identifying mistakes and adjusting their approach accordingly.

Out of 300 students, only 44 (14.67%) answered correctly, while 51 (17%) provided partially correct answer of part (a); 95 students (about 32%) gave either correct or partially correct answer. Of these 95 students, only 20 students (6.6%) demonstrated a clear understanding of the connection between the equation and its graph. The majority—205 students (68%)—showed little understanding of the question or its graphical representation. Many students were confused by the missing coefficients of x and y in the equation. Some identified the midpoint of the given line as the solution, while others suggested that a complete equation would allow them to check the points on the graph by substitution, though they failed to recognize that these points are solutions to the equation. In part (b), which requires an algebraic approach—use of *slope-point formula* of a straight line—to find the missing coefficients, about 55% students—significantly higher than in part (a)—answered either correctly or partially correctly, —reflecting their strong reliance on algebraic skills in problem-solving.

Many students believed that finding the "*obvious*" solutions required solving for the coefficients, with one student commenting, “Without solving, we can’t find the missing coefficients.” This reliance on algebra is further highlighted by a student who wrote, “We can’t find the obvious solution without the missing coefficients, as they determine the constant nature of the graph.” Interestingly, a few students first found the equation of the line using the *two-point formula* and then verified the given points on the graph, demonstrating their heavy reliance on algebraic skills.

Students showed more confidence with algebraic methods than with interpreting graphs. While their metacognitive skills helped with algebra, they struggled to reflect on and understand graphical representations, which caused confusion. This suggests a need for more training to encourage flexible problem-solving and improve graph interpretation. The bar graph below (Figure 5) summarizes the results for question 2.

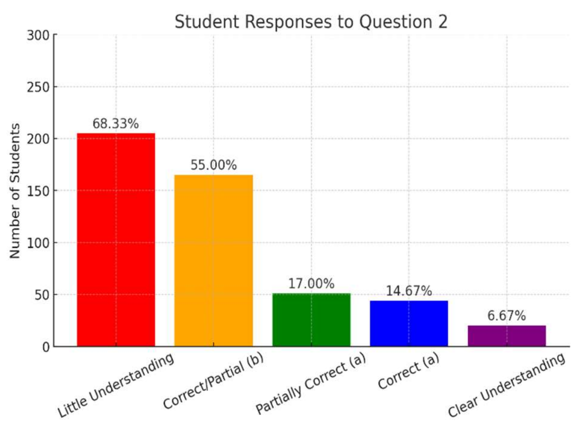

Figure 5: Students Responses to Question 2.

The following Figure 6 (from left to right) includes responses from three students. The first student demonstrates a clear understanding of *solution points*. The second student shows some confusion about the concept, while the third student completely misunderstands the idea of *solution points*.

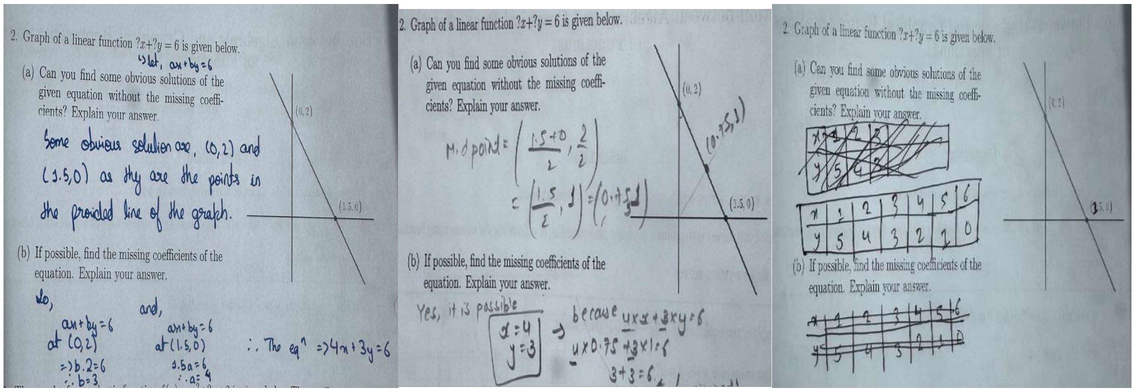

Figure 6. Sample Responses to Question 2.

Question 3

Refer to the graph below in Figure 7 to answer the questions.

a) $f(4) = g(-2)$
b) $f(4) = g(4)$
c) $f(-2) = g(-2)$
d) More information is necessary to say something definitive about $f(x)$ and g(x).

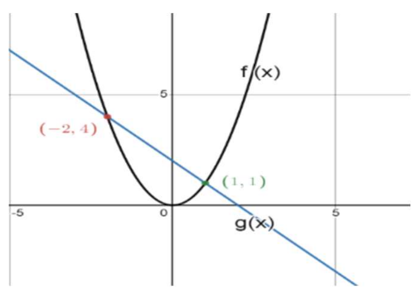

Figure 7. Graph of a straight line and parabola.

The question requires students to interpret the graph of two functions, $f(x)$ and $g(x)$ in Figure 7 and make connections between their values at specific points. According to the *constructivist* framework, students will build their understanding through interaction with the graph, recognizing how the two functions relate to one another. The focus would be on their ability to construct meaning from the graph and draw conclusions based on the points of intersection.

Out of the 300 students, only 31 (about 10%) answered the question correctly. A surprising 52% of students were completely unsure of what the question was asking, with one student commenting, "I don't know. Very bad question." Additionally, 66 students (22%) selected choice (d), believing that there was insufficient information to answer the question. Around

12% chose answers (a) or (b). In total, 90% of students either selected an incorrect answer, gave an unclear response, or did not attempt the question at all. This outcome is particularly surprising, given that many students struggled to understand the *domain* and *solution points* of a function, even when a graphical representation was provided. Some students even attempted to find the algebraic form of the function from the graph to determine the correct answer. Despite the relatively low level of difficulty of the problem, the percentage of correct responses was alarmingly low. This result mirrors the trends observed in previous problems, where students showed a strong reluctance to use graphical representations and instead relied heavily on algebraic methods. The bar graph below (Figure 8) gives an outlook of the results of question 3.

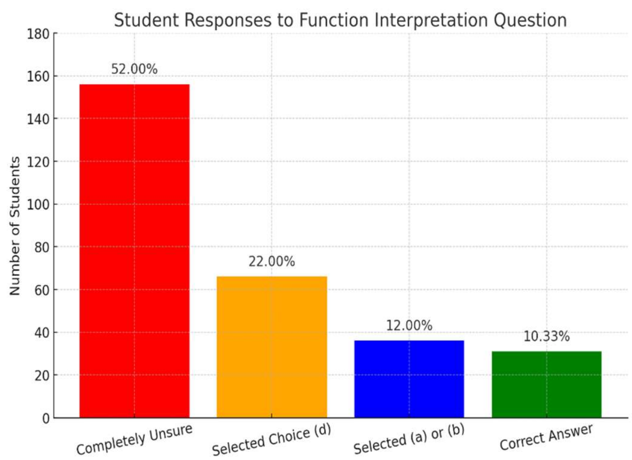

Figure 8: Students' Responses to Question 3.

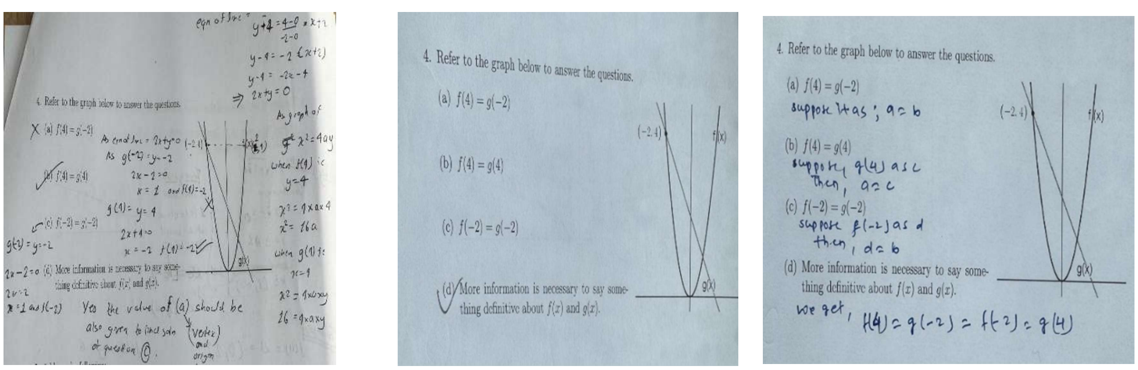

Figure 9. Sample responses to question 3.

Figure 9 (left to right) presents responses from three students, each displaying different approaches to the problem. The students either provide incorrect answers or attempt to solve the problem algebraically, despite a graphical representation of the function being provided. The first and third responses involve algebraic methods, while the middle response is an incorrect answer. All three responses fail to demonstrate the necessary *metacognitive regulation* in problem-solving.

Question 4

The graph of the function $f(x) = \sqrt{(x-1)}$ is given alongside in Figure 10. Answer the following questions.

a) Find the domain of f. Demonstrate or explain how you found it.
b) Find $f(0), f(1), f(5), f(6)$.
c) Find the x values on the domain of f that correspond to the points A, B, C, D. Include details in your answer.

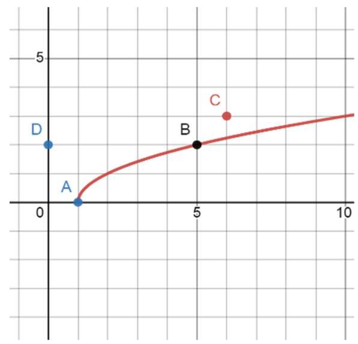

Figure 10. Graph of a square root function.

This task's problem analysis framework examines how students approach problem-solving, focusing on identifying key strategies and evaluating various methods. Students must apply their knowledge of domains, evaluate function values, and understand the connection between algebraic expressions and their graphs—essential concepts in high school mathematics (National Council of Teachers of Mathematics [NCTM], 2000; Common Core State Standards Initiative [CCSSI], 2010). These concepts are introduced in grade 10 and 11 in Nepal and must be carried forward to grade 12.

Based on students' responses in part (a), only 42 out of 300 students (14%) understood the domain and attempted the problem graphically. In contrast, 79 students (26%) understood the concept but approached the problem algebraically. However, 179 students (60%) had no clear understanding of how to approach the problem. This highlights a significant gap in students' understanding of fundamental concepts like domain, even when different function representations are provided.

The aim of the question in part (b) was to assess whether students recognize that $x = 0$ is excluded from the domain found in part (a) and to evaluate their willingness to use the graph (Figure 10) to find values at various points within the domain. The survey results show that only 22 students (about 7.5%) correctly identified that $f(0)$ is undefined, as $x = 0$ is not part of the domain. Meanwhile, three out of four students attempted the problem without understanding this concept, simply plugging in x-values into the equation. Many of these students recognized that $f(0)$ resulted in an imaginary number and responded with terms like "undefined," "impossible," or "infinity." However, there was insufficient evidence to suggest they understood that $x = 0$ is not part of the domain, and thus $f(0)$ cannot be computed either algebraically or graphically. In contrast, more than half of the students—189 out of 300—had no idea how to approach the problem. They either did not attempt it or provided incorrect answers.

As seen in part (b), a similar pattern emerged in part (c) too. Only 16 students (about 5.5%) met the expectations of the problem and answered correctly. In contrast, 103 students (approximately 34%) answered the question without realizing that points C and D cannot correspond to any x-values, as they are not *solution points* of the equation. One student wrote, “By studying the graph, I found the values of x for points A, B, C, and D,” despite not understanding that C and D are not on the graph. Meanwhile, 63% of students either had no idea how to approach the problem or gave incorrect answers. Below is an example of responses from three students.

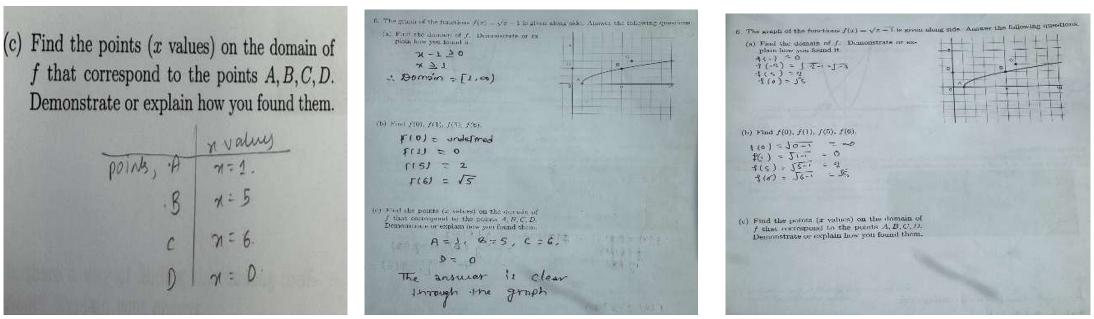

Figure 11. Sample Responses to Question 4.

Figure 11 (left to right) displays three responses to Question 5, illustrating that the students struggled to understand the domain of the function, despite the graphical representation provided. The second and third responses include the correct answer to part (a), showing sufficient evidence of using the algebraic equation rather than the graph.

In conclusion, there is a significant gap in students' understanding of the domain of the function and how to evaluate points on the graph. At the same time, a large portion of students failed to recognize key *domain restrictions* and the inability of certain points (C and D) to correspond to any x-values. Also, many students struggled to use the graph effectively, and there was widespread confusion around function evaluation and domain concepts. The bar graph below (Figure 12) summarizes the outcome of the responses to the question.

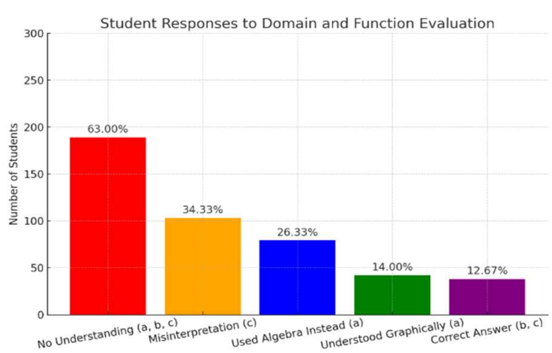

Figure 12. Students’ Responses to Question 4.

Question 5

Answer the following questions. Try to include as much detail as possible in your answer.

a) Find the domain of the function $f(x) = \frac{x-2}{x+2}$.
b) Find the domain of the function whose graph is given below in Figure 13.

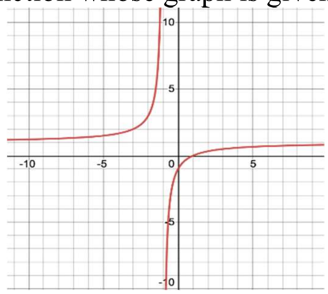

Figure 13. Graph of a Rational Function.

c) Which question (a) or (b) did you find easier to answer? Explain.

The objective of this task is to assess students' understanding of the domain of function, including *restrictions* caused by undefined values. The *cognitive demand* requires algebraic reasoning to determine where $f(x)$ is undefined, interpretating a graphical representation to determine *domain restriction*, and evaluates students' perceived difficulty in switching between algebraic and graphical methods.

The outcome of the survey indicates, only 75 students (25%) correctly identified the domain in part (a), and an unexpectedly low number—only 25 students (about 9%)—were able to identify the correct domain in part (b) where a graphical representation (Figure 13) of the function was provided. This low number is striking, especially given that 275 students struggled to approach the problem in part (b). In part (c), 27% of students felt comfortable finding the domain when presented with the algebraic form, while only about 10% found the graphical method easier. Surprisingly, 62% of respondents did not select any method as a compatible approach for finding the domain, and only 2% felt equally comfortable with both methods.

These results suggest that the majority of students lack a clear understanding of the domain of a function, regardless of how the function is represented. A clear pattern emerged showing that more students were comfortable determining the domain when given the algebraic representation. Many students noted that part (a) was easier because the equation was directly provided. The following are a few student comments from part (c) that reflect this preference:

- "(a) is a bit easier than (b) because the direct function is given in this question. But in (b), the function should first be determined."
- "(a) is easier to answer since the function is straightforward, but (b) is more difficult since the function in the graph is not easy to identify."
- "(a) was easier because the graph is difficult to understand."

From these comments, it's clear that many students find it easier to determine the domain when the function is given algebraically. They expressed difficulty in part (b) because they first needed to determine the function from the graph before identifying its domain.

Additionally, some students commented on how the education system in Nepal places greater emphasis on algebraic and computational problem-solving methods, leaving students less comfortable with graphical approaches:

- "I found (a) easier because we have done this type of problem before using just one method. We are not as familiar with the graphical approach since our education system focuses more on algebraic methods."
- "(a) is easier because solving algebraically is easier, and we're not used to graphical assignments."
- "I found (a) easier because it was taught more in class, and the teacher focused more on part (a) than part (b)."

Students showed a strong preference for algebraic methods over graphical ones when determining the domain of a function. While 25% correctly answered part (a) algebraically, only 9% succeeded with the graphical approach in part (b). Many struggled with graphs due to limited exposure, as their education emphasized algebraic computation. This highlights the need for greater focus on graphical reasoning to improve students' ability to interpret functions across different representations. Figure 14 presents a bar graph summarizing the responses to Question 5

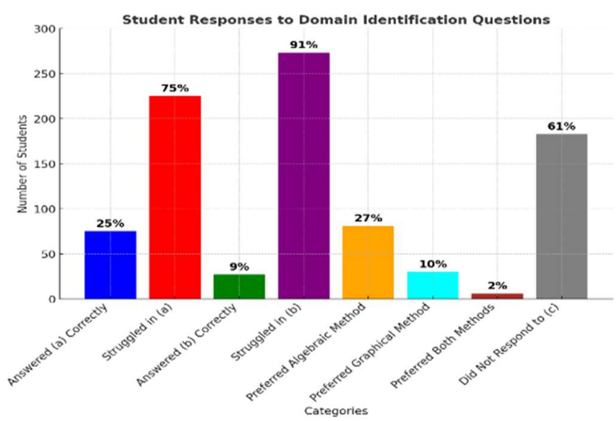

Figure 14. Students' Responses to Question 5.

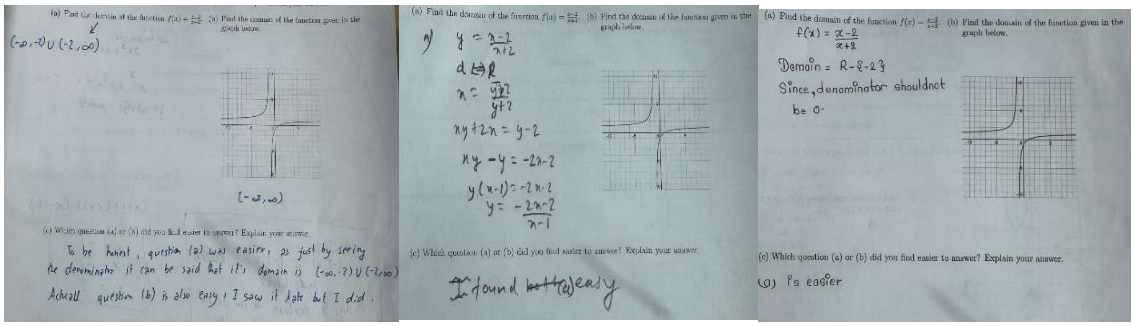

Figure 15. Sample Responses to Question 5.

The responses in Figure 15 (left to right) show that none of the three students answered part (b). They seem to believe that determining the domain is easier when an algebraic equation is provided, rather than when the information is presented graphically. In the second response, the student deviates from the question's intent by applying an algebraic method typically used to find the inverse of a function. Despite this, the student still finds part (a) easier than part (b).

DISCUSSION

The findings of this study reveal a consistent pattern in students' problem-solving approaches: a strong preference for algebraic methods and limited comfort with graphical representations. This tendency was evident across all five questions administered to grade 12 STEM students in Nepal. Even when graphical approaches were potentially more intuitive or efficient, students overwhelmingly defaulted to symbolic procedures.

In Question 1, although both methods were viable, 67% of students chose algebraic solutions, and nearly half were unaware of graphical alternatives. Responses such as "There is no other method" or "I have no clue" suggest a lack of exposure to graph-based reasoning and highlight limited *metacognitive regulation*—students' ability to reflect on and evaluate their strategies (Hacker et al., 2009). These findings align with Duval's (1999, 2017) notion that students struggle to convert between *semiotic registers*, particularly from algebra to graph and vice versa.

The difficulty interpreting graphs was more pronounced in Questions 2 and 3. In Question 2, which asked students to identify "obvious" solutions from a graph, only 14.67% answered correctly. Many were confused by missing coefficients, indicating a rigid reliance on equation-based reasoning. In Question 3, where students compared values of two functions at specific points on a graph, only 10% responded correctly, while over half expressed uncertainty or frustration. These results are surprising given the relatively low cognitive demand of the tasks and suggest that even when visual information is clearly presented, many students default to symbolic thinking or abandon the task altogether.

Performance on Questions 4 and 5 reinforced this trend. When asked to determine the domain of a function, 25% answered correctly using algebraic expressions, but only 9% succeeded when a graph was provided. Many students attempted to reconstruct the function algebraically before answering, reinforcing their discomfort with visual reasoning. These behaviors are consistent with international findings (e.g., Knuth, 2000a; Knuth, 2000b; Van Dyke & White, 2004), but the lower success rates observed here suggest deeper instructional gaps in the Nepali context.

Moreover, many students appeared to have forgotten key concepts introduced in Grade 11, such as *domain restrictions, function evaluation,* and *solution points*. This supports concerns about knowledge decay and lack of review, as discussed in studies on learning retention (Bjork & Bjork, 2011; Brown et al., 2014; Rohrer & Pashler, 2007). While students' algebraic competence seemed relatively stronger, their graphical reasoning and ability to coordinate representations were notably underdeveloped.

Taken together, the results suggest that many students show a strong preference for symbolic methods and have difficulty using or interpreting graphs. Most were not aware that problems could be solved in different ways and did not reflect on which method might be more efficient or appropriate. This points to limited *metacognitive engagement*—students rarely questioned their own approaches or considered alternatives. These patterns highlight a need to better support students in making connections between representations and in thinking more deeply about their problem-solving strategies. They may also reflect broader challenges in teaching practices and curriculum design that deserve further attention.

CONCLUSIONS

The findings of this study point to several important takeaways for mathematics education, especially in countries like Nepal, where instruction and curriculum have long focused more on symbolic procedures than on helping students build conceptual understanding. If we want students to develop flexible thinking and strong problem-solving skills, it's crucial to better balance algebraic and graphical reasoning in how we teach functions.

Balanced Curriculum Design

Students' strong preference for algebra—even when using a graph would have been easier—suggests that current textbooks and lessons in Nepal place a heavy emphasis on procedural fluency. A recent informal analysis of Grade 9–12 mathematics textbooks found that over 84% of the problems focus mainly on symbolic computation, with very few involving graphs. This imbalance limits students' opportunities to think visually and understand functions in multiple ways (Booth & Newton, 2012; Fey & Harel, 2004; Hiebert & Carpenter, 1992). To address this, the curriculum should incorporate more graph-based problems, not just equations; include tasks that encourage students to move between different representations (Duval, 1999); and provide opportunities for students to explain and compare different solution methods. Visual thinking should not be treated as an optional extra but should be integrated into regular lessons from the early grades onward.

Metacognitive Instruction

Many students did not seem to realize that problems could be solved in more than one way. Even when asked directly, they tended to stick with familiar methods and showed little reflection on their choices. This observation aligns with research indicating that students do not automatically develop metacognitive skills unless those skills are explicitly taught and practiced (Hacker et al., 2009; Mainali, 2021; Orhun, 2012; Schoenfeld, 1992; Tiew et al., 2023). Teachers can help build metacognition into regular lessons by asking students to explain why they chose a particular method, encouraging them to compare algebraic and graphical strategies, and using simple prompts such as “Could there be a faster way to do this?” Modeling their own thinking and decision-making during lessons is another powerful tool. In addition, teacher training should include Duval’s (1999, 2017) framework on switching between representations, so teachers are better equipped to support students who struggle to connect different forms of mathematical thinking.

Use of Visualization Tools

Visualization tools like *Desmos* and *GeoGebra* can be very effective in helping students understand the behavior of functions, such as *intercepts*, *symmetry*, and overall trends (Artigue, 2009; Larkin & Simon, 1987). These tools make it easier for students to connect equations with their graphical representations and develop visual understanding. However, simply providing access to these tools is not enough; teachers need to design purposeful activities that encourage students to predict what a graph will look like before plotting it, explore how changing parameters affect the graph’s shape, and reflect on when a visual approach might be more appropriate than an algebraic method.

Assessment Reform

Currently, most mathematics tests reward students primarily for obtaining the correct answer through algebraic methods, which discourages the use and appreciation of visual reasoning. To address this, assessments should include problems that integrate both equations and graphs, require students to interpret graphs directly, and award credit for explaining their problem-solving strategies rather than just for arriving at the correct solution. Implementing these changes would send a clear message that understanding and reasoning are valued over mere computation (NCTM, 2000; Schoenfeld, 1992).

Support Retention

Many students struggled to recall or apply basic concepts like domain and function evaluation, despite having learned them in grades 10 and 11. This aligns with research showing that if topics are not revisited regularly, students tend to forget important material (Bjork & Bjork, 2011; Rohrer & Pashler, 2007). To address this issue, teachers can incorporate spaced practice and weave review activities into different units, design cumulative reviews that connect earlier ideas to new contexts, and begin the school year with short assessments to identify areas that need refreshing.

Local Challenges, Global Relevance

Although this study focused on Nepal, the issues are familiar to educators in many countries. Students often favor algebra over graphs because that's what they're used to (Knuth, 2000a; Van Dyke & White, 2004). What seems different here is just how rarely students successfully used graphs, which may be a result of fewer opportunities to practice or less exposure in class. Future research could look at how different teaching practices and educational systems affect students' ability to move between representations. Lessons from other countries might help inform what works best in different contexts.

In the end, these results show that improving students' understanding of functions isn't just about teaching more content—it's about teaching differently. We need to help students see that mathematics can be approached in more than one way, and that being flexible in how they think is just as important as getting the right answer. Fixing these issues will take coordinated changes to curriculum, instruction, assessment, and teacher support—but the payoff is students who are better prepared to think deeply and solve problems in a connected, meaningful way.

REFERENCES

[1] Arnal-Palacián, M., Baeza, M. Á., & Claros-Mellado, J. (2022). Quadratic Functions Representation: Pencil and Paper vs GeoGebra. *International Journal for Technology in Mathematics Education*, *29*(3), 165-178.

[2] Artigue, M. (2009). Didactical design in mathematics education. *Nordic Studies in Mathematics Education, 14*(1), 7-18.

[3] Bjork, R. A., & Bjork, E. L. (2011). Making things hard on yourself, but in a good way: Creating desirable difficulties to enhance learning. In M. A. Gernsbacher et al. (Eds.), *Psychology and the real world: Essays illustrating fundamental contributions to society* (pp. 56-64). Worth Publishers.

[4] Booth, L. (1981). Graphs in Mathematics and Science. *Mathematics in School, 10*(4), 2–5

[5] Booth, J. L., & Newton, K. J. (2012). Conceptualizing algebraic reasoning: Developing the capacity for multiple representations. In L. R. Wixted & S. D. Zannetti (Eds.), *Teaching and learning mathematics through multiple representations* (pp. 181-206). Springer.

[6] Brown, P. C., Roediger, H. L., & McDaniel, M. A. (2014). *Make it stick: The science of successful learning.* Harvard University Press.

[7] Common Core State Standards Initiative. (2010). *Common Core state standards for mathematics*. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

[8] Dubinsky, E., & Guershon Harel. (1992). *The Concept of function: aspects of epistemology and pedagogy.* Mathematical Association of America.

[9] Durant, K., & Garofalo, J. (1994). Teaching functions and graphing in pre-calculus mathematics. *Journal of Developmental Education, 18*(1), 18.

[10] Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. In F. Hitt & M. Santos (Eds.), *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 3–26). Cuernavaca, Morelos, México

[11] Duval, R. (2017). Understanding the mathematical way of thinking: The registers of semiotic representations. Springer International Publishing. https://doi.org/10.1007/978-3-319-56910-9.

[12] Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics, 21*(6), 521–544. https://doi.org/10.1007/bf00315943

[13] Fey, J. T., & Harel, G. (2004). The roles of algebraic and graphical representations in understanding functions. In G. Harel & E. Dubinsky (Eds.), *Proceedings of the 12th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 109-114).

[14] Hacker, D. J., Dunlosky, J., & Graesser, A. C. (2009). *Metacognition in Educational Theory and Practice*Kotovsky (Eds.). In *Complex information processing: The impact of Herbert A. Simon* (pp. 285–318). Lawrence Erlbaum Associates, Inc.

[15] Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65-97). Macmillan Publishing.

[16] Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. *The Journal of Mathematical Behavior, 17*(1), 123–134. https://doi.org/10.1016/s0732-3123(99)80064-9

[17] Knuth, E. J. (2000a). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education, 31*(4), 500. https://doi.org/10.2307/749655

[18] Knuth, E. J. (2000b). Understanding connections between equations and graphs. *The Mathematics Teacher*, *93*(1), 48-53.

[19] Lobato, J., Hohensee, C., Rhodehamel, B., & Diamond, J. (2012). Using student reasoning to inform the development of conceptual learning goals: The case of quadratic functions. *Mathematical Thinking and Learning*, *14*(2), 85-119.

[20] Larkin, J., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*(1), 65–99. https://doi.org/10.1111/j.1551-6708.1987.tb00863.x

[21] Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, Graphs, and Graphing: Tasks, Learning, and Teaching. *Review of Educational Research, 60*(1), 1. https://doi.org/10.2307/1170224

[22] Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33–40). Lawrence Erlbaum Associates.

[23] Markovits, Z., Eylon, Bat Sheva, & Bruckheimer, M. (1988). Difficulties students have with the function concept. In *The ideas of algebra, K12: 1988 yearbook* (pp. 43–60). The National Council of Teachers of Mathematics.
[24] Mainali, B. (2021). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 9*(1), 1-21. https://doi.org/10.46328/ijemst.1111
[25] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. NCTM.
[26] Orhun, Nevin. (2012). Graphical Understanding in Mathematics Education: Derivative Functions and Students' Difficulties. *Procedia - Social and Behavioral Sciences, 55*, 679–684. https://doi.org/10.1016/j.sbspro.2012.09.551
[27] Pavlopoulou, K. (1993). Un problème décisif pour l'apprentissage de l'algèbre linéaire : La coordination des registres de représentation. In R. Duval (Éd.), *Annales de didactique et de sciences cognitives*, 5, 67–93. IREM de Strasbourg.
[28] Piaget, J. (1972). *The psychology of the child*. Basic Books.
[27] Romberg, T. A., Fennema, E., & Carpenter, T. P. (2012). Towards a common research perspective. In *Integrating Research on the Graphical Representation of Functions* (pp. 1–9). Routledge.
[29] Rohrer, D., & Pashler, H. (2007). Increasing retention without increasing study time. *Current Directions in Psychological Science, 16*(4), 183-186.
[30] Schliemann, A. D., Carraher, D. W., & Teixidor-i-Bigas, M. (2022). Teacher development structured around reasoning about functions. *International Journal of Science and Mathematics Education*, *20*(4), 793-816.
[31] Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). Macmillan Publishing.
[32] Schoenfeld, A. H., Smith, J. P., III, & Arcavi, A. (1993). Learning: The microgenetic analysis of one student's evolving understanding of a complex subject matter domain. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 4, pp. 55–175). Erlbaum.
[33] Shuard, H., & Neill, H. (1977). *From Graphs to Calculus (The mathematics curriculum: a critical review).* Blackie For the Schools Council.
[34] Steffe, L. P., & Gale, J. (Eds.). (1995). *Constructivism in education*. Lawrence Erlbaum Associates.
[35] Tiew, N., Teoh, S., Singh, P., Walida, S., & Surya, F. (2023). An analysis of students' understanding of algebraic concepts. *Jurnal Pendidikan Sains dan Matematik Malaysia, 13*, 86-95. https://doi.org/10.37134/jpsmm.vol13.2.8.2023
[36] Van Dyke, F., & White, A. (2004). Examining students' reluctance to use graphs. *The Mathematics Teacher*, *98*(2), 110-11.
[37] Wilkie, K. J. (2024). Coordinating visual and algebraic reasoning with quadratic functions. *Mathematics Education Research Journal*, *36*(1), 33-69.