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Ryan L. Cerveza, Issue 62 Editor

Don Mariano Marcos Memorial State University, Philippines

What makes mathematics teaching truly meaningful and effective in today's diverse classrooms? In this issue, we invite you to explore fifteen insightful, research-based articles that tackle this question from multiple perspectives. Spanning educational levels, instructional contexts, and cultural settings, these studies illuminate the dynamic landscape of mathematics teaching, learning, and research. Organized into seven thematic categories—foundational studies on mathematical thinking and proof, teachers' knowledge and beliefs, curriculum design and instructional models, problem-solving and mathematical communication, integration of technology, mathematical representations and literacy, and assessment and evaluation—this collection offers readers a rich and multifaceted view of contemporary mathematics education.

Conducted by authors from Cyprus, Indonesia, the Philippines, Rwanda, Russia, Japan, Nepal, and South Korea, the research exemplifies MTRJ's dedication to scholarship that not only advances theory but also provides practical insights to inform and inspire classroom practice. Central to MTRJ's mission is the belief that meaningful mathematics education emerges from the intersection of theoretical insight, empirical research, and instructional practice.

The articles in this issue collectively reinforce this principle by demonstrating how mathematical teaching can be cultivated through purposeful task design, reflective teaching, thoughtful assessment, and the strategic use of technology. The editorial team sincerely thanks all the researchers who contributed their valuable articles to this issue. Your dedication, insights, and scholarly work have greatly enriched the content and purpose of this publication.

Taken as a whole, this issue offers a rich, research-grounded exploration of mathematical thinking and reasoning as lived experiences in real classrooms. Readers will gain insights into how students think, how teachers teach, how curricula are designed, and how assessment and technology can be leveraged to support deeper understanding. Each article contributes a distinct perspective, yet together they form a coherent narrative about the possibilities and challenges of advancing mathematics education through research.

We invite readers—researchers, teacher educators, curriculum developers, and classroom practitioners alike—to engage deeply with the studies presented in this issue. By doing so, readers will not only encounter innovative ideas and empirical findings but also gain practical insights that can inform teaching, learning, and future research. This collection exemplifies the spirit of *MTRJ*: scholarship that **illuminates mathematical teaching and transforms mathematics classroom practice**.

Reasoning in Mathematics. The first set foregrounds the intellectual core of mathematics education: *how learners reason, coordinate ideas, and construct proofs*. **Aristidou's** study in Cyprus illustrates how mistakes and conjectural thinking in student projects can serve as catalysts for mathematical discovery. This work has significant implications for classroom practice, suggesting that teachers should create environments where exploration, error analysis, and conjecturing are valued as legitimate components of the learning process in mathematics.

Studies from Indonesia by **Pangadongan et al**. and **Widadah et al**. further deepen this perspective by examining students' cognitive coordination and reasoning strategies in advanced topics such as derivatives and proof. These findings imply that instruction should move beyond procedural fluency toward explicit support for abstraction, reflection, and reasoning, particularly in secondary and tertiary mathematics classrooms.

The Story of a "Theorem": From Mistakes to New Mathematics (pp. 7 - 22)

Michael Aristidou (Cyprus)

This article highlights the productive role of errors in mathematical discovery. Through a student project on quaternions, it shows how incorrect theorems and omissions can lead to new mathematical insights, reinforcing exploratory learning and authentic mathematical practice.

A Framework for Mathematical Proof: A Combination of Deduction, Induction, and Abduction (pp. 23 - 41)

Soffil Widadah, Tatag. Y. E. Siswono, and Rooselyna Ekawati (Indonesia)

This research examines how students use multiple forms of reasoning in constructing mathematical proofs. It emphasizes the complementary roles of abduction, deduction, and induction in developing rigorous yet flexible proof strategies.

How Students Think About a Reverse Problem of Derivative: A Study of Arnon's Coordination in Mental Mechanism (pp. 42 - 73)

Fara Virgianita Pangadongan, Toto Nusantara, and I Made Sulandra (Indonesia)

This study investigates students' mental coordination when solving reverse problems involving derivatives and antiderivatives. It contributes to cognitive theory by detailing how encapsulation, assimilation, and

accommodation operate in advanced mathematical thinking. This study served as a reference for educators when designing mathematical problem-solving exercises and lessons.

Teaching Knowledge and Practice. The studies by **Indartiningsih et al**. (Indonesia) and **Habiyaremye et al.** (Rwanda) emphasize that teachers' understanding of mathematical thinking directly shapes classroom opportunities for reasoning. Indartiningsih et al.'s research on primary mathematics teachers' conceptions of critical thinking shows that stronger conceptions are associated with more effective instructional design, highlighting the need for professional development focused on pedagogical reasoning. Similarly, Habiyaremye et al.'s quasi-experimental study demonstrates that integrating TPACK (Technological Pedagogical Content Knowledge) into lesson planning enhances tutors' mathematical knowledge for teaching, promoting more open-ended questioning and deeper student reasoning.

These findings suggest that teacher education programs and in-service training should prioritize pedagogical reasoning, reflective practice, and research-based instructional frameworks.

Classroom Practices Mirroring Mathematical Knowledge for Teaching in Rwandan Teacher-Training Colleges: TPACK-Informed Lesson Planning (pp. 74 - 95)

Hashituky Telesphore Habiyaremye, Celestin Ntivuguruzwa, and Philothere Ntawiha (Rwanda)

This quasi-experimental study demonstrates how integrating TPACK (Technological Pedagogical Content Knowledge) into lesson planning enhances tutors' mathematical knowledge for teaching. It documents shifts toward more open-ended questioning and deeper student reasoning.

Teachers' Conceptions of Critical Thinking in Mathematics Teaching (pp. 96 - 123)

Duhwi Indartiningsih, Tatag Yuli Eko Siswono, Wiryanto, and Suryanti (Indonesia)

This mixed-methods study examines how primary mathematics teachers conceptualize critical thinking. It shows that stronger conceptions are associated with better instructional design, highlighting the need for professional development focused on pedagogical reasoning.

Design-Based Instruction in Mathematics. Design research studies in this issue demonstrate the power of hypothetical learning trajectories, realistic mathematics education, and project-based learning in addressing students' learning obstacles. The works of **Zafirah et al**. and **Nurhikmayati et al**., conducted in Indonesian classrooms, show how carefully sequenced instructional designs can support conceptual understanding in social arithmetic and geometry.

For curriculum developers and classroom teachers, these studies highlight the importance of research-informed curriculum planning that anticipates students' difficulties and scaffolds mathematical thinking over time.

These studies emphasize the critical role of research-based instructional strategies in fostering both conceptual understanding and effective mathematical communication.

Design and Implementation of Hypothetical Learning Trajectory on the Geometry of Cubes and Cuboids using Project-Based Learning (pp. 124 - 150)

Iik Nurhikmayati, Sufyani Prabawanto, Darhim, Jarnawi Afgani Dahlan (Indonesia)

This article shows how HLT combined with project-based learning helps overcome epistemological, ontogenetic, and didactic learning obstacles in geometry.

Discovering the Power of Realistic Mathematics Education: A Hypothetical Learning Trajectory for Teaching Social Arithmetic (pp. 151 - 194)

Afifah Zafirah, Ahmad Fauzan, Yerizon, Randika Irwa Risky, and Fardatil Aini Agusti (Indonesia)

This design research study illustrates how a contextualized HLT grounded in realistic mathematics education supports students' understanding of profit and loss concepts.

Mathematical Discourse and Problem Solving. Several articles emphasize that mathematical thinking is made visible through problem-solving, problem-posing, and communication. A study from the Philippines reveals both the potential and the challenges faced by students and pre-service teachers in articulating mathematical ideas clearly and meaningfully. Complementing these findings, **Samura et al.'s** (Indonesia) quasi-experimental study demonstrates that problem-based learning significantly improves junior high school students' mathematical communication skills, enabling them to express ideas clearly and coherently.

The positive impact of problem-based learning suggests that classrooms should provide structured opportunities for discussion, justification, and critique, helping learners develop both mathematical understanding and communicative competence.

The Application of Problem-Based Learning in Improving Junior High School Students' Mathematical Communication Skills (pp. 195 - 222)

Asri Ode Samura, In Hi Abdullah, Muhammad Daut Siagian, Habibi Ratu Perwira Negara (Indonesia)

This quasi-experimental study demonstrates that problem-based learning significantly improves students' ability to communicate mathematical ideas clearly and coherently.

Well-Stated Problem: What is the Level of Problem-Posing Performance

of Pre-Service Mathematics Teachers? (pp. 223 - 247)

John Rey Oficiar, Jerwin Reonisto, Anthony Bactol (Philippines)

This study evaluates the quality of problems posed by pre-service teachers and identifies common weaknesses such as ambiguity and misalignment of representations, underscoring the pedagogical importance of problem posing.

Technology-Enhanced Mathematics Learning. Technology-focused studies in this issue illustrate how digital tools can enhance conceptual understanding and learner engagement when grounded in sound pedagogical design. Research on augmented reality in Indonesia and Japan shows improvements in students' geometric understanding, while Moodle-based adaptation tools in Russia highlight the role of technology in supporting diverse and multilingual learners. Moreover, **Tristanti et al.**'s study demonstrates that digital worksheets are more effective than printed materials in developing students' mathematical argumentation.

These findings imply that technology integration in mathematics education should be intentional, research-driven, and inclusive, reinforcing rather than replacing mathematical reasoning. Read on!

Technology-Based Worksheets to Develop Students' Mathematical Argumentation Skills (pp. 248 - 277)

Lia Budi Tristanti, Syarifatul Maf'ulah, M.Bahrul Subkhi (Indonesia)

This research compares digital and printed worksheets, showing that technology-based materials significantly enhance students' mathematical argumentation using Toulmin's framework.

Boosting Junior High School Students' Mathematical Proficiency Utilizing the Integration of AR in the Classroom (pp. 278 - 298)

Wanda Nugroho Yanuarto, Elfis Suanto, Ira Hapsari, Masanori Fukui (Indonesia/ Japan)

This experimental study demonstrates that augmented reality applications improve students' understanding of geometry and foster positive attitudes toward learning mathematics.

Joint Research and Design of Educational Adaptation Tools in Mathematics for Foreign Students in the LMS Moodle (pp. 299 - 310)

Veronika I. Belousova, Alla A. Knysh, Kseniia S. Potorochina (Russia)

This article focuses on blended learning and digital adaptation tools designed to support foreign students in learning mathematics, emphasizing inclusivity and bilingual approaches.

Representation and Contextualization. The next set addresses the role of representations and real-world

contexts in developing mathematical literacy. Studies from Nepal and Indonesia show that students often struggle to connect symbolic, graphical, and contextual representations.

The implication for classroom teaching is clear: mathematics instruction must deliberately foster representational flexibility and contextual reasoning, enabling students to move meaningfully between abstract symbols and real-world applications.

Jambi Batik Pattern: Developing PISA-Like Mathematics Problems (pp. 311 - 339)

Tria Gustiningsi, Feri Tiona Pasaribu, and Yelli Ramalisa (Indonesia)

This design research study develops culturally contextualized PISA-like problems, demonstrating their validity, practicality, and potential to enhance students' mathematical literacy.

Bridging the Gap: Understanding Students' Struggles with Algebraic and Graphical Representations of Functions (pp. 340 - 359)

Santosh Pathak (South Korea/ Nepal)

This study reveals students' strong preference for algebraic methods and limited awareness of graphical strategies, highlighting the need for balanced representational instruction.

Assessment in Mathematics Education. Assessment-related research in this issue highlights the importance of diagnostic and interpretive assessment practices. The Rasch analysis conducted by **Balasabas and Arnado** in the Philippines demonstrates how assessment data can reveal mismatches between test difficulty and student ability, as well as the influence of affective factors such as test anxiety.

For mathematics educators, this emphasizes the need to view assessment not merely as a grading tool but as a research-informed mechanism for improving instruction, identifying learning gaps, and supporting students' mathematical confidence.

Analyzing Mathematics 10 First Quarter Examination and Students' Test Taking Skill Through the Rasch Model (pp. 360 - 382)

Jay A. Balasabas, Alvic A. Arnado (Philippines)

Using the Rasch model, this study evaluates test item difficulty and student response patterns. It provides insights into test validity, student misfit behavior, and the role of affective factors such as test anxiety in mathematics assessment.

The Problem Corner (pp. 383 - 387)

Ivan Retamoso