

## The Effect of GeoGebra-Assisted Discovery Learning Considering Emotional Stability on the Achievement of Mathematical Reasoning

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*Abstract: This study aims to describe and examine the effect of GeoGebra-assisted discovery learning on students' mathematical reasoning achievement in 3D geometry lessons. A Quasi-experimental research design with a post-test-only control group was used to conduct this study. This study involved 108 twelfth-grade students from a public senior high school in West Java, who were selected through purposive sampling. A valid and reliable mathematical reasoning test and emotional stability questionnaire, was used to collect the data. Effect size was computed using Hedges' equation, and two-way ANOVA was used to test group differences and interaction. Results showed that GeoGebra-assisted discovery learning significantly enhanced students' mathematical reasoning, exhibiting a strong positive effect ( $g = 1.247$ ). Furthermore, significant differences in mathematical reasoning achievements were found among students exposed to GeoGebra-assisted discovery learning, discovery learning alone, and directed learning. However, no significant differences were found in the mathematical reasoning achievements among students with high, moderate, and low emotional stability. Additionally, there was no significant interaction between the learning model and emotional stability in affecting students' mathematical reasoning. These findings suggest that GeoGebra-assisted discovery learning is an effective intervention to enhance Indonesian students' mathematical reasoning.*

Keywords: discovery learning, emotional stability, 3D geometry, GeoGebra, mathematical reasoning.

### INTRODUCTION

Mathematical reasoning is central to mathematical education, encapsulating ideas, processes, and the essence of human thought. This process is recognized as a critical skill for students' development. It is emphasized by the Indonesian Ministry of Education and Culture and was identified by the National Council of Teachers of Mathematics (2000) as a key goal and one of the four pillars

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of mathematics education (Sukirwan et al., 2018). These endorsements highlight the crucial role of reasoning in enabling students to solve mathematical problems effectively. Mathematical reasoning is defined as the process of deriving solutions from presented problems. It not only enhances problem-solving skills but also makes complex mathematical concepts more accessible and understandable for students (Sunay & Erdogan, 2024). Additionally, it enhances students' ability to formulate logical arguments, thereby enriching their overall understanding of mathematics. By promoting mathematical reasoning, educational strategies aim to prepare students better to engage and overcome with challenging mathematical content, ensuring their proficiency and conceptual clarity.

The importance of reasoning starkly contrasts with the actual educational outcomes observed. According to a survey by OECD using the Programme for International Student Assessment (PISA), there are significant gaps in students' mathematical reasoning abilities. Since its inception in 2000, PISA has aimed to assess the knowledge, skills, well-being, and equity of 15-year-old students globally, with mathematical reasoning as one of the main evaluated competencies (Wijaya et al., 2024). In the 2022 PISA assessment, Indonesia's average mathematics score significantly decreased by 13 points to 366, down from 379 in 2018 (Wijaya et al., 2024). Additionally, numerous empirical studies have shown that Indonesian students' achievements in mathematical reasoning remain suboptimal (Lestari, 2019). Further, empirical evidence indicates that the mathematical reasoning of Indonesian high school students is also notably weak (Johar et al., 2017). This decline underscores an urgent need for targeted interventions to enhance mathematical reasoning skills among students in Indonesia.

Mathematical reasoning, a cornerstone of mathematical proficiency, extends beyond mere calculation abilities. It is a comprehensive cognitive skill that includes analysis, generalization, integration, and solving non-standard problems (Suparman et al., 2021). Regrettably, many students exhibit mathematical reasoning skills that are largely imitative, showing a dependency on memorization over genuine understanding. This approach restricts their capacity to apply learned concepts in new contexts (Marasabessy & Hasanah, 2021). To overcome these obstacles, there is a critical need for pedagogical strategies to improve students' analytical, reasoning skills, and boost their confidence in handling the complexities of mathematics. An integrative approach is crucial for fostering a deeper and more flexible understanding of mathematical concepts among students.

Research on three-dimensional (3D) geometry highlights the difficulties students face with abstract mathematical concepts, particularly their struggles with visualizing and describing 3D shapes effectively (İbili et al., 2020). The complexity of 3D geometry requires not only psychomotor skills but also critical thinking and reasoning abilities (Fujita et al., 2020). These demands often lead to difficulties in understanding and solving 3D geometry problems, emphasizing the need for improved educational strategies (Hallowell et al., 2015). Additionally, empirical studies have shown that students commonly make conceptual and procedural errors in 3D geometry (Moss & Lamberg,

2019). These errors are primarily due to limited abstraction and visualization skills, which are essential for accurately describing spatial relationships and calculating distances between points and area in three-dimensional spaces. Such findings underscore the importance of instructional methods that effectively enhance these foundational skills.

Similarly, GeoGebra, has emerged as a leading technology in the new generation of mathematics learning tools. It has transformed the landscape of mathematics education, providing easy and free access to mathematical processes and tools for communities worldwide (Tamam & Dasari, 2021). According to (Celen, 2020), various studies have demonstrated the ability of GeoGebra to assist in fostering discovery and experimentation processes among students in classrooms. Its visualization features effectively assist students in formulating various mathematical conjectures, pointing towards its significant role in promoting an interactive and student-centered learning environment. Moreover, discovery learning is a mindset in which students incorporate concepts through observation, classification, hypothesizing, explaining, measuring, and concluding (Druckman & Ebner, 2018). Additionally, discovery learning is described as a model and strategy that emphasizes student activity, offering learning opportunities through exploration (Tran et al., 2014). It can be facilitated through various strategies in the classroom, highlighting its adaptability and effectiveness in fostering a deeper understanding and engagement among students. This model not only improves the learning experience but also prepares students for more complex problem-solving and knowledge application in real-world contexts (Simamora et al., 2018). Therefore, GeoGebra software supported the discovery, which can be applied in mathematics teaching to promote students' enhanced mathematical reasoning ability in geometry topics.

Numerous empirical studies have examined the use of GeoGebra software in discovery learning (Marion, Abdullah, & Rahman, 2023; Murni et al., 2017). The implementation of GeoGebra-assisted discovery learning significantly improves students' understanding of geometrical concepts (Marion, Abdullah, & Rahman, 2023). Additionally, discovery learning facilitated by GeoGebra software substantially enhances students' mathematical problem-solving and representation skills (Murni et al., 2017). Furthermore, GeoGebra-assisted discovery learning positively affects students' critical and creative mathematical thinking (Batubara, 2019). Other studies have also shown that integrating GeoGebra software with discovery learning significantly improves students' spatial visualization skills (Suparman et al., 2021).

Similarly, Thapa et al. (2022) demonstrated that the integration of GeoGebra in mathematics instruction fosters meaningful learning, promotes student motivation, and enhances active engagement, particularly in understanding geometric concepts such as circles. Moreover, Triet et al., (2024) showed that the use of a GeoGebra-supported 5E learning model significantly improves students' comprehension and retention of geometric concepts, such as the area of a trapezium, while also increasing engagement and positive attitudes towards mathematics. In addition, Nguyen et al. (2023) found that combining the flipped classroom model with GeoGebra software

significantly enhances students' problem-solving abilities and learning attitudes, further supporting the role of GeoGebra in creating an effective, student-centered learning environment.

While there is a growing body of research on the use of technology in mathematics education, particularly with tools like GeoGebra, most studies primarily focus on its effectiveness in enhancing conceptual understanding, visualization, and problem-solving skills. Previous research has demonstrated that GeoGebra supports students in exploring mathematical concepts dynamically, yet limited studies specifically examine its role in developing mathematical reasoning, particularly in the domain of 3D geometry. Since mathematical reasoning is a critical competency in higher-order thinking, it is essential to investigate how GeoGebra-assisted discovery learning impacts this skill beyond general problem-solving.

Additionally, while discovery learning has been widely studied as an instructional approach that fosters student engagement and inquiry-based learning, few studies explore its interaction with students' emotional stability. Emotional stability is known to influence cognitive processes, learning motivation, and academic performance, but there is a lack of research on how it affects students' ability to engage in mathematical reasoning, especially in a technology-assisted learning environment.

The present study examines the effectiveness of discovery learning supported by GeoGebra on Indonesian high school students' mathematical reasoning achievement in 3D geometry, considering this aspect of students' emotional stability. The following research questions were projected to achieve the purpose of this study.

1. How is the profile of students' mathematical reasoning viewed by the learning model and emotional stability?
2. What is the size of the effect GeoGebra assisted discovery learning, and only discovery learning, has on students' mathematical reasoning acquisition?
3. Is there any significant difference in mathematical reasoning achievement between students who learn using only discovery learning, GeoGebra-assisted discovery learning, and directed learning?
4. Is there any significant difference in mathematical reasoning achievement between students who have high, moderate, and low emotional stability?
5. Is there any significant interaction effect between the learning model and emotional stability on the acquisition of students' mathematical reasoning?

## CONCEPTUAL FRAMEWORK

### Mathematical Reasoning

Reasoning is defined as a process of thought used to make statements and reach conclusions in problem-solving that is not strictly based on formal logic and is not limited to evidence (Brown et al., 2010). Specifically, mathematical reasoning involves the ability to analyze, generalize, synthesize/integrate, provide accurate reasoning, and solve non-routine problems (Jeannotte & Kieran, 2017). (Hjelte et al., 2020) argue that mathematical reasoning is a fundamental objective of mathematics education. It represents a high level of mathematical thinking characterized by the capacity to think logically and systematically (Mueller et al., 2014). This implies that mastery of mathematics depends on mathematical reasoning, as it enables the generation of ideas and accurate interpretation of mathematical concepts.

Moreover, research suggests that emotional stability plays a crucial role in mathematical reasoning. Individuals with high emotional stability are more likely to exhibit logical and systematic thinking, as they can regulate negative emotions that may arise when faced with complex mathematical challenges (Hjelte et al., 2020). Conversely, individuals with low emotional stability are more prone to experiencing math anxiety, stress, and cognitive overload, which can hinder their ability to engage in effective reasoning (Pekrun & Linnenbrink-Garcia, 2012). Therefore, understanding the relationship between emotional stability and mathematical reasoning can provide insights into how affective factors influence students' cognitive abilities in mathematics.

### Emotional Stability

Emotion is defined as the distinct feelings and thoughts that emerge in an individual in response to perceived changes in their body when confronted with specific situations (Pekrun & Linnenbrink-Garcia, 2012). These alterations arise from a complex assessment carried out by the individual's nervous system in response to internal or external stimuli. Emotions represent psychological states or feelings that vary across different circumstances. These emotional shifts can significantly influence an individual's behavior and activities, highlighting the importance of emotional stability in effectively managing these changes.

Emotional stability is defined as the condition in which an individual, upon exposure to emotional stimuli from external sources, does not succumb to emotional disturbances like depression or anxiety (Chaturvedi & Chander, 2010). Emotional stability can be observed through an individual's daily behavior, evident during social interactions, which include work performance, creativity, and tendencies toward worry. This trait involves the capability to manage and regulate one's emotions, incorporating the skills to recognize, understand, and control negative emotions such as fear, anger, and sadness, as well as to sustain positive emotions like joy and calmness (Barrick & Mount,

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2012). Individuals with high emotional stability are generally better equipped to cope with emotional fluctuations.

Emotional stability includes three factors such as emotional adequacy, emotional maturity, and emotional control (Li & Ahlstrom, 2016). Particularly, emotional adequacy refers to the appropriateness of emotional reactions to received stimuli, including the type, content, and direction emotions. Additionally, emotional maturity is characterized by the ability to respond emotionally in an appropriate manner in unpleasant situations and under certain conditions. Subsequently, emotional control refers to a crucial part of self-adjustment and mental health. Emotional maturity, control, and adequacy of functioning well and healthily indicate good emotional health and adjustment, ultimately supporting the achievement of stable emotions.

Emotional stability has significant implications for academic performance, particularly in subjects that require logical reasoning, such as mathematics. Students with high emotional stability are more likely to persist through challenging problems, remain calm under pressure, and effectively process information, which are essential qualities for mathematical reasoning (Li & Ahlstrom, 2016). In contrast, students with low emotional stability often experience higher levels of anxiety and frustration, which can impair their ability to think logically and systematically (Barrick & Mount, 2012). Research has also linked emotional stability to mathematics anxiety, where students with lower emotional stability tend to experience greater fear and apprehension when solving mathematical problems, negatively affecting their problem-solving abilities (Pekrun & Linnenbrink-Garcia, 2012).

Understanding the connection between emotional stability and mathematical reasoning is crucial in designing learning environments that cater to students' affective and cognitive needs. Investigating how emotional stability interacts with mathematical reasoning will provide insights into how students' affective states influence their ability to engage with complex mathematical problems and how educators can develop strategies to support students in overcoming emotional barriers to learning mathematics.

## Discovery Learning

Discovery learning is a cognitive process where students acquire concepts through activities such as observation, classification, hypothesizing, explaining, measuring, and concluding (Druckman & Ebner, 2018). Tran et al., (2014) identified three main characteristics of discovery learning: (1) activities that promote the assimilation of new information into students' prior knowledge bases, (2) interest-based activities that are student-driven, allowing students to determine the sequence and frequency of their learning tasks, and (3) problem-solving activities that stimulate the creation, integration, and generalization of knowledge. Discovery learning is part of heuristic learning, an educational approach that utilizes methods designed to engage students in a process-oriented

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approach, encouraging them to lead, discover independently, and reflect throughout their learning activities.

In the context of discovery learning, emotional stability plays a critical role in how students respond to the open-ended nature of this learning approach. Since discovery learning requires active exploration, self-directed problem solving, and independent thinking, students must be able to manage frustration, persist through uncertainty, and regulate their emotions when facing difficulties (Druckman & Ebner, 2018). Students with high emotional stability are more likely to embrace challenges, stay engaged, and remain resilient when encountering difficult concepts. In contrast, students with low emotional stability may experience heightened stress, discouragement, or frustration when confronted with the ambiguity and cognitive demands of discovery-based tasks (Tran et al., 2014).

Thus, emotional stability serves as an essential factor in determining how well students adapt to discovery learning environments. By understanding his relationship, educators can develop strategies to support students who struggle with emotional regulation, ensuring that all learners can fully engage with discovery-based mathematics instruction and maximize their mathematical reasoning potential.

During the problem formulation phase, students identify various issues within the content. The data acquisition phase allows students to explore these problems by gathering information from online and offline sources, observing objects, conducting interviews, and performing experiments. In the data processing phase, students organize, classify, tabulate, and analyze the data. During the verification phase, educators help students validate their hypotheses based on their data analysis. Finally, in the generalization phase, students draw conclusions based on their verification results, incorporating feedback from educators and peers. These phases of discovery learning, especially when supported by GeoGebra software, are effectively used in geometry lessons.

GeoGebra was developed by Markus Hohenwarter in 2001. According to (Hohenwarter et al., 2008), GeoGebra represents a software program designed to integrate geometry, algebra, and calculus within a single dynamic environment. This program is available to download for free from [www.geogebra.org](http://www.geogebra.org). GeoGebra is a Java-based software that requires the Java Runtime Environment (JRE) to be installed on a computer as a prerequisite. It is a dynamic geometry system capable of constructing points, vectors, segments, lines, and conic sections, as well as functions, and altering these constructions subsequently in which equations and coordinates can be input directly (Celen, 2020). One of GeoGebra's unique features is that an object in the geometry window correlates to an equation in the algebra window and vice versa (Zengin et al., 2012). Hence, GeoGebra can handle variables for vectors, numbers, points, derivatives, and integrals of functions; it can even provide commands such as root or extremum.

The GeoGebra program offers substantial benefits for both teachers and students (Hohenwarter et al., 2008). GeoGebra can be installed on personal computers and is available for usage at any time and location, in contrast to commercial software, which is usually only accessible at school. Teachers can utilize GeoGebra to construct dynamic online learning environments where students actively explore a variety of mathematical concepts. Additionally, the use of GeoGebra in mathematics learning offers several advantages: Firstly, it enables the quick and accurate production of geometric drawings, surpassing traditional tools like pencils, rulers, or compasses. Secondly, its animation and manipulation (dragging) capabilities offer students a clearer visual understanding of geometric concepts. Thirdly, it serves as a tool for feedback/evaluation, helping to verify the accuracy of the drawings produced. Fourthly, GeoGebra simplifies the process of investigating or demonstrating the properties of geometric objects, therefore improving the learning environment for both educators and students.

## METHODOLOGY

### Research Design and Approach

This study used a quantitative, quasi-experimental methodology to examine how GeoGebra-assisted discovery learning affected students' mathematical reasoning achievement. Numerous studies advocate for employing a quasi-experimental design within a quantitative framework as an effective approach to test and address hypotheses linked to theoretical assertions. Additionally, this study employed a post-test-only control group design. A post-test-only control group design was implemented to minimize potential threats to internal validity that may arise from pretest sensitization effects (Campbell & Stanley, 1963).

In this setup, two experimental classrooms and one control classroom were assigned as follows:

1. The GeoGebra-assisted discovery learning (DL+G) group engaged in interactive and visual exploration of 3D geometry concepts using GeoGebra software.
2. The standard discovery learning (DL) group participated in similar exploratory activities but without GeoGebra, relying solely on traditional learning tools.
3. The directed learning (DI-Control Group) group followed a teacher-led instructional approach, where concepts were introduced explicitly, and students were guided step-by-step through structured problem-solving activities.

The intervention was conducted over three sessions, with each classroom receiving ninety minutes of instruction per sessions to ensure comparability across groups. The instructional process within each condition followed a structured lesson plan, ensuring consistency in content while varying the mode of instruction.

### *Use of GeoGebra for Analyzing Problems*

To further support students in visualizing and analyzing geometric problems, GeoGebra was integrated into the lessons to enhance conceptual understanding. The specific usage included:

- *Provision of Figures:* The teacher initially provided problem scenarios, but students actively constructed and manipulated figures within GeoGebra.
- *Manipulatable Features:* Students were encouraged to drag points, adjust angles, and measure segment lengths to dynamically observe changes.
- *Key GeoGebra Features Used:* Measure Tool, Point Movement, and Dynamic Angle Adjustments

### *Differences in Learning Approaches*

#### 1. GeoGebra-assisted Discovery Learning (DL+G)

Students actively manipulated objects, formulated conjectures, and tested hypotheses using GeoGebra (Hardi et al., 2021). The phases for implementing the discovery learning model include stimulation, problem formulation, data acquisition, data processing, verification, and generalization. In the stimulation phase, educators identify relevant content or pose questions to guide students in formulating problems. Exploring distance and angle in 3D geometry using the discovery learning model with GeoGebra as follows:

##### (1) The distance between points or between a point and a line in 3D space.

The following activities illustrates how students explore the concept of distance and angle in 3D space using GeoGebra. Specifically, they learn how to determine the distance between two points, or between a point and a line or plane, through step-by-step visual construction.

##### a. Stimulation

How do we measure the distance between two points in 3D space?

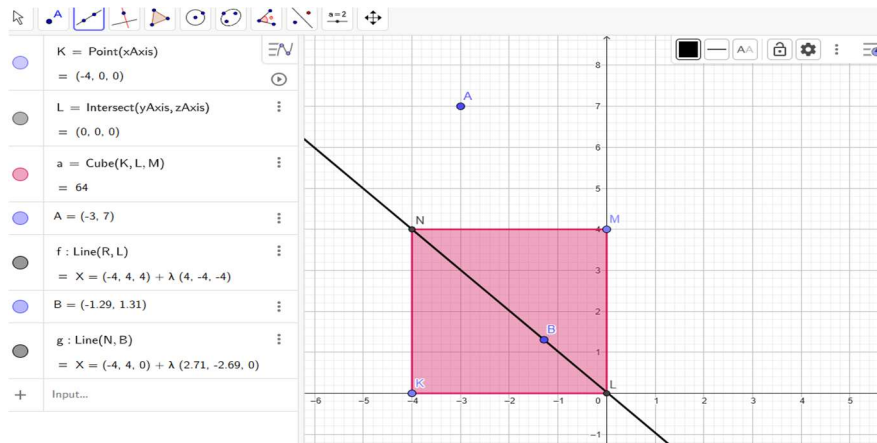


Figure 1. Students Begin Constructing 3D Representations by Defining Points, Lines, and Planes on a Two-Dimensional GeoGebra Interface

In figure 1, students begin by using GeoGebra to define basic geometric elements such as points and lines. Although the interface shown is two-dimensional, this step serves as a foundation for constructing three-dimensional representations. Students identify points (e.g., A, B, and C) and connect them with lines, preparing for spatial reasoning in the following stages.

b. Problem Statement, data collection, data processing and verification

After the instruction contained in the step “a” is done, then the result will look like in the following view

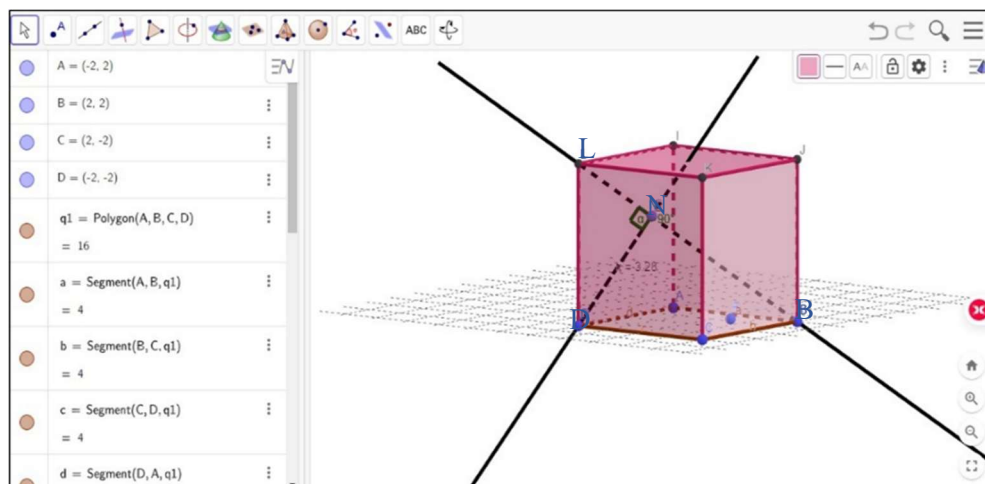


Figure 2. Outcome view of the distance between point and line

Figure 2 displays the outcome of measuring the distance between a point and a line. In this stage, students will have completed the previous steps and are now able to visualize the relationships between geometric objects in 3D space. As shown in the figure, a cube is displayed with line LB intersecting with point D. The next task involves measuring the perpendicular distance between point D and line LB, as indicated by the highlighted perpendicular segment. This allows students to apply the concepts of distance and geometric relationships to a practical problem.

c. Generalization

At this stage, each group is asked determining the distance between point D and line LB with a side length of 4 cm by identifying the perpendicular line segment connecting point D to line LB. After this, the method involves using the ratio of the areas of two triangles to calculate the distance. This approach ensures accuracy in finding the shortest distance. The calculation follows these steps: Establish a perpendicular segment from D to LB. Relate the distances using geometric properties of triangles and apply proportionality of their areas. Derive the exact distance based on the given edge length of 4 cm.

$$\text{Area}(\triangle DBL) = \text{Area}(\triangle BLD)$$

$$\frac{1}{2} \cdot DB \cdot LD = \frac{1}{2} \cdot BL \cdot DN$$

$$\frac{1}{2} \cdot 4\sqrt{2} \cdot 4 = \frac{1}{2} \cdot 4\sqrt{3} \cdot DN$$

$$DN = \frac{4\sqrt{6}}{3}$$

Here, DN represents the distance from point D to the line LB, as illustrated in Figure 2. This is the shortest distance and is determined by constructing a perpendicular segment from D to LB.

2. Discovery Learning Only (DL)

Students used paper-based tools for visualization and problem solving instead of GeoGebra, relying on conventional methods for exploration. This method follows structured phases of Discovery Learning as outlined by (Hardi et al., 2021), which include stimulation, problem formulation, data collection, data processing, verification, and generalization. It emphasizes self-guided learning, logical reasoning, and manual problem-solving without instant digital feedback. By following the six phases of discovery learning, students actively construct their knowledge while strengthening their ability to work with mathematical concepts systematically.

3. Directed Learning (DI)

The teacher controlled the demonstration, and students observed the effects rather than exploring independently. The implementation of Directed Learning follows key phases of instruction, which include stimulation, explanation, demonstration, guided practice, independent practice, and assessment.

This implementation ensured that students not only visualized mathematical problems but also actively engaged in mathematical reasoning through interactive exploration.

To ensure the validity of findings, students were assigned to their respective classroom based on purposive sampling, with a focus on selecting those with lower early mathematics achievement to assess the impact of instructional interventions on students who typically struggle with mathematical reasoning. The effectiveness of each instructional method was measured through post-test scores, assessing students' ability to apply reasoning skills in solving complex mathematical problems.

This differentiation in instructional methods allows for a clear attribution of effects to the learning interventions, ensuring methodological rigor and facilitating replicability in future research.

## Participants

This study involved 108 twelfth-grade students of a public senior high school in West Java, Indonesia. They were distributed in two experimental classrooms and one control classroom, with each classroom consisting of 36 students. Participants were selected using purposive sampling in collaboration with the school, which identified three intact Grade-12 classes that met the study's inclusion criteria (Etikan, 2016). To ensure equivalence between groups, we considered students' prior mathematics achievement based on their previous semester grades. This selection criterion ensured that both groups had similar ability before the intervention. Additionally, all students were from the same school and grade level.

Additionally, they were chosen as the participants in that the average early mathematics achievement of those classrooms was less than other classrooms in twelfth grade, ensuring that the intervention targeted students who might benefit the most from GeoGebra-assisted discovery learning. This selection was also intended to examine whether students with lower mathematical proficiency could improve their reasoning skills through the intervention while controlling for prior mathematical ability to reduce variability in the sample.

## Instrument

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The instruments employed in this study were a mathematical reasoning test and an emotional stability questionnaire. The mathematical reasoning test consisted of four mathematical problems designed to assess core aspects of mathematical reasoning indicators, including: spatial reasoning, logical thinking, and generalization. (see Appendix 1). The maximum possible score for the test was 16 points (4 points per problem) (Saputri et al., 2017). The scoring rubric accounted for accuracy of the solution, depth of reasoning, and clarity of explanation.

This test was validated and proven reliable, as indicated by Pearson's correlation ( $r \geq 0.75$ ) for each item and a Cronbach's Alpha of 0.91. The emotional stability questionnaire contained three indicators: emotional adequacy, emotional maturity, and emotional control. It was specifically used to measure students' emotional stability in the context of 3-D geometry learning (see Appendix 2). This questionnaire, based on the theory proposed by Hurlock (1978), which outlines the three components of emotional stability, was also validated and proven reliable, evidenced by Pearson's correlation ( $r \geq 0.70$ ) for each item and a Cronbach's Alpha of 0.83. According to Özdemir et al. (2019), a Pearson's correlation  $r$  above 0.70 for each item indicates validity. Additionally, Bonett & Wright (2014) assert that a Cronbach's Alpha above 0.75 confirms the reliability of the instrument. These validations confirm that the mathematical reasoning test and emotional stability questionnaire are suitable tools for measuring students' mathematical reasoning and emotional stability, respectively.

### Data Collection and Analysis

A mathematical reasoning test was administrated after the intervention in the GeoGebra-assisted discovery learning, discovery learning, and directed learning. Additionally, the questionnaire on emotional stability was also administered to students during the intervention process. Hedges'  $g$  equation was applied to measure the effect size of discovery learning promoted by GeoGebra on the acquisition of students' mathematical reasoning (Jaya & Suparman, 2021). Moreover, the value of Hedges'  $g$  was classified as  $0.00 \leq g \leq 0.20$  (weak),  $0.21 \leq g \leq 0.50$  (modest),  $0.51 \leq g \leq 1.00$  (moderate), and  $g > 1.00$  (strong) (Cohen et al., 2002). In addition, descriptive statistics were used to describe students' emotional stability in which classified students' emotional stability as  $x < \bar{x} - s$  (low),  $\bar{x} - s \leq x \leq \bar{x} + s$  (moderate), and  $x \geq \bar{x} + s$  (high) (Azwar, 2022). Inferentially, a two ways ANOVA test was used to examine students' different mathematical reasoning viewed by learning model and emotional stability. According to Rutherford (2011), the data is assumed to be normally distributed, which justifies the application of a two-way ANOVA test.

## RESULTS

## The Profile of Students' Mathematical Reasoning Viewed by Learning Model and Emotional Stability (RQ1)

Students' mathematical reasoning achievement from learning 3-D geometry for each classroom is shown in Table 1. It shows that the mathematical reasoning of students who learn using GeoGebra-assisted discovery learning was higher than the mathematical reasoning of students who learn using discovery learning and directed learning.

Classroom	Sample Size	Mean	Deviation Standard
Discovery Learning	36	5.86	2.66
GeoGebra-assisted Discovery Learning	36	9.22	3.13
Directed Learning	36	5.53	2.71

Table 1: The acquisition of students' mathematical reasoning for each learning model

Additionally, the acquisition of students' mathematical reasoning from learning 3-D geometry for each level of students' emotional stability is shown in Table 2. It indicates that students with high emotional stability have better mathematical reasoning scores compared to students with moderate and low emotional stability. This finding suggests that there are differences in students' mathematical reasoning scores across emotional stability levels, with higher scores observed among students with high emotional stability.

Level of Emotional Stability	Sample Size	Mean	Deviation Standard
High	19	7.74	2.96
Moderate	74	6.59	3.29
Low	15	5.17	3.58

Table 2: The acquisition of students' mathematical reasoning viewed by emotional stability

Based on examples of students' responses to the mathematical reasoning ability test categorized by emotional stability, the results are presented as follows:

### 1. High Emotional Stability

Students are able to accurately illustrate contextual problems using diagrams. They are also able to write down important information such as sides' length, as well as, the positions of the lamp and switch, that is a basis for determining distance. The calculation step uses the Pythagorean theorem that has been explained, showing the relationship between side length in the G'OG triangle and obtaining precise results.

Answers from students with high emotional stability showed clarity, systematic structure, precision and consistency in solving problems sequentially, resulting in accurate answers. This indicates excellent focus during the analysis stage. The solution to the answer above is presented in the image below:

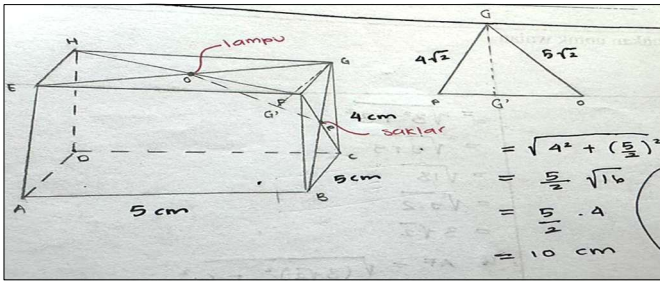


Figure 3. Sample Solution of a Student with High Emotional Stability in the Mathematical Reasoning Test

## 2. Moderate Emotional Stability

Students are able to illustrate contextual problems with diagrams and outline the calculation steps. However, there are minor errors, such as inputting incorrect values in the calculations, resulting in less accurate outcomes. Answers from students with moderate emotional stability show clarity, systematic organization, and detail, even though the results are incorrect. The problem-solving steps are sufficiently detailed, indicating good focus during the analysis stage, despite potential pressure while solving the problem. The solution to the answer above is presented in the image below:

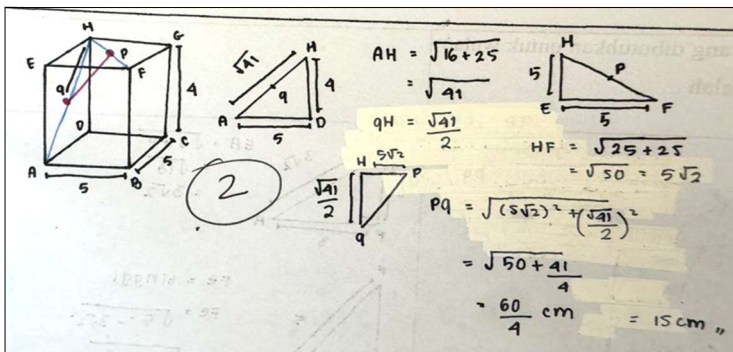


Figure 4. Sample Solution of a Student with Moderate Emotional Stability in the Mathematical Reasoning Test

## 3. Low Emotional Stability

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Students are able to illustrate contextual problems in the form of a rectangular prism and mark a point within the prism as the point whose distance to the space diagonal is calculated. Students apply the Pythagorean theorem but provide very minimal additional explanations. Students with low emotional stability still manage to complete the problem, even though the result is incorrect. This demonstrates an effort to maintain focus, prioritizing speed over elaborating the process. The solution to the answer above is presented in the image below:

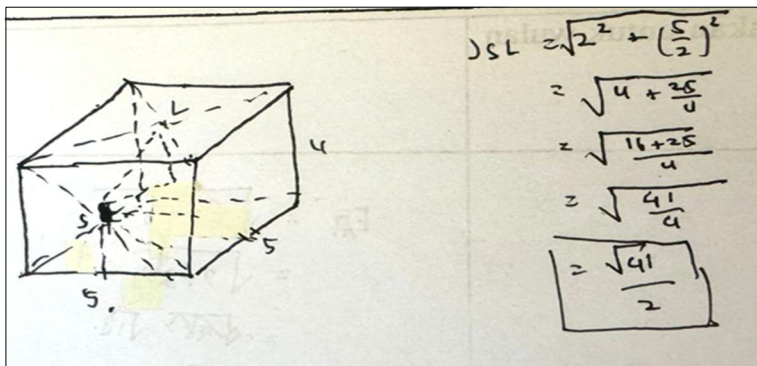


Figure 5. Sample Solution of a Student with Low Emotional Stability in the Mathematical Reasoning Test

### The Effect Size of Experimental Intervention on the Acquisition of Students' Mathematical Reasoning (RQ2)

The effect size using Hedge's equation of experimental interventions, such as GeoGebra-assisted discovery learning and only discovery learning, on the acquisition of students' mathematical reasoning is shown in Table 4. It reveals that the implementation of GeoGebra-assisted discovery learning had a significantly strong positive effect on students' mathematical reasoning acquisition. In contrast, discovery learning alone showed a weak positive effect on students' mathematical reasoning. It indicates that GeoGebra-assisted discovery learning is more effective in optimizing students' mathematical reasoning compared to using discovery learning alone.

Experimental Intervention	Effect Size	Z-value	Sig.
GeoGebra-Discovery Learning	1.247	4.88	0.00
Discovery Learning	0.122	0.52	0.60

Table 4: The effect size of experimental intervention on the acquisition of mathematical reasoning

### Students' Mathematical Reasoning Viewed by Learning Model and Emotional Stability (RQ3, RQ4, & RQ5)

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According to Rutherford (2011), it is assumed that the data are normally distributed, allowing for the application of a two-way ANOVA test. Consequently, a normality test using the Kolmogorov-Smirnov framework was performed (see Table 5). It shows that the significance values from the Kolmogorov-Smirnov normality test for each category exceeded 0.05, indicating that the data concerning students' mathematical reasoning, as viewed by both learning model and emotional stability, are normally distributed. Therefore, the precondition for normality has been met, justifying the use of a two-way ANOVA test.

Mathematical Reasoning	Category	Stat.	N	Sig.
Learning Model	GeoGebra-assisted discovery learning	0.938	36	0.201
	Discovery learning	0.959	36	0.194
	Directed learning	0.949	36	0.171
Emotional Stability	High	0.927	19	0.153
	Moderate	0.897	74	0.227
	Low	0.951	15	0.539

Table 5: The results of the normality test using the framework of Kolmogorov-Smirnov

The results of the two-way ANOVA are presented in Table 6. It indicates that the significance value of the  $F$  test for the learning model variable was less than 0.05, suggesting a significant difference in mathematical reasoning among students learning 3-D geometry through GeoGebra-assisted discovery learning, only discovery learning, and directed learning. Furthermore, the results of the post-hoc test using LSD, presented in Table 7, reveal significant differences in mathematical reasoning between students using GeoGebra-assisted discovery learning and those using only discovery learning, as well as between students using GeoGebra-assisted discovery learning and directed learning. However, Table 6 also indicates that students learning through only discovery learning and directed learning do not significantly differ in their mathematical reasoning.

Source	Mean Square	df	F-value	Sig.
Learning Model	96.845	2	12.174	0.000
Emotional Stability	11.282	2	1.418	0.247
Learning Model * Emotional Stability	9.912	4	1.246	0.297

Table 6: The results of two ways ANOVA test

Table 6 indicates that the significance value of the  $F$  test for the source of emotional stability was greater than 0.05, suggesting that there is no significant difference in mathematical reasoning between students with high, moderate, and low emotional stability. Additionally, Table 7 reveals that the significance value of the  $F$  test for the interaction between the learning model and emotional

stability was also above 0.05, indicating that there is no significant interaction effect between the learning model and emotional stability on the acquisition of students' mathematical reasoning.

Learning Model	Mean Difference	Sig.
GeoGebra-assisted Discovery Learning – Only Discovery Learning	3.630	0.00
GeoGebra-assisted Discovery Learning – Directed Learning	3.443	0.00
Directed Learning – Only Discovery Learning	0.187	0.84

Table 7: The results of post-hoc test using LSD

## DISCUSSION

### The Effectiveness of GeoGebra-assisted Discovery Learning on the Achievement of Students' Mathematical Reasoning Considering Emotional Stability

This study examines the effect of GeoGebra-supported discovery learning on mathematical reasoning achievement among Indonesian high school students, with a particular focus on 3D geometry. Through dynamic visualization and interaction, GeoGebra has been recognized as a significant educational tool for fostering a deeper understanding of mathematical concepts (Celen, 2020; Zengin et al., 2012). We intend to explain the findings of the present study, shedding light on the implications of discovery learning supported by GeoGebra for improving students' mathematical reasoning skills.

A key finding of this study is that GeoGebra-assisted discovery learning significantly improves students' mathematical reasoning skills. Students in this group scored higher on average than those in traditional discovery learning and direct learning. Previous studies also support this finding (Marion, Abdullah, & Rahman, 2023). This outcome aligns with previous studies showing students' positive response to GeoGebra-assisted discovery learning, with significant improvements across learning cycles. It notably enhances students' mathematical reasoning in Cartesian coordinates. In his theoretical framework, Van Joolingen (1999) argues that discovery learning is a vital tool for promoting cognitive development within educational settings. The findings from this research support this theory, demonstrating that GeoGebra-assisted discovery learning significantly enhances mathematical reasoning with a strong effect size of 1.24. When GeoGebra is incorporated into discovery learning frameworks, it significantly boosts students' mathematical reasoning abilities. This substantial impact underscores the effectiveness of integrating GeoGebra into discovery learning environments.

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Furthermore, the study found no significant differences in mathematical reasoning abilities among students with varying levels of emotional stability. Although Cheng et al. (2022) suggest a positive correlation between emotional stability and academic achievement, the results indicate that GeoGebra-assisted discovery learning improves mathematical reasoning regardless of students' emotional stability levels. These findings highlight that while high emotional stability typically correlates with better academic outcomes, GeoGebra's influence on mathematical reasoning is independent of emotional stability.

### **Implications to Mathematics Learning**

These findings have significant practical implications. The positive impact of GeoGebra-assisted discovery learning on mathematical reasoning suggests its effective use as an alternative instructional approach. The GeoGebra program enhances knowledge construction and reasoning skills by enabling students to visually explore three-dimensional concepts. Participants reported that GeoGebra simplifies the visualization of spatial relationships, transforming their perception of mathematics from challenging to accessible. This shift can motivate students to engage more deeply with mathematical concepts, leading to improved learning outcomes.

### **Limitations and Suggestions**

The research, however, encountered limitations relating to GeoGebra-assisted learning's technological requirements. Since students were unable to bring their laptops to class, their dependency on laptops for implementing the learning process posed a challenge, inhibiting their full participation in learning activities. Due to this limitation, education institutions must ensure that students have adequate access to computer lab facilities or laptops in order to fully take advantage of GeoGebra-assisted discovery learning.

### **CONCLUSION**

This study shows that learning through GeoGebra-based discovery learning significantly enhances the mathematical reasoning abilities of students compared to discovery learning without GeoGebra or direct instruction. GeoGebra offers a dynamic, visual, and interactive platform by which learners can investigate mathematical ideas in an easier and more meaningful way. This interactive approach not only strengthens students' conceptual understanding but also fosters critical thinking and reasoning skills. On the other hand, even though discovery learning without GeoGebra encourages active participation and exploration of concepts, it does not avail the same cognitive support like visualization facilities offered by GeoGebra. Meanwhile, direct instruction, which

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focuses on teacher-led instruction, reflects a lower impact on developing students' reasoning abilities.

Furthermore, this study focused on the effect of emotional stability on reasoning in mathematics. The students in the category of high emotional stability always outperformed others in the categories of medium and low emotional stability. This indicates that emotional stability and concentration is important in solving problems and deducing logically. However, the differences among groups of emotional stability are not statistically significant; hence, it indicates that there might be other factors which mediate the relationship between emotional stability and reasoning ability.

Interestingly, the findings did not show any significant interaction between emotional stability and the learning model on students' mathematical reasoning ability. This shows that the advantages of GeoGebra-based learning can be universally applied to students with varying levels of emotional stability. These findings indicate the importance of incorporating technology-enhanced learning tools, such as GeoGebra, to foster mathematical reasoning, while also taking into account individual emotional influences in student learning outcomes. Further studies may be designed to focus on the long-term impact of the proposed intervention and study how different types of students interact with technology in various educational settings.

## ACKNOWLEDGMENTS

The authors express their gratitude to the Indonesian high school students from a public high school in West Java, Indonesia, who participated in this experiment, as well as to the educational authorities in West Java, Indonesia for granting permission to conduct the study.

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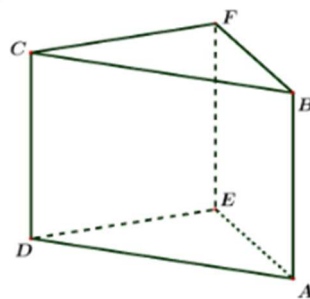
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## APPENDIX

### Appendix 1. Mathematical Reasoning Test

1. A child is assigned to observe a building from two locations. First, the child observes the building from a park with an elevation angle of  $45^\circ$ . Then, the child moves to a cake shop and observes the building with an elevation angle of  $30^\circ$ . The park is located south of the building, and the cake shop is east of the park. Given that the distance from the park to the cake shop is 100 meters, the height of the building is ....
2. Wynn's room is a block with length: width: height = 5:5:4, on the ceiling of the room, there is a lamp located the right at the center of the ceiling plane. On one wall of the room, there is a switch located the right in the center of the wall. Calculate the distance from the lamp to the switch and explain how to determine the distance!
3. Wulan will decorate her new cafe with decorative lights. The decorative lights will be installed in the center of the ceiling. To add the beauty, she plans to add LED lights installed from each corner of the ceiling to the center of the decorative lights. After conducting a price survey in the market-place it turns out that the LED lights sold in the market are in the form of rollers, where 1 roller is 3 m long with a price of 35,000 IDR/ roller. If the new cafe is 8 m x 8 m x 8 m, then it can be concluded that the amount of funds needed for her to buy LED lights is....
4. In prism ADE.BCF,  $DA = 6$  cm and the distance of E to AD is 3 cm. What is the height of the prism? Decide whether the following statement (1) and (2) are sufficient to answer the question and give your reason!
  - a. Volume of prism is  $180 \text{ cm}^3$
  - b.  $AF = 5$  cm



### Appendix 2. Emotional Stability Questionnaire

Every statement is scaled from 1 to 5 representing strongly disagree (1), disagree (2), adequate agree (3), agree (4), and strongly agree (5).

*Emotional Adequacy*

1. I try to be someone who is liked by friends and teachers
2. I try to be a good listener to whoever I am talking to
3. I try to comfort my fellow friends who are experiencing sadness
4. I will care about friends who are experiencing a disaster
5. I always prioritize my interests over others
6. I will let my friends who are absent from school do the work given by the teacher
7. I feel that I do many things that are beneficial to others

*Emotional Maturity*

8. It doesn't take me long to adjust to something new
9. I am easily stressed if I get into trouble in my neighborhood
10. I get angry easily when my friends criticize me
11. During the learning process in class, I will study hard
12. I feel that the environment around me can accept me for who I am
13. I am aware of my strengths and weaknesses
14. I try to be ready and calm in facing tomorrow
15. I feel confident that I will succeed in what I am doing
16. I never make the same mistake a second time in carrying out a task
17. I often get tense and emotional when my work is criticized
18. I am easily offended when insulted by others
19. I feel that I am better than others
20. I feel anxious when I am among people I don't know
21. I often experience mood swings
22. I feel inferior when I hang out with friends
23. I feel worried about many things

*Emotional Control*

24. I am easily discouraged by problems in the environment around me.
25. When I get angry, I can immediately think of the consequences or risks that arise when I get angry
26. I try to hold my anger even though someone always harms me.
27. Many things in my environment change my mood.
28. I feel excessive tension every time I complete an assignment or test given by my teacher
29. I rush to make decisions because of my emotions
30. I feel sad when I fail continuously
31. I am easily offended by anything that annoys me
32. If I get angry at a friend who offends me, I usually try to control my anger first
33. Even when I'm upset, I can still do my assignments well
34. I get caught up in problems that happen to me
35. I tend to show my anger in front of people I don't like
36. I panic easily if something happens to me
37. I am still kind to my friends even though they have done something wrong to me
38. Although I have a lot of assignments, I can still excel as a student
39. I try to relax and distract myself from problems that make me tense
40. I am easily distracted by the commotion around me