

Transforming Geometry Learning with an Interactive E-Module Integrating Ethno-Realistic Mathematics Education: A Design-Based Approach to Enhancing Numeracy Skills

I Putu Ade Andre Payadnya¹, Kadek Rahayu Puspawati¹, Gde Iwan Setiawan²

¹Universitas Mahasaraswati Denpasar, Bali, Indonesia

²Universitas PGRI Mahadewa Indonesia, Bali, Indonesia

adeandre@unmas.ac.id

Abstract: Integrating cultural contexts into mathematics education, mainly through Ethno-Realistic Mathematics Education (Ethno-RME), presents a significant opportunity to enhance students' numeracy skills in geometry. However, challenges persist in effectively incorporating cultural elements into curricula while maintaining rigorous and modern mathematical instruction. This study addresses this gap by designing a learning trajectory for teaching geometry through the lens of Ethno-RME with E-Module as technology integration. The research is guided by design-based research methods, encompassing preliminary design, experimental design, and retrospective analysis stages. Primary data include qualitative descriptive and quantitative data collected through Focus Group Discussions (FGD), observations of student activities, field notes, student worksheets, video analysis, tests, and interviews. Participants included students from four schools, with a total of 123 students. The findings indicate that the implementation of the Hypothetical Learning Trajectory (HLT) significantly improved students' numeracy skills, increasing the average pass rate across four schools from 55.25% in the pre-test to 86.2% in the post-test, with all schools showing substantial progress. Students demonstrated enhanced problem-solving skills, critical thinking, and creative reasoning. This research underscores the value of blending cultural relevance with digital-based mathematical instruction, providing a novel approach to designing a curriculum that is both educationally impactful and culturally meaningful.

Keywords: Ethno-Realistic Mathematics Education, Design-Based Research, Numeracy Skills, Cultural Contexts, Geometry Learning Trajectory

INTRODUCTION

Students' numeracy skills in Indonesia play a crucial role in improving the quality of education and preparing a globally competitive future generation (Trisnawati et al., 2022). Within the context of the Merdeka Curriculum plan, enhancing numeracy skills has become one of the key pillars to

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empower students to compete internationally (Nelisiwe & Yu, 2023). However, the lack of numeracy skills among Indonesian students remains a serious concern. The 2022 PISA survey results show that Indonesia scored 366 in mathematics, a decrease of 13 points from the 2018 average score of 379 (OECD, 2023). These results reveal that Indonesian students have a weak understanding of geometry concepts, particularly in recognizing various shapes and using visual information to make conclusions (Laksmiwati et al., 2023). Many students also feel disconnected from mathematics, largely due to the lack of teaching approaches that are relevant to their everyday lives. While ethnomathematics has been widely studied, there is limited research exploring the integration of ethnomathematics with digital technologies to address these issues, particularly in the Indonesian context.

The Ethno-Realistic Mathematics Education (Ethno-RME) approach provides a distinctive framework for mathematics instruction by blending cultural and social contexts with realistic mathematics learning (Prahmana, 2022). By embedding cultural elements and students' everyday experiences into geometry lessons, Ethno-RME facilitates students' recognition of geometric patterns in their environment and enhances their understanding of the real-world applications of geometric concepts. This approach connects mathematical ideas with students' cultural backgrounds, making learning more relevant and engaging. However, the effective integration of technology into this culturally based learning is crucial, as technology itself reflects cultural advancements and practices. Thus, incorporating interactive digital learning media and tools becomes essential to support this integration and enhance the educational experience.

Interactive E-Modules represent a significant advancement in educational technology, transforming traditional learning methods by moving educational content to digital platforms (Setiyani et al., 2022). These interactive modules are designed to provide stimulating and immersive learning experiences, allowing students to engage actively with content. By presenting geometry materials rooted in cultural contexts through an interactive digital format, E-Modules enable students to explore geometric concepts in depth while maintaining a connection to their cultural heritage. The integration of technology in this manner not only boosts students' interest in learning but also aids in their comprehension of complex mathematical ideas, making the learning process both more dynamic and effective (Hwang et al., 2023).

While ethnomathematics research often emphasizes cultural relevance in learning, few studies have effectively merged this approach with digital tools like interactive E-Modules. This study addresses this research gap by designing a Geometry Learning approach based on Ethno-RME supported by Interactive E-Modules. This dual focus on culture and technology is intended to enhance students' numeracy skills in geometry and provide a practical and innovative teaching model for educators. The study seeks to answer the following research questions: 1) How can Ethno-Realistic Mathematics Education be integrated with Interactive E-Modules to improve students' numeracy skills in geometry? 2) What are the effects of this integrated approach on students' numeracy skills in geometric concepts and their connection to cultural contexts?

This research emphasizes the urgent need to improve students' numeracy skills in geometry, an essential aspect of mathematics education that is often underdeveloped. Ethno-RME integrates local cultural aspects into the context of mathematics education, providing students with a more relevant and meaningful learning experience (Payadnya et al., 2024). Additionally, the use of Interactive E-Modules as a teaching aid reflects the state of the art in educational technology, a method that has not been widely applied in previous ethnomathematics research (Fouze & Amit, 2023). Consequently, this research not only offers a novel approach that combines culture and technology but also demonstrates significant potential in enhancing the effectiveness of learning and improving students' numeracy skills in geometry.

In the following section, we explore the research methodology, providing a detailed description of the research design and the three key stages involved: preliminary design, design experiment, and retrospective analysis. These stages are crucial in forming a coherent learning trajectory. The third section presents the results of implementing and analyzing the developed curriculum. It assesses the improvement in students' numeracy skills, offering insights into their progress, key findings, and student feedback from each activity. The conclusion then discusses the study's limitations and suggests directions for future research.

LITERATURE REVIEW

Ethnomathematics

Ethnomathematics explores the interconnections between mathematics and culture, demonstrating how mathematical concepts are deeply rooted in cultural practices, artifacts, and traditions. D'Ambrosio (Hamidah et al., 2024), a pioneer in this field, emphasized that recognizing these connections can make mathematics more engaging and meaningful for learners. This approach not only contextualizes mathematics in real-life settings but also fosters students' appreciation of their cultural heritage.

Research in ethnomathematics has explored its potential to bridge the gap between formal mathematics and students' everyday experiences. Studies have shown that incorporating cultural artifacts like batik patterns, woven textiles, and traditional architecture into mathematics instruction enhances student engagement and understanding. For instance, Prahmana & D'Ambrosio (2020) demonstrated how exploring symmetrical motifs in batik patterns in Yogyakarta, deepens students' understanding of geometry. Suherman and Vidákovich (2022) highlighted that the traditional "Tapis Lampung" fabric can serve as a medium to promote Indonesian local wisdom globally while also providing a contextual foundation for teaching mathematics in both rural and urban schools.

Despite these promising findings, ethnomathematics remains underutilized in formal classrooms. Challenges include a lack of teacher expertise in integrating cultural contexts into lessons, limited instructional resources, and insufficient research on the use of digital tools to support

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ethnomathematics. This study addresses these gaps by designing and testing digital instructional materials that integrate ethnomathematics, aiming to make its application more accessible and scalable across diverse educational settings.

Local Instructional Theory (LIT)

Local Instructional Theory (LIT) is a framework developed to design and analyze teaching strategies and materials for specific mathematical topics. Unlike general teaching theories, LIT is iterative and context-specific, enabling educators to refine instructional methods based on empirical evidence (Gravemeijer, 2020). It is particularly useful in addressing challenges in teaching complex mathematical topics like geometry, where students often struggle with abstraction.

LIT has been widely used to develop instructional sequences for various mathematical topics. Paoletti et al. (2023) highlighted its application in geometry education, where teaching sequences were designed to build students' understanding of abstract concepts like the properties of shapes and spatial reasoning. Gravemeijer (2020) emphasized the role of LIT in iterative development, where real-world contexts are used to scaffold students' conceptual understanding. Studies in Indonesia (Meryansumayeka et al., 2022) have also shown how LIT can support the teaching of geometry, but these studies are often limited to small-scale implementations without leveraging modern technologies.

While LIT has been effective in targeted interventions, there is limited research on its integration with cultural contexts and digital tools. This study seeks to address this gap by combining LIT with ethnomathematics and interactive e-modules, creating a framework that is both culturally responsive and technologically enhanced.

Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME) is an approach that prioritizes the use of real-world problems as a basis for teaching mathematics. Developed in the Netherlands, RME has significantly enhanced students' mathematical reasoning and problem-solving skills. The approach emphasizes active learning, where students construct their understanding through guided discovery in realistic contexts (Gravemeijer, 2020).

RME has been extensively studied for its impact on student learning. In Indonesia, research by Prahmana (2022) demonstrated how RME-based geometry lessons improved students' ability to analyze and solve problems in real-life scenarios, such as calculating the dimensions of traditional houses or analyzing the shapes and areas of rice fields. Altiner et al. (2023) reported similar findings, showing that RME improved student engagement and conceptual understanding by

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grounding abstract concepts in familiar, everyday situations. However, studies often highlight challenges in implementation, such as the lack of teacher training and resources, particularly in developing countries.

Although RME has proven effective, its potential for integration with ethnomathematics and digital tools remains underexplored. This study seeks to fill this gap by developing e-modules that combine RME with ethnomathematics, offering a scalable solution for improving geometry instruction in culturally diverse and resource-limited settings.

Research Contribution

By addressing the identified gaps in the literature—limited integration of ethnomathematics, LIT, and RME with digital tools—this study aims to develop a comprehensive instructional framework. The framework leverages culturally relevant contexts and technology to enhance student engagement and understanding, particularly in geometry education. The study's findings will contribute to the broader discourse on culturally responsive and technology-enhanced mathematics education.

METHOD

Research Design

This study employed a design research approach aimed at developing a Local Instructional Theory (LIT) in the form of a learning trajectory (Bakker & van Eerde, 2015). The LIT framework was operationalized through the integration of Realistic Mathematics Education (RME) principles, emphasizing contextual relevance, active student participation, and the use of mathematical models to bridge informal and formal understanding. The research process consisted of three key stages: preliminary design, design experiment, and retrospective analysis. The focus of this research was on geometry, a subject area that represented a significant weakness in numeracy among Indonesian students and offered extensive potential for ethnomathematical exploration. Geometry was chosen for this study due to its foundational role in mathematics, its close connection to real-world applications, and its alignment with RME principles, which facilitate the integration of realistic and cultural contexts into mathematical learning.

Guiding Principles and Criteria for Integration

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The integration of relevant materials and Realistic Mathematics Education (RME) into the lessons was guided by three key principles: contextual relevance, cognitive load, and engagement through interactivity. Contextual relevance ensures that learning materials align with Balinese ethnomathematics, allowing students to connect abstract mathematical concepts with real-world cultural contexts. This approach is supported by Prahmana and D'Ambrosio's (2020) ethnomathematics framework, which emphasizes the significance of integrating cultural elements into mathematics education to enhance students' understanding and engagement.

Cognitive load is another crucial consideration in instructional design, as it ensures that learning materials are developmentally appropriate and scaffold students' understanding. According to Klepsch and Seufert (2020), Cognitive Load Theory, instructional materials should be structured to balance intrinsic, extraneous, and germane cognitive loads, enabling students to progressively develop their mathematical reasoning. Additionally, structured guidance in learning is more effective than minimally guided instruction, reinforcing the need for a well-sequenced approach that gradually increases complexity.

Finally, engagement and interactivity play a vital role in fostering student motivation and participation. Research by García-López, et al. (2023) indicates that interactive learning environments significantly enhance student engagement by incorporating multimedia elements such as simulations and gamified assessments. By integrating these guiding principles, this study ensures that the incorporation of Ethno-RME and digital learning tools is both pedagogically sound and culturally meaningful.

Research Location, Participants, and Time

The study was conducted in four middle schools, both public and private, located in Bali. One class from each school, specifically 8th-grade students, participated in the study. The selection of locations and schools was based on previous research that highlighted extensive ethnomathematical exploration within Balinese cultural contexts. Additionally, the involvement of two schools from each region provided comparative data, thereby enhancing the validity of the research findings. The selected schools were those with established collaboration agreements with the research institution, facilitating coordination and the provision of support such as teachers, students, classrooms, and necessary research equipment. Participants included students from four schools, with 35 students from School 1, 28 from School 2, 30 from School 3, and 30 from School 4. This research was carried out in the academic year of 2023/2024.

Data Collection Techniques

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The primary data for this research included both qualitative, descriptive, and quantitative data. Qualitative data were obtained through the development of the learning process using Focus Group Discussions (FGD), observation of student activities, field notes, analysis of student worksheets, video analysis of the learning process, tests, and interviews. Unstructured observations and open-ended interviews were employed (Taherdoost, 2022). Quantitative data on students' numeracy skills were collected through pre-test and post-test assessments using essay questions as the evaluation instrument.

The selected mathematical content for this study was plane geometry, chosen due to its importance in the middle school curriculum as it laid the foundation for understanding more complex geometric concepts at higher educational levels. Additionally, plane geometry allowed for the integration of cultural aspects through Ethno-Realistic Mathematics Education and supported the use of interactive technology such as Interactive E-Modules to visualize concepts engagingly and clearly. This made the content highly relevant and supportive of the innovative approach employed in this research.

Validation of HLT

The Hypothetical Learning Trajectory (HLT) was validated by two experts: one specializing in mathematics education content and the other in digital media. A validation tool consisting of criteria such as content accuracy, cultural relevance, alignment with Realistic Mathematics Education (RME) principles, and the effectiveness of digital tools was employed.

The content expert recommended integrating more interactive and culturally relevant materials to better align with ethnomathematical principles, while the digital media expert suggested enhancing the visualization of geometric concepts by replacing static 2D images with interactive 3D models created using Assembly Studio, providing students with a more immersive learning experience. Additionally, they advised incorporating Quizizz for assessments due to its gamified and interactive features, which effectively engage students. Based on their feedback, the lessons were revised by replacing regular 2D pictures with 3D models to allow for a more dynamic exploration of geometric concepts and transitioning from traditional assessment tools to Quizizz for both formative and summative evaluations, ensuring immediate feedback and improved student engagement.

These changes were incorporated into the HLT and validated again by the experts to ensure their appropriateness and effectiveness in achieving the study's objectives.

Research Procedure

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As previously mentioned, this research was conducted through three general stages: preliminary design, design experiment, and retrospective analysis carried out over four months to produce a Hypothetical Learning Trajectory (HLT) in the initial stage and a Local Instructional Theory (LIT) in the final stage. Before this study, the researcher had already conducted various preliminary studies related to the topic. The detailed activities at each stage are presented in Table 1 below.

No	Stage	Activities
1	Preliminary design	This stage focused on analyzing the situation and issues in mathematics education at schools. Several activities were conducted during this stage to analyze the situation and design the initial plan, including observing classroom teaching, interviewing mathematics teachers, analyzing the current mathematics curriculum, and reviewing relevant references and studies. Subsequently, the HLT and learning tools such as modules, student worksheets (LKPD), learning media, and instruments were developed. Instrument testing was conducted using the content validity index, content validity scale, and instrument reliability. Additionally, a pre-test was conducted to assess students' initial numeracy skills.
2	Design experiment	The HLT was implemented in the research subject classes, with observations of the teaching activities, field notes, and multi-directional documentation of the learning process. This implementation was conducted in four research schools, each using one regular class (30-35 students). The results obtained from each school were compared to obtain more valid and informative data.
3	Retrospective analysis	Data collected during the field implementation were analyzed qualitatively using the constant comparison method, following these steps: (1) reviewing the learning recordings, (2) creating a general transcript, (3) marking important segments, (4) completing the transcript of those segments, (5) seeking confirmation or contradiction in other segments, and (6) discussing findings with colleagues for validation. Quantitative data were analyzed using a mean difference test to assess improvements in students' numeracy skills.

Table 1. Research Stages

After the retrospective analysis, the data obtained, which included scores, students' worksheets, and student responses, were used to assess the development of students' numeracy skills and to inform future improvements to the Hypothetical Learning Trajectory (HLT).

Pre-Test and Post-Test Content

The pre-test and post-test were designed to measure students' numeracy skills. They included questions on fundamental geometric concepts such as understanding the properties of shapes, calculating areas and perimeters, and applying geometric reasoning to solve real-world problems. Ethnomathematical elements were integrated into some questions to assess students' ability to connect mathematical concepts to cultural contexts. The results of these tests were compared, particularly in terms of achieving the minimum mastery criteria.

RESULTS AND DISCUSSION

Results

In the preliminary phase of the study on "Designing Geometry Instruction with Ethno-Realistic Mathematics Education and Interactive E-Modules: A Research-Based Approach to Enhancing Numeracy Skills," the researcher conducted several crucial activities, including a comprehensive literature review, interviews with mathematics teachers, and the development of the Hypothetical Learning Trajectory (HLT) with guidance from a supervisor. The literature review focused on integrating cultural contexts and realistic problem-solving approaches in geometry, emphasizing Ethno-Realistic Mathematics Education (Ethno-RME) and the use of interactive digital tools such as e-modules. Interviews with three mathematics teachers revealed that students often relied on memorizing formulas and solving procedural problems without fully grasping the underlying concepts, especially in tasks involving culturally relevant or irregular shapes. To address these challenges, the HLT was designed over two months and incorporated cultural elements along with interactive applications. The HLT was then validated by experts in ethnomathematics and realistic mathematics education, ensuring a comprehensive approach that integrates both cultural and technological aspects into the geometry curriculum. Here is the summary of the HLT design which includes learning activities along with key findings observed in each stage.

No.	Learning Objective	Main Activity	Key Outcome
1	Understanding Sanggah Cucuk	<ul style="list-style-type: none"> Watch videos and analyze images of Sanggah Cucuk. 	<ul style="list-style-type: none"> Identify Sanggah Cucuk as a triangular prism.

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	Students recognize the shape and function of Sanggah Cucuk.	<ul style="list-style-type: none"> Identify geometric shapes and materials needed. 	<ul style="list-style-type: none"> Determine the geometric components (e.g., Klakat shapes).
2	Designing Sanggah Cucuk Students create a design for Sanggah Cucuk.	<ul style="list-style-type: none"> Design Sanggah Cucuk virtually. Calculate and record required materials. 	<ul style="list-style-type: none"> Complete a design with accurate quantities (e.g., 3 square Klakats, 1 triangular Klakat).
3	Building Sanggah Cucuk Students construct Sanggah Cucuk.	<ul style="list-style-type: none"> Construct the model using provided materials. Compare with other groups and reflect on challenges. 	<ul style="list-style-type: none"> Build Sanggah Cucuk as planned. Reflect on design and material challenges.
4	Applying Mathematical Concepts Students calculate and compare surface area and materials.	<ul style="list-style-type: none"> Calculate the surface area and total bamboo needed. Compare with a standard triangular prism. 	<ul style="list-style-type: none"> Accurately calculate the surface area and material needed. Understand the connection between math concepts and the design.

Table 2. Hypothetical Learning Trajectory (HLT)

The HLT design outlined in Table 2 provides a detailed overview of the learning activities, capturing key insights and findings observed across each stage. These findings have been synthesized from data gathered at four participating schools, offering a comprehensive look into the outcomes and challenges encountered during the learning process. By analyzing the data from these different contexts, the summary highlights patterns, variations, and effective strategies within the HLT framework, offering valuable implications for refining instructional practices. Here are the findings in each activity.

The object used in the HLT is “Sanggah Cucuk” as shown in Figure 1 below:



Figure 1. Sanggah Cucuk (Source: <https://paduarsana.com/2013/05/21/sanggah-cucuk-dan-tawur-agung-kesanga/> & https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcRpXuNyHnN59up9wHEnM68G1SIvM_T45HG1wg&)

The "Sanggah Cucuk" is a traditional Balinese shrine made of bamboo, typically constructed for specific ceremonies, such as the Tawur Agung Kesanga, which marks the ritual purification preceding the Balinese New Year (Nyepi). This sacred structure serves as a temporary altar to place offerings, symbolizing a bridge between humans and the divine. Its design incorporates both 2D and 3D geometry concepts, reflecting the mathematical precision involved in Balinese craftsmanship. As a cultural object, the Sanggah Cucuk embodies the spirituality, artistry, and mathematical ingenuity of the Balinese people.

Learning Activity

Data collected from the four schools, consisting of students' responses to each activity in the worksheets, were systematically gathered, reduced, and categorized based on the key findings from each learning activity. To facilitate collaborative learning, students were grouped into teams of 5-7 members in each class, allowing them to work on the worksheet through discussion and cooperation. This data was then analyzed to assess students' numeracy skills and their comprehension of each segment of the material. The process involved identifying patterns and trends within the students' work to evaluate their progress in both understanding the geometric concepts and applying numeracy skills across the different instructional phases. The analysis not only provided insights into the overall effectiveness of the Ethno-Realistic Mathematics Education approach but also highlighted specific areas where students demonstrated significant gains or encountered challenges. By categorizing the data into relevant themes and aligning them with the instructional objectives, the study was able to draw meaningful conclusions regarding the integration of cultural context and digital tools in enhancing students' numeracy skills.

Learning Activity 1

As outlined in the table, the first activity centers on students analyzing the structure of the Sanggah Cucuk, a traditional Balinese shrine, through an instructional video embedded within the E-Module. The activity is designed to engage students in observing and understanding the geometric shapes that form the Sanggah Cucuk. Through guided exploration, students recognize that the structure is an equilateral triangular prism, consisting of three rectangular faces and one equilateral triangular face. This exercise helps students relate mathematical concepts to real-world cultural artifacts, bridging the gap between abstract geometry and tangible, culturally relevant examples.

In analyzing the video, students demonstrated their ability to break down complex structures into fundamental geometric shapes, reflecting a deeper understanding of both geometry and its applications. The recognition of Sanggah Cucuk's components—three squares or rectangles and one equilateral triangle—shows that students can connect their theoretical knowledge to practical contexts. By engaging with a culturally significant object, students not only reinforced their grasp of geometric concepts but also developed an appreciation for how these concepts manifest in everyday life. The following example of a student's response highlights their ability to accurately describe the structure, thereby demonstrating the effectiveness of using culturally embedded resources to enhance both comprehension and engagement in mathematics.

3) Apa saja yang perlu kita persiapkan untuk membuat Sanggah Cucuk?

<p>Bahan-bahan :</p> <ol style="list-style-type: none"> 1. Bambu 2. Tali 3. potongan bambu sebanyak 50 4. Klakat sebanyak 4 buah. (persegi ada 3, segitiga ada 1) 	<p>Alat - Alat :</p> <ol style="list-style-type: none"> 1. Pisau 2. Kain putih.
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Translation:

3) What do we need to prepare for making a Sanggah Cucuk?*

Materials

1. Bamboo
2. Rope
3. 50 pieces of bamboo
4. 4 Klakat (3 square-shaped, 1 triangular-shaped)

Tools:

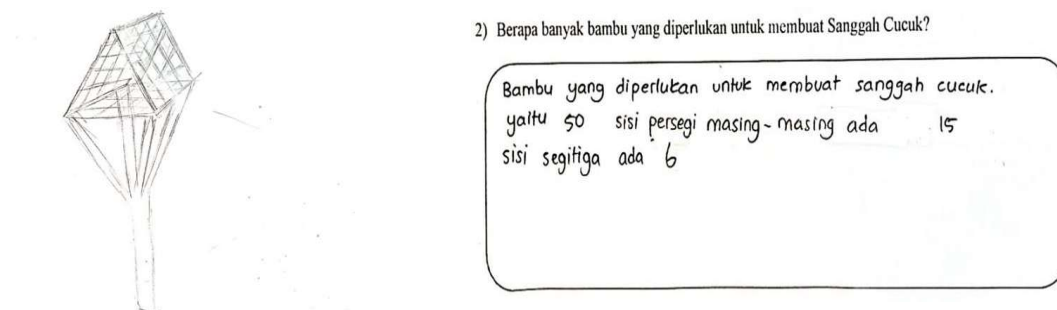
1. Knife
2. White cloth

Figure 2. Student's Answer on Activity 1

The observations from Figure 2 reveal that students demonstrated realistic thinking and accurately calculated the required materials for constructing a Sanggah Cucuk. In responding to question number 3, students provided precise explanations of the materials needed by drawing connections to their understanding of Sanggah Cucuk's components. Despite never having constructed a Sanggah Cucuk themselves, students utilized prior knowledge and experiences to deliver well-reasoned answers. This illustrates how integrating cultural experiences and real-world contexts into learning enhances students' numeracy skills while reinforcing their understanding of ethnomathematical concepts and realistic mathematics. By linking prior experiences with mathematical tasks, students effectively navigated abstract ideas and applied them to practical problem-solving. These findings highlight the effectiveness of Ethno-Realistic Mathematics Education in fostering meaningful engagement with mathematical content, promoting reasoning, accurate calculations, and connections between mathematical concepts and their real-world applications (Apriatni et al., 2022).

Learning Activity 2

In the second learning activity, students were tasked with designing their version of a Sanggah Cucuk. Before this, they had watched a 3D design demonstration in the E-Module. Building on this visual guidance, students analyzed the design and then proceeded to create their sketches. The hands-on nature of this task, combined with the interactive 3D model, sparked enthusiasm among the students, as they engaged actively in manipulating the 3D design. This approach not only allowed students to exercise their creativity but also deepened their spatial reasoning and geometric understanding. The following example showcases a student's original design, reflecting both their engagement with the cultural context and their grasp of geometric principles.



Translation:

How much bamboo is needed to make a Sanggah Cucuk?

The bamboo needed to make a Sanggah Cucuk includes 50 square sides, each with 15 sides, and 6 triangular sides.

Figure 3. Example of Student's Design on Sanggah Cucuk and Student's Answer on a Question in Activity 2

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Figure 3 shows that students effectively designed the Sanggah Cucuk, showcasing their representational skills and spatial awareness. Their ability to visualize and accurately depict geometric structures demonstrates a strong grasp of the concepts being taught. However, challenges emerged in the follow-up question, where students were required to calculate the quantity of bamboo needed for construction. The varying results among groups revealed inconsistencies in their estimation and calculation processes. These discrepancies highlight the need for further instruction to strengthen students' mathematical reasoning and problem-solving skills, particularly when applying theoretical knowledge to practical, real-world scenarios (Firdausy & Indriati, 2021).

To address these inconsistencies, the researcher emphasized that each calculated quantity must be supported by detailed reasoning and accurate calculations. The rule that students can only take the exact amount of bamboo they recorded for the next activity reinforces the importance of accuracy and discourages intentional overestimation. This approach not only fosters careful mathematical planning but also introduces students to the importance of resource management, linking mathematical concepts to real-world applications in a culturally relevant context.

Learning Activity 3

This activity was the most engaging and lively of all the sessions, as students were directly involved in constructing the Sanggah Cucuk. The high level of enthusiasm among the students indicates that project-based learning using real objects and incorporating cultural contexts can significantly boost student motivation. The hands-on experience allowed students to connect mathematical concepts with tangible outcomes, reinforcing their understanding while making the learning process more enjoyable and meaningful.



Figure 4. Students Working on Sanggah Cucuk

Figure 4 highlights the students' notably positive responses during the third activity, as they proudly presented their Sanggah Cucuk creations derived from their mathematical calculations. This enthusiasm underscores the effectiveness of hands-on, project-based learning within a cultural context, which makes mathematical concepts more accessible and engaging. However, when analyzing and comparing their designs with those of their peers, many students struggled to identify design differences but were more successful in recognizing calculation errors. The leftover bamboo observed in most groups points to inconsistencies in their numeracy skills, particularly in applying accurate calculations to practical tasks.

[awalnya mengambil 52 bambu
namun hanya menggunakan 46 pcs
jadi sisa bambu adalah 6]

Translation:

Initially, 52 bamboo pieces were taken, but only 46 pieces were used, so the remaining bamboo is 6 pieces.

Figure 5. Students Realize Their Calculation Mistake

Figure 5 highlights a positive outcome, as students successfully identified calculation errors when determining the amount of bamboo needed. They quickly devised corrective measures and reconciled their results with those of their peers, demonstrating improved problem-solving abilities. This achievement reflects the effectiveness of a learning experience enriched with ethnomathematics and cultural relevance, which helped students enhance both accuracy and consistency in their work. By incorporating realistic ethnomathematical contexts and familiar cultural elements, the approach not only improved numeracy skills but also fostered a deeper understanding and quicker correction of errors, resulting in more uniform outcomes among students (Supriyadi et al., 2022).

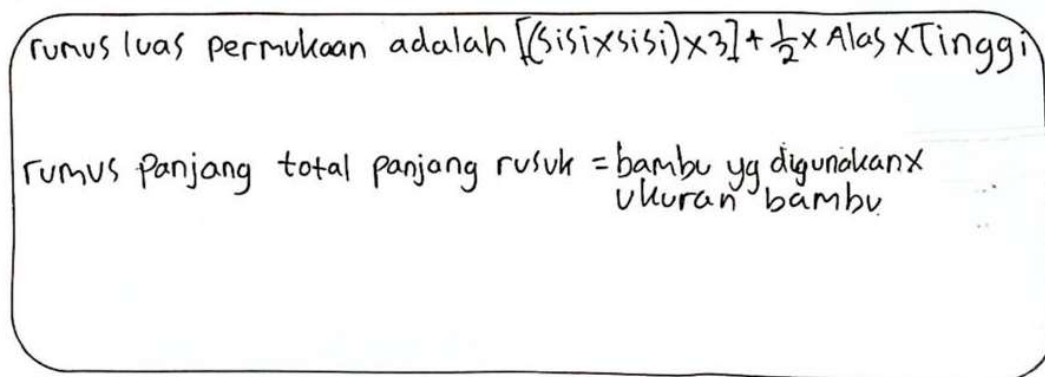
Learning Activity 4

In this activity, students were presented with several problems related to the Sanggah Cucuk and triangular prisms. Specifically, they were tasked with solving questions about edge lengths related to the amount of bamboo needed and surface area related to the covering fabric. The students successfully addressed these challenges, accurately deriving the formula for the surface area of a prism and applying it to calculate the total length of bamboo and the fabric required for the Sanggah Cucuk. This performance demonstrates a significant improvement in their numeracy skills, particularly in applying geometric formulas to solve real-world problems.

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3) Bagaimana rumus luas permukaan dan panjang total rusuk prisma segitiga?



Rumus luas permukaan adalah $[(sisi \times sisi) \times 3] + \frac{1}{2} \times \text{Alas} \times \text{Tinggi}$

Rumus panjang total panjang rusuk = bambu yg digunakan x ukuran bambu

Translation:

The formula for Surface Area is $(side \times side) \times 3 + \frac{1}{2} \times base \times height$.

The formula for Total Edge Length is bamboo used \times bamboo size.

Figure 6. Students Able to Find the Surface Area Formula of a Prism Based on Activities Participated In

The success of the activity depicted in Figure 6 highlights the effectiveness of the Ethno-Realistic Mathematics Education (Ethno-RME) approach. By situating mathematical problems within the culturally relevant and tangible context of the Sanggah Cucuk, students were able to bridge abstract geometric concepts with practical applications. This integration of cultural elements not only deepened their understanding of mathematical principles but also enhanced their problem-solving skills. The student's ability to apply mathematical formulas to real-life scenarios demonstrates the success of Ethno-RME, emphasizing that culturally grounded education can significantly improve numeracy skills while creating a more engaging and impactful learning experience.

The difficulty observed in students' responses to the mathematical modeling question underscores a challenge in their ability to connect geometric concepts with algebraic modeling using variables. This issue can be attributed to the question's lack of clear structure and its misalignment with the student's cognitive development stages. According to Piaget's Theory of Cognitive Development, students progress from concrete operational thinking, which involves tangible problem-solving, to formal operational thinking, which encompasses abstract reasoning and the use of variables (Rabindran & Madanagopal, 2020). In this instance, while students effectively handled concrete geometric problems related to the Sanggah Cucuk, they struggled with more abstract tasks requiring algebraic modeling.

- 5) Bagaimana model matematika dari pembuatan Sanggah Cucuk berkaitan dengan panjang dan banyak total bambu serta luas kain total yang diperlukan?

Sanggah cucuk memiliki 2 bangun datar yaitu persegi dan segitiga, untuk mengetahui berapa panjang total kain yg digunakan. Di gunakan rumus luas permukaan permukaan segitiga dan luas permukaan persegi.

Translation:

5) What is the mathematical model for making a Sanggah Cucuk about the total length and amount of bamboo and the total area of cloth needed?

The Sanggah Cucuk consists of 2 shapes: squares and triangles. To determine the total length of cloth used, the formulas for the surface area of a triangle and the surface area of a square are applied.

Figure 7. Students Answer Questions About the Mathematical Model Incorrectly

The difficulty highlighted in Figure 7 emphasizes the critical role of mathematical connection skills, which involve linking various mathematical concepts and procedures to solve complex problems. Many students struggle to integrate geometric understanding with algebraic methods, as this demands cognitive maturity and the ability to connect abstract and concrete ideas. For example, some students' responses indicate a tendency to treat algebraic and geometric concepts in isolation, resulting in incomplete or incorrect solutions. To bridge this gap, tasks should be scaffolded to align with students' developmental stages, providing step-by-step support to help them progress from foundational to advanced problem-solving. Structured questions that build on prior knowledge and foster the integration of mathematical concepts can enhance students' ability to apply algebraic techniques effectively in solving geometric problems (Ukobizaba et al., 2021).

Students' Test and Interview Results

Pre-Test Result

Before implementing the HLT (Hypothetical Learning Trajectory), the researcher administered a pre-test on numeracy skills to the students to assess their initial capabilities. The results of the pre-test are as follows:

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School	Number of Students	Percentage of Students Passing (>80)	Number of Students Passing
School 1	35	50%	30
School 2	28	60%	25
School 3	30	55%	24
School 4	30	55%	27

Table 3. Students' Pre-Test Score in Each School

Table 3 shows the overall performance of students is below expectations, with only 50% to 60% of students achieving scores above the 80% threshold in each school. School 1 has the lowest pass rate at 50%, with only 17 out of 35 students passing. Schools 3 and 4 have a slightly higher pass rate at 55%, with 16 out of 30 students passing. School 2, though higher than the others, still only reaches a 60% pass rate. This indicates that numeracy skills among the students are insufficient and there is a clear need for improvement in the instructional strategies to enhance students' numeracy proficiency.

Post-Test Result

The researcher administered a post-test on numeracy skills at the end of HLT implementation to evaluate the overall learning achievement and determine the percentage of students who met the score threshold of 80, as outlined in the table below. Here is the table showing the percentage of students who achieved scores above 80 in four schools:

School	Number of Students	Percentage of Students Passing (>80)	Number of Students Passing
School 1	35	85%	30
School 2	28	90%	25
School 3	30	80%	24
School 4	30	90%	27

Table 4. Students' Post-Test Score in Each School

A total of 123 students, with 106 students (approximately 86.2%) scoring 80 or above. Table 4 presents the achievement levels of students in four schools involved in the study, indicating the percentage and number of students who scored above the threshold of 80. School 2 and School 4 had the highest percentage of students meeting this criterion, with 90% of their students passing. Meanwhile, School 1 and School 3 had slightly lower pass rates at 85% and 80%, respectively. Overall, the majority of students across all schools demonstrated strong performance, with more than 86% of the total students achieving scores above the set threshold, reflecting the effectiveness of the implemented instructional strategies.

The comparison between the pre-test and post-test results reveals a clear improvement in students' numeracy skills after the implementation of the Hypothetical Learning Trajectory (HLT). In the pre-test, the total average percentage of students passing the 80 threshold across all four schools was 55.25%, indicating that a majority of students struggled to meet the target. Specifically, the pass rates ranged from 50% in School 1 to 60% in School 2. This highlighted a need for stronger instructional strategies to address gaps in students' numeracy skills. However, after the implementation of the HLT, the post-test results showed a significant improvement, with the average pass rate rising to approximately 86.2%. In the post-test, the percentage of students who scored above 80 increased across all schools, with the highest improvements observed in Schools 2 and 4, both achieving 90% pass rates. Schools 1 and 3 also showed positive changes, with pass rates reaching 85% and 80%, respectively. This substantial increase in the pass rate across all schools suggests that the HLT effectively addressed the deficiencies in numeracy skills, indicating that the learning strategies implemented had a significant impact on enhancing students' understanding and performance in numeracy. The comparison demonstrates that the HLT contributed to a remarkable improvement in student achievement, highlighting the importance of well-designed instructional frameworks in boosting students' academic outcomes.

Interview Result

In addition to the test, the researcher conducted interviews with two representative students from each school to gain deeper insights into their experiences and perceptions of the learning activities. The selection criteria for these students were based on their active participation during the lessons and the diversity of their academic performance, ensuring that both high-achieving and average students were included. This approach allowed the researcher to capture a broad range of perspectives on the effectiveness of the learning model. The interviews were designed to explore how the integration of project-based activities and cultural elements influenced their motivation and engagement in mathematics.

The findings from these interviews revealed that students felt highly motivated throughout the learning process. They appreciated the hands-on, digital, and project-based nature of the activities, which allowed them to actively engage with the subject matter. Additionally, the incorporation of cultural context through traditional elements like the Sanggah Cucuk provided a unique learning

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experience that extended beyond mathematics. Students expressed that, alongside developing their mathematical skills, they also gained valuable knowledge about their cultural heritage. This dual focus not only enriched their learning but also made the material more relatable and meaningful, leading to a more positive and memorable educational experience (Jia & Zhang, 2023).

Discussion

The study reveals several significant insights into the impact of the Ethno-Realistic Mathematics Education (Ethno-RME) approach on students' numeracy skills and conceptual understanding. By integrating cultural artifacts, such as the Sanggah Cucuk, with mathematical instruction, the study demonstrated that students can effectively link abstract geometric concepts with real-world applications, enhancing both engagement and comprehension. This approach aligns with Piaget's theory of cognitive development, which suggests that connecting concrete experiences to abstract concepts can facilitate understanding, particularly for students in the concrete operational stage (Oogarah-Pratap et al., 2020).

The activities highlighted varying levels of student proficiency in applying mathematical concepts. While students showed enthusiasm and creativity in tasks involving the design and construction of the Sanggah Cucuk, issues emerged in areas requiring precise calculations and mathematical modeling. The discrepancies in bamboo estimation and challenges in algebraic modeling reflect Vygotsky's Social Development Theory, which emphasizes the role of social interaction in cognitive growth. Collaborative learning environments helped students engage deeply with the material but also exposed areas where additional support is needed (Thompson, 2022). This suggests that future iterations of the Hypothetical Learning Trajectory (HLT) should include more structured guidance to enhance mathematical reasoning and problem-solving skills.

Students' ability to apply geometric formulas in practical contexts demonstrated the effectiveness of Ethno-RME in reinforcing their understanding of mathematical principles. However, difficulties with abstract tasks, such as algebraic modeling, indicated a gap in students' transition from concrete to formal operational thinking, as outlined by Piaget (Sibgatullin et al., 2022). This underscores the need for developmentally appropriate scaffolding to support students in bridging this gap. By aligning instructional activities with cognitive development stages and providing targeted support, educators can better facilitate students' transition to more abstract mathematical reasoning.

The integration of culturally relevant and hands-on learning experiences, as seen in the study, significantly enhanced student motivation and problem-solving abilities. Dewey's Experiential Learning Theory supports this finding, highlighting the benefits of experiential and contextually grounded learning in fostering a deeper understanding of mathematical concepts (Uyen et al., 2022). The positive outcomes observed in students' engagement and accuracy in practical tasks underscore the value of incorporating cultural contexts into mathematical instruction.

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In conclusion, the study affirms the effectiveness of Ethno-RME in improving students' numeracy skills and understanding by providing culturally relevant, hands-on learning experiences. To further enhance this approach, future revisions to the HLT should focus on addressing gaps in mathematical reasoning and supporting the transition from concrete to abstract thinking. By incorporating structured support and developmentally appropriate tasks, educators can build on the successes observed and continue to refine the integration of cultural contexts and digital tools in mathematics education.

Practical Implications

This research contributes to the growing body of literature on culturally responsive mathematics education by integrating Ethno-Realistic Mathematics Education (Ethno-RME) with digital tools to enhance students' numeracy skills. Previous studies have emphasized the importance of contextualizing mathematical concepts within students' cultural experiences to foster engagement, understanding, and learning outcomes (Mairing & Nini, 2023; Orozco & Pasia, 2021). However, many of these studies have focused primarily on theoretical frameworks without offering practical applications that directly connect cultural elements with formal mathematics. This research fills that gap by presenting a tangible model where cultural artifacts, such as the Sanggah Cucuk, are used to bridge abstract mathematical concepts and real-world applications. In doing so, it complements previous work by demonstrating how culturally relevant pedagogy can be operationalized effectively in classroom settings. Additionally, this study identifies the challenges in applying abstract mathematical reasoning and highlights the need for more structured guidance to support students in developing their mathematical problem-solving skills. The findings align with existing theories on the role of cultural context in learning and underscore the need for developmentally appropriate scaffolding to guide students through cognitive transitions (Osher et al., 2020).

Moreover, this study advocates for incorporating more structured guidance to enhance mathematical reasoning and problem-solving skills within the Hypothetical Learning Trajectory (HLT). Structured guidance in the form of step-by-step problem-solving techniques, scaffolding questions, and visual representations can provide the necessary support for students to bridge the gap between concrete and abstract thinking. For example, teachers can implement scaffolded questioning techniques such as, "What happens when we apply this rule in a real-world context?" or "Can you explain how this cultural artifact helps us understand this mathematical concept?" These prompts help students connect abstract concepts with tangible cultural experiences, thereby facilitating deeper comprehension. Additionally, teachers can guide students through the process of analyzing a cultural artifact and identifying the mathematical properties within it. For instance, when exploring the Sanggah Cucuk, students could be prompted to identify patterns, shapes, or symmetries that relate to geometric concepts, thereby making abstract math concepts more accessible. Teachers could also encourage students to create mathematical models based on cultural artifacts and apply mathematical reasoning to understand the artifact's function and significance.

Another example of structured guidance is the use of interactive digital tools, such as virtual manipulatives, to help students visualize mathematical concepts in real time. These tools can be programmed to provide instant feedback, guide students through progressively complex tasks, and suggest hints when students encounter difficulties. For example, a virtual geometry tool could allow students to manipulate shapes inspired by cultural designs and explore their properties, helping them connect the geometry of cultural artifacts with formal mathematical concepts (Payadnya et al., 2023). Furthermore, scaffolded worksheets can be used, with progressively challenging tasks that build students' problem-solving abilities. For example, initial tasks might involve simple arithmetic operations related to cultural elements, while later tasks could require students to apply abstract algebraic concepts to these real-world contexts.

The practical implications of this study are particularly relevant for curriculum designers and educators seeking to enhance mathematics instruction by making it more culturally inclusive and engaging. The integration of Ethno-RME with interactive digital modules provides a promising framework for future curriculum development, illustrating how cultural elements can be seamlessly woven into modern pedagogical practices.

By addressing both the strengths and areas for improvement in students' numeracy skills, this research offers actionable insights for refining instructional strategies, particularly in regions where cultural heritage plays a central role in everyday life (Thi et al., 2023). The research emphasizes the importance of structured questioning and scaffolded support, which can significantly improve students' ability to transition from concrete to abstract mathematical thinking (DeJarnette & Hord, 2022). This approach not only enriches the learning experience but also has the potential to improve student outcomes in mathematical problem-solving, making it a valuable resource for advancing mathematics education in diverse cultural contexts.

CONCLUSIONS

The findings of this research highlight the significant positive impact of integrating Ethno-Realistic Mathematics Education (Ethno-RME) with digital media on enhancing students' numeracy skills and their understanding of geometric concepts. The learning activities, which incorporated culturally relevant contexts such as the Sanggah Cucuk, resulted in high levels of student numeracy skills. Students exhibited a strong grasp of geometric principles and were able to connect these concepts to real-world applications grounded in their cultural heritage. The project-based and collaborative nature of the tasks further reinforced students' numeracy skills, fostering their problem-solving abilities, critical thinking, and teamwork. These outcomes underscore the effectiveness of combining ethnomathematics with realistic mathematics education to provide a comprehensive learning experience that is both culturally meaningful and educationally impactful.

However, it is important to acknowledge the primary limitation of this study, which is the restricted sample size and the specific cultural context in which the research was conducted. The study was

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limited to four schools within a particular geographical area, which may constrain the generalizability of the findings to broader educational contexts. The cultural homogeneity of the student sample also limits the exploration of diverse ethnomathematical practices. Future research should aim to include a more diverse range of cultural contexts and larger sample sizes. Such an approach would allow for a more thorough understanding of how different cultural perspectives can be integrated into mathematics education, leading to the development of more inclusive and adaptable teaching methods across various educational settings.

Based on these conclusions, it is recommended that educators and curriculum developers incorporate culturally relevant content into mathematics education. By aligning mathematical instruction with students' cultural backgrounds and real-world experiences, teachers can create more engaging and relatable learning environments that enhance students' cognitive and affective outcomes. Additionally, adopting structured questions and scaffolded tasks, as highlighted by the study's findings, can help address gaps in understanding and support the gradual development of mathematical thinking from concrete to abstract reasoning. Incorporating these approaches not only promotes better comprehension of mathematical concepts but also contributes to the development of a more inclusive, culturally responsive, and student-centered mathematics education framework.

ACKNOWLEDGMENTS

The authors express their gratitude to Direktorat Jenderal Pendidikan Tinggi Kementerian Pendidikan dan Kebudayaan Republik Indonesia (Directorate General of Higher Education, Ministry of Education and Culture, Republic of Indonesia), for providing funding for this research through Hibah Penelitian Fundamental - Reguler Tahun 2024 (Fundamental Research Grant - Regular Program 2024) with research contract number: K. 1201/B.07.01/Unmas/VI/2024.

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APPENDIX

Pre-Test and Post-Test Questions

“Transforming Geometry Learning with Interactive E-Module Integrating Ethno-Realistic Mathematics Education: A Design-Based Approach to Enhancing Numeracy Skills”

Pre-Test Questions

1. Understanding Shapes in Culture

In Balinese traditional architecture, many temples feature triangular and square patterns in their designs. Describe the properties of squares, triangles, and prisms, and explain how these shapes contribute to the stability of traditional structures.

Answer:

- Square: Has four equal sides and four right angles; used in foundations for stability.
- Triangle: Three sides with different types (equilateral, isosceles, scalene); strong structure for roofs.
- Prism: 3D shape with two parallel bases; used in pillars for support.
- Application in Balinese Architecture: Triangular roofs allow water drainage, square bases ensure balance and prism-shaped columns provide durability.

2. Calculating Area and Perimeter

A traditional woven mat (kain tenun) has a square design with a side length of 1.5 meters. Calculate its perimeter and area.

Answer:

- Perimeter = $4 \times \text{side} = 4 \times 1.5 = 6$ meters
- Area = $\text{side} \times \text{side} = 1.5 \times 1.5 = 2.25$ square meters.

3. Applying Geometric Reasoning

A farmer in Bali wants to divide his triangular rice field into two equal sections. The field has a base of 20 meters and a height of 15 meters. How much land will each section have after division?

Answer:

- Total Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 20 \times 15 = 150$ square meters
- Each Section's Area = $150 \div 2 = 75$ square meters.

Post-Test Questions

1. Understanding Shapes in Culture

In traditional Indonesian architecture, many houses and temples incorporate square, triangular, and prism-like structures. Explain the properties of these shapes and how they contribute to real-world stability and design.

Answer:

- Square: Provides structural strength in walls and floors.
- Triangle: Distributes weight evenly, making roofs more durable.
- Prism: Used in beams and pillars for strong support.
- Application in Architecture: Squares form stable building bases, triangular roofs prevent water accumulation, and prisms strengthen the framework of structures.

2. Calculating Area and Perimeter

A square-shaped window in a traditional house has a side length of 2 meters. Calculate its perimeter and area.

Answer:

- Perimeter = $4 \times \text{side} = 4 \times 2 = 8$ meters
- Area = $\text{side} \times \text{side} = 2 \times 2 = 4$ square meters.

3. Applying Geometric Reasoning

A farmer builds a triangular shelter for livestock with a base of 18 meters and a height of 12 meters. If he divides it into two equal parts, how much area will each part have?

Answer:

- Total Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 18 \times 12 = 108$ square meters
- Each Section's Area = $108 \div 2 = 54$ square meters