Exploration of Primary School Students' Metacognitive Thinking Processes in Solving Problems

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Abstract: This study aims to investigate the thinking process of primary school students when solving mathematical problems. It used a qualitative case study approach. The researcher administered a metacognitive test to 30 fifth-grade students in Yogyakarta and selected three focal participants based on their performance in planning, monitoring, and evaluating metacognitive problem-solving indicators. Data were collected through students' metacognitive problem-solving tasks and interviews, employing mathematical metacognitive capability tests and semi-structured interview guidelines. After students completed the given problems, cognitive interviews were conducted to capture their thinking processes during problem-solving. These interviews were conducted individually, involving only the researcher and the participant. The researcher employed cross-sectional data analysis and focus group discussions (FGDs) to ensure data validity. The results indicate that students exhibit two distinct thinking processes when solving complex mathematical problems. Students undergo metacognitive thinking processes in problem-solving, which can result in either correct or incorrect answers. Students with high metacognitive abilities demonstrate effective use of metacognitive skills, as evidenced by clear and structured problem-solving strategies. Conversely, those with low metacognitive abilities struggle to apply these skills effectively.

Keywords: metacognitive, problem-solving, indicator metacognitive, thinking process, mathematics

INTRODUCTION

Mathematics is fundamental in primary education and the foundation of scientific disciplines. It cultivates creative, logical, rational, critical, and systematic thinking through interconnected concepts. As a core curriculum component, mathematics enhances students' ability to analyze phenomena logically and achieve their educational goals (Bature & Atweh, 2016; Milovanovic et al., 2013; Mwakapenda, 2004). According to the 2018 PISA, Indonesia ranked 72nd out of 78 countries, indicating a weak mathematical performance. Similarly, the 2022 results indicate that Indonesian students perform significantly below average in reading, mathematics, and science (OECD, 2023).

Mathematics is fundamental across scientific disciplines, fostering creativity, logic, reasoning, critical thinking, and analytical skills (Skemp, 1971; Suneetha et al., 2011). However, the mathematics study often misrepresents problem-solving methods, a phenomenon known as cognitive conflict (Madu & Orji, 2015; Obreque & Andalón, 2021; Radovic S. & Pampaka, 2023). The Fourth Industrial Revolution and 21st-century demands require skill development within a new framework (Siregar et al., 2020). Technological advancements necessitate adapting cognitive processes to evolving complexities. Children exhibit diverse traits, including difficulties in mathematics, which may indicate cognitive limitations.

Engaging in metacognitive thinking processes enhances students' metacognitive skills, enabling faster information acquisition and retention, which supports their academic and professional growth (Schunk, 2012). The importance of metacognition is particularly pronounced among primary students (Bogdanović et al., 2015; Hoffman & Spatariu, 2008). Metacognitive capacity is essential in learning, allowing students to regulate cognitive processes, refine strategies, and identify strengths and areas for improvement (Bagci & Unveren, 2020). All learners can engage in metacognition, enabling them to reflect on their learning, reading, and skill development. Metacognition includes key skills for education and professional settings. Objective assessment measures are crucial for effective diagnosis and intervention (Castillo Diaz & Gomes, 2021).

Student metacognition remains underdeveloped, especially in mathematics education. Key components are required to strengthen cognitive skills and academic performance (Branigan, 2019; Tandean et al., 2020; Yilmaz et al., 2010). Metacognitive proficiency is a key predictor of academic success, fostering learning and advanced cognitive abilities (Kent, 2022). Metacognition is a higher-order thinking skill involving planning, monitoring, and evaluating of cognitive processes (Flavell, 1976; Karaali, 2015). Metacognition involves three key activities: planning, monitoring, and evaluation (Bakkaloglu, 2020; Gusmão & Moll, 2022). Table 1 below presents the result of studies on low metacognitive abilities in Indonesia.

Studies	Findings	Evidence
Anjelina et al., (2020)	Most students achieved the planning indicator of metacognitive abilities but not for monitoring and evaluation.	A descriptive analysis of 28 students in Banda Aceh showed partial achievement in metacognitive indicators.
OECD (2023)	Metacognition positively influences mathematics achievement, but overall achievement remains low	Analysis of the 2022 PISA data indicated that higher metacognition correlates with better math achievement.
Aminah et al., (2018)	Metacognitive teaching-learning (MTL) improved self-regulated learning but insignificant in logical thinking.	Quasi-experiment with 7- tenth-grade students in Sumedang showed low scores in logical thinking despite MTL.
Supriatna et al., (2019)	Higher learning methods improved problem-solving skills, but overall skills remain low.	ANOVA analysis of the fifth-grade students in West Kalimantan indicated better skills with advanced methods

Table 1: Study data on metacognitive abilities in Indonesia.

Table 1 indicates that students' metacognitive skills in mathematical problem-solving remain low. Therefore, these skills should be improved early in primary school to help students increase their problem-solving skills, academic achievement, critical thinking skills, and self-confidence. Research shows that metacognitive abilities develop with age and are shaped by multiple factors (Kennedy et al., 2008). Metacognition typically emerges around age eight, aligning with elementary school years (Kent, 2022; Tandean et al., 2020). Introducing metacognitive concepts early is vital for effective knowledge management. Therefore, exploring methods to develop children's metacognitive skills in early self-directed learning is essential (Khan, 2015). Exploring strategies to foster metacognitive approaches in elementary students is essential (Duman & Semerci, 2019). Early engagement in metacognitive activities enhances students' metacognitive skills. Consequently, many developing countries, including Indonesia, have integrated metacognitive elements into competency standards for elementary graduates. Student awareness progresses through four levels: tactical, aware, strategic, and perceptive (Schraw et al., 2006). As a core subject, mathematics fosters higher-order thinking by promoting cognitive regulation and reasoning (Kennedy et al., 2008; Khan, 2015).

Previous research, especially in Indonesia, indicates that learners' abilities remain insufficient (Hastuti et al., 2020). Even among highly capable elementary school educators, performance remains below standard (Wilhelmi et al., 2021). This gap is significant, as metacognitive activity is a key indicator of cognitive development and academic motivation (Schneider & Artelt, 2010). Insufficient metacognitive skills in early education can negatively impact students' abilities in higher education, making early intervention essential (Schneider & Lockl, 2002). Metacognition is crucial for mathematical problem-solving at all educational levels, and its absence hinders reasoning and cognitive processes. Research indicates that students often struggle to achieve desired success rates in problem-solving due to weak metacognitive skills (Alzahrani, 2017; Bakkaloglu, 2020; Özsoy & Ataman, 2009). The literature highlights metacognition as a critical factor in academic success (Desoete, 2001), with strong correlations between cognitive processes, problem-solving skills, and metacognition. Students with well-developed metacognitive abilities tend to perform better in problem-solving tasks (Özsoy & Ataman, 2009). Understanding cognitive processes enhances strategic thinking and academic performance.

Students' low metacognitive skills partly originate from teacher-centered learning activities emphasizing cognitive aspects over active engagement (Hastuti et al., 2020). Routine tasks that lack authentic problem-solving opportunities further hinder the development of higher-order thinking skills. Such approaches encourage passive learning, limiting students' metacognitive engagement (Veenman et al., 2006). As a result, students often struggle to reflect on and assess their cognitive understanding (Desoete, 2001). Metacognition is closely linked to problem-solving and is most evident when individuals face unfamiliar challenges, uncertainties, or dilemmas (Du Toit & Kotze, 2009).

These challenges are primarily classroom-related. Research shows that students across Indonesia continue to face educational disadvantages (Akshay et al., 2021). They encounter obstacles in learning, which can be both engaging and tedious, often due to difficulties in understanding essential concepts (Anintia et al., 2017). Additionally, limited prior knowledge contributes to low metacognitive capacity, poor information organization, uncertainty in strategy use, ineffective learning methods, and difficulty applying previously acquired knowledge (Shamir et al., 2008).

Students with low metacognitive abilities are often unable to solve mathematical problems effectively. This happened due to students being faced with difficult tasks and require critical thinking ability or unreadily available answers, they tend to be lazy about them. Even if students can work on the problems correctly, they fail to solve daily life problems (Martínez Zarzuelo et al., 2020; Peake et al., 2021). Students have difficulty understanding a problem, such as what is known, the underlying issues, and what strategies must be used to solve contextual problems (Dixon, 2017; Komalasari, 2009; Yansen et al., 2019). The student's lack of ability to monitor his or her learning purposes (Karakoc, 2016). Students never plan the time to be used to complete the tasks given by the teacher, so students feel the need for time.

Metacognitive development can enhance students' self-literacy, thereby increasing their motivation to learn (Bandura, 1997; Oliveira et al., 2022; Wang et al., 2017). A good study skill can become a thinking habit applied in a wide range of contexts (Pratiwi et al., 2019). Critical and reflective problem-solving and decision-making skills are essential for students, as they enhance self-regulation in the learning process, increase motivation, and support emotional management even in challenging situations. These competencies also enable students to navigate complex issues, resolve conflicts, and become active learners capable of thinking critically, reflecting deeply, solving problems effectively, and making informed decisions (Denizli & Erdoğan, 2018; Gambrill & Gibbs, 2009; Wallace, 2015). Research shows that good metacognition will enable students to solve problems appropriately (Adnyani, 2020).

Research indicates that metacognitive experience enhances students' understanding of knowledge. Students with strong metacognitive skills demonstrate superior critical thinking, problem-solving, and decision-making skills (Adnyani, 2020; Fajari et al., 2020). Individuals with more complex cognitive processes tend to possess stronger metacognitive skills (Duman & Semerci, 2019). Students who receive metacognitive instruction learn more effectively than those who do not (O'Neil & Brown, 1997). Additionally, metacognitive abilities help students recognize and correct mistakes, evaluate their work, and assess the effectiveness of different learning strategies.

Metacognitive thinking enhances learning ability and influences task completion in the learning process (Cañada & Arumí, 2012). It also impacts students' academic performance. Therefore, integrating metacognitive skills into classroom activities and curricula at all educational levels is essential (Farida Jamil et al., 2023). Metacognitive skills can be taught and developed. This study examines elementary school students' thinking processes in mathematical problem-solving and project-based learning.

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+-----------------+      +-----------------+      +-----------------+
|    Analyzing    | ---> |   Identifying   | ---> |   Evaluating    |
+-----------------+      +-----------------+      +-----------------+
       |                        |                        |
       v                        v                        v
+-----------------+      +-----------------+      +-----------------+
|    Planning     | ---> |   Monitoring   | ---> |   Evaluation    |
+-----------------+      +-----------------+      +-----------------+
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Figure 1: Diagram of the Metacognitive Thinking Trajectory

The metacognitive thinking trajectory in mathematical problem-solving involves a series of interconnected cognitive processes that enhance reasoning and self-regulation. This trajectory begins with a thorough problem analysis, followed by identifying relevant concepts and strategies (Flavell, 1979). In the evaluation phase, students critically assess the feasibility and accuracy of their approaches before applying them. Next, they engage in a structured planning phase, outlining their steps toward a solution, and then moving to the monitoring phase, where they track their progress and make necessary adjustments. Finally, during the evaluation stage, students carefully review the entire process to ensure accuracy and effectiveness. This systematic approach enhances cognitive regulation and problem-solving skills while fostering critical thinking, ultimately promoting independent learning in mathematics. The selection of this context is based on results of FGD with primary students in Yogyakarta to explore their thinking processes and metacognition. Considering primary school students' low metacognitive ability to solve math problems, this study aims to investigate their thinking processes when solving mathematical problems. The research question is: How does the exploration of primary school students' metacognitive thinking process in solving problems?

THEORETICAL FRAMEWORK

Metacognition

Metacognition involves planning, monitoring, and reflecting on problem-solving processes (Alzahrani, 2017). It encompasses knowledge, experiences, goals, and strategies (Flavell, 1979) enabling individuals to reflect on, understand, and regulate their learning (Cahayasti & Indrasari, 2018). Metacognition includes self-reflection on strengths, weaknesses, and learning strategies (Anggo et al., 2015). It is a cognitive tool that enhances problem-solving, decision-making, critical thinking, and creativity, ultimately improving learning outcomes (Hendriana et al., 2020; Karakoc, 2016; Wallace, 2015). Closely linked to cognitive control and higher-order thinking, metacognition fosters self-regulated learning (Prabawanto, 2019). It allows individuals to manage cognition effectively, assess challenges, evaluate their understanding, utilize information to achieve goals, and track progress (Schneider & Artelt, 2010b).

Metacognitive thinking ability primarily involves metacognitive regulation, which is essential for self-directed learning and influences actions during the learning process (Schraw et al., 2006; Whitebread et al., 2009). It encompasses planning, monitoring, and evaluation (Schraw et al., 2006), each with multiple components. Figure 2 provides a detailed overview of these aspects.

Figure 2. Metacognitive Thinking Process Scheme

	Description	Operational
Planning	A systematic preparation of activities that will be done to achieve a specific goal. Select the right strategy, the purpose, and the allocation time. Planning involves a strategic approach to the source of data location.	Planning is a way of thinking about problems. Planning is always future-oriented. Planning takes note of the relationship between achieving goals and decision-making, and planning presents comprehensive policies and programs.
Monitoring	A regular process of data collection and measurement of progress over the objective of the program and monitoring the changes that focus on the process and output. Students' acquired ability in carrying out an action. These abilities are periodic	Compliance monitoring works to ensure expectations/plans. The spectacle monitoring serves to assess the organization's progress based on the achievement of the expected target. Typically, the output of monitoring is a progress report. Such output is described in both descriptive and non-descriptive terms. The output monitoring is to know if the corresponding process is already in place. The output monitoring is useful in repairing the monitoring process/activity mechanisms.
Evaluation	Activities to gather and provide information for the decision-maker. Assessment of results and efficiency in a student's learning ability and the process of student learning. Conducting review and revise purposes.	Selection, collecting, analysis, and presentation of information that can be used as a basis for decision-making and subsequent compiling of programs

Table 2. Metacognitive Thinking Process

Measuring Metacognitive Thinking Capacity

Positive thinking capacity can be measured through tests, observations, and questionnaires, including reflective assessment, which evaluates metacognitive thinking ability. Reflective appraisal is a metacognitive process, and individuals with advanced reflective assessment skills tend to be better learners (Gravill et al., 2009; Van Opstal & Daubenmire, 2015). This formative approach places students at the center of assessment practice (Alzahrani, 2017). Reflective assessment is rooted in strong theoretical foundations of John Dewey's philosophy (Dewey, 1966) and cognitive constructivist learning theories. It promotes critical thinking and well-researched cognitive development (Johnson & Johnson, 2002; OECD, 2019; Wallace, 2015).

Metacognitive control is applying knowledge to regulate cognitive processes, aiding self-management and learning (Duman & Semerci, 2019). Metacognition is measured through three key indicators: planning, monitoring, and evaluation (Schraw et al., 2006). Planning involves setting learning objectives, considering learning strategies, and managing time effectively. It helps students anticipate challenges and develop result-oriented strategies (Veenman et al., 2006). Monitoring requires continuous awareness of comprehension and performance, including periodic self-assessments during learning (Schneider & Artelt, 2010b; Schraw et al., 2006). Evaluation enables students to assess their task performance, compare results with peers, and identify weaknesses in their problem-solving strategies (Schraw et al., 2006).

Experts reveal that planning process involves identifying key information and setting goals for problem-solving while understanding the necessary steps. The monitoring process includes accurately applying formulas, ensuring logical progression, and overseeing problem-solving steps. The evaluation process requires students to review their steps for consistency with the given information and verify their solutions.

Metacognition in Mathematical Problem-Solving

Metacognitive and problem-solving skills are essential for students to engage with mathematical advancements effectively. Problem-solving involves identifying and resolving issues, while metacognition enables awareness and regulation of cognitive processes (Scheibe et al., 2023). These skills enhance mathematical comprehension, logical reasoning, and critical thinking, supporting academic success and lifelong learning. This study explores metacognition's role in mathematical problem-solving, particularly in collaborative settings, highlighting its impact on cognitive regulation and learning strategies. It also examines the link between metacognitive strategies and problem-solving abilities, emphasizing the importance of problem-based learning. Further research is needed to examine this relationship more deeply and to enhance teaching methodologies in mathematics education.

Polya's four-stage problem-solving approach is understanding the problem, devising a plan, carrying out the plan, and looking back. Metacognition significantly enhances it, which fosters critical thinking and problem-solving abilities. Metacognitive strategies aid students in comprehending problems effectively, as research indicates that metacognitive frameworks improve understanding, planning, and overall problem-solving performance (Barokah et al., 2020; Lee et al., 2014). Developing a plan relies on metacognitive knowledge and self-regulation, helping students formulate effective strategies. During the execution phase, metacognitive monitoring ensures that students stay on track and adjust their approach when necessary. The last is reflection or looking back to consolidate learning, reinforce understanding, and enhance future problem-solving skills (Scheibe et al., 2023). In conclusion, integrating metacognitive strategies into

Pólya's model promotes self-awareness, strategic thinking, and adaptability, ultimately improving students' problem-solving abilities and deepening their learning experience.

Metacognition is crucial in enhancing students' problem-solving abilities by improving their awareness of cognitive processes. Research indicates that metacognitive instruction significantly boosts performance by helping students effectively plan, monitor, and evaluate their actions (Safari & Meskini, 2016; Zhao et al., 2019). Additionally, metacognition fosters self-regulated learning, enabling students to take control of their learning process, build confidence, and develop perseverance in problem-solving tasks (Fazira et al., 2021; Lee et al., 2014). Task-relevant monitoring further supports problem-solving by encouraging students to check their work and consider alternative strategies, leading to more accurate solutions (Scheibe et al., 2023). According to Pólya's problem-solving stages, one effective metacognitive framework is the Problem Wheel. This tool guides students in assessing their thinking throughout the problem-solving process by prompting self-reflective questioning, thereby enhancing their metacognitive skills.

METHOD

Research Design

This study aims to examine students' mathematical thinking processes. It employed a qualitative research design using a case study approach. Qualitative research is a method for understanding human or social issues by constructing a comprehensive and detailed narrative, presenting findings descriptively, gathering information from multiple sources, and conducting the study in a natural context (Creswell, 2014).

Sample and Data Collection

The study involved 30 primary school students in Yogyakarta, particularly the fifth-grade students. Four participants were selected based on their classification according to the correctness of their responses to planning, monitoring, and evaluation indicators. The researcher used a purposive sampling technique to select school samples by considering the accreditation, the number of students and teachers, and willingness to participate in the research. The students voluntarily participated in the study by signing informed consent and their information and responses were kept confidential.

Using a case study approach, data were collected through students' mathematical problem-solving tests and interviews. The research instruments included a mathematical metacognitive ability test and semi-structured cognitive interview guidelines. After students completed the given problems, cognitive interviews were conducted to capture their thinking processes during problem-solving. These interviews were conducted individually, involving only the student and the researcher.

The data were categorized based on three metacognitive indicators: planning, monitoring, and evaluation. From each category, one correct response was selected for further analysis. Students' focus was assessed using metacognitive indicators, as outlined in Table 3 below.

Process	Indicators
Planning	Identifying desired goals Important information to be used in problem solving Students understand the steps that will be done to solve the problem.
Monitoring	Students are able to write the formula correctly and precisely The final step made by student. Students control or monitor the completion steps of given information (known).
Evaluation	Students recheck the steps outlined whether consistent with the given information (known) or not. Verifying that what's been done is right

Source: Schraw et al., (2006)

Table 3: Metacognitive Indicators

Data Analysis

Data analysis was based on three aspects of metacognition: planning, monitoring, and evaluation. The study examined how students demonstrated these metacognitive skills: (1) planning, (2) monitoring, and (3) evaluation. Researchers investigated the impact of different learning approaches on higher-order thinking skills, including metacognition, by comparing the metacognitive abilities of elementary school students who learned through a guided learning model with those who followed a conventional learning model. The analysis focused on students' responses according to specific metacognitive indicators.

The interview data were analyzed using students' thinking process schematics to assess their metacognitive abilities and understand their thought processes while solving problems. The analysis categorized metacognitive thinking into three stages: planning, monitoring, and evaluation. To ensure the validity of the research data, several methods were employed: (1) focus group discussions (FGD) with university leaders to verify research findings, (2) cross-examination with each respondent to maintain accurate interview transcripts, and (3) a review of relevant literature to support the study.

The study included cognitive interviews with students to validate their problem-solving approaches. These interviews were conducted with representative participants to reflect the broader student population. The primary objective was to gain deeper insights into students' cognitive processes when solving mathematical problems. The final stage involved documentation, which was started with collecting students' preliminary academic data from class teachers before formal data collection. To ensure data validity, researchers employed multiple methods, including cross-sectional data analysis and FGDs. A cross-sectional analysis was conducted with class teachers to assess whether students' thinking aligned with their previous learning experiences. Meanwhile, FGDs with school principals, teachers, and mathematics educators were conducted to verify the appropriateness of research instruments and methodologies in addressing the study's objectives.

Data were analyzed qualitatively as proposed by Miles, Huberman, and Saldana which consisted of three stages: data reduction, data display, and conclusion (Miles et al., 2014). The analysis was carried out through interviews, observations, and literature studies. The analysis of interview data was based on Creswell & Creswell (2018) covering raw data, transcription, and interpretation. Then, didactical design validation analysis was conducted through a focus group discussion between teachers and researchers before and after the teaching experiment for further design improvement. The method of examining learning impediments that meet information alludes to the strategy (Creswell, 2013), beginning with raw information, translation, and interpretation. The validity test used source and technical triangulation. The validation of *metacognitive problem-solving tasks used an FGD between teachers and researchers.*

RESULTS

Researchers found that all 30 students employed various planning, monitoring, and evaluation strategies when solving mathematical problems. Their problem-solving performance is presented in Figure 3.

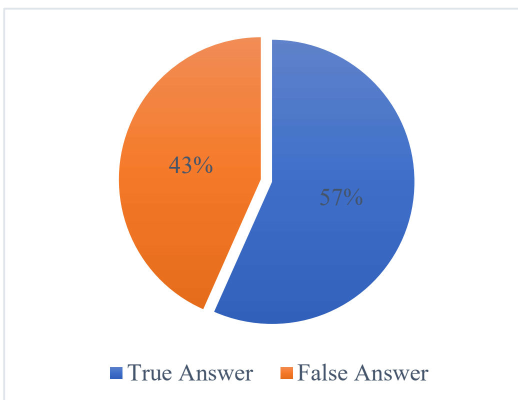


Figure 3. General Results

The results of the analysis indicate that students employ two types of thinking processes when solving complex mathematical problems. Among the 30 student responses, 17 (57%) provided correct answers, while the rest (43%) provided incorrect answers. All correct responses followed the structured planning, monitoring, and evaluation process.

Analysis of Correct Answers

Among the 30 students who participated in the mathematical metacognitive capability test, 17 provided correct answers, while the rest provided incorrect answers. Figure 4 presents an example of a student's correct response.

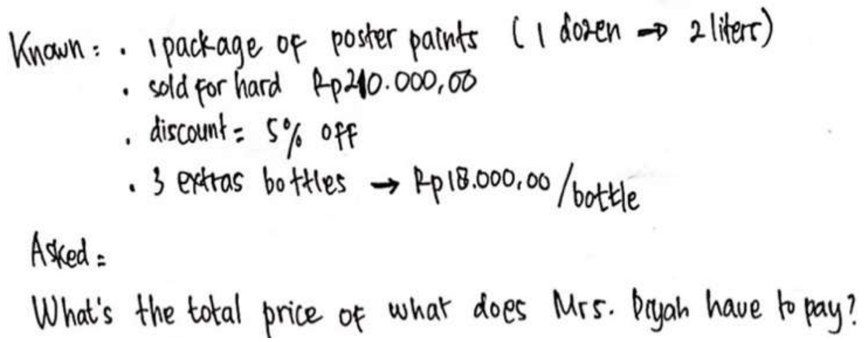


Figure 4. An Example of a Correct Answer

Figure 4 demonstrates that students with strong metacognitive abilities effectively apply metacognitive skills when solving problems. This is evident in their ability to clearly and accurately outline their problem-solving process. The figure above indicates that the student exhibits strong metacognitive skills. Students with high metacognitive skills systematically write down given information (known) and correctly state the problem (asked). Additionally, they use visual aids such as diagrams or annotations to enhance their understanding and facilitate problem-solving. Their solution plans are typically concise and often presented as formulas without detailed steps or justifications for their chosen concepts.

In terms of execution, these students approach problems systematically and accurately. Furthermore, they engage in self-verification by reviewing their answers to ensure correctness. Figure 5 illustrates the complete thought process behind a correct answer.

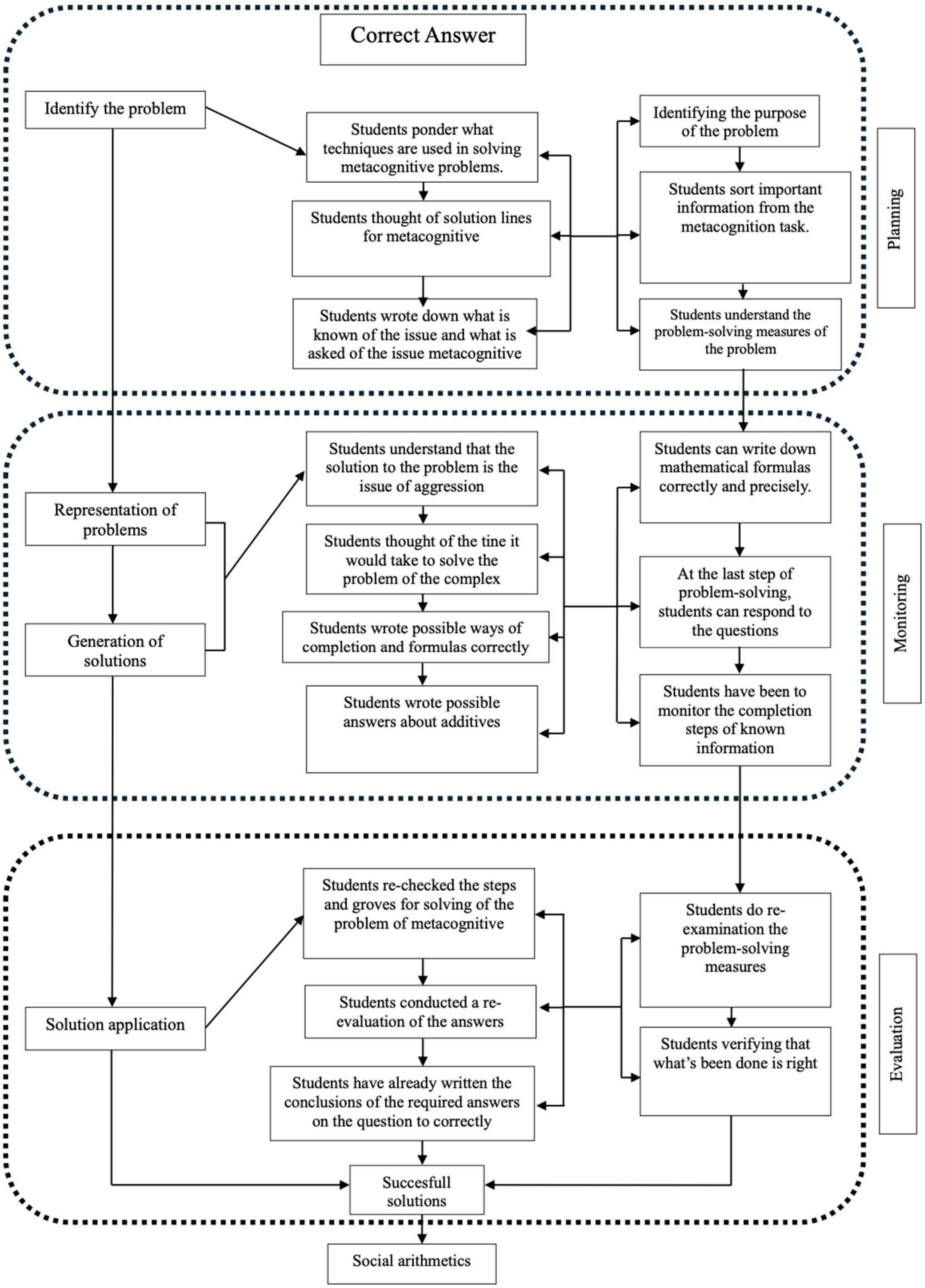
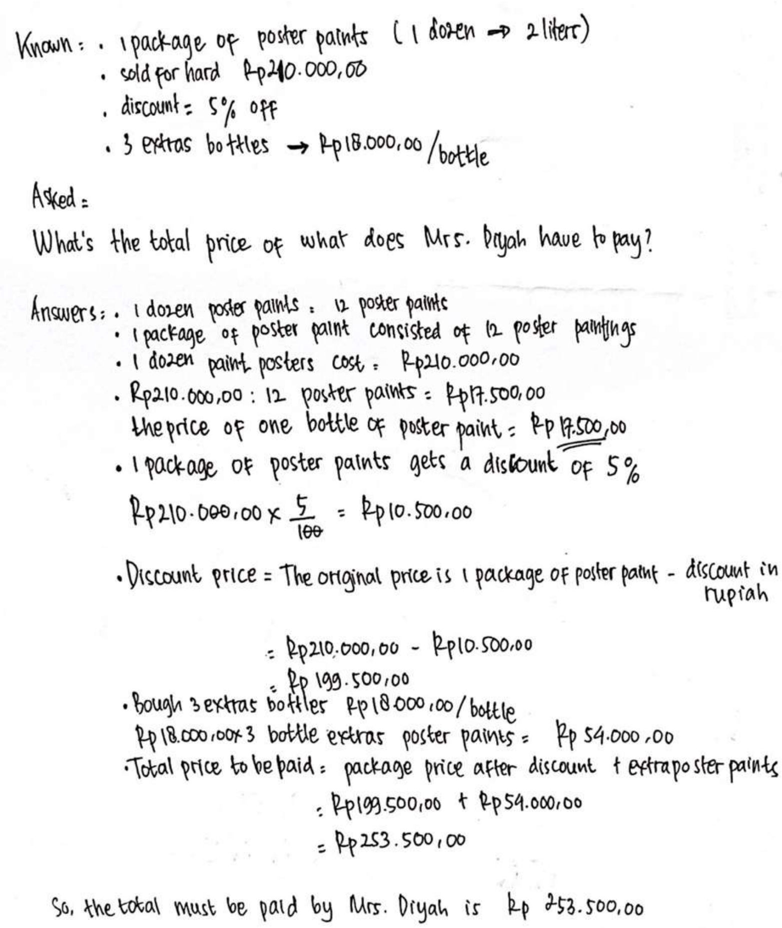


Figure 5. A Mathematics Metacognitive Scheme of the Correct Answer Analysis


Known : • 1 package of poster paints (1 dozen → 2 liters)
• sold for hard Rp210.000,00
• discount = 5% off
• 3 extras bottles → Rp18.000,00 /bottle

Asked :
What's the total price of what does Mrs. Diyah have to pay?

Answers : • 1 dozen poster paints = 12 poster paints
• 1 package of poster paint consisted of 12 poster paintings
• 1 dozen paint posters cost = Rp210.000,00
• Rp210.000,00 : 12 poster paints = Rp17.500,00
the price of one bottle of poster paint = Rp 17.500,00
• 1 package of poster paints gets a discount of 5%
Rp210.000,00 x 5/100 = Rp10.500,00
• Discount price = The original price is 1 package of poster paint - discount in rupiah
= Rp210.000,00 - Rp10.500,00
= Rp 199.500,00
• Bough 3 extras bottles Rp18.000,00 / bottle
Rp18.000,00 x 3 bottle extras poster paints = Rp 54.000,00
• Total price to be paid = package price after discount + extra poster paints
= Rp199.500,00 + Rp54.000,00
= Rp253.500,00

So, the total must be paid by Mrs. Diyah is Rp 253.500,00

Figure 6: The Complete Version of a Correct Answer

Figure 6 shows that students who provided correct answers have gone through the planning step. The student has done the identification phase. This can be seen from the following conversation transcript (R is a researcher, while S1 and S2 are students).

R: Can you understand the problem presented, and how do you analyze the problem?

S1: Yes, I understand the question. What I need to understand first is the purpose of the question, and classify the "given information (known), problem (asked), answer and so on". Then, write the answers in the available space. In the planning phase, the student first identified the given information, such as the cost of one package of poster paints, the discount percentage, and the price of additional bottles. The problem was then broken down into sequential steps to determine the total cost.

S1: Yes, and the student also established sub-goals, such as calculating the price of one bottle, applying the discount, and determining the cost of extra bottles. This demonstrates a clear problem-solving strategy before performing calculations.

R: Excellent observation. Now, let's discuss the monitoring phase. How do you ensure accuracy while solving the problem?

S2: I systematically monitored the calculations using proper mathematical operations. For instance, when applying the 5% discount. The discount was then subtracted from the original price to ensure the correct discounted amount. Additionally, when calculating the cost of extra bottles, I verified the multiplication:

R: That's a key metacognitive strategy—constantly checking one's work. In the evaluation phase, did you review and validate your final answer?

S2: Yes, after summing the discounted package price and the cost of extra bottles, the final total was clearly stated as Rp253,500.00. I also provided a concluding statement, reinforcing the accuracy of the solution. One improvement could be rechecking whether the initial assumption about the package price was correct. Meanwhile, I stated that one package costs Rp210,000.00 instead of Rp240,000.00 as initially given, this discrepancy should be reviewed for consistency.

This is seen from the student who wrote down the given information (known) and the question. When students write down the given information and the question, it indicates that they can identify the given information on the question. In this case, it is based on the student's analysis of answers. When students write down the given information and the question using the symbol, it indicates that they understand the given information, the problem, and the steps that will be taken to solve the problem.

Figure 7. Documentation of Thinking Processes of Students with Correct Answers

This section discusses student's identification and problem-solving processes in metacognitive tasks. Students have successfully identified the objective of the metacognitive problem. At this stage, their responses indicate that they can recognize and extract relevant information from the problem. They are first required to comprehend the metacognitive question presented in the scenario and determine the answer to the question: *"What is the total amount of money to be paid by Mrs. Diyah?"*

The students' responses demonstrate an understanding of the necessary steps to solve the problem. They can identify and categorize information as "known systematically*"* and *"asked."* Additionally, students show an awareness of the planning stages, which involve outlining the steps to be done after identifying the given information (*known*), understanding the problem statement (*asked*), solving the problem (*answer*), and drawing conclusions (*final result*).

This structured approach reflects students' ability to engage in metacognitive thinking, effectively plan their problem-solving process, and apply logical reasoning to reach a solution.

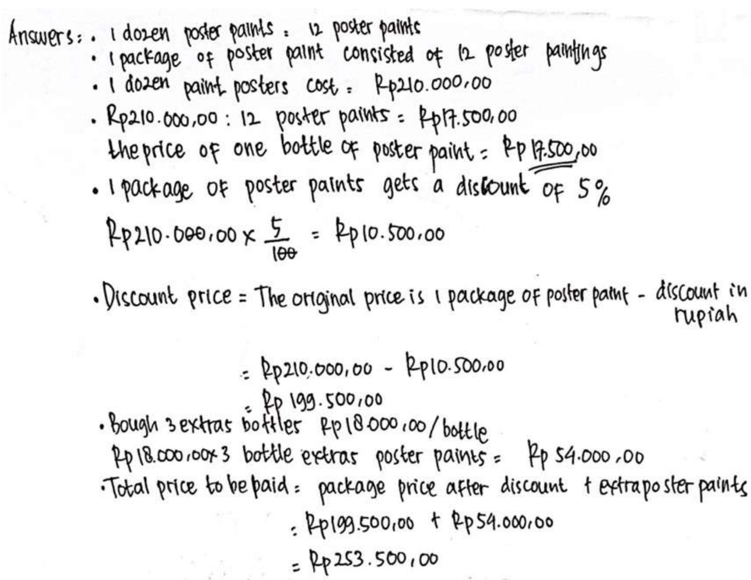


Figure 8: Monitoring Stage

Figure 8 shows student's attempts to solve mathematical metacognitive problems by answering and performing calculations. At this stage, they monitor, as evidenced by their efforts to write down possible solutions and relevant formulas. The formulas provided indicate that students can devise a structured solution plan. However, some students exhibit uncertainty despite correctly writing the formula, particularly in assigning values. For instance, one student notes that the price of a single unit is Rp210.000,00, but the notation appears unclear, suggesting possible confusion. This lack of precision may lead to errors in problem-solving.

Furthermore, the correct formatting of currency values should follow the General Guidelines for Indonesian Spelling (PUEBI), which specify that (1) the symbol Rp (Indonesian Rupiah) should be used to denote the currency; (2) no space should be placed between Rp and the numerical value; (3) a period (.) should be used to separate thousands, e.g., Rp18,000.00; (4) a comma (,) should be used as a decimal separator; (5) a slash (/) can be used as a unit separator, e.g., Rp210.000/pack. By adhering to these formatting rules, students can enhance the clarity and accuracy of their mathematical responses.

Based on the results of the students' mathematical metacognitive test, it was observed that in the written responses, they solved the addition problem using visual aids such as lines. This is evident in their answers, where they initially calculated the total number of poster paints in one dozen, determining that one packet contained 12 poster paints.

The results of the study:

Students correctly identified that one dozen poster paints cost Rp210,000.00. They then proceeded to calculate the price per unit: Rp210.000,00: 12 poster paints = Rp17.500,00. This calculation indicates that the price of one bottle of poster paint within the package is Rp17.500,00. Additionally, the problem states that each package of poster paints receives a 5% discount, which the students must also consider in their calculations.

Thus, $Rp210.000 \times \frac{5}{100} = Rp\ 10.500,00$

Students only write =

Discount price = The original price is 1 package of poster paints - discount in rupiah

= Rp210.000,00 – Rp10.500,00

= Rp199.500,00

Later, Mrs Diyah purchased three additional bottles at the regular price of Rp18.000,00 per bottle.The students then calculated the total cost accordingly,

Rp18,000,0000x 3 bottles of additional poster paint = Rp54.000,00

The next step requires students to calculate the total amount Mrs. Diyah needs to pay.

Total price to be paid = package price after discount + extra-posters paint

= Rp199.500,00 + Rp54.000,00

= Rp253.500,00

The student's response also demonstrates their approach to determining the price of three additional bottles. The correct price is Rp18.000,00 per bottle; however, the student wrote ",00" slightly lower than Rp18.000,00. During the interview, the student explained that they had mistakenly omitted ",00" due to its close placement near the multiplication symbol but had already accounted for it in their formula. In the context of numerical notation within Indonesia's financial system, the use of ",00" following a nominal value reflects the formal standard for representing amounts in the Indonesian currency, the rupiah. Additionally, in writing the equal sign (=), the student used dots, making it resemble a division symbol. Despite this, the student successfully followed the planned steps, as evident from their accurate mathematical calculations based on the previously established formula. The student's computations were correct, demonstrating their ability to implement the planning stage effectively. Moreover, students with high metacognitive skills understand the correct procedures for solving the problem. Their results are accurate, and they can clearly explain the strategies used to reach a solution.

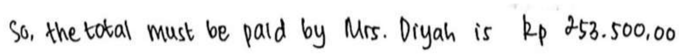

So, the total must be paid by Mrs. Diyah is Rp 253.500,00

Figure 9: Evaluation stage

The evaluation process consists of two key indicators: students reexamine whether their steps align with the given information and verify the accuracy of their solution. Based on Figure 9, students have demonstrated the ability to evaluate and review their answers. Students analyze, select, and present relevant information as a foundation for decision-making and further problem-solving. They have rechecked their steps in solving the metacognitive problem to ensure correct responses. Since the student's answers appeared fragmented, they modified their response to ensure accuracy. Moreover, students provided a conclusion by writing, *"So the total amount to be paid by Mrs. Diyah is Rp253.500,00."* However, the student made a minor formatting error by inserting an unnecessary space between "Rp" and "253.500,00," writing it as "Rp 253.500,00" instead of the correct format *"Rp253.500,00."* While this did not affect the numerical result, following the standard Indonesian writing conventions would be more appropriate. Furthermore, the student mixed their conclusion with the calculated result and the information in the problem. Overall, students managed to complete the evaluation phase by reviewing and verifying their solution.

Analysis of Incorrect Answers

Among the 30 students who participated in the mathematical metacognitive capability test, 17 provided correct answers, while the rest provided incorrect answers. Figure 10 presents an example of a student's incorrect response.

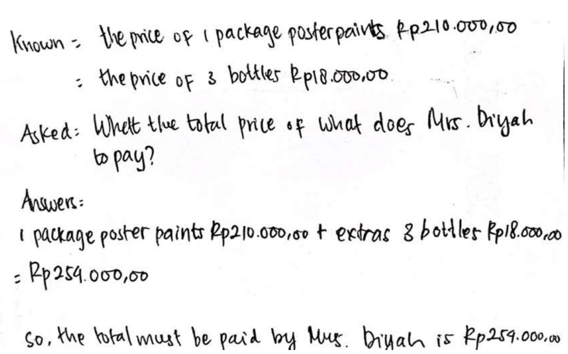

Known : the price of 1 package posterpaints Rp210.000,00
: the price of 3 bottles Rp18.000,00
Asked: What the total price of what does Mrs. Diyah to pay?
Answers:
1 package poster paints Rp210.000,00 + extras 3 bottles Rp18.000,00
= Rp259.000,00
So, the total must be paid by Mrs. Diyah is Rp259.000,00

Figure 10: Evaluation Stage

Almost all students with low metacognitive abilities do not use addition during problem-solving. This is evident in the students' vague work of writing out the problem-solving process. Examples of students' work with low metacognitive abilities can be seen from the following conversation transcript (R is a researcher, while S3 and S4 are students).

R: Can you understand the problem presented?
S3: I understood a little about it but was still confused about doing it. I could not sort out all the information and not write them all down.
R: How was the "asked" part for you to answer correctly?
S3: Since in the "asked" section, we only looked at the familiar issues by looking for a sentence using a question mark, it was the question.
R: How do you answer the problem of this mathematical metacognitive test? Do you know the formula?
S4: I answered it simply by adding up what I wrote in the 'known' section, and I was still confused about the formula when counting on the cost discount and the additional items.
R: Why?
S4: Because I feel rushed or overwhelmed. I do not write down the answers carefully and systematically. I do not have a habit of rechecking answers and I think this can be a contributing factor because I was often unaware of errors in calculations or explanations.

The complete thinking process of students with incorrect answers is presented in Figure 11.

Figure 11 shows some key factors contributing to students' lack of accuracy in solving metacognitive mathematical essay questions. One primary cause is their limited metacognitive ability to plan, monitor, and evaluate their thinking processes during problem-solving. Students who do not habitually reflect on their problem-solving steps often struggle to understand questions, select appropriate strategies, or include all necessary components in their answers. Additionally, the higher complexity of metacognitive problems compared to standard ones can cause students to feel rushed or overwhelmed, leading to imprecise and disorganized solutions. A lack of habit in reviewing answers further contributes to errors, as students frequently overlook miscalculations or inaccuracies in their reasoning. To improve accuracy in solving metacognitive mathematical essay problems, students should receive training in metacognitive skills, including strategic planning, monitoring their thought processes, and evaluating their responses before finalizing their answers.

Students with low metacognitive abilities recorded the given information and problem requirements but failed to develop a structured problem-solving plan. However, despite the lack of a clear plan, some students solved the problem systematically and accurately. Observations indicate that students who provided incorrect answers attempted the planning stage but did not execute it effectively. This is evident in their approach as they misidentified or inaccurately recorded key information and problem requirements. Additionally, they struggled with using appropriate mathematical symbols and failed to comprehend the necessary steps to solve the problem correctly and fully. A more in-depth analysis of their planning process is needed to understand better the challenges they face in structuring their problem-solving approach.

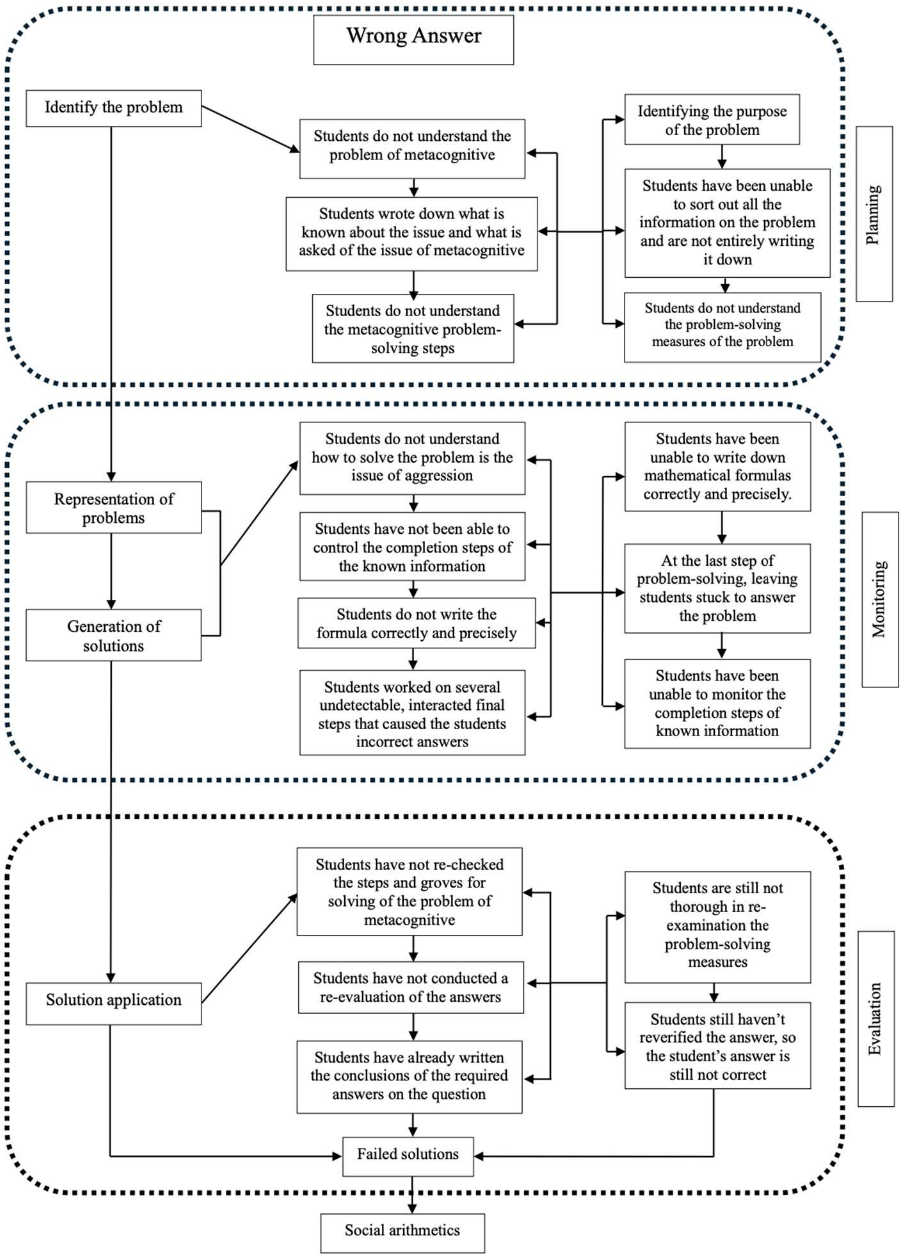


Figure 11: A Metacognitive Analysis Scheme of Incorrect Answers

Figure 12: Documentation of Thinking Processes of Students with Incorrect Answers

Figure 12 shows the documented response of a student who attempted to solve a mathematics problem but provided an incorrect solution. The student's approach to the metacognitive task was also flawed, indicating the absence of monitoring indicators in their problem-solving process. One of the monitoring indicators, as they failed to write the formula correctly and accurately. Specifically, the student wrote the number 1 in the section Rp210.000,00**,** while elsewhere, they wrote the number 4**,** creating ambiguity that could lead to confusion. Additionally, in their written response, the student demonstrated inconsistencies, which may indicate a lack of clarity or understanding in their problem-solving approach.

In the final step, students failed to verify their problem-solving process appropriately. They struggled to oversee the completion of the given information, as indicated by their prior use of the scaffolding technique, without careful attention to detail. This led to errors in solving the problem, preventing students from completing the given task successfully. One key mistake was calculating the discount for a package of poster paints. Students did not clearly outline the necessary steps, such as: (a) recognizing that one dozen consists of 12 bottles (indicating that Mrs. Diyah initially purchased one package containing 12 bottles); (b) identifying that one dozen poster paints cost Rp210.000,00.; (c) calculating the price per bottle as Rp210.000,00 ÷ 12 = Rp17.500,00; and (d) acknowledging that one package of poster paints receives a 5% discount.

The research findings:

The lack of clarity in these calculations suggests that students need further guidance in systematically applying problem-solving steps and verifying their solutions.

Thus, Rp 210.000 x $\frac{5}{100}$ *= Rp 10.500,00*

Students only wrote down the following information:

1 package of poster paints Rp210.00,00 + extras 3 bottles Rp18.000,00

= Rp254.000,00

Students failed to complete certain essential steps, leading to incorrect answers. They immediately stated that the price of one bottle of poster paint was Rp210.000,00 without accounting for the final step of applying the 5% discount to the total package price of Rp210.000,00. This suggests that students were unable to effectively control or monitor their problem-solving process, particularly in verifying and completing key calculations. As a result, they made errors in formulating their final answers to the metacognitive problem.

Students lacked precision in selecting formulas and monitoring their calculations, leading to errors in solving the metacognitive problem. Some students did not explicitly write the required formula in their responses. Instead, they stated the given information as follows: *"1 package of poster paints Rp210.000,00 + 3 additional bottles Rp18.000,00 = Rp254.000,00."* However, the student mistakenly wrote *"an additional 3 bottles of Rp18.000,00,"* suggesting a misunderstanding of the total cost calculation. The problem is that each additional bottle costs Rp18.000,00, but the student incorrectly presented the price for all three bottles as Rp18.000,00 instead of calculating the correct total. This error indicates that the student failed to fully process and apply the given information which leads to incorrect answers in the metacognitive problem.

The evaluation process includes two key indicators: (1) students reexamining whether their problem-solving steps align with the given information and (2) verifying the accuracy of their solution. Based on the study results, students who provided incorrect answers did not properly evaluate or review their responses. They failed to recheck the steps taken to solve the metacognitive problem, as their answers remained unverified and uncorrected. Additionally, students did not modify their responses to ensure accuracy. Despite this, the student wrote a conclusion stating, *"So the total cost is Rp254.000,00,"* although the answer was incorrect. This indicates a lack of thorough evaluation and self-monitoring in their problem-solving approach.

All student activities related to problem-solving in metacognitive abilities involve metacognitive regulatory components. These components are essential as they help students organize, control, connect, assess, and refine their thinking strategies throughout the problem-solving process. Ideally, these activities occur continuously and are closely linked to students' metacognitive knowledge during problem-solving. Metacognitive regulation consists of three key components: planning, monitoring, and evaluation.

Based on metacognitive regulation analysis, not all students effectively apply counter-regulation when solving problems. Students employ different strategies for monitoring and regulating their thinking, depending on their cognitive abilities. Research findings indicate that students with strong metacognitive skills effectively utilize these abilities in mathematical problem-solving, leading to better performance than their peers. Observations showed that students who apply metacognitive strategies successfully solve problems and achieve higher test scores. Additionally, they demonstrate the ability to systematically and accurately document problem-solving steps. This aligns with previous studies (Duman & Semerci, 2019 and Özsoy & Ataman, 2009) that students with high metacognitive abilities outperform those with lower metacognitive skills in mathematical problem-solving. Students who practice mindfulness experience lower levels of academic burnout due to increased self-awareness and a non-judgmental approach to challenges (Sapancı, 2023). Therefore, examining the mediating role of mindfulness in the relationship between dysfunctional metacognitions is essential for further understanding.

In mathematical problem-solving, metacognitive knowledge and metacognitive regulation are crucial in helping students analyze and resolve problems effectively. When faced with a problem, students recall and connect relevant material explanations to the given task. Their prior knowledge and experiences with similar problems further enhance their ability to understand and apply appropriate problem-solving strategies. Additionally, students' awareness of planning, monitoring, evaluating, and revising their thought processes helps minimize errors. By continuously monitoring and controlling their cognitive processes, students can identify mistakes and retrace their steps to correct them when encountering difficulties. This highlights the relationship between metacognitive knowledge and regulation in conflict resolution during problem-solving. Cognitive knowledge and cognitive regulation strongly correlate (Schraw et al., 2006). Therefore, metacognitive awareness and regulation significantly contribute to students' ability to develop independent learning skills.

DISCUSSION

Based on this analysis, metacognitive capability during problem-solving is directly associated with students' performance outcomes. Encouraging students to engage in problem-solving activities can enhance their metacognitive skills, ultimately fostering greater independence in learning. Additionally, metacognitive strategies improve students' comprehension (Wicaksono et al., 2021). This aligns with a previous study (Bogdanović et al., 2015) which shows that the more students understand their cognitive processes, the more effective their learning and academic achievement will be. Furthermore, a metacognitive approach enhances both learning outcomes and student assessment practices (Alzahrani, 2017; Shamir et al., 2008). A mathematical problem involving discounts and additional purchases requires students to utilize critical thinking and problem-solving skills. In metacognitive assessment, this problem is a tool to evaluate students' ability to plan, monitor, and evaluate their thought processes. A metacognitive test instrument for data analysis includes assessment tools that measure students' awareness and regulation of their cognitive processes, such as structured problem-solving tasks, think-aloud protocols, and reflective assessments. Students identify key information, track their calculations, and verify their solutions through this process. By integrating metacognitive test instruments, researchers can analyze students' problem-solving strategies, identify errors, and enhance their mathematical reasoning skills.

Strengthening metacognitive skills to develop students' competence in mathematical problem-solving is vital (Rey et al., 2024). This study found that high-achieving students effectively utilize advanced planning, monitoring, and evaluation skills. In contrast, students with moderate mathematical abilities do not yet fully utilize metacognitive planning and evaluation skills, though they demonstrate strong monitoring capabilities (Wakhata et al., 2023). Meanwhile, students with lower mathematical abilities struggle to apply effective planning, monitoring, and evaluation strategies. Research suggests that the metacognitive approach can enhance problem-solving skills through three key aspects: planning, monitoring, and evaluation, which collectively support the development of students' metacognitive abilities in mathematics (Thomas & Chantharanuwong, 2022). Therefore, continuous interventions to track students' progress in mathematics classes to enhance their problem-solving skills are important.

Metacognitive aspects, including metacognitive knowledge and self-regulation, enhance mathematical performance (Schneider & Artelt, 2010a). Metacognition helps students develop a structured approach to solving mathematical problems (Du Toit & Kotze, 2009). Additionally, encouraging students to engage in problem-solving can promote the use of written metacognitive strategies during the process (Farida Jamil et al., 2023).

Regularly applying metacognitive strategies helps students become more adept at identifying errors by revisiting initial steps, allowing them to correct mistakes and refine their approach. To develop effective and sustainable problem-solving skills, students must be aware of their mathematical knowledge and learn to plan, monitor, and regulate their cognitive processes. Therefore, mathematical understanding must be supported by metacognitive processes to ensure problem-solving quality (Yıldız & Öztürk,2023).

In this context, research highlights a strong relationship between metacognition and success in mathematics courses. Thus, teachers and students must possess strong knowledge of metacognitive strategies to facilitate self-directed learning. A pedagogical approach that integrates metacognition is essential in fostering students' awareness and control over their learning processes.

CONCLUSIONS

Based on the results of the analysis and discussions, metacognition emerges around the age of 8-10 and students apply all the planning, monitoring, and evaluating stages. Students who apply metacognitive closeness as an awareness of one's cognition or process in which one thinks about thinking to build a strategy to solve problems. The metacognitive strategy refers to raising awareness about the thinking process that needs to increase student metacognitive awareness. When this consciousness is realized, one can control his mind by planning, monitoring, evaluating, and judging his learning. Teachers can build student's consciousness so they know and realize their weaknesses and strengths, and even plan, monitor, and evaluate what they have done. A teacher needs to strategize in math so students can design, monitor, control, and evaluate their actions. This is evidenced by the fact that the teacher guides students in planning a study plan before attempting metacognitive test problems. This process involves reflective questioning about their strategies for understanding mathematical concepts and overcoming difficulties. Students must then document their study plans, selecting appropriate methods such as rereading, summarizing, or engaging in discussions. "How did you approach problem-solving in the metacognitive test?" The teacher provides guidance in revising these plans as needed. Through this approach, students are expected to assess the effectiveness of their learning strategies, identify weaknesses such as procrastination or ineffective methods, and recognize their strengths, such as improved comprehension through discussion or problem-solving exercises. In the monitoring stage, the teacher encourages students to assess their comprehension during the learning process by answering reflective questions and using a self-reflection checklist. This approach helps students identify learning difficulties, evaluate the effectiveness of their study strategies, and recognize the need for additional support, promoting a more self-regulated learning process. In the evaluation stage, the teacher guides students in reflecting on their learning outcomes and processes after completing assignments or tests. Students complete a reflection journal and engage in class discussions to assess their study strategies, problem-solving approaches, and overall learning experiences. This process helps students identify effective learning methods, understand their strengths and weaknesses, and develop strategies for improvement, ultimately fostering independent and effective learning.

Metacognitive abilities are strongly correlated with academic performance, particularly in problem-solving. Encouraging students to engage in structured problem-solving activities enhances their metacognitive skills, promoting independent learning and deeper comprehension. Research supports that increased metacognitive awareness leads to better learning outcomes and achievements. Furthermore, metacognitive strategies are essential in student learning and assessment, particularly in mathematical problem-solving. High-achieving students effectively utilize planning, monitoring, and evaluation skills, whereas lower-proficiency students face difficulties in these areas. This highlights the importance of metacognitive approaches in improving problem-solving abilities. Metacognitive interventions should be integrated into daily learning to optimize student success. Self-regulation and metacognitive knowledge significantly affect mathematical performance, helping students identify and correct errors systematically. Developing metacognitive awareness enables students to revise their problem-solving strategies, supporting long-term academic growth. Therefore, fostering metacognitive skills among both teachers and students is crucial for enhancing self-directed learning and overall educational success.

Furthermore, students can select a type of representation and modify it to be a more useful support in creating mathematical models. Students with high mathematical skills can use advanced planning, monitoring, and comprehensive skills and vice versa. Students with low mathematical skills cannot optimally utilize their skills in the planning and evaluation stages. However, they perform optimally in the monitoring stage. Moreover, they cannot use high-value planning, monitoring, and evaluation skills. It can be said that the metacognitive approach can solve the problem in the planning, monitoring, and evaluation stages.

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REFERENCES

[1] Adnyani, L. P. A. P. (2020). Applying cognitive conflict strategy to develop mathematical critical thinking ability and character of students. *Journal of Mathematics Education*.

[2] Akshay, C., Visagaperumal, D., & Chandy, V. (2021). Relevance of educational thoughts of Paulo Freire. *International Journal of Research Publication and Reviews*, *2*(8), 386–392.

[3] Alzahrani, K. S. (2017). *Metacognition and its role in mathematics learning: an exploration of the perceptions of a teacher and students in a secondary school*.

[4] Aminah, M., Kusumah, Y. S., Suryadi, D., & Sumarmo, U. (2018). The effect of metacognitive teaching and mathematical prior knowledge on mathematical logical thinking ability and self-regulated learning. *International Journal of Instruction*, *11*(3), 45–62. https://doi.org/10.12973/iji.2018.1134a

[5] Anggo, M., Ode, L., Jazuli, A., Eka, Y., & Kansil, Y. (2015). Metacognitive strategies on mathematics learning to improve student's environmental awareness. In *International Journal of* Education *and Research* (Vol. 3, Issue 4). www.ijern.com

[6] Anintia, R., Sadhu, S., & Annisa, D. (2017). Identify students' concept understanding using t-Tier Multiple Choice Questions (TTMCs) on stoichiometry. *International Journal of Science and Applied Science: Conference Series*, *2*(1), 308. https://doi.org/10.20961/ijsascs.v2i1.16734

[7] Anjelina, Z., Usman, & Marwan. (2020). Students' metacognitive abilities in solving derivative problems. *Journal of Physics: Conference Series*, *1460*(1). https://doi.org/10.1088/1742-6596/1460/1/012044
[8] Bagci, H., & Unveren, D. (2020). Investigation the relationship between metacognitive awareness of reading strategies and self-efficacy perception in reading comprehension in mother-tongue: Sample of 8th graders. *International Journal of Educational Methodology*, *6*(1), 83–98. https://doi.org/10.12973/ijem.6.1.83
[9] Bakkaloglu, S. (2020). Analysis of metacognitive awareness of primary and secondary school students in terms of some variables. *Journal of Education and Learning*, *9*(1), 156. https://doi.org/10.5539/jel.v9n1p156
[10] Bandura, A. (1997). *Self-efficacy - The Exercise of Control*. W.H. Freeman and Company.
[11] Barokah, I., Budiyono, B., & Saputro, D. R. S. (2020). Students metacognition in solving mathematical problems based on gender differences. *Journal of Physics: Conference Series*, *1469*(1). https://doi.org/10.1088/1742-6596/1469/1/012158
[12] Bature, I. J., & Atweh, B. (2016). Achieving quality mathematics classroom instruction through productive pedagogies. *International Journal of Educational Methodology*, *2*(1), 1–18. https://doi.org/10.12973/ijem.2.1.1
[13] Bloom, B. S. (1978). *Taxonomy of educational objectives (the clasification of educational goals) handbook 1 cognitive domain*. Longman.
[14] Bogdanović, I., Obadović, D. Ž., Cvjetićanin, S., Segedinac, M., & Budić, S. (2015). Students' metacognitive awareness and physics learning efficiency and correlation between them. *European Journal Of Physics Education*, *6*(2). https://doi.org/10.20308/ejpe.96231
[15] Branigan, H. E. (2019). *Exploring metacognition in primary school classrooms*. *April*.
[16] Cahayasti, I. T., & Indrasari, S. Y. (2018). *Metacognitive strategy on completion of mathematics word problems and mathematics achievement among 3 rd-grade elementary students*.
[17] Cañada, M. D., & Arumí, M. (2012). Self-regulating activity: Use of metacognitive guides in the interpreting classroom. *Educational Research and Evaluation*, *18*(3), 245–264. https://doi.org/10.1080/13803611.2012.661934
[18] Castillo Diaz, M. A., & Gomes, C. M. A. (2021). Presenting the meta-performance test, a metacognitive battery based on performance. *International Journal of Educational Methodology*, *7*(2), 289–303. https://doi.org/10.12973/ijem.7.2.289
[19] Creswell, John. W. (2013). *Research Design: Qualitative, Quantitative, and Mixed Methods Approches*. SAGE Publications, Inc .
[20] Creswell, John. W. (2014). *Research Design: Qualitative, Quantitative, and Mixed Methods Approches*. SAGE Publications, Inc. https://doi.org/10.1016/S0953-7562(10)80014-0
[21] Denizli, Z. A., & Erdoğan, A. (2018). Development of a three dimensional geometric thinking test for early graders. *Journal on Mathematics Education*, *9*(2), 213–226.

[22] Desoete, A. (2001). Off-line metacognition in children with mathematics learning disabilities. *Ghent, Faculty of Psychology and Educational Sciences, Ghent, Belgium*, *May*, 160. https://biblio.ugent.be/publication/522137
[23] Dewey, J. (1966). *Democracy and education*. The Free Press.
[24] Dixon, P. A. (2017). Extensive listening , teacher proficiency and 21 st century skills : interview with Dr Willy A. Renandya. *RELC Journal*. https://doi.org/10.1177/0033688217695658
[25] Du Toit, S., & Kotze, G. (2009). Metacognitive strategies in the teaching and learning of mathematics. *Pythagoras*, *70*, 57–67. https://doi.org/10.4102/pythagoras.v0i70.39
[26] Duman, B., & Semerci, Ç. (2019). The effect of a metacognition-based instructional practice on the metacognitive awareness of the prospective teachers. *Universal Journal of Educational Research*, *7*(3), 720–728. https://doi.org/10.13189/ujer.2019.070311
[27] Fajari, L. E. W., Sarwanto, & Chumdari. (2020). Improving elementary school ' s critical thinking skills through three different PBL-Assisted Learning Media Viewed from learning styles. *Journal of E-Learningg And Knowledge Society*, *16*(01), 55–64. https://doi.org/https://doi.org/10.20368/1971-8829/1135193 1.
[28] Farida Jamil, A., Yuli, T., Siswono, E., & Setianingsih, R. (2023). The Emergence and Form of Metacognitive Regulation: Case Study of More and Less Successful Outcome Groups in Solving Geometry Problems Collaboratively. *Mathematics Teaching Research Journal*, *15*(1).
[29] Fazira, S. K., Sukoriyanto, S., & Rahardjo, S. (2021). Identification of young students' metacognition in mathematical problem solving using the multi-method interview approach. *AIP Conference Proceedings*, *2330*. https://doi.org/10.1063/5.0043385
[30] Flavell, J. H. (1976). *Metacognitive aspects of problem solving* (In L. B. Resnick. The nature of intelligence, Ed.).
[31] Flavell, J. H. (1979). *Metacognituve and Cognitive Monitoring : a new area of cognitive developmental inquiry*. Americant pcychologist.
[32] Gambrill, E., & Gibbs, L. (2009). *Critical Thinking for Helping Professionals A Skolls Based Workbook*. Oxford University Press.
[33] Gravill, J. I., Compeau, D. R., & Marcolin, B. L. (2009). Metacognition and IT: the influence of self-efficacy and self-awareness. *Journal of Strength and Conditioning Research / National Strength & Conditioning Association*, *16*(4), 1042–1049.
[34] Gusmão, T. C. R. S., & Moll, V. F. (2022). Metacognitive analysis of a mathematics class on surface measurement. *Revista Latinoamericana de Investigacion En Matematica Educativa*, *25*(2), 169–196. https://doi.org/10.12802/relime.22.2522
[35] Hastuti, I. D., Surahmat, Sutarto, & Dafik. (2020). The effect of guided inquiry learning in improving metacognitive skill of elementary school students. *International Journal of Instruction*, *13*(4), 315–330. https://doi.org/10.29333/iji.2020.13420a
[36] Hendriana, H., Setiawan, W., & Aripin, U. (2020). The effectiveness of geogebra's metacognitive guidance to develop critical thinking skills. *International Journal on Emerging Mathematics Education*, *3*(2), 129. https://doi.org/10.12928/ijeme.v3i2.13806

[37] Hoffman, B., & Spatariu, A. (2008). The influence of self-efficacy and metacognitive prompting on math problem-solving efficiency. *Contemporary Educational Psychology*, *33*(4), 875–893. https://doi.org/10.1016/j.cedpsych.2007.07.002

[38] Johnson, D.W., & Johnson, R, T. (2002). *Meaningfull assessment: A manageable and cooperative process*. Allyn Bacon.

[39] Karaali, G. (2015). Metacognition in the Classroom: Motivation and Self- Awareness of Mathematics Learners. *Mathematics & Statistics Journals*, *April*, 37–41. https://doi.org/10.1080/10511970.2015.1027837

[40] Karakoc, M. (2016). The significance of critical thinking ability in terms of education. *International Journal of Humanities and Social Science Invention*, *6*(7), 81–84.

[41] Kennedy, L., M., Tipps, S., & Johnson, A. (2008). *Guinding children's learning of mathematics* (7th ed.). Thomson Hinger Education.

[42] Kent. (2022). *Metacognition in the Primary Classroom*. Educational Psychology Service.

[43] Khan, L. A. (2015). What is mathematics? *Mathematics, Poetry and Beauty*, *May*, 159–163. https://doi.org/10.1142/9789814602952_0028

[44] Komalasari, K. (2009). The effect of contextual learning in civic education on students' civic competence. *Journal of Social Sciences*, *5*(4), 261–270. https://doi.org/https://doi.org/10.3844/jssp.2009.261.270

[45] Lee, N. H., Yeo, D. J. S., & Hong, S. E. (2014). A metacognitive-based instruction for Primary Four students to approach non-routine mathematical word problems. *ZDM - International Journal on Mathematics Education*, *46*(3), 465–480. https://doi.org/10.1007/s11858-014-0599-6

[46] Madu, B. C., & Orji, E. (2015). Effects of cognitive conflict instructional strategy on students ' conceptual change in temperature and heat. *SAGE Opens*. https://doi.org/10.1177/2158244015594662

[47] Martínez Zarzuelo, A., Rodríguez Mantilla, J. M., Roanes Lozano, E., & Fernández Díaz, M. J. (2020). Effect of scratch in the learning of geometric concepts of future primary school teachers. *Revista Latinoamericana de Investigacion En Matematica Educativa*, *23*(3), 357–386. https://doi.org/10.12802/relime.20.2334

[48] Miles, Matthew. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative Data Analysis: A methods Sourcebook*. SAGE Publications. http://library1.nida.ac.th/termpaper6/sd/2554/19755.pdf

[49] Milovanovic, M., Obradovic, J., & Milajic, A. (2013). Application of interactive multimedia tools in teaching mathematics--Examples of Lessons from Geometry. *Turkish Online Journal of Educational Technology*, *12*(1), 19–31.

[50] Mwakapenda, W. (2004). Understanding student understanding in mathematics. *Pythagoras*, 28–35.

[51] Obreque, K. S., & Andalón, J. L. (2021). Teachers´ epistemology of school mathematical knowledge: a case study. *Revista Latinoamericana de Investigacion En Matematica Educativa*, *24*(2), 177–206. https://doi.org/10.12802/RELIME.21.2423

[52] OECD. (2019). *PISA 2018: Assessment and Analytical Framework*.

[53] OECD. (2023). *PISA 2022 Results*. https://oecdch.art/a40de1dbaf/C108.

[54] Oliveira, W., Hamari, J., Joaquim, S., Toda, A. M., Palomino, P. T., Vassileva, J., & Isotani, S. (2022). The effects of personalized gamification on students' flow experience, motivation, and enjoyment. *Smart Learning Environments*, *9*(1). https://doi.org/10.1186/s40561-022-00194-x

[55] O'Neil, Jr. H. F., & Brown, S. R. (1997). *Differential effect of question formats in math assessment on metacognition and affect*. CRESSTCSE University of California.

[56] Özsoy, G., & Ataman, A. (2009). The effect of metacognitive strategy training on mathematical problem solving achievement. In *International Electronic Journal of Elementary Education* (Vol. 1, Issue 2). www.iejee.com

[57] Peake, C., Alarcón, V., Herrera, V., & Morales, K. (2021). Development of early numerical ability: contributions from cognitive psychology to initial mathematics education. *Revista Latinoamericana de Investigacion En Matematica Educativa*, *24*(3), 299–326. https://doi.org/10.12802/RELIME.21.2433

[58] Prabawanto, S. (2019). Enhancement of students' mathematical communication under metacognitive scaffolding approach. *Infinity Journal*, *8*(2), 117. https://doi.org/10.22460/infinity.v8i2.p117-128

[59] Pratiwi, E., Nusantara, T., & Muksar, M. (2019). Students' thinking process when experiencing cognitive conflict. *International Journal of Innovation, Creativity and Change*. www.ijicc.net

[60] Radovic S., D., & Pampaka, M. (2023). Relationship between students' perceptions of mathematical teaching, sex and mathematical attitudes. *Revista Latinoamericana de Investigacion En Matematica Educativa*, *25*(3), 311–340. https://doi.org/10.12802/RELIME.22.2533

[61] Rey, J., Oficiar, G., Ibañez, E. D., & Pentang, J. T. (2024). Metacognitive awareness as a predictor of mathematical modeling competency among preservice elementary teachers. *International Journal of Educational Methodology*, *10*(2), 279–292. https://doi.org/10.12973/ijem.10.2.1079

[62] Safari, Y., & Meskini, H. (2016). The Effect of Metacognitive Instruction on Problem Solving Skills in Iranian Students of Health Sciences. *Global Journal of Health Science*, *8*(1), 150–156. https://doi.org/10.5539/gjhs.v8n1p150

[63] Sapancı, A. (2023). From metacognition to academic burnout in university students: The mediating role of mindfulness. *Journal of Pedagogical Research*, *7*(4), 356–368. https://doi.org/10.33902/JPR.202313708

[64] Scheibe, D. A., Was, C. A., Dunlosky, J., & Thompson, C. A. (2023). Metacognitive Cues, Working Memory, and Math Anxiety: The Regulated Attention in Mathematical Problem Solving (RAMPS) Framework. *Journal of Intelligence*, *11*(6). https://doi.org/10.3390/jintelligence11060117

[65] Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. *ZDM - International Journal on Mathematics Education*, *42*(2), 149–161. https://doi.org/10.1007/s11858-010-0240-2

[66] Schneider, W., & Lockl, K. (2002). *The development of metacognitive knowledge in children and adolescents* (In Perfect). University Press.

[67] Schraw, G., Crippen, K. J., & Hartley, K. (2006). Promoting self-regulation in science education: Metacognition as part of a broader perspective on learning. *Research in Science Education*, *36*(1–2), 111–139. https://doi.org/10.1007/s11165-005-3917-8

[68] Schunk, D. H. (2012). *Learning Theories An Educational Perspective* (6th ed.). Pearson Education, Inc.

[69] Shamir, A., Zion, M., & Levy, O. S. (2008). Peer tutoring, metacognitive processes and multimedia problem-based learning: The effect of mediation training on critical thinking. *Journal of Science Education and Technology*, *17*(4), 384–398. https://doi.org/10.1007/s10956-008-9108-4

[70] Sintema, E. J., & Mosimege, M. (2023). High School Students' Beliefs about Mathematical Problem Solving: A Cluster Analysis. *Mathematics Teaching Research Journal*, *15*(1).

[71] Siregar, Y. E. Y., Rahmawati, Y., & Suyono. (2020). Elementary school teacher's perspectives towards developing mathematics literacy through a STEAM-based approach to learning. *Journal of Physics: Conference Series*, *1460*(1), 012030. https://doi.org/10.1088/1742-6596/1460/1/012030

[72] Skemp, R. R. (1971). *The Psychology of Learning Mathematics*. Richard Clay (The Causer Press) Ltd.

[73] Suneetha, E., Rao, R. S., & Rao, D. B. (2011). *Methods of learning mathematics*. SAGE Publications.

[74] Supriatna, I., Asmahasanah, S., Rachmadtullah, R., Khair Asdar, A., Fahrudin, & Rasmitadila. (2019). The effect of learning methods and self regulation on problem-solving ability of mathematics in elementary school. *Journal of Physics: Conference Series*, *1175*(1). https://doi.org/10.1088/1742-6596/1175/1/012139

[75] Tandean, D., Sharon, M., & Scool, C. (2020). *Metacognitive strategies, reading comprehension, young learners*. 124–139.

[76] Thomas, G. P., & Chantharanuwong, W. (2022). Factor structure and dimensionality of an instrument designed to measure the metacognitive orientation of Thai science classroom learning environments. *International Journal of Educational Methodology*, *8*(4), 805–818. https://doi.org/10.12973/ijem.8.4.805

[77] Ulfa, M., & Purnamaningsih, I. R. (2022). Innovation of Learning Media Through Information and Communication Technology in the Revolution Era 4.0. *PROJECT : Professional Journal of English Education*, *5*(4), 885–890.

[78] Van Opstal, M. T., & Daubenmire, P. L. (2015). Extending students' practice of metacognitive regulation skills with the science writing heuristic. *International Journal of Science Education*, *37*(7), 1089–1112. https://doi.org/10.1080/09500693.2015.1019385

[79] Veenman, M. V. J., Van Hout-Wolters, B. H. A. M., & Afflerbach, P. (2006). Metacognition and learning: Conceptual and methodological considerations. *Metacognition and Learning*, *1*(1), 3–14. https://doi.org/10.1007/s11409-006-6893-0

[80] Wakhata, R., Balimuttajjo, S., & Mutarutinya, V. (2023). Building on Students' Prior Mathematical Thinking: Exploring Students' Reasoning Interpretation of Preconceptions in Learning Mathematics. *Mathematics Teaching Research Journal*, *15*(1).

[81] Wallace, E. D. (2015). Developing critical thinking skill: assessing the effectiveness of workbook exercise. *Journal of College Teaching & Learning*, *12*(56).

[82] Wang, C., Serena, S., & Christopher, S. (2017). Achievement goals , motivational self-talk , and academic engagement among Chinese students. *Asia Pacific Education Review*, *18*(3), 295–307. https://doi.org/10.1007/s12564-017-9495-4

[83] Whitebread, D., Coltman, P., Pasternak, D. P., Sangster, C., Grau, V., Bingham, S., Almeqdad, Q., & Demetriou, D. (2009). The development of two observational tools for assessing metacognition and self-regulated learning in young children. *Metacognition and Learning*, *4*(1), 63–85. https://doi.org/10.1007/s11409-008-9033-1

[84] Wicaksono, A. B., Suwandi, S., & Andayani, A. (2021). Application of heuristic methods in developing metacognitive knowledge in Indonesian language teaching materials for foreign speakers. *ACM International Conference Proceeding Series*, *553*(Iclp 2020), 54–58. https://doi.org/10.1145/3516875.3516989

[85] Wilhelmi, M. R., Godino, J. D., & Lacasta, E. (2021). Didactic effectiveness of mathematical definitions: The case of the absolute value. *International Electronic Journal of Mathematics Education*, *2*(2), 72–90. https://doi.org/10.29333/iejme/176

[86] Yansen, D., Ilma Indra Putri, R., & Fatimah, S. (2019). Developing pisa-like mathematics problems on uncertainty and data using asian games football context. *Journal on Mathematics Education*, *10*(1), 37–46.

[87] Yilmaz, Ç., Altun, S. A., & Olkun, S. (2010). Factors affecting students' attidude towards Math: ABC theory and its reflection on practice. *Procedia - Social and Behavioral Sciences*, *2*(2), 4502–4506. https://doi.org/10.1016/j.sbspro.2010.03.720

[88] Yıldız, Kübra. A., & Öztürk, Mesut. (2023). Relationship between middle school students metacognition and problem-posing performance: A serial mediation analysis. *Journal of Pedagogical Research*. https://doi.org/10.33902/jpr.202323665

[89] Zhao, N., Teng, X., Li, W., Li, Y., Wang, S., Wen, H., & Yi, M. (2019). A path model for metacognition and its relation to problem-solving strategies and achievement for different tasks. *ZDM - Mathematics Education*, *51*(4), 641–653. https://doi.org/10.1007/s11858-019-01067-3

APPENDIX

Metacognitive Definition	Synthesis Result	Observed Variable	Measurable Variable	Indicators	Number of Task
Metacognition would improve the student's metacognitive skills. Metacognitive skills help students learn information quickly and information for educational or professional development of students (Schunk, 2012). Metacognition is important for primary students (Bogdanović et al., 2015; Hoffman & Spatariu, 2008). As can be seen, metacognitive capacity is important, especially in the learning environment, since students can manage their cognitive and learning processes and identify their strengths and the areas necessary to improve by developing new cognitive skills (Bagci & Unveren, 2020). Each learner can metacognition, which means each student can think about learning, reading, or displaying their skills. Castillo Diaz & Gomes (2021) state that metacognition incorporates a set of capacities with imperative applications within the instructive and proficient settings requested by today's society. Creating objective assessment measures that permit the nature of the build to be evaluated is exceedingly important for conducting demonstrative and mediation forms.	Metacognition is the student's high-level thinking process, which controls students' knowledge of the process of thinking and cognitive forms and engages active controls of students' cognitive activities such as learning and solving day-to-day problems of mathematical learning. Additives include understanding and management of mathematical thought and learning. This involves students' self-awareness of how they process mathematical information, understand concepts, and solve problems.	Metacognitive Regulation	1. Planning	1. Identifying the purpose of the problems 2. Sorting important information 3. Students control the completion steps of given information	1
			2. Monitoring	1. Students can write the formula correctly and precisely 2. The final step made by students fallen 3. Students control the completion steps of given information	
			3. Evaluation	1. Students do a reexamination of the problem settlement measures 2. Verifying that what's done is right	

Table 4. The Guidelines of Mathematical Metacognitive Ability Test

The Mathematical Metacognitive Test

Mrs. Diyah purchased a packet of posters containing a dozen two-liter [2 liters] paint bottles. The package was supposed to be sold for Rp240.000,00, but Mrs Diyah received a 5% discount.

Mrs Diyah also purchased three additional bottles at a regular price: Rp18.000,00 each.

What's the total cost?

Student Answer Sheet:

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