Mathematics Teaching Strategies: A Reflection on Pedagogical Content Knowledge (PCK) for Improving Teaching Quality

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Abstract: This study investigates elementary school teachers' mastery of mathematical content and pedagogical content knowledge (PCK) in Eastern Indonesia, a region bordering other countries. Specifically, the research explores the relationship between teachers' knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) and how these factors impact their teaching strategies. The central research question is: How does elementary school teachers' mastery of mathematical content and PCK influence their teaching strategies? Using a qualitative approach, data were collected from 77 elementary school teachers through descriptive tests, in-depth interviews, and classroom observations. The findings indicate that teachers' overall PCK, including KCS, KCT, and KCC, remains at a low level. Many teachers demonstrate content knowledge that is disconnected from foundational mathematical concepts, which negatively impacts their pedagogical approaches and their ability to select appropriate teaching strategies. This study highlights the need for further research on improving teachers' content knowledge and pedagogical approaches to enhance the quality of mathematics instruction.

Keywords: mathematics education, PCK, teachers, teaching strategy, teaching quality

INTRODUCTION

Diverse educational sectors endeavor to provide students with competencies for the 4.0 Era. This era is characterized by the integration of digital technologies, automation, artificial intelligence (AI), big data, and the Internet of Things (IoT) into various aspects of life, including education (Celik et al., 2022). In the context of education, the 4.0 Era demands a shift from traditional teaching methods to technology-driven, student-centered learning models that foster critical thinking, problem-solving, and digital literacy. The primary focus of education in Indonesia nowadays encompasses learning model innovation in the digitalization era (Erhan et al., 2022), differentiated

learning (Yu & Canton, 2023), and the growth of reading and numeracy (Piper et al., 2018). Global competition increases the need to improve the quality of education in Indonesia. Several regions in Indonesia with qualified teacher resources are trying to update the teaching model, improve teaching quality, and use an independent curriculum.

However, in the border areas, Indonesia has various unique challenges in terms of education. Border areas in Indonesia face various challenges, such as access to resources, school locations, and differences in language and culture, which greatly affect the quality of education (Magayang et al., 2020). The teacher is the main pillar and the main source of learning in improving the quality of education. The quality of education is determined by the competence and resources of teachers (Carroll, 2020). The main tasks and responsibilities of the teacher are the main axis of teaching activities. The teacher's ability to convey teaching material is a professional competency that the teacher must fulfill (Dawson & Shand, 2019). With various limitations, the teacher should have good pedagogical skills so students can capture the knowledge the teacher conveys.

Additionally, teachers encounter numerous challenges (Febriana et al., 2018), particularly in instructing mathematics at the basic level. The proficiency of teachers in mathematics influences their approach to developing students' mathematical knowledge and skills (Coskun, 2020). They are required to consistently enhance their proficiency in math's. Mastery of mathematics influences teachers' daily thought processes (Castro et al., 2021).

This research is urgent because it fills the void of previous research. Previous research has discussed teacher limitations in teaching (Murniarti et al., 2021; Nguyen et al., 2020) but has yet to discuss teacher content and pedagogical knowledge in teaching mathematics in limited areas. This study is related to several studies that explain that elementary school students in border areas have low knowledge and problem-solving abilities (Fredy et al., 2021).

This study discusses the teacher's strategy for working on questions and problems. The aim is to identify the mastery of mathematics teaching strategies of the teachers at elementary schools in the border areas of Indonesia based on reflections on their PCK. This study is expected to become a recommendation for improving the quality of teacher resources in elementary schools so that specific and targeted improvement efforts can be identified.

Pedagogical Content Knowledge (PCK) is a critical component of teacher professionalization, particularly in the context of mathematics education. PCK is defined as the integration of content knowledge, pedagogical knowledge, and an understanding of students' learning processes (Gladys et al., 2024). This unique blend enables educators to tailor their teaching strategies to meet the diverse needs of their students, thereby fostering improved learning outcomes (Hill et al., 2008; Ball et al., 2008).

The KCS domain emphasizes the necessity for teachers to understand their students' pre-existing knowledge and learning difficulties, which is crucial for designing effective instructional strategies (D. Celik et al., 2022; Mitra, 2024). The KCT domain focuses on the pedagogical strategies that

best convey specific content, while the KCC domain pertains to the alignment of teaching with curricular standards (Bognar et al., 2024). Research indicates that teachers with a strong foundation in PCK are more adept at implementing inquiry-based and student-centered teaching methods. These educators are better equipped to adapt their instructional strategies based on their understanding of both the subject matter and their students' learning needs (Bales, 2019; Mdaka et al., 2023; Pilous & Leuders, 2023).

The development of PCK is influenced by teaching experience, with studies showing that novice teachers often struggle to integrate these components effectively compared to their more experienced counterparts (Evens et al., 2018). The urgency of assessing PCK in relation to teaching effectiveness cannot be overstated. The application of PCK directly correlates with student learning outcomes, making it essential for teacher education programs to emphasize the development of PCK among future educators (Koopman et al., 2019). This focus on PCK not only prepares teachers to deliver content effectively but also equips them with the skills necessary to foster a supportive learning environment that acknowledges and addresses the diverse needs of their students (Donnelly & Berry, 2019).

This multifaceted approach to teaching not only enhances the teacher's ability to deliver content but also significantly impacts student engagement and achievement (ŞEN, 2021; Sie & Agyei, 2024). The urgency of assessing PCK in relation to teaching effectiveness cannot be overstated. As noted by various scholars, the way teachers apply their PCK directly correlates with student learning outcomes, making it essential for teacher education programs to emphasize the development of PCK among future educators (Hassan et al., 2018; Metz, 2018). This focus on PCK not only prepares teachers to deliver content effectively but also equips them with the skills necessary to foster a supportive learning environment that acknowledges and addresses the diverse needs of their students (Sarıçoban & Kirmizi, 2020).

METHOD

Participant and Context of the Study

This study aimed to find PCK patterns in rural teachers by exploring elementary school teachers' thinking skills. A total of 77 teachers participated to accomplish this goal. The subjects comprised 53 individuals from public schools and 24 from private schools in rural regions of Indonesia. The research subject criteria comprised 58 male and 19 female teachers aged 30-56. The participants of this study were teachers who had taught for at least three years in elementary schools and had a diploma education background in the elementary school teacher education study program. Some teachers had to teach three classes because only three were in one school. The conditions of the place where they taught were far from adequate facilities because they were in the outermost and furthest areas from the city center. Access to schools needed to be improved, and it tends to be difficult to pass with steep roads and minimal vehicles. Learning infrastructure and facilities still need to be improved to reach optimal effectiveness, especially the learning media. Students' school

attendance was still low because they helped their parents meet their daily needs. Challenges related to students' cultural and economic backgrounds limited their opportunities to enhance their knowledge and teaching competencies.

The research methodology involved 77 elementary school teachers who cooperated on six questions concerning KCC, KCS, and KCT. The sampling technique used was purpose sampling by selecting teachers who had filled in the answers. We only included 27 teachers in the subsequent study process because of this test, as the remaining teachers were required to provide a response or provided responses that were not as pertinent to the extant questions. We subsequently selected three teachers who satisfactorily answered each question from the 27 teachers. Three teachers were the interview subjects, and the criteria included 1) answering all the wrong answers with the wrong understanding of the questions, 2) answering all the answers wrong but still understanding the question's meaning, and 3) teachers with unique answers. We involved the three subjects in in-depth interviews.

Data and Measurement

This study employed three main instruments: a test, interviews, and classroom observations to assess teachers' Pedagogical Content Knowledge (PCK) in elementary mathematics.

Test

The data was collected using an ability test on mathematics teaching strategies in elementary schools, adapted from Ball (Hill et al., 2008). This method was used because the teachers convey that not all math material is well mastered. The first data collection was done using a test. The test was followed by 77 teachers, with 27 teachers collecting answers. The test was carried out for 1 week because it was adjusted to the management of teachers to become educators and ease of signal access. The working technique was in the form of open answers according to the teacher's understanding. Consisting of 6 questions with 1 KCC question, 2 KCS questions, and 3 KCT questions. Each question has 10 points for the correct score and 0 for the wrong score, with 60 points. The results of this test were used to identify interview subjects.

Category	Item	Questions
KCC	1a	What is the learning media for fraction operation?
	1b	How do you apply realistic mathematics?
KCS	2a	How important is it to teach geometry concepts by linking contextual problems?
	2b	Write down one way to teach geometry to grade 6 students!
KCT	3a	Explain whether algebraic material is in elementary school!
	3b	Give one example of algebraic material for lower-grade students!
KCT	4a	$\frac{1}{3} : \frac{1}{6} = 2$ Prove with a picture!
	4b	Make a simple plan on how to teach the material!
KCT	5a	(-5) - (-9) = 4

Category	Item	Questions
		Explain the correct solution to teach the problem!
	5b	Should the teacher use media for this solution? Explain!
KCS		125 : 5 = ...
	6a	What is the concept of teaching division operations to minimize student errors?
	6b	Write down an alternative to teaching this material!

Table 1: Questions of teacher's PCK test

Each correct answer earned 5 points, with a maximum score of 60. Conducted over a week, the test accommodated teachers' schedules and internet access limitations. Validity was ensured through expert reviews and pilot testing, while reliability was confirmed via inter-rater scoring consistency.

Interview

Interviews were used to obtain complete data on the teacher's thinking in developing mathematics teaching strategies for students. Interviews were also conducted with lecturers who taught mathematics in elementary school. The in-depth interview was conducted over four meetings. Three participants were interviewed separately. The interview consisted of 3 PCK domains: KCS with two questions, KCT with four questions, and KCC with 1 question. The interview questions were also adjusted to the answers to the test questions that had been worked on. The data obtained from the interview were transcripts of answers and descriptions of teacher abilities. Validity was ensured through expert review and member checking, while reliability was maintained via coding triangulation. The interview and observation instrument used was adapted by Hill et al. (2008) as shown in Table 2.

Category	Domain
(KCS)	More studies concerning learning difficulties in mathematics
	More skills concerned with teaching students at different levels
(KCT)	More courses about didactic mathematics
	More courses about how to differentiate teaching
	More skills to motivate students in mathematics
	More studies about problem-solving
(KCC)	More courses about using technology in teaching mathematics

Table 2: The instrument PCK Domain

Observations

Observations were also carried out to observe teacher performance. Observation supports data and is an important instrument in providing a perspective on research results. The observation position was carried out during eight weekly meetings to balance the test and interview data. The teacher conducted the teaching and learning process and then used a laptop or mobile phone setup so that the observer could monitor activities online. The objects observed were mathematics ability, the

teacher's teaching method, classroom management, student activities, school environment, and student enthusiasm for learning. The researcher conducted observations as a mathematics lecturer. The observation results were descriptions of activities and findings based on learning activities. A structured rubric, reviewed by experts, ensured validity, while observer agreement checks enhanced reliability. This multi-method approach strengthened the accuracy and credibility of the study's findings.

Data Analysis

Data analysis was conducted by data collection, reduction, and conclusion. The analytical data pertained to triangulation through the cross-validation of conclusions from several data sources. The research guaranteed robustness and reliability. To ensure validity, the interview questions were reviewed by mathematics education experts to confirm alignment with the test results and PCK framework. Member checking was employed, where interview transcripts were shared with participants for verification. To ensure reliability, coding consistency was maintained through triangulation, where two researchers independently analyzed interview transcripts and resolved discrepancies through discussion.

The data regarding test results on the efficacy of teaching strategies could be validated using qualitative insights from interviews and case studies. Qualitative data offered context and depth to the observed numerical trends. Qualitative data can elucidate the reasons and mechanisms underlying its practical application. Qualitative research insights emphasize increased attention to test outcomes.

Data analysis in this study included analysis of teacher answers using tests, in-depth interviews on three aspects of PCK, and observation as supporting data. Analysis of teacher test results in solving PCK questions on mathematics material was done by first identifying each open-ended answer item. The answer items were explained to obtain a score for the distribution of answers to each question. This score was used to select three research participants who were interview subjects. The data was qualitative at the in-depth interview stage, so data reduction was needed to obtain answer codes and the relationship between each answer. Not only that, at the interview stage, it was supported by authentic evidence of the interview subject's answers. The observation data described the conditions appropriate for the test and interview data.

The analysis results at the test stage were presented in a graph of the demographic results of the teacher's answers. The next result was a table of teacher answer results and a table containing the results of teacher interviews, overall coding results, and observation results. The results of the various data collections were checked for validity by the researcher and one expert with triangulation, namely, comparing data with other sources. In this process, the researcher conducted a thematic analysis to identify patterns that emerged in the puzzle of teacher knowledge, curriculum, and challenges of teaching mathematics in elementary schools in rural areas. In this analysis, results about teacher stories and experiences are explored in depth. The narrative of the teacher's

answers provides a picture of the conditions that influence the learning process and teacher interaction with students. The results obtained from this triangulation process are PCK patterns related to KCC, KCS, and KCT in mathematics learning by elementary school teachers in rural areas. The coding results at the final stage are in the form of analytics presented in patterns and relationships for each PCK domain, including KCC, KCS, and KCT.

RESULTS

Test Result

Test results were available at eight weeks. Figure 1 shows the results of the teacher's PCK test.

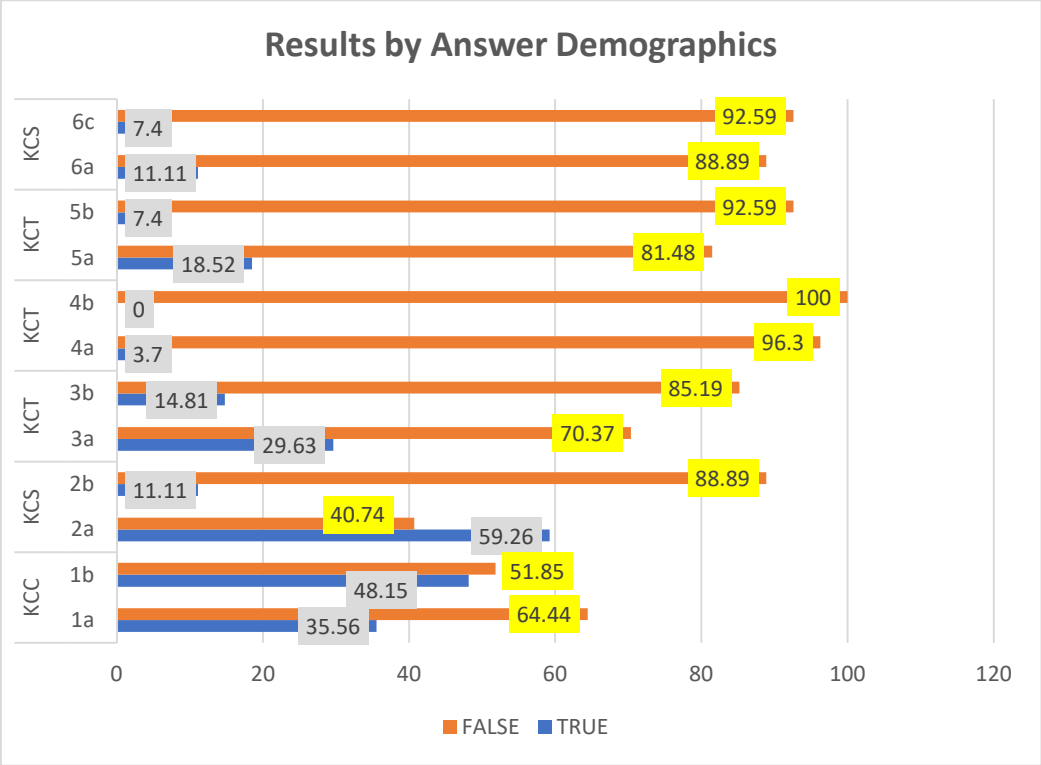


Figure 1: Results by Answer Demographics

The data in Figure 1 above obtained from 77 teachers who worked on the questions. The teachers gave answers that needed to match the questions. The following discussion is about each category of the questions.

Teacher's Knowledge of Content and Curriculum (KCC)

For question no. 1, most of the teachers explained the answers incorrectly. The teachers that answered incorrectly were 64.44% because the teachers did not understand the meaning of the questions correctly. The teacher gave answers that did not follow the given instructions. Of the 12 teachers who gave wrong answers, 10 answered questions far from those given, for example, in the answer in Table 3.

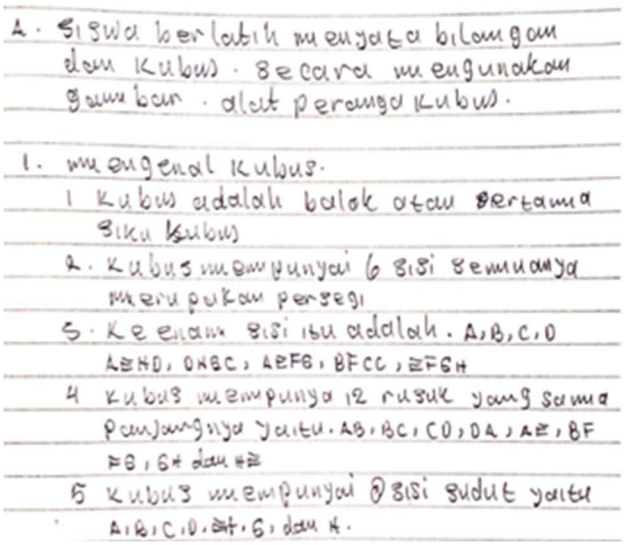
The Answer	Translate
	A. Students practice expressing numbers and cubes. By using pictures. Cube tool. 1. Get to know the cube a. The cube is the first block or corner cube. 2. A cube has six sides, all of which are square 3. The six sides are A, B, C, and D. AEND, OHEC, AEFG, BFCC, EFGH 4. The cube has 12 edges of the same length, namely AB, BC, CD, DA, AE, BF, FG, GH, and HF 5. The cube has eight angles, namely A, B, C, D, E, F, G, and H

Table 3: The Answer to Question 1 KCC

The initial teacher's response pertains to the cube's material and its characteristics. These answers are a sample of 10 teachers whose answers are far from far from correct. During the interviews, the teachers were given an opportunity to rework the same task, however, their responses remained incorrect.

Category	Item	Answers
Interview (T for Teacher)	1	T 1: "I have difficulty understanding the meaning of the question, which I understand like that." T 2: "I have tried to fix the answer, as far as I know." T 3: "We do not use media, so we do not understand."
Interview with Lecturer		The answer to using learning media is monotonous due to limited access in the area. They have no desire to create media and only use existing media. Not only that, but they need help understanding various learning methods and media. So, the student learning model is still teacher-centered.
Observation results		At this stage, the teacher needed to explore the answers more, and the answers were based on personal experience. There are limited uses of media and learning supports. Teachers also need help to make students active because teachers tend to be teacher-centered. Some teachers have used learning aids such as marbles, sago caterpillars, and fruit to learn numbers, so the technology in this study is devoted to aids to help convey material.

Table 4: The Answer to Interview of KCC

To clarify the picture of teachers at the KCC stage, we adapted the KCC pattern from (Toyinbo, 2018) with adjustments to the results in the field, namely as follows:

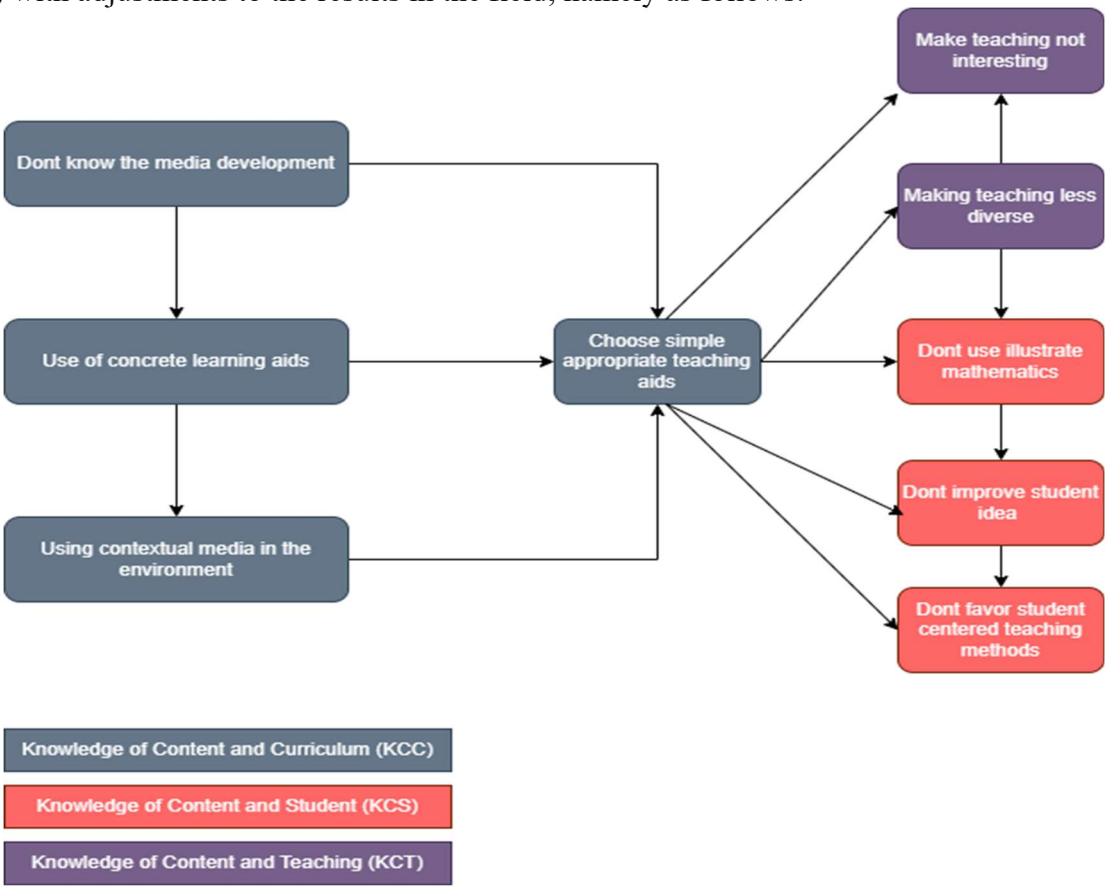


Figure 2. The pattern of relationship between KCC with KCT and KCS

Teacher's Knowledge of Content and Students (KCS)

The discussion of KCS examines how the teacher anticipates student answers and difficulties. Questions 2a and 2b discuss material on geometry material and geometry teaching techniques to anticipate student answers. There were 11 (40.74%) wrong answers in question 2a and 24 (88.89%) in question 2b. In question 2a, the error lies in how the teacher discusses geometry material. For questions 2b and 24, all the 27 teachers answered wrong. The teacher's answer is shown in Table 5 below.

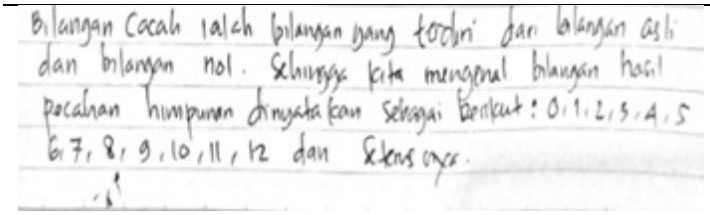
Answer	Translate
	b. Whole numbers are numbers that consist of natural numbers and zero. So that we know the resulting set fractions are expressed as follows: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on

Table 5: The Answer to KCS

The question is about how the teacher teaches geometry material. The teacher answered with a very far-reaching answer. He answered by explaining the meaning of whole numbers. Many answers need to match the questions. Question 6 discusses material about division to explore the teacher's technique in anticipating student answers and difficulties. There were 24 (88.89%) errors in question 6a, which discussed the teacher's technique of minimizing student errors in division material, and 25 (92.59%) errors in question 6b concerning alternative ways of calculating division without using a tiered division technique.

Category	Item	The Answers
Interview (T: Teacher)	2	T 1: "It is hard to understand the answer." T 2: "I know how to do it but am confused about how to teach it to students." T 3: "The answer is area or circumference, right?"
	6	T 1: "Multiplication and division material for students is very difficult." T 2: "I have no way to teach divisions 1-10 to students; I also find division difficult."
Interview with Lecturer		Some teachers did not master understanding mathematical concepts. In classroom learning, teachers tend to explain very basic things, so they need help representing mathematics in a relevant way. This impacts how they determine how to solve the problem. Over time, we teach in the same way. Most of the teachers answered the questions wrong. Deficiencies in the initial calculations make the teachers unable to correctly apply the tiered counting technique.
Observation results		Teacher errors occur due to a need for more understanding of the questions. Teachers' limited ability to understand the principles of geometric material and arithmetic operations impacts their determination of learning methods. Because of their limited abilities, teachers are not optimal in explaining how to teach the material.

Table 6: The Answer to Data of KCS

Based on answers from various sources, we tried to create a teacher answer pattern based on a model adapted from Toyinbo (2018). This pattern describes the condition of KCS in the border area in Figure 3 below:

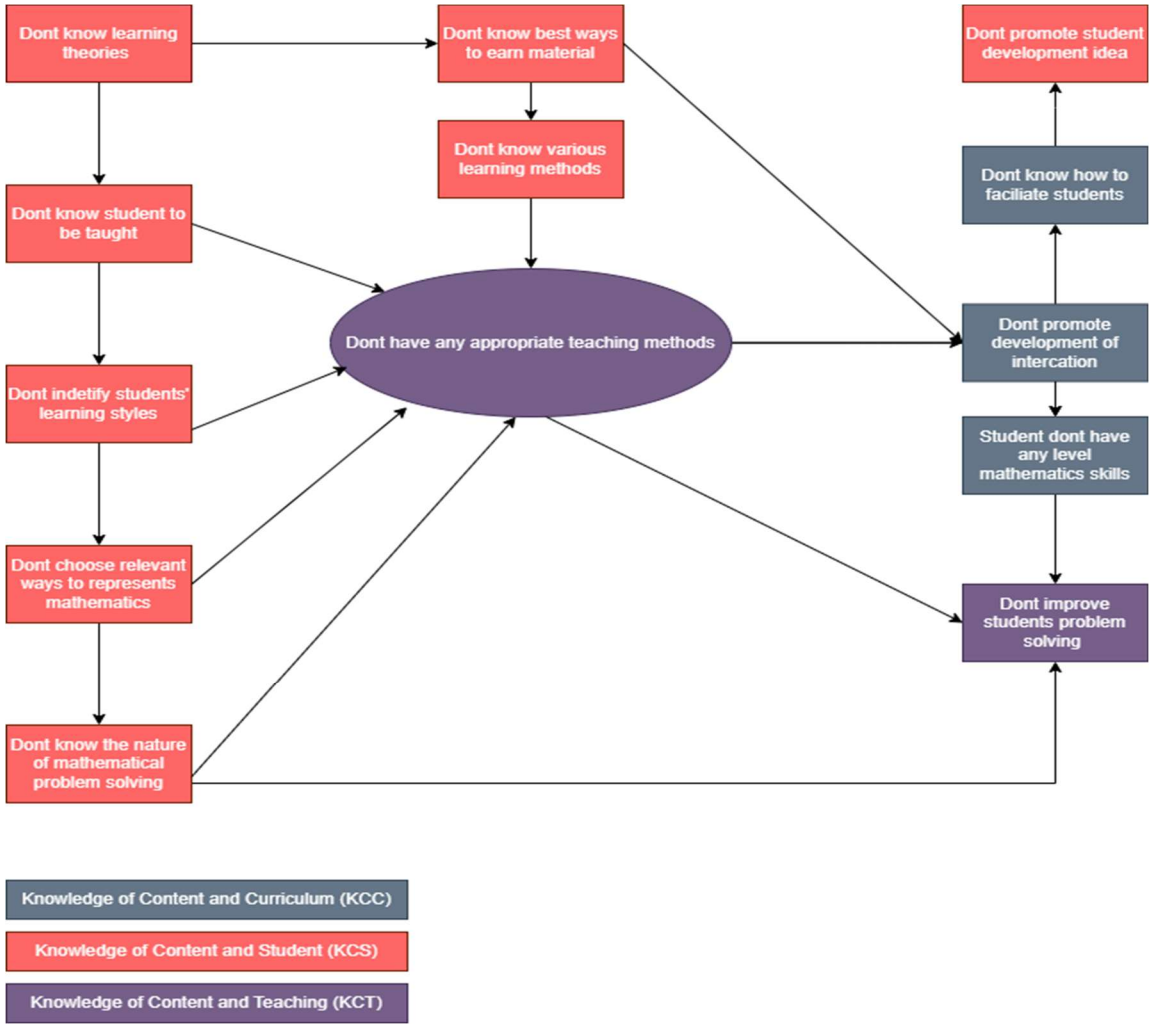


Figure 3: Pattern of relationship between KCS with KCT and KCC

Knowledge of Content and Teaching (KCT)

The problems presented are in questions 3a to 5b. Question 3a discusses algebra and the scope of algebraic material in basic education units. It was found that 19 (70.37%) teachers answered incorrectly on the meaning of algebra, and 23 (85.19%) teachers answered incorrectly to give an example of algebra in elementary school. The teachers still need to understand the concept or scope of algebra material for elementary school students. Other findings are in question 5a, about calculating integer operations, and 5b, about how to teach integer operations to students. There were 22 (81.48%) teachers who answered question 5a incorrectly, and 25 (92.59%) answered question 5b incorrectly.

The highlight of the KCT results is question number 4. Questions 4a and 4b discuss fractions. The teachers gave various interesting answers to this question. Problem 4a is about how to divide fractions with pictures. As many as 26 (96.3%) teachers answered this question incorrectly. Question 4b is about how to teach question 4a to elementary school students, and all 27 teachers (100%) answered it incorrectly. The teacher's answer is presented in Table 7 below.

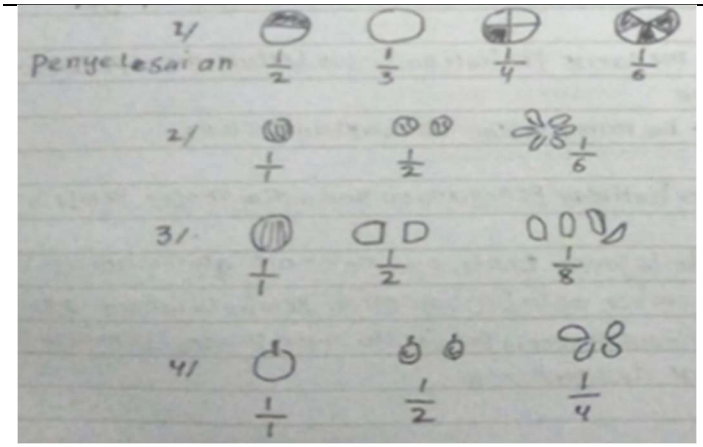
Answer	Translate
	This answer does not match the question. The depiction of fractions does not match the picture.

Table 7. The Answer to Data of KCT

The teachers' way of thinking in the sample answers is very interesting. The findings from their answers are 1) they have not been able to understand the purpose of the questions; 2) they have not been able to draw fractions according to their value; 3) they do not understand the concept of fractions; 4) they use the formula that in dividing fractions it must be reversed to multiplication; 5) they have not been able to project the distribution of fractions correctly.

Category	Item	The Answers
Interview (T: Teacher)	3	T 1: "I did not know algebra was in elementary school materials." T 2: "I just looked for the answer on Google and copied it because I did not understand." T 3: "I have never heard of the term algebra."
	4	T 1: "Fraction material is the most difficult. Making children understand is also very difficult." T 2: "Have difficulty drawing fractions. I can count but not draw; I never studied fractions with pictures." T 3: "I know what the problem means, but doing fractions by drawing is difficult. Fraction questions are difficult problems."
	5	T 1: "Do it by going into debt or going into debt with a larger figure." T 2: "The answer must have a fraction because it is a fraction question." T 3: "I still often make mistakes on this question."

Category	Item	The Answers
Interview with Lecturer		For teachers, the most difficult material is fractions. So, it is natural that they make many mistakes. In the beginning, learning the material for drawing simple fractions was still difficult. Especially in teaching fractions, you can imagine they cannot draw 1/4 fractions how they teach these fractions. That is an illustration of their abilities in fractional material
Observation Results		The recurring problem is that the teachers need help understanding the purpose of the questions. Many answers need to be revised. Teachers felt less thorough, and the answers they counted were the questions presented. They said that they had difficulty teaching mathematics. Doing math problems for them is a big scourge. The questions take a long time. Besides that, sometimes they are not focused, so they often add questions according to their wishes.

Table 8: The Answer to Data of KCT

Teachers do not try to find out in-depth because they think this material is not essential and does not need to be taught. KCT has an important role because the teacher teaches material to students. For clarification, we compiled a KCT pattern related to KCS and KCC with an adaptation of the KCT pattern (Toyinbo, 2018), like Figure 8 below.

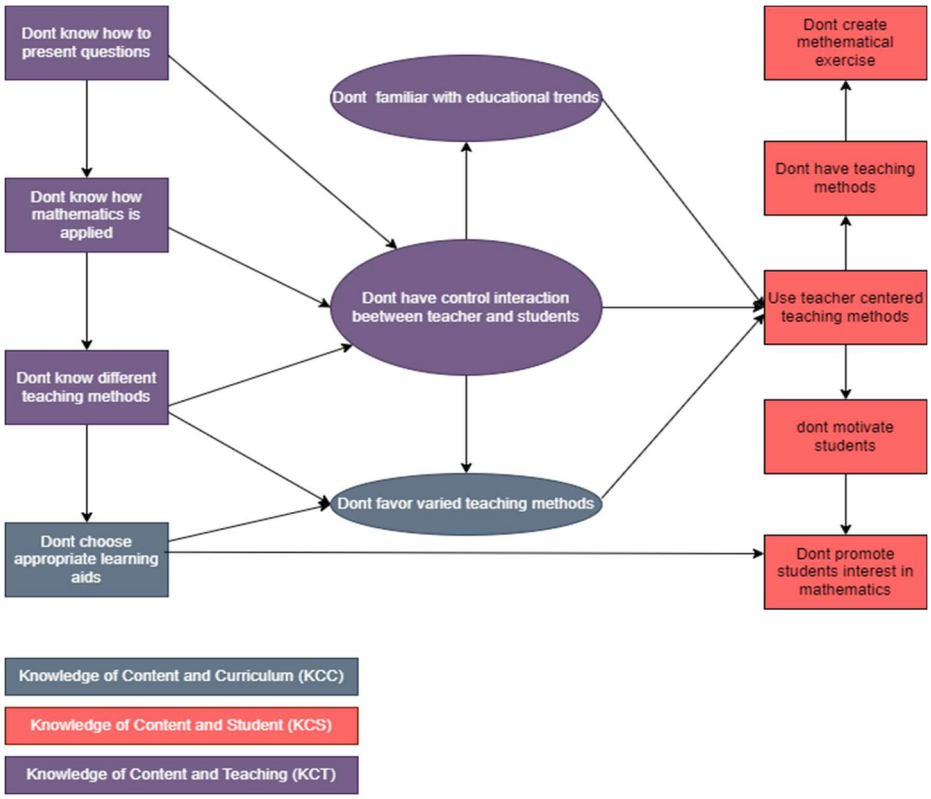


Figure 4: Pattern of relationship between KCT with KCS and KCC

DISCUSSION

Pedagogic Content Knowledge (PCK)

The main research finding is that teachers teach based on their understanding, which results in low student learning achievement. Based on the results obtained, teachers' content and pedagogic competencies are still low, meaning low success in mathematics. These results are based on previous research, which states that if teacher content knowledge is low, mathematics learning success is low (Saran, 2018).

Another major finding is that the teachers' answers are unique and interesting, thus triggering further research to find out the teachers' thoughts in depth. Out of the 27 teachers who submitted responses, 10 teachers received a score of 0. This was because they answered questions that were not relevant to the given test items. Additionally, some teachers created their own questions instead of responding to the provided ones, which led to incorrect or unscorable answers. Besides, teachers often add their questions according to their wishes. In PCK, teachers face various challenges, which must be adapted to the situations and conditions of students and their learning environment. This challenge causes teachers in the KCC domain to find it difficult. Environmental, cultural, and social problems of students who participate in learning problems for students also take up the teacher's time.

In the KCC domain, the most prominent is technology or digital literacy. Some teachers have used learning aids such as marbles, sago caterpillars, and fruit to learn numbers, so the technology in this study is devoted to aids to help convey material. This fact shows that teachers' use of curriculum, learning technology, and teaching techniques still needs to catch up. Research (Mailizar & Fan, 2020) examines the six abilities needed for the current 4.0 Era. However, related to the conditions for teachers at the border, the six capabilities of the digital era are difficult to reach.

In the KCS domain, the findings are that teachers also have not mastered the concept properly. For example, the concept of drawing simple fractions. Most of the teachers answered incorrectly, and various unique answers emerged. Teachers need assistance in fractional material as they explain that fractional material is very difficult for themselves and their students. This finding follows research that has proven that fractions are difficult material (Agathangelou & Charalambous, 2021).

Several teachers in various regions of Indonesia have started to develop teaching quality (Rahmadi & Lavicza, 2021). However, this study's findings show that teachers are unwilling to improve students' understanding of content. Based on the teachers' answers, the way the teacher explores students' abilities in mathematics still needs to be visible. When students answer incorrectly, the teacher must try to discover the mistakes and difficulties students are experiencing.

Based on the findings and results of this study, PCK greatly affects students' abilities. PCK plays an important role in the teacher's capacity to teach mathematics and help students learn mathematics (Golding & Batiibwe, 2020). This result illustrates that the students' math scores in the low border areas are due to the low PCK results of border teachers. This PCK plays an important role in building students' understanding of topics and methods for effective learning (D. Celik et al., 2022). However, the results of this study show that teachers in the border areas need to teach more effectively because their PCK level still needs to be higher.

Pedagogical Content Knowledge (PCK) is a critical component of teacher professionalization, particularly in the context of mathematics education. PCK is defined as the integration of content knowledge, pedagogical knowledge, and an understanding of students' learning processes. This unique blend enables educators to tailor their teaching strategies to meet the diverse needs of their students, thereby fostering improved learning outcomes (Hill et al., 2008; Ball et al., 2008). PCK is categorized into three primary domains: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). The KCS domain emphasizes the necessity for teachers to understand their students' pre-existing knowledge and learning difficulties, which is crucial for designing effective instructional strategies (Mason, 2018; Hill, Rowan, & Ball, 2005).

The KCT domain focuses on the pedagogical strategies that best convey specific content, while the KCC domain pertains to the alignment of teaching with curricular standards (Mason, 2018; Ball et al., 2008). Research indicates that teachers with a strong foundation in PCK are more adept at implementing inquiry-based and student-centered teaching methods. These educators are better equipped to adapt their instructional strategies based on their understanding of both the subject matter and their students' learning needs (Mason, 2018; Smith & Neale, 2017). Furthermore, the development of PCK is influenced by teaching experience, with studies showing that novice teachers often struggle to integrate these components effectively compared to their more experienced counterparts (Evens et al., 2018). The urgency of assessing PCK in relation to teaching effectiveness cannot be overstated. The application of PCK directly correlates with student learning outcomes, making it essential for teacher education programs to emphasize the development of PCK among future educators (Koopman et al., 2019; Smith et al., 2017). This focus on PCK not only prepares teachers to deliver content effectively but also equips them with the skills necessary to foster a supportive learning environment that acknowledges and addresses the diverse needs of their students (Donnelly & Berry, 2019).

The most influential PCK in this study is the KCT section, namely how the teacher conveys the material to students. The teacher's strategy still needs to be revised because the teacher's level of understanding still needs to be higher. The mathematics teaching techniques and strategies presented by the teacher needed to be more appropriate. Teachers who do not have content knowledge will have difficulty teaching knowledge to students (Klemer et al., 2019; Özpınar & Arslan, 2022). This is very consistent with the results of this study, which state that content knowledge is needed to match a teacher's pedagogy.

Teachers who have the characteristics of teaching successful mathematics understand the material, can teach with the right technique, and apply discipline (M. Park & Kim, 2023). The results of this study show these successful characteristics have yet to appear in Indonesia. Teachers still need to implement disciplined learning techniques for students. The use of problem-solving techniques presented could be more appropriate and suitable. There are still limitations to how teachers think and how to present solutions to students. The lack of teacher content knowledge is the most important obstacle in this study.

Research findings that are no less interesting are that of the 77 teachers who received the questions, only 27 teachers worked on them. The 50 teachers who had been waiting to work on and were contacted one by one to submit still need to show that the commitment of teachers in this border area still needs to be higher. The quality of education is heavily affected by teaching competence, learning innovation, and teacher commitment (Asiyah et al., 2021). Many teachers do not want to work on the problem and do not provide information. Every effort was made, for example, asking the class leader to take the questions directly to the house and accompanying them to work on them. However, they refused to do it without giving any reason. The commitment of teachers who have no desire to improve is a big challenge for developing the quality of teacher resources in border areas.

Teacher Errors in Answering Questions

Various findings in this study are unique. There is a question about proving the results of dividing fractions with media images. All teachers answered the question incorrectly. This error arises because the teacher needs more content knowledge and critical thinking. Mistakes often occur when teachers need to think critically in mathematics, especially because they lack knowledge and motivation (Novitasari et al., 2024; Ridwan et al., 2022). This is due to various errors caused by the teacher's lack of mathematics knowledge.

The teachers under study stated that mathematics was difficult for them to understand and teach. This finding is in line with previous studies showing that mathematics is challenging both to learn and to teach (Copur-Gencturk et al., 2019). This follows various interview results showing that teachers struggle to learn mathematics. Various errors in calculations occur because of views on mathematics. In the previous discussion, fractions were found to be the most feared math problem by teachers and students. Fractions are considered difficult math material (Empson et al., 2020). This result is consistent with the finding that fractional questions were answered by most of the teachers wrongly, and there were various unexpected answers to the questions.

The teachers' unexpected responses indicated that their ways of thinking about mathematics, sense of mathematics, and overall understanding of mathematical concepts still need to be improved (Greenstein & Fernández, 2023; Isnani et al., 2019). The teachers' thinking about mathematics is very far from the mathematics concept. For example, when asked how to apply realistic mathematics education media (question 1b), several teachers explained topics related to the scope of learning in elementary schools rather than describing how the media could be used.

The teachers' sense of mathematics here still needs to improve. The sense here is interpreted as how the teacher understands the content of mathematics and uses it in solving problems. They still need to work on problems correctly, for example, in algebra questions and examples of algebra in learning. Many teachers cannot provide examples of algebra material in elementary school learning. Their level of understanding when working on questions needs to be improved. Various inappropriate answers arise because they need to understand the intent of the questions properly.

Teachers find it difficult to teach geometry material because of its broad scope. Teachers need to gain more content knowledge of geometry, and this is illustrated in the geometry problem material in questions 2a and 2b. Their pedagogical content helps students learn geometry because it can reduce misconceptions and increase student learning motivation. In this discussion, the teachers must be aware of geometry material. They need help understanding the questions' meaning and the geometry material's scope. They cannot distinguish flat and geometric shapes and answer with answers far from the question. In the division material in questions 6a and 6b, the teachers experienced difficulties because they needed help to carry out the division operation. They have limitations that can only do divisions 1–10. In the context of tiered division, several teachers got the answer right. However, they needed help in teaching division techniques to students.

Distribution remains a serious challenge for students and teachers, who still require guidance in carrying out division and understanding mathematical operations related to comprehension. Students need help to divide and read comprehension operations (Fuchs & Malone, 2021). In previous research, students found it difficult, whereas in this study, the teachers found it difficult to divide numbers into hundreds. The teachers felt that they needed help to carry out the distribution process. The tool that is often used is a calculator. In every discussion of the problem, there is an interesting condition in the teachers. Many teachers have not yet memorized the basic multiplication facts from 1 to 10. This lack of automatic recall makes it difficult for them to carry out division operations, as division relies heavily on the understanding and quick retrieval of multiplication relationships.

The knowledge of mathematics content of the teachers at the border still has various problems in the field. The basic abilities that teachers should have become the main problem for them. This result is in line with previous research by Lo (2020), who found that only 6% of teachers correctly answered the mathematics questions in his reseach. The questions presented are about solving problems and presenting solutions. This finding reflects that the teachers' content knowledge and problem-solving skills still need improvement, which significantly impacts the quality of educational resources, particularly in border areas.

The findings of the study on Pedagogical Content Knowledge (PCK) underscore its critical importance in shaping effective teaching practices and, consequently, student learning outcomes. The evidence suggests that teachers with higher levels of PCK are more capable of implementing inquiry-based and student-centered teaching strategies, which are essential for fostering deeper understanding and engagement among students (Krijan et al., 2018). This is particularly relevant in

mathematics and science education, where the complexity of concepts necessitates a nuanced approach to teaching that considers both the content and the learners' cognitive processes (S. Park et al., 2010). The study's findings highlight that strong PCK enables teachers to not only convey content effectively but also to adapt their teaching methods based on their understanding of students' prior knowledge and learning difficulties. This adaptability is crucial in addressing the diverse needs of students, particularly in heterogeneous classrooms where learners may have varying levels of readiness and understanding.

For instance, teachers who possess a robust understanding of how students' reason through mathematical concepts can better anticipate misconceptions and tailor their instructional strategies, accordingly, leading to improved student outcomes. Moreover, the implications of the study extend beyond individual classrooms. By demonstrating the link between PCK and student achievement, the findings advocate for the integration of PCK development into teacher education programs. This integration is essential for preparing future educators to meet the demands of modern classrooms and to implement effective teaching practices that promote student learning (Sun et al., 2024). Continuous professional development focused on enhancing PCK can lead to sustained improvements in teaching quality and student performance over time.

CONCLUSION

Teachers' mastery of mathematics teaching strategies in elementary schools is viewed in terms of PCK with three domains: KCS, KCT, and KCC. In the KCS domain, the identification and anticipation of misconceptions in students' answers are low. Various answers given by teachers do not follow the questions and concepts given. In the KCT domain, namely regarding teacher strategies, methods, techniques, and pedagogy in teaching, the teachers are in a low category. The indicator is that the teacher cannot teach students learning strategies for mastering mathematics. The teachers do not have self-concepts and solutions to bridge students in several materials, including algebra, fractions, and number operations. In the KCC domain, teachers are in the lowest category regarding classroom management and the use of technology in learning activities. However, in this domain, based on their experience, the teachers can apply objects around them as learning media.

The outcome of the research reflects the circumstances of elementary school teachers in border areas. The teachers' PCK is significantly influenced by their mentality, environmental conditions, commitment, and ability. The teachers are still unable to accurately interpret and comprehend the queries. Their capacity to interpret inquiries is quite unanticipated, as they provide a variety of responses that are inconsistent with the context of the inquiries. The knowledge and understanding of the mathematical concepts of teachers at the border are still mostly inaccurate. Their low content knowledge makes their pedagogy in determining mathematics teaching strategies inappropriate. Human resource development in border areas is urgently needed for elementary school teachers. Further research on human development is urgently needed, but what is no less important is to explore in depth the reasons and main factors for fatal errors in teachers' thinking and problem-solving.

To enhance student achievement in mathematics, improving teachers' Pedagogical Content Knowledge (PCK) is crucial for strengthening classroom instruction. Professional development programs should focus on refining teachers' conceptual understanding and pedagogical strategies to ensure effective mathematics teaching. Training should include workshops on teaching mathematics strategies to help teachers select and implement appropriate instructional methods, improving their Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS). Additionally, curriculum-based training is essential to align classroom content with educational standards, ensuring students receive structured and coherent instruction. Integrating technology can further support conceptual understanding through interactive digital tools, enhancing engagement with mathematical concepts. By closing these knowledge gaps, teachers can better facilitate student-centered learning, address misconceptions, and create a more effective classroom environment for mathematics instruction. Future research should explore how strengthening PCK influences student learning outcomes, particularly in problem-solving, conceptual mastery, and engagement in mathematical reasoning.

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APPENDIX

Test Instrument

A six-question open-ended test assessed three Pedagogical Content Knowledge (PCK) domains:

a. KCC (1 question): Curriculum knowledge and learning media.
b. KCS (2 questions): Student difficulties and teaching approaches.
c. KCT (3 questions): Teaching strategies, problem-solving, and algebra concepts.

Each correct response earned 5 points, with a total possible score of 60 points.

Category	Item	Questions
KCC	1a	What is the learning media for fraction operation?
	1b	How do you apply realistic mathematics?
KCS	2a	How important is it to teach geometry concepts by linking contextual problems?
	2b	Write down one way to teach geometry to grade 6 students!

KCT	3a	Explain whether algebraic material is in elementary school!
	3b	Give one example of algebraic material for lower-grade students!
KCT	4a	$\frac{1}{3} : \frac{1}{6} = 2$ Prove with a picture!
	4b	Make a simple plan on how to teach the material!
KCT	5a	(-5) - (-9) = 4 Explain the correct solution to teach the problem!
	5b	Should the teacher use media for this solution? Explain!
KCS	6a	125 : 5 = ... What is the concept of teaching division operations to minimize student errors?
	6b	Write down an alternative to teaching this material!

Interview Instrument

Interviews explored teachers' decision-making in mathematics instruction:

a. KCS (2 questions): Student learning challenges and adaptation strategies.
b. KCT (4 questions): Differentiated instruction, motivation, and problem-solving.
c. KCC (1 question): Use of technology in teaching.

Category	Domain
(KCS)	More studies concerning learning difficulties in mathematics More skills concerned with teaching students at different levels
(KCT)	More courses about didactic mathematics More courses about how to differentiate teaching More skills to motivate students in mathematics More studies about problem-solving
(KCC)	More courses about using technology in teaching mathematics

Table 2: The instrument PCK Domain

Observation Instrument

a. Mathematical ability (clarity and accuracy of explanations).
b. Teaching methods (strategies and instructional effectiveness).
c. Classroom management (organization and student engagement).

A structured rubric ensured validity, and inter-rater reliability was applied.
This appendix provides a summary of the study's data collection process.