

Technology-Based Worksheets to Develop Students' Mathematical Argumentation Skills

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Abstract: This study examines the need for and effectiveness of technology-based worksheets in improving students' mathematical argumentation skills. The worksheets were developed based on the Toulmin argumentation model, including mathematical problems, data, claims, warrants, qualifiers, rebuttals, and backing to guide students in constructing logical mathematical arguments. A pretest–posttest design with two unequal experimental groups was used. The population consisted of 200 prospective mathematics teachers in East Java, Indonesia, and 80 students were randomly selected and divided into two groups. Experimental Group 1 used digital-based worksheets, while Experimental Group 2 used printed worksheets. Data were collected through questionnaires and tests. Initial findings showed that although lecturers allowed students to express opinions, debate, and challenge peers' ideas, the worksheets commonly used did not promote structured mathematical argumentation or integrate technology. Therefore, technology-based worksheets were needed. The results showed that the average post-test score of the digital worksheet group (82.88) was significantly higher than that of the printed worksheet group (69.22). Statistical testing ($p < 0.05$) confirmed a significant difference between the two groups. It can be concluded that digital-based worksheets have a positive and significant effect on students' mathematical argumentation abilities.

Keywords: mathematical argumentation skills, printed worksheets, technology-based worksheets

INTRODUCTION

One challenge encountered when teaching mathematics at the university level is the insufficient proficiency among students in constructing mathematical arguments, particularly when tackling argumentation problems. Within the realm of mathematical argumentation, certain students occasionally neglect to employ deductive reasoning (Inglis et al., 2007; Trisanti, 2019a; Trisanti et al., 2016). However, deductive reasoning stands as the sole valid form of argumentation, as its

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premises derive from verified definitions, theorems, and/or facts (Harel, 2001; Lodder, 2004; Tall, 2004).

The inadequate proficiency of students in mathematical argumentation stems from their insufficient grasp of the significance of argumentation. Additionally, they lack proper training in executing effective arguments (Harel & Sowder, 1998). Enhancing their argumentation skills necessitates a transition from non-deductive to deductive argumentation. Hence, it is crucial for the teaching objectives to be clearly defined to progressively enhance students' competency in mathematical argumentation, ultimately revealing formal evidence (Harel, 2001). Teachers should guide students in constructing and evaluating mathematical arguments (Ball et al., 2008).

Consideration of the use of worksheets to improve mathematical argumentation skills is important. This is in accordance with the suggestions given by Pruitt (2015) and NGSS (2013), which emphasize that the teacher's role is not only limited to guiding students in understanding concepts and skills, but also facilitating students' abilities in conducting investigations, building models, arguing, and achieving certainty in the context of science and mathematics. Learning approaches that integrate collaborative components and software technology are seen as effective strategies for improving students' argumentation skills (Fukawa-Connelly & Silverman, 2015). Kartika et al. (2023) emphasize the importance of structured teaching activity design. Integrating technology into worksheets can enhance interaction and collaboration, as well as help students better understand and apply mathematical concepts.

One effort to optimize learning activities is to empower students through the use of worksheets (Mutia & Prasetyo, 2018). Considering the teacher's role as a facilitator, worksheets enable students to become more active in learning. Students can use worksheets as a guide in understanding concepts and problem-solving, developing skills, and providing opportunities for students to demonstrate their knowledge (Karsli & Şahin, 2009; Utami et al., 2016). The advancement of technology today encourages researchers to utilize it in creating worksheets. In this study, worksheets are designed in such a way with the help of technology to enhance students' mathematical argumentation abilities. One of the goals of combining worksheets with technology is to motivate students in learning, as the appearance of the worksheets becomes more engaging and less monotonous.

Well-designed Student Worksheets (LKS) can promote active engagement and reflective thinking in students during mathematics learning (Suryawati et al., 2020). Along with the increasing use of technology in education, several studies have shown that integrating digital tools in LKS can enrich the learning experience and facilitate the development of higher-order thinking skills, including mathematical argumentation (Drijvers, 2013; Zbiek & Hollebrands, 2008). However, most of these studies focus on the general effectiveness of technology without specifically developing or evaluating teaching tools designed directly to build students' argumentation skills. Therefore, this study

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aims to describe the needs and outcomes of implementing technology-based worksheets specifically designed to improve students' mathematical argumentation skills as a contribution to the development of evidence-based learning design

The components in the technology-based worksheet refer to the Toulmin scheme. The Toulmin scheme is an argumentation model developed by philosopher Stephen Toulmin to analyze and describe the structure of arguments in detail (Toulmin, 2003). This scheme consists of six main components consisting of data, claim, warrant, backing, disclaimer and qualifier. Data is evidence or reasons that support a claim. Data provides the factual basis for claims. A claim is a statement that you want to prove or support in an argument. This is the main conclusion drawn from the argument. Warrants are a logical bridge that connects data with claims. The Warrant explains why the data is relevant to support the claim. Backing is additional support that strengthens the warrant. Backing provides further justification and is often based on deeper principles or data. A disclaimer is an acknowledgment of a potential objection or condition that could weaken a claim. Rebuttals help make arguments more realistic and credible by considering exceptions. Qualifiers are words or phrases that indicate the extent to which a claim applies or introduces uncertainty. Qualifiers help set claim limits.

Previous studies have shown that technology integration in mathematics education can enhance students' engagement, conceptual understanding, and higher-order thinking skills (Drijvers et al., 2010; Zbiek et al., 2007). Specifically, digital tools such as dynamic geometry software, online collaborative platforms, and interactive simulations have been found to support the development of mathematical reasoning and discourse (Hoyles & Noss, 2016; Kieran et al., 2015). However, much of the existing literature focuses on general reasoning skills or problem-solving abilities, with limited emphasis on structured mathematical argumentation as a distinct competency. Tristani & Nusantara (2021), Tristani et al. (2024), and Tristani & Nusantara (2023) have explored students' mathematical argumentation from both content-based and semiotic perspectives, identifying challenges and variations in students' abilities to construct valid arguments across different topics and learning models. Nevertheless, their studies primarily focus on face-to-face instruction and do not address the integration of digital tools in facilitating argumentation. Furthermore, while some research has explored the use of digital worksheets or scaffolding tasks (e.g., Niess et al., 2013), these often emphasize content transmission rather than engaging students in constructing and critiquing mathematical arguments. In contrast, this study offers a more targeted approach by designing technology-based worksheets explicitly aimed at promoting students' ability to construct logical, evidence-based arguments in mathematics. By aligning the worksheet design with Toulmin's model of argumentation and integrating interactive features that prompt justification and reflection, this study addresses a gap in the literature and contributes a novel instructional tool that blends pedagogical theory with practical classroom technology.

Research related to learning to develop mathematical argumentation skills includes applying the infusion learning model (L. B. Tristani & Nusantara, 2021), applying PBL and CIRC (L. B. Tristani & Nusantara, 2022a), developing student argumentation through small group collaboration using the dynamic geometry software (DGS) software environment (Fukawa-Connelly & Silverman, 2015), and developing a web-based multimedia whiteboard system to assist students in learning mathematical problem solving where students are asked to criticize solutions and reply to other people's arguments (Hwang et al., 2006). It appears that no previous research has used technology-based worksheets to develop students' mathematical argumentation abilities. Therefore, this research was carried out with the aim of:

- A. Describing the need for technology-based worksheets to develop students' mathematical argumentation abilities.
- B. Describing the results of applying technology-based worksheets to develop students' mathematical argumentation abilities

LITERATURE REVIEW

Mathematical Arguments Worksheet

Worksheets or student worksheets are defined as sheets containing guidance for students to perform tasks or activities related to learning (Yaden, 2017). Worksheets can help improve students' understanding of subject matter, enhance learning motivation, support active student engagement in the learning process, assist students in organizing information, and enhance critical thinking skills (Basuki & Wijaya, 2018; Kaymakci, 2012). According to Abd-El Khalick et al. (2010) and Susantini et al. (2016), worksheets are part of teaching materials that can be used to develop students' thinking skills, pose and answer questions, make connections between concepts, and assess students' learning progress.

The application of worksheets in mathematics education has a positive impact on students' learning outcomes (Nurhidayah et al., 2020). Worksheets help improve conceptual understanding, critical thinking skills, and students' ability to solve mathematical problems (Novitasari et al., 2022). Therefore, it is recommended that teachers use worksheets as part of their mathematics teaching strategies to enhance students' learning outcomes

Perangkat Teknologi Argumen **Koreksi Jawaban Siswa** Suharnanik ▾

KUNCI JAWABAN

Tanggal Jawab Siswa 02/02/2022

Nama Kelompok Kelompok A

Nama Siswa Regita

Isi Soal Sebuah toko menjual pensil seharga Rp 500 per pensil dan penghapus seharga Rp 1.000 per penghapus. Jika total penjualan pensil dan penghapus adalah 1 dengan harga Rp 8.500, berapa jumlah pensil dan penghapus yang terjual?

Data

harga satu pensil adalah Rp 500 dan harga satu penghapus adalah Rp 1000 jumlah total pensil dan penghapus yang terjual adalah 10

Langkah Penyelesaian	Alasan	Dasar Teori
<input type="checkbox"/> jumlah pensil yang terjual adalah x dan jumlah penghapus yang terjual adalah y	<input type="checkbox"/> Mengubah permasalahan realistic ke kalimat matematika	<input type="checkbox"/> Aturan variabel
<input type="checkbox"/> Harga total pensil yang terjual adalah 500x	<input type="checkbox"/> Mengubah permasalahan realistic ke kalimat matematika	<input type="checkbox"/> Data: harga satu pensil adalah Rp 500
<input type="checkbox"/> $x + y = 10$	<input type="checkbox"/> Model matematika dari masalah	<input type="checkbox"/> Data: jumlah total pensil dan penghapus yang terjual adalah 10

Kesimpulan

jumlah pensil yang terjual 3 dan penghapus 7

Kekuatan Penyelesaian

Yakin benar karena langkah penyelesaian disesuaikan dengan konsep SPLDV metode substitusi

Komentar

Masukkan Komentar

Deskripsi Penilaian

Masukkan Data Deskripsi Penilaian

Deskripsi Penilaian

Data - Data yang digunakan benar dan sesuai dengan fakta.

Klaim - Klaim dinyatakan dengan jelas dan tidak ambigu.

Warrant - Warrant menunjukkan alasan atau prinsip yang menghubungkan data dan klaim secara masuk akal

Backing - Mengutip Aksioma, Postulat, Definisi, Teorema, Dalil, Lema, Korolari, dan Konjektur dari sumber yang kredible untuk memperkuat validitas wan

Sanggahan - Menunjukkan kondisi tertentu di mana klaim tidak berlaku atau kurang kuat

Qualifier - Menggunakan kata-kata seperti "mungkin benar", "jelas benar", "bias jadi", "kemungkinan" atau kata yang lain untuk menyatakan tingkat ket

Figure 1. Mathematical Argument Worksheet Framework

Mathematical Argument Worksheet is a worksheet used to help students develop and organize mathematical arguments systematically and logically. This worksheet contains a series of tasks or questions designed to lead students through the process of critical thinking and mathematical problem solving, with a focus on forming valid and coherent arguments. The components in the

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mathematical argumentation worksheets refer to the components of Toulmin's argument scheme (Toulmin, 2003), which include:

- (1) Mathematical problem or statement, which presents a mathematical problem to be solved or a question to be answered.
- (2) Data, which requires students to provide a factual basis for proving claims.
- (3) Claim, which requires students to convey the statement they want to prove or support in an argument.
- (4) Warrants, which require students to create a logical bridge/completion steps that connect the data with the claim, and explain why the data is relevant to support the claim.
- (5) Backing, which requires students to provide additional support that strengthens the warrant.
- (6) Rebuttal, which requires students to consider exceptions makes the argument more realistic and credible.
- (7) Qualifiers, which require students to express words or phrases that indicate the extent to which a claim applies or introduces uncertainty.
- (8) Description of mathematical argumentation abilities, in which the lecturer conveys mathematical argumentation skills based on filling in data, claims, warrants, qualifiers, rebuttals and backing in student worksheets.

Figure 1 shows the framework of the Mathematical Argument Worksheet. In this research, mathematical argument worksheets are packaged in digital-based worksheets and printable worksheets. Digital-based worksheets can be accessed via a link <https://argumen.oldmapps.com/>.

Mathematical Argumentation Skills

Mathematical argumentation is defined as a process that involves making claims, using evidence to support those claims, and presenting and evaluating arguments in a mathematical context (Wells, 2014). Mathematical argumentation ability refers to a person's ability to use logic and mathematical evidence in explaining, defending, and convincing others about the truth of a mathematical statement or concept (Tristani & Nusantara, 2022b). Mastering mathematical argumentation is a valuable skill in mathematics education, which not only enhances academic understanding but also prepares students for challenges beyond the academic environment (Sriraman & Umland, 2020). Table 1 describes the elements of student competence in mathematical argumentation.

Component	Indicators
Data	<ol style="list-style-type: none"> 1. The data used must be correct and in accordance with the facts. 2. Data must be directly related to the claim being made. 3. All data necessary to support the claim must be included. 4. Data comes from reliable and valid sources. 5. Data is presented clearly and easy to understand. 6. The data logically supports the claims made without any contradiction.
Claim	<ol style="list-style-type: none"> 1. Claims are stated clearly and unambiguously. 2. Claims must be specific and focused on the topic discussed. 3. Claims can be proven or supported with strong evidence and reasons. 4. Claims are relevant to the context of the argument and discussion topic. 5. Claims are directly related to the data and evidence presented 6. Claims are based on logical and rational thinking.
Warrants	<ol style="list-style-type: none"> 1. Warrant shows reasons or principles that make a reasonable connection between data and claims 2. The explanation in the warrant must be acceptable to the audience and based on logic or strong evidence 3. There is a specific explanation of the data that supports the claim, not just an assumption. 4. Warrants are based on general principles, theories, or verifiable evidence
Backing	<ol style="list-style-type: none"> 1. Quote axioms, postulates, definitions, theorems, propositions, lemmas, correlations and conjectures from credible sources to strengthen the validity of the warrant 2. Include widely accepted theories or principles to strengthen the basis of the warrant. 3. Provide concrete examples or illustrations that support the warrant.
Rebuttal	<ol style="list-style-type: none"> 1. Indicates certain conditions where the claim is not valid or lacks strength 2. Present data or evidence to weaken the claim
Qualifiers	Using words such as "probably true", "obviously true", "could be", "likely" or other words to express the level of truth of the claim

Table 1. Competence In Mathematical Argumentation

METHOD

Research Design

This study utilized a quantitative experimental method to determine the impact of learning with technology-based worksheets on students' mathematical argumentation skills. The research employed a pretest-posttest, nonequivalent multiple group design. The research design of this study is illustrated in Figure 2.

Group	Beginning	Treatment	End
Experiment 1	Y_1	T_a	Y_2
Experiment 2	Y_1	T_b	Y_2

Figure 2. Research Design

Y_1 is the pretest of students' mathematical argumentation skills, T_a is the treatment with digital-based worksheets, T_b is the treatment with printable worksheets, Y_2 is the post-test of students' mathematical argumentation skills.

The treatment is considered to have a significant effect on students' mathematical argumentation skills if a significant difference in mathematical argumentation skills is found between experimental group 1 and experimental group 2. The learning process with digital-based worksheets was conducted for experimental group 1 over three sessions, with the post-test administered in the fourth session. Each session lasted 100 minutes. For experimental group 2, students received the same material as experimental group 1. However, they did not use digital-based worksheets as their treatment; instead, they used printable worksheets. Both groups were then given the same validated post-test, which was a mathematical argumentation test.

Population and Sample

The population of this study consisted of 200 prospective math teacher students at a tertiary institution in East Java, Indonesia, all approximately 20 years old. The sample for this study included 80 prospective math teacher students, divided into an experimental group 1 (40 students) and an experimental group 2 (40 students). The experimental group 1 comprised 15 males and 25 females, while the experimental group 2 comprised 16 males and 24 females.. This selection was based on preliminary test results provided to all classes, ensuring that both groups had equivalent initial abilities. This is also supported by the results of interviews with teachers in both groups, which indicated that both groups have equal mathematical ability and activity (homogeneous). In terms of gender, both groups have a comparable number of male students, with 15 male students in Experiment group 1 and 16 male students in Experiment group 2. Both groups also have a comparable number of female students, with 25 female students in Experiment group 1 and 24 female students in Experiment group 2. This indicates that the researcher implemented strict control over external variables that could affect the study's results.

Worksheet Design

In this study, the worksheets used consist of two formats: digital format and printed format. The digital format worksheets are designed using interactive learning software that allows students to interact directly with the content. This format includes multimedia elements, such as explanatory videos, interactive images, and quizzes that can be completed online. Meanwhile, the printed

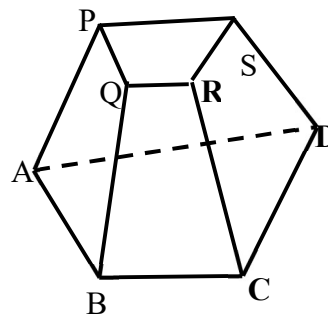
worksheets are the physical version of the same material, which includes text, static images, and clear instructions for students to complete on paper.

The content of both formats is identical in terms of topics and learning objectives. Each worksheet is designed to cover the same concepts and achieve the desired learning outcomes. However, the digital format allows for a more dynamic and engaging presentation of content. Several adaptations are made to ensure that the digital worksheets are more effective and interactive. These include: the addition of interactive elements, such as quizzes and tasks that can be completed online; the integration of multimedia, such as videos or animations, that support concept understanding; and adjustments to instructions to ensure that students understand how to use the interactive features available in the digital format. Figure 3 illustrates a task used in the worksheet for spatial geometry content.

Task 1

Information:

The following image represents a pyramid with its top cut off



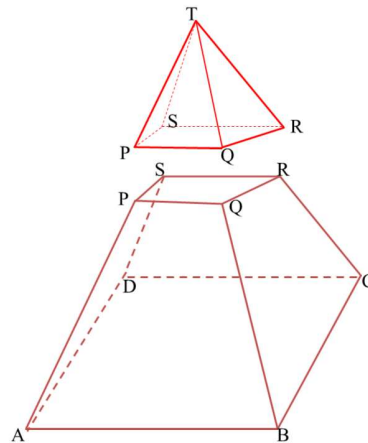
Introductory Questions:

1. Identify the characteristics of image!
2. Name and define the image according to the characteristics you have described!

Problem 1:

Prove that the bases of a frustum of a pyramid are similar!

Task 2



Prove that $V_{Frustum} = \frac{1}{3} t (A + B + \sqrt{AB})$.

Here t is the height of the frustum, A is the area of the base of the frustum, and B is the area of the top of the frustum!

Figure 3. Argumentation Ability Task

The spatial geometry task have the potential to be used in learning that develops students' mathematical argumentation skills, as these tasks require students to prove a formula or theorem, which involves logical reasoning steps and mathematical argumentation.

The spatial geometry task is important in students' learning as it involves fundamental geometric concepts, such as volume and surface area, which are essential for understanding three-dimensional structures. Proving the volume formula of a frustum deepens students' understanding of spatial geometry, which is a crucial foundation in advanced mathematics. Furthermore, the proof process enhances students' skills in deductive reasoning and formal logic, which are essential abilities for higher-level mathematics learning.

This task also challenges students to think about the relationships between various geometric elements, helping them develop analytical and problem-solving skills. The task supports creative mathematical thinking by asking students not only to understand the formula but also to prove and explain its origins. Creativity emerges when students seek different ways to approach the proof, such as using visual methods, algebra, or a combination of both. Creative thinking can be expected when students try to connect this task with the concept of the volume of a pyramid or use substitution techniques from simpler problems. Creativity may also arise in how students organize their arguments or create illustrations that aid in the proof. Thus, this task provides space for exploration and the discovery of new solutions, supporting the development of creative thinking skills in mathematics.

The tasks designed in the worksheets were implemented through small-group learning activities consisting of 3-4 students per group. Students were asked to collaboratively discuss and solve the problems, particularly those that required constructing mathematical arguments. The teacher acted as a facilitator, monitoring group discussions, providing limited guidance when necessary, and occasionally posing prompting questions to encourage the development of arguments. After working in groups, selected students were invited to present their group's solutions to the class. This was followed by whole class discussions involving responses from both peers and the teacher, thereby fostering a learning environment that supports idea exchange and the strengthening of mathematical argumentation.

Instrument

This instrument consists of a questionnaire and a test. Two types of questionnaires were used: the digital worksheet needs questionnaire and the digital worksheet assessment questionnaire. The digital worksheet needs questionnaire was filled out by students before they participated in learning using the digital worksheet. The digital worksheet assessment questionnaire was filled out by students after they participated in learning using the digital worksheet. Both questionnaires were provided on Google Forms. The assessment results from the respondents were analyzed by calculating the average score obtained for each component of the digital worksheet design, then converted into qualitative statements according to Table 2.

Score Interval	Decision
4.20 - 5.00	Digital worksheet design is very appropriate
3.40 - 4.19	Digital worksheet design is appropriate
2.60 - 3.39	Digital worksheet design is somewhat appropriate
1.80 - 2.59	Digital worksheet design is inappropriate
1.00 - 1.79	Digital worksheet design is very inappropriate

Table 2. Score Conversion

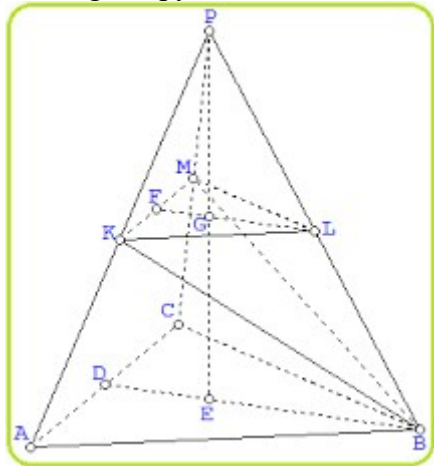
This test was a mathematical argumentation test used to collect data about students' mathematical argumentation skills. The data on students' mathematical argumentation competence were analyzed quantitatively by assigning scores to each of the elements. The scoring guidelines were defined as follows: a score of 2 was assigned if students correctly identified the elements, a score of 1 if students incorrectly identified the elements, and a score of 0 if students did not identify any elements.

MATHEMATICAL ARGUMENTATION TEST

Instructions:

- a. Fill in your Name, Class, and Phone Number in the provided columns!
- b. Read the instructions carefully first!
- c. Show all your ideas and solutions!
- d. If there are any incorrect answers, cross them out without erasing!

1. Investigate the truth of this statement!
 In every simple triangular pyramid, the area of the top face and the base area of the pyramid are proportional to the square of the height of the upper pyramid with the height of the original pyramid.



2. Based on the answer to question 1, create a mathematical argumentation scheme!

Figure 4. Mathematical Argumentation Test

The test was in a descriptive format and included two evidentiary problems. Before administering it to the students, the test was validated by experts, specifically two mathematics education specialists and one mathematician. The purpose of this validation was to ensure the test met content and construct validation criteria. These criteria were assessed in three components: the appropriateness of the material content, the construction of the material, and the language suitability. If there were any revisions, the instrument was revised according to the suggestions provided by the validators. Once the validators declared the instrument fit for use, it was administered to the students to assess their mathematical argumentation skills. Figure 4 below is a mathematical argumentation test.

Essentially, both groups received the same learning process, including the teaching strategies, the teacher, the material taught, the content of the worksheets, and the duration of the learning time.

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The only difference lies in the type of worksheet used. Experiment group 1 utilized digital-based worksheets, while Experiment group 2 used printable worksheets.

Research Hypothesis

The treatment is considered to have a significant effect on students' mathematical argumentation skills if a significant difference in mathematical argumentation skills is found between experimental group 1 and experimental group 2. The research hypotheses are as follows:

- H₀: There is no difference in mathematical argumentation skills between experimental group 1 and experimental group 2.
- H₁: There is a difference in mathematical argumentation skills between experimental group 1 and experimental group 2.

Analysis of Data

The research data, comprising the post-test scores of students, underwent statistical analysis. A t-test was conducted to assess the significance of the treatment, specifically the impact of learning with digital-based worksheets. The treatment is deemed significant if there is a notable difference in post-test scores between experimental group 1 and experimental group 2.

The criterion for supporting H₀ states, "H₀ is supported if the Sig. value ≥ 0.05 ," indicating no significant difference exists in post-test scores between experimental group 1 and experimental group 2. Absence of a significant difference suggests the treatment lacks a substantial effect on the experimental group. Here, the treatment involves implementing

Before conducting the t-test analysis, it is necessary to perform tests for normality and homogeneity. The normality test assesses whether the data distribution follows a normal distribution, where a distribution is considered normal if the asymptotic significance value is > 0.05 . The homogeneity test examines the similarity between the experimental-1 and experimental-2 groups, specifically comparing their variances. Comparison between the groups is valid only if their variances are similar. Homogeneity is confirmed if the Sig. value > 0.05 , indicating no significant difference in variance at the 5% significance level.

When the data is not normally distributed or when the assumption of homogeneity of variances is not met, the data is analyzed using the Mann-Whitney test. The results of the Mann-Whitney test are confirmed with the statistical significance value from the Z test in the Mann-Whitney test.

RESULTS

Preliminary research results

The preliminary research results were based on the analysis of the digital worksheet needs, filled out by students through Google Forms as shown in Table 3.

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No	Statement	Percentage
1	Lecturers use worksheets in learning	20
2	The worksheets used by lecturers are fun and make students actively learn	80
3	Lecturers use worksheets that are appropriate to the learning material	90
4	The worksheet encourages students to construct mathematical arguments based on the Toulmin scheme	15
5	Lecturers utilize technology in using worksheets	10
6	Lecturers use worksheets to introduce technology	10
7	Lecturers use worksheets to guide students to develop mathematical argumentation skills	45
8	Lecturers use worksheets to allow students to evaluate observations	85
9	Lecturers use worksheets to enable students to create problem models and predictions	85
10	Lecturers use worksheets to make it easier for students to design steps to compose mathematical arguments	60
11	Lecturers use worksheets to facilitate students in finding answers	95
12	Teachers use worksheets to integrate technology and mathematics in the learning process, making it easier for students to understand concepts	45
13	The lecturer gives students the opportunity to refute their friends' statements	100
14	Lecturers provide opportunities for students to express their opinions	100
15	Lecturers give students the opportunity to debate with their friends	100

Table 3. Needs Analysis Results for Digital Worksheets

Based on Table 3, four important points are evident. First, not all lecturers use worksheets. This is in line with statement 1, which indicates that only 20% of lecturers use worksheets in their teaching. Second, the worksheets used by the lecturers are enjoyable and encourage active learning among students, are aligned with the learning materials, enable students to evaluate observations, help students create problem models and predictions, facilitate students in designing steps to construct mathematical arguments, and assist students in finding answers. Third, lecturers provide opportunities for students to challenge their peers' statements, express opinions, and engage in debates with their peers. Fourth, the worksheets used by lecturers have not yet encouraged students to construct mathematical arguments based on the Toulmin model, have not utilized technology in the use of worksheets, and have not guided students in developing their mathematical argumentation skills. As a result, students' ability to provide answers and reasons for solving problems remains weak. This is also supported by research from Tristanti Nusantara (insert citation year), which indicates that, overall, the argumentation skills demonstrated by students, particularly in refutation skills, are still in the poor category.

Many students have not yet mastered the mathematical concepts they are studying. A strong grasp of these concepts is essential for making counterarguments, as it allows students to provide alternative answers (Dinantika et al., 2020). Trisanti (2019) indicates that students tend to focus on empowering claims without utilizing all elements of argumentation skills. This leads to a weak understanding of the concepts of floating and submerged materials. Ideally, a comprehensive empowerment of argumentation skills would enhance students' conceptual understanding. The findings of this research serve as a reference for making improvements in future studies to achieve positive changes by empowering argumentation skills, thereby directly enhancing students' conceptual understanding. Therefore, technology-based worksheets are needed to develop students' mathematical argumentation skills.

The design of the worksheets refers to the components of the Toulmin model. The worksheets are available in both digital and printable formats. The structure and components of the digital and printable worksheets are the same; the difference lies in their usage. Digital worksheets are used on electronic devices such as computers or smartphones. Users can fill out and save their answers directly on the device, and the worksheets can be used for online learning. Printable worksheets, on the other hand, are printed on paper and used manually, requiring writing tools to complete them.

Mathematical Argumentation Ability Results

After the mathematics lessons using worksheets, students in experiment group 1 and experiment group 2 were given a posttest to measure their achievement in mathematical argumentation ability. The posttest data were analyzed for normality, homogeneity, and t-tests. The following are the results of the normality test, homogeneity test, and t-test on the students' mathematical argumentation ability data.

Test of Normality

Based on the data processing results of the posttest scores for experiment group 1 and experiment group 2, with the number of participants being $n_1 = 40$, $n_2 = 40$ respectively, the mean scores obtained were a $\bar{x}_1 = 82.8750$, $\bar{x}_2 = 69.2188$, with standard deviations of $s_1 = 10.10828$, $s_2 = 8.66485$. To determine the effect of digital worksheets on students' mathematical argumentation abilities, several statistical tests need to be performed. First, a normality test was conducted using SPSS software, which produced the Shapiro-Wilk normality test output for the posttest scores, as shown in Table 4 below.

Group	n	Mean	Std. Deviation	Shapiro-Wilk Statistic	Shapiro-Wilk Sig.
Experiment-1	40	82.8750	10.10828	0.945	0.069
Experiment-2	40	69.2188	8.66485	0.927	0.031

Table 4. Shapiro-Wilk Normality Test Output For The Posttest Scores

The results indicate that the posttest scores for both experiment groups were tested for normality using the Shapiro-Wilk test. For experiment group 1, the Shapiro-Wilk statistic is 0.945 with a significance value of 0.069, indicating that the data is normally distributed. For experiment group 2, the Shapiro-Wilk statistic is 0.927 with a significance value of 0.031, indicating that the data is not normally distributed.

Homogeneity of Variance Test

Since the posttest scores for one of the groups are not normally distributed, the next step is to perform a homogeneity of variance test using Levene's test for the posttest scores. Table 5 below shows the results of Levene's test for the homogeneity of variance for the posttest scores.

Levene Statistic	df1	df2	Sig.
2.135	1	78	0.148

Table 5. Results of Levene's Homogeneity of Variance Test for Posttest Scores

The results of Levene's test indicate that the significance value (Sig.) is 0.148. Since this value is greater than 0.05, it suggests that the variances of the posttest scores for both groups are homogeneous, meaning there is no significant difference in the variances of the two groups. Given these results, further analysis can be conducted using appropriate statistical methods that do not assume normal distribution, such as the Mann-Whitney U test, to compare the posttest scores of the two groups.

In the experiment-1 group, the posttest scores do not follow a normal distribution. Therefore, the Mann-Whitney test was used to compare the means of the experiment-1 and experiment-2 groups. The results of the data analysis using SPSS software, including the R_1 and R_2 values, are presented in Table 6 below.

	Group	N	Mean Rank	Sum of Ranks
Posttest Value	Experimental 1	40	43.89	1404.50
	Experimental 2	40	21.11	675.50
	Total	80		

Table 6. R_1 dan R_2 Values from the Mann-Whitney U Test

Based on the data analysis results in Table 6, the values obtained are $R_1 = 1404.50$ and $R_2 = 675.50$. The Experimental-1 group has a higher posttest score based on the average rank (43.89) compared to the Experimental-2 group (21.11). This may indicate that the Experimental-1 group tends to perform better in the posttest compared to the Experimental-2 group; however, this result is confirmed by examining the statistical significance value from the Mann-Whitney test. The next step

involves using SPSS software to perform the Mann-Whitney-Wilcoxon test, as shown in Table 7 below.

	Posttest Value
Mann-Whitney-Wilcoxon	147.500
Wilcoxon W	675.500
Z	4.902
Asymp. Sig. (2-tailed)	0.000

Table 7. Mann-Whitney-Wilcoxon Test

Based on the output in Table 7, the calculated Z value is 147.500 and the significance (Sig) value is 0.000. Since $\text{Sig} < 0.05$, the null hypothesis (H_0) stating $\mu_1 = \mu_2$ is rejected. It can be concluded that the average post-test scores for the experiment-1 and experiment-2 groups are not the same. Therefore, it can be concluded that there is an effect of using digital worksheet-assisted mathematics learning on students' mathematical argumentation skills

Digital Worksheet Design Assessment Results

The assessment of the digital worksheet design was conducted to evaluate the suitability of the digital worksheet and the design of learning activities aimed at developing students' argumentation skills. This assessment process ensures that the digital worksheet is appropriate and effective for use in learning activities designed to enhance students' argumentation abilities. The results of the digital worksheet design assessment are presented in Table 8.

Based on the assessment results in Table 8, all evaluated components received scores ranging from 4.20 to 5.00. According to Table 2, the digital worksheet design is deemed very appropriate and can be used to develop students' mathematical argumentation skills. Figure 5 represents the results of students' mathematical arguments through digital worksheets.

No	Digital-based worksheet design	Score
1	Suitability of digital-based worksheets	
	a. Suitability of activities on digital-based worksheets with Toulmin scheme components	4.30
	b. Images, writing, symbols and animations presented are appropriate to the context of the material	4.45
2	Learning Activities	
	a. Through digital-based experimental activities and worksheets, students are able to train in collecting and presenting data correctly and in accordance with the facts	4.60
	b. Activities on digital-based worksheets can train students in making claims that are relevant to the context of arguments and discussion topics	4.65
	c. Activities on digital-based worksheets can train students in making warrants that show reasons or principles that connect data and claims in a reasonable way	4.50
	d. Activities on digital-based worksheets can train students in citing axioms, postulates, definitions, theorems, postulates, lemmas, correlations and conjectures from credible sources as backing to strengthen the validity of the warrant.	4.40
	e. Activities on digital-based worksheets can train students to indicate certain conditions where a claim does not apply or is not strong enough	4.30
	f. Activities on digital-based worksheets can train students to use words such as "probably true", "obviously true", "could be", "possible" or other words to express the level of truth of a claim.	4.50
	g. Activities on digital-based worksheets can educate students' skills in expressing objections to something that is different from what they understand	4.35

Table 8. Digital Worksheet Design Assessment Results

Data

Limas terpancung adalah bentuk tiga dimensi yang terbentuk dari limas yang dipotong sejajar dengan alasnya.

Volume dari limas penuh dapat dihitung dengan rumus umum: $\frac{1}{3} \times t \times (A + B + \sqrt{AB})$

Jawaban Langkah

No	Langkah Penyelesaian	Alasan	Dasar Teori
1	$t_2 = \frac{\sqrt{B}}{\sqrt{A}} \times t_1$	Limas penuh sebangun dengan limas kecil, maka rasio tinggi antara dua limas yang berhubungan dengan luas alasnya	Kesebangunan
2	$t_1 = \frac{t}{1 - \frac{\sqrt{B}}{\sqrt{A}}}$	hubungan antara tinggi t1 dari limas penuh, dan tinggi t dari limas terpancung, serta luas alas besar A, dan luas alas kecil B	Definisi limas terpancung dan kesebangunan
3	Volume Limas Terpancung = Volume Limas Penuh - Volume Limas Kecil	Limas terpancung terbentuk dari limas yang dipotong sejajar dengan alasnya.	Definisi Limas terpancung
4	Volume Limas Terpancung $= \frac{1}{3} \times A \times t_1 - \frac{1}{3} \times B \times t_2$	substitusi rumus volume limas penuh dan volume limas kecil	Rumus volume limas
5	$= \frac{1}{3} \frac{t}{1 - \frac{\sqrt{B}}{\sqrt{A}}} \left(\frac{A\sqrt{A} - B\sqrt{B}}{\sqrt{A}} \right)$	suntitisi langkah 1 dan langkah 2	konsep aljabar

(a)

	Penyederhanaan	Metode	Konsep
6	$= \frac{1}{3} \left(\frac{t}{\sqrt{A}-\sqrt{B}} \right) \left(\frac{A\sqrt{A}-B\sqrt{B}}{\sqrt{A}} \right)$	menyamakan penyebut	konsep aljabar
7	$= \frac{1}{3} \frac{t}{\sqrt{A}\sqrt{B}} (A\sqrt{A}-B\sqrt{B})$	manipulasi aljabar	konsep aljabar
8	$= \frac{1}{3} t \frac{(A + \sqrt{AB} + B)(\sqrt{A} - \sqrt{B})}{\sqrt{A} - \sqrt{B}}$	pefaktoran	konsep aljabar
9	$= \frac{1}{3} t (A + B + \sqrt{AB})$	manipulasi aljabar	konsep aljabar

Kesimpulan

Volume Limas terpancung = $\frac{1}{3}t(A + B + \sqrt{AB})$

Kekuatan Penyelesaian

Jawaban ini kuat karena didukung oleh dasar teori yang jelas (kesebangunan, rumus volume limas), pemanfaatan metode geometris yang benar (pengurangan volume), dan penyederhanaan yang logis. Penjelasan ini terstruktur dengan baik dan benar

Sanggahan

Limas dengan alas yang lebih kompleks (misalnya, poligon tidak beraturan atau permukaan yang melengkung), pendekatan ini memerlukan penyesuaian.

[Kembali](#)

(b)

Figure 5. The results of students' mathematical arguments through digital worksheets

Figure 6 represents the results of students' mathematical arguments through printable worksheets.

1. Berdasarkan masalah, tulis data yang diketahui! (Data merupakan rangkaian fakta-fakta yang digunakan untuk membuktikan)

$$\text{Volume Limas terpancung} = \text{Volume Limas Peruh} - \text{Volume Limas kecil}$$

2. Tulis klaim! (Klaim merupakan pernyataan yang akan dibuktikan)

$$\text{Volume Limas terpancung} = \frac{1}{3} \cdot t (A + B + \sqrt{AB})$$

3. Tulis langkah penyelesaian, alasan dan dasar teori

Langkah Penyelesaian	Alasan	Dasar Teori
$t_2 = \frac{\sqrt{B}}{\sqrt{A}} \cdot t_1$	$t_2 =$ tinggi Limas kecil $t_1 =$ tinggi Limas Peruh	Kecebangunan
$t_1 = t : \left(1 - \frac{\sqrt{B}}{\sqrt{A}}\right)$		Kecebangunan
$V_{\text{Limas terpancung}} = V_{\text{Limas Peruh}} - V_{\text{Limas kecil}}$		konsep limas terpancung
$= \frac{1}{3} \cdot A \cdot t_1 - \frac{1}{3} \cdot B \cdot t_2$		Volume Limas
$= \frac{1}{3} \cdot \frac{t}{1 - \frac{\sqrt{B}}{\sqrt{A}}} \left(A - \frac{B\sqrt{B}}{\sqrt{A}}\right)$		Langkah 1 & 2
$= \frac{1}{3} \cdot \frac{t}{1 - \frac{\sqrt{B}}{\sqrt{A}}} \left(\frac{A\sqrt{A} - B\sqrt{B}}{\sqrt{A}}\right)$	menyamakan penyebut	
$= \frac{1}{3} \cdot \frac{t\sqrt{A}}{\frac{\sqrt{A}-\sqrt{B}}{\sqrt{A}}} \left(\frac{A\sqrt{A} - B\sqrt{B}}{\sqrt{A}}\right)$	menyamakan penyebut	
$\frac{1}{3} t \frac{(A + \sqrt{A}\sqrt{B} + B)(\sqrt{A} - \sqrt{B})}{\sqrt{A} - \sqrt{B}}$		Pemfaktoran
$\frac{1}{3} t (A + B + \sqrt{AB})$		

4. Tulis Kesimpulan!

$$\text{Volume Limas terpancung} = \frac{1}{3} \cdot t (A + B + \sqrt{AB})$$

5. Bagaimana dengan tingkat kekuatan (*qualifier*) dari kesimpulan?

paeti benar karena di dasarkan pada langkah logis dan konsep kecebangunan

6. Tulis sanggahan dari kesimpulan?

Tidak ada.

Figure 6. The results of Students' Mathematical Arguments through Printable Worksheets

Based on the results of students' answers in Figures 5 and 6, it appears that students fill in all argument components on digital worksheets. However, students do not fill in all argument

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components on Printable Worksheets, where students do not fill in all the reasons and theoretical bases used when compiling mathematical arguments as in Figure 6.

During the lesson on the volume of a frustum of a cone, formulated by the equation $V_{\text{Frustum}} = \frac{1}{3} t (A + B + \sqrt{AB})$, students showed various responses to the tasks in the digital worksheets. Some students actively discussed in their groups to construct logical arguments, linking the frustum's height t , the areas of the large base A , the small base B , and the square root of the product of both base areas \sqrt{AB} . Meanwhile, other students required prompting from the teacher to clarify the relationship between the elements of the formula and how to apply them in calculating the volume of the frustum of a cone. For example, in one task, there was a group that initially presented an argument based on simple geometric calculations, such as estimating the volume without considering the deeper relationship between the base areas and the height of the frustum. After receiving a prompting question from the teacher, such as "How do you relate the values of A and B with the frustum's height t in the volume formula?", the group began to develop a deeper understanding and referred to basic concepts of geometry and calculus to ensure that the formula $V_{\text{Frustum}} = \frac{1}{3} t (A + B + \sqrt{AB})$ could be applied correctly. The teacher provided selective feedback to maintain students' independent thinking, such as suggesting they review their calculations or compare arguments between groups regarding the appropriate way to calculate the base areas and height. During the class discussion, students commented on each other's group presentations, both in the form of agreement and rebuttal, supported by mathematical reasoning. This not only strengthened their understanding of the frustum volume formula but also created opportunities for deeper development of mathematical argumentation. Thus, the use of digital worksheets in this learning process not only promotes individual cognitive activity but also enhances social interactions that support the development of mathematical thinking in an authentic learning context. Students are not only learning how to calculate volume but also developing the ability to argue and discuss mathematical concepts more critically and in greater depth.

DISCUSSION

The findings of this study indicate that mathematics learning assisted by digital worksheets has a positive impact on students' mathematical argumentation skills. This is consistent with previous research that emphasizes digital-based learning to enhance cognitive abilities and academic performance (Shapiro, 2006). Engaging students in activities that integrate authentic projects and online learning can promote active learning, critical thinking, and collaborative problem-solving, which are essential skills for success in the 21st-century workforce (Hasnunidah et al., 2023). Learning models that implement argumentation have a significant impact on improving students' argumentation abilities (Nussbaum, 2008). Additionally, the use of e-worksheets makes learning

more engaging and encourages students to be more active during the learning process (Kanselaar et al., 2002)

The mathematical argumentation skills of students in the experiment-1 group show a higher average compared to the experiment-2 group (mean scores of $\bar{x}_1 = 82.8750$, $\bar{x}_2 = 69.2188$, respectively). This indicates that students in the experiment-1 group find the interactive electronic worksheets more engaging and easier to understand. With electronic-based worksheets, students not only learn theory but also develop relevant practical skills (Hafsah et al., 2024). Creating clear, accurate, and simple teaching materials to ensure student comprehension is crucial, and this is a primary need in the development of e-worksheets (Prastowo, 2019; Sadjati, 2012). The use of e-worksheets facilitates teachers in presenting teaching materials and assignments, and it allows students to complete tasks online anytime and anywhere, significantly impacting students' cognitive learning outcomes (Kim & Hannafin, 2011; Mispa et al., 2022).

The results of this study indicate that digital-based worksheet-assisted mathematics learning has an effect on students' mathematical argumentation skills. Therefore, teachers must adapt to digital tools for teaching, communication, evaluation, and problem-solving in the context of education. Joshi et al. (2021) suggest that teachers should possess digital pedagogy skills. In the era of digitalization, these digital pedagogy skills help teachers maximize the use of technology to support the teaching and learning process. Nursyahidah and Albab (2021) also argue that mathematics teachers can integrate cultural contexts into mathematics learning and enhance students' understanding of mathematical concepts through the use of educational technology to make learning more effective. Overall, this research highlights the importance of integrating digital tools into mathematics learning to enhance students' mathematical argumentation skills. With appropriate adaptation to technology, teachers can not only enrich students' learning experiences but also create a more inclusive and responsive learning environment that meets students' needs in this digital era

Conversely, the limitation of the experiment-2 group, which uses printable worksheets for mathematics learning, lies in the static and non-interactive nature of printable worksheets. This restricts students' ability to interact dynamically with the material, thereby limiting their mathematical argumentation skills. Students learn more effectively when they can interact with the material through various forms of representation and manipulation (Van de Walle et al., 2003). Printable worksheets cannot provide immediate feedback to students, whereas Hattie & Timperley (2007) emphasize the importance of immediate feedback for improving students' understanding and skills. According to Prabjandee (2023), one of the main weaknesses of printable worksheets is the lack of interactivity, which can lead to students losing motivation and engagement in the learning process.

In the use of digital tools for learning, there are several potential biases that need to be considered, such as accessibility issues and varying levels of digital literacy. Not all students have equal access to digital devices and internet connections. Students from economically disadvantaged

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backgrounds struggle to access the digital tools and resources needed, which can result in inequality in learning experiences. Additionally, students with special needs face challenges in using digital tools if they are not designed with accessibility in mind, such as the lack of support for assistive devices.

Students also have varying levels of digital literacy. Some students are very proficient in using digital tools, while others struggle and lack confidence in utilizing the technology. This can affect how they interact with learning materials and contribute to group activities. Variations in digital literacy can also impact students' ability to search for, evaluate, and use information obtained online, which is important in the learning context.

Research findings indicate that mathematics learning supported by digital worksheets (LKS digital) has a positive impact on students' mathematical argumentation skills. This result aligns with Piaget's constructivist theory, which emphasizes the importance of active student engagement in constructing knowledge through meaningful learning experiences (Piaget, 1970). Digital worksheets enable students to explore mathematical concepts interactively and provide them with opportunities to reflect on and independently develop logical arguments.

Furthermore, these findings are consistent with Vygotsky's theory of the zone of proximal development (ZPD), in which social interaction through discussion and collaboration can foster the development of higher-order thinking skills, including argumentation (Vygotsky, 1978). In the context of digital worksheets, students can work in groups or pairs to solve problems, explain their reasoning for choosing certain strategies, and critique each other's arguments processes that support the development of strong mathematical argumentation.

The instructional implication of these findings is the crucial role of teachers in designing digital worksheets that not only present practice problems but also facilitate open-ended questions, problem-solving scenarios, and tasks that encourage students to express reasoning, provide justification, and defend their opinions logically. This is supported by findings from (Trisanti et al., 2024), which demonstrate that digital tools based on problem-solving tasks can significantly enhance students' mathematical communication and argumentation skills. Thus, digital worksheets have the potential to serve as strategic tools in creating a mathematics learning environment that is more reflective, interactive, and conceptually grounded, rather than merely procedural.

The worksheets used in this study were designed to develop students' mathematical argumentation skills in the topic of frustum volume. However, these worksheets can also be applied to other mathematical topics for the same purpose. The components within these worksheets are general and flexible, making them easily adaptable by other educators or researchers for different topics or teaching contexts. Therefore, although this study focuses on a specific topic, its findings can contribute to the broader development of mathematics education.

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CONCLUSIONS

The research results show that not all lecturers use worksheets in their teaching. However, during the learning process, lecturers always provide opportunities for students to challenge their peers' statements, express opinions, and engage in debates. When lecturers do use worksheets, these worksheets do not encourage students to construct mathematical arguments based on Toulmin's schema, do not leverage technology, and do not guide students in developing their mathematical argumentation skills. Therefore, technology-based worksheets are needed to enhance students' mathematical argumentation skills, adhering to Toulmin's schema.

The average post-test score for the experiment-1 group is 82.8750, while for the experiment-2 group it is 69.2188. With a calculated Z value of 147.500 and a significance (Sig) value of 0.000, which is less than 0.05, the null hypothesis (H_0) stating that the means of the two groups are equal ($\mu_1 = \mu_2$) is rejected. Thus, it can be concluded that mathematics learning using digital worksheets has a positive effect on students' mathematical argumentation skills.

The assessment of the digital worksheet design was conducted to evaluate its suitability and the effectiveness of the learning activity design in developing students' argumentation skills. This assessment aims to ensure that the digital worksheets are indeed appropriate and effective for use in learning aimed at enhancing students' argumentation skills. The assessment results show that all evaluated components received scores ranging from 4.20 to 5.00. In other words, the digital worksheet design is deemed very appropriate and effective for developing students' mathematical argumentation skills.

Although the findings of this study indicate an improvement in students' mathematical argumentation skills through the use of digital worksheets, we acknowledge several limitations that warrant consideration. First, the successful implementation of this approach largely depends on adequate access to technology and students' levels of digital literacy, which may vary across educational institutions. Second, it is possible that part of the observed improvement in student performance was influenced by the novelty effect of the digital format, suggesting the need for long-term investigation to assess the consistency of outcomes. Therefore, further research is recommended to explore the long-term impact and effectiveness of this approach across diverse contexts with varying levels of technological access and digital readiness.

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REFERENCES

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- [1] Abd-El Khalick, F., BouJaoude, S., Duschl, R., Lederman, N. G., Anderman, L. H., & Midgley, C. (2010). Effectiveness of genetics student worksheet to improve creative thinking skills of teacher candidate students Eficacia de la hoja de trabajo para mejorar las habilidades de pensamiento creativo de los estudiantes de genética. *Educational Leadership*, 68(1), 34–37.
https://www.researchgate.net/publication/310254538_Effectiveness_of_genetics_student_worksheet_to_improve_creative_thinking_skills_of_teacher_candidate_students
- [2] Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge For Teaching: What Makes It Special? *Journal Of Teacher Education*, 59(5), 389–407.
<https://doi.org/10.1177/0022487108324554>
- [3] Basuki, W. A., & Wijaya, A. (2018). The development of student worksheet based on realistic mathematics education. *Journal of Physics: Conference Series (Vol. 1097, No. 1, P. 012112)*. IOP Publishing. <https://iopscience.iop.org/article/10.1088/1742-6596/1097/1/012112/pdf>
- [4] Dinantika, H. K., Viyanti, & Suyatna, A. (2020). Design of Student Worksheet to Accommodate Learning Style and Initial Knowledge and Reduce The Differences of Scientific Reasoning and Argumentation Performance. *Proceedings of the 4th International Conference on Learning Innovation and Quality Education*, 1–7.
<https://dl.acm.org/doi/pdf/10.1145/3452144.3453752>
- [5] Drijvers, P. (2013). Digital technology in mathematics education: Why it works (or doesn't). In *Selected Regular Lectures from the 12th International Congress on Mathematical Education*, 135–151.
https://dspace.library.uu.nl/bitstream/handle/1874/322431/978-3-319-17187-6_8.pdf?sequence=2
- [6] Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75, 213–234.
<https://link.springer.com/content/pdf/10.1007/s10649-010-9254-5.pdf>
- [7] Fukawa-Connelly, T., & Silverman, J. (2015). The Development of Mathematical Argumentation in an Unmoderated, Asynchronous Multi-User Dynamic Geometry Environment. *Contemporary Issues in Technology and Teacher Education*, 15(4), 445–488. <https://www.learntechlib.org/d/150824/>
- [8] Hafisah, D. S., Aznam, N., & Rohaeti, E. (2024). The Development of Electronic Student Worksheets Based on Problem-Based Learning and Its Impact on Motivation and Problem-Solving Skills. *Jurnal Penelitian Pendidikan IPA*, 10(9), 6984–6992.
<https://jppipa.unram.ac.id/index.php/jppipa/article/view/8873>
- [9] Harel, G. (2001). The Development of Mathematical Induction as a Proof Scheme : A Model for DNR-Based Instruction. *Learning and Teaching Number Theory, Journal of Mathematical Behavior*, 185–212. https://mathweb.ucsd.edu/~harel/The_Development_of_Mathematical_Induction_as_a_Proof_Scheme_-_A_Model_for_DNR-Based_Instruction.pdf
- [10] Harel, G., & Sowder, L. (1998). Students' Proof Schemes: Results from Exploratory Studies. In *CBMS Research in Collegiate Mathematics Education. III* (Vol. 7, pp. 234–

- 283). <https://mathweb.ucsd.edu/~harel/Students' Proof Schemes.pdf>
- [11] Hasnunidah, N., Rosidin, U., Maulina, D., & Ismi, R. (2023). Students' Argumentation Skillstowards Using Biology e-Worksheetbased on Project-Argumentative Learning Model. *Eurasian Journal of Educational Research*, 103(2), 341–361. <http://repository.lppm.unila.ac.id/52431/1/1194-Article Text-2468-1-10-20230821.pdf>
- [12] Hattie, J., & Timperley, H. (2007). The Power of Feedback. *Review of Educational Research*, 77(1), 81–112. <https://blog.intuyuconsulting.com.au/wp-content/uploads/2012/03/Power-of-Feedback-Article.pdf>
- [13] Hoyles, C., & Noss, R. (2016). Mathematics and digital technology: challenges and examples from design research. In: *Proceedings of the 13th International Congress on Mathematical Education. International Commission on Mathematical Instruction: Hamburg, Germany*, 1–8. [https://discovery.ucl.ac.uk/id/eprint/1476986/1/TSG36Finalv2CHRN\[1\].pdf](https://discovery.ucl.ac.uk/id/eprint/1476986/1/TSG36Finalv2CHRN[1].pdf)
- [14] Hwang, W. Y., Chen, N. S., & Hsu, R. L. (2006). Development And Evaluation Of Multimedia Whiteboard System For Improving Mathematical Problem Solving. *Computers & Education*, 46(2), 105–121. <https://doi.org/10.1016/j.compedu.2004.05.005>
- [15] Inglis, M., Mejia-Ramos, J. P., & Simpson, A. (2007). Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics*, 66(1), 3–21. <https://doi.org/10.1007/s10649-006-9059-8>
- [16] Joshi, D. R., Neupane, U., & Joshi, P. R. (2021). Synthesis Review of Digital Frameworks and DEPSWALIC Digital Competency Framework for Teachers from Basic to University Level. *Mathematics Teaching Research Journal*, 13(2), 108–136. <https://files.eric.ed.gov/fulltext/EJ1383146.pdf>
- [17] Kanselaar, G., Erkens, G., Prangma, M. E., & Jaspers, J. G. M. (2002). Computer support for collaborative and argumentative writing. *Rapport ISOR*, 2, 1–214. <https://www.kanselaar.net/wetenschap/files/cosarboek.pdf>
- [18] Karsli, F., & Şahin, Ç. (2009). Developing worksheet based on science process skills: Factors affecting solubility. In *Asia-Pacific Forum on Science Learning and Teaching*, 10(1), 1–12. https://www.eduhk.hk/apfslt/download/v10_issue1_files/sahin.pdf
- [19] Kartika, H., Budiarto, M. T., Fuad, Y., Clark, L. J., & Jeonghyeon, K. (2023). Comparative Analysis of Students' Argumentation Patterns in the Context of Algebraic Problems. *Mathematics Teaching Research Journal*, 15(5), 105–128. <https://files.eric.ed.gov/fulltext/EJ1412239.pdf>
- [20] Kaymakci, S. (2012). A Review of Studies on Worksheets in Turkey. *Online Submission. Retrieved from ERIC Database (ED529088)*. <https://files.eric.ed.gov/fulltext/ED530699.pdf>
- [21] Kieran, C., Doorman, M., & Ohtani, M. (2015). Frameworks and principles for task design. *Task Design in Mathematics Education: An ICMI Study*, 22(19-81). https://doi.org/10.1007/978-3-319-09629-2_13
- [22] Kim, M. C., & Hannafin, M. J. (2011). Scaffolding problem solving in technology-enhanced learning environments (TELEs): Bridging research and theory with practice. *Computers & Education*, 56(2), 403–417. <https://doi.org/10.1016/j.compedu.2010.08.024>

- [23] Lodder, A. R. (2004). Law, Logic, Rhetoric: A Procedural Model Of Legal Argumentation. *Logic, Epistemology, and the Unity of Science*, 569–588. <https://papers.ssrn.com/sol3/Delivery.cfm?abstractid=1312456>
- [24] Mispa, R., Putra, A. P., & Zaini, M. (2022). Penggunaan E-LKPD berbasis live worksheet pada konsep protista terhadap hasil belajar peserta didik kelas X SMAN 7 Banjarmasin. *Jurnal Pendidikan Indonesia*, 3(1), 1–12. <https://media.neliti.com/media/publications/424625-none-a9684bff.pdf>
- [25] Mutia, N. B., & Prasetyo, Z. K. (2018). The effectiveness of students' worksheet based on multiple representations to increase creative thinking skills. *Journal of Education and Learning (EduLearn)*, 12(4), 631–637. <http://edulearn.intelektual.org/index.php/EduLearn/article/view/8487>
- [26] NGSS, L. S. (2013). *Next Generation Science Standards: For States, by States*. The National Academies Press.
- [27] Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C., & Kersaint, G. (2013). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, 13(1), 38–70. <https://citejournal.org/volume-9/issue-1-09/mathematics/mathematics-teacher-tpack-standards-and-development-model/>
- [28] Novitasari, M., Narimo, S., Fajri, D. N., & Raisia, A. (2022). Critical Thinking Skills Through Literacy and Numeration Oriented Mathematics Student Worksheet. *Jurnal Basicedu*, 6(4), 5775–5784. <https://jbasic.org/index.php/basicedu/article/view/3173>
- [29] Nurhidayah, N., Ninawati, M., & Soro, S. (2020). The influence of worksheet implementation towards students mathematics learning outcomes. *Indonesian Journal of Elementary Teachers Education*, 1(1). <https://journal.uniku.ac.id/index.php/ijete/article/view/2874>
- [30] Nursyahidah, F., & Albab, I. U. (2021). Learning Design on Surface Area and Volume of Cylinder Using Indonesian Ethno-Mathematics of Traditional Cookie Maker Assisted by GeoGebra. *Mathematics Teaching Research Journal*, 13(4), 79–98. <https://files.eric.ed.gov/fulltext/EJ1332350.pdf>
- [31] Nussbaum, E. M. (2008). Using argumentation vee diagrams (AVDs) for promoting argument-counterargument integration in reflective writing. *Journal of Educational Psychology*, 100(3), 549–565. <https://psycnet.apa.org/record/2008-10939-005>
- [32] Piaget, J. (1970). *Science of Education and the Psychology of the Child*. New York: Viking Press.
- [33] Prabjandee, D. (2023). A Review of the Website Liveworksheets.com. *Computer Assisted Language Learning*, 24(1), 269–279. <https://old.callej.org/journal/24-1/Le-Prabjandee2023.pdf>
- [34] Prastowo, A. (2019). *Panduan kreatif membuat bahan ajar inovatif menciptakan metode pembelajaran yang menarik dan menyenangkan*. Diva Press. <https://ecampus-fip.umj.ac.id/repo/handle/123456789/5218>
- [35] Pruitt, S. L. (2015). The Next Generation Science Standards. *Science and Children*, 52(9), 7. <https://files.eric.ed.gov/fulltext/EJ1153875.pdf>

- [36] Sadjati, I. M. (2012). *Pengembangan Bahan Ajar*. Universitas Terbuka.
<http://repository.ut.ac.id/4157>
- [37] Shapiro, N. S. (2006). Learning communities: Reforming undergraduate education. *The Journal of Higher Education*, 77(3), 550–553.
<https://doi.org/https://doi.org/10.1080/00221546.2006.11778940>
- [38] Sriraman, B., & Umland, K. (2020). Argumentation in mathematics education. In *Encyclopedia of mathematics education* (pp. 63–66).
https://link.springer.com/rwe/10.1007/978-94-007-4978-8_11
- [39] Suryawati, E., Suzanti, F., Zulfarina, Z., Putriana, A. R., & Febrianti, L. (2020). The implementation of local environmental problem-based learning student worksheets to strengthen environmental literacy. *Jurnal Pendidikan IPA Indonesia*, 9(2), 169–178.
<https://journal.unnes.ac.id/nju/jpii/article/view/22892>
- [40] Susantini, E., Isnawati., & Lisdiana, L. (2016). Effectiveness of genetics student worksheet to improve creative thinking skills of teacher candidate students Eficacia de la hoja de trabajo para mejorar las habilidades de pensamiento creativo de los estudiantes de genética. *Research in Science & Technological Education*, 30(1), 65–80.
https://www.researchgate.net/publication/310254538_Effectiveness_of_genetics_student_worksheet_to_improve_creative_thinking_skills_of_teacher_candidate_students
- [41] Tall, D. (2004). Introducing Three Worlds of Mathematics. *For the Learning of Mathematics*, 23(3), 29–33. <https://www.semanticscholar.org/paper/Introducing-Three-Worlds-of-Mathematics-Tall/1a91ed4cb77347755cc37344d6eb3ecf94b69f41>
- [42] Toulmin, S. (2003). *The uses of argument*. Cambridge University Press.
<https://doi.org/10.2307/2183556>
- [43] Trisanti, L. B. (2019). The Process Of Thinking By Prospective Teachers Of Mathematics In Making Arguments. *Journal of Education and Learning (EduLearn)*, 13(1), 17–24. <https://doi.org/http://dx.doi.org/10.11591/edulearn.v13i1.6853>
- [44] Trisanti, L. B., & Nusantara, T. (2021). Improving students' mathematical argumentation skill through infusion learning strategy. *Journal of Physics: Conference Series*, 1783(1), 012103. <https://doi.org/10.1088/1742-6596/1783/1/012103>
- [45] Trisanti, L. B., & Nusantara, T. (2022a). The advantage and impact of CIRC-typed and problem-based cooperative learning models on students' mathematical argument. *2nd International Conference on Education and Technology (ICETECH 2021)*, 172–178.
<https://www.atlantispress.com/proceedings/icetech-21/125968176>
- [46] Trisanti, L. B., & Nusantara, T. (2022b). The influence of infusion learning strategy on students' mathematical argumentation skill. *International Journal of Instruction*, 15(2), 277–292. <https://e-iji.net/ats/index.php/pub/article/view/384>
- [47] Trisanti, L. B., Nusantara, T., & Maf'ulah, S. (2024a). Development of digital tools to improve mathematical argumentation skills. *Journal of Hunan University Natural Sciences*, 51(10). <https://jonuns.com/index.php/journal/article/view/1662>
- [48] Trisanti, L. B., Nusantara, T., & Maf'ulah, S. (2024b). Implementation of inquiry learning model in collaboration with pbl to improve student understanding in number theory course. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, 8(1), 150–163.

- <https://journal.ummat.ac.id/index.php/jtam/article/view/17421>
- [49] Trisanti, L. B., Sutawidjaja, A., As'ari, A. R., & Muksar, M. (2016). The construction of deductive warrant derived from inductive warrant in preservice-teacher mathematical argumentations. *Educational Research and Reviews*, 11(17), 1696–1708.
<https://doi.org/https://doi.org/10.5897/ERR2016.2872>
- [50] Trisanti, L., & Nusantara, T. (2023). The Effectiveness of Infusion Learning Model in Linear Algebra Course. *Education Research International*, 2023(9004072), 1–10.
<https://doi.org/https://doi.org/10.1155/2023/9004072>
- [51] Utami, W. S., Ruja, I. N., & Utaya, S. (2016). The Effectiveness of Geography Student Worksheet to Develop Learning Experiences for High School Students. *Journal of Education and Learning*, 5(3), 315–321. <https://files.eric.ed.gov/fulltext/EJ1111407.pdf>
- [52] Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2003). *Elementary and Middle School Mathematics: Teaching Developmentally*. Pearson Education.
https://api.pageplace.de/preview/DT0400.9781292331447_A39573276/preview-9781292331447_A39573276.pdf
- [53] Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Cambridge, MA: Harvard University Press.
https://w.pauldowling.me/rtf/2021.1/readings/LSVygotsky_1978_MindinSocietyDevelopmentofHigherPsycholo.pdf
- [54] Wells, J. (2014). Developing argumentation in mathematics: The role of evidence and context. In Retrieved from <https://core.ac.uk/download/pdf/43064325.pdf>.
<https://core.ac.uk/download/pdf/43359810.pdf>
- [55] Yaden, Z. (2017). A Development of students' worksheet based on contextual teaching and learning. *International Journal of Learning, Teaching and Educational Research*, 16(6), 64–79. <https://www.ijlter.org/index.php/ijlter/article/view/951>
- [56] Zbiek, R. M., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In *Second handbook of research on mathematics teaching and learning*, 2 (pp. 1169–1207).
<https://www.scirp.org/journal/paperinformation?paperid=103439>
- [57] Zbiek, R. M., & Hollebrands, K. (2008). A research-informed view of the process of incorporating mathematics technology into classroom practice by in-service and prospective teachers. *Research on Technology and the Teaching and Learning of Mathematics*, 1, 287–344. <https://www.emerald.com/books/edited-volume/19473/chapter-abstract/104103027/A-Research-Informed-View-of-the-Process-of?redirectedFrom=fulltext>