Information and Communication Technology to Enhance Students' Conceptual Understanding and Reasoning Skills in Mathematics: A Case from Bangladesh

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Abstract: Nowadays, it is widely accepted that Information and Communication Technology (ICT) can promote positive changes in the teaching and learning process. This study explores how ICTs can be used in teaching and learning to enhance students' conceptual understanding and reasoning skills in mathematics. A mixed method case study (convergent parallel) approach was employed for this study, and the case was selected from a government school in Dhaka. The case constitutes an ICT-facilitated Grade 10 mathematics classroom, one mathematics teacher, and students of that classroom. Through a classroom-based intervention study, teachers' teaching practice with ICTs and students' conceptual understanding and adaptive reasoning in mathematics were investigated. The mathematical software GeoGebra was considered as the catalyst for the intervention, while other technologies were also used. It is found that students' mathematical concepts and reasoning ability increase if ICT is effectively applied in the teaching-learning process. Besides, different teaching-learning practices using ICTs were identified as effective pedagogical approaches to develop conceptual understanding and reasoning skills in mathematics. Finally, a few implications (theories and practices) for the findings have been articulated.

Keywords: ICT-facilitated classroom, conceptual understanding, reasoning skill, case study, intervention

INTRODUCTION

The world in the 21st century is changing fast due to the continuous advancement of technology. All such advancements are somehow connected to mathematics. Mathematics is no longer perceived as a subject for basic computations. Rather, mathematics knowledge is considered as one of the key catalysts of such advancement. According to Redecker and Johannessen (2013), students must be prepared with sufficient mathematical skills and competencies to cope with this 21stcentury world. That indicates students must have to learn mathematics with proper understanding. Research shows that students fail to develop a meaningful understanding of mathematics if it is taught with an emphasis on drill and practice and rote learning of formulas (Hellum-Alexander, 2010). It is argued that learning mathematics becomes effective when the teaching and learning approach aims to develop students' conceptual understanding rather than focusing on the correct solution to a problem (Sultana et al., 2016).

In Bangladesh, students learn mathematics from primary to secondary levels as a compulsory subject. Thus, it is desired that they will grow up with good mathematical knowledge. In reality, students' performance in mathematics is inferior (Morshed, 2013). The study also showed that most Bangladeshi students of Grade 10 fail to represent a problem in multiple ways. Instead, they can identify information and perform routine procedures according to direct instructions only (Sultana et al., 2020). One of the primary reasons behind this could be the existing teaching practice. Most mathematics teachers have limited knowledge about modern teaching strategies and practice traditional approaches to teaching (Barua et al., 2020). In a recent study, Kabir and Jalali (2021) found that Bangladeshi teachers' focus on teaching is to disseminate mathematical knowledge to the students instead of making math meaningful, and they (teachers) fail to engage students in logical and systematic reasoning. As a result, students' ability to adaptively reason does not develop, and they fail to transform real-world problems into math problems.

Information and Communication Technology (ICT) refers to all technologies used for processing information and communication. Nowadays, there has been widespread interest in innovative projects leveraging ICT for delivering services (Khan, 2017; Zhang et al., 2025), for transforming organizations (Khan et al., 2015) and governance (Bukth & Khan, 2024), for promoting positive changes in the teaching and learning process, and whatnots. Like other countries, Bangladesh has also greatly emphasized incorporating ICTs in education. Bangladesh's government has taken several initiatives, such as developing multimedia classrooms, well-equipped ICT labs, a teachers' portal, digital content, students' e-portals, etc. (Islam & Ferdosh, 2019). Despite several initiatives, research shows no remarkable change between students' performance in general and multimedia classrooms (Imon, 2017). Besides, teachers still struggle to properly use digital materials (Ropum, 2022). That is why they are found not to regularly use digital devices in their classroom teaching-learning process (Khan et al., 2012). The use of ICT in Bangladesh is mainly limited to using scientific and ordinary calculators and PowerPoint slides in mathematics classrooms (Sultana et al., 2016). They found that most teachers use ICT tools to show pictures, video clips, and diagrams related to a topic to stimulate and engage students in the lesson rather than focusing on conceptual understanding and reasoning skills. Teachers are unaware of how to teach with ICT tools to develop a concept, and there are no specific guidelines on how to practice ICTs to build students' mathematical knowledge with proper understanding (Sultana & Khan, 2017). Besides, though the potential of ICTs (e.g., multimedia, math software, etc.) for mathematics teaching is well accepted for enhancing students' performance, their impact in developed countries may not be the same as in developing countries like Bangladesh due to contextual differences. Thus, the study aims to explore how ICTs can be applied in the mathematics teaching-learning process so that students' mathematical concepts, as well as reasoning abilities, are enhanced. The research questions the study seeks to address:

RQ1. Are there significant changes in students' conceptual understanding and adaptive reasoning due to the ICT-facilitated teaching-learning process?
RQ2. How does the ICT-facilitated teaching-learning process promote students' conceptual understanding and reasoning skills in mathematics?

In this study, the mathematics lesson topic was algebra (e.g., functions and linear equations). Given the broad scope of functions and linear equations, *basic concepts of functions and linear functions, slope and graphs of linear equations, and use of linear equations in real life* (all these topics are in line with the national curriculum for Grade 10) were considered.

Research Gap

Though the National Education Policy 2010 of Bangladesh emphasizes the use of ICT in science and mathematics, no specific approach has been suggested to be followed to develop students' mathematical concepts and reasoning skills. Thus, teachers in Bangladesh are unaware of how to develop students' mathematical concepts and reasoning skills with the help of ICTs (Sultana & Khan, 2017). In addition, limited studies have focused on the pedagogical approaches using ICTs to promote students' conceptual understanding and reasoning abilities, especially in the context of developing countries like Bangladesh. As such, by drawing on a case in Bangladesh, this study aims to add new knowledge to the existing literature on the mathematics teaching-learning process using ICT in a developing country context.

LITERATURE REVIEW

Students with proper conceptual understanding and adaptive reasoning in mathematics are vital in this fast-changing world. Conceptual Understanding (CU) is one of the mathematical abilities that enables students to organize their knowledge. Mathematical understanding means knowledge about unrelated concepts and the ability to explain their relationship (Kilpatrick et al., 2001; Kenedi et al., 2019). Students with CU can apply and adapt acquired mathematical ideas to explain new mathematical concepts (Kilpatrick et al., 2001). The authors claimed that students could use several mathematical representations and communicate their ideas if they have a conceptual understanding. Adaptive reasoning (AR) is thinking logically, estimating the answer with proper justification, and judging mathematical truth (Milgram, 2007). Consequently, students with adaptive reasoning skills have the ability to estimate, support their claims with arguments or proof, and draw inferences (Kilpatrick et al., 2001). In a study, Marasabessy (2021) claimed that students' logical thinking abilities will be formed by involving them in non-routine tasks. In this situation, students connect concepts and context by their logical thinking capability. Besides, reflective thinking skills (e.g., the ability to explain, the ability to justify) are also generated by logical thinking. Students' adaptive reasoning ability is vital as it helps them cope with real-life situations (Marasabessy, 2021).

Extant literature showed that ICT with appropriate pedagogical approaches greatly influences the development of students' higher-order thinking (Laborde, 2001). The use of ICT in the teaching-learning process not only develops students' mathematical procedural skills but also develops problem-solving, reasoning, and justifying skills if it is used strategically (Pierce & Stacey, 2010). While ICT has been seen to positively influence the quality of pedagogy and students' learning, its potential would be of no use if it is used only as a technological possibility rather than in the service of educational needs (Cuban, 2001). Thus, teachers must be skilled in determining when and how technology can enhance students' learning (International Society for Technology in Education [ISTE], 2008). When teachers use technology strategically in mathematics teaching, it helps to develop students' mathematical concepts, stimulate their interest, and increase their reasoning skills in mathematics (National Council of Teachers of Mathematics [NCTM], 2011; Das, 2019). Das (2019) stated that various types of ICT tools such as graphic calculators, specialized software (e.g., GeoGebra, SymPy, Maxima), spreadsheets, databases, etc., can be used for teaching mathematics, and with the effective use of these tools, the impact of ICT in mathematics education can be maximized (Becta, 2003). Again, Smith et al. (2018) claimed that to develop students' conceptual understanding of mathematics, teachers must create an environment for students to get involved in the discussions. The focus of the discussion should be reasoning rather than simply asking for the answer (Department of Basic Education [DBE], 2018). Since using mathematical language and explaining students' ideas behind their mathematical work are vital aspects of building students' conceptual understanding, teachers should provide opportunities for the learners to speak mathematically and explain their answers with proper justifications (DBE, 2018). However, teachers should explore students' misconceptions and use them as building blocks for deeper mathematical understanding (Anthony & Walshaw, 2009). During the question-and-answer session, teachers should create an environment where students have to respond by thinking critically and with proper justification (Anthony & Walshaw, 2009), and such initiatives ultimately promote students' adaptive reasoning ability. They also argued that when students struggle to deal with a problem, effective teachers should support them rather than provide complete solutions and encourage them to search for more information, try another method, discuss the problem with peers, and reflect on their own learning. In several studies (DBE, 2018; Wassie & Zergaw, 2018), researchers have demonstrated the potential of dynamic software and apps like GeoGebra and Desmos to enhance learners' conceptual understanding. These tools are well-recognized for their ability to reinforce mathematical concepts, improve visualization, and rectify misconceptions.

Conceptual Framework of the Study

The study considered the Technology Integration Panel (TIP) proposed by Li and Dawley (2019) as a theoretical base. This framework acknowledged the multifaceted nature of teaching-learning with technology while providing a clear outline to guide educators in their journey toward achieving their goals. Unlike conventional rubrics that oversimplify technology integration by equating it with either learning or teaching, TIP adopts a three-dimensional approach encompassing pedagogy, learning context, and access to technology (Li & Dawley, 2019). The feature "pedagogy" covers the learning objectives and the instructional design. According to Li and Dawley (2019), learning objectives are the instructional goals teachers emphasize in their teaching practices, such as whether

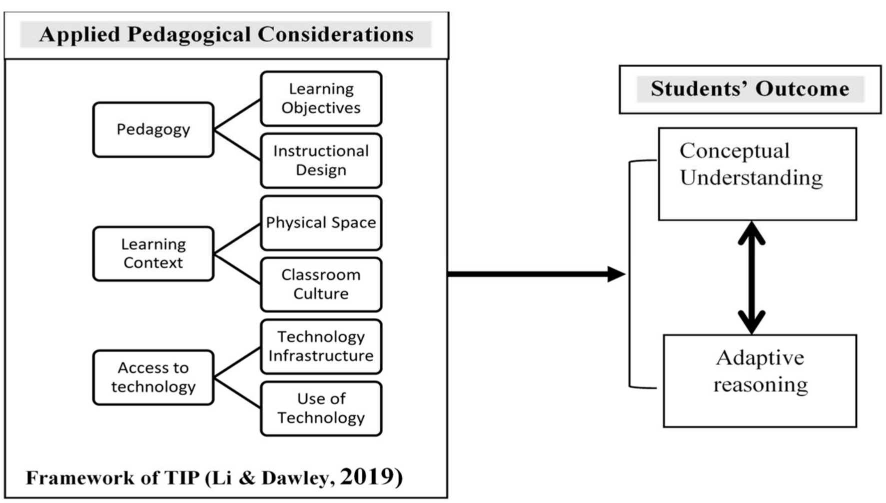


Figure 1: Conceptual Framework of the study

learning would be rote learning or meaningful learning/discovery learning. In contrast, instructional design could be student-centered or scaffolding. The "learning context" aspect includes physical space and classroom culture. In a technology-supported learning environment, classroom culture should be collaborative, and the classroom space should be designed to support a flexible and collaborative learning environment (Li & Dawley, 2019). The aspect of "access to technology" encompasses the availability of appropriate technical infrastructure and the use of technology. Based on these three-dimensional approaches, this study tried to explore the effective pedagogical considerations with ICTs to enhance students' mathematical concepts and reasoning skills, which are interconnected. The figure above (Figure 1) represents the conceptual framework of this study.

METHODOLOGY

This study employed a case study to understand the holistic and meaningful features of a complex real-life phenomenon (Yin, 2014). Again, to address the two research questions, a mixed method approach was adopted where the qualitative and quantitative data were collected simultaneously and analyzed separately to draw a more vivid comprehension of the phenomena (Creswell & Guetterman, 2019). Case study experts have recommended integrating qualitative and quantitative research in investigating the case (Yin, 2014). Thus, this study uses a *case study mixed method research approach* where the mixed method is specified as convergent parallel mixed methods (Guetterman & Mitchell, 2018). The case was chosen from a government school in Dhaka, in which an ICT-facilitated classroom, math teacher, and students of that classroom were the unit of analysis.

Research Design

An intervention was designed for the mathematics teacher in an ICT-facilitated classroom environment and the students of that class. The intervention process applied a cyclic approach of design, evaluation, and revision phases (McKenney et al., 2006). The aim of the design process was to understand teachers' mathematical practice while conducting classes with ICTs and improve the instructional sequence (if required) to enhance students' mathematical conceptual understanding and reasoning skills. The design process of the intervention is shown in Figure 2.

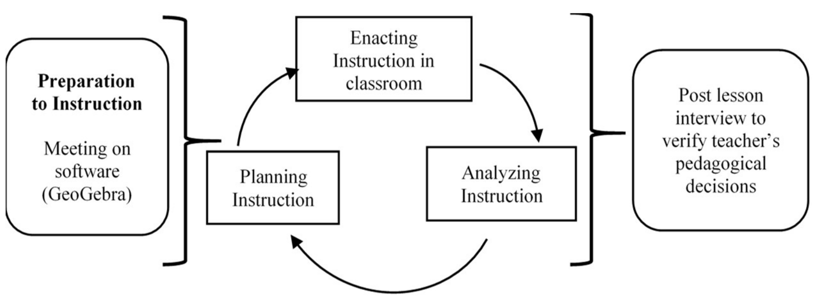


Figure 2: The design process of the intervention

In this design process, the planning stage is the session for determining the instructional goals, selecting tasks, designing and developing ICT-supported materials and activity sheets, and structuring classroom action. In contrast, the enacting session is the actual instructional sequence in the classroom. During the planning sessions, the researcher clearly explained to the teacher the primary goal of the research study. The researcher and the teacher worked as a team to design the instructional materials for the enacting sessions. The teacher's implementation of instructions was analyzed at the last stage of this mini-cycle, and further improvement was suggested for the next class (if required). After completing each mini-cycle, post-lesson interviews were conducted to evaluate the instruction and investigate the teacher's mathematical practices. In this study, GeoGebra, along with other ICT tools, was used as a primary teaching-learning instrument since they have been found effective for the topics of study (e.g., function and linear equation) (Reezan, 2013).Prior to the cyclic approach of the intervention, the researcher arranged several meetings with the teacher to let him know about the usefulness, properties, and different functional options of GeoGebra software and provided some of the pre-prepared GeoGebra materials resource links so that he could get the idea to develop the GeoGebra supported instructional materials for the intervention sessions. The researcher also assisted the teacher in developing the materials when he asked for help.

Data Collection Method

Since the teacher could play a critical role in this research, the case selection depended on the mathematics teacher. This study considered one mathematics teacher and 30 Grade 10 students as sample. The participant teacher was selected purposively as the teacher had experience teaching with technology in the actual classroom setting and was willing to conduct the classes with the support of the mathematical software GeoGebra. Students were chosen by stratified random sampling to ensure all levels (based on academic performance) of students' representation. The total number of students was divided into three strata, and then 10 students were chosen randomly from each stratum. The whole process of data collection (pilot study and main study) was done from September 2022 to March 2023. A total of 6 classes were observed during the intervention. Each class was 50 minutes, and the post-lesson interview session was about 1 hour after each class.

Data were collected in 3 stages (before the intervention, during the intervention, and after the intervention). Before the intervention, a paper-pencil test was administered to determine the baseline understanding of students' mathematical concepts and reasoning skills. Classroom observations and semi-structured teacher interviews were conducted during the intervention. After the intervention, a paper-pencil test and FGD were done. The paper-pencil test and FGD supported the researcher in understanding whether students' CU and AR in mathematics developed or not. In contrast, classroom observation, semi-structured interviews, and FGD guided the researcher in finding out how students' CU and AR developed in an ICT-facilitated TL environment. Table 1 shows the research questions of this study, the data type, the source of data, the number of respondents, sampling techniques, and the instrument used for the respective questions in the study.

Research Questions	Data type	Source of data	Sampling techniques	Instrument
Are there significant changes in students' conceptual understanding and adaptive reasoning due to the ICT-facilitated teaching-learning process?	Quan. Qual.	30 Students of Grade 10 8 Students of Grade 10	Stratified random sampling	Paper-pencil test FGD (1)
How does the ICT-facilitated teaching-learning process promote students' conceptual understanding and reasoning skills in mathematics?	Qual.	6 Math classes of Grade 10 1 Math teacher of Grade 10 8 Students of Grade 10	Purposive sampling	Classroom observation Post lesson interview (6) FGD (1)

Table 1: Methodology matrix of the study

The interview questions were mainly open-ended and developed from existing literature on mathematical conceptual understanding, reasoning skills, and usage of ICT in mathematics TL. A "Tools validation workshop" was organized, where the participants were experts in the relevant field of this study. The experts' opinions were taken into account for further modifications and adjustments based on a pilot study. Besides, all interviews and observed classes were recorded (audio and video records, respectively), and field notes were taken wherever required during the observation.

A paper-pencil test was conducted before the intervention to measure students' baseline understanding of conceptual understanding (CU) and adaptive reasoning (AR). After the intervention, the test was again conducted to examine their level of proficiency in CU and AR enhancement. A total of five items (two for CU and three for AR) were administered in each paper-pencil test. All the test items were set based on the indicators (proposed by Kilpatrick et al., 2001; Milgram, 2007, Table 2) for the CU and AR skills in mathematics. The test scores range from 1 to 10 for CU and AR separately and were measured by comparing them with the standardized rubrics proposed by Lerís et al. (2017) (Table 2). As the marks of each component in the paper-pencil test were 10, to ensure an equal interval length for the proficiency levels, the range for the intervals was 2.5 (for details, please see Table 3). The test items before intervention and the test items after intervention were similar but not the same. The researcher used several data sources and data gathering methods for data triangulation to validate the data. The response validation technique was also used to validate the data.

Components	Indicator (Kilpatrick et al., 2001; Milgram, 2007)	Excellent	Good	Moderate	Poor
Conceptual Understanding (CU)	Ability to connect math concept to another one math concept/setting	Correct & complete	Correct & incomplete	Less complete	Incorrect/ not perform
	Ability to represent math in various ways	Correct & complete	Correct & incomplete	Less complete	Incorrect
Adaptive Reasoning (AR)	Ability to estimate the answer with proper justification	Correct & complete	Correct & incomplete	Less complete	Incorrect
	Ability to explain and draw conclusion	Correct & complete	Correct & incomplete	Managed to conclude	Incorrect

Table 2: Rubrics of the conceptual understanding and adaptive reasoning (Adapted from Lerís et al., 2017)

Proficiency level	Range of scores
Excellent	> 7.5 to10
Good	> 5.0 to 7.5
Moderate	> 2.5 to 5
Poor	0 to 2.5

Table 3: Proficiency level associated with the score range

Data Analysis

The quantitative data were analyzed using descriptive statistics (Tavakol & Dennick, 2011) to measure the average conceptual understanding and adaptive reasoning skills score. A paired sample t-test was also conducted to compare the pre- and post-test scores. Qualitative data were analyzed following the analytical approach used by Powell et al. (2003). The following seven sequential interacting phases were considered in analyzing the recorded (both audio and video) data (Figure 3).

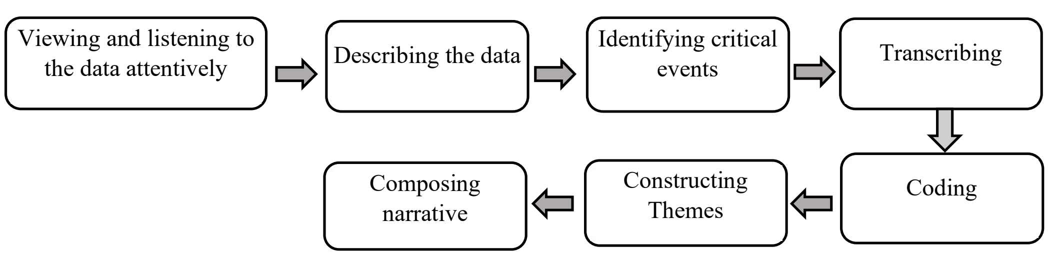


Figure 3: Sequential analytical approach for recorded data

RESULTS AND DISCUSSION

Are there significant changes in students' conceptual understanding and adaptive reasoning due to the ICT-facilitated teaching-learning process?

Evidence from the paper-pencil test

From the analysis (Table 4) of the paper-pencil test, it is observed that before the intervention, the mean score of conceptual understanding in mathematics (i.e.*ability to connect math in a new situation* and *represent math in various ways*) was very low having a value of 2.07 (SD=1.26), whereas the mean score 3.63 (SD=2.48) for the adaptive reasoning (i.e., *ability to estimate the answer with proper justification* and *ability to conclu*de) is at a moderate level. However, after the intervention, students' CU and AR increased and reached 6.13 (SD=1.99) and 6.73 (SD=2.27), respectively.

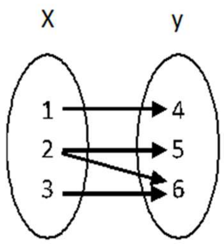
Components	Test	Mean	Std. Deviation	Mean Difference	t-value	p-value
Conceptual Understanding	Pre-test	2.07	1.26	-4.06	-9.80	<.001
	Post-test	6.13	1.99			
Adaptive Reasoning	Pre-test	3.63	2.48	-3.10	-5.94	<.001
	Post-test	6.73	2.27			

Table 4: Paired sample t-test for the conceptual understanding and adaptive reasoning

It is observed that there is a significant change ($p<0.05$) in the mean scores of the pre-test and post-test for both CU and AR (Table 4). This change means that students' ability of math conceptual understanding and reasoning skills have enhanced after attending the six experimental classes.

It was found that students' responses in two test items (Table 5, Q2 (b) and Q3) of the paper-pencil test improved significantly after the intervention. For test item Q2, students could identify the correct answer before the intervention but failed to explain the reason behind the answer. On the other hand, after the intervention, they could choose the correct answer with proper justification, which reflects their improvement in reasoning ability. Again, from the responses to test item Q3, it is observed that after the intervention, students could link mathematics to real-life situations and can apply the concept of mathematics in the new situation, which was not the case before the intervention.

Q2: a) Which of the following is not true for function? (Please √ the correct answer(s))

ii)


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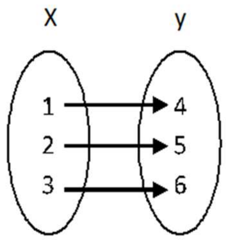

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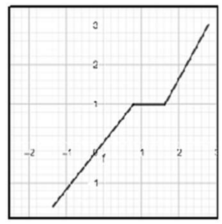

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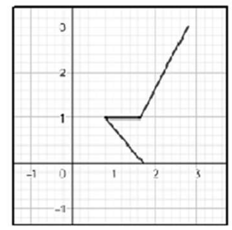

b) Explain why they are not function.

Q3: The following graph shows the distance vs time relationship for journey of 2 people Alam and Belal.

Tell a story about the Belal's Journey including specific detail about time and distance.

Table 5: Two items of paper-pencil test

Evidence from Focus Group Discussion (FGD)

Students' responses from FGD also reflect their development of CU and AR. During the FGD, the researcher asked them several questions about their experience in six experimental classes and provided them with a contextual problem (Figure 4) on "function". By analyzing their responses, it was found that students' conceptual understanding and reasoning skills developed very well after the intervention. They could explain the problem in different ways by connecting one concept to another, reflecting their CU. Besides, they could explain the reason behind their answer with proper justification and draw reasonable conclusions that reflect their ability of reasoning skills. For example, the FGD responses in Table 6 reflect students' CU and AR.

FGD Question	**FGD Responses (R=researcher, S=students)**
Mili and Shajib started from their home to the train station at the same time, and Ripon started 3 minutes later. The following graph shows the relation between the distance and time of their travels.	**R**: Can you talk about Mili's Journey? Here, the x-axis is in time(Minute), and the y-axis is in distance(meter). **S1:** Mili moves from a static position to 4 meters in 2 minutes with uniform velocity. Then there is no change at 4 meters for 2 minutes. That means she was not in motion for 2 minutes. After that, she went again with uniform velocity from 4 m to 7 m. **R:** Why uniform velocity, not acceleration? **S1:** Here, the line is straight. If the motion is accelerated, then the graph should be in curve form. **R:** Any other opinion? **S2:** I think it is acceleration as the curve is increasing.

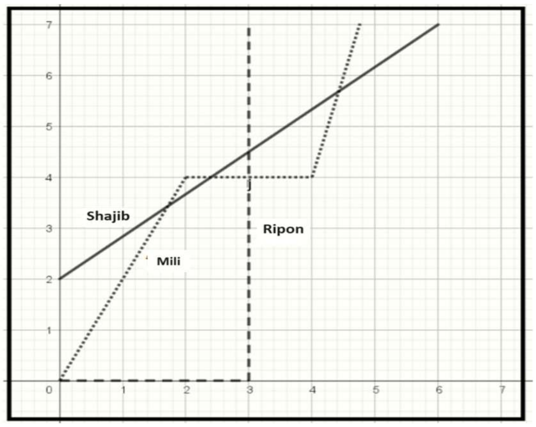

Figure 4: Distance vs time graph

S3: No, I think S1 is right. If it is acceleration, then velocity would change with time. But here, distance changes with time uniformly.
S2: Oh, yes. I did not notice that. It should be uniform velocity.

Table 6: A portion of FGD responses to Question

Thus, the paper-pencil test and FGD show that after experiencing classes in an ICT-facilitated environment, students' mathematical concepts and reasoning ability significantly increased. This finding conforms with the findings of Granberg and Olsson (2015), who reported that ICT tools assist students in engaging in creative reasoning to develop a solid conceptual understanding.

How does the ICT-facilitated teaching-learning process promote students' conceptual understanding and adaptive reasoning in mathematics?

By analyzing the data collected from classroom observation, teacher interviews, and FGD, the researcher identified several pedagogical approaches that foster students' CU and AR. She observed the pattern in the identified approaches and grouped similar patterns into 5 themes (Miles & Huberman, 1994). These themes collectively show the pedagogical considerations that the teacher uses when teaching mathematics in an ICT-facilitated environment.

This study found that a single approach cannot develop students' CU and AR. It is found that some of the approaches adopted by teachers directly enhance specific attributes while some have indirect influence. For instance, students develop their strong conceptual understanding while engaging in collaborative work. Thus, the pedagogical approach- "*Providing scope of- peer discussion, groupwork, whole class discussion"* has a direct influence on developing conceptual understanding, whereas the rearrangement of classroom setup to "*support collaborative learning space"* ultimately opens up scope for smooth collaboration and has an indirect effect on students' conceptual development. The pedagogical considerations for enhancing CU and AR are shown in Table 7.

Dimensions	Applied Pedagogies for CU	Applied Pedagogies for AR
Learning Context	***Rearranging classroom amenities*** • Physical space support- collaborative learning • Physical space support- observation learning	***Rearranging classroom amenities*** • Physical space support- collaborative learning
Access to technology	***Ensuring Technology Accessibility and Its Appropriate Usage*** • Selection of tools- presentation, Geogebra, spreadsheet, Desmose • Involving learners directly in ICT • Using ICT (screen) to perform the task accurately	
Pedagogy	***Adopting different strategies to make learning meaningful*** • Linking and connecting math with real-life • Providing scope to apply acquired knowledge in a new situation • Exploring linking math with prior knowledge • Providing multiple ways of representation • Providing scope of- peer discussion, group work, whole class discussion • Providing the opportunity to speak/ use mathematical language • Offering challenging tasks and a variety of examples ***Offering multi-channel feedback*** • Feedback by the teacher, feedback by peer • Student's self-reflection ***Offering opportunities to identify error/ imprecision*** • Exploring misconception • Identify mistakes (by peer, by learner's own) • Providing counter example	***Adopting different strategies to make learning meaningful*** • Providing scope of- peer discussion, group work, whole class discussion • Providing a variety of examples Offering challenging tasks • Providing scope to explain students' thoughts with proper justification ***Offering multi-channel feedback*** • Feedback from the teacher • Feedback by peer • Student's Self-reflection

Table 7: Pedagogical considerations for developing Conceptual Understanding and Adaptive Reasoning

Pedagogical Considerations for Enhancing Conceptual Understanding

This study found that *classroom rearrangement* is an important pedagogical consideration to enhance students' conceptual development in mathematics. For conducting classes with ICT, the teacher used a portable whiteboard to link the screen. During the post-lesson interview session, he stated:

> *[…] Only one board was in the class. There was no extra facility to project the screen. So, I brought an extra portable board so that students could work on the board. Umm.. I also brought the board to explain some of my work. […] Sometimes, I use the board to*

> *explain more about what was projected on the computer screen to give students a clear concept.*

It was also found that the teacher kept some extra chairs and a table in the class to engage students in group discussion. The study shows that if teachers intend to create an interactive and collaborative learning environment, all the features of *classroom rearrangement are vital* (e.g., board to link with the projected screen, round shape or face-to-face sitting arrangement, certain space among the groups, placement of board, projector screen and lighting arrangement, etc.). Such a supportive learning environment (observation and collaboration) creates scope for constructive discussions and helps develop students' CU. This finding is supported by Ozdemir and Pape (2012) as they stated that while students work collaboratively, they get the opportunity to explore the limits of their own understanding by dialogic talk (Wegerif, 2010).

The selection of appropriate ICT tools and their proper utilization for math teaching-learning is also found to be crucial for developing students' mathematical concepts. With the support of ICT, teachers can present and discuss mathematical topics in-depth, ultimately deepening and extending students' mathematical understanding. It was found that the teacher used the digital screen (i.e., displayed graph paper on the screen) to perform tasks. His intention behind this action was to allow students to draw the diagram by thinking from different perspectives so that they understand the concept more clearly. As he expressed:

> *Since the image of the graph paper was on the screen, there was no chance of error [...] they can realize how the diagram was drawn in different ways, they understood what happens when the angle is acute, what happens when the angle is obtuse, they can compare these things. These things they quickly understood.*

The study also shows that while students learn mathematics by connecting it to real-life situations, learning becomes more authentic. To connect mathematics with real-life situations, teachers provide familiar contextual examples and use ICT so that learners can visualize the concept in real-life. While observing the class, it was found that the teacher used animated diagrams (Figure 5) to make the learning more visible, interesting, and concrete. Table 8 shows such a classroom context.

Such an approach helps students to deepen their mathematical concepts. According to the teacher:

> *When ICT tools are used for showing the pictures [...], we can see the real shape [...], and I think when students learn in this way, their understanding becomes concrete and long-lasting.*

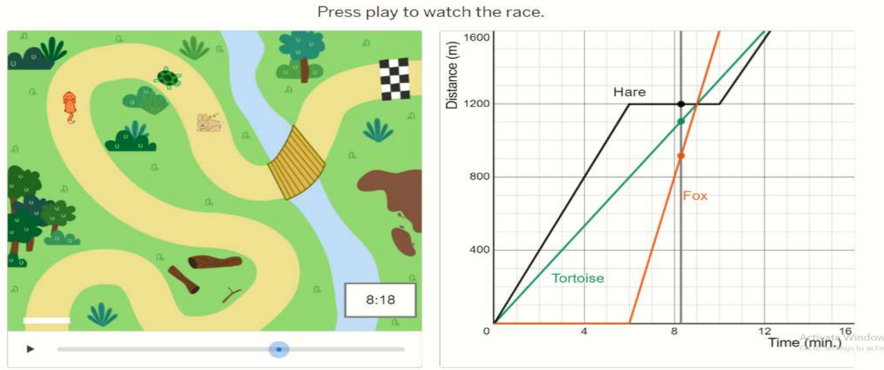

Figure 5: Use simulation to link math with real context

T: Here, you see three graphs (the teacher displays Figure 5 on the screen): one for the hare, one for the tortoise, and one for the fox. Now, I will play the button, and then you can see the real meaning of these graphs.
(teacher pressed the play button)
T: Where did the three animals meet?
S: at 1200 m
T: The graph of the hare is parallel to the x-axis for some time- what is the meaning of this?
S: The hare is at rest at that time, which means it has no motion.

Table 8: Classroom context for linking mathematical abstract concepts with real-life

Like the teacher, students during the FGD expressed their positive responses regarding the use of simulation as they found it helpful to visualize the meaning of mathematics in a real-life context. One of the students stated:

> *The example we have seen for uniform velocity or constant acceleration. In this case, we saw the animation, the graph changing with time, I think it was not possible in analog without a digital system. So that was very helpful to build up our concept.*

This finding is consistent with the findings of Wassie and Zergaw (2018), who reported that dynamic software like GeoGebra has the ability to enhance students' mathematical concepts. In addition, the study suggested that learners have to provide the scope to use mathematical language to make sense of the connection between mathematics and real-life. This viewpoint is well reported in the works (DBE, 2018) that ICT helps to visualize and connect math with the real world. Besides, *exploring students*' *prior knowledge, linking math with prior knowledge,* and *providing scopes to apply the acquired knowledge in new situations* are considered effective strategies to develop students' conceptual understanding. Similar findings were reported by Balka et al. (2010), who claimed that if teachers intend todevelop students' solid understanding of math concepts, they need to construct new knowledge with prior knowledge and utilize the new knowledge to solve problems in a new situation.

The study also found that students need to introduce or experience several examples from various dimensions. Working with problems sequencing from easier to harder helps learners interlink among the contents and ultimately enhances their mathematical concepts. This is consistent with the findings by Wolfram (2010), who found that harder problems, along with simpler ones, could develop students' conceptual understanding. In this study, *engaging learners in peer discussion, group work, and whole class discussion* are also found to be effective approaches for developing conceptual understanding, and the use of ICT in the TL process also saves time to do all those activities smoothly. Regarding this issue, the teacher stated, "*[...] if we can save that amount of time through digital technology, extra time can be given for group study*". It has been found that when students get involved in discussion and collaborative work, the concepts become clearer to them, and learners can develop their understanding of math. During the FGD, regarding this issue, one of the students argued:

> *Everyone does not think in the same way. I may know one rule to solve a problem, my friend may know another rule, and another may think of another way to solve the problem. So when we solve that problem together, we can enhance our knowledge by getting opinions from each other.*

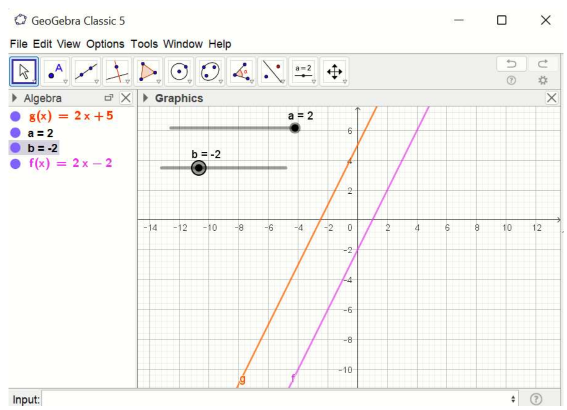

Figure 6: Exploring parallel line using slider

This finding is confirmed by researchers (DBE, 2018; Ozdemir & Pape, 2012), who showed that teachers should encourage and create the environment so that students can be involved in the discussion and progress with a solid understanding. This study also shows that *offering unfamiliar, non-routine, challenging tasks* is considered to be an effective strategy for developing a *conceptual understanding* of mathematics. During the classroom observation, it was found that the teacher offered students to draw a parallel line of $y=2x+5$, and students explored this issue by moving the slider (Figure 6) with proper understanding. In line with the work of Sultana et al. (2017), this study found that by dealing with unfamiliar, non-routine, and challenging tasks, students will be ready for realistic obstacles that exist in the real world.

The findings of this study show that multi-channel feedback (e.g., feedback by teacher, Peer, and Students' self-reflection) can be applied in an ICT-facilitated teaching-learning environment to increase conceptual understanding. To ensure multi-channel feedback, the teacher guided students to the correct path rather than directly mentioning the right or wrong answer, and created the scope to open feedback by peers, which encourages them to think more. Regarding this issue, the teacher stated:

> *[...] When I give feedback to my students, I try to give precise feedback so they are not confused. I try to expose their wrong idea or mistakes through conversation. I think when students identify their mistakes in this way, their understanding becomes more concrete.*

Besides, students' self-reflection helps them to identify their own mistakes. In addition, exploration of students' misconceptions, encouragement to deliver counterexamples, and identifying mistakes are also found effective for developing conceptual understanding in this study. An ICT-facilitated teaching-learning environment opens up the scope to apply these strategies effectively. Many other studies support this finding (e.g., Granberg & Olsson, 2015; Ogwel, 2008), where it is argued that ICT offers challenging tasks, alleviates students' misconceptions, and provides the opportunity to show counterexamples.

Pedagogical Considerations for Enhancing Adaptive Reasoning

The study found that classroom rearrangement, suitable for collaboration, indirectly increases students' adaptive reasoning. During group work, peer work, and whole-class discussions, students engaged in arguments and explained their ideas to the group members or peers with proper logic, which enhanced their reasoning skills. This is well supported by Fang (2021), who reported that group discussions are a fruitful instructional design for enhancing students' reasoning ability. Though extant literature does not stress providing diverse dimensional problems for building adaptive reasoning, this study argues that various problems foster students' multi-dimensional thinking as students attempt to solve the problem using proper logic rather than rote learning. Again, challenging, non-routine, and a variety of problems create the scope for attempting multi-dimensional tasks from a logical point of view. This finding is consistent with the work of (Marasabessy, 2021), who stated that students' ability to think logically will be formed by dealing with non-routine tasks. In this situation, students apply logical thinking to connect concepts and contexts. Besides, learners need to get involved in creative reasoning and critical thinking to explain their thoughts with proper justifications. In every class, the teacher encouraged students to speak in mathematical language and asked them to talk about the reason behind their answers. That is, the teacher was not only concerned about right or wrong answers but also tried to understand the reason behind the students' thoughts. He often asked students to justify their answers. Regarding this issue, the teacher stated:

> *[…] When a student delivers a correct result, it does not mean he understands the problem properly. So, it is essential to know how he solves the problem and why he does it that way, and he needs to understand the logic behind his work.*

This viewpoint is well reported in the works (DBE, 2018), that teachers should encourage students to use mathematical language and explain their own ideas with logic. Besides,while teachers provide feedback, the instructions should be clear and lead students to think about the reason behind the mistakes and what needs to be done. Similar findings were reported by Shute (2008) that feedback is more effective when it not only provides information about the correctness of the answer but also elaborates on the qualities of student work or how to improve. Supporting the findings of Thonney and Montgomery (2019), this study also found that peer feedback makes students logical and constructive. The study found that in almost every class, teachers invited students to provide feedback to their peers with proper justifications, ultimately enhancing their reasoning skills. A sample of such a classroom situation is shown in Table 9.

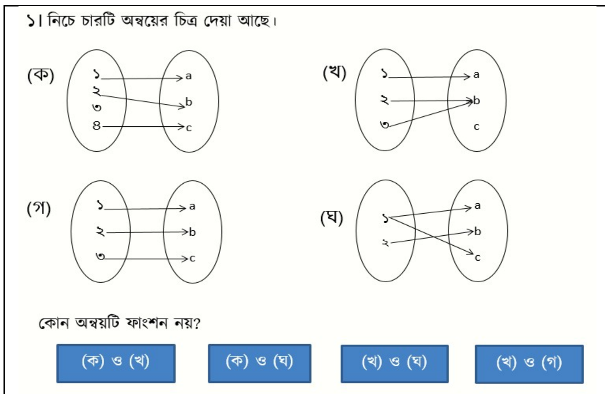	**T:** S7, tell us if S11's answer is correct or not. **S7:** No sir. I think S11 is partially correct. The 4th diagram is also not a function. **T:** Explain to us why. **S7:** To be a function, every input must have exactly one output. But here, one input has two outputs, i.e., 'a' and 'c', which is not true for function. (Here, **T**=teacher, **S**=students)

Table 9: Sample of classroom scenario of peer feedback

The study's findings show that both the teacher and students showed positive views about peer feedback. During FGD, one of the students argued:

> *When our teacher told us to say what was right or wrong with our friend's work, we thought and tried to identify what mistakes he had made. I believe it insisted on us thinking more.*

The study shows that students' self-reflection on their work also enhances their reasoning skill as this approach insists them to re-check their performed tasks and to think more with proper logic. It is consistent with the findings of Colley et al. (2012) where they suggested that reflection enhances students' knowledge system of thinking.

IMPLICATIONS AND LIMITATIONS

The findings of this study have a few implications for both theory and practice. By identifying several pedagogical approaches under five major themes that enhance students' conceptual understanding and reasoning skills in mathematics, this study adds new knowledge to the extant literature of the ICT-based mathematics teaching-learning process, especially in a developing country's context like Bangladesh. Besides, the study endorses that the policymakers should take the necessary initiatives to redesign the curriculum so that ICT-supported effective pedagogy for developing CU and AR could be addressed in the curriculum. Besides, the findings of this study would be helpful to teachers by providing an appropriate guideline for teaching ICT for students' conceptual development and reasoning skills in mathematics beyond the traditional approach. Moreover, the findings could be utilized by the teachers' trainer to develop the teachers' ICT-based professional skills, focusing on developing math concepts and reasoning skills.

While the findings of the study have several implications, the study has a few limitations that should be recognized before applying the findings to other situations. The case chosen for this study was from a Govt. school in Dhaka. Drawing on cases from different contexts (e.g., rural and urban contexts, Bangla medium and English medium schools, and high-tech and low-tech schools) would have strengthened the study's findings more. Thus, further study has been suggested for different settings. Besides, the sample size is limited to one mathematics teacher and 30 students of Grade 10. Though the researcher has collected data from different sources (e.g., classroom observations, post-lesson interviews,FGDs, and semi-structured interviews) to get a rigorous view and a clear essence of the scenario, more samples would have made the findings of the study more reliable and representative. Furthermore, the mathematics lesson topic for the study is limited to teaching *linear functions, slopes, and graphs of linear equations and real-life application*s. Thus, further study can be undertaken to see whether the findings of this study can be generalized for other mathematics content.

CONCLUSIONS

The study explored the effective pedagogical approaches to develop students' conceptual understanding and reasoning skills in mathematics. By applying a classroom-based (ICT-facilitated) intervention, the study intended to examine whether students' CU and reasoning skills in mathematics are enhanced. The study found that if teachers apply ICT effectively in their teaching, it will enhance students' mathematical concepts and reasoning abilities. The study highlighted some important pedagogical considerations such as rearranging classroom amenities to support collaborative and observation learning, ensuring technology accessibility as well as its proper use, adopting different strategies to make learning meaningful, offering multi-channel feedback and opportunities to identify imprecision for promoting students' conceptual understanding and adaptive reasoning skills. This in-depth exploration provides valuable insights into how teachers can adapt their teaching practices by integrating ICTs to develop students' conceptual understanding and adaptive reasoning in mathematics.

ACKNOWLEDGMENTS

The article is developed based on the author's PhD thesis (Sultana, T., 2024). The author is grateful to the supervisor, Prof. Dr. Md. Abdul Halim, and Co-supervisor, Prof. Mohammad Nure Alam Siddique, for their valuable insights and guidance.

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