

## Cultural Relevance in Mathematics Education Using Indigenous Patterns for Teaching Algebra

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*Abstract: The study identified the algebraic patterns present in the cultural artifacts in the mountain ranges of northeastern Luzon, Philippines, and explored their integration into teaching mathematics. Culturally responsive teaching is a practice applied both locally and internationally, designed to strengthen the link between students' cultural context and their academic learning. Qualitative research was applied through an ethnographic approach since it highlights the indigenous community's extensive collection of tangible cultural artifacts. To precisely identify cultural artifacts and their mathematical patterns, ethnographic research uses contextual interviews, passive observation, and archival analysis. It was found that bead patterns, baskets, and pendants all symbolize personal ornaments, livelihood activities, and status symbols, respectively. Additionally, gongs are employed in ceremonies and festivities, and baskets display regional expertise and practical artistry. The community's relationship with the land and sustainable living methods is reflected in architectural styles such as traditional homes. Further, by employing a mathematical investigation approach, the study found algebraic patterns such as sequences and series. The study also established a procedure for integrating the cultural artifacts' patterns into teaching mathematics, particularly in the teacher-education curriculum.*

Keywords: Algebraic patterns, ethnomathematics, mathematics education, mathematical investigation

### INTRODUCTION

The rich cultural heritage of the indigenous peoples in the mountain ranges of northeastern Luzon, Philippines, offers a unique and valuable perspective on the interplay between mathematics and culture. Indigenous groups such as the *Ifugao*, *Kalinga*, *Mountain Province*, and *Isnag* have developed sophisticated systems of knowledge that are reflected in their art, architecture, textiles, and other cultural artifacts (Capua et al., 2022; Hortelano & Lapinid, 2024; Abbacan-Tuguic, 2016). These artifacts often contain intricate patterns and designs that, upon closer examination,

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reveal deep mathematical principles, including algebraic patterns. Despite the evident presence of mathematics in these cultural forms, there is limited exploration into how these patterns can be integrated into contemporary educational practices, particularly in the curriculum for pre-service mathematics teachers (Hortelano & Roleda, 2024).

The study is motivated by the desire to bridge cultural knowledge with formal mathematical education, enhancing the learning experience for future educators (Howley-Rouse, 2023; Llaneta, 2017). Incorporating students' cultural practices into mathematics instruction fosters belonging and enhances understanding, as mathematics is a product of human activity influenced by cultural values (Danoebroto & Suyata, 2024).

Internationally, integrating cultural knowledge into mathematics education is recognized as a way to make learning more meaningful and relevant (Danoebroto & Suyata, 2024). Culturally responsive teaching uses familiar cultural resources to teach knowledge, skills, and attitudes, supporting students in understanding the world around them. Learning becomes more accessible when situated within a student's frame of reference, achieving greater success when taught through their cultural lens (Parker et al., 2017). For instance, research indicates that many families possess mathematical knowledge and skills related to cooking, weaving, and cultural activities, which can provide relevant contexts for teaching mathematics (Howley-Rouse, 2023). In Southeast Asia, AI (Artificial Intelligence) tools in mathematics education are being customized to align with students' cultural backgrounds, including language preferences and traditional learning practices (Payadnya et al, 2024). Ethnomathematics, which connects mathematics and daily life activities, recognizes mathematics as part of a broader knowledge system that helps people understand and interact with the world (Arya et al., 2024; Hamidah et al., 2024). This approach expands mathematical authorship and strengthens the relationship between learners and mathematics by drawing from diverse community resources.

In the Philippines, ethnomathematics is increasingly recognized as a valuable framework for culturally relevant mathematics education (Alangui, 2017). Studies continue to document the ethnomathematics of indigenous communities, such as the *Agta Tabangnon* (one of the communities in the Philippines), highlighting the mathematical concepts embedded in their cultural practices (Briones et al, 2023). Recent research emphasizes the potential of ethnomathematics to create culturally relevant and engaging learning activities, contributing to a more inclusive understanding of mathematics (Abay & Parola, 2024). For example, ethnomathematical concepts found in traditional games can be integrated into mathematics instruction to foster a deeper understanding and appreciation of mathematics among students (Abay & Parola, 2024; Villarin et al, 2024). This aligns with the broader goal of promoting equity and inclusion in mathematics education by connecting mathematical concepts to students' cultural backgrounds and lived experiences.

Culturally responsive mathematics teaching improves student performance, interest, problem-solving persistence, and confidence (Krings, 2019). For pre-service teachers, developing a culturally based approach to mathematical investigation is crucial for understanding how diverse students engage with mathematical concepts (Aga, 2024; Harris et al., 2024). Exposure to culturally responsive teaching methods helps them recognize and value cultural backgrounds while acquiring

skills to integrate culturally relevant strategies into their practices. Artifacts from various cultures, which often feature intricate patterns and structures, provide valuable starting points for mathematical investigations (Thomas et al., 2024). These patterns, found in crafts, architecture, and indigenous art, offer real-world contexts to explore mathematical concepts like algebraic sequences and series (Lyublinskaya et al., 2024). Incorporating such patterns deepens students' understanding of abstract concepts and makes learning more inclusive and engaging for diverse groups. In culturally rich environments, this approach ensures that lessons are relevant and resonate with students from various backgrounds, promoting equitable learning (Thomas et al., 2024). Although pre-service teachers have few inadequacies in mathematical content knowledge (Dewantara et al., 2024), pre-service teachers who embrace this method are better prepared to create inclusive classrooms where all students feel represented and empowered in their mathematical learning (Aga, 2024; Thomas & Berry, 2019; Rigney et al., 2020).

This research aims to integrate algebraic patterns found in indigenous cultural artifacts into math teaching, offering a culturally relevant context for algebra. By incorporating local knowledge, it enhances the curriculum, highlighting mathematics as a dynamic, culturally connected discipline. With a focus on culturally responsive teaching, the study explores how these patterns can be included in training pre-service math teachers to promote a more inclusive and meaningful approach to education.

The study is guided by the following research questions:

1. What types of cultural artifacts are produced by the indigenous people in the mountain ranges of northeastern Luzon, Philippines?
2. What algebraic patterns can be observed in these cultural artifacts?
3. What material can be designed as a guide for teachers to integrate these culture-based algebraic patterns in teaching mathematical investigation?

Recent research in mathematics education highlights the importance of integrating local and cultural contexts to make math more meaningful for students. Studies have demonstrated the value of incorporating indigenous knowledge, particularly in areas like geometry and number systems. However, effectively integrating these insights into the formal curriculum remains challenging, especially in countries like the Philippines, where many indigenous cultures are underrepresented (Curtis et al., 2021).

## REVIEW OF LITERATURE

### Ethnomathematics and Indigenous Knowledge

Ethnomathematics provides a framework for understanding how cultural practices, including those of indigenous communities, reflect mathematical concepts (Mairing & Nini, 2024; Alangu, 2017). This approach recognizes that mathematics is not just an abstract discipline but is embedded in various cultural activities and artifacts (Hamidah et al., 2024; Batiibwe, 2024). In the context of indigenous communities in the Philippines, ethnomathematics can be a tool for culturally relevant

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mathematics education (Alangui, 2017; Briones et al., 2023). This involves exploring the mathematical ideas and practices within these communities and integrating them into the formal mathematics curriculum (Magni, 2017; Danoebroto & Suyata, 2024).

### Mathematical Patterns in Indigenous Art and Textiles

Indigenous art and textiles often showcase complex geometric and algebraic patterns (Capua et al., 2022; Abbacan-Tuguic, 2016; Golding, 2018). For example, the traditional clothing of the *Ifugao* people in the Eastern Cordillera region features frieze group designs, demonstrating the weavers' ability to create geometric shapes without formal mathematical knowledge (Capua et al., 2022). Similarly, *Kalinga* tattoo designs introduce geometrical concepts such as points, lines, and polygons (Abbacan-Tuguic, 2016). The *Karara* designs of Maranao weaving incorporate algebra, number properties, operations, and geometry (Bitiibwe, 2024). Analyzing fractals in *Navajo* weaving reveals patterns in the Chief's Blanket, Two Grey Hills, and Ganado Red styles.

### Integrating Indigenous Knowledge into Mathematics Education

Integrating indigenous knowledge into mathematics education can enhance student engagement, problem-solving skills, and cultural appreciation (Bitiibwe, 2024). By connecting mathematical concepts to familiar cultural contexts, educators can make mathematics more relevant and accessible to students (Hortelano & Roleda, 2024; Mairing & Nini, 2024; Alangui, 2017). This approach aligns with culturally responsive teaching, which uses cultural resources to support students' understanding of the world. For instance, ethnomathematical concepts found in traditional games, such as *Dakon* or *Congklak* (board games generally known as count-and-capture games), can be integrated into mathematics instruction to foster a deeper understanding and appreciation of mathematics among students (Bitiibwe, 2024).

A key challenge is connecting abstract mathematical concepts with the lived experiences of indigenous students. While indigenous students may be familiar with mathematical patterns in cultural artifacts, presenting them to a broader audience requires balancing cultural relevance with mathematical rigor (Sari et al., 2024). Additionally, pre-service math teachers need to develop the pedagogical skills to effectively integrate this approach in the classroom (Çilingir Altıner et al., 2023).

Furthermore, the methodology section in this study details data collection from indigenous communities in northeastern Luzon, analysis of cultural artifacts, and the framework for incorporating these findings into pre-service teacher training. The findings section identifies algebraic patterns in the artifacts and offers curriculum integration recommendations. Finally, the discussion addresses the study's implications for math education and teacher training, suggesting directions for future research and practices in culturally responsive teaching.

In sum, this study aims to uncover the algebraic patterns embedded in the cultural artifacts of Indigenous peoples in the region and to explore how these patterns can enrich the curriculum for pre-service mathematics teachers, contributing to a more inclusive, culturally responsive approach

to teaching mathematics. The output of this research, although it was not yet implemented as part of the study, was a teacher's guide in integrating culture in teaching mathematical investigation.

## METHODS

This section presents the research design, the sources of data, the different instruments and the procedure of collecting the data, and the methods in the analysis of the data.

### Research Design

This study used a qualitative research approach. Specifically, an ethnographic design was applied to explore and describe the algebraic patterns in tangible artifacts. This design is well-suited to the study, as it focuses on the region's rich collection of physical cultural artifacts. Ethnographic research involves archival investigation, passive observation, and contextual interviews to accurately uncover cultural artifacts and their mathematical patterns.

### Source of Data

The study took place in the mountain ranges of northeastern Luzon, Philippines, focusing on the provinces of Ifugao and Mountain Province. It included the municipalities of Alfonso Lista, Aguinaldo, Mayoyao, and Banaue in Ifugao, and Paracelis in Mountain Province. The study examined physical artifacts from these regions, such as pendants, musical instruments, architectural designs, baskets, and beadwork, to investigate their patterns.

The study conducted interviews with local individuals, including *crafting center owners* and *elders*, and *cultural experts*, to validate observations made during data collection. The elders and experts were senior community members and specialized in local traditions, while the crafting center owners created cultural items like native clothing, wine, tattoos, and traditional houses. The study included participants from dominant ethnolinguistic groups recognized by the National Commission on Indigenous Peoples (NCIP), with 25 participants interviewed. The study identified 25 participants through the NCIP, a government agency that protects and promotes the rights and welfare of Indigenous Cultural Communities and Indigenous Peoples, respecting their beliefs, customs, traditions, and ancestral domains while granting approval.

### Instrumentation and Data Collection

The study's main goal was to find and examine physical artifacts by employing precise measurements and reliable measurement methods with instruments like protractors, calipers, and rulers. The researchers labeled sketches or diagrams, recorded these measurements, and examined dimensions, angles, and proportions. To find significant patterns that provide insights into cultural prac-

tices, craftsmanship methods, and functional roles, they also took into account the larger archaeological context. This methodical approach provided a strong basis for interpreting the cultural significance of these artifacts, and it was reinforced by meticulous documentation and contextual analysis.

Additionally, photographs of the cultural artifacts and observation notes were taken to supplement the interview process. The researchers took a cautious approach to ethical issues, making sure to get people's permission before taking their pictures and honoring their desires for the use and distribution of their photos. The participants were not objectified in any manner, and they were treated with respect.

The study emphasized the importance of respecting cultural context and avoiding misrepresentation or exoticization of a community's identity. The researcher took responsibility for sharing photographs, considering their potential impact, and refraining from exploiting them. This ethical approach contributed to a more inclusive, respectful, and accurate portrayal of documented communities. A Certification Precondition from the National Commission on Indigenous Peoples (NCIP) was obtained for the study's conduct and dissemination.

Moreover, to triangulate the study's findings regarding the artifacts, participants were interviewed. A semi-structured questionnaire was employed to interview the owners of cultural artifact centers, elders, and culture experts. This approach allowed for a more personalized and flexible interview process, facilitating a deeper understanding of the subject matter.

The interview protocol was developed through a thorough process, starting with a thorough understanding of the research objectives and themes. Key areas were identified and participants' information was collected. Questions were designed to be clear, concise, and relevant to the research goals. A pilot interview was conducted, followed by debriefing sessions and revisions. Ethical considerations were taken into account, resulting in a comprehensive tool for gathering valuable data.

Prior to the interview, participants were provided with a letter detailing the nature and purpose of the study, and their consent to participate was obtained.

## Data Analysis

A thematic approach was employed to systematically identify, analyze, and report recurring patterns (themes) within the dataset, which included sources such as interviews, surveys, and textual documents.

The mathematical investigation approach was used to analyze algebraic patterns in cultural artifacts from Northeastern Luzon. Also, the thematic analysis of the interview results involves examining recurring patterns and mathematical relationships, integrating archaeology, anthropology, and mathematics to understand their cultural significance. The process begins by compiling a va-

riety of cultural items, systematically exploring algebraic patterns, and deriving conclusions. Attention is given to algebraic relationships, progressions, and formulas, ensuring no detail is overlooked. The concept of mathematical sequences and series is also considered in the analysis.

## RESULTS AND DISCUSSION

The next section shows the summary of the descriptions of the different cultural artifacts found in the regions and the algebraic patterns explored. Also, at the end of this section is the integration of these findings into mathematical investigations for pre-service teachers.

### The Cultural Artifacts

**Pendants.** The *Buong or Linglingo* (Lucky Charm Pendant), as shown in Figure 1, is a handmade artifact found at Barangay Bocos, Banaue, Ifugao. The *Buong or Linglingo* is a centuries-old symbol of fertility and luck since the Metal Age of the Philippines from 500-1000 AD. Individually made, each one has slight differences and individual markings. Figure 1(a) is bronze made from extra-small (XS) to extra-large (XL) in size with the measurement in diameter. Figure 1(b) is the *Linglingo* with the mold, and Figure 1(c) is the finished product of silver.

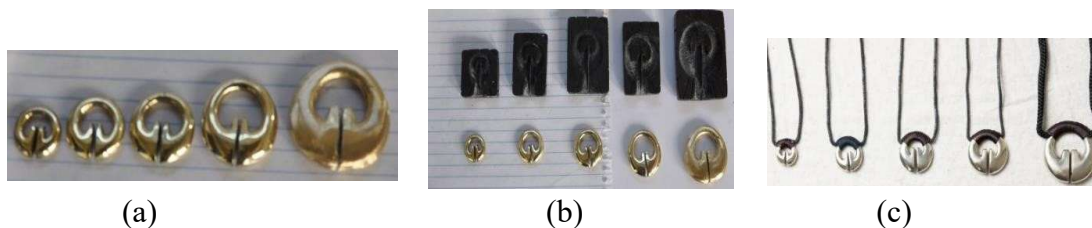


Figure 1: The Buong or Linglingo of Banaue, Ifugao

**Musical Instruments.** Figure 2 shows the *Gangha* or Gong artifact of Banaue, Ifugao. According to the respondents, the *Gangha* was made of pure bronze metal and silver nickel used for rituals and celebrations. They made a set of 3, a set of 4, and a set of 7. The set of seven signifies the ‘do, re, mi, fa, so, la, ti, do’ sequence of making the *Gangha*. Moreover, it has been mentioned by the maker that the first *Gangha* obtained a thickness of 2.5 mm and the rest measured a thickness of 2mm.



Figure 2: The *Gangha* or Gong of Banaue, Ifugao

**Basketry.** In Figure 3, it shows the *Pallungan* or a rectangular winnowing tool crafted from rattan or *anoh* (a type of climbing plant from the palm family). Its dimensions span 58 cm x 58 cm at the top and 47 cm x 47 cm at the bottom. This essential implement is employed by residents to exactly cleanse rice by removing impurities and husks. The top view (Figure 3(a)), side view (Figure 3(b)), and focused image (Figure 3(c)) provide insight into the intricate weaving technique employed in its construction, showcasing the skilled craftsmanship involved. The use of rattan or *anoh* underscores not only the resourcefulness of indigenous materials but also the preservation of traditional techniques passed down through generations. The *Pallungan* serves as a tangible link between past practices and contemporary household needs, embodying both practical functionality and cultural significance.

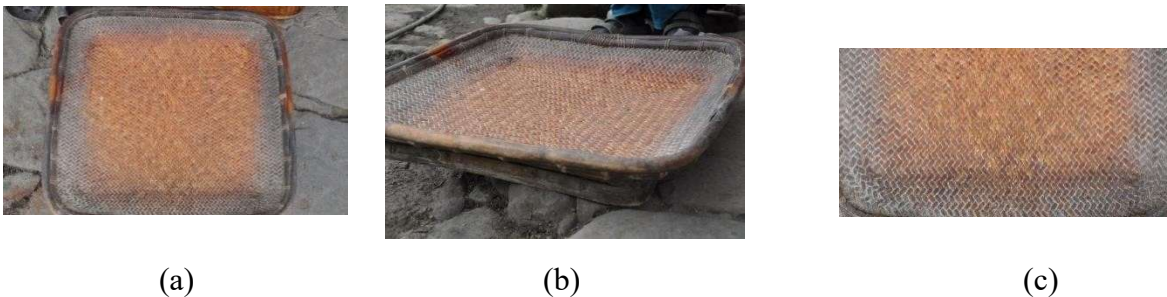


Figure 3: The *Pallungan* or a rectangular winnower of Barangay Maga

Additionally, there are other baskets with the same pattern found in both places Mayoyao and Aguineldo such as the *Taga'e* (basket plate), *Hape'eng* (ritual object), *Labfa* (necklace), *Inomon Hape'eng* (traditional backpack), *Ho-op* (woven basket), *Inomon Pasikeng* (traditional backpack), and *Upping* (lunch box).

**Architectural Designs.** Figure 4 shows the modernized native house/ native house or *baleh* at Mayoyao, Ifugao. Figure 4(a) reflects the complete image of the modernized/ galvanized native

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house at a distance. Figure 4(b) and Figure 4(c) depict the surroundings of the native house. Figure 6(d) shows where in the attic of the native house. There are three levels of the *baleh*. The first level is the *daulon* or ground floor. Moreover, it has a multi-function. It is an open area within the level of the yard or *dolah*. The area for pounding rice is beside this *daulon* shaded by the roof. The *luhung* or mortar and *al-u* or pestle are used for pounding rice. The second level is the *bohongna* or the one-room quarter, which is accessible through a removable ladder. The family cooks, eats, and socializes inside this area. The third level is the *palah* or attic above the floor, purposely for *palay* (rice plant) storage. However, the researcher found the *patye* - a shelf extending from the wall boards to the underside of the roof. It serves to safely keep and organize the things and gadgets of the family, as well as provide structural support to the roof.

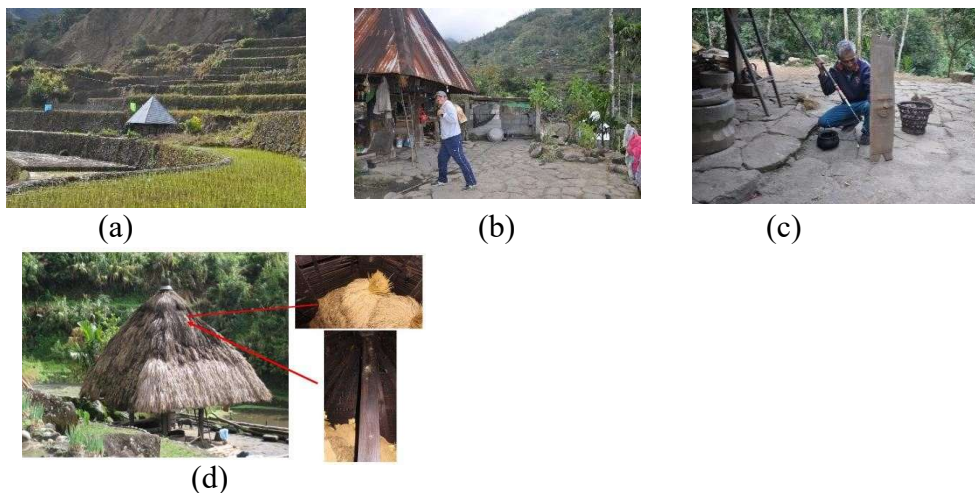


Figure 4. The Native House of Barangay Maga

The outer part of the house is embellished with rows of skulls of animals offered to the gods during their annual rituals, which are also a source of pride to the homeowner. The *baleh* type is considered the typical Ifugao native house. Residents mentioned the rice terraces and *baleh* caught the attention of pioneering Western anthropologists who came to the Philippines even before the Second World War.

Using Geometry and Trigonometry, indigenous peoples in the province determined the appropriate shapes of the structure of the house, such as a pyramid-shaped roof that lets strong typhoons pass by without impact on the structure, as well as the conservation of energy. The appropriate angle of inclination of the wall of about 30 degrees, and the roof that varies from 45 degrees to 60 degrees, were special angles of significance to the Egyptians when building their pyramids. They observed the perpendicularity of the king post to the center beam for the stability of the roof. The squareness of the *ambubulan* (central roof hub) near the apex of the pyramid roof has to be established and well-fastened to resist typhoons from twisting the roof frames since all the rafters are fastened to it. The strength of the four *bughol* (the main rafters) that formed the edges of the pyramid must be well fastened at the apex.

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Moreover, measurements of lengths, heights, and appropriate distances, for instance, of the rafters and the purlins, were estimated. The construction of the roof or the putting of the *runo* leaves (reed leaves) of the roof follows a mathematical pattern of sequence and series from the base until it reaches the apex of the pyramid. The *runo* or cogon is tied up in smaller bundles and fastened on the purlins (Figure 5).



Figure 5: The Roof of Native House in Mayoyao, Ifugao

**Beadworks.** The beadworks of the *Ga'dang* Community in Paracelis, Mountain Province contain double *ansulam* or *antalsik* (full diamond) design and *ansulam* or *antalsik* (half diamond). Each full and half diamond is formed by positioning the beads per row, making a minimum of 2 rows for the smallest half diamond and a minimum of 3 rows for the full-figured diamond.

The *Bakwat* (usually a belt) as shown in Figure 6 is a textile to enclose the skirt so that it will not be fall and designed with colorful beads. The colors of the beads are red, yellow, white, and black. Figure 6(a) shows the whole artifact of the *Bakwat*. Also, Figure 6(b) is the focus design of the *ansulam/antalsik* beads in its center and Figure 6(c) shows both ends of the design.

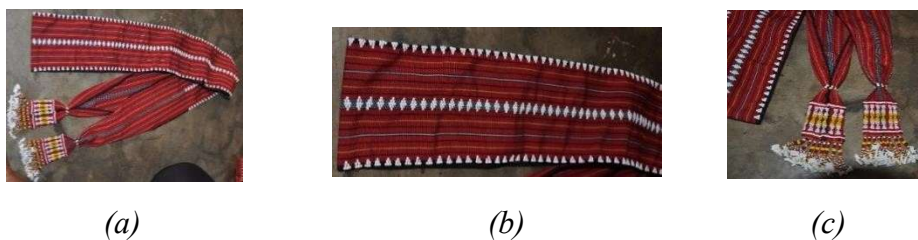


Figure 6: The Bakwat of the *Ga'dang* in Paracelis

Figure 7 shows the *Hambag* (bag) of the *Ga'dang* in Paracelis, Mountain Province. The Figure 7(a) is the pouch used where they can put facial make-up paraphernalia, whereas Figure 7(b) is the body bag used to put cellphones and other personal materials. Both of the bags embedded

beads with geometric designs such as *ansulam/antallasik* (triangular), double *ansulam/antallasik* (diamond), and *lappaw*-circular beads formation.



Figure 7: The Hambag of Ga'dang in Paracelis, Mt. Province

Furthermore, other similar patterns of beads from Alfonso Lista and Paracelis are *Abag* (loincloth/use by males to cover their private parts), *Kuton* (male vest and it is shorter than cheleko), *Barangal* (look like a handkerchief use for headdress or to wrap in the shoulder for males), *Tapit* (cape/kappa that wears by the males which signifies that he is a king or prince), *Aken (Innammata and lallad)* (skirt used by females), *Barawasi (Innammata and lallad)* (blouse use by females with long sleeve), *Sayay* (betel nut pouch use both men and women), *Singat* (forehead dress worn by ladies), *Tarabbung* (modernized hat for male and female for sun protection), *Bayoyung* (head band used by females), and *Cheleko* (modernized vest/ dress for males. It is longer than a kuton).

Indigenous artifacts showcase craftsmanship, creativity, and worldview, connected to environment, spiritual beliefs, and social organization (Cameron, 2024). Integrating these cultural elements into classrooms can make mathematics more relevant for indigenous students (Kadonsi, 2024; Sari et al., 2024). Teachers can highlight the connection between everyday cultural practices and mathematical concepts through patterns like symmetry, geometry, and counting systems (Agbata et al., 2024).

Indigenous students benefit from incorporating their cultural heritage into their education, as it validates their identity and makes learning more meaningful (Lunda et al., 2024). Further, by analyzing geometric patterns and mathematical concepts, they see the practical application of these concepts in their lives, fostering a sense of pride and active participation (Agbata et al., 2024; Xu & Ball, 2024). Additionally, incorporating local cultural artifacts into the curriculum enhances cultural awareness, inclusiveness, and appreciation for indigenous cultures, promoting understanding and appreciation among non-indigenous students (Kurniawan et al., 2023).

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## Algebraic Patterns

In Table 1, the algebraic patterns are embedded in the different cultural artifacts, specifically on pendants, musical instruments, baskets, architectural roofing, and bead designs.

The Algebraic Patterns	Places and the Artifacts
$a_n = a_1 + (n-1)d$ where $n \in [1, +\infty)$	<b>Banaue</b> Buung/ Linglingo
$a_n = a_1 + (n-1)d$ where $(1 \leq n \leq 7)$	<b>Banaue</b> Gangha (Gong)
$3 + 5 + 7 + \dots + (2n-1) = n^2 + 2$	<b>Mayoyao</b> Pallungan (winnower), Taga'e (Basket Plate), Hape'eng, Labfa, Inomon Hape'eng, Hag'cha, Ho-op, Uppig (Lunch box)  <b>Aguinaldo</b> Pallungan (rectangular and circular winnower), Taga'e, Inamon/Inomon Pasikeng
$3 + 5 + 7 + \dots + (2n+1) = n^2 + 2n$	<b>Mayoyao, Ifugao</b> Native House Baleh
Half Diamond (ansulam/antalsik design) $1+2+3+4+5+ \dots +n = \frac{n(n+1)}{2}$	<b>Alfonso Lista</b> Abag (Loincloth), Barangal, Tapit, Aken, Barawasi Bakwat, Sayay (betel nut pouch)
Full Diamond (double ansulam/antalsik design) $1 + 3 + 5 + \dots + (2n-1) = n^2$	<b>Paracelis</b> Abag (Loincloth), Kuton, Barangal, Tapit, Aken (Innammata and lallad), Barawasi (Innammata and lallad), Bakwat (Innammata and lallad), Sayay, Singat, Tarabbung, Bayoyung, Cheleko, Camisola, Hambag (pouch/body bag)

Table 1: The Algebraic Patterns that are found in the Artifacts of the Region

It is evident how deeply ingrained mathematical knowledge is in indigenous practices when these algebraic patterns are found in cultural artifacts. These trends are not random, rather, they represent the community's perceptions of time, social structures, and the natural world (Sari et al., 2024; Xu & Ball, 2024). Indigenous peoples have long used mathematical reasoning in their cultural practices, albeit in ways that are frequently disregarded in formal education systems, as evidenced by the use of such patterns in artifacts (Huencho & Chandia, 2023).

## Development of a Culturally Based Approach to Mathematical Investigation

The next section provides the sample procedure of this study for using mathematical investigation to come up with a conjecture from the cultural artifacts. It was found that the patterns explored can be a basis for integrating culture when teaching in the classroom. According to Kurniawan et al. (2023), patterns found in cultural artifacts can be used as a basis to integrate culture into classroom teaching.

On pendants, Table 2 shows the first five sizes and the diameters in inches of the *Linglingo*. The table presents the expanded form using the common difference from the 1<sup>st</sup> size  $a_1$  to the 5<sup>th</sup> size  $a_5$ .

$a_n$ (where $n$ is the size)	Diameter in inches	Expanded form	$a_n$ with respect to $n$ and $d$
$a_1$	$\frac{3}{8}$	$\frac{3}{8}$	$3 + \text{left (0 right) left } \left\{ \frac{1}{8} \right\}$
$a_2$	$\frac{5}{8}$	$\frac{3}{8} + \left\{ \frac{1}{4} \right\}$	$3 + \text{left (1 right) left } \left\{ \frac{1}{8} \right\}$
$a_3$	$\frac{7}{8}$	$\frac{3}{8} + \frac{1}{4}$	$3 + \text{left (2 right) left } \left\{ \frac{1}{8} \right\}$
$a_4$	$1\frac{1}{8}$	$\frac{3}{8} + \frac{1}{4} + \frac{1}{4}$	$3 + \text{left (3 right) left } \left\{ \frac{1}{8} \right\}$
$a_5$	$1\frac{3}{8}$	$\frac{3}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$	$3 + \text{left (4 right) left } \left\{ \frac{1}{8} \right\}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_n$			$a_1 + (n-1)d$

Table 2: Sample exploration of the data gathered from the *buung* or *linglingo* pendants

Based on the exploration made, the following conjecture was found:



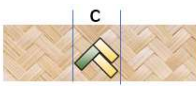



*Let  $a_n$  be the  $n$ th size of the Linglingo,  $a_1$  be the first size of the buung or linglingo,  $n$  be the rank of the linglingo (XS-XL...) or  $n \in [1, +\infty)$ , and  $d$  is the common difference of the sizes. The  $n$ th size of the buung/linglingo equals the first linglingo plus one less than the order of the linglingo times the common difference. Its algebraic notation is  $a_n = a_1 + (n-1)d$ .*

Thus, the sequence  $a_n = a_1 + (n-1)d$  where  $n \in [1, +\infty)$ , is apparent in *Buong* or *Linglingo*, or a lucky charm pendant.

By exploring the other patterns in these artifacts, students deepen their understanding of math while gaining insight into the cultural significance behind the artifacts. This approach not only enhances engagement but also fosters cultural awareness, showing how math is used across different cultures and making learning more inclusive and relevant (Putra & Mahmudah, 2021; Arya et al., 2024; Danoebroto & Suyata, 2024).

Moreover, according to Pradhan (2020), cultural artifacts embed mathematical ideas and knowledge. Table 3 presents a comprehensive summary of artifacts, their associated investigative tasks, and the conjectures explored through them. These investigative tasks can be assigned by

teachers to mathematics pre-service teachers as part of an activity focused on teaching mathematical investigation. This approach not only models the process of integrating culture into teaching but also immerses students in mathematical investigation within a context that resonates with their lived experiences (Mairing & Nini, 2024; Hunter & Miller, 2022). According to Magni (2017), these practices provide opportunities for pre-service teachers to develop mathematical concepts beyond the classroom (Çilingir Altner et al., 2023). By studying artifacts, pre-service teachers explore mathematical ideas and appreciate how mathematics is developed and used across cultures (Kurniawan et al., 2023). With the teacher acting as a facilitator, students will be guided through the process of formulating conjectures (Wang, 2024).

Cultural Artifacts	Investigative Tasks	Conjecture
The Buung or Linglingo of Banaue, Ifugao	The <i>Buung</i> or <i>Linglingo</i> , of Banaue, is a circular pendant with increasing sizes from extra small (XS), small (S), medium (M), large (L), and extra-large (XL) and a diameter in inches 0.375, 0.625, 0.875, 1.125, and 1.375 respectively. Investigate the diameter as the size of the pendant increases! 	Let $a_n$ be the $n$ th size of the Linglingo, $a_1$ be the first size of the buung or linglingo, $n$ be the rank of the linglingo (XS-XL...) or $n \in [1, +\infty)$ , and $d$ is the common difference of the sizes. The $n$ th size of the buung/linglingo equals to the first linglingo plus one less than the order of the linglingo times the common difference. It is algebraic notation is $a_n = a_1 + (n-1)d$ .
Gangha or Gong artifact of Banaue, Ifugao	One of the musical instrument found in Banaue, Ifugao is the <i>Gangha</i> or commonly known as Gong. The first four sizes of the Gangha in inches are 13, 12.5, 12, and 11.5. These sizes are aligned to the first four pitches <i>do</i> , <i>re</i> , <i>mi</i> , and <i>fa</i> . As the size becomes smaller, the pitch also becomes higher. Investigate! 	Let $a_n$ be the $n$ th diameter size of the gangha $a_1$ be the first size of the gangha, $n$ be the order/rank of the gangha (s. t. $1 \leq n \leq 7$ ), and $d$ is the common difference of the sizes. The $n$ th size of the gangha equals to the first gangha size plus one less than the order of the gangha times the common difference. Its algebraic notation is $a_n = a_1 + (n - 1)d$
<i>Pallungan</i> or a rectangular winnower	Among the baskets found in Mayoyao and Aguinaldo, a rectangular winnower called <i>Pallungan</i> , was crafted from split bamboo poser ( <i>anoh</i> ) and featured a rattan rim constructed in a one-over-two twill pattern. 	Let  be $x$ and let  be $y$ . Let  be $x \oplus y_n$ , $n \in \{1, 2\}$ , where there are two $y$ 's overlapping in $x$ . The columns ( $c$ ) in the design will be defined as the arrangement of $x \oplus y_n$ vertically. If $x \oplus y_n$ is arranged by tiling it in columns where the number of $x$ increases by 1 as the number of columns ( $n$ ) increases from $n > 1$ , then the total number of $x$ and $y$ is $3+5+7+\dots+(2n+1)=n^2+2n$

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Modernized native house/  
native house *baleh* at  
Mayoyao Ifugao

$\oplus$  is used to get the sum of  $x$ 's and  $y$ 's as it is tiled to form the basket. Arranged  $x \oplus y_n$  by tiling it in columns where the number of  $x$  increases by 1 as the number of columns ( $n$ ) increases from  $n > 1$ . Investigate the arrangement!

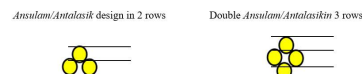
The *baleh* type is considered as the typical Ifugao native house. A runo was used in its roof. The runo is tied up in smaller bundles and fastened on the purlins.




Let  be  $x$  where  $x$  is a small bundle of cogon or runo leaves. The roof of the native house *baleh* is constructed through small bundles of *runo* or cogon leaves (Note: we let 3 small bundles start from the apex down). Starting from the first and the highest row, down to the 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, and 5<sup>th</sup> have a total number of bundled *runos* of 3, 8, 15, 24, and 35 respectively. Investigate the number of small bundles of *runo* leaves in each row.


*Bakwat and Hambag* in  
Paracelis, Mountain  
Province

The beadworks of the *Ga'dang* Community in Paracelis Mountain Province contain double *ansulam* or *antalsik* design (full diamond) and *ansulam* or *antalsik* (half diamond). Each full and half diamond is formed by positioning the beads per row, making a minimum of 2 rows for the smallest half diamond and a minimum of 3 rows for the full-figured diamond (Note: We let 1 bead to form the smallest half and full diamond).



As the textile enlarges depending on the person wearing the material, the number of rows will also be increased as possible. Investigate the number of beads required by the *ansulam* or *antalsik* and double *ansulam* or *antalsik* constructed in the textile as the number of rows increases.

Let  be  $x$ -small bundle of cogon or runo

leaves. Let  be the three small bundles of cogon or runo leaves where the  $x$ 's first arranged in a row  $x \oplus x_n$ . The row in the design will be defined as the arrangement of  $x \oplus x_n$  horizontally. If  $x \oplus x_n$  is arranged by connecting it in rows where the number of rows ( $n$ ) increases from  $n > 1$ , then the total number of  $x$ -small bundle of cogon or runo leaves is

$$3+5+7+\dots+(2n+1)=n^2+2n$$

a. Let  $n$  be the number of rows and  $h$  for the total number of beads for a half diamond (*ansulam/antalsik* design). If the beads are arranged in rows, where  $n$  increases from  $n \geq 1$ , then the total number of beads for half diamond (*ansulam/antalsik* design) ( $h$ ) is

$$1+2+3+4+5+\dots+n=\frac{n(n+1)}{2}$$

b. Let  $n$  be the number of rows, and  $f$  for the total number of beads for a full diamond (double *ansulam/antalsik*). If the beads are arranged in rows, where  $n$  increases from  $n \geq 1$ , then the total number of beads for a full diamond (double *ansulam/antalsik*) ( $f$ ) is

$$1+3+5+\dots+(2n+1)=n^2$$

Table 3: The summary of artifacts with their corresponding investigative tasks and the conjectures investigated

## The Teacher's Guide for a Culturally-Based Mathematical Investigation

The findings of this study on the algebraic patterns found from the different artifacts were used in generating mathematical investigation tasks with their corresponding conjectures. Now, these concepts were concretized by converting them into a Teacher's Guide (see Appendix). The preliminary part gives information about the subject code, title, topic, faculty, and time frame. The preliminary part is followed by a 3-part format: Objectives, Procedure, and Evaluation. The Objectives tell what the students should learn after the lesson, and the Evaluation gives the students assessment activities. Further, the Procedure is divided into motivation, discussion, and generalization. Under the discussion part, the teacher will initiate a discussion about the nature of the artifacts, enabling students to engage meaningfully with the topic. Following this, the teacher will present and explain the investigative task, linking it to the prior discussion of the artifacts. Afterward, the students will be allowed to conduct the investigative activity, where they will organize and analyze the data to identify patterns. The patterns they uncover will guide them in formulating conjectures (Wang, 2024).

Next, because students often show difficulty in proving conjectures, the teacher will facilitate this process, ensuring that students understand the steps involved in mathematical reasoning (Hartono et al., 2024; Farkash et al., 2024). Finally, upon validating the conjecture, the teacher will encourage students to extend their findings, applying their insights to further explore the artifacts under investigation. This approach fosters a deeper understanding of both the mathematical concepts and the cultural context embedded in the study of the artifacts (Wang, 2024). In the generalization part, the students will be asked different questions about the artifacts discussed and the patterns discovered during the sessions.

## CONCLUSIONS

The artifacts explored in the study are all important symbols of a community's identity, values, and traditions. Pendants, bead designs, and baskets serve as fortune bringers, personal adornments and status markers, and represent livelihood activities, respectively. Musical instruments like gongs are used in rituals and celebrations, while baskets showcase local knowledge and functional artistry. Architectural designs like traditional houses reflect the community's relationship with the land and sustainable living practices.

In different areas of the region, artifacts such as pendants and musical instruments were found to exhibit consistent and repeating patterns that align with mathematical sequences. For example, the geometric arrangements in beadwork in half and full diamonds often followed a predictable order, with colors or shapes repeating in a manner that mirrors the arithmetic series. Likewise, the intricate patterns in woven baskets often adhered to principles of symmetry and repeated units, which can be linked to concepts of the sum of odd numbers ( $n \geq 3$ ). A similar pattern was explored in the roofing of the native houses.

The results describe a teaching approach that integrates cultural artifacts with mathematical investigation, making math more engaging and meaningful by linking it to students' cultural backgrounds. Teachers lead students through discussions of the artifacts, introduce investigative tasks, and guide them in analyzing data to uncover patterns that form the basis for conjectures. Through supporting the process of proving conjectures and expanding the investigation, teachers help students strengthen their critical thinking and mathematical reasoning, while deepening their appreciation for both mathematics and the cultural relevance of the artifacts. This method creates a dynamic, context-rich learning environment that promotes both mathematical exploration and cultural understanding.

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## APPENDIX

### Sample Teacher's Guide in Integrating the Cultural Artifacts into Teaching Mathematical Investigation

1

#### Teacher's Guide 1<sup>st</sup> Semester, S.Y 2024, 2025

**Subject:** SEME 109. Problem-solving, mathematical investigation, and modeling

**Topic:** Mathematical Investigation (*Basketry of Mayoyao and Aguinaldo*)

**Faculty:** TBA

**Time Frame:** 2 sessions/ 1 hour per session

#### I. Objectives:

At the end of the lesson, the students should be able to:

1. describe the native baskets of Mayoyao and Aguinaldo;
2. identify distinct patterns found in the baskets of the region; and
3. apply the mathematical investigation activity to the algebraic pattern found in the baskets.

#### II. Procedure:

##### *Session 1*

##### A. Motivation

**Review.** Ask the following questions:

1. Provide a description of the Gangha from Banaue, Ifugao. What materials was it crafted from? How many pieces typically make up a set of Gangha? Share details about its sizes.
2. What pattern has been identified in the Gangha? Presentation of the topic.  
Present the topic Mathematical Investigation of the Basketry of Mayoyao and Aguinaldo

##### B. Discussion

*Localizing/ contextualizing*

1. Distribute a copy of the material on the Basketry of Mayoyao and Aguinaldo (see Annex A) to each student and have them read and study it.
2. After the students read the material, ask the following questions (*try to catch their interest by relating the material to their experience in their community*):
  - a. What types of baskets are commonly found in Mayoyao and Aguinaldo?
  - b. How are each of these baskets used in daily life? What roles do they serve within the community?
  - c. What materials are typically used to make these baskets? How are they crafted? Do you know anyone in your community who makes these baskets, and have you ever observed the process?
  - d. Which of these baskets can you find in your own home? How do you use them, and who in your household typically uses them?

*Data gathering*

3. Show common images found in the baskets:



- a. Instruct the students to examine the images and identify any common patterns among the baskets (guide them through this activity). Allow time for them to analyze the images individually. Afterward, pair them up so they can discuss their observations and share initial thoughts. Next, have each student find a new partner

to further discuss and refine their ideas. Finally, ask 3 to 5 students to share the patterns they've identified with the class, and encourage the others to reflect on whether they observed similar patterns.

b. Help them convert their ideas into mathematical expressions.

4. Evaluation (for session 1)

Essay: In 200 words, describe one type of basket found in the locality and discuss their roles in the community.




**Session 2**

*Presentation of the Investigative task:*

5. Present to the students the following task:





*“Among the baskets found in Mayoyao and Aguinaldo, a rectangular winnower called Pallungan, was crafted from split bamboo poser (anoh) and featured a rattan rim constructed in a one-over-two twill pattern.*



Let  be  $x$  and let  be  $y$ . Let  be  $x \oplus y_n$ ,  $n = \{1, 2\}$ , where there are two  $y$ 's overlapping in  $x$ . The columns ( $c$ ) in the design will be defined as the arrangement of  $x \oplus y_n$  vertically. The symbol  $\oplus$  is used to get the sum of  $x$ 's and  $y$ 's as it is tilled to form the basket. Arranged  $x \oplus y_n$  by tilling it in columns where the number of  $x$  increases by 1 as the number of columns ( $n$ ) increases from  $n > 1$ . Investigate the arrangement!”

6. *Exploring systematically*

Lead the students to come up with this observation (facilitate this activity):

Number of columns ( $n$ )	Figure formed	$x \oplus y_n$
1		3
2		8
3		15
4		24
⋮	⋮	⋮




Guide the students to complete the table where  $n \geq 1$ :

Number of columns ( $n$ )	Sum per column	$x \oplus y_n$
1	$3 = 3$	$1^2 + 2(1) = 3$
2	$3 + 5 = 8$	$2^2 + 2(2) = 8$
3	$3 + 5 + 7 = 15$	$3^2 + 2(3) = 15$
4	$3 + 5 + 7 + 9 = 24$	$4^2 + 2(4) = 24$
$\vdots$	$\vdots$	$\vdots$
$n$	$3 + 5 + 7 + \dots + 2n + 1$ $= n^2 + 2n$	$n^2 + 2n$

Ask the students to find  $x \oplus y_n$ , when  $n = \{5, 6, 7\}$ , and try to verify them by drawing the figure formed by  $x$ 's and  $y$ 's.

### 7. Writing the conjecture

Guide the students to come up with the conjecture:

Let  be  $x$  and let  be  $y$ . Let  be  $x \oplus y_n$ ,  $n = \{1, 2\}$ , where there are two  $y$ 's overlapping in  $x$ . The columns ( $c$ ) in the design will be defined as the arrangement of  $x \oplus y_n$  vertically. If  $x \oplus y_n$  is arranged by tiling it in columns where the number of  $x$  increases by 1 as the number of columns ( $n$ ) increases from  $n > 1$ , then the total number of  $x$  and  $y$  is

$$3 + 5 + 7 + \dots + (2n + 1) = n^2 + 2n$$

### 8. Proving the conjecture:

Ask the students how they would go about proving the conjecture. Guide them through the process of proving it using mathematical induction (*remind them of the steps involved*). Begin with the base case, demonstrating that the statement holds true for the initial value, typically  $n = 1$ . Next, lead them through the inductive step, where they assume the statement is true for  $n = k$  (*the induction hypothesis*), and then show that it must also be true for  $n = k + 1$ .

Ask them to provide the conclusion.

### C. Generalization

Ask the following (*provide metacognitive prompts to guide the students in answering these questions*):

1. Name a type of basket found in either Mayoyao or Aguinaldo and describe it in three sentences. What are its key characteristics, and what purpose does it serve in the community?
2. Explain the pattern observed in the baskets, focusing on the techniques and processes used in their creation. How does the design or structure of the baskets reveal this pattern?
3. What algebraic patterns can be identified in the design or construction of these baskets? Are there recurring numerical or algebraic relationships?
4. Is this pattern valid for all values of  $n$ ? How was the pattern rigorously proven, and what steps were taken to verify its general applicability?
5. Describe the process of the investigation activity that led to the discovery and proof of the algebraic pattern present in the baskets. How did you explore and confirm this pattern through observation and analysis?

### III. Evaluation

Do the following task in a group with 3 members.

#### Investigation Task:

Investigate the arithmetic sequence  $4, 9, 14, 19, \dots$  by identifying the common difference and deriving the formula for the  $n^{\text{th}}$  term. Using this formula, calculate the  $15^{\text{th}}$  term and the sum of the first 8 terms. Afterward, create a new arithmetic sequence with a negative common difference, and calculate the 6th term and the sum of the first 12 terms. Analyze how the negative common difference changes the behavior of the sequence and its sum.