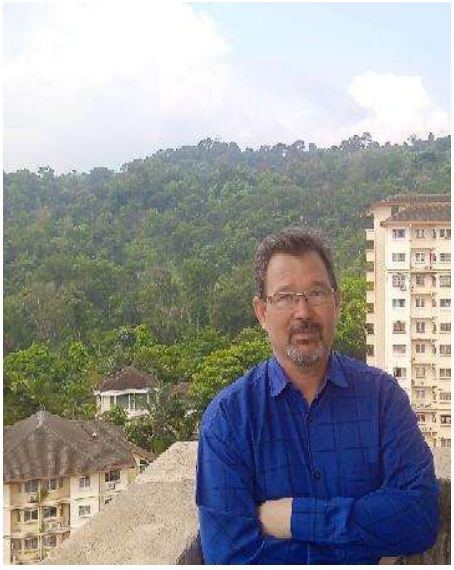
Editorial of the Fall Issue TR 61, Vol 17 No 5

Hosseinali Gholami

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The current issue includes articles employing various study methods: six qualitative, three quantitative, and four mixed-method studies from Australia, Bangladesh, Indonesia, Kazakhstan, the Philippines, the Slovak Republic, and the USA. The articles in this issue explore mathematics education across a range of educational levels, including primary, middle, high school, and university. In detail, four articles concentrate on primary school mathematics, three investigate middle school mathematics, five address key topics in high school mathematics, and one focuses on university-level mathematics education. This distribution demonstrates a broad and balanced coverage of the educational continuum, offering valuable research contributions that are relevant to both educators and researchers working within these critical stages of mathematical development.

The article by **Viona Adelia**, **Ratu Ilma Indra Putri**, **and Zulkardi** examines grade 4 students' conceptual understanding of fractions through the use of appropriate learning materials. It emphasizes that (1) mastering the part-whole subconstruct requires progressively building on students' existing fraction knowledge across its essential elements, and (2) engaging with incomplete fraction scenarios enables students to deepen their grasp of valid fractional representations. These findings provide important guidance for enhancing teaching strategies related to the part-whole concept and overcoming related learning difficulties.

Exploring the Part-Whole Subconstruct in Fractions Learning: Insights from A Design Research Study by **Viona Adelia**, **Ratu Ilma Indra Putri**, and **Zulkardi,** (Indonesia) ……...Page 5

The teaching approach presented in the article by **I Putu Ade Andre Payadnya, Kadek Rahayu Puspadewi, and Gde Iwan Setiawan** led to significant improvements in primary students'

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problem-solving abilities, critical thinking, and creative reasoning within geometry concepts. Additionally, another study by **Wahyu Purwaningsih, Sugiman Sugiman, and Haryanto Haryanto** explores the cognitive processes involved in problem-solving at the primary education level.

Transforming Geometry Learning with An Interactive E-Module Integrating Ethno-Realistic Mathematics Education: A Design-Based Approach to Enhancing Numeracy Skills by **I Putu Ade Andre Payadnya, Kadek Rahayu Puspadewi, and Gde Iwan Setiawan,** (Indonesia)……..…Page 24

Exploration of Primary School Students' Metacognitive Thinking Processes in Solving Problems by **Wahyu Purwaningsih, Sugiman Sugiman, and Haryanto Haryanto,** (Indonesia) … ……...…Page 52

Enhancing both content knowledge (CK) and pedagogical content knowledge (PCK) among teachers is essential for effective teaching. An article by **Murni Sianturi and Dessy Rizki Suryani,** involving a large participant group, focuses on improving primary teachers' mathematical content knowledge through in-service training programs. Similarly, a study by **Yeni Rakhmawati, Heri Retnawati, Yoppy Wahyu Purnomo, Uzak K. Zhapbasbayev, and Gulzhaina K. Kassymova** addresses the improvement of teaching quality among primary educators by developing their pedagogical content knowledge in mathematics.

The Knowledge of Indigenous West Papuan Pre-service Teachers about Primary School Mathematical Contents: Strengths and Weaknesses by **Murni Sianturi, and Dessy Rizki Suryani, (**Australia, Indonesia) …….. Page 85

Mathematics Teaching Strategies: A Reflection on Pedagogical Content Knowledge (PCK) for Improving Teaching Quality by **Yeni Rakhmawati, Heri Retnawati, Yoppy Wahyu Purnomo, Uzak K. Zhapbasbayev, and Gulzhaina K. Kassymova,** (Indonesia, Kazakhstan) .....Page 104

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Among the three articles on middle school mathematics education, one study by **Laela Sagita, Ratu Ilma Indra Putri, Zulkardi, and Rully Charitas Indra Prahmana** highlights a financial literacy learning environment model as a professional development program. This study emphasizes enhancing teachers' abilities to design math lessons that integrate financial literacy tailored to students' needs. Another article by **Valeria Suryani Kurnila, Dwi Juniati, and Agung Lukito** explores enhancing communication and connection skills through differentiated instruction and Writing to Learn strategies in a bilingual context, presenting innovative approaches to math teaching. The third article, by **Amihan M. Ulpindo and Rommel S. De Gracia,** is a comparative study on Process-Oriented Guided Inquiry Learning (POGIL) aimed at improving students' problem-solving abilities and higher-order thinking skills.

Bridging mathematics and financial literacy through flipped classroom by **Laela Sagita, Ratu Ilma Indra Putri, Zulkardi, and Rully Charitas Indra Prahmana,** (Indonesia) …. …Page 128

Enhancing Communication and Connection Abilities through Differentiated Instruction and Writing to Learn in a Bilingual Context by **Valeria Suryani Kurnila, Dwi Juniati, Agung Lukito,** (Indonesia) …….Page 156

Proficiency Level of Grade 9 Students on Quadratic Functions Through Process-Oriented Guided Inquiry Learning (POGIL) by **Amihan M. Ulpindo, and Rommel S. De Gracia,** (Philippines)……Page 197

Among the four featured articles on high school mathematics education, four emphasize teaching mathematics by connecting concepts to real-world contexts. These studies highlight how this approach not only helps students appreciate the value of mathematics but also equips them with practical skills to apply mathematical reasoning in solving real-life problems. One notable article by **Wulan Resti Oktaviani, Darhim, Nurjanah, Kusnandi, and Suparman** focuses specifically on teaching geometry using GeoGebra software. This technology-based method facilitates students' understanding of three-dimensional mathematical problems more effectively than traditional teaching approaches, enhancing their spatial visualization and engagement.

Cultural Relevance in Mathematics Education Using Indigenous Patterns for Teaching Algebra by **Joseph B. Tandas, and Ryan L. Cerveza,** (Philippines) …….Page 237
A Realistic Approach to Solving Math Problems: A Case from "Stand and Deliver" by **Jakub Lipták,** (Slovak Republic) ……..…Page 262

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Information and Communication Technology to Enhance Students' Conceptual Understanding and Reasoning Skills in Mathematics: A Case from Bangladesh by **Tamanna Sultana,** (Bangladesh) ……Page 282

The Effect of GeoGebra-Assisted Discovery Learning Considering Emotional Stability on the Achievement of Mathematical Reasoning by **Wulan Resti Oktaviani, Darhim, Nurjanah, Kusnandi, Suparman,** (Indonesia) …….....Page 307

The notable study conducted by **Abraham Ayebo** thoroughly examines the attitudes of undergraduate health science students toward statistics, both prior to and following the completion of an introductory statistics course. This research provides valuable insights into how students initially perceive statistics and how their perspectives evolve as a result of formal instruction in the subject. The findings hold significant relevance for educators, highlighting the critical need to design statistics courses that not only enhance students' technical competence and understanding but also actively foster more positive attitudes and greater confidence toward learning statistics.

Examining the Attitudes of Students Before and After a Course in Introductory Statistics by **Abraham Ayebo,** (USA) ……..……..Page 334

No edition would feel whole without the ever-engaging Problem Corner, thoughtfully curated by our dedicated Problem Corner editor, **Dr. Ivan Retamoso,** (USA) ……. Page 355

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Exploring the Part-Whole Subconstruct in Fractions Learning: Insights from a Design Research Study

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Abstract: The part-whole relationship is sometimes overlooked in fractions learning due to its perceived limitations. However, prior studies have shown that students often struggle to master the part-whole subconstruct and encounter difficulties. Despite these challenges, the part-whole subconstruct plays a critical role in supporting students' subsequent fractions learning. This study aimed to explore how students understand and reason about the part-whole subconstruct across its key components. Using a design research approach, a teaching experiment was conducted with 38 fourth-grade students. Students' written works, transcripts from the teaching experiment recordings, and observational notes were retrospectively analyzed to examine the learning trajectory. Several key findings emerged from the teaching experiment, each corresponding to a component of the part-whole subconstruct. We highlight that (1) learning the part-whole subconstruct involves progressively building upon students' prior fractions concepts across its key components and (2) working with incomplete situations helped students refine their understanding of what constitutes a valid fractional representation. These findings offer valuable insights for improving instruction on the part-whole subconstruct and addressing challenges associated with it. The study concludes with recommendations for future research based on the findings.

Keywords: Design Principle, Design Research, Fraction, Part-Whole, Teaching Experiment

INTRODUCTION

Prior research suggests that the usefulness of the part-whole subconstruct is limited. For example, it interferes with understanding of improper fractions. The part-whole subconstruct is described as a situation in which a whole is divided into equal parts, with a fraction representing how many of these parts are being considered (Charalambous & Pitta-Pantazi, 2006). Thus, given that a fraction is a part of a whole, the numerator of a fraction must be smaller than or equal to the denominator. The part-whole subconstruct relies on mental actions of partitioning and dis-

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embedding and misses out the mental action of iterating (Wilkins & Norton, 2018). Also, the part-whole subconstruct involves equal parts (one slice out of four and one t-shirt out of four t-shirts) and equal wholes (1/4 of a pie is the same as 2/8 of a pie); without a proper understanding students might answer that 1/4 of a family-sized pizza and 1/4 of a regular-sized pizza are the same (Pedersen & Bjerre, 2021). The part-whole subconstruct is fundamental in developing understanding of the other four subconstructs (measure, ratio, quotient, and operator) (Charalambous & Pitta-Pantazi, 2006), which also plays a role in students' subsequent understanding, such as equivalent fractions (Pedersen & Bjerre, 2021). Thus, the part-whole subconstruct of fractions is also fundamental to overall students' conceptual understanding of fractions.

Previous research indicated that by the end of the fifth grade, students have yet to attain mastery of the part-whole subconstruct of fractions and encounter difficulties in associating congruency with parts of the whole. Their common errors include: 1) failing to associate fractions with equal divisions of a whole; 2) linking fractions solely to congruent segments of the whole; and 3) identifying the portion of the whole that is represented (Hodnik Čadež and Manfreda Kolar, 2018). It is important to pay attention to these errors for excellent student understanding of the part-whole subconstruct.

Most prior studies on the part-whole subconstruct have primarily focused on equal partitioning (Cramer & Wyberg, 2009; Torres-Peña et al., 2024) or general partitioning (Purnomo et al., 2021). However, these components alone do not fully capture the complexity involved in understanding the part-whole subconstruct. There are several other components in the part-whole subconstruct that need to be accounted as well. These aspects are often overlooked in the literature, which tends to emphasize other subconstructs, like measure, ratio, quotient, or operator (Purnomo et al., 2021).

While some studies have questioned the effectiveness of the part-whole subconstruct to introduce fractions (Purnomo et al., 2021; Simon et al., 2018; Watanabe, 2006; Wilkins & Norton, 2018), it is important to acknowledge that subconstructs of fractions are interrelated (Kieren, 1976) and mutually supportive (Charalambous & Pitta-Pantazi, 2006). Each subconstruct contributes in a distinct and essential way to building a comprehensive understanding of fractions. For example, although the equal partitioning, unitizing, and reunitizing components of the part-whole subconstruct may be addressed through the partitioning and iterating components of the measure subconstruct (Purnomo et al., 2021), the inclusion component of the part-whole subconstruct supports students in distinguishing between fractions as part-whole relationships and as ratios, as in understanding the difference between $\frac{2}{5}$ and $\frac{2}{3}$ (Charalambous & Pitta-Pantazi, 2006). Moreover, development of fraction subconstruct in isolation does not guarantee in understanding of the other concepts (Brousseau et al., 2004).

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Theoretical Framework

The theoretical framework is structured around the lens of a deeper examination of the conceptual understanding required to develop students' understanding of the part-whole subconstruct to guide the present study.

Understanding the Part-Whole Subconstruct

Charalambous and Pitta-Pantazi (2006) have outlined several ideas or concepts that students need to develop to master the part-whole subconstruct of fractions. First, they proposed that students must comprehend that the parts into which the whole is partitioned must be of equal size, and they should be capable of partitioning a continuous area and a discrete set into equal parts, as well as determining whether the whole has been partitioned in equal parts. This statement highlights that equal partitioning is an important component of the part-whole subconstruct. However, the subconstruct entails more than simply partitioning the whole into equal parts. Other aspects must be taken into account, like recognizing different types of wholes and comparing parts in relation to the whole. Next, they talked about the idea of inclusion or embeddedness and they provided an illustration of this component: unless pupils develop this understanding, they might erroneously count some parts twice—once for the numerator and again for the denominator. For example, when asked to identify 2/3, students are likely to choose a representation illustrating 2/5. They emphasized the importance of understanding that the parts are to be considered as integral components of the whole. Furthermore, they also underscored that the part-whole relationship should remain consistent despite changes in the representation of the parts. Finally, they concluded the ideas of mastering the part-whole subconstruct with the component of unitizing and reunitizing—a component that allows students to fully understand the part-whole subconstruct. This statement emphasizes the necessity of reconstructing the whole based on its parts. The first four points in Table 1 summarizes these ideas.

In addition to Charalambous and Pitta-Pantazi's (2006) ideas on mastering the part-whole subconstruct, we also see the potential to support students' understanding through the problem-solving component. Pramudiani et al. (2022) found that the ambiguity regarding what constitutes the whole can be used to foster students' understanding of the part-whole relationship. They proposed a task designed to stimulate students' reasoning about the part-whole relationship in an incomplete situation, posing the question: "Would students understand a fraction as a part of a whole when one part is missing?". We see the potential of this problem to further support students' understanding of the part-whole relationship by encouraging them to recognize the relationship even in less straightforward contexts. The last point in Table 1 summarizes this idea.

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No	Components	Description	Source
1	Equal partitioning of the whole	Students should understand that the parts into which the whole is partitioned must be of equal size.	Charalambous and Pitta-Pantazi, 2006.
2		Student should be able to partition a continuous area or a discrete set into equal parts and discern whether the whole has been partitioned in equal parts.	Charalambous and Pitta-Pantazi, 2006.
3	Inclusion	Students should develop the idea of inclusion or embeddedness, according to which parts are considered as components of the whole.	Charalambous and Pitta-Pantazi, 2006.
4	Unitizing and reunitizing	Students develop unitizing and reunitizing abilities, which allow them to reconstruct the whole based on its parts.	Charalambous and Pitta-Pantazi, 2006.
5	Part-whole relationship in incomplete situations	Students' understanding of fractions should be well addressed with challenging problems on the part-whole relationship, in this case in incomplete situations.	Pramudiani et al., 2022.

Table 1: The Framework of the Part-Whole Subconstruct

Hence, this study differs from prior studies in the aspect of building a strong foundation of the part-whole subconstruct by integrating a series of questions that contain important part-whole components. This study contributes to both the practical design of instructional tasks and the theoretical development of learning trajectories in fractions. Therefore, the main research question addressed in this study is "How do students understand and reason about the key components of the part-whole subconstruct in fractions?".

In order to help students understand and develop an ability to reason about the key components of the part-whole subconstruct, we have formulated the following specific sub-research questions:

R1. How are previous fractional concepts used to understand new ones?
R2. How incomplete problem situations can expand insights into fractions learning, particularly in terms of the part-whole subconstruct?

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METHOD

Research Procedure

Design research seeks to develop theories with a focus on specific learning processes (Bakker, 2018). It aims to establish a trajectory for students to transition from their existing knowledge to the desired learning objectives (Putri et al., 2021). Therefore, this study adopted design research as its research methodology to answer the research question by enabling the students to understand fractions as parts of a whole through a series of activities such as equal partitioning, unitizing, and solving problems involving incomplete partitions. The design research methodology consists of three phases: 1) preparation for experiment, 2) teaching experiment, and 3) retrospective analysis (Bakker, 2018).

Participants and Context of the Study

This study was conducted in a public elementary school in a suburb area in Palembang, Indonesia. The participants of the teaching experiment were 38 fourth-grade students in a whole class setting with heterogeneity in their mathematical performance, as informed by the teacher. The selection of the class was influenced by the teacher's willingness to participate in the study. The teaching experiment involved two lessons over two weeks. The classroom setting was equipped with basic teaching tools, such as a whiteboard and visual aids, supplemented with hands-on materials—printed pictures of objects, such as *martabak*, Indonesian stuffed sweet pancake—to engage students through concrete representations of fractions. This context was chosen for reasons of its alignment with the study's aim of integrating students' informal knowledge into the teaching of fractions as parts of a whole.

Task Design

The task employed in this study was designed based on the components of the part-whole subconstruct, as reviewed in the literature (see Table 1). The first component is the equal partitioning of the whole. Charalambous and Pitta-Pantazi (2006) proposed that students must comprehend that the parts into which the whole is partitioned must be of equal size and they should be capable of partitioning a continuous area and a discrete set into equal parts as well as determining whether the whole has been partitioned in equal parts. In the teaching experiment conducted by Yoshida and Sawano (2002), the importance of equal partitioning and equal whole was emphasized for understanding fractions. Their findings showed that students who received an instruction focused on these two aspects outperformed the control group in solving non-routine fractions tasks. Simon (2006) further explained that in equal partitioning, students need to understand that the fraction produced represents a new unit whose size is relative to the original unit (whole). Therefore, we present questions 1, 3, and 5 to address the first component (see Table 2). These questions address students' understanding of equal partitioning, adapted from the study by Pantziara and Phillipou (2011). Question 1 focuses on students' ability to determine whether the

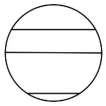
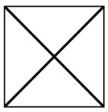
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whole has been partitioned equally. Question 3 assesses students' ability to reunitize the whole to produce equal partitions. Finally, question 5 requires students to partition a discrete set into equal parts.

The next component is inclusion. This component helps students develop the idea of inclusion or embeddedness (Charalambous & Pitta-Pantazi, 2006). Lamon (2020) further suggested using discrete units of various types, including units composed of more than one object, since children often struggle with units that consist of multiple objects. Incorporating this component is essential for advancing students' part-whole understanding—particularly in recognizing and identifying the correct whole in more complex representations. Therefore, we adapted a task from Lamon (2020) that addresses students' understanding of inclusion for question 4.

The component of unitizing and reunitizing refers to the ability to reconstruct the whole based on its part (Charalambous and Pitta-Pantazi, 2006). Bruce et al. (2023) defined a unit fraction as the representation of the smallest segment of a fraction. Unit fractions can be combined or iterated to generate larger quantities, or, conversely, the quantities can be deconstructed to generate the underlying unit fractions. They further explained that engaging with unit fractions helps pupils internalize the meaning of fractions as quantities. To engage students with this component, we came up with 6 questions. According to Simon et al. (2018), this type of task facilitates the process of building students' understanding of fractions based on composite unit concepts. This enables students to coordinate their composite unit concepts with reunitizing to construct a unit system for measurement. We believe that this kind of task would be beneficial for students' fractions understanding.

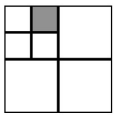
In terms of the problem-solving in fractions as parts of the whole subconstruct, Pramudiani et al. (2022) found that comprehending the relationship of fractions as parts of the whole should be well addressed. They designed tasks to stimulate students' reasoning regarding the part-whole relationship. Therefore, we adapted their task of the part-whole relationship in incomplete situations to address students' understanding of fractions as parts of the whole in question 2 by encouraging students to recognize the part-whole relationship in less clear situations. Table 2 encompasses all the questions utilized in this study.

No.	Question
1	Which of the following figures represents the fraction $\frac{1}{4}$? a.  b.  c.  d. 
2	Write the fraction that is represented by the cheese-topped *martabak.*

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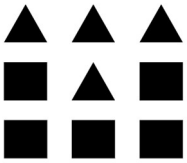


3 Write the fraction that is represented by of the shaded area.

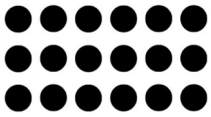
a. 

b. 

4 What fraction is represented by the triangle?



5 Circle $\frac{2}{3}$ of these objects.



6 a. The fraction of the following figure represents the fraction $\frac{2}{3}$. Determine what the figure would look like if the fraction is $\frac{3}{3}$.



b. The fraction of the following figure represents the fraction is $\frac{2}{3}$. Determine what the figure would look like if the fraction is $\frac{4}{3}$.

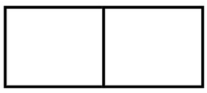


Table 2: The Questions on the Part-Whole Subconstruct

Conjectures of Students' Thinking

It was conjectured that for question 1 students could choose figure a. We conjectured for question 2 that students might answer $\frac{1}{7}$ because they had not yet been exposed to the part-whole rela-

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tionship in incomplete situations. For question 3a, students could draw another four small squares in each big square to discern whether the whole had been partitioned in equal parts, then give $\frac{1}{16}$ as the answer. Meanwhile, students might find question 3b difficult since the already-partitioned parts are not congruent. Question 3b assesses students' ability to combine certain processes and exhibit signs of conceptual understanding to a certain extent (Pantziara & Phillipou, 2012). For question 4, we conjectured that students would understand that when they were asked to determine the fractions, they would be able to identify the whole being referred to. For question 5, we conjectured that students could circle 12 objects. Lastly, for question 6, we expected that some students would come up with the idea of finding the unit fraction and then re-unitizing it to determine the desired fraction.

Data Sources and Analytical Procedure

The data sources for this study comprised students' written works, recordings of the teaching experiment, and observational notes from whole-class conversations. The data analysis technique used involved a retrospective analysis by comparing the hypothetical learning trajectory with the actual learning to determine its alignment with the learning objective. The retrospective analysis entailed: 1) examining students' works, 2) transcribing classroom discussions and scrutinizing the observational notes, 3) describing students' understanding and reasoning during whole-class discussions recorded in the observational notes, and 4) evaluating the match of learning hypotheses and students' actual learning. The selection of students' answers or responses for analysis, described in the Results and Discussion section, was done by purposive sampling. Specifically, examples were chosen to represent the range of students' understanding. This approach allowed the study to capture diverse reasoning patterns, difficulties encountered during the learning process, and how to tackle the difficulties. While all the 38 students participated in the teaching experiment, selected students' works and dialogues were highlighted to illustrate key aspects of learning related to each component of the part-whole subconstruct.

RESULTS AND DISCUSSION

Equal Partitioning of the Whole

For question 1, it was not hard for students to circle figure a. However, there were some students who circled figure b as well. What one student knew, for example, was that to determine the fraction, all he had to do was count the shaded parts and the unshaded parts. Below is an excerpt of the teacher and the student (S1) (see Figure 1).

S1: I think figure b is also (the answer).

T: And why is that?

S1: Because there is one shaded part

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T: And then?

S1: Hmm... I don't know. It's usually like that.

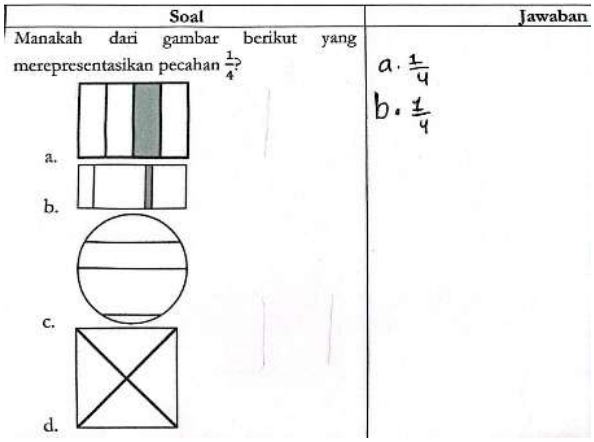


Figure 1: Students' representation of $\frac{1}{4}$

The teacher then asked students to continue to task 3a. Unfortunately, the real process missed the conjecture. No students were able to repartition another three big squares into four smaller squares. Instead, they answered $\frac{1}{7}$. The teacher then returned to question 1 and asked the students what the difference is between figure a and b.

S2: The sizes (of the parts) are different.

T: And are the parts in task 3a different (in size)?

S2: yes, so the answer is not $\frac{1}{7}$. It's $\frac{1}{4}$.

T: Where did you get $\frac{1}{4}$?

S2: One, two, three, four (counting the four smaller squares in the one big square).

T: The object is not only these (the four small squares) but all of this (the whole figure).

S2: (repartitioning the object like in figure 2)

T: Can you name the fraction of the shaded part now?

S2: It's one-sixteenth.

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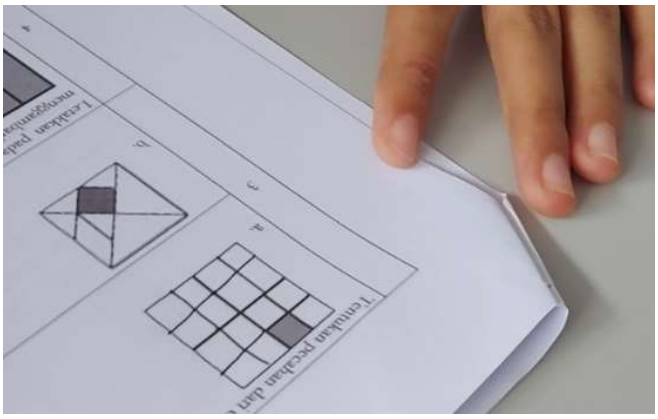


Figure 2: Student's repartitioning the figure in question 3a

Unlike task 3a, task 3b was quite difficult to be repartitioned. However, S2 understood that the repartition should be in the triangle (see the red circle in figure 2) even though it was still hard for her since she mistakenly drew a wrong triangle (see the green circle in figure 3). To support the student, the teacher gave her a hint to look at the shape she thought the partitions would be and ensured that all the partitions she made look exactly alike. On the other side, task c was quite easy for students to solve. They were all able to circle 12 circles objects to mark $\frac{2}{3}$ of all the objects.

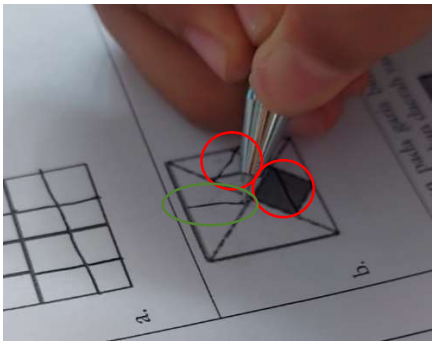


Figure 3: Student's attempt to repartition the figure in question 3b

The teaching experiment on equal partitioning tasks revealed insights into students' understanding related to fractions as parts of a whole. For question 1, while most students correctly identified figure a as representing $\frac{1}{4}$, some students demonstrated a procedural approach to fractions. S1 relied on counting shaded and unshaded parts without considering equal partitioning. Prior studies, such as one by Hodnik Čadež and Manfreda Kolar (2018), have highlighted that children often associate the partitioning of a whole with dividing it into congruent parts, which is a fundamental aspect of understanding fractions. However, our findings suggest that some students may not even recognize the necessity of congruent parts in determining fractions, reflecting a more

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profound gap in their conceptual understanding. This highlights a prevalent issue in fractions learning: a focus on part-whole relationships without verifying congruence.

In question 3a, students faced challenges with repartitioning unequal parts into equal partitions. Contrary to the conjecture, no student initially attempted to subdivide the larger squares into smaller, congruent parts. This underscores a gap in students' reasoning, highlighting the importance of explicitly teaching strategies for creating equal partitions. Question 5, in contrast, was relatively straightforward, with all students successfully circling 12 out of 36 circles objects to represent $\frac{2}{3}$.

Inclusion

In Question 4, the whole is 9, as the figure contains 9 objects. Half of the students were able to answer $\frac{4}{9}$, but the other half gave $\frac{4}{5}$ as an answer, as in ratio. See the excerpt bellow between the teacher and a student who answered $\frac{4}{5}$ and the answer (figure 4).

T: Why did you answer $\frac{4}{5}$?

S3: Because (the question is) asking what fraction is represented by the triangle. There are 4 triangles and there are 5 squares.

T: That's why your answer is $\frac{4}{5}$?

S3: Yes, Ma'am.

T: What if…. The question is "What fraction is represented by the triangle compared to all of the objects?"

S3: $\frac{4}{5}$?

T: Are you sure?

S3: No… I guess it should be $\frac{4}{9}$, because you said compared to all of the objects.

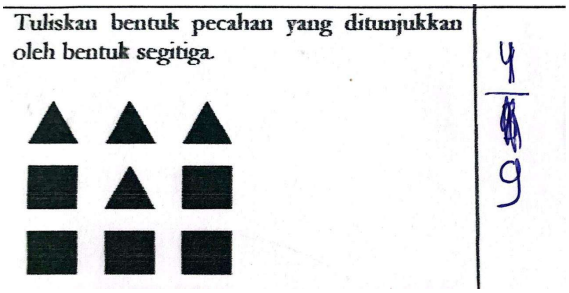


Figure 4: Student's changing his answer

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S3's initial struggle aligns with Lamon's (2020) observation that children often have difficulties working with units composed of multiple objects. From another perspective, Möhring et al. (2016) explained that spatial proportional reasoning differs between part-whole and part-part reasoning. This means proportions can be represented as either part-whole relations or part-part relations. Their review also revealed that children's thinking about proportions and their approach to solving such problems changes with age and depends on how the problems are framed. To address this issue, it may be beneficial to articulate questions more explicitly and align them with students' current level of understanding, particularly by clearly defining what is considered the whole.

Unitizing and Reunitizing

For question 6, the idea is to partition the given area to obtain the unit fraction and then iterate it until the desired fraction is achieved. We expected that some students would be able to come up with the idea of finding the unit fraction and then reunitizing it to determine the desired fraction. However, no students were able to generate this idea without additional scaffolding by the teacher. Here is the excerpt of the teacher and one student.

T: Could you draw for me the representation of the fraction $\frac{2}{3}$?

S4: Yes, ma'am (see figure 5a).

T: Now, could you draw for me the representation of the asked fraction?

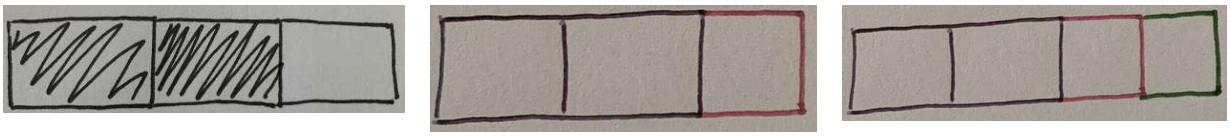
S4: $\frac{3}{3}$? (then she shaded one more unit).

T: Okay, now could you answer this question (question 6)?

S4: So… I have to draw another rectangle (see figure 5b).

T: How about question 6b?

S4: I think I should add one more rectangle to it (see figure 5c).



a b C

Figure 5: Student's unitizing and reunitizing

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A similar case was observed in the teaching experiment conducted by Simon et al. (2018). They emphasized the importance of balancing students' exploration with sufficient support to facilitate breakthroughs in understanding. After receiving additional guidance, S4 iterated the unit fraction to generate the requested fraction. Through this process, she learned how fractions function both as parts of a whole and as measures, which will support her future learning of fractions (Simon et al., 2018).

Part-Whole Relationship in Incomplete Situations

When students have a solid understanding of the part-whole relationship, Hodnik Čadež and Manfreda Kolar (2018) concluded, they must be able to identify parts of an equally divided but incomplete whole. Pramudiani et al. (2022) designed tasks to address this issue and found that incomplete situation problems have the potential to encourage mathematical reasoning and deepen the understanding of fractions. Their results also revealed that, when determining a fraction, students sometimes count only the physically visible parts and struggle to refer to the correct whole.

Our findings align with the conjecture for question 2: all students answered $\frac{1}{7}$. Based on the conversation between the teacher and the students, it is evident that they counted only the visible parts of the *martabak* as the whole. To challenge their reasoning, the teacher asked one question: "Would you be okay if you received this kind of *martabak* in this condition from the seller?" See the excerpt below.

> T: Would you be okay if you received this kind of *martabak* in this condition from the seller?
>
> S: No…
>
> T: Why?
>
> S5: Because it's incomplete, I would be mad if I pay full.
>
> T: Then, how many parts are there if it's complete?
>
> S5: 8? Because I think it's missing one slice.
>
> T: Therefore, what is the fraction of that is represented by the cheese-topped *martabak*?
>
> S5: $\frac{1}{8}$, ma'am.

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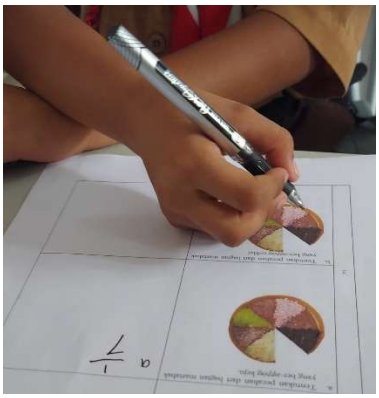


Figure 6: Student's attempt to referring the correct whole

Based on the findings of Pramudiani et al. (2022), students' ability to determine a fraction in an incomplete situation largely depends on their comprehension of fractions as parts of a whole. They suggest stimulating students' persistence in understanding fractions by providing appropriate aids. In this study, the aim was to help students recognize the incompleteness of the *martabak* slices and understand how it affects the fraction that is representeds it. Fosnot and Dolk (2002) observed that such challenges help children build a deeper understanding of fractions by encouraging them to reason through the relationships.

The result of the teaching experiment led to several key findings. The first relates to the component of equal partitioning of the whole. The most critical aspect is ensuring that students comprehend how to determine a fraction, specifically that the whole must be partitioned into equal-sized parts. Although prior studies have found that constructing a fraction is generally considered the easiest aspect of fractions learning (Jiang et al., 2020; Pantziara & Philippou, 2012), this was not observed in the present study. A lack of understanding in this component can hinder students' ability to perform fractional operations and leave them vulnerable to perceptual distractors, especially for those with an unstable understanding of fractions (Noordin et al., 2012). As observed, some students considered figure b in question 1 as a representation of $\frac{1}{4}$. They were also unable to repartition the figures in questions 3a and 3b to ensure the parts were of equal size. Therefore, students' understanding of constructing fractions based on equal partitioning should be carefully emphasized in instructional practices.

The next key finding relates to the component of inclusion. A discrete unit consisting of multiple objects may lead students to interpret fractions as ratios. This observation aligns with the findings of Pramudiani et al. (2022), who noted that students often equate comparing (to the whole) to ratio. Being explicit in such cases is essential, as how children perceive a problem depends on how the teacher presents it (Möhring et al., 2016).

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The third key finding relates to the component of unitizing and reunitizing. It is worth noting that unitizing to generate the desired fraction may not come naturally to students. Providing fractional representations that they are familiar with can be beneficial. This also aligns with the finding of Pramudiani et al. (2022). In such cases, engaging with unitizing and reunitizing can help extend students' understanding of the measurement subconstruct of fractions (Simon et al., 2018).

The final key finding pertains to problem-solving within the part-whole relationship in incomplete situations. Previous research has emphasized the interrelation between students' ability to determine a fraction in an incomplete whole and their part-whole comprehension (Hodnik Čadež & Manfreda Kolar, 2018; Pramudiani et al., 2022). Such problems have the potential to stimulate students' reasoning in fractions learning, highlighting the interplay between conceptual understanding, persistence, and the use of aids to support their learning.

However, there is a limitation to the current study. This study focused on advancing students' understanding and reasoning about the part-whole subconstruct only in incomplete situations. Further research on problems related to the part-whole relationship that can potentially expand insights into the part-whole subconstruct, such as those involving more complex scenarios of multiple wholes, could provide deeper insights into fractions learning, particularly within the part-whole subconstruct. Additionally, this study did not systematically examine students' mastery of each component of the part-whole subconstruct as a sequential learning process. Future research could focus on treating each component as a milestone to better understand the progression of students' fractional reasoning.

CONCLUSIONS

The study aimed to explore how students understand and reason about the key components of the part-whole subconstruct in fractions. To provide more comprehensive insights, we divided this aim into two sub-questions: the first focused on how students utilize previous fractional concepts to understand new ones, and the second focused on how incomplete problem situations can expand students' insights into the part-whole subconstruct. In the following, we reflect on how the results obtained contribute to answering these sub-questions.

This study found that students' understanding of the part-whole relationship—particularly the idea that a whole must be partitioned into equal-sized parts—played a crucial role in learning more advanced components of the part-whole subconstruct. However, some students overlooked whether the parts are actually equal, which shows the importance of emphasizing that fractions must come from equally sized parts of a whole.

Students' experiences with part-whole representations also supported their understanding of the inclusion component. For example, their ability to recognize a fraction as a part of a whole helped them begin to distinguish between situations requiring comparison to the whole (part-whole) and those requiring comparison between parts (ratio). This distinction was not always

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clear to students, especially when the whole is composed of multiple or composite units. Moreover, students' recall of earlier representations helped them make sense of unit fractions. In one case, a student identified two-thirds as a composite unit, and correctly extended it by drawing an additional rectangle to represent three-thirds and four-thirds. This demonstrated the use of previous understanding (i.e., identifying parts of a whole and interpreting them visually) to support learning about improper fractions and iteration. These findings suggest that learning the part-whole subconstruct involves building upon foundational concepts in a progressive manner. Prior knowledge acts as a stepping stone that helps students navigate increasingly complex components, including inclusion and unitizing, thereby strengthening their overall understanding of fractions.

This study also found that presenting students with incomplete problem situations—where certain parts of the whole are missing or not explicitly shown—encouraged them to think more deeply about the part-whole relationship. These situations required students to infer the missing parts and consider what the complete whole should look like, which prompted them to reason beyond surface-level representations.

By recognizing the incompleteness of a situation, students began to understand that a fraction cannot be determined solely based on the visible parts; it also depends on knowing the total number of equal parts that make up the whole. This awareness helped them refine their understanding of what constitutes a valid fraction representation. For instance, they learned that if a whole is divided into unequal parts or if some parts are missing, they need to think more carefully about what the fraction is actually represented. Therefore, incomplete situations can serve as a productive challenge that pushes students to revisit and solidify their foundational understanding of the part-whole relationship. They also support the development of critical reasoning skills, particularly in identifying the whole and verifying the basis of comparison in fraction tasks.

This study focused specifically on students' understanding and reasoning in incomplete problem situations related to the part-whole subconstruct. However, it did not explore other types of problem-solving contexts within the part-whole relationship. Future research is recommended to investigate more complex scenarios, such as tasks involving multiple wholes, to further expand our understanding of fractions learning within this subconstruct. Additionally, future studies may benefit from treating each component of the part-whole subconstruct as a milestone that students need to comprehend before progressing to the next. Experimental research comparing control and intervention groups could provide deeper insights into how students build a more robust and sequential understanding of each component.

ACKNOWLEDGMENTS

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Transforming Geometry Learning with an Interactive E-Module Integrating Ethno-Realistic Mathematics Education: A Design-Based Approach to Enhancing Numeracy Skills

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Abstract: Integrating cultural contexts into mathematics education, mainly through Ethno-Realistic Mathematics Education (Ethno-RME), presents a significant opportunity to enhance students' numeracy skills in geometry. However, challenges persist in effectively incorporating cultural elements into curricula while maintaining rigorous and modern mathematical instruction. This study addresses this gap by designing a learning trajectory for teaching geometry through the lens of Ethno-RME with E-Module as technology integration. The research is guided by design-based research methods, encompassing preliminary design, experimental design, and retrospective analysis stages. Primary data include qualitative descriptive and quantitative data collected through Focus Group Discussions (FGD), observations of student activities, field notes, student worksheets, video analysis, tests, and interviews. Participants included students from four schools, with a total of 123 students. The findings indicate that the implementation of the Hypothetical Learning Trajectory (HLT) significantly improved students' numeracy skills, increasing the average pass rate across four schools from 55.25% in the pre-test to 86.2% in the post-test, with all schools showing substantial progress. Students demonstrated enhanced problem-solving skills, critical thinking, and creative reasoning. This research underscores the value of blending cultural relevance with digital-based mathematical instruction, providing a novel approach to designing a curriculum that is both educationally impactful and culturally meaningful.

Keywords: Ethno-Realistic Mathematics Education, Design-Based Research, Numeracy Skills, Cultural Contexts, Geometry Learning Trajectory

INTRODUCTION

Students' numeracy skills in Indonesia play a crucial role in improving the quality of education and preparing a globally competitive future generation (Trisnawati et al., 2022). Within the context of the Merdeka Curriculum plan, enhancing numeracy skills has become one of the key pillars to

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empower students to compete internationally (Nelisiwe & Yu, 2023). However, the lack of numeracy skills among Indonesian students remains a serious concern. The 2022 PISA survey results show that Indonesia scored 366 in mathematics, a decrease of 13 points from the 2018 average score of 379 (OECD, 2023). These results reveal that Indonesian students have a weak understanding of geometry concepts, particularly in recognizing various shapes and using visual information to make conclusions (Laksmiwati et al., 2023). Many students also feel disconnected from mathematics, largely due to the lack of teaching approaches that are relevant to their everyday lives. While ethnomathematics has been widely studied, there is limited research exploring the integration of ethnomathematics with digital technologies to address these issues, particularly in the Indonesian context.

The Ethno-Realistic Mathematics Education (Ethno-RME) approach provides a distinctive framework for mathematics instruction by blending cultural and social contexts with realistic mathematics learning (Prahmana, 2022). By embedding cultural elements and students' everyday experiences into geometry lessons, Ethno-RME facilitates students' recognition of geometric patterns in their environment and enhances their understanding of the real-world applications of geometric concepts. This approach connects mathematical ideas with students' cultural backgrounds, making learning more relevant and engaging. However, the effective integration of technology into this culturally based learning is crucial, as technology itself reflects cultural advancements and practices. Thus, incorporating interactive digital learning media and tools becomes essential to support this integration and enhance the educational experience.

Interactive E-Modules represent a significant advancement in educational technology, transforming traditional learning methods by moving educational content to digital platforms (Setiyani et al., 2022). These interactive modules are designed to provide stimulating and immersive learning experiences, allowing students to engage actively with content. By presenting geometry materials rooted in cultural contexts through an interactive digital format, E-Modules enable students to explore geometric concepts in depth while maintaining a connection to their cultural heritage. The integration of technology in this manner not only boosts students' interest in learning but also aids in their comprehension of complex mathematical ideas, making the learning process both more dynamic and effective (Hwang et al., 2023).

While ethnomathematics research often emphasizes cultural relevance in learning, few studies have effectively merged this approach with digital tools like interactive E-Modules. This study addresses this research gap by designing a Geometry Learning approach based on Ethno-RME supported by Interactive E-Modules. This dual focus on culture and technology is intended to enhance students' numeracy skills in geometry and provide a practical and innovative teaching model for educators. The study seeks to answer the following research questions: 1) How can Ethno-Realistic Mathematics Education be integrated with Interactive E-Modules to improve students' numeracy skills in geometry? 2) What are the effects of this integrated approach on students' numeracy skills in geometric concepts and their connection to cultural contexts?

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This research emphasizes the urgent need to improve students' numeracy skills in geometry, an essential aspect of mathematics education that is often underdeveloped. Ethno-RME integrates local cultural aspects into the context of mathematics education, providing students with a more relevant and meaningful learning experience (Payadnya et al., 2024). Additionally, the use of Interactive E-Modules as a teaching aid reflects the state of the art in educational technology, a method that has not been widely applied in previous ethnomathematics research (Fouze & Amit, 2023). Consequently, this research not only offers a novel approach that combines culture and technology but also demonstrates significant potential in enhancing the effectiveness of learning and improving students' numeracy skills in geometry.

In the following section, we explore the research methodology, providing a detailed description of the research design and the three key stages involved: preliminary design, design experiment, and retrospective analysis. These stages are crucial in forming a coherent learning trajectory. The third section presents the results of implementing and analyzing the developed curriculum. It assesses the improvement in students' numeracy skills, offering insights into their progress, key findings, and student feedback from each activity. The conclusion then discusses the study's limitations and suggests directions for future research.

LITERATURE REVIEW

Ethnomathematics

Ethnomathematics explores the interconnections between mathematics and culture, demonstrating how mathematical concepts are deeply rooted in cultural practices, artifacts, and traditions. D'Ambrosio (Hamidah et al., 2024), a pioneer in this field, emphasized that recognizing these connections can make mathematics more engaging and meaningful for learners. This approach not only contextualizes mathematics in real-life settings but also fosters students' appreciation of their cultural heritage.

Research in ethnomathematics has explored its potential to bridge the gap between formal mathematics and students' everyday experiences. Studies have shown that incorporating cultural artifacts like batik patterns, woven textiles, and traditional architecture into mathematics instruction enhances student engagement and understanding. For instance, Prahmana & D'Ambrosio (2020) demonstrated how exploring symmetrical motifs in batik patterns in Yogyakarta, deepens students' understanding of geometry. Suherman and Vidákovich (2022) highlighted that the traditional "Tapis Lampung" fabric can serve as a medium to promote Indonesian local wisdom globally while also providing a contextual foundation for teaching mathematics in both rural and urban schools.

Despite these promising findings, ethnomathematics remains underutilized in formal classrooms. Challenges include a lack of teacher expertise in integrating cultural contexts into lessons, limited instructional resources, and insufficient research on the use of digital tools to support

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ethnomathematics. This study addresses these gaps by designing and testing digital instructional materials that integrate ethnomathematics, aiming to make its application more accessible and scalable across diverse educational settings.

Local Instructional Theory (LIT)

Local Instructional Theory (LIT) is a framework developed to design and analyze teaching strategies and materials for specific mathematical topics. Unlike general teaching theories, LIT is iterative and context-specific, enabling educators to refine instructional methods based on empirical evidence (Gravemeijer, 2020). It is particularly useful in addressing challenges in teaching complex mathematical topics like geometry, where students often struggle with abstraction.

LIT has been widely used to develop instructional sequences for various mathematical topics. Paoletti et al. (2023) highlighted its application in geometry education, where teaching sequences were designed to build students' understanding of abstract concepts like the properties of shapes and spatial reasoning. Gravemeijer (2020) emphasized the role of LIT in iterative development, where real-world contexts are used to scaffold students' conceptual understanding. Studies in Indonesia (Meryansumayeka et al., 2022) have also shown how LIT can support the teaching of geometry, but these studies are often limited to small-scale implementations without leveraging modern technologies.

While LIT has been effective in targeted interventions, there is limited research on its integration with cultural contexts and digital tools. This study seeks to address this gap by combining LIT with ethnomathematics and interactive e-modules, creating a framework that is both culturally responsive and technologically enhanced.

Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME) is an approach that prioritizes the use of real-world problems as a basis for teaching mathematics. Developed in the Netherlands, RME has significantly enhanced students' mathematical reasoning and problem-solving skills. The approach emphasizes active learning, where students construct their understanding through guided discovery in realistic contexts (Gravemeijer, 2020).

RME has been extensively studied for its impact on student learning. In Indonesia, research by Prahmana (2022) demonstrated how RME-based geometry lessons improved students' ability to analyze and solve problems in real-life scenarios, such as calculating the dimensions of traditional houses or analyzing the shapes and areas of rice fields. Altıner et al. (2023) reported similar findings, showing that RME improved student engagement and conceptual understanding by

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grounding abstract concepts in familiar, everyday situations. However, studies often highlight challenges in implementation, such as the lack of teacher training and resources, particularly in developing countries.

Although RME has proven effective, its potential for integration with ethnomathematics and digital tools remains underexplored. This study seeks to fill this gap by developing e-modules that combine RME with ethnomathematics, offering a scalable solution for improving geometry instruction in culturally diverse and resource-limited settings.

Research Contribution

By addressing the identified gaps in the literature—limited integration of ethnomathematics, LIT, and RME with digital tools—this study aims to develop a comprehensive instructional framework. The framework leverages culturally relevant contexts and technology to enhance student engagement and understanding, particularly in geometry education. The study's findings will contribute to the broader discourse on culturally responsive and technology-enhanced mathematics education.

METHOD

Research Design

This study employed a design research approach aimed at developing a Local Instructional Theory (LIT) in the form of a learning trajectory (Bakker & van Eerde, 2015). The LIT framework was operationalized through the integration of Realistic Mathematics Education (RME) principles, emphasizing contextual relevance, active student participation, and the use of mathematical models to bridge informal and formal understanding. The research process consisted of three key stages: preliminary design, design experiment, and retrospective analysis. The focus of this research was on geometry, a subject area that represented a significant weakness in numeracy among Indonesian students and offered extensive potential for ethnomathematical exploration. Geometry was chosen for this study due to its foundational role in mathematics, its close connection to real-world applications, and its alignment with RME principles, which facilitate the integration of realistic and cultural contexts into mathematical learning.

Guiding Principles and Criteria for Integration

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The integration of relevant materials and Realistic Mathematics Education (RME) into the lessons was guided by three key principles: contextual relevance, cognitive load, and engagement through interactivity. Contextual relevance ensures that learning materials align with Balinese ethnomathematics, allowing students to connect abstract mathematical concepts with real-world cultural contexts. This approach is supported by Prahmana and D'Ambrosio's (2020) ethnomathematics framework, which emphasizes the significance of integrating cultural elements into mathematics education to enhance students' understanding and engagement.

Cognitive load is another crucial consideration in instructional design, as it ensures that learning materials are developmentally appropriate and scaffold students' understanding. According to Klepsch and Seufert (2020), Cognitive Load Theory, instructional materials should be structured to balance intrinsic, extraneous, and germane cognitive loads, enabling students to progressively develop their mathematical reasoning. Additionally, structured guidance in learning is more effective than minimally guided instruction, reinforcing the need for a well-sequenced approach that gradually increases complexity.

Finally, engagement and interactivity play a vital role in fostering student motivation and participation. Research by García-López, et al. (2023) indicates that interactive learning environments significantly enhance student engagement by incorporating multimedia elements such as simulations and gamified assessments. By integrating these guiding principles, this study ensures that the incorporation of Ethno-RME and digital learning tools is both pedagogically sound and culturally meaningful.

Research Location, Participants, and Time

The study was conducted in four middle schools, both public and private, located in Bali. One class from each school, specifically 8th-grade students, participated in the study. The selection of locations and schools was based on previous research that highlighted extensive ethnomathematical exploration within Balinese cultural contexts. Additionally, the involvement of two schools from each region provided comparative data, thereby enhancing the validity of the research findings. The selected schools were those with established collaboration agreements with the research institution, facilitating coordination and the provision of support such as teachers, students, classrooms, and necessary research equipment. Participants included students from four schools, with 35 students from School 1, 28 from School 2, 30 from School 3, and 30 from School 4. This research was carried out in the academic year of 2023/2024.

Data Collection Techniques

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The primary data for this research included both qualitative, descriptive, and quantitative data. Qualitative data were obtained through the development of the learning process using Focus Group Discussions (FGD), observation of student activities, field notes, analysis of student worksheets, video analysis of the learning process, tests, and interviews. Unstructured observations and open-ended interviews were employed (Taherdoost, 2022). Quantitative data on students' numeracy skills were collected through pre-test and post-test assessments using essay questions as the evaluation instrument.

The selected mathematical content for this study was plane geometry, chosen due to its importance in the middle school curriculum as it laid the foundation for understanding more complex geometric concepts at higher educational levels. Additionally, plane geometry allowed for the integration of cultural aspects through Ethno-Realistic Mathematics Education and supported the use of interactive technology such as Interactive E-Modules to visualize concepts engagingly and clearly. This made the content highly relevant and supportive of the innovative approach employed in this research.

Validation of HLT

The Hypothetical Learning Trajectory (HLT) was validated by two experts: one specializing in mathematics education content and the other in digital media. A validation tool consisting of criteria such as content accuracy, cultural relevance, alignment with Realistic Mathematics Education (RME) principles, and the effectiveness of digital tools was employed.

The content expert recommended integrating more interactive and culturally relevant materials to better align with ethnomathematical principles, while the digital media expert suggested enhancing the visualization of geometric concepts by replacing static 2D images with interactive 3D models created using Assembly Studio, providing students with a more immersive learning experience. Additionally, they advised incorporating Quizizz for assessments due to its gamified and interactive features, which effectively engage students. Based on their feedback, the lessons were revised by replacing regular 2D pictures with 3D models to allow for a more dynamic exploration of geometric concepts and transitioning from traditional assessment tools to Quizizz for both formative and summative evaluations, ensuring immediate feedback and improved student engagement.

These changes were incorporated into the HLT and validated again by the experts to ensure their appropriateness and effectiveness in achieving the study’s objectives.

Research Procedure

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As previously mentioned, this research was conducted through three general stages: preliminary design, design experiment, and retrospective analysis carried out over four months to produce a Hypothetical Learning Trajectory (HLT) in the initial stage and a Local Instructional Theory (LIT) in the final stage. Before this study, the researcher had already conducted various preliminary studies related to the topic. The detailed activities at each stage are presented in Table 1 below.

No	Stage	Activities
1	Preliminary design	This stage focused on analyzing the situation and issues in mathematics education at schools. Several activities were conducted during this stage to analyze the situation and design the initial plan, including observing classroom teaching, interviewing mathematics teachers, analyzing the current mathematics curriculum, and reviewing relevant references and studies. Subsequently, the HLT and learning tools such as modules, student worksheets (LKPD), learning media, and instruments were developed. Instrument testing was conducted using the content validity index, content validity scale, and instrument reliability. Additionally, a pre-test was conducted to assess students' initial numeracy skills.
2	Design experiment	The HLT was implemented in the research subject classes, with observations of the teaching activities, field notes, and multi-directional documentation of the learning process. This implementation was conducted in four research schools, each using one regular class (30-35 students). The results obtained from each school were compared to obtain more valid and informative data.
3	Retrospective analysis	Data collected during the field implementation were analyzed qualitatively using the constant comparison method, following these steps: (1) reviewing the learning recordings, (2) creating a general transcript, (3) marking important segments, (4) completing the transcript of those segments, (5) seeking confirmation or contradiction in other segments, and (6) discussing findings with colleagues for validation. Quantitative data were analyzed using a mean difference test to assess improvements in students' numeracy skills.

Table 1. Research Stages

After the retrospective analysis, the data obtained, which included scores, students' worksheets, and student responses, were used to assess the development of students' numeracy skills and to inform future improvements to the Hypothetical Learning Trajectory (HLT).

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Pre-Test and Post-Test Content

The pre-test and post-test were designed to measure students' numeracy skills. They included questions on fundamental geometric concepts such as understanding the properties of shapes, calculating areas and perimeters, and applying geometric reasoning to solve real-world problems. Ethnomathematical elements were integrated into some questions to assess students' ability to connect mathematical concepts to cultural contexts. The results of these tests were compared, particularly in terms of achieving the minimum mastery criteria.

RESULTS AND DISCUSSION

Results

In the preliminary phase of the study on "Designing Geometry Instruction with Ethno-Realistic Mathematics Education and Interactive E-Modules: A Research-Based Approach to Enhancing Numeracy Skills," the researcher conducted several crucial activities, including a comprehensive literature review, interviews with mathematics teachers, and the development of the Hypothetical Learning Trajectory (HLT) with guidance from a supervisor. The literature review focused on integrating cultural contexts and realistic problem-solving approaches in geometry, emphasizing Ethno-Realistic Mathematics Education (Ethno-RME) and the use of interactive digital tools such as e-modules. Interviews with three mathematics teachers revealed that students often relied on memorizing formulas and solving procedural problems without fully grasping the underlying concepts, especially in tasks involving culturally relevant or irregular shapes. To address these challenges, the HLT was designed over two months and incorporated cultural elements along with interactive applications. The HLT was then validated by experts in ethnomathematics and realistic mathematics education, ensuring a comprehensive approach that integrates both cultural and technological aspects into the geometry curriculum. Here is the summary of the HLT design which includes learning activities along with key findings observed in each stage.

No.	Learning Objective	Main Activity	Key Outcome
1	**Understanding Sanggah Cucuk**	• Watch videos and analyze images of Sanggah Cucuk.	• Identify Sanggah Cucuk as a triangular prism.

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	Students recognize the shape and function of Sanggah Cucuk.	• Identify geometric shapes and materials needed.	• Determine the geometric components (e.g., Klakat shapes).
2	**Designing Sanggah Cucuk** Students create a design for Sanggah Cucuk.	• Design Sanggah Cucuk virtually. • Calculate and record required materials.	• Complete a design with accurate quantities (e.g., 3 square Klakats, 1 triangular Klakat).
3	**Building Sanggah Cucuk** Students construct Sanggah Cucuk.	• Construct the model using provided materials. • Compare with other groups and reflect on challenges.	• Build Sanggah Cucuk as planned. • Reflect on design and material challenges.
4	**Applying Mathematical Concepts** Students calculate and compare surface area and materials.	• Calculate the surface area and total bamboo needed. • Compare with a standard triangular prism.	• Accurately calculate the surface area and material needed. • Understand the connection between math concepts and the design.

Table 2. Hypothetical Learning Trajectory (HLT)

The HLT design outlined in Table 2 provides a detailed overview of the learning activities, capturing key insights and findings observed across each stage. These findings have been synthesized from data gathered at four participating schools, offering a comprehensive look into the outcomes and challenges encountered during the learning process. By analyzing the data from these different contexts, the summary highlights patterns, variations, and effective strategies within the HLT framework, offering valuable implications for refining instructional practices. Here are the findings in each activity.

The object used in the HLT is “Sanggah Cucuk” as shown in Figure 1 below:

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Figure 1. Sanggah Cucuk (Source: https://paduarsana.com/2013/05/21/sanggah-cucuk-dan-tawur-agung-kesanga/ & https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcRpXuNyHnN59up9wHEnM68G1SIvM_T45HG1wg&)

The "Sanggah Cucuk" is a traditional Balinese shrine made of bamboo, typically constructed for specific ceremonies, such as the Tawur Agung Kesanga, which marks the ritual purification preceding the Balinese New Year (Nyepi). This sacred structure serves as a temporary altar to place offerings, symbolizing a bridge between humans and the divine. Its design incorporates both 2D and 3D geometry concepts, reflecting the mathematical precision involved in Balinese craftsmanship. As a cultural object, the Sanggah Cucuk embodies the spirituality, artistry, and mathematical ingenuity of the Balinese people.

Learning Activity

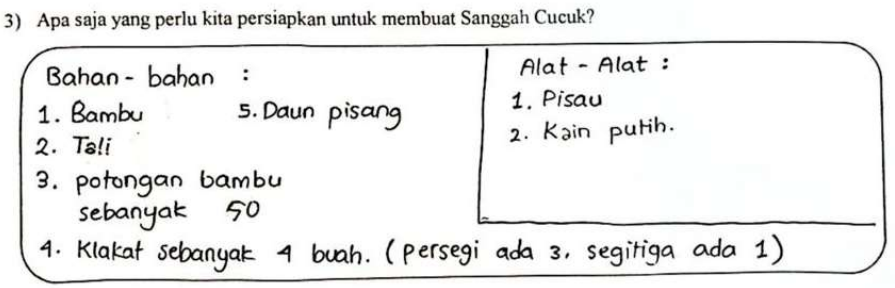
Data collected from the four schools, consisting of students' responses to each activity in the worksheets, were systematically gathered, reduced, and categorized based on the key findings from each learning activity. To facilitate collaborative learning, students were grouped into teams of 5-7 members in each class, allowing them to work on the worksheet through discussion and cooperation. This data was then analyzed to assess students' numeracy skills and their comprehension of each segment of the material. The process involved identifying patterns and trends within the students' work to evaluate their progress in both understanding the geometric concepts and applying numeracy skills across the different instructional phases. The analysis not only provided insights into the overall effectiveness of the Ethno-Realistic Mathematics Education approach but also highlighted specific areas where students demonstrated significant gains or encountered challenges. By categorizing the data into relevant themes and aligning them with the instructional objectives, the study was able to draw meaningful conclusions regarding the integration of cultural context and digital tools in enhancing students' numeracy skills.

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Learning Activity 1

As outlined in the table, the first activity centers on students analyzing the structure of the Sanggah Cucuk, a traditional Balinese shrine, through an instructional video embedded within the E-Module. The activity is designed to engage students in observing and understanding the geometric shapes that form the Sanggah Cucuk. Through guided exploration, students recognize that the structure is an equilateral triangular prism, consisting of three rectangular faces and one equilateral triangular face. This exercise helps students relate mathematical concepts to real-world cultural artifacts, bridging the gap between abstract geometry and tangible, culturally relevant examples.

In analyzing the video, students demonstrated their ability to break down complex structures into fundamental geometric shapes, reflecting a deeper understanding of both geometry and its applications. The recognition of Sanggah Cucuk's components—three squares or rectangles and one equilateral triangle—shows that students can connect their theoretical knowledge to practical contexts. By engaging with a culturally significant object, students not only reinforced their grasp of geometric concepts but also developed an appreciation for how these concepts manifest in everyday life. The following example of a student's response highlights their ability to accurately describe the structure, thereby demonstrating the effectiveness of using culturally embedded resources to enhance both comprehension and engagement in mathematics.



Translation:

3) What do we need to prepare for making a Sanggah Cucuk?**

Materials
1. Bamboo
2. Rope
3. 50 pieces of bamboo

4. 4 Klakat (3 square-shaped, 1 triangular-shaped)

Tools:
1. Knife
2. White cloth

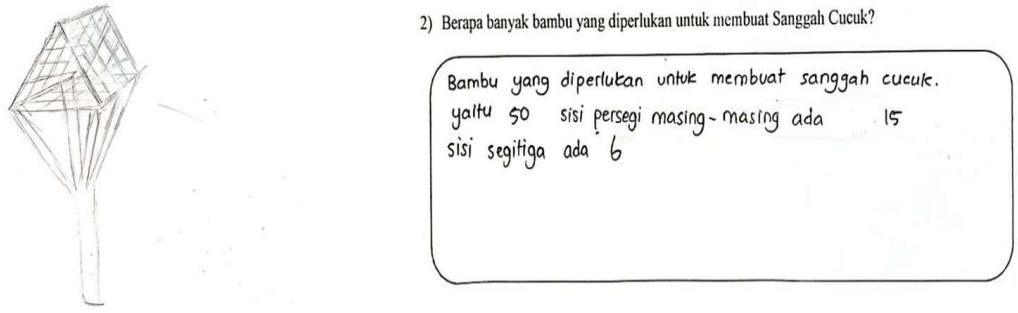
Figure 2. Student's Answer on Activity 1

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The observations from Figure 2 reveal that students demonstrated realistic thinking and accurately calculated the required materials for constructing a Sanggah Cucuk. In responding to question number 3, students provided precise explanations of the materials needed by drawing connections to their understanding of Sanggah Cucuk's components. Despite never having constructed a Sanggah Cucuk themselves, students utilized prior knowledge and experiences to deliver well-reasoned answers. This illustrates how integrating cultural experiences and real-world contexts into learning enhances students' numeracy skills while reinforcing their understanding of ethnomathematical concepts and realistic mathematics. By linking prior experiences with mathematical tasks, students effectively navigated abstract ideas and applied them to practical problem-solving. These findings highlight the effectiveness of Ethno-Realistic Mathematics Education in fostering meaningful engagement with mathematical content, promoting reasoning, accurate calculations, and connections between mathematical concepts and their real-world applications (Apriatni et al., 2022).

Learning Activity 2

In the second learning activity, students were tasked with designing their version of a Sanggah Cucuk. Before this, they had watched a 3D design demonstration in the E-Module. Building on this visual guidance, students analyzed the design and then proceeded to create their sketches. The hands-on nature of this task, combined with the interactive 3D model, sparked enthusiasm among the students, as they engaged actively in manipulating the 3D design. This approach not only allowed students to exercise their creativity but also deepened their spatial reasoning and geometric understanding. The following example showcases a student's original design, reflecting both their engagement with the cultural context and their grasp of geometric principles.



<u>*Translation:*</u>

How much bamboo is needed to make a Sanggah Cucuk?

The bamboo needed to make a Sanggah Cucuk includes 50 square sides, each with 15 sides, and 6 triangular sides.

Figure 3. Example of Student's Design on Sanggah Cucuk and Stundent's Answer on a Question in Activity 2

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Figure 3 shows that students effectively designed the Sanggah Cucuk, showcasing their representational skills and spatial awareness. Their ability to visualize and accurately depict geometric structures demonstrates a strong grasp of the concepts being taught. However, challenges emerged in the follow-up question, where students were required to calculate the quantity of bamboo needed for construction. The varying results among groups revealed inconsistencies in their estimation and calculation processes. These discrepancies highlight the need for further instruction to strengthen students' mathematical reasoning and problem-solving skills, particularly when applying theoretical knowledge to practical, real-world scenarios (Firdausy & Indriati, 2021).

To address these inconsistencies, the researcher emphasized that each calculated quantity must be supported by detailed reasoning and accurate calculations. The rule that students can only take the exact amount of bamboo they recorded for the next activity reinforces the importance of accuracy and discourages intentional overestimation. This approach not only fosters careful mathematical planning but also introduces students to the importance of resource management, linking mathematical concepts to real-world applications in a culturally relevant context.

Learning Activity 3

This activity was the most engaging and lively of all the sessions, as students were directly involved in constructing the Sanggah Cucuk. The high level of enthusiasm among the students indicates that project-based learning using real objects and incorporating cultural contexts can significantly boost student motivation. The hands-on experience allowed students to connect mathematical concepts with tangible outcomes, reinforcing their understanding while making the learning process more enjoyable and meaningful.

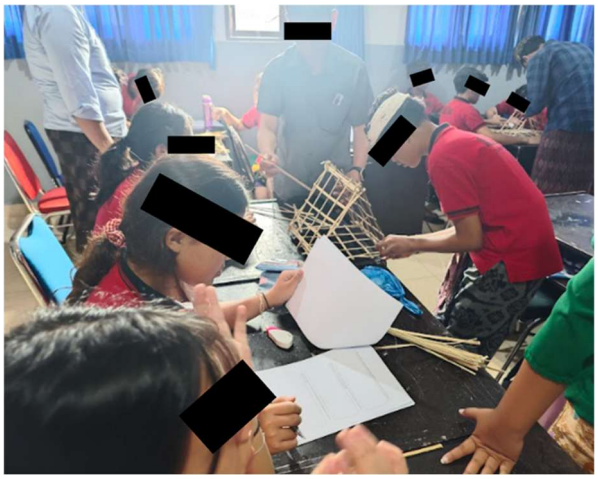
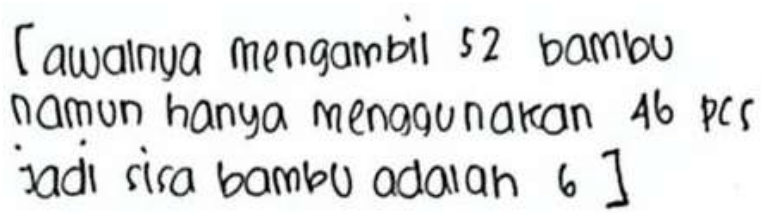


Figure 4. Students Working on Sanggah Cucuk

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Figure 4 highlights the students' notably positive responses during the third activity, as they proudly presented their Sanggah Cucuk creations derived from their mathematical calculations. This enthusiasm underscores the effectiveness of hands-on, project-based learning within a cultural context, which makes mathematical concepts more accessible and engaging. However, when analyzing and comparing their designs with those of their peers, many students struggled to identify design differences but were more successful in recognizing calculation errors. The leftover bamboo observed in most groups points to inconsistencies in their numeracy skills, particularly in applying accurate calculations to practical tasks.



Translation:

Initially, 52 bamboo pieces were taken, but only 46 pieces were used, so the remaining bamboo is 6 pieces.

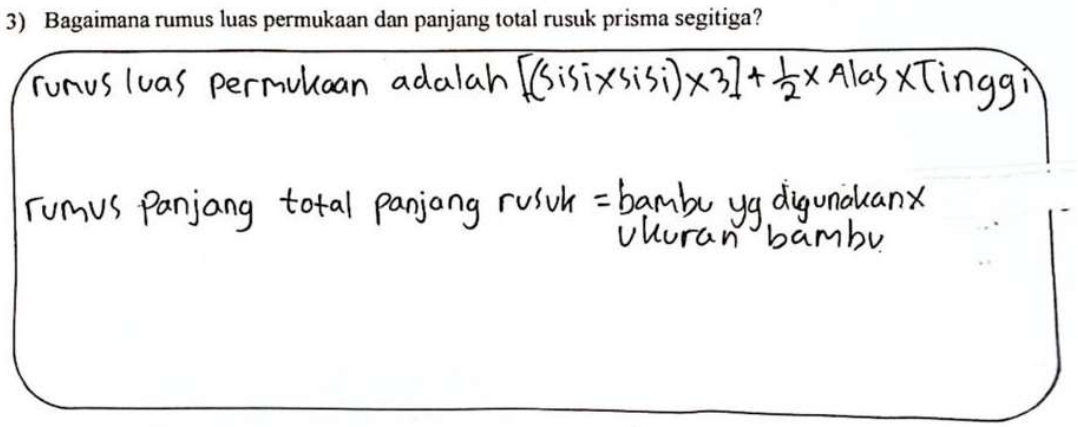
Figure 5. Students Realize Their Calculation Mistake

Figure 5 highlights a positive outcome, as students successfully identified calculation errors when determining the amount of bamboo needed. They quickly devised corrective measures and reconciled their results with those of their peers, demonstrating improved problem-solving abilities. This achievement reflects the effectiveness of a learning experience enriched with ethnomathematics and cultural relevance, which helped students enhance both accuracy and consistency in their work. By incorporating realistic ethnomathematical contexts and familiar cultural elements, the approach not only improved numeracy skills but also fostered a deeper understanding and quicker correction of errors, resulting in more uniform outcomes among students (Supriyadi et al., 2022).

Learning Activity 4

In this activity, students were presented with several problems related to the Sanggah Cucuk and triangular prisms. Specifically, they were tasked with solving questions about edge lengths related to the amount of bamboo needed and surface area related to the covering fabric. The students successfully addressed these challenges, accurately deriving the formula for the surface area of a prism and applying it to calculate the total length of bamboo and the fabric required for the Sanggah Cucuk. This performance demonstrates a significant improvement in their numeracy skills, particularly in applying geometric formulas to solve real-world problems.

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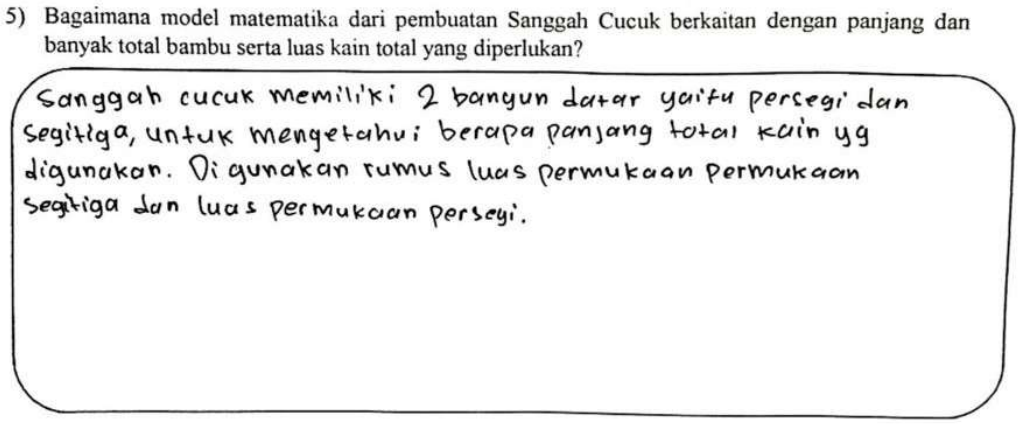
Translation:
The formula for Surface Area is (side x side) x 3) + ½ x base x height.
The formula for Total Edge Length is bamboo used x bamboo size.

Figure 6. Students Able to Find the Surface Area Formula of a Prism Based on Activities Participated In

The success of the activity depicted in Figure 6 highlights the effectiveness of the Ethno-Realistic Mathematics Education (Ethno-RME) approach. By situating mathematical problems within the culturally relevant and tangible context of the Sanggah Cucuk, students were able to bridge abstract geometric concepts with practical applications. This integration of cultural elements not only deepened their understanding of mathematical principles but also enhanced their problem-solving skills. The student's ability to apply mathematical formulas to real-life scenarios demonstrates the success of Ethno-RME, emphasizing that culturally grounded education can significantly improve numeracy skills while creating a more engaging and impactful learning experience.

The difficulty observed in students' responses to the mathematical modeling question underscores a challenge in their ability to connect geometric concepts with algebraic modeling using variables. This issue can be attributed to the question's lack of clear structure and its misalignment with the student's cognitive development stages. According to Piaget's Theory of Cognitive Development, students progress from concrete operational thinking, which involves tangible problem-solving, to formal operational thinking, which encompasses abstract reasoning and the use of variables (Rabindran & Madanagopal, 2020). In this instance, while students effectively handled concrete geometric problems related to the Sanggah Cucuk, they struggled with more abstract tasks requiring algebraic modeling.

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Translation:

5) What is the mathematical model for making a Sanggah Cucuk about the total length and amount of bamboo and the total area of cloth needed?

The Sanggah Cucuk consists of 2 shapes: squares and triangles. To determine the total length of cloth used, the formulas for the surface area of a triangle and the surface area of a square are applied.

Figure 7. Students Answer Questions About the Mathematical Model Incorrectly

The difficulty highlighted in Figure 7 emphasizes the critical role of mathematical connection skills, which involve linking various mathematical concepts and procedures to solve complex problems. Many students struggle to integrate geometric understanding with algebraic methods, as this demands cognitive maturity and the ability to connect abstract and concrete ideas. For example, some students' responses indicate a tendency to treat algebraic and geometric concepts in isolation, resulting in incomplete or incorrect solutions. To bridge this gap, tasks should be scaffolded to align with students' developmental stages, providing step-by-step support to help them progress from foundational to advanced problem-solving. Structured questions that build on prior knowledge and foster the integration of mathematical concepts can enhance students' ability to apply algebraic techniques effectively in solving geometric problems (Ukobizaba et al., 2021).

Students' Test and Interview Results

Pre-Test Result

Before implementing the HLT (Hypothetical Learning Trajectory), the researcher administered a pre-test on numeracy skills to the students to assess their initial capabilities. The results of the pre-test are as follows:

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School	Number of Students	Percentage of Students Passing (>80)	Number of Students Passing
School 1	35	50%	30
School 2	28	60%	25
School 3	30	55%	24
School 4	30	55%	27

Table 3. Students' Pre-Test Score in Each School

Table 3 shows the overall performance of students is below expectations, with only 50% to 60% of students achieving scores above the 80% threshold in each school. School 1 has the lowest pass rate at 50%, with only 17 out of 35 students passing. Schools 3 and 4 have a slightly higher pass rate at 55%, with 16 out of 30 students passing. School 2, though higher than the others, still only reaches a 60% pass rate. This indicates that numeracy skills among the students are insufficient and there is a clear need for improvement in the instructional strategies to enhance students' numeracy proficiency.

Post-Test Result

The researcher administered a post-test on numeracy skills at the end of HLT implementation to evaluate the overall learning achievement and determine the percentage of students who met the score threshold of 80, as outlined in the table below. Here is the table showing the percentage of students who achieved scores above 80 in four schools:

School	Number of Students	Percentage of Students Passing (>80)	Number of Students Passing
School 1	35	85%	30
School 2	28	90%	25
School 3	30	80%	24
School 4	30	90%	27

Table 4. Students' Post-Test Score in Each School

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A total of 123 students, with 106 students (approximately 86.2%) scoring 80 or above. Table 4 presents the achievement levels of students in four schools involved in the study, indicating the percentage and number of students who scored above the threshold of 80. School 2 and School 4 had the highest percentage of students meeting this criterion, with 90% of their students passing. Meanwhile, School 1 and School 3 had slightly lower pass rates at 85% and 80%, respectively. Overall, the majority of students across all schools demonstrated strong performance, with more than 86% of the total students achieving scores above the set threshold, reflecting the effectiveness of the implemented instructional strategies.

The comparison between the pre-test and post-test results reveals a clear improvement in students' numeracy skills after the implementation of the Hypothetical Learning Trajectory (HLT). In the pre-test, the total average percentage of students passing the 80 threshold across all four schools was 55.25%, indicating that a majority of students struggled to meet the target. Specifically, the pass rates ranged from 50% in School 1 to 60% in School 2. This highlighted a need for stronger instructional strategies to address gaps in students' numeracy skills. However, after the implementation of the HLT, the post-test results showed a significant improvement, with the average pass rate rising to approximately 86.2%. In the post-test, the percentage of students who scored above 80 increased across all schools, with the highest improvements observed in Schools 2 and 4, both achieving 90% pass rates. Schools 1 and 3 also showed positive changes, with pass rates reaching 85% and 80%, respectively. This substantial increase in the pass rate across all schools suggests that the HLT effectively addressed the deficiencies in numeracy skills, indicating that the learning strategies implemented had a significant impact on enhancing students' understanding and performance in numeracy. The comparison demonstrates that the HLT contributed to a remarkable improvement in student achievement, highlighting the importance of well-designed instructional frameworks in boosting students' academic outcomes.

Interview Result

In addition to the test, the researcher conducted interviews with two representative students from each school to gain deeper insights into their experiences and perceptions of the learning activities. The selection criteria for these students were based on their active participation during the lessons and the diversity of their academic performance, ensuring that both high-achieving and average students were included. This approach allowed the researcher to capture a broad range of perspectives on the effectiveness of the learning model. The interviews were designed to explore how the integration of project-based activities and cultural elements influenced their motivation and engagement in mathematics.

The findings from these interviews revealed that students felt highly motivated throughout the learning process. They appreciated the hands-on, digital, and project-based nature of the activities, which allowed them to actively engage with the subject matter. Additionally, the incorporation of cultural context through traditional elements like the Sanggah Cucuk provided a unique learning

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experience that extended beyond mathematics. Students expressed that, alongside developing their mathematical skills, they also gained valuable knowledge about their cultural heritage. This dual focus not only enriched their learning but also made the material more relatable and meaningful, leading to a more positive and memorable educational experience (Jia & Zhang, 2023).

Discussion

The study reveals several significant insights into the impact of the Ethno-Realistic Mathematics Education (Ethno-RME) approach on students' numeracy skills and conceptual understanding. By integrating cultural artifacts, such as the Sanggah Cucuk, with mathematical instruction, the study demonstrated that students can effectively link abstract geometric concepts with real-world applications, enhancing both engagement and comprehension. This approach aligns with Piaget's theory of cognitive development, which suggests that connecting concrete experiences to abstract concepts can facilitate understanding, particularly for students in the concrete operational stage (Oogarah-Pratap et al., 2020).

The activities highlighted varying levels of student proficiency in applying mathematical concepts. While students showed enthusiasm and creativity in tasks involving the design and construction of the Sanggah Cucuk, issues emerged in areas requiring precise calculations and mathematical modeling. The discrepancies in bamboo estimation and challenges in algebraic modeling reflect Vygotsky's Social Development Theory, which emphasizes the role of social interaction in cognitive growth. Collaborative learning environments helped students engage deeply with the material but also exposed areas where additional support is needed (Thompson, 2022). This suggests that future iterations of the Hypothetical Learning Trajectory (HLT) should include more structured guidance to enhance mathematical reasoning and problem-solving skills.

Students' ability to apply geometric formulas in practical contexts demonstrated the effectiveness of Ethno-RME in reinforcing their understanding of mathematical principles. However, difficulties with abstract tasks, such as algebraic modeling, indicated a gap in students' transition from concrete to formal operational thinking, as outlined by Piaget (Sibgatullin et al., 2022). This underscores the need for developmentally appropriate scaffolding to support students in bridging this gap. By aligning instructional activities with cognitive development stages and providing targeted support, educators can better facilitate students' transition to more abstract mathematical reasoning.

The integration of culturally relevant and hands-on learning experiences, as seen in the study, significantly enhanced student motivation and problem-solving abilities. Dewey's Experiential Learning Theory supports this finding, highlighting the benefits of experiential and contextually grounded learning in fostering a deeper understanding of mathematical concepts (Uyen et al., 2022). The positive outcomes observed in students' engagement and accuracy in practical tasks underscore the value of incorporating cultural contexts into mathematical instruction.

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In conclusion, the study affirms the effectiveness of Ethno-RME in improving students' numeracy skills and understanding by providing culturally relevant, hands-on learning experiences. To further enhance this approach, future revisions to the HLT should focus on addressing gaps in mathematical reasoning and supporting the transition from concrete to abstract thinking. By incorporating structured support and developmentally appropriate tasks, educators can build on the successes observed and continue to refine the integration of cultural contexts and digital tools in mathematics education.

Practical Implications

This research contributes to the growing body of literature on culturally responsive mathematics education by integrating Ethno-Realistic Mathematics Education (Ethno-RME) with digital tools to enhance students' numeracy skills. Previous studies have emphasized the importance of contextualizing mathematical concepts within students' cultural experiences to foster engagement, understanding, and learning outcomes (Mairing & Nini, 2023; Orozco & Pasia, 2021). However, many of these studies have focused primarily on theoretical frameworks without offering practical applications that directly connect cultural elements with formal mathematics. This research fills that gap by presenting a tangible model where cultural artifacts, such as the Sanggah Cucuk, are used to bridge abstract mathematical concepts and real-world applications. In doing so, it complements previous work by demonstrating how culturally relevant pedagogy can be operationalized effectively in classroom settings. Additionally, this study identifies the challenges in applying abstract mathematical reasoning and highlights the need for more structured guidance to support students in developing their mathematical problem-solving skills. The findings align with existing theories on the role of cultural context in learning and underscore the need for developmentally appropriate scaffolding to guide students through cognitive transitions (Osher et al., 2020).

Moreover, this study advocates for incorporating more structured guidance to enhance mathematical reasoning and problem-solving skills within the Hypothetical Learning Trajectory (HLT). Structured guidance in the form of step-by-step problem-solving techniques, scaffolding questions, and visual representations can provide the necessary support for students to bridge the gap between concrete and abstract thinking. For example, teachers can implement scaffolded questioning techniques such as, "What happens when we apply this rule in a real-world context?" or "Can you explain how this cultural artifact helps us understand this mathematical concept?" These prompts help students connect abstract concepts with tangible cultural experiences, thereby facilitating deeper comprehension. Additionally, teachers can guide students through the process of analyzing a cultural artifact and identifying the mathematical properties within it. For instance, when exploring the Sanggah Cucuk, students could be prompted to identify patterns, shapes, or symmetries that relate to geometric concepts, thereby making abstract math concepts more accessible. Teachers could also encourage students to create mathematical models based on cultural artifacts and apply mathematical reasoning to understand the artifact's function and significance.

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Another example of structured guidance is the use of interactive digital tools, such as virtual manipulatives, to help students visualize mathematical concepts in real time. These tools can be programmed to provide instant feedback, guide students through progressively complex tasks, and suggest hints when students encounter difficulties. For example, a virtual geometry tool could allow students to manipulate shapes inspired by cultural designs and explore their properties, helping them connect the geometry of cultural artifacts with formal mathematical concepts (Payadnya et al., 2023). Furthermore, scaffolded worksheets can be used, with progressively challenging tasks that build students' problem-solving abilities. For example, initial tasks might involve simple arithmetic operations related to cultural elements, while later tasks could require students to apply abstract algebraic concepts to these real-world contexts.

The practical implications of this study are particularly relevant for curriculum designers and educators seeking to enhance mathematics instruction by making it more culturally inclusive and engaging. The integration of Ethno-RME with interactive digital modules provides a promising framework for future curriculum development, illustrating how cultural elements can be seamlessly woven into modern pedagogical practices.

By addressing both the strengths and areas for improvement in students' numeracy skills, this research offers actionable insights for refining instructional strategies, particularly in regions where cultural heritage plays a central role in everyday life (Thi et al., 2023). The research emphasizes the importance of structured questioning and scaffolded support, which can significantly improve students' ability to transition from concrete to abstract mathematical thinking (DeJarnette & Hord, 2022). This approach not only enriches the learning experience but also has the potential to improve student outcomes in mathematical problem-solving, making it a valuable resource for advancing mathematics education in diverse cultural contexts.

CONCLUSIONS

The findings of this research highlight the significant positive impact of integrating Ethno-Realistic Mathematics Education (Ethno-RME) with digital media on enhancing students' numeracy skills and their understanding of geometric concepts. The learning activities, which incorporated culturally relevant contexts such as the Sanggah Cucuk, resulted in high levels of student numeracy skills. Students exhibited a strong grasp of geometric principles and were able to connect these concepts to real-world applications grounded in their cultural heritage. The project-based and collaborative nature of the tasks further reinforced students' numeracy skills, fostering their problem-solving abilities, critical thinking, and teamwork. These outcomes underscore the effectiveness of combining ethnomathematics with realistic mathematics education to provide a comprehensive learning experience that is both culturally meaningful and educationally impactful.

However, it is important to acknowledge the primary limitation of this study, which is the restricted sample size and the specific cultural context in which the research was conducted. The study was

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limited to four schools within a particular geographical area, which may constrain the generalizability of the findings to broader educational contexts. The cultural homogeneity of the student sample also limits the exploration of diverse ethnomathematical practices. Future research should aim to include a more diverse range of cultural contexts and larger sample sizes. Such an approach would allow for a more thorough understanding of how different cultural perspectives can be integrated into mathematics education, leading to the development of more inclusive and adaptable teaching methods across various educational settings.

Based on these conclusions, it is recommended that educators and curriculum developers incorporate culturally relevant content into mathematics education. By aligning mathematical instruction with students' cultural backgrounds and real-world experiences, teachers can create more engaging and relatable learning environments that enhance students' cognitive and affective outcomes. Additionally, adopting structured questions and scaffolded tasks, as highlighted by the study's findings, can help address gaps in understanding and support the gradual development of mathematical thinking from concrete to abstract reasoning. Incorporating these approaches not only promotes better comprehension of mathematical concepts but also contributes to the development of a more inclusive, culturally responsive, and student-centered mathematics education framework.

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APPENDIX

Pre-Test and Post-Test Questions
"Transforming Geometry Learning with Interactive E-Module Integrating Ethno-Realistic Mathematics Education: A Design-Based Approach to Enhancing Numeracy Skills"

Pre-Test Questions

1. Understanding Shapes in Culture
In Balinese traditional architecture, many temples feature triangular and square patterns in their designs. Describe the properties of squares, triangles, and prisms, and explain how these shapes contribute to the stability of traditional structures.

Answer:
- Square: Has four equal sides and four right angles; used in foundations for stability.
- Triangle: Three sides with different types (equilateral, isosceles, scalene); strong structure for roofs.
- Prism: 3D shape with two parallel bases; used in pillars for support.
- Application in Balinese Architecture: Triangular roofs allow water drainage, square bases ensure balance and prism-shaped columns provide durability.

2. Calculating Area and Perimeter
A traditional woven mat (kain tenun) has a square design with a side length of 1.5 meters. Calculate its perimeter and area.

Answer:
- Perimeter = 4 × side = 4 × 1.5 = 6 meters
- Area = side × side = 1.5 × 1.5 = 2.25 square meters.

3. Applying Geometric Reasoning
A farmer in Bali wants to divide his triangular rice field into two equal sections. The field has a base of 20 meters and a height of 15 meters. How much land will each section have after division?

Answer:
- Total Area = ½ × base × height = ½ × 20 × 15 = 150 square meters
- Each Section's Area = 150 ÷ 2 = 75 square meters.

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Post-Test Questions

1. Understanding Shapes in Culture
In traditional Indonesian architecture, many houses and temples incorporate square, triangular, and prism-like structures. Explain the properties of these shapes and how they contribute to real-world stability and design.

Answer:
- Square: Provides structural strength in walls and floors.
- Triangle: Distributes weight evenly, making roofs more durable.
- Prism: Used in beams and pillars for strong support.
- Application in Architecture: Squares form stable building bases, triangular roofs prevent water accumulation, and prisms strengthen the framework of structures.

2. Calculating Area and Perimeter
A square-shaped window in a traditional house has a side length of 2 meters. Calculate its perimeter and area.

Answer:
- Perimeter = 4 × side = 4 × 2 = 8 meters
- Area = side × side = 2 × 2 = 4 square meters.

3. Applying Geometric Reasoning
A farmer builds a triangular shelter for livestock with a base of 18 meters and a height of 12 meters. If he divides it into two equal parts, how much area will each part have?

Answer:
- Total Area = ½ × base × height = ½ × 18 × 12 = 108 square meters
- Each Section's Area = 108 ÷ 2 = 54 square meters

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Exploration of Primary School Students' Metacognitive Thinking Processes in Solving Problems

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Abstract: This study aims to investigate the thinking process of primary school students when solving mathematical problems. It used a qualitative case study approach. The researcher administered a metacognitive test to 30 fifth-grade students in Yogyakarta and selected three focal participants based on their performance in planning, monitoring, and evaluating metacognitive problem-solving indicators. Data were collected through students' metacognitive problem-solving tasks and interviews, employing mathematical metacognitive capability tests and semi-structured interview guidelines. After students completed the given problems, cognitive interviews were conducted to capture their thinking processes during problem-solving. These interviews were conducted individually, involving only the researcher and the participant. The researcher employed cross-sectional data analysis and focus group discussions (FGDs) to ensure data validity. The results indicate that students exhibit two distinct thinking processes when solving complex mathematical problems. Students undergo metacognitive thinking processes in problem-solving, which can result in either correct or incorrect answers. Students with high metacognitive abilities demonstrate effective use of metacognitive skills, as evidenced by clear and structured problem-solving strategies. Conversely, those with low metacognitive abilities struggle to apply these skills effectively.

Keywords: metacognitive, problem-solving, indicator metacognitive, thinking process, mathematics

INTRODUCTION

Mathematics is fundamental in primary education and the foundation of scientific disciplines. It cultivates creative, logical, rational, critical, and systematic thinking through interconnected concepts. As a core curriculum component, mathematics enhances students' ability to analyze phenomena logically and achieve their educational goals (Bature & Atweh, 2016; Milovanovic et al., 2013; Mwakapenda, 2004). According to the 2018 PISA, Indonesia ranked 72nd out of 78 countries, indicating a weak mathematical performance. Similarly, the 2022 results indicate that Indonesian students perform significantly below average in reading, mathematics, and science (OECD, 2023).

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Mathematics is fundamental across scientific disciplines, fostering creativity, logic, reasoning, critical thinking, and analytical skills (Skemp, 1971; Suneetha et al., 2011). However, the mathematics study often misrepresents problem-solving methods, a phenomenon known as cognitive conflict (Madu & Orji, 2015; Obreque & Andalón, 2021; Radovic S. & Pampaka, 2023). The Fourth Industrial Revolution and 21st-century demands require skill development within a new framework (Siregar et al., 2020). Technological advancements necessitate adapting cognitive processes to evolving complexities. Children exhibit diverse traits, including difficulties in mathematics, which may indicate cognitive limitations.

Engaging in metacognitive thinking processes enhances students' metacognitive skills, enabling faster information acquisition and retention, which supports their academic and professional growth (Schunk, 2012). The importance of metacognition is particularly pronounced among primary students (Bogdanović et al., 2015; Hoffman & Spatariu, 2008). Metacognitive capacity is essential in learning, allowing students to regulate cognitive processes, refine strategies, and identify strengths and areas for improvement (Bagci & Unveren, 2020). All learners can engage in metacognition, enabling them to reflect on their learning, reading, and skill development. Metacognition includes key skills for education and professional settings. Objective assessment measures are crucial for effective diagnosis and intervention (Castillo Diaz & Gomes, 2021).

Student metacognition remains underdeveloped, especially in mathematics education. Key components are required to strengthen cognitive skills and academic performance (Branigan, 2019; Tandean et al., 2020; Yilmaz et al., 2010). Metacognitive proficiency is a key predictor of academic success, fostering learning and advanced cognitive abilities (Kent, 2022). Metacognition is a higher-order thinking skill involving planning, monitoring, and evaluating of cognitive processes (Flavell, 1976; Karaali, 2015). Metacognition involves three key activities: planning, monitoring, and evaluation (Bakkaloglu, 2020; Gusmão & Moll, 2022). Table 1 below presents the result of studies on low metacognitive abilities in Indonesia.

Studies	Findings	Evidence
Anjelina et al., (2020)	Most students achieved the planning indicator of metacognitive abilities but not for monitoring and evaluation.	A descriptive analysis of 28 students in Banda Aceh showed partial achievement in metacognitive indicators.
OECD (2023)	Metacognition positively influences mathematics achievement, but overall achievement remains low	Analysis of the 2022 PISA data indicated that higher metacognition correlates with better math achievement.
Aminah et al., (2018)	Metacognitive teaching-learning (MTL) improved self-regulated learning but insignificant in logical thinking.	Quasi-experiment with 7- tenth-grade students in Sumedang showed low scores in logical thinking despite MTL.
Supriatna et al., (2019)	Higher learning methods improved problem-solving skills, but overall skills remain low.	ANOVA analysis of the fifth-grade students in West Kalimantan indicated better skills with advanced methods

Table 1: Study data on metacognitive abilities in Indonesia.

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Table 1 indicates that students' metacognitive skills in mathematical problem-solving remain low. Therefore, these skills should be improved early in primary school to help students increase their problem-solving skills, academic achievement, critical thinking skills, and self-confidence. Research shows that metacognitive abilities develop with age and are shaped by multiple factors (Kennedy et al., 2008). Metacognition typically emerges around age eight, aligning with elementary school years (Kent, 2022; Tandean et al., 2020). Introducing metacognitive concepts early is vital for effective knowledge management. Therefore, exploring methods to develop children's metacognitive skills in early self-directed learning is essential (Khan, 2015). Exploring strategies to foster metacognitive approaches in elementary students is essential (Duman & Semerci, 2019). Early engagement in metacognitive activities enhances students' metacognitive skills. Consequently, many developing countries, including Indonesia, have integrated metacognitive elements into competency standards for elementary graduates. Student awareness progresses through four levels: tactical, aware, strategic, and perceptive (Schraw et al., 2006). As a core subject, mathematics fosters higher-order thinking by promoting cognitive regulation and reasoning (Kennedy et al., 2008; Khan, 2015).

Previous research, especially in Indonesia, indicates that learners' abilities remain insufficient (Hastuti et al., 2020). Even among highly capable elementary school educators, performance remains below standard (Wilhelmi et al., 2021). This gap is significant, as metacognitive activity is a key indicator of cognitive development and academic motivation (Schneider & Artelt, 2010). Insufficient metacognitive skills in early education can negatively impact students' abilities in higher education, making early intervention essential (Schneider & Lockl, 2002). Metacognition is crucial for mathematical problem-solving at all educational levels, and its absence hinders reasoning and cognitive processes. Research indicates that students often struggle to achieve desired success rates in problem-solving due to weak metacognitive skills (Alzahrani, 2017; Bakkaloglu, 2020; Özsoy & Ataman, 2009). The literature highlights metacognition as a critical factor in academic success (Desoete, 2001), with strong correlations between cognitive processes, problem-solving skills, and metacognition. Students with well-developed metacognitive abilities tend to perform better in problem-solving tasks (Özsoy & Ataman, 2009). Understanding cognitive processes enhances strategic thinking and academic performance.

Students' low metacognitive skills partly originate from teacher-centered learning activities emphasizing cognitive aspects over active engagement (Hastuti et al., 2020). Routine tasks that lack authentic problem-solving opportunities further hinder the development of higher-order thinking skills. Such approaches encourage passive learning, limiting students' metacognitive engagement (Veenman et al., 2006). As a result, students often struggle to reflect on and assess their cognitive understanding (Desoete, 2001). Metacognition is closely linked to problem-solving and is most evident when individuals face unfamiliar challenges, uncertainties, or dilemmas (Du Toit & Kotze, 2009).

These challenges are primarily classroom-related. Research shows that students across Indonesia continue to face educational disadvantages (Akshay et al., 2021). They encounter obstacles in

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learning, which can be both engaging and tedious, often due to difficulties in understanding essential concepts (Anintia et al., 2017). Additionally, limited prior knowledge contributes to low metacognitive capacity, poor information organization, uncertainty in strategy use, ineffective learning methods, and difficulty applying previously acquired knowledge (Shamir et al., 2008).

Students with low metacognitive abilities are often unable to solve mathematical problems effectively. This happened due to students being faced with difficult tasks and require critical thinking ability or unreadily available answers, they tend to be lazy about them. Even if students can work on the problems correctly, they fail to solve daily life problems (Martínez Zarzuelo et al., 2020; Peake et al., 2021). Students have difficulty understanding a problem, such as what is known, the underlying issues, and what strategies must be used to solve contextual problems (Dixon, 2017; Komalasari, 2009; Yansen et al., 2019). The student's lack of ability to monitor his or her learning purposes (Karakoc, 2016). Students never plan the time to be used to complete the tasks given by the teacher, so students feel the need for time.

Metacognitive development can enhance students' self-literacy, thereby increasing their motivation to learn (Bandura, 1997; Oliveira et al., 2022; Wang et al., 2017). A good study skill can become a thinking habit applied in a wide range of contexts (Pratiwi et al., 2019). Critical and reflective problem-solving and decision-making skills are essential for students, as they enhance self-regulation in the learning process, increase motivation, and support emotional management even in challenging situations. These competencies also enable students to navigate complex issues, resolve conflicts, and become active learners capable of thinking critically, reflecting deeply, solving problems effectively, and making informed decisions (Denizli & Erdoğan, 2018; Gambrill & Gibbs, 2009; Wallace, 2015). Research shows that good metacognition will enable students to solve problems appropriately (Adnyani, 2020).

Research indicates that metacognitive experience enhances students' understanding of knowledge. Students with strong metacognitive skills demonstrate superior critical thinking, problem-solving, and decision-making skills (Adnyani, 2020; Fajari et al., 2020). Individuals with more complex cognitive processes tend to possess stronger metacognitive skills (Duman & Semerci, 2019). Students who receive metacognitive instruction learn more effectively than those who do not (O'Neil & Brown, 1997). Additionally, metacognitive abilities help students recognize and correct mistakes, evaluate their work, and assess the effectiveness of different learning strategies.

Metacognitive thinking enhances learning ability and influences task completion in the learning process (Cañada & Arumí, 2012). It also impacts students' academic performance. Therefore, integrating metacognitive skills into classroom activities and curricula at all educational levels is essential (Farida Jamil et al., 2023). Metacognitive skills can be taught and developed. This study examines elementary school students' thinking processes in mathematical problem-solving and project-based learning.

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Figure 1: Diagram of the Metacognitive Thinking Trajectory

The metacognitive thinking trajectory in mathematical problem-solving involves a series of interconnected cognitive processes that enhance reasoning and self-regulation. This trajectory begins with a thorough problem analysis, followed by identifying relevant concepts and strategies (Flavell, 1979). In the evaluation phase, students critically assess the feasibility and accuracy of their approaches before applying them. Next, they engage in a structured planning phase, outlining their steps toward a solution, and then moving to the monitoring phase, where they track their progress and make necessary adjustments. Finally, during the evaluation stage, students carefully review the entire process to ensure accuracy and effectiveness. This systematic approach enhances cognitive regulation and problem-solving skills while fostering critical thinking, ultimately promoting independent learning in mathematics. The selection of this context is based on results of FGD with primary students in Yogyakarta to explore their thinking processes and metacognition. Considering primary school students' low metacognitive ability to solve math problems, this study aims to investigate their thinking processes when solving mathematical problems. The research question is: How does the exploration of primary school students' metacognitive thinking process in solving problems?

THEORETICAL FRAMEWORK

Metacognition

Metacognition involves planning, monitoring, and reflecting on problem-solving processes (Alzahrani, 2017). It encompasses knowledge, experiences, goals, and strategies (Flavell, 1979) enabling individuals to reflect on, understand, and regulate their learning (Cahayasti & Indrasari, 2018). Metacognition includes self-reflection on strengths, weaknesses, and learning strategies (Anggo et al., 2015). It is a cognitive tool that enhances problem-solving, decision-making, critical thinking, and creativity, ultimately improving learning outcomes (Hendriana et al., 2020; Karakoc, 2016; Wallace, 2015). Closely linked to cognitive control and higher-order thinking, metacognition fosters self-regulated learning (Prabawanto, 2019). It allows individuals to manage cognition effectively, assess challenges, evaluate their understanding, utilize information to achieve goals, and track progress (Schneider & Artelt, 2010b).

Metacognitive thinking ability primarily involves metacognitive regulation, which is essential for self-directed learning and influences actions during the learning process (Schraw et al., 2006; Whitebread et al., 2009). It encompasses planning, monitoring, and evaluation (Schraw et al., 2006), each with multiple components. Figure 2 provides a detailed overview of these aspects.

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Figure 2. Metacognitive Thinking Process Scheme

	Description	Operational
Planning	A systematic preparation of activities that will be done to achieve a specific goal. Select the right strategy, the purpose, and the allocation time. Planning involves a strategic approach to the source of data location.	Planning is a way of thinking about problems. Planning is always future-oriented. Planning takes note of the relationship between achieving goals and decision-making, and planning presents comprehensive policies and programs.
Monitoring	A regular process of data collection and measurement of progress over the objective of the program and monitoring the changes that focus on the process and output. Students' acquired ability in carrying out an action. These abilities are periodic	Compliance monitoring works to ensure expectations/plans. The spectacle monitoring serves to assess the organization's progress based on the achievement of the expected target. Typically, the output of monitoring is a progress report. Such output is described in both descriptive and non-descriptive terms. The output monitoring is to know if the corresponding process is already in place. The output monitoring is useful in repairing the monitoring process/activity mechanisms.
Evaluation	Activities to gather and provide information for the decision-maker. Assessment of results and efficiency in a student's learning ability and the process of student learning. Conducting review and revise purposes.	Selection, collecting, analysis, and presentation of information that can be used as a basis for decision-making and subsequent compiling of programs

Table 2. Metacognitive Thinking Process

Measuring Metacognitive Thinking Capacity

Positive thinking capacity can be measured through tests, observations, and questionnaires, including reflective assessment, which evaluates metacognitive thinking ability. Reflective appraisal is a metacognitive process, and individuals with advanced reflective assessment skills tend to be better learners (Gravill et al., 2009; Van Opstal & Daubenmire, 2015). This formative approach places students at the center of assessment practice (Alzahrani, 2017). Reflective as-

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sessment is rooted in strong theoretical foundations of John Dewey's philosophy (Dewey, 1966) and cognitive constructivist learning theories. It promotes critical thinking and well-researched cognitive development (Johnson & Johnson, 2002; OECD, 2019; Wallace, 2015).

Metacognitive control is applying knowledge to regulate cognitive processes, aiding self-management and learning (Duman & Semerci, 2019). Metacognition is measured through three key indicators: planning, monitoring, and evaluation (Schraw et al., 2006). Planning involves setting learning objectives, considering learning strategies, and managing time effectively. It helps students anticipate challenges and develop result-oriented strategies (Veenman et al., 2006). Monitoring requires continuous awareness of comprehension and performance, including periodic self-assessments during learning (Schneider & Artelt, 2010b; Schraw et al., 2006). Evaluation enables students to assess their task performance, compare results with peers, and identify weaknesses in their problem-solving strategies (Schraw et al., 2006).

Experts reveal that planning process involves identifying key information and setting goals for problem-solving while understanding the necessary steps. The monitoring process includes accurately applying formulas, ensuring logical progression, and overseeing problem-solving steps. The evaluation process requires students to review their steps for consistency with the given information and verify their solutions.

Metacognition in Mathematical Problem-Solving

Metacognitive and problem-solving skills are essential for students to engage with mathematical advancements effectively. Problem-solving involves identifying and resolving issues, while metacognition enables awareness and regulation of cognitive processes (Scheibe et al., 2023). These skills enhance mathematical comprehension, logical reasoning, and critical thinking, supporting academic success and lifelong learning. This study explores metacognition's role in mathematical problem-solving, particularly in collaborative settings, highlighting its impact on cognitive regulation and learning strategies. It also examines the link between metacognitive strategies and problem-solving abilities, emphasizing the importance of problem-based learning. Further research is needed to examine this relationship more deeply and to enhance teaching methodologies in mathematics education.

Polya's four-stage problem-solving approach is understanding the problem, devising a plan, carrying out the plan, and looking back. Metacognition significantly enhances it, which fosters critical thinking and problem-solving abilities. Metacognitive strategies aid students in comprehending problems effectively, as research indicates that metacognitive frameworks improve understanding, planning, and overall problem-solving performance (Barokah et al., 2020; Lee et al., 2014). Developing a plan relies on metacognitive knowledge and self-regulation, helping students formulate effective strategies. During the execution phase, metacognitive monitoring ensures that students stay on track and adjust their approach when necessary. The last is reflection or looking back to consolidate learning, reinforce understanding, and enhance future problem-solving skills (Scheibe et al., 2023). In conclusion, integrating metacognitive strategies into

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Pólya's model promotes self-awareness, strategic thinking, and adaptability**,** ultimately improving students' problem-solving abilities and deepening their learning experience.

Metacognition is crucial in enhancing students' problem-solving abilities by improving their awareness of cognitive processes. Research indicates that metacognitive instruction significantly boosts performance by helping students effectively plan, monitor, and evaluate their actions (Safari & Meskini, 2016; Zhao et al., 2019). Additionally, metacognition fosters self-regulated learning**,** enabling students to take control of their learning process, build confidence, and develop perseverance in problem-solving tasks (Fazira et al., 2021; Lee et al., 2014). Task-relevant monitoring further supports problem-solving by encouraging students to check their work and consider alternative strategies, leading to more accurate solutions (Scheibe et al., 2023). According to Pólya's problem-solving stages, one effective metacognitive framework is the Problem Wheel**.** This tool guides students in assessing their thinking throughout the problem-solving process by prompting self-reflective questioning, thereby enhancing their metacognitive skills.

METHOD

Research Design

This study aims to examine students' mathematical thinking processes. It employed a qualitative research design using a case study approach. Qualitative research is a method for understanding human or social issues by constructing a comprehensive and detailed narrative, presenting findings descriptively, gathering information from multiple sources, and conducting the study in a natural context (Creswell, 2014).

Sample and Data Collection

The study involved 30 primary school students in Yogyakarta, particularly the fifth-grade students. Four participants were selected based on their classification according to the correctness of their responses to planning, monitoring, and evaluation indicators. The researcher used a purposive sampling technique to select school samples by considering the accreditation, the number of students and teachers, and willingness to participate in the research. The students voluntarily participated in the study by signing informed consent and their information and responses were kept confidential.

Using a case study approach, data were collected through students' mathematical problem-solving tests and interviews. The research instruments included a mathematical metacognitive ability test and semi-structured cognitive interview guidelines. After students completed the given problems, cognitive interviews were conducted to capture their thinking processes during problem-solving. These interviews were conducted individually, involving only the student and the researcher.

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The data were categorized based on three metacognitive indicators: planning, monitoring, and evaluation. From each category, one correct response was selected for further analysis. Students' focus was assessed using metacognitive indicators, as outlined in Table 3 below.

Process	Indicators
Planning	Identifying desired goals Important information to be used in problem solving Students understand the steps that will be done to solve the problem.
Monitoring	Students are able to write the formula correctly and precisely The final step made by student. Students control or monitor the completion steps of given information (known).
Evaluation	Students recheck the steps outlined whether consistent with the given information (known) or not. Verifying that what's been done is right

Source: Schraw et al., (2006)

Table 3: Metacognitive Indicators

Data Analysis

Data analysis was based on three aspects of metacognition: planning, monitoring, and evaluation. The study examined how students demonstrated these metacognitive skills: (1) planning, (2) monitoring, and (3) evaluation. Researchers investigated the impact of different learning approaches on higher-order thinking skills, including metacognition, by comparing the metacognitive abilities of elementary school students who learned through a guided learning model with those who followed a conventional learning model. The analysis focused on students' responses according to specific metacognitive indicators.

The interview data were analyzed using students' thinking process schematics to assess their metacognitive abilities and understand their thought processes while solving problems. The analysis categorized metacognitive thinking into three stages: planning, monitoring, and evaluation. To ensure the validity of the research data, several methods were employed: (1) focus group discussions (FGD) with university leaders to verify research findings, (2) cross-examination with each respondent to maintain accurate interview transcripts, and (3) a review of relevant literature to support the study.

The study included cognitive interviews with students to validate their problem-solving approaches. These interviews were conducted with representative participants to reflect the broader student population. The primary objective was to gain deeper insights into students' cognitive processes when solving mathematical problems. The final stage involved documentation, which was started with collecting students' preliminary academic data from class teachers before formal data collection. To ensure data validity, researchers employed multiple methods, including cross-sectional data analysis and FGDs. A cross-sectional analysis was conducted with class teachers

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to assess whether students' thinking aligned with their previous learning experiences. Meanwhile, FGDs with school principals, teachers, and mathematics educators were conducted to verify the appropriateness of research instruments and methodologies in addressing the study's objectives.

Data were analyzed qualitatively as proposed by Miles, Huberman, and Saldana which consisted of three stages: data reduction, data display, and conclusion (Miles et al., 2014). The analysis was carried out through interviews, observations, and literature studies. The analysis of interview data was based on Creswell & Creswell (2018) covering raw data, transcription, and interpretation. Then, didactical design validation analysis was conducted through a focus group discussion between teachers and researchers before and after the teaching experiment for further design improvement. The method of examining learning impediments that meet information alludes to the strategy (Creswell, 2013), beginning with raw information, translation, and interpretation. The validity test used source and technical triangulation. The validation of *metacognitive problem-solving tasks used an FGD between teachers and researchers.*

RESULTS

Researchers found that all 30 students employed various planning, monitoring, and evaluation strategies when solving mathematical problems. Their problem-solving performance is presented in Figure 3.



Figure 3. General Results

The results of the analysis indicate that students employ two types of thinking processes when solving complex mathematical problems. Among the 30 student responses, 17 (57%) provided correct answers, while the rest (43%) provided incorrect answers. All correct responses followed the structured planning, monitoring, and evaluation process.

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Analysis of Correct Answers

Among the 30 students who participated in the mathematical metacognitive capability test, 17 provided correct answers, while the rest provided incorrect answers. Figure 4 presents an example of a student's correct response.

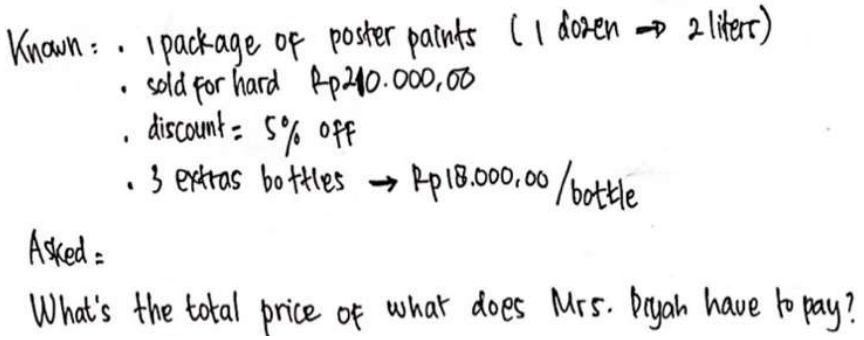


Figure 4. An Example of a Correct Answer

Figure 4 demonstrates that students with strong metacognitive abilities effectively apply metacognitive skills when solving problems. This is evident in their ability to clearly and accurately outline their problem-solving process. The figure above indicates that the student exhibits strong metacognitive skills. Students with high metacognitive skills systematically write down given information (known) and correctly state the problem (asked). Additionally, they use visual aids such as diagrams or annotations to enhance their understanding and facilitate problem-solving. Their solution plans are typically concise and often presented as formulas without detailed steps or justifications for their chosen concepts.

In terms of execution, these students approach problems systematically and accurately. Furthermore, they engage in self-verification by reviewing their answers to ensure correctness. Figure 5 illustrates the complete thought process behind a correct answer.

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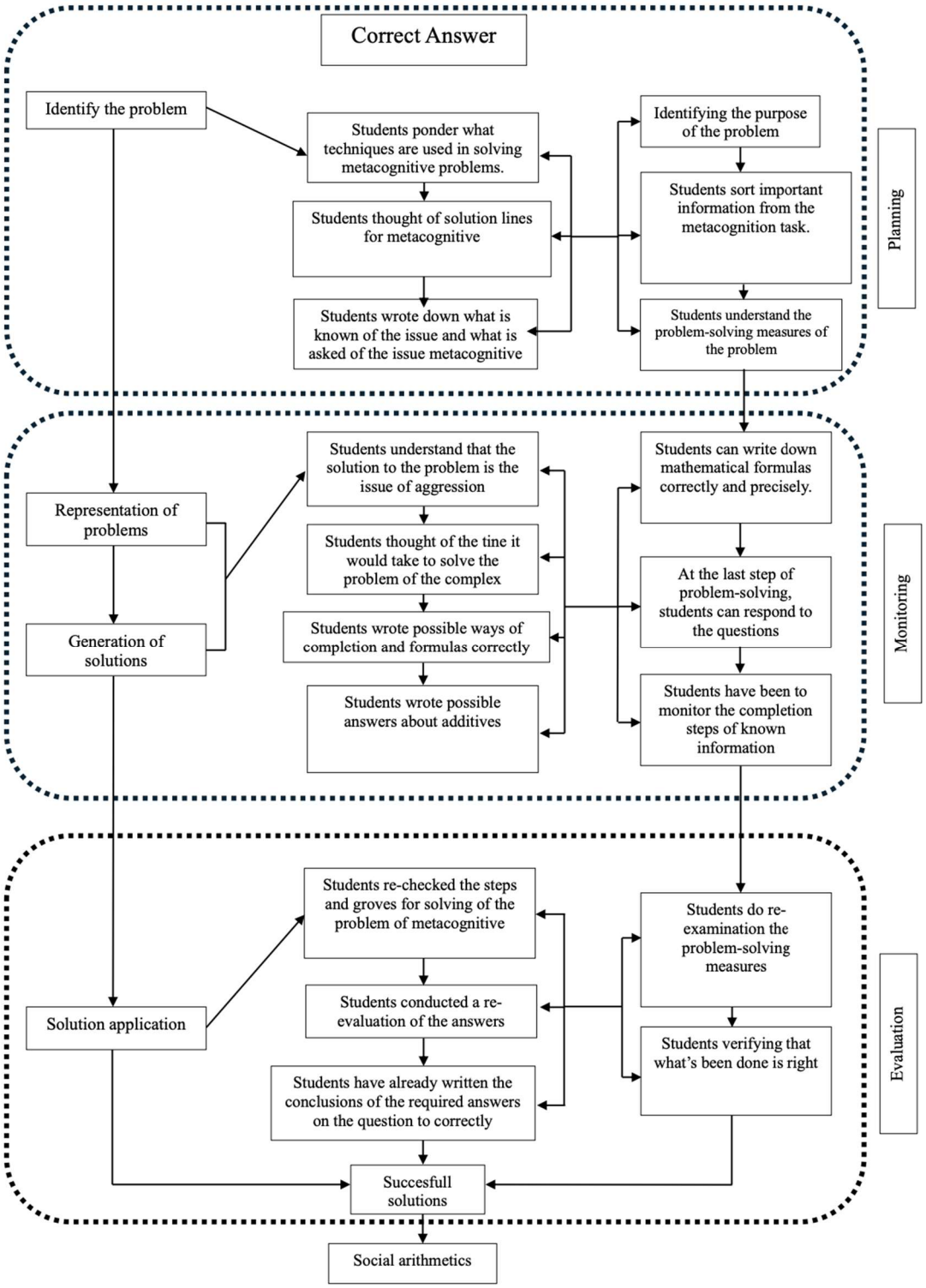


Figure 5. A Mathematics Metacognitive Scheme of the Correct Answer Analysis

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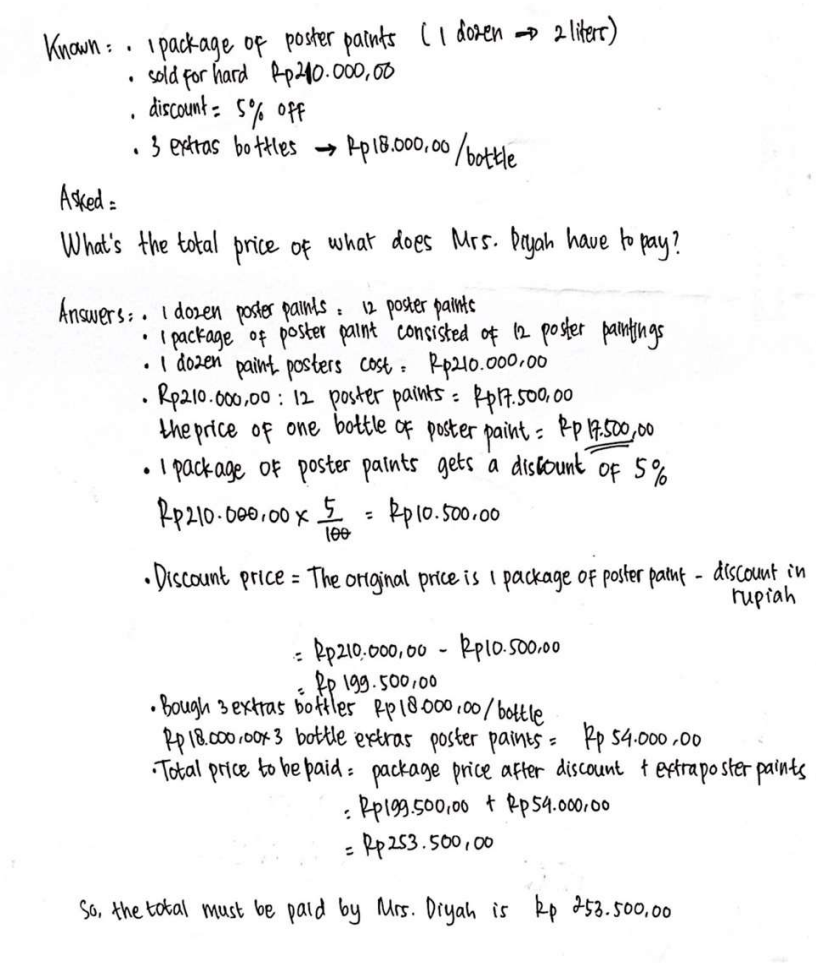


Figure 6: The Complete Version of a Correct Answer

Figure 6 shows that students who provided correct answers have gone through the planning step. The student has done the identification phase. This can be seen from the following conversation transcript (R is a researcher, while S1 and S2 are students).

R: Can you understand the problem presented, and how do you analyze the problem?

S1: Yes, I understand the question. What I need to understand first is the purpose of the question, and classify the "given information (known), problem (asked), answer and so on". Then, write the answers in the available space. In the planning phase, the student first identified the given information, such as the cost of one package of poster paints, the discount percentage, and the price of additional bottles. The problem was then broken down into sequential steps to determine the total cost.

S1: Yes, and the student also established sub-goals, such as calculating the price of one bottle, applying the discount, and determining the cost of extra bottles. This demonstrates a clear problem-solving strategy before performing calculations.

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R: Excellent observation. Now, let's discuss the monitoring phase. How do you ensure accuracy while solving the problem?

S2: I systematically monitored the calculations using proper mathematical operations. For instance, when applying the 5% discount. The discount was then subtracted from the original price to ensure the correct discounted amount. Additionally, when calculating the cost of extra bottles, I verified the multiplication:

R: That's a key metacognitive strategy—constantly checking one's work. In the evaluation phase, did you review and validate your final answer?

S2: Yes, after summing the discounted package price and the cost of extra bottles, the final total was clearly stated as Rp253,500.00. I also provided a concluding statement, reinforcing the accuracy of the solution. One improvement could be rechecking whether the initial assumption about the package price was correct. Meanwhile, I stated that one package costs Rp210,000.00 instead of Rp240,000.00 as initially given, this discrepancy should be reviewed for consistency.

This is seen from the student who wrote down the given information (known) and the question. When students write down the given information and the question, it indicates that they can identify the given information on the question. In this case, it is based on the student's analysis of answers. When students write down the given information and the question using the symbol, it indicates that they understand the given information, the problem, and the steps that will be taken to solve the problem.



Figure 7. Documentation of Thinking Processes of Students with Correct Answers

This section discusses student's identification and problem-solving processes in metacognitive tasks. Students have successfully identified the objective of the metacognitive problem. At this stage, their responses indicate that they can recognize and extract relevant information from the

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problem. They are first required to comprehend the metacognitive question presented in the scenario and determine the answer to the question: *"What is the total amount of money to be paid by Mrs. Diyah?"*

The students' responses demonstrate an understanding of the necessary steps to solve the problem. They can identify and categorize information as "known systematically*"* and *"asked."* Additionally, students show an awareness of the planning stages, which involve outlining the steps to be done after identifying the given information (*known*), understanding the problem statement (*asked*), solving the problem (*answer*), and drawing conclusions (*final result*).

This structured approach reflects students' ability to engage in metacognitive thinking, effectively plan their problem-solving process, and apply logical reasoning to reach a solution.

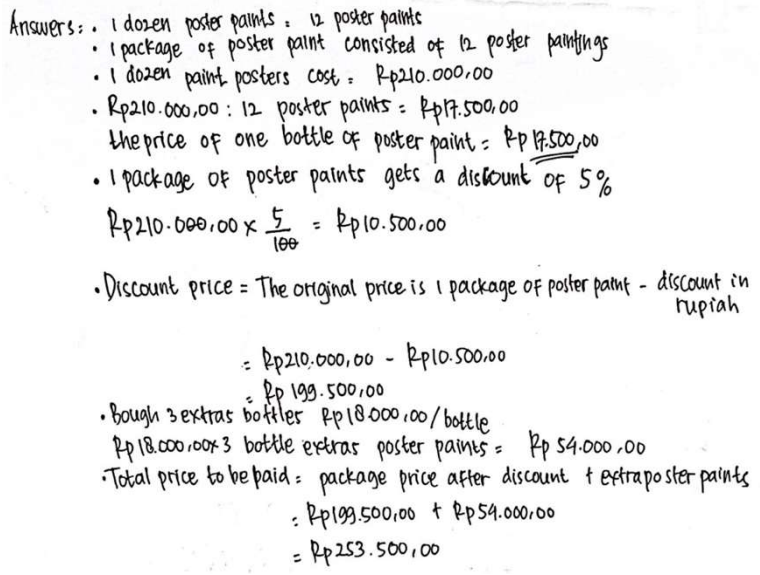


Figure 8: Monitoring Stage

Figure 8 shows student's attempts to solve mathematical metacognitive problems by answering and performing calculations. At this stage, they monitor, as evidenced by their efforts to write down possible solutions and relevant formulas. The formulas provided indicate that students can devise a structured solution plan. However, some students exhibit uncertainty despite correctly writing the formula, particularly in assigning values. For instance, one student notes that the price of a single unit is Rp210.000,00, but the notation appears unclear, suggesting possible confusion. This lack of precision may lead to errors in problem-solving.

Furthermore, the correct formatting of currency values should follow the General Guidelines for Indonesian Spelling (PUEBI), which specify that (1) the symbol Rp (Indonesian Rupiah) should be used to denote the currency; (2) no space should be placed between Rp and the numerical val-

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ue; (3) a period (.) should be used to separate thousands, e.g., Rp18,000.00; (4) a comma (,) should be used as a decimal separator; (5) a slash (/) can be used as a unit separator, e.g., Rp210.000/pack. By adhering to these formatting rules, students can enhance the clarity and accuracy of their mathematical responses.

Based on the results of the students' mathematical metacognitive test, it was observed that in the written responses, they solved the addition problem using visual aids such as lines. This is evident in their answers, where they initially calculated the total number of poster paints in one dozen, determining that one packet contained 12 poster paints.

The results of the study:

Students correctly identified that one dozen poster paints cost Rp210,000.00. They then proceeded to calculate the price per unit: Rp210.000,00: 12 poster paints = Rp17.500,00. This calculation indicates that the price of one bottle of poster paint within the package is Rp17.500,00. Additionally, the problem states that each package of poster paints receives a 5% discount, which the students must also consider in their calculations.

Thus, $Rp210.000 \times \frac{5}{100} = Rp\ 10.500,00$

Students only write =

Discount price = The original price is 1 package of poster paints - discount in rupiah

= Rp210.000,00 – Rp10.500,00

= Rp199.500,00

Later, Mrs Diyah purchased three additional bottles at the regular price of Rp18.000,00 per bottle.The students then calculated the total cost accordingly,

Rp18,000,0000x 3 bottles of additional poster paint = Rp54.000,00

The next step requires students to calculate the total amount Mrs. Diyah needs to pay.

Total price to be paid = package price after discount + extra-posters paint

= Rp199.500,00 + Rp54.000,00

= Rp253.500,00

The student's response also demonstrates their approach to determining the price of three additional bottles. The correct price is Rp18.000,00 per bottle; however, the student wrote ",00" slightly lower than Rp18.000,00. During the interview, the student explained that they had mistakenly omitted ",00" due to its close placement near the multiplication symbol but had already accounted for it in their formula. In the context of numerical notation within Indonesia's financial system, the use of ",00" following a nominal value reflects the formal standard for representing

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amounts in the Indonesian currency, the rupiah. Additionally, in writing the equal sign (=), the student used dots, making it resemble a division symbol. Despite this, the student successfully followed the planned steps, as evident from their accurate mathematical calculations based on the previously established formula. The student's computations were correct, demonstrating their ability to implement the planning stage effectively. Moreover, students with high metacognitive skills understand the correct procedures for solving the problem. Their results are accurate, and they can clearly explain the strategies used to reach a solution.

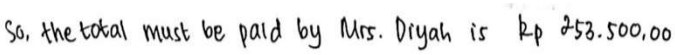


Figure 9: Evaluation stage

The evaluation process consists of two key indicators: students reexamine whether their steps align with the given information and verify the accuracy of their solution. Based on Figure 9, students have demonstrated the ability to evaluate and review their answers. Students analyze, select, and present relevant information as a foundation for decision-making and further problem-solving. They have rechecked their steps in solving the metacognitive problem to ensure correct responses. Since the student's answers appeared fragmented, they modified their response to ensure accuracy. Moreover, students provided a conclusion by writing, *"So the total amount to be paid by Mrs. Diyah is Rp253.500,00."* However, the student made a minor formatting error by inserting an unnecessary space between "Rp" and "253.500,00," writing it as "Rp 253.500,00" instead of the correct format *"Rp253.500,00."* While this did not affect the numerical result, following the standard Indonesian writing conventions would be more appropriate. Furthermore, the student mixed their conclusion with the calculated result and the information in the problem. Overall, students managed to complete the evaluation phase by reviewing and verifying their solution.

Analysis of Incorrect Answers

Among the 30 students who participated in the mathematical metacognitive capability test, 17 provided correct answers, while the rest provided incorrect answers. Figure 10 presents an example of a student's incorrect response.

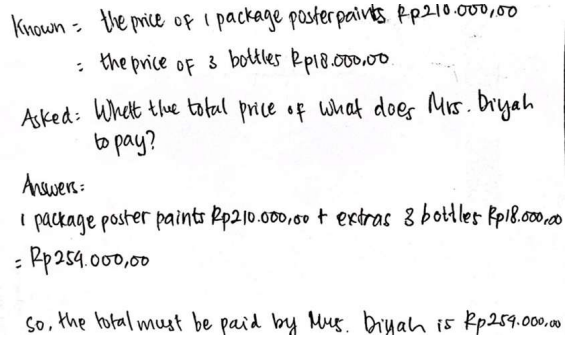


Figure 10: Evaluation Stage

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Almost all students with low metacognitive abilities do not use addition during problem-solving. This is evident in the students' vague work of writing out the problem-solving process. Examples of students' work with low metacognitive abilities can be seen from the following conversation transcript (R is a researcher, while S3 and S4 are students).

R: Can you understand the problem presented?
S3: I understood a little about it but was still confused about doing it. I could not sort out all the information and not write them all down.
R: How was the "asked" part for you to answer correctly?
S3: Since in the "asked" section, we only looked at the familiar issues by looking for a sentence using a question mark, it was the question.
R: How do you answer the problem of this mathematical metacognitive test? Do you know the formula?
S4: I answered it simply by adding up what I wrote in the 'known' section, and I was still confused about the formula when counting on the cost discount and the additional items.
R: Why?
S4: Because I feel rushed or overwhelmed. I do not write down the answers carefully and systematically. I do not have a habit of rechecking answers and I think this can be a contributing factor because I was often unaware of errors in calculations or explanations.

The complete thinking process of students with incorrect answers is presented in Figure 11.

Figure 11 shows some key factors contributing to students' lack of accuracy in solving metacognitive mathematical essay questions. One primary cause is their limited metacognitive ability to plan, monitor, and evaluate their thinking processes during problem-solving. Students who do not habitually reflect on their problem-solving steps often struggle to understand questions, select appropriate strategies, or include all necessary components in their answers. Additionally, the higher complexity of metacognitive problems compared to standard ones can cause students to feel rushed or overwhelmed, leading to imprecise and disorganized solutions. A lack of habit in reviewing answers further contributes to errors, as students frequently overlook miscalculations or inaccuracies in their reasoning. To improve accuracy in solving metacognitive mathematical essay problems, students should receive training in metacognitive skills, including strategic planning, monitoring their thought processes, and evaluating their responses before finalizing their answers.

Students with low metacognitive abilities recorded the given information and problem requirements but failed to develop a structured problem-solving plan. However, despite the lack of a clear plan, some students solved the problem systematically and accurately. Observations indicate that students who provided incorrect answers attempted the planning stage but did not execute it effectively. This is evident in their approach as they misidentified or inaccurately recorded key information and problem requirements. Additionally, they struggled with using appropriate

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mathematical symbols and failed to comprehend the necessary steps to solve the problem correctly and fully. A more in-depth analysis of their planning process is needed to understand better the challenges they face in structuring their problem-solving approach.

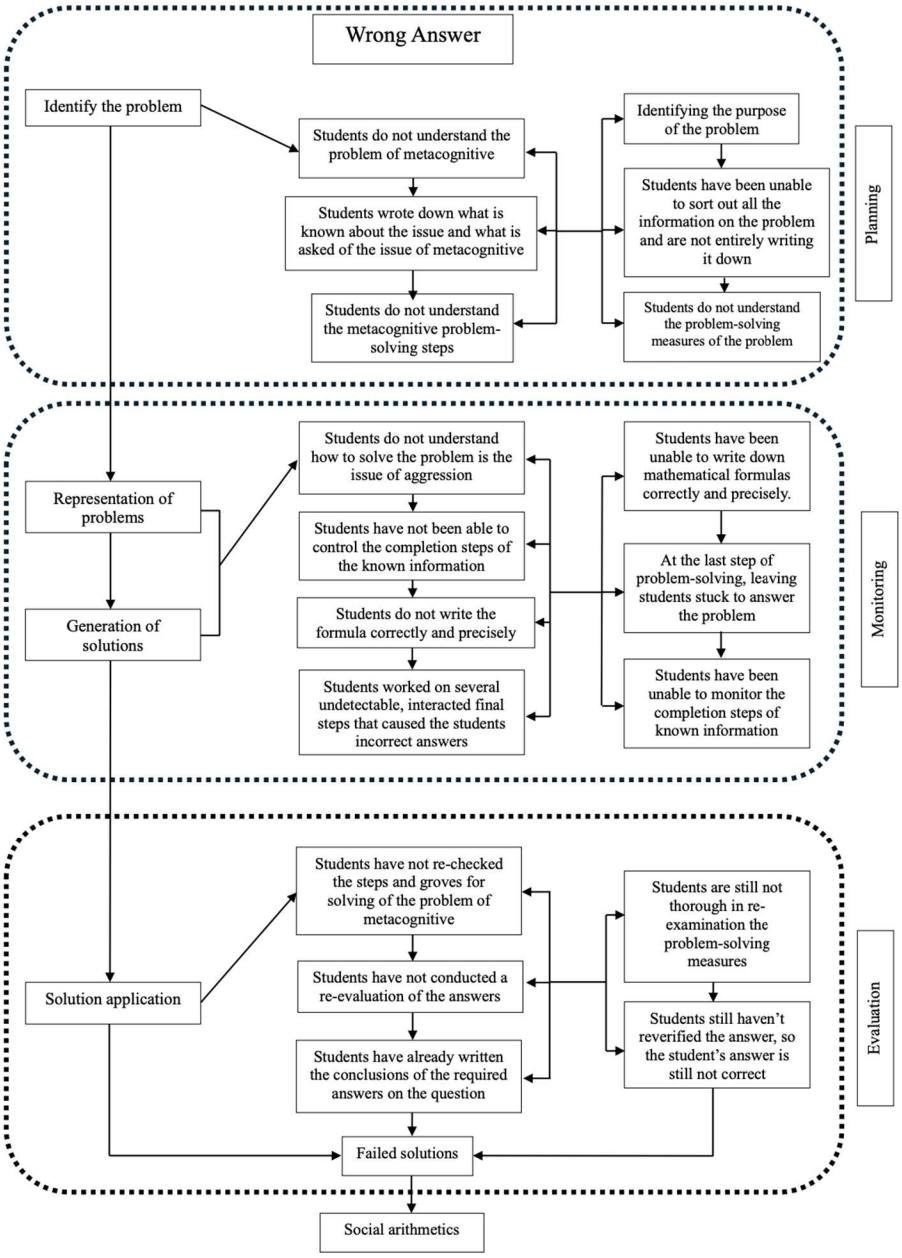


Figure 11: A Metacognitive Analysis Scheme of Incorrect Answers

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Figure 12: Documentation of Thinking Processes of Students with Incorrect Answers

Figure 12 shows the documented response of a student who attempted to solve a mathematics problem but provided an incorrect solution. The student's approach to the metacognitive task was also flawed, indicating the absence of monitoring indicators in their problem-solving process. One of the monitoring indicators, as they failed to write the formula correctly and accurately. Specifically, the student wrote the number 1 in the section Rp210.000,00**,** while elsewhere, they wrote the number 4**,** creating ambiguity that could lead to confusion. Additionally, in their written response, the student demonstrated inconsistencies, which may indicate a lack of clarity or understanding in their problem-solving approach.

In the final step, students failed to verify their problem-solving process appropriately. They struggled to oversee the completion of the given information, as indicated by their prior use of the scaffolding technique, without careful attention to detail. This led to errors in solving the problem, preventing students from completing the given task successfully. One key mistake was calculating the discount for a package of poster paints. Students did not clearly outline the necessary steps, such as: (a) recognizing that one dozen consists of 12 bottles (indicating that Mrs. Diyah initially purchased one package containing 12 bottles); (b) identifying that one dozen poster paints cost Rp210.000,00.; (c) calculating the price per bottle as Rp210.000,00 ÷ 12 = Rp17.500,00; and (d) acknowledging that one package of poster paints receives a 5% discount.

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The research findings:

The lack of clarity in these calculations suggests that students need further guidance in systematically applying problem-solving steps and verifying their solutions.

Thus, Rp 210.000 x $\frac{5}{100}$ *= Rp 10.500,00*

Students only wrote down the following information:

1 package of poster paints Rp210.00,00 + extras 3 bottles Rp18.000,00

= Rp254.000,00

Students failed to complete certain essential steps, leading to incorrect answers. They immediately stated that the price of one bottle of poster paint was Rp210.000,00 without accounting for the final step of applying the 5% discount to the total package price of Rp210.000,00. This suggests that students were unable to effectively control or monitor their problem-solving process, particularly in verifying and completing key calculations. As a result, they made errors in formulating their final answers to the metacognitive problem.

Students lacked precision in selecting formulas and monitoring their calculations, leading to errors in solving the metacognitive problem. Some students did not explicitly write the required formula in their responses. Instead, they stated the given information as follows: *"1 package of poster paints Rp210.000,00 + 3 additional bottles Rp18.000,00 = Rp254.000,00."* However, the student mistakenly wrote *"an additional 3 bottles of Rp18.000,00,"* suggesting a misunderstanding of the total cost calculation. The problem is that each additional bottle costs Rp18.000,00, but the student incorrectly presented the price for all three bottles as Rp18.000,00 instead of calculating the correct total. This error indicates that the student failed to fully process and apply the given information which leads to incorrect answers in the metacognitive problem.

The evaluation process includes two key indicators: (1) students reexamining whether their problem-solving steps align with the given information and (2) verifying the accuracy of their solution. Based on the study results, students who provided incorrect answers did not properly evaluate or review their responses. They failed to recheck the steps taken to solve the metacognitive problem, as their answers remained unverified and uncorrected. Additionally, students did not modify their responses to ensure accuracy. Despite this, the student wrote a conclusion stating, *"So the total cost is Rp254.000,00,"* although the answer was incorrect. This indicates a lack of thorough evaluation and self-monitoring in their problem-solving approach.

All student activities related to problem-solving in metacognitive abilities involve metacognitive regulatory components. These components are essential as they help students organize, control, connect, assess, and refine their thinking strategies throughout the problem-solving process. Ideally, these activities occur continuously and are closely linked to students' metacognitive

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knowledge during problem-solving. Metacognitive regulation consists of three key components: planning, monitoring, and evaluation.

Based on metacognitive regulation analysis, not all students effectively apply counter-regulation when solving problems. Students employ different strategies for monitoring and regulating their thinking, depending on their cognitive abilities. Research findings indicate that students with strong metacognitive skills effectively utilize these abilities in mathematical problem-solving, leading to better performance than their peers. Observations showed that students who apply metacognitive strategies successfully solve problems and achieve higher test scores. Additionally, they demonstrate the ability to systematically and accurately document problem-solving steps. This aligns with previous studies (Duman & Semerci, 2019 and Özsoy & Ataman, 2009) that students with high metacognitive abilities outperform those with lower metacognitive skills in mathematical problem-solving. Students who practice mindfulness experience lower levels of academic burnout due to increased self-awareness and a non-judgmental approach to challenges (Sapancı, 2023). Therefore, examining the mediating role of mindfulness in the relationship between dysfunctional metacognitions is essential for further understanding.

In mathematical problem-solving, metacognitive knowledge and metacognitive regulation are crucial in helping students analyze and resolve problems effectively. When faced with a problem, students recall and connect relevant material explanations to the given task. Their prior knowledge and experiences with similar problems further enhance their ability to understand and apply appropriate problem-solving strategies. Additionally, students' awareness of planning, monitoring, evaluating, and revising their thought processes helps minimize errors. By continuously monitoring and controlling their cognitive processes, students can identify mistakes and retrace their steps to correct them when encountering difficulties. This highlights the relationship between metacognitive knowledge and regulation in conflict resolution during problem-solving. Cognitive knowledge and cognitive regulation strongly correlate (Schraw et al., 2006). Therefore, metacognitive awareness and regulation significantly contribute to students' ability to develop independent learning skills.

DISCUSSION

Based on this analysis, metacognitive capability during problem-solving is directly associated with students' performance outcomes. Encouraging students to engage in problem-solving activities can enhance their metacognitive skills, ultimately fostering greater independence in learning. Additionally, metacognitive strategies improve students' comprehension (Wicaksono et al., 2021). This aligns with a previous study (Bogdanović et al., 2015) which shows that the more students understand their cognitive processes, the more effective their learning and academic achievement will be. Furthermore, a metacognitive approach enhances both learning outcomes and student assessment practices (Alzahrani, 2017; Shamir et al., 2008). A mathematical problem involving discounts and additional purchases requires students to utilize critical thinking and

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problem-solving skills. In metacognitive assessment, this problem is a tool to evaluate students' ability to plan, monitor, and evaluate their thought processes. A metacognitive test instrument for data analysis includes assessment tools that measure students' awareness and regulation of their cognitive processes, such as structured problem-solving tasks, think-aloud protocols, and reflective assessments. Students identify key information, track their calculations, and verify their solutions through this process. By integrating metacognitive test instruments, researchers can analyze students' problem-solving strategies, identify errors, and enhance their mathematical reasoning skills.

Strengthening metacognitive skills to develop students' competence in mathematical problem-solving is vital (Rey et al., 2024). This study found that high-achieving students effectively utilize advanced planning, monitoring, and evaluation skills. In contrast, students with moderate mathematical abilities do not yet fully utilize metacognitive planning and evaluation skills, though they demonstrate strong monitoring capabilities (Wakhata et al., 2023). Meanwhile, students with lower mathematical abilities struggle to apply effective planning, monitoring, and evaluation strategies. Research suggests that the metacognitive approach can enhance problem-solving skills through three key aspects: planning, monitoring, and evaluation, which collectively support the development of students' metacognitive abilities in mathematics (Thomas & Chantharanuwong, 2022). Therefore, continuous interventions to track students' progress in mathematics classes to enhance their problem-solving skills are important.

Metacognitive aspects, including metacognitive knowledge and self-regulation**,** enhance mathematical performance (Schneider & Artelt, 2010a). Metacognition helps students develop a structured approach to solving mathematical problems (Du Toit & Kotze, 2009). Additionally, encouraging students to engage in problem-solving can promote the use of written metacognitive strategies during the process (Farida Jamil et al., 2023).

Regularly applying metacognitive strategies helps students become more adept at identifying errors by revisiting initial steps**,** allowing them to correct mistakes and refine their approach**.** To develop effective and sustainable problem-solving skills**,** students must be aware of their mathematical knowledge and learn to plan, monitor, and regulate their cognitive processes**.** Therefore, mathematical understanding must be supported by metacognitive processes to ensure problem-solving quality (Yıldız & Öztürk,2023).

In this context, research highlights a strong relationship between metacognition and success in mathematics courses**.** Thus, teachers and students must possess strong knowledge of metacognitive strategies to facilitate self-directed learning. A pedagogical approach that integrates metacognition is essential in fostering students' awareness and control over their learning processes**.**

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CONCLUSIONS

Based on the results of the analysis and discussions, metacognition emerges around the age of 8-10 and students apply all the planning, monitoring, and evaluating stages. Students who apply metacognitive closeness as an awareness of one's cognition or process in which one thinks about thinking to build a strategy to solve problems. The metacognitive strategy refers to raising awareness about the thinking process that needs to increase student metacognitive awareness. When this consciousness is realized, one can control his mind by planning, monitoring, evaluating, and judging his learning. Teachers can build student's consciousness so they know and realize their weaknesses and strengths, and even plan, monitor, and evaluate what they have done. A teacher needs to strategize in math so students can design, monitor, control, and evaluate their actions. This is evidenced by the fact that the teacher guides students in planning a study plan before attempting metacognitive test problems. This process involves reflective questioning about their strategies for understanding mathematical concepts and overcoming difficulties. Students must then document their study plans, selecting appropriate methods such as rereading, summarizing, or engaging in discussions. "How did you approach problem-solving in the metacognitive test?" The teacher provides guidance in revising these plans as needed. Through this approach, students are expected to assess the effectiveness of their learning strategies, identify weaknesses such as procrastination or ineffective methods, and recognize their strengths, such as improved comprehension through discussion or problem-solving exercises. In the monitoring stage, the teacher encourages students to assess their comprehension during the learning process by answering reflective questions and using a self-reflection checklist. This approach helps students identify learning difficulties, evaluate the effectiveness of their study strategies, and recognize the need for additional support, promoting a more self-regulated learning process. In the evaluation stage, the teacher guides students in reflecting on their learning outcomes and processes after completing assignments or tests. Students complete a reflection journal and engage in class discussions to assess their study strategies, problem-solving approaches, and overall learning experiences. This process helps students identify effective learning methods, understand their strengths and weaknesses, and develop strategies for improvement, ultimately fostering independent and effective learning.

Metacognitive abilities are strongly correlated with academic performance, particularly in problem-solving. Encouraging students to engage in structured problem-solving activities enhances their metacognitive skills, promoting independent learning and deeper comprehension. Research supports that increased metacognitive awareness leads to better learning outcomes and achievements. Furthermore, metacognitive strategies are essential in student learning and assessment, particularly in mathematical problem-solving. High-achieving students effectively utilize planning, monitoring, and evaluation skills, whereas lower-proficiency students face difficulties in these areas. This highlights the importance of metacognitive approaches in improving problem-solving abilities. Metacognitive interventions should be integrated into daily learning to optimize student success. Self-regulation and metacognitive knowledge significantly affect mathematical performance, helping students identify and correct errors systematically. Developing metacognitive awareness enables students to revise their problem-solving strategies, supporting long-term

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academic growth. Therefore, fostering metacognitive skills among both teachers and students is crucial for enhancing self-directed learning and overall educational success.

Furthermore, students can select a type of representation and modify it to be a more useful support in creating mathematical models. Students with high mathematical skills can use advanced planning, monitoring, and comprehensive skills and vice versa. Students with low mathematical skills cannot optimally utilize their skills in the planning and evaluation stages. However, they perform optimally in the monitoring stage. Moreover, they cannot use high-value planning, monitoring, and evaluation skills. It can be said that the metacognitive approach can solve the problem in the planning, monitoring, and evaluation stages.

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APPENDIX

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Metacognitive Definition	Synthesis Result	Observed Variable	Measurable Variable	Indicators	Number of Task
Metacognition would improve the student's metacognitive skills. Metacognitive skills help students learn information quickly and information for educational or professional development of students (Schunk, 2012). Metacognition is important for primary students (Bogdanović et al., 2015; Hoffman & Spatariu, 2008). As can be seen, metacognitive capacity is important, especially in the learning environment, since students can manage their cognitive and learning processes and identify their strengths and the areas necessary to improve by developing new cognitive skills (Bagci & Unveren, 2020). Each learner can metacognition, which means each student can think about learning, reading, or displaying their skills. Castillo Diaz & Gomes (2021) state that metacognition incorporates a set of capacities with imperative applications within the instructive and proficient settings requested by today's society. Creating objective assessment measures that permit the nature of the build to be evaluated is exceedingly important for conducting demonstrative and mediation forms.	Metacognition is the student's high-level thinking process, which controls students' knowledge of the process of thinking and cognitive forms and engages active controls of students' cognitive activities such as learning and solving day-to-day problems of mathematical learning. Additives include understanding and management of mathematical thought and learning. This involves students' self-awareness of how they process mathematical information, understand concepts, and solve problems.	Metacognitive Regulation	1. Planning	1. Identifying the purpose of the problems	1
				2. Sorting important information	
				3. Students control the completion steps of given information	
			2. Monitoring	1. Students can write the formula correctly and precisely	
				2. The final step made by students fallen	
				3. Students control the completion steps of given information	
			3. Evaluation	1. Students do a reexamination of the problem settlement measures	
				2. Verifying that what's done is right	

Table 4. The Guidelines of Mathematical Metacognitive Ability Test

The Mathematical Metacognitive Test

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Mrs. Diyah purchased a packet of posters containing a dozen two-liter [2 liters] paint bottles. The package was supposed to be sold for Rp240.000,00, but Mrs Diyah received a 5% discount.



Mrs Diyah also purchased three additional bottles at a regular price: Rp18.000,00 each.



What's the total cost?

Student Answer Sheet:

.....
.....
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.....

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The Knowledge of Indigenous West Papuan Pre-service Teachers about Primary School Mathematical Contents: Strengths and Weaknesses

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Abstract: Teachers' knowledge about mathematical content affects students' mathematics learning outcomes. This study aimed to identify the knowledge of pre-service primary school teachers about mathematical contents. Within a quantitative methodological approach, 150 respondents (58 Papuans and 92 non-Papuans) were involved in this study. An evaluation was developed to measure their knowledge of mathematics topics at the primary level, particularly the content area of number. Furthermore, strengths and weaknesses within each basic arithmetic operation for Papuan pre-service teachers were also identified. It comprised 30 items that covered fundamental numeracy and problem-solving. This study found that the pre-service teachers' knowledge of mathematical content was low—not exceeding 50 on the scale. Mixed operations and division were challenging for all Indigenous West Papuan pre-service teachers. The teaching department could assist Indigenous pre-service teachers in increasing their capability of mathematical content knowledge. Historical and cultural factors should be considered when designing teaching programs.

Keywords: Indigenous; pre-service teachers; mathematical content knowledge

INTRODUCTION

Data collected from multiple assessment sources indicates low mathematics performance of West Papuan students at all educational levels. There are a variety of tools available for assessing mathematics outcomes, including small-class size, large-scale observational studies, National Examinations, and internationally representative tests, e.g., PISA and TIMSS (Dirhantoro & Paramita, 2014; Manibuy et al., 2014; Ministry of Education, 2019; Sianturi et al., 2018; The Indonesian

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Bureau of Statistics, 2019). Despite the diversity of methodologies and frameworks used, there is a close review with resounding consensus that West Papuans perform very poorly and differ greatly from their peers.

The continued low performance of students in recent years has prompted a growing interest in exploring how teachers' commitment, content knowledge, and beliefs might play a role in this low performance (Sianturi et al., 2018; Werang et al., 2017). On the other hand, the proficiency of teachers in West Papua is worrying. This is reinforced by a report from the Office Head of the Study on the Papua Province Education Quality Assurance which revealed that the quality of teachers in Papua Province was poor (Costa, 2018). The latest teacher competency test report in 2017 described 2,401 of 2,564 teacher applicants (94%) did not pass the assessment, as their content knowledge acquisition was below the standard (Costa, 2018). Provinces of Papua and West Papua are the two provinces with the highest ranking for teachers who did not pass the teacher efficacy assessment (Costa, 2018). In fact, a focal point across this literature is a high concentration of teachers to come to the field with little understanding of the fundamental principles of mathematics. Although local Papuan governments organise teacher professional development programs to improve in-service teachers' mathematic skills (Ismail & Imawan, 2021; Weking, 2025), more relevant programs in teacher education are needed to prepare the prospective mathematics teachers.

Here, we investigated one particular aspect of teacher quality, i.e., mathematical content knowledge, specifically knowledge about the primary school mathematics curriculum. Accordingly, a research question for this study was designed to understand Indigenous West Papuan pre-service teachers' knowledge of primary school mathematics curricula. The research questions guiding this study include:

1. What is the level of Indigenous West Papuan pre-service primary teachers' knowledge of basic mathematical content?
2. Which mathematical content areas are the strongest and weakest for Indigenous West Papuan pre-service teachers?

We intend to investigate these research questions to develop a conceptual model of mathematics teacher development, which might serve as a guide to our existing policy analysis. These investigations could provide empirical evidence that supports or disproves assumptions currently used in explaining how teacher preparation programs are conducted, accreditation requirements for such programs, and teacher professional development initiatives.

In Indonesia, teacher candidates begin developing their mathematical content knowledge through a developmental education program in the teacher education department. To prepare pre-service teachers to have sufficient competence upon completing their teaching program, the Faculty of Education and Teacher Training develop courses relevant to their qualifications. Regarding mathematical content knowledge, especially for students majoring in primary school teacher education, the faculty members ensure that the mathematical courses cover all the mathematical domains at the primary school level. Moreover, to scaffold the mathematical content knowledge of pre-service

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teachers, mentor teachers play a crucial role in providing them with guidance and offering feedback.

LITERATURE REVIEW

Teachers' Mathematical content knowledge

Across the international literature on mathematics teacher education, there is a consensus that the most basic requirement for teaching mathematics is to have a complete mastery of the content knowledge needed to teach mathematics (Bowie et al., 2019). Mathematics knowledge is required for teachers to teach math to primary students (Livy et al., 2019). A teacher who is proficient in teaching school mathematics has a comprehensive and profound understanding of school mathematics, such as an understanding of whole ideas, proper methods for introducing students to new mathematical concepts, and engaging ways to create a sense of wonder that students will take away from the experience (Llinares & Chapman, 2019).

"Content knowledge (CK) is the teacher's understanding of the material" (Suharta & Parwati, 2020, p. 293). In teaching mathematics, Ball et al., (2008, p. 399) define mathematical knowledge as mathematical knowledge "entailed by teaching"—in other words, mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students. There is a link between the measure of teachers' mathematical content knowledge (MCK) and students' mathematics outcomes (Browning et al., 2014). A previous empirical study discovered a positive relationship between teachers' content knowledge and student mathematics learning achievement in Indonesia (Suharta & Parwati, 2020). There is an assumption if a student's mathematical ability is low, it might be influenced by the teacher's understanding of mathematics (Suharta & Parwati, 2020).

The effect of this content knowledge on a teacher's competence is considered critical. The need for solid content knowledge as a key indicator of the four teachers' core competencies has resulted in its inclusion in Indonesian governmental legislation, such as Law Number 20 of 2003 (2003) and Law Number 14 of 2005 (2005), Government Regulation Number 19 of 2005 (2005), Government Regulation Number 74 of 2008 (2008), and Government Regulation Number 16 of 2007 (2007). The four teachers' competencies include pedagogical, personal, professional, and social competencies. *Pedagogical competency* relates to teachers' ability to manage learning effectively. *Personal competency* concerns stability, integrity, and exemplary character. *Social competency* emphasizes effective communication and collaboration with students. Among these, *professional competency* which consists of the mastery of subject matter and continuous professional development, directly reflects teachers' content knowledge, serving as the foundation for effective teaching. Therefore, according to these legislations, teachers must hold a standard academic qualification in which they have excellent content knowledge competence to teach in the classroom.

Research on teaching mathematics indicates the importance of supporting pre-service teachers' MCK to enhance their mathematics and teaching (Kula Ünver, 2018). Ball et al., (2008, p. 399-402) introduced five domains of mathematical knowledge, including common content knowledge

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(CCK), specialised content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). Ball et al.'s (2008) framework of mathematical knowledge for teaching offers a clear and structured understanding of the different kinds of knowledge required for effective mathematics instruction. The distinction between CCK and SCK emphasises that teaching mathematics involves more than just solving problems—it requires a deep understanding of mathematical concepts in ways that support student learning. SCK, for instance, includes the ability to interpret various models of operations like subtraction or division, which may not be necessary in other professions but are crucial for clarifying student misconceptions. This nuanced approach elevates the professional nature of teaching and highlights the unique intellectual demands it places on educators.

However, the boundaries between the domains—especially between KCS and KCT—can be difficult to distinguish in practice. Teaching is inherently dynamic, and a teacher's response to a student's misunderstanding often draws simultaneously on knowledge of mathematics, student thinking, and instructional strategies. Furthermore, while the framework effectively captures cognitive aspects of teaching, it under-represents the affective and cultural dimensions, such as building student confidence or addressing equity in mathematics classrooms. As such, while Ball et al.'s model is a valuable tool for teacher education and research, its practical application requires adaptation to diverse teaching contexts and a broader consideration of the socio-emotional realities of the classroom.

Bognar et al. (2024) present a model of professional development (PD) that strongly aligns with Ball et al.'s framework (2008), particularly in its emphasis on enhancing pedagogical content knowledge (PCK) through structured interventions, coaching, and the integration of technology. Their findings support the idea that improving PCK—especially KCS and KCT—has a greater impact on student outcomes than CK alone, reinforcing the view that teaching demands more than procedural fluency. However, the near absence of general pedagogical knowledge in these PD programs raises concerns about a potentially narrow focus. While specialised knowledge is critical, effective teaching also relies on classroom management, student motivation, and differentiated instruction—areas typically covered under pedagogical knowledge. Therefore, a more balanced PD approach that integrates mathematical knowledge for teaching with foundational pedagogical skills may better support teachers in diverse educational contexts.

The Urgency of Indigenous Teacher Professional Development for the Future and Well-Being of Indigenous Students

Indigenous educators can foster valuable community and cultural knowledge of educational delivery services that can contribute to the empowerment of marginalized Indigenous populations, which is noteworthy for such communities as a whole (Thornton et al., 2011). Research has indicated that Indigenous students require a more culturally appropriate learning environment, and Indigenous teachers can create such learning environments (Emekauw, 2004; O'Connor, 2010; Sianturi et al., 2018; Yen et al., 2013). Since Indigenous teachers must demonstrate cultural sensitivity to the needs of their students and understand Indigenous protocols to teach their students

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effectively, their roles are considered crucial within this context because those teachers have a profound knowledge of the context of their Indigenous students.

It was evident throughout the lesson that an Indigenous educator in Thornton et al.'s (2011) study was confident in her ability to teach mathematics and to find a way to perform well. Indeed, when she taught small groups earlier in her career, she frequently paused to ask the mentor teacher to elaborate on relatively simple mathematical concepts. Over time, making a connection between symbols, places, and words was an easy task for her. She could bridge the gap between the children's lived experiences and the resources they were familiar with by creating that connection and using ways they preferred. No wonder the children had a general sense of involvement with the lesson, as they scrambled to match cards, joined in the race, called answers out, and enthusiastically drew their representations of what was happening.

According to Sianturi et al. (2018), having an Indigenous West Papuan teacher in the classroom who was willing to infuse Indigenous knowledge and stories in his teaching generated a new nuance to the classroom in which students were incredibly enthusiastic about engaging with his teaching approach. The teacher's belief in mastering mathematics material in accordance with the applicable curriculum by integrating the material with Indigenous cultural content knowledge not only prompted the improvement of Indigenous students' mathematics learning achievement but also for himself, it sparked strong enthusiasm that he could become a role model for his community.

Developing this type of confidence can be challenging as part of any pre-service teacher education program within a remote community (Thornton et al., 2011). The problem manifests even more acutely when we consider the harsh challenges of remote Indigenous culture. Refuting any unwritten regulations regarding who should be in charge in the classroom will be tough. However, there are a number of supporting factors in this case: the support provided by the mentor teacher, the university lecturer, the school-based coordinator, and the peers in the education program (Thornton et al., 2011).

Systematic Inequality in Education in West Papua

The development of Indigenous teachers in West Papua must be understood within a broader context of historical and structural marginalisation. Educational inequities in the region are not merely technical issues of access or quality but are deeply embedded in colonial legacies, state policies, and sociopolitical hierarchies that continue to shape educational outcomes for Indigenous Papuans (Kudiai, 2015; Sianturi & Hurit, 2024; Sianturi et al., 2018; 2023). These systemic forces have not only limited the opportunities for Indigenous communities to access quality education but have also impeded the development of culturally grounded and confident Indigenous teachers. Understanding the intersecting impacts of these structural barriers is essential in order to challenge deficit narratives and work toward a more just and culturally responsive teacher education system in West Papua.

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The long history of colonialism, educational system, cultural background, sociopolitical and socio-economic factors indirectly contribute to the mathematical content knowledge of Indigenous West Papuan pre-service teachers. Most Indigenous West Papuan pre-service teachers have undergone systematic inequality in education.

Scholars have argued that the existence of political legacy, violence, and international influences under Indonesian state colonial rules (Kudiai, 2015; Sugandi, 2008; Upton, 2009; Webb-gannon, 2020). Such organic developments, prejudice against the low quality of Papuan students and stereotypes that label them with primitiveness have indirectly hindered numerous possibilities for the Papuan students to leverage their potential (Macleod, 2011; Sianturi et al., 2022a). People from outside Papua have transmigrated and dominated customary Papuan lands (Upton, 2009; Widjojo et al., 2010). Now they are a minority in their land, and Javanese have become predominant in those lands (Siadari, 2017; Wardana, 2017).

More than 200 Indigenous Papuan tribes, which are scattered in coastal and highland areas, inhabit the West Papuan region. The geographical condition of this region is another factor that is also an obstacle to the limited access and education services (Kristiansen, 2006). Therefore, this condition causes the learning experience of Indigenous students to differ geographically.

Another cause is the lack of teachers' commitment and quality. A study conducted in the southern region of Papua stated that one of the biggest educational problems is the low commitment and quality of primary school teachers (Werang et al., 2017). Interviews with the Education and Teaching Office of Southern Papua (2017) found that the highest teacher absenteeism rates occurred in the outskirts of southern Papua, especially among primary school teachers. Some teachers often do not come to class, so learning stops, and students do not obtain educational services like other students. Meanwhile, due to school age, Indigenous students are forced to be promoted to the next grade, even though they have not mastered the material in previous grades. Moreover, the low quality of teachers causes students' learning experiences to be less than optimal. As mentioned earlier, most teachers in Papua cannot achieve the standard of teacher quality (Costa, 2018).

Another problem found to be a cause of systematic inequality in education in West Papua is the design and implementation of a curriculum that is irrelevant to Papuan children. The curriculum currently implemented, namely the 2013 national curriculum, is seen as not relevant to the educational needs of Papuan children (Sianturi et al., 2018; 2022a). There is a separation between children's life experiences and cultural context with the material presented and the presentation of the material does not consider Indigenous ways of knowing (Sianturi et al., 2025). In mainstream education, learning emphasises western mindsets and measures learning outcomes in a standard way (Rigney et al., 2020; Sianturi et al., 2024). Though international and local literature has demonstrated that Indigenous people have a learning system that is different from western patterns, and learning will be more fun for Indigenous children through a cultural approach (e.g., a Harris, 1984; Matang & Owens, 2014; Owens, 2012; Yunkaporta, 2009). That is why students often have difficulty understanding learning.

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Apart from the government and policymakers who have the authority to reform curriculum, this is also inseparable from the responsibility of tertiary institutions, which have a role to play in preparing prospective teachers. Prospective teachers must be equipped with knowledge and skills to incorporate the Papuan cultural context into learning. However, this is absent in the structure of the courses at tertiary institutions across West Papua.

The Context of the Study

The level of understanding of mathematics material affects a prospective teacher's readiness and confidence to teach mathematics material. Several prospective teachers do not master the mathematics material, and mistakes often occur in solving math problems (Sumartini, 2020). The mistakes occur due to pre-service teachers' lack of mathematical concepts (Sumartini, 2020). Prospective teachers with inadequate mastery of mathematics will think they are not good enough to teach mathematics (Sumartini, 2020).

Pre-service primary school teachers in Papua often experience the same thing. A lack of understanding of the material often causes anxiety in entering a teaching internship program. We limited our study to investigating the level of knowledge of prospective primary school teachers about mathematics material in primary schools. Mathematics topics taught at the primary level comprise five content areas: number, shape and space, measurement, data handling, and algebra. In this study, we focused on the content area of "number" which covers whole numbers, fractions, decimals, percentages, and basic arithmetic operations. Moreover, strengths and weaknesses within each basic arithmetic operation for Papuan pre-service teachers were also explored. Since this was a preliminary study in this region for the tertiary level, the results provided an overview of areas of improvement for teacher education programs. Our next project would be conducted qualitatively to discover the causes and strategies (including historical and sociocultural factors) that could be conducted both from the perspectives of pre-service teachers and instructors at the teaching department and supported by higher education curriculum analysis.

METHOD

We identified the mathematical abilities of pre-service primary school teachers. Participants were given a mathematical test. Computational fluency and mathematical interpretation and modeling skills were highlighted in the test. To accomplish this investigation, this study employed a quantitative methodological approach (Creswell, 2009). Ethical clearance was obtained from the Human Research Committee at a local university, and all participants provided us with informed consent.

Research Participants

This study involved 150 pre-service teachers in years 1, 2, and 3. The demography of the respondents is provided in Table 1. The participants came from diverse tribal backgrounds (58 Papuans and 92 non-Papuans). For the convenience of the report, we grouped some tribes based on the

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similarity of regional origin. Ambonese, Kei, and Tanimbar tribes were grouped into Maluku, as they were originally from Maluku Island. Makassar, Bugis and Toraja tribes were grouped into Makassar as they were originally from Sulawesi Island. Meanwhile, the ethnic groups of NTT, Batak, NTB, and other ethnic groups were grouped into others. The number of respondents from the Javanese tribe was dominant.

Criteria	Category	Frequency (*n*)
Year of study	Year 1	30
	Year 2	46
	Year 3	84
Gender	Female	110
	Male	40
Tribe	Asmat	11
	Dani	2
	Muyu	16
	Mandobo	9
	Auyu	4
	Marind	13
	Serui	3
	Java	37
	Makasar	11
	Maluku	27
	Others	17

Table 1. The demography of participants

Instruments and Data Analysis

Data was collected through a survey sheet consisting of two parts, the first part was a mathematical test sheet, and the second part was respondents' information. The test given consisted of simple arithmetic operations (e.g., addition, subtraction, multiplication, division, and mixed operations) and problem-solving (e.g., word problems). Problem-solving questions were addressed to identify their analytical thinking. The test comprised 30 items (24 multiple-choice items and six essay items), in which 4 multiple-choice items and 1 essay question represented each topic of number content area. Table 2 provides some examples of test items. We analysed data quantitatively. This test was developed by adapting the basic mathematics questions distributed in the latest National Examination at the primary level and from textbooks used in primary schools across Papua. The rationale for this test construction choice was that most of the questions came from these two sources which are also often a scourge for pre-service primary school teachers.

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Item number	Test items	English translation
8	$\frac{5}{4} - \frac{7}{9} = \ldots$ a. $\frac{2}{5}$ b. $\frac{1}{18}$ c. $\frac{12}{13}$ d. $\frac{17}{36}$ e. $\frac{35}{36}$	$\frac{5}{4} - \frac{7}{9} = \ldots$ a. $\frac{2}{5}$ b. $\frac{1}{18}$ c. $\frac{12}{13}$ d. $\frac{17}{36}$ e. $\frac{35}{36}$
15	Faktor Persekutuan Terbesar (FPB) dari 36, 48, dan 72 adalah.... a. 36 b. 24 c. 12 d. 8 e. 6	The Greatest Common Factor (GPA) of 36, 48, and 72 is.... a. 36 b. 24 c. 12 d. 8 e. 6
17	$135 - 27 \div 9 \times 25 = \ldots$ a. $\frac{108}{225}$ b. 60 c. 300 d. 2.700 e. 3.300	$135 - 27 \div 9 \times 25 = \ldots$ a. $\frac{108}{225}$ b. 60 c. 300 d. 2,700 e. 3,300
24	Bibi memiliki beras $1\frac{3}{4}$ kg. Kemudian Bibi membeli lagi $2\frac{1}{2}$ kg. Beras tersebut dimasak 0,2 kg. Sisa beras Bibi sekarang ...kg. a. $3\frac{3}{20}$ b. $3\frac{1}{20}$ c. $4\frac{1}{20}$ d. $3\frac{3}{20}$ e. $4\frac{9}{20}$	My auntie has $1\frac{3}{4}$ kg of rice. She buys again $2\frac{1}{2}$ kg of rice. Then she cooks 0.2 kg of rice. Her leftover rice now is ...kg. a. $3\frac{3}{20}$ b. $3\frac{1}{20}$ c. $4\frac{1}{20}$ d. $3\frac{3}{20}$ e. $4\frac{9}{20}$

Table 2. Examples of test items

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Two experts validated this test to measure whether the test items represented each topic. The results gained from experts indicated that all items described each topic of the content area measured. The experts also provided corrections regarding word choices in the test items. After revising the test, we tested it on 113 pre-service teachers of years 1, 2, and 3 from the Indonesian language and literature department. The test reliability was measured by using Cronbach's alpha value. The value of Cronbach's alpha was 0.58. This means that the test was reliable. The data was analyzed using SPSS, employing descriptive analysis. The paper-pencil test was administered and completed in 90 minutes.

RESULTS

The results of this study comprised four parts, namely: (1) the overview of mathematical content knowledge of Papuan pre-service primary school teachers; (2) the strengths and weaknesses of Papuan pre-service primary school teachers in mastering mathematical knowledge for each topic. A normality test was measured to determine whether the data was normally distributed. The statistical results showed a significance value of 0.181 greater than 0.05 (Appendix A), which means that the data were normally distributed. Thus, statistical tests could be measured parametrically.

The Overview of Pre-service Teachers' Mathematical Content Knowledge

Table 3 describes pre-service primary school teachers' knowledge about mathematics topics. None of the pre-service teachers answered all the questions correctly. The highest score was 19, and the lowest score was 1. Overall, their performance was 43.17 on a scale of 100. That is, their capability to master fundamental mathematical knowledge did not even exceed 50%. The tribes with the lowest average scores were Asmat pre-service primary school teachers, and the highest was Javanese pre-service primary school teachers. For the Papuan tribe itself, the Serui and Auyu tribes had higher performance than other Papuan tribes.

Tribe	N	Minimum	Maximum	Mean	%
Asmat	11	4	11	5.82	24.25
Dani	2	6	7	6.50	27.08
Muyu	16	1	17	7.31	30.46
Mandobo	9	3	12	6.33	26.38
Auyu	4	8	16	10.50	43.75
Marind	13	1	18	8.46	35.25
Serui	3	9	15	12.33	51.38
Jawa	37	1	19	13.35	55.63
Makassar	11	2	19	12.19	50.79
Maluku	27	1	16	10.16	42.33
Others	17	7	19	12.41	51.71
Total	150	1	19	10.36	43.17

Table 3. The pre-service primary school teachers' knowledge about mathematics topics

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The Strengths and Weaknesses of Indigenous Papuan Pre-Service Teachers in Mastering Mathematical Content for Each Topic

Asmat and Mandobo pre-service primary school teachers did not master 50% of any operation (Table 4). For the Dani tribe itself, none of them could solve mathematical word problems. On the other hand, Muyu and Marind pre-service primary school teachers performed better in completing mathematical word problems, obtaining the highest correct percentage. Mixed counting and division operations were complex operations for all Papuan tribes. Meanwhile, the Dani, Muyu, and Mandobo tribes experienced difficulties in subtraction operations. The addition operation was a relative operation that almost all Papuan pre-service primary school teachers could solve.

Arithmetic Operation	Asmat		Dani		Muyu		Mandobo		Auyu		Marind		Serui	
	Mean	%	Mean	%	Mean	%	Mean	%	Mean	%	Mean	%	Mean	%
Addition	1.55	38.75	2.00	50.00	1.63	40.75	1.78	44.50	2.75	68.75	1.62	40.5	2.33	58.25
Subtraction	1.00	25.00	1.00	25.00	0.75	18.75	0.78	19.50	1.50	37.50	1.15	28.75	3.00	75.00
Multiplication	1.00	25.00	2.00	50.00	1.31	32.75	1.44	36.00	2.25	56.25	1.38	34.50	2.00	50.00
Division	0.64	16.00	0.50	12.50	1.13	28.25	0.78	19.50	1.00	25.00	1.69	42.25	2.00	50.00
Mixed operations	0.36	9.00	1.00	25.00	0.50	12.50	0.67	16.75	1.00	25.00	0.54	13.50	1.00	25.00
Word problems	1.27	31.75	0.00	0.00	2.00	50.00	0.89	22.25	2.00	50.00	2.08	58.00	2.00	50.00

Table 4. Pre-service teachers' mathematical content knowledge by Papuan ethnicity

DISCUSSION

Results revealed that pre-service primary school teachers' overall knowledge of primary school mathematics materials was deficient. This implies that most of them struggled to master mathematical materials at the primary school level. Some of them completed fundamental mathematical courses in years 2 and 3 of their study, including multiplication, division, and word problems.

This result was a little worrying for the future generation of Papuan children. How will Papuan education improve if the quality of pre-service primary school teachers, especially Indigenous Papuan pre-service teachers, does not meet the expected standards? Remarkably for such marginalised communities, the literature describes Indigenous educators as having the potential to foster meaningful community and cultural knowledge of educational delivery services (Thornton et al., 2011). Several researchers have recommended that teachers shift the learning environment for Indigenous students to be more culturally relevant (Emekauw, 2004; O'Connor, 2010; Sianturi et al., 2018; Yen et al., 2013). Since teaching Indigenous students requires cultural sensitivity to their multiple and diverse needs and an understanding of Indigenous protocols, Indigenous teachers'

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roles are crucial because they have a profound knowledge of their Indigenous context. Sianturi et al. (2018) found that the presence of an Indigenous West Papuan teacher who committed to adopting Indigenous knowledge and stories in his teaching created a new nuance to his classroom, in which students were incredibly enthusiastic about learning with his approach. The results of this study, which was a little discouraging, provided a picture of the quality of the developmental education program in the teaching and education department. Therefore, the teaching and education department must consider how to increase Indigenous Papuan pre-service teachers. It was essential for the government and the Faculty of Education and Teacher Training to give more attention to Indigenous Papuan pre-service teachers. In order to understand the kind of attention most beneficial to these tribes, the faculty first needed to give them a voice. The strategy conducted has not addressed this challenge because they were completely silent on how these voices were considered. For example, a comprehensive bill, the Teacher Law, aimed at raising the quality of teachers through teacher certification was passed by the government in 2005 in an effort to improve education quality; however, the teacher certification has not improved student learning outcomes nor increased the competencies of teachers (The World Bank, 2014). A commitment to provide teachers with training related to the content knowledge will increase their teaching practices (Habiyaremye et al., 2022).

The strategy should be more related to the historical and cultural factors, and our findings provided a discussion that can be helpful. The results also described that five Indigenous West Papuan tribes performed differently in each topic of mathematics materials. Asmat and Mandobo pre-service teachers encountered difficulties in mastering all basic mathematical knowledge, including addition, subtraction, multiplication, division, mixed operations, and problem-solving. Auyu tribe is a tribe that originally came from the same region or adjacent to where Mandobo, Asmat, Muyu, and Marind tribes came from. However, the Auyu tribe seemed to perform better than them. For Muyu and Marind students, solving math problems was the most effortless operation. Based on history, the Muyu tribe had always known the barter system with *ot* (seashell) and *mindit* (animal teeth), which remain in use today (Jerat Papua, 2014; Mei, 2016). Additionally, the Muyu tribe is also known for having good arithmetic and social skills (Mei, 2016). This is why their capability to complete mathematical problem-solving operations was much better than other operations. Future research could usefully explore the impact of stereotype threat on mathematical achievement across Indigenous groups to better understand sociocultural dimensions of performance disparities.

The results revealed notable weaknesses in the understanding of mixed operations and division, which were the most challenging for Indigenous Papuan pre-service teachers. These areas showed high error rates and a lack of procedural fluency and conceptual clarity. Pre-service teachers often struggle with conceptual understanding of elementary number theory, showing a tendency towards procedural approaches (Zazkis & Campbell, 1996). For instance, in our study, many respondents struggled to apply the correct order of operations in word problems or made errors in multi-step division problems. Research on pre-service teachers' mathematical understanding also indicates significant challenges in understanding division concepts (Simon, 1993), including difficulties with division by zero (Crespo & Nicol, 2006). Furthermore, recent research highlights difficulties in identifying errors and comprehending content in mathematical word

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problems, with pre-service teachers often focusing on solutions rather than critically evaluating problem statements (Abdullah et al., 2024). These studies consistently demonstrate gaps in conceptual understanding, particularly in connecting mathematical concepts to real-world contexts. Common themes include challenges in explaining mathematical reasoning, limited depth of understanding in seemingly basic operations, and difficulties in applying mathematical knowledge to diverse situations. These findings underscore the need for enhanced teacher education programs that emphasize conceptual understanding and critical thinking in mathematics.

Other research has also found that Indigenous people's way of learning tends to apply a whole pattern rather than a sequence form (Harris, 1984; Hughes & More, 1997; Yunkaporta, 2009). This resulted in Indigenous students encountering difficulties in completing sequence arithmetic operations like mixed arithmetic operations and division (Hughes & More, 1997). Therefore, when deciding programs for teaching department, taking cultural factors into consideration enables educational authorities to provide a transformed learning experience (Sianturi et al., 2018). Contextual mathematics should be made more visible to Indigenous students, including the incorporation of cultural attributes into mathematics (Bishop, 2002; Showalter, 2013; Yao, 2014).

The observed weaknesses in mathematical knowledge cannot be fully understood without considering the cultural and historical context of Indigenous education in West Papua. Indigenous students often come from families and communities with limited access to quality primary and secondary mathematics education due to systemic inequities, under-resourced schools, and language barriers. International literature indicates an essential issue of the role of culture in our (mis)understanding of "mathematical ability" itself (Antone, 2012; Forgarty & Schwab, 2012). The mathematical ability of Indigenous people is developed within their cultural context (Owens, 2012). Therefore, assessing a mathematical ability through a standardised test might not be a good choice (Rigney et al., 2020; Sianturi et al., 2024). Our study used a standardised test, which might influence Papuan pre-service teachers to understand the test. A study by Sianturi et al. (2025) revealed that when a test was modified from a standardised form into an item that incorporated Indigenous cultural context, Papuan students (junior and high school levels) completed the test easier and faster. Knowing this might help the government and the Faculty of Education and Teacher Training design effective strategies for effective learning, designing professional teacher programs and assessing teachers' mathematical ability.

Our study presented pre-service teachers' knowledge of mathematics materials in primary schools, but we recommend readers not interpret these results for political or other non-research purposes. The study has several limitations that must be acknowledged. First, the focus on number content excludes other mathematical domains such as geometry, algebra, or measurement, which may have revealed different strengths and weaknesses. Second, the sample is limited to pre-service teachers in one province, which may not represent all Indigenous Papuan groups. Therefore, future researchers can consider a larger sample size. Third, the study relies on quantitative test results, which—while useful—do not fully capture the cultural reasoning or informal mathematical strat-

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egies employed by participants. Lastly, the study does not explore in depth the pedagogical approaches used in the teacher education programs, which may have influenced the outcomes. Further research is required to determine the impact of history and culture on their pre-service teachers' mathematical ability, particularly Indigenous Papuan pre-service teachers. In addition, it is also necessary to investigate qualitatively the strengths and weaknesses of Indigenous Papuan pre-service teachers that affect their mastery of mathematical content knowledge. As we mentioned earlier, we intended to conduct our following research qualitatively to identify the causes and strategies (including historical and sociocultural lenses) from the perspectives of pre-service teachers and instructors of the teaching department and complemented by a curriculum analysis of higher education. Future research may also adopt mixed-methods designs, combining standardised assessments with interviews or classroom observations, to better understand the cognitive and contextual processes behind correct and incorrect responses. In doing so, researchers can move beyond test scores to grasp the full picture of mathematical learning among Indigenous populations. While our study highlighted structural and historical factors that might influence Indigenous Papuan pre-service teachers' mathematical content knowledge, it did not extensively explore these factors and other potential contributors such as pedagogical approaches, curriculum design, or disparities in access to educational resources, which represent limitations to the current analysis. Future research may investigate these potential contributors specifically.

CONCLUSION

The mathematical content knowledge of pre-service teachers at the primary school teacher education department is one indicator of core competencies for a teacher. Our study reveals that the competence of mathematical content knowledge of such teacher candidates is low. The average level has not exceeded 50 on the scale, which is below the standard. Asmat pre-service teachers have the lowest performance among other pre-service teachers. Mixed operations and division are challenging for all Indigenous West Papuan pre-service teachers.

This present study provides some recommendations. The Faculty of Teacher Education and Training might provide more assistance in equipping their Indigenous West Papuan pre-service teachers. In the first semester, matriculation in fundamental mathematics might be given to increase their mathematical knowledge. The most critical factor that the faculty should consider is historical and sociocultural factors. In designing teaching programs for Indigenous Papuan pre-service teachers, the faculty should incorporate cultural backgrounds into their curricula and teaching practices. It is also worth considering providing Indigenous West Papuan pre-service teachers with a space to share their insights and inspirations on how the program and Indigenous culture-based instruction can be developed to better support them.

The findings of this study have also direct implications for teacher education policies and curriculum design. First, mathematics education courses for pre-service teachers in Papua should include diagnostic assessments to identify specific content gaps early in the program. Second, targeted interventions—such as bridging courses focused on number concepts and problem-solving—

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should be integrated into the curriculum. Third, teacher education programs should adopt culturally relevant pedagogies that contextualise mathematical concepts within Indigenous students' lived experiences, supporting both understanding and retention.

Furthermore, policymakers should consider the systemic challenges Indigenous students face and implement affirmative support mechanisms, such as mentorship, additional academic tutoring, and community-based learning strategies. These would help ensure that Indigenous pre-service teachers not only improve their MCK but are also empowered to become effective educators in their own communities.

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APPENDIX

Appendix A. Analysis of normal distribution test

N		150
Normal Parameters[b]	Mean	10.3600
	Std. Deviation	4.61145
Most Extreme Differences	Absolute	0.090
	Positive	0.087
	Negative	-0.090
Kolmogorov-Smirnov Z		1.096
Asymp. Sig. (2-tailed)		0.181

[b] *Normal distribution is assumed*

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Mathematics Teaching Strategies: A Reflection on Pedagogical Content Knowledge (PCK) for Improving Teaching Quality

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Abstract: This study investigates elementary school teachers' mastery of mathematical content and pedagogical content knowledge (PCK) in Eastern Indonesia, a region bordering other countries. Specifically, the research explores the relationship between teachers' knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC) and how these factors impact their teaching strategies. The central research question is: How does elementary school teachers' mastery of mathematical content and PCK influence their teaching strategies? Using a qualitative approach, data were collected from 77 elementary school teachers through descriptive tests, in-depth interviews, and classroom observations. The findings indicate that teachers' overall PCK, including KCS, KCT, and KCC, remains at a low level. Many teachers demonstrate content knowledge that is disconnected from foundational mathematical concepts, which negatively impacts their pedagogical approaches and their ability to select appropriate teaching strategies. This study highlights the need for further research on improving teachers' content knowledge and pedagogical approaches to enhance the quality of mathematics instruction.

Keywords: mathematics education, PCK, teachers, teaching strategy, teaching quality

INTRODUCTION

Diverse educational sectors endeavor to provide students with competencies for the 4.0 Era. This era is characterized by the integration of digital technologies, automation, artificial intelligence (AI), big data, and the Internet of Things (IoT) into various aspects of life, including education (Celik et al., 2022). In the context of education, the 4.0 Era demands a shift from traditional teaching methods to technology-driven, student-centered learning models that foster critical thinking, problem-solving, and digital literacy. The primary focus of education in Indonesia nowadays encompasses learning model innovation in the digitalization era (Erhan et al., 2022), differentiated

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learning (Yu & Canton, 2023), and the growth of reading and numeracy (Piper et al., 2018). Global competition increases the need to improve the quality of education in Indonesia. Several regions in Indonesia with qualified teacher resources are trying to update the teaching model, improve teaching quality, and use an independent curriculum.

However, in the border areas, Indonesia has various unique challenges in terms of education. Border areas in Indonesia face various challenges, such as access to resources, school locations, and differences in language and culture, which greatly affect the quality of education (Magayang et al., 2020). The teacher is the main pillar and the main source of learning in improving the quality of education. The quality of education is determined by the competence and resources of teachers (Carroll, 2020). The main tasks and responsibilities of the teacher are the main axis of teaching activities. The teacher's ability to convey teaching material is a professional competency that the teacher must fulfill (Dawson & Shand, 2019). With various limitations, the teacher should have good pedagogical skills so students can capture the knowledge the teacher conveys.

Additionally, teachers encounter numerous challenges (Febriana et al., 2018), particularly in instructing mathematics at the basic level. The proficiency of teachers in mathematics influences their approach to developing students' mathematical knowledge and skills (Coskun, 2020). They are required to consistently enhance their proficiency in math's. Mastery of mathematics influences teachers' daily thought processes (Castro et al., 2021).

This research is urgent because it fills the void of previous research. Previous research has discussed teacher limitations in teaching (Murniarti et al., 2021; Nguyen et al., 2020) but has yet to discuss teacher content and pedagogical knowledge in teaching mathematics in limited areas. This study is related to several studies that explain that elementary school students in border areas have low knowledge and problem-solving abilities (Fredy et al., 2021).

This study discusses the teacher's strategy for working on questions and problems. The aim is to identify the mastery of mathematics teaching strategies of the teachers at elementary schools in the border areas of Indonesia based on reflections on their PCK. This study is expected to become a recommendation for improving the quality of teacher resources in elementary schools so that specific and targeted improvement efforts can be identified.

Pedagogical Content Knowledge (PCK) is a critical component of teacher professionalization, particularly in the context of mathematics education. PCK is defined as the integration of content knowledge, pedagogical knowledge, and an understanding of students' learning processes (Gladys et al., 2024). This unique blend enables educators to tailor their teaching strategies to meet the diverse needs of their students, thereby fostering improved learning outcomes (Hill et al., 2008; Ball et al., 2008).

The KCS domain emphasizes the necessity for teachers to understand their students' pre-existing knowledge and learning difficulties, which is crucial for designing effective instructional strategies (D. Celik et al., 2022; Mitra, 2024). The KCT domain focuses on the pedagogical strategies that

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best convey specific content, while the KCC domain pertains to the alignment of teaching with curricular standards (Bognar et al., 2024). Research indicates that teachers with a strong foundation in PCK are more adept at implementing inquiry-based and student-centered teaching methods. These educators are better equipped to adapt their instructional strategies based on their understanding of both the subject matter and their students' learning needs (Bales, 2019; Mdaka et al., 2023; Pilous & Leuders, 2023).

The development of PCK is influenced by teaching experience, with studies showing that novice teachers often struggle to integrate these components effectively compared to their more experienced counterparts (Evens et al., 2018). The urgency of assessing PCK in relation to teaching effectiveness cannot be overstated. The application of PCK directly correlates with student learning outcomes, making it essential for teacher education programs to emphasize the development of PCK among future educators (Koopman et al., 2019). This focus on PCK not only prepares teachers to deliver content effectively but also equips them with the skills necessary to foster a supportive learning environment that acknowledges and addresses the diverse needs of their students (Donnelly & Berry, 2019).

This multifaceted approach to teaching not only enhances the teacher's ability to deliver content but also significantly impacts student engagement and achievement (ŞEN, 2021; Sie & Agyei, 2024). The urgency of assessing PCK in relation to teaching effectiveness cannot be overstated. As noted by various scholars, the way teachers apply their PCK directly correlates with student learning outcomes, making it essential for teacher education programs to emphasize the development of PCK among future educators (Hassan et al., 2018; Metz, 2018). This focus on PCK not only prepares teachers to deliver content effectively but also equips them with the skills necessary to foster a supportive learning environment that acknowledges and addresses the diverse needs of their students (Sarıçoban & Kirmizi, 2020).

METHOD

Participant and Context of the Study

This study aimed to find PCK patterns in rural teachers by exploring elementary school teachers' thinking skills. A total of 77 teachers participated to accomplish this goal. The subjects comprised 53 individuals from public schools and 24 from private schools in rural regions of Indonesia. The research subject criteria comprised 58 male and 19 female teachers aged 30-56. The participants of this study were teachers who had taught for at least three years in elementary schools and had a diploma education background in the elementary school teacher education study program. Some teachers had to teach three classes because only three were in one school. The conditions of the place where they taught were far from adequate facilities because they were in the outermost and furthest areas from the city center. Access to schools needed to be improved, and it tends to be difficult to pass with steep roads and minimal vehicles. Learning infrastructure and facilities still need to be improved to reach optimal effectiveness, especially the learning media. Students' school

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attendance was still low because they helped their parents meet their daily needs. Challenges related to students' cultural and economic backgrounds limited their opportunities to enhance their knowledge and teaching competencies.

The research methodology involved 77 elementary school teachers who cooperated on six questions concerning KCC, KCS, and KCT. The sampling technique used was purpose sampling by selecting teachers who had filled in the answers. We only included 27 teachers in the subsequent study process because of this test, as the remaining teachers were required to provide a response or provided responses that were not as pertinent to the extant questions. We subsequently selected three teachers who satisfactorily answered each question from the 27 teachers. Three teachers were the interview subjects, and the criteria included 1) answering all the wrong answers with the wrong understanding of the questions, 2) answering all the answers wrong but still understanding the question's meaning, and 3) teachers with unique answers. We involved the three subjects in in-depth interviews.

Data and Measurement

This study employed three main instruments: a test, interviews, and classroom observations to assess teachers' Pedagogical Content Knowledge (PCK) in elementary mathematics.

Test

The data was collected using an ability test on mathematics teaching strategies in elementary schools, adapted from Ball (Hill et al., 2008). This method was used because the teachers convey that not all math material is well mastered. The first data collection was done using a test. The test was followed by 77 teachers, with 27 teachers collecting answers. The test was carried out for 1 week because it was adjusted to the management of teachers to become educators and ease of signal access. The working technique was in the form of open answers according to the teacher's understanding. Consisting of 6 questions with 1 KCC question, 2 KCS questions, and 3 KCT questions. Each question has 10 points for the correct score and 0 for the wrong score, with 60 points. The results of this test were used to identify interview subjects.

Category	Item	Questions
KCC	1a	What is the learning media for fraction operation?
	1b	How do you apply realistic mathematics?
KCS	2a	How important is it to teach geometry concepts by linking contextual problems?
	2b	Write down one way to teach geometry to grade 6 students!
KCT	3a	Explain whether algebraic material is in elementary school!
	3b	Give one example of algebraic material for lower-grade students!
KCT	4a	$\frac{1}{3} : \frac{1}{6} = 2$ Prove with a picture!
	4b	Make a simple plan on how to teach the material!
KCT	5a	(-5) - (-9) = 4

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Category	Item	Questions
		Explain the correct solution to teach the problem!
	5b	Should the teacher use media for this solution? Explain!
KCS		125 : 5 = ...
	6a	What is the concept of teaching division operations to minimize student errors?
	6b	Write down an alternative to teaching this material!

Table 1: Questions of teacher's PCK test

Each correct answer earned 5 points, with a maximum score of 60. Conducted over a week, the test accommodated teachers' schedules and internet access limitations. Validity was ensured through expert reviews and pilot testing, while reliability was confirmed via inter-rater scoring consistency.

Interview

Interviews were used to obtain complete data on the teacher's thinking in developing mathematics teaching strategies for students. Interviews were also conducted with lecturers who taught mathematics in elementary school. The in-depth interview was conducted over four meetings. Three participants were interviewed separately. The interview consisted of 3 PCK domains: KCS with two questions, KCT with four questions, and KCC with 1 question. The interview questions were also adjusted to the answers to the test questions that had been worked on. The data obtained from the interview were transcripts of answers and descriptions of teacher abilities. Validity was ensured through expert review and member checking, while reliability was maintained via coding triangulation. The interview and observation instrument used was adapted by Hill et al. (2008) as shown in Table 2.

Category	Domain
(KCS)	More studies concerning learning difficulties in mathematics
	More skills concerned with teaching students at different levels
(KCT)	More courses about didactic mathematics
	More courses about how to differentiate teaching
	More skills to motivate students in mathematics
	More studies about problem-solving
(KCC)	More courses about using technology in teaching mathematics

Table 2: The instrument PCK Domain

Observations

Observations were also carried out to observe teacher performance. Observation supports data and is an important instrument in providing a perspective on research results. The observation position was carried out during eight weekly meetings to balance the test and interview data. The teacher conducted the teaching and learning process and then used a laptop or mobile phone setup so that the observer could monitor activities online. The objects observed were mathematics ability, the

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teacher's teaching method, classroom management, student activities, school environment, and student enthusiasm for learning. The researcher conducted observations as a mathematics lecturer. The observation results were descriptions of activities and findings based on learning activities. A structured rubric, reviewed by experts, ensured validity, while observer agreement checks enhanced reliability. This multi-method approach strengthened the accuracy and credibility of the study's findings.

Data Analysis

Data analysis was conducted by data collection, reduction, and conclusion. The analytical data pertained to triangulation through the cross-validation of conclusions from several data sources. The research guaranteed robustness and reliability. To ensure validity, the interview questions were reviewed by mathematics education experts to confirm alignment with the test results and PCK framework. Member checking was employed, where interview transcripts were shared with participants for verification. To ensure reliability, coding consistency was maintained through triangulation, where two researchers independently analyzed interview transcripts and resolved discrepancies through discussion.

The data regarding test results on the efficacy of teaching strategies could be validated using qualitative insights from interviews and case studies. Qualitative data offered context and depth to the observed numerical trends. Qualitative data can elucidate the reasons and mechanisms underlying its practical application. Qualitative research insights emphasize increased attention to test outcomes.

Data analysis in this study included analysis of teacher answers using tests, in-depth interviews on three aspects of PCK, and observation as supporting data. Analysis of teacher test results in solving PCK questions on mathematics material was done by first identifying each open-ended answer item. The answer items were explained to obtain a score for the distribution of answers to each question. This score was used to select three research participants who were interview subjects. The data was qualitative at the in-depth interview stage, so data reduction was needed to obtain answer codes and the relationship between each answer. Not only that, at the interview stage, it was supported by authentic evidence of the interview subject's answers. The observation data described the conditions appropriate for the test and interview data.

The analysis results at the test stage were presented in a graph of the demographic results of the teacher's answers. The next result was a table of teacher answer results and a table containing the results of teacher interviews, overall coding results, and observation results. The results of the various data collections were checked for validity by the researcher and one expert with triangulation, namely, comparing data with other sources. In this process, the researcher conducted a thematic analysis to identify patterns that emerged in the puzzle of teacher knowledge, curriculum, and challenges of teaching mathematics in elementary schools in rural areas. In this analysis, results about teacher stories and experiences are explored in depth. The narrative of the teacher's

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answers provides a picture of the conditions that influence the learning process and teacher interaction with students. The results obtained from this triangulation process are PCK patterns related to KCC, KCS, and KCT in mathematics learning by elementary school teachers in rural areas. The coding results at the final stage are in the form of analytics presented in patterns and relationships for each PCK domain, including KCC, KCS, and KCT.

RESULTS

Test Result

Test results were available at eight weeks. Figure 1 shows the results of the teacher's PCK test.

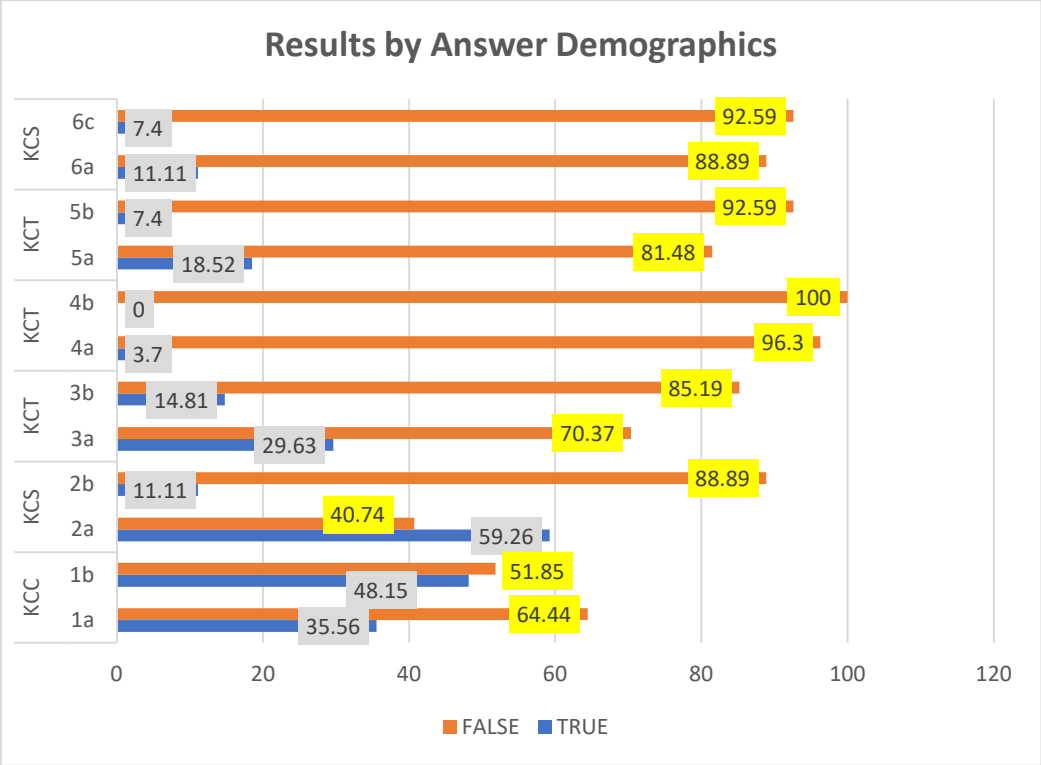


Figure 1: Results by Answer Demographics

The data in Figure 1 above obtained from 77 teachers who worked on the questions. The teachers gave answers that needed to match the questions. The following discussion is about each category of the questions.

Teacher's Knowledge of Content and Curriculum (KCC)

For question no. 1, most of the teachers explained the answers incorrectly. The teachers that answered incorrectly were 64.44% because the teachers did not understand the meaning of the questions correctly. The teacher gave answers that did not follow the given instructions. Of the 12 teachers who gave wrong answers, 10 answered questions far from those given, for example, in the answer in Table 3.

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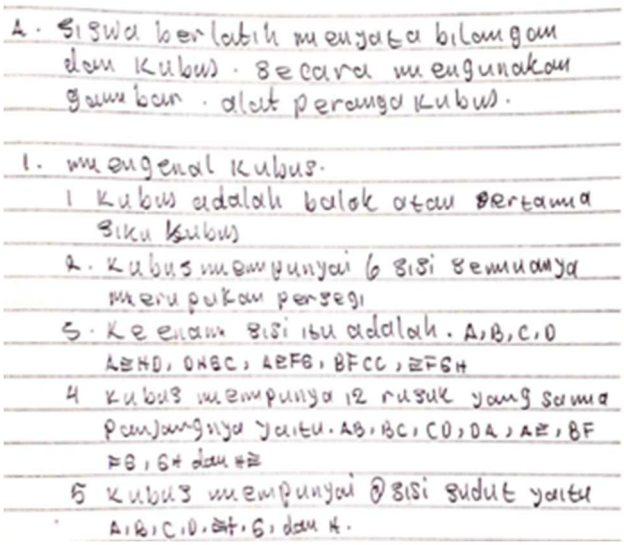
The Answer	Translate
	A. Students practice expressing numbers and cubes. By using pictures. Cube tool. 1. Get to know the cube a. The cube is the first block or corner cube. 2. A cube has six sides, all of which are square 3. The six sides are A, B, C, and D. AEND, OHEC, AEFG, BFCC, EFGH 4. The cube has 12 edges of the same length, namely AB, BC, CD, DA, AE, BF, FG, GH, and HF 5. The cube has eight angles, namely A, B, C, D, E, F, G, and H

Table 3: The Answer to Question 1 KCC

The initial teacher's response pertains to the cube's material and its characteristics. These answers are a sample of 10 teachers whose answers are far from far from correct. During the interviews, the teachers were given an opportunity to rework the same task, however, their responses remained incorrect.

Category	Item	Answers
Interview (T for Teacher)	1	T 1: "I have difficulty understanding the meaning of the question, which I understand like that." T 2: "I have tried to fix the answer, as far as I know." T 3: "We do not use media, so we do not understand."
Interview with Lecturer		The answer to using learning media is monotonous due to limited access in the area. They have no desire to create media and only use existing media. Not only that, but they need help understanding various learning methods and media. So, the student learning model is still teacher-centered.
Observation results		At this stage, the teacher needed to explore the answers more, and the answers were based on personal experience. There are limited uses of media and learning supports. Teachers also need help to make students active because teachers tend to be teacher-centered. Some teachers have used learning aids such as marbles, sago caterpillars, and fruit to learn numbers, so the technology in this study is devoted to aids to help convey material.

Table 4: The Answer to Interview of KCC

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To clarify the picture of teachers at the KCC stage, we adapted the KCC pattern from (Toyinbo, 2018) with adjustments to the results in the field, namely as follows:

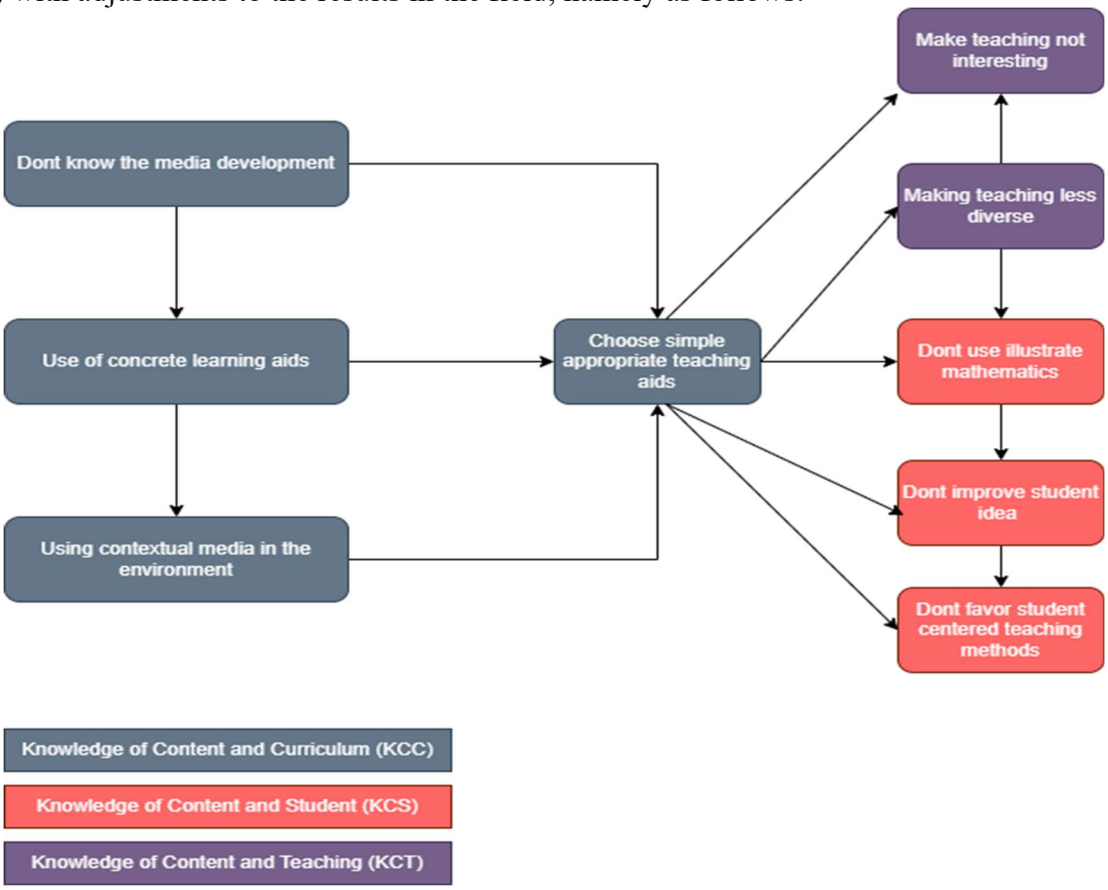


Figure 2. The pattern of relationship between KCC with KCT and KCS

Teacher's Knowledge of Content and Students (KCS)

The discussion of KCS examines how the teacher anticipates student answers and difficulties. Questions 2a and 2b discuss material on geometry material and geometry teaching techniques to anticipate student answers. There were 11 (40.74%) wrong answers in question 2a and 24 (88.89%) in question 2b. In question 2a, the error lies in how the teacher discusses geometry material. For questions 2b and 24, all the 27 teachers answered wrong. The teacher's answer is shown in Table 5 below.

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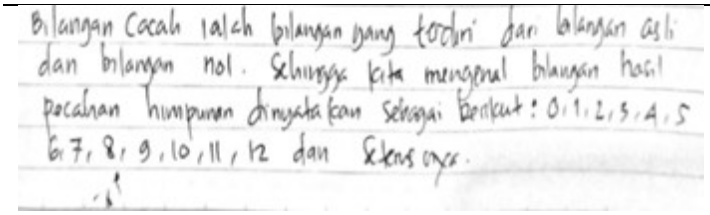
Answer	Translate
	b. Whole numbers are numbers that consist of natural numbers and zero. So that we know the resulting set fractions are expressed as follows: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on

Table 5: The Answer to KCS

The question is about how the teacher teaches geometry material. The teacher answered with a very far-reaching answer. He answered by explaining the meaning of whole numbers. Many answers need to match the questions. Question 6 discusses material about division to explore the teacher's technique in anticipating student answers and difficulties. There were 24 (88.89%) errors in question 6a, which discussed the teacher's technique of minimizing student errors in division material, and 25 (92.59%) errors in question 6b concerning alternative ways of calculating division without using a tiered division technique.

Category	Item	The Answers
Interview (T: Teacher)	2	T 1: "It is hard to understand the answer." T 2: "I know how to do it but am confused about how to teach it to students." T 3: "The answer is area or circumference, right?"
	6	T 1: "Multiplication and division material for students is very difficult." T 2: "I have no way to teach divisions 1-10 to students; I also find division difficult."
Interview with Lecturer		Some teachers did not master understanding mathematical concepts. In classroom learning, teachers tend to explain very basic things, so they need help representing mathematics in a relevant way. This impacts how they determine how to solve the problem. Over time, we teach in the same way. Most of the teachers answered the questions wrong. Deficiencies in the initial calculations make the teachers unable to correctly apply the tiered counting technique.
Observation results		Teacher errors occur due to a need for more understanding of the questions. Teachers' limited ability to understand the principles of geometric material and arithmetic operations impacts their determination of learning methods. Because of their limited abilities, teachers are not optimal in explaining how to teach the material.

Table 6: The Answer to Data of KCS

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Based on answers from various sources, we tried to create a teacher answer pattern based on a model adapted from Toyinbo (2018). This pattern describes the condition of KCS in the border area in Figure 3 below:

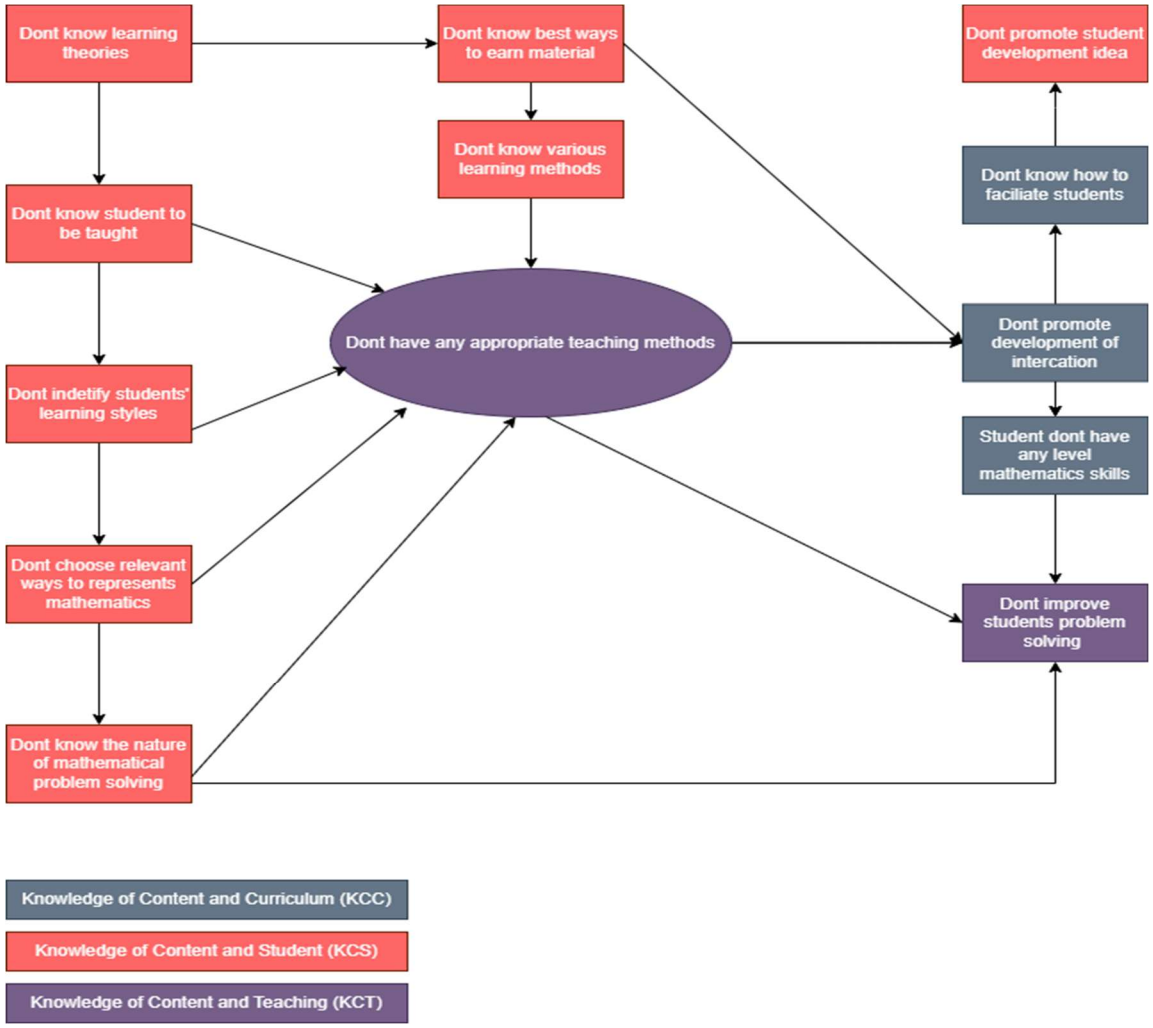


Figure 3: Pattern of relationship between KCS with KCT and KCC

Knowledge of Content and Teaching (KCT)

The problems presented are in questions 3a to 5b. Question 3a discusses algebra and the scope of algebraic material in basic education units. It was found that 19 (70.37%) teachers answered incorrectly on the meaning of algebra, and 23 (85.19%) teachers answered incorrectly to give an example of algebra in elementary school. The teachers still need to understand the concept or scope of algebra material for elementary school students. Other findings are in question 5a, about calculating integer operations, and 5b, about how to teach integer operations to students. There were 22 (81.48%) teachers who answered question 5a incorrectly, and 25 (92.59%) answered question 5b incorrectly.

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The highlight of the KCT results is question number 4. Questions 4a and 4b discuss fractions. The teachers gave various interesting answers to this question. Problem 4a is about how to divide fractions with pictures. As many as 26 (96.3%) teachers answered this question incorrectly. Question 4b is about how to teach question 4a to elementary school students, and all 27 teachers (100%) answered it incorrectly. The teacher's answer is presented in Table 7 below.

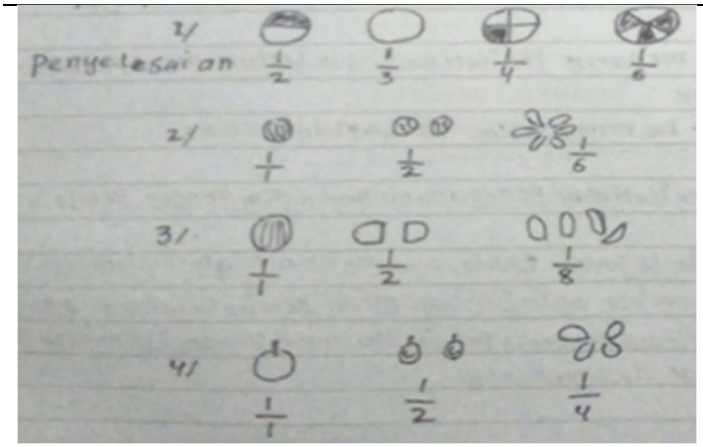
Answer	Translate
	This answer does not match the question. The depiction of fractions does not match the picture.

Table 7. The Answer to Data of KCT

The teachers' way of thinking in the sample answers is very interesting. The findings from their answers are 1) they have not been able to understand the purpose of the questions; 2) they have not been able to draw fractions according to their value; 3) they do not understand the concept of fractions; 4) they use the formula that in dividing fractions it must be reversed to multiplication; 5) they have not been able to project the distribution of fractions correctly.

Category	Item	The Answers
Interview (T: Teacher)	3	T 1: "I did not know algebra was in elementary school materials." T 2: "I just looked for the answer on Google and copied it because I did not understand." T 3: "I have never heard of the term algebra."
	4	T 1: "Fraction material is the most difficult. Making children understand is also very difficult." T 2: "Have difficulty drawing fractions. I can count but not draw; I never studied fractions with pictures." T 3: "I know what the problem means, but doing fractions by drawing is difficult. Fraction questions are difficult problems."
	5	T 1: "Do it by going into debt or going into debt with a larger figure." T 2: "The answer must have a fraction because it is a fraction question." T 3: "I still often make mistakes on this question."

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Category	Item	The Answers
Interview with Lecturer		For teachers, the most difficult material is fractions. So, it is natural that they make many mistakes. In the beginning, learning the material for drawing simple fractions was still difficult. Especially in teaching fractions, you can imagine they cannot draw 1/4 fractions how they teach these fractions. That is an illustration of their abilities in fractional material
Observation Results		The recurring problem is that the teachers need help understanding the purpose of the questions. Many answers need to be revised. Teachers felt less thorough, and the answers they counted were the questions presented. They said that they had difficulty teaching mathematics. Doing math problems for them is a big scourge. The questions take a long time. Besides that, sometimes they are not focused, so they often add questions according to their wishes.

Table 8: The Answer to Data of KCT

Teachers do not try to find out in-depth because they think this material is not essential and does not need to be taught. KCT has an important role because the teacher teaches material to students. For clarification, we compiled a KCT pattern related to KCS and KCC with an adaptation of the KCT pattern (Toyinbo, 2018), like Figure 8 below.

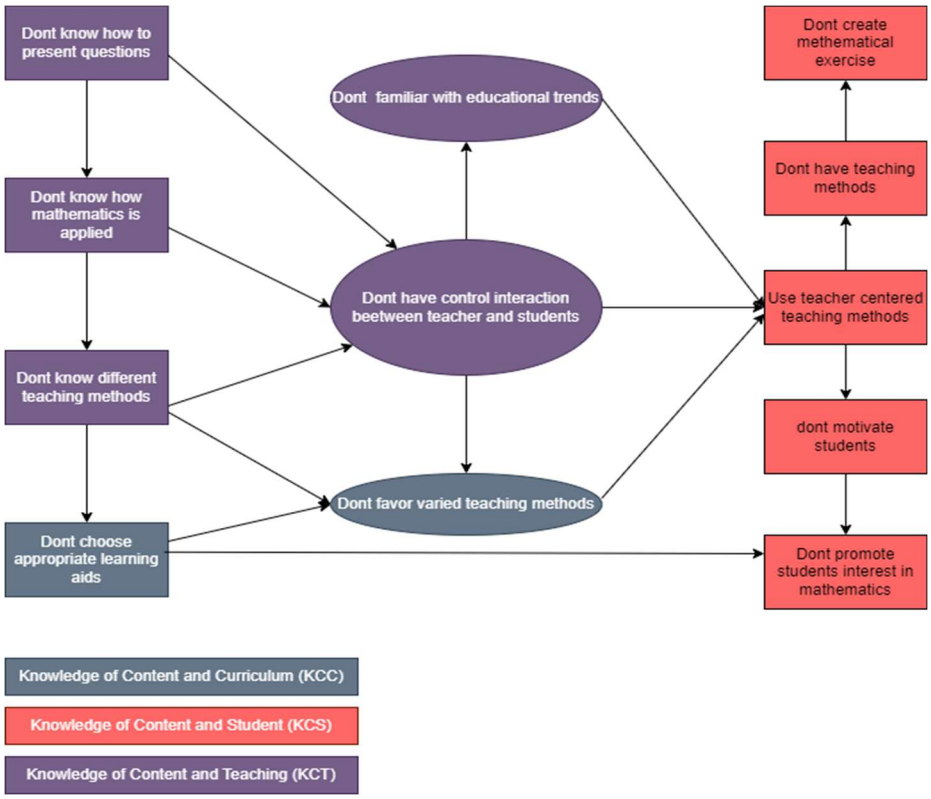


Figure 4: Pattern of relationship between KCT with KCS and KCC

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DISCUSSION

Pedagogic Content Knowledge (PCK)

The main research finding is that teachers teach based on their understanding, which results in low student learning achievement. Based on the results obtained, teachers' content and pedagogic competencies are still low, meaning low success in mathematics. These results are based on previous research, which states that if teacher content knowledge is low, mathematics learning success is low (Saran, 2018).

Another major finding is that the teachers' answers are unique and interesting, thus triggering further research to find out the teachers' thoughts in depth. Out of the 27 teachers who submitted responses, 10 teachers received a score of 0. This was because they answered questions that were not relevant to the given test items. Additionally, some teachers created their own questions instead of responding to the provided ones, which led to incorrect or unscorable answers. Besides, teachers often add their questions according to their wishes. In PCK, teachers face various challenges, which must be adapted to the situations and conditions of students and their learning environment. This challenge causes teachers in the KCC domain to find it difficult. Environmental, cultural, and social problems of students who participate in learning problems for students also take up the teacher's time.

In the KCC domain, the most prominent is technology or digital literacy. Some teachers have used learning aids such as marbles, sago caterpillars, and fruit to learn numbers, so the technology in this study is devoted to aids to help convey material. This fact shows that teachers' use of curriculum, learning technology, and teaching techniques still needs to catch up. Research (Mailizar & Fan, 2020) examines the six abilities needed for the current 4.0 Era. However, related to the conditions for teachers at the border, the six capabilities of the digital era are difficult to reach.

In the KCS domain, the findings are that teachers also have not mastered the concept properly. For example, the concept of drawing simple fractions. Most of the teachers answered incorrectly, and various unique answers emerged. Teachers need assistance in fractional material as they explain that fractional material is very difficult for themselves and their students. This finding follows research that has proven that fractions are difficult material (Agathangelou & Charalambous, 2021).

Several teachers in various regions of Indonesia have started to develop teaching quality (Rahmadi & Lavicza, 2021). However, this study's findings show that teachers are unwilling to improve students' understanding of content. Based on the teachers' answers, the way the teacher explores students' abilities in mathematics still needs to be visible. When students answer incorrectly, the teacher must try to discover the mistakes and difficulties students are experiencing.

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Based on the findings and results of this study, PCK greatly affects students' abilities. PCK plays an important role in the teacher's capacity to teach mathematics and help students learn mathematics (Golding & Batiibwe, 2020). This result illustrates that the students' math scores in the low border areas are due to the low PCK results of border teachers. This PCK plays an important role in building students' understanding of topics and methods for effective learning (D. Celik et al., 2022). However, the results of this study show that teachers in the border areas need to teach more effectively because their PCK level still needs to be higher.

Pedagogical Content Knowledge (PCK) is a critical component of teacher professionalization, particularly in the context of mathematics education. PCK is defined as the integration of content knowledge, pedagogical knowledge, and an understanding of students' learning processes. This unique blend enables educators to tailor their teaching strategies to meet the diverse needs of their students, thereby fostering improved learning outcomes (Hill et al., 2008; Ball et al., 2008). PCK is categorized into three primary domains: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). The KCS domain emphasizes the necessity for teachers to understand their students' pre-existing knowledge and learning difficulties, which is crucial for designing effective instructional strategies (Mason, 2018; Hill, Rowan, & Ball, 2005).

The KCT domain focuses on the pedagogical strategies that best convey specific content, while the KCC domain pertains to the alignment of teaching with curricular standards (Mason, 2018; Ball et al., 2008). Research indicates that teachers with a strong foundation in PCK are more adept at implementing inquiry-based and student-centered teaching methods. These educators are better equipped to adapt their instructional strategies based on their understanding of both the subject matter and their students' learning needs (Mason, 2018; Smith & Neale, 2017). Furthermore, the development of PCK is influenced by teaching experience, with studies showing that novice teachers often struggle to integrate these components effectively compared to their more experienced counterparts (Evens et al., 2018). The urgency of assessing PCK in relation to teaching effectiveness cannot be overstated. The application of PCK directly correlates with student learning outcomes, making it essential for teacher education programs to emphasize the development of PCK among future educators (Koopman et al., 2019; Smith et al., 2017). This focus on PCK not only prepares teachers to deliver content effectively but also equips them with the skills necessary to foster a supportive learning environment that acknowledges and addresses the diverse needs of their students (Donnelly & Berry, 2019).

The most influential PCK in this study is the KCT section, namely how the teacher conveys the material to students. The teacher's strategy still needs to be revised because the teacher's level of understanding still needs to be higher. The mathematics teaching techniques and strategies presented by the teacher needed to be more appropriate. Teachers who do not have content knowledge will have difficulty teaching knowledge to students (Klemer et al., 2019; Özpınar & Arslan, 2022). This is very consistent with the results of this study, which state that content knowledge is needed to match a teacher's pedagogy.

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Teachers who have the characteristics of teaching successful mathematics understand the material, can teach with the right technique, and apply discipline (M. Park & Kim, 2023). The results of this study show these successful characteristics have yet to appear in Indonesia. Teachers still need to implement disciplined learning techniques for students. The use of problem-solving techniques presented could be more appropriate and suitable. There are still limitations to how teachers think and how to present solutions to students. The lack of teacher content knowledge is the most important obstacle in this study.

Research findings that are no less interesting are that of the 77 teachers who received the questions, only 27 teachers worked on them. The 50 teachers who had been waiting to work on and were contacted one by one to submit still need to show that the commitment of teachers in this border area still needs to be higher. The quality of education is heavily affected by teaching competence, learning innovation, and teacher commitment (Asiyah et al., 2021). Many teachers do not want to work on the problem and do not provide information. Every effort was made, for example, asking the class leader to take the questions directly to the house and accompanying them to work on them. However, they refused to do it without giving any reason. The commitment of teachers who have no desire to improve is a big challenge for developing the quality of teacher resources in border areas.

Teacher Errors in Answering Questions

Various findings in this study are unique. There is a question about proving the results of dividing fractions with media images. All teachers answered the question incorrectly. This error arises because the teacher needs more content knowledge and critical thinking. Mistakes often occur when teachers need to think critically in mathematics, especially because they lack knowledge and motivation (Novitasari et al., 2024; Ridwan et al., 2022). This is due to various errors caused by the teacher's lack of mathematics knowledge.

The teachers under study stated that mathematics was difficult for them to understand and teach. This finding is in line with previous studies showing that mathematics is challenging both to learn and to teach (Copur-Gencturk et al., 2019). This follows various interview results showing that teachers struggle to learn mathematics. Various errors in calculations occur because of views on mathematics. In the previous discussion, fractions were found to be the most feared math problem by teachers and students. Fractions are considered difficult math material (Empson et al., 2020). This result is consistent with the finding that fractional questions were answered by most of the teachers wrongly, and there were various unexpected answers to the questions.

The teachers' unexpected responses indicated that their ways of thinking about mathematics, sense of mathematics, and overall understanding of mathematical concepts still need to be improved (Greenstein & Fernández, 2023; Isnani et al., 2019). The teachers' thinking about mathematics is very far from the mathematics concept. For example, when asked how to apply realistic mathematics education media (question 1b), several teachers explained topics related to the scope of learning in elementary schools rather than describing how the media could be used.

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The teachers' sense of mathematics here still needs to improve. The sense here is interpreted as how the teacher understands the content of mathematics and uses it in solving problems. They still need to work on problems correctly, for example, in algebra questions and examples of algebra in learning. Many teachers cannot provide examples of algebra material in elementary school learning. Their level of understanding when working on questions needs to be improved. Various inappropriate answers arise because they need to understand the intent of the questions properly.

Teachers find it difficult to teach geometry material because of its broad scope. Teachers need to gain more content knowledge of geometry, and this is illustrated in the geometry problem material in questions 2a and 2b. Their pedagogical content helps students learn geometry because it can reduce misconceptions and increase student learning motivation. In this discussion, the teachers must be aware of geometry material. They need help understanding the questions' meaning and the geometry material's scope. They cannot distinguish flat and geometric shapes and answer with answers far from the question. In the division material in questions 6a and 6b, the teachers experienced difficulties because they needed help to carry out the division operation. They have limitations that can only do divisions 1–10. In the context of tiered division, several teachers got the answer right. However, they needed help in teaching division techniques to students.

Distribution remains a serious challenge for students and teachers, who still require guidance in carrying out division and understanding mathematical operations related to comprehension. Students need help to divide and read comprehension operations (Fuchs & Malone, 2021). In previous research, students found it difficult, whereas in this study, the teachers found it difficult to divide numbers into hundreds. The teachers felt that they needed help to carry out the distribution process. The tool that is often used is a calculator. In every discussion of the problem, there is an interesting condition in the teachers. Many teachers have not yet memorized the basic multiplication facts from 1 to 10. This lack of automatic recall makes it difficult for them to carry out division operations, as division relies heavily on the understanding and quick retrieval of multiplication relationships.

The knowledge of mathematics content of the teachers at the border still has various problems in the field. The basic abilities that teachers should have become the main problem for them. This result is in line with previous research by Lo (2020), who found that only 6% of teachers correctly answered the mathematics questions in his reseach. The questions presented are about solving problems and presenting solutions. This finding reflects that the teachers' content knowledge and problem-solving skills still need improvement, which significantly impacts the quality of educational resources, particularly in border areas.

The findings of the study on Pedagogical Content Knowledge (PCK) underscore its critical importance in shaping effective teaching practices and, consequently, student learning outcomes. The evidence suggests that teachers with higher levels of PCK are more capable of implementing inquiry-based and student-centered teaching strategies, which are essential for fostering deeper understanding and engagement among students (Krijan et al., 2018). This is particularly relevant in

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mathematics and science education, where the complexity of concepts necessitates a nuanced approach to teaching that considers both the content and the learners' cognitive processes (S. Park et al., 2010). The study's findings highlight that strong PCK enables teachers to not only convey content effectively but also to adapt their teaching methods based on their understanding of students' prior knowledge and learning difficulties. This adaptability is crucial in addressing the diverse needs of students, particularly in heterogeneous classrooms where learners may have varying levels of readiness and understanding.

For instance, teachers who possess a robust understanding of how students' reason through mathematical concepts can better anticipate misconceptions and tailor their instructional strategies, accordingly, leading to improved student outcomes. Moreover, the implications of the study extend beyond individual classrooms. By demonstrating the link between PCK and student achievement, the findings advocate for the integration of PCK development into teacher education programs. This integration is essential for preparing future educators to meet the demands of modern classrooms and to implement effective teaching practices that promote student learning (Sun et al., 2024). Continuous professional development focused on enhancing PCK can lead to sustained improvements in teaching quality and student performance over time.

CONCLUSION

Teachers' mastery of mathematics teaching strategies in elementary schools is viewed in terms of PCK with three domains: KCS, KCT, and KCC. In the KCS domain, the identification and anticipation of misconceptions in students' answers are low. Various answers given by teachers do not follow the questions and concepts given. In the KCT domain, namely regarding teacher strategies, methods, techniques, and pedagogy in teaching, the teachers are in a low category. The indicator is that the teacher cannot teach students learning strategies for mastering mathematics. The teachers do not have self-concepts and solutions to bridge students in several materials, including algebra, fractions, and number operations. In the KCC domain, teachers are in the lowest category regarding classroom management and the use of technology in learning activities. However, in this domain, based on their experience, the teachers can apply objects around them as learning media.

The outcome of the research reflects the circumstances of elementary school teachers in border areas. The teachers' PCK is significantly influenced by their mentality, environmental conditions, commitment, and ability. The teachers are still unable to accurately interpret and comprehend the queries. Their capacity to interpret inquiries is quite unanticipated, as they provide a variety of responses that are inconsistent with the context of the inquiries. The knowledge and understanding of the mathematical concepts of teachers at the border are still mostly inaccurate. Their low content knowledge makes their pedagogy in determining mathematics teaching strategies inappropriate. Human resource development in border areas is urgently needed for elementary school teachers. Further research on human development is urgently needed, but what is no less important is to explore in depth the reasons and main factors for fatal errors in teachers' thinking and problem-solving.

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To enhance student achievement in mathematics, improving teachers' Pedagogical Content Knowledge (PCK) is crucial for strengthening classroom instruction. Professional development programs should focus on refining teachers' conceptual understanding and pedagogical strategies to ensure effective mathematics teaching. Training should include workshops on teaching mathematics strategies to help teachers select and implement appropriate instructional methods, improving their Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS). Additionally, curriculum-based training is essential to align classroom content with educational standards, ensuring students receive structured and coherent instruction. Integrating technology can further support conceptual understanding through interactive digital tools, enhancing engagement with mathematical concepts. By closing these knowledge gaps, teachers can better facilitate student-centered learning, address misconceptions, and create a more effective classroom environment for mathematics instruction. Future research should explore how strengthening PCK influences student learning outcomes, particularly in problem-solving, conceptual mastery, and engagement in mathematical reasoning.

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APPENDIX

Test Instrument

A six-question open-ended test assessed three Pedagogical Content Knowledge (PCK) domains:

a. KCC (1 question): Curriculum knowledge and learning media.
b. KCS (2 questions): Student difficulties and teaching approaches.
c. KCT (3 questions): Teaching strategies, problem-solving, and algebra concepts.

Each correct response earned 5 points, with a total possible score of 60 points.

Category	Item	Questions
KCC	1a	What is the learning media for fraction operation?
	1b	How do you apply realistic mathematics?
KCS	2a	How important is it to teach geometry concepts by linking contextual problems?
	2b	Write down one way to teach geometry to grade 6 students!

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KCT	3a	Explain whether algebraic material is in elementary school!
	3b	Give one example of algebraic material for lower-grade students!
KCT	4a	$\frac{1}{3} : \frac{1}{6} = 2$ Prove with a picture!
	4b	Make a simple plan on how to teach the material!
KCT	5a	(-5) - (-9) = 4 Explain the correct solution to teach the problem!
	5b	Should the teacher use media for this solution? Explain!
KCS	6a	125 : 5 = ... What is the concept of teaching division operations to minimize student errors?
	6b	Write down an alternative to teaching this material!

Interview Instrument

Interviews explored teachers' decision-making in mathematics instruction:

a. KCS (2 questions): Student learning challenges and adaptation strategies.
b. KCT (4 questions): Differentiated instruction, motivation, and problem-solving.
c. KCC (1 question): Use of technology in teaching.

Category	Domain
(KCS)	More studies concerning learning difficulties in mathematics More skills concerned with teaching students at different levels
(KCT)	More courses about didactic mathematics More courses about how to differentiate teaching More skills to motivate students in mathematics More studies about problem-solving
(KCC)	More courses about using technology in teaching mathematics

Table 2: The instrument PCK Domain

Observation Instrument

a. Mathematical ability (clarity and accuracy of explanations).
b. Teaching methods (strategies and instructional effectiveness).
c. Classroom management (organization and student engagement).

A structured rubric ensured validity, and inter-rater reliability was applied.
This appendix provides a summary of the study's data collection process.

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Bridging Mathematics and Financial Literacy Through Flipped Classroom

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Abstract: Integrating financial literacy into mathematics education is essential for helping students make informed financial decisions. However, many teachers struggle to design lessons that meaningfully connect mathematical concepts with financial contexts, often due to limited content and pedagogical knowledge. This study addresses this issue by developing a flipped classroom–based model, the Math Fina-Flip Classroom, to enhance teachers' professionalism in creating financial literacy–integrated activities. Using a design research approach, the study followed three phases: preliminary, prototyping, and assessment. The model incorporates key features—content, coherence, active learning, duration, collective participation, and ICT integration—that structure a flipped learning environment. Teachers engage across three stages: before class, during class, and after class. Findings show improvements in teachers' content knowledge and pedagogical content knowledge, including a 19% increase in understanding and applying financial contexts in mathematics lessons. Collaborative learning communities further support professional growth. However, limited time in the "during class" phase indicates a need for extended reflective and practice-based activities. This study provides a practical framework for integrating financial literacy into mathematics education and responds to professional development challenges faced by Indonesian teachers. Future research should refine and expand the model for broader implementation and long-term sustainability.

Keywords: flipped classroom, financial literacy, teacher professionalism, blended learning, mathematics education.

INTRODUCTION

Nowadays, several countries emphasize the importance of financial education in school curricula, particularly in the field of mathematics, such as Australia, Canada, Denmark, Estonia, Finland, Japan, Korea, Norway, and the United Kingdom (OECD, 2019). Financial literacy is a complex

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and multidimensional concept involving various elements and disciplines. In assessing 15-year-old students, the OECD simplified this concept by selecting the most relevant components. Financial literacy is defined as the knowledge, skills, motivation, and confidence to apply financial understanding in making effective decisions, improving personal and societal financial well-being, and participating fully in economic life (OECD, 2019). As an interdisciplinary field, financial literacy is closely related to other competencies and domains, such as reading, numeracy, mathematical literacy, social studies, and civic education. Recently, the OECD expanded its study of financial literacy through the 2021 framework, highlighting the intersection between financial literacy and mathematical literacy. This intersection includes basic arithmetic tasks that require students to apply their knowledge in everyday financial contexts.

Mathematics tasks with financial contexts are not only used to provide a relevant context for learning mathematics. At the same time, students also gain knowledge and attitude about finance. Sawatzki provides an alternative for teachers to use financial literacy contexts in mathematics lessons by using problems that students can imagine (Sawatzki, 2017; Sawatzki & Sullivan, 2018). Preliminary research indicates that teachers deal with challenges when designing and developing lesson plans. Considering that mathematical knowledge starts from problems, the teacher's ability to build problems is very important (Leavy & Hourigan, 2020). When teaching mathematical concepts using a financial context, it is important for teachers to base the context on students' daily activities to ensure it is meaningful and not merely superficial (Savard, 2022; Savard & Polotskaia, 2017). Learning that involves such contexts may require a more careful approach or instructional design to ensure its effectiveness (Burgos et al., 2024). In doing so, students will make mistakes in making financial decisions, for example not knowing the application of percentage discounts (Sagita et al., 2023; Savard, 2022).

Teacher Professional Development (TPD) programs that focus on financial literacy can significantly improve teachers' self-efficacy and their capacity to integrate financial literacy into classroom lessons (Blue & Grootenboer, 2019). Unfortunately, these programs are often not widely accessible, thereby leading to inequality in the level of preparation among educators (Jorgensen & Savla, 2010). For example, short workshops and seminars are not enough to change teachers' teaching practices. It is optimal for educators to engage in collaborative learning, sharing, collaboration, and support in order to enhance the quality of their instruction. The establishment of professional learning communities is one way to achieve this goal.

TPD is designed to enhance the competence and expertise of teachers, with the goal of maximizing their positive influence. Currently, there is a growing acknowledgment that teacher learning is a continuous and integrated process, which is shaped by the environment in which professional development occurs (Guskey, 2000; Timperley et al., 2008). This suggests that there is no universally-effective TPD (Teacher Professional Development) approach that can be applied in every country, school, or classroom. Instead, professional development efforts should develop to the particular content, process, and context (Guskey, 2000).

Prior studies in the general professional development process developed a framework or key features to effective TPD. Desimone and Garet outlined five core features of effective professional

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development: content focus, active learning, coherence, duration, and collective participation (Desimone & Garet, 2015). Building on this general framework, Compen et al. adapted and refined these components to develop a specialized TPD model tailored to financial education, which serves as an important theoretical foundation for this study (Compen et al., 2019). In line with Burn, there are six characteristics that a TPD learning environment should have, including developing teachers' competencies, based on an understanding of teachers' needs and their work environment, focusing on deepening teachers' content knowledge and pedagogical skills, solving important problems related to the teaching and learning process, and building a professional learning community (Burn et al., 2023).

TPD frameworks that have been generated from previous research do not specifically include the use of technology as a supporting component, indicating a potential area that is less explored. Meanwhile, some argue that such technologies have the potential to transform classrooms (Goos et al., 2020). Online learning, e-learning, and blended learning in TPD mathematics education are relatively new areas of research (Goos et al., 2020). Moreover, the flipped classroom model as one of blended learning approach is an innovative approach to teaching and learning that involves a pedagogical shift from the traditional academic process. In this model, students are introduced to the subjects outside the physical confines of the classroom, before engaging in face-to-face instruction (Lopes & Soares, 2018).

Potential future studies are recommended to examine the implementation of various TPD initiatives using the key features of the prior literature (Compen et al., 2019; Desimone & Garet, 2015). Our objective is to create a model for mathematics teacher professionalism in the development of mathematics activities that incorporate financial literacy concepts. This model encompasses various essential elements that can enhance teachers' expertise in both content and pedagogy. These elements include the provision of comprehensive content, the promotion of active learning, the establishment of coherence, the consideration of duration, and the encouragement of collective participation. Furthermore, the incorporation of technology is essential to enable universal access for all educators.

The importance of this research is that teachers, as facilitators, must be able to design appropriate learning for students according to the characteristics of their students. However, when teachers lack sufficient knowledge of mathematics and pedagogical skills, particularly in integrating financial literacy, they may utilize financial contexts merely as a backdrop, without fully engaging with the meaning or relevance of the content in the learning process. This can lead to student errors in applying mathematics to solve financial problems. Furthermore, it is important for teachers to create effective and efficient learning. This research further applied a framework for developing teacher professionalism by Compen et al., (2019) and Desimone & Garet (2015), designed with blended learning and optimizing the learning community at school. Based on empirical data and previous research studies, this study formulated a research question: 'What are the flipped classroom stages of a mathematics teacher professionalism training program in designing student activities with financial literacy content in mathematics learning?'.

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METHOD

Research Procedure

The design research procedure was adopted to address the research questions formulated in this study. Design research aims to develop high-quality interventions through an iterative cycle of design, implementation, and refinement (Plomp, 2013). A high-quality intervention can be achieved when the research systematically follows key stages, namely preliminary research, a development or prototyping phase, and an assessment phase (Plomp & Nieveen, 2010; Plomp, 2013). The preliminary research stage involved analyzing the problem and conducting a comprehensive literature review covering the domains of financial literacy, mathematics teacher professional development, mathematics content for first-grade junior high school, and the identification of mathematics learning objectives relevant to financial literacy. This literature review was not limited to textual sources but was enriched through the collection of additional supporting information, including interviews and discussions with mathematics teachers. The outcomes of this stage provided the theoretical underpinnings necessary for designing the core components of a financial literacy learning environment model. Building upon the preliminary research, the prototyping phase focused on developing a prototype of a mathematics teacher professional development program. This phase involved iterative cycles of testing and refinement across different groups of participants to enhance the quality and relevance of the intervention. The final stage, the assessment phase, aimed to systematically evaluate the extent to which the developed intervention met the specified design criteria and objectives.

Formative Evaluation

The intervention process emphasized the importance of formative evaluation as a key activity for gradually developing and refining the intervention (Plomp, 2013). To support this process, three main criteria were used to assess the quality of the intervention: validity, practicality, and effectiveness (Ploom & Nieveen, 2010). The formative evaluation in this study followed several stages based on Tessmer (2013), including self-evaluation, expert review, one-to-one evaluation, small group evaluation, and field testing, as illustrated in Figure 1.

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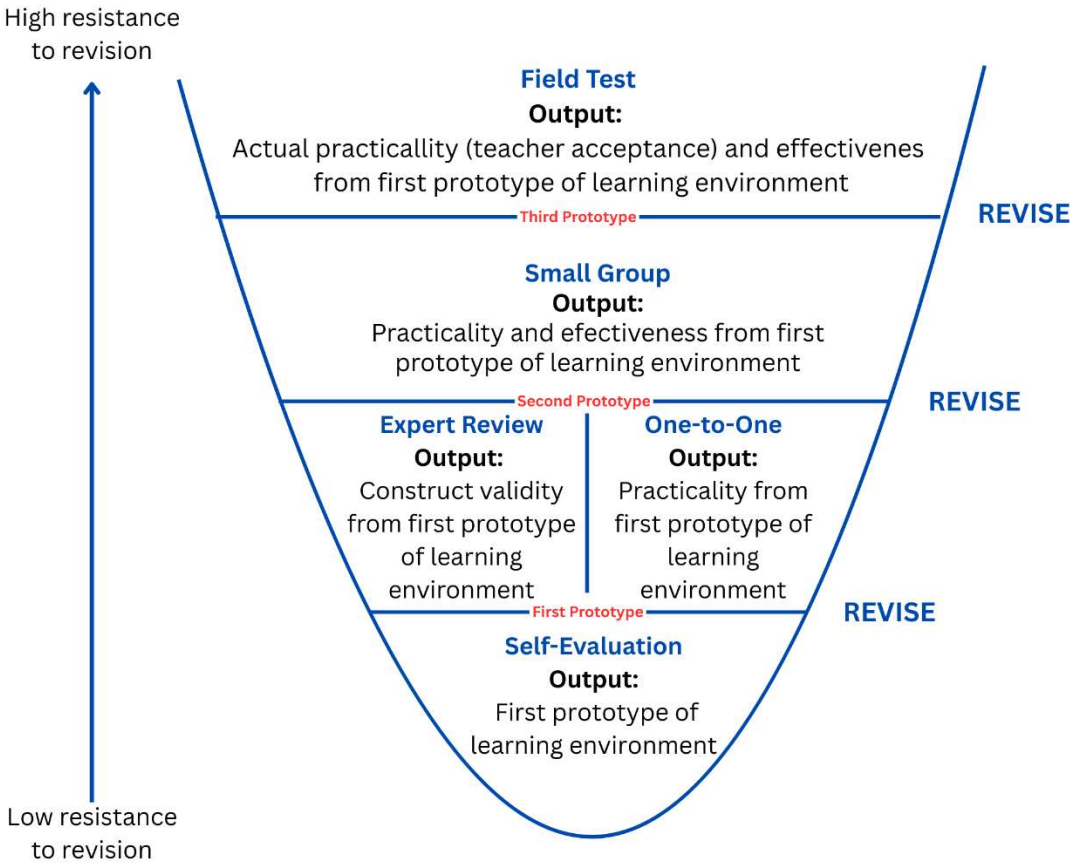


Figure 1: Layers of Tessmer's formative evaluation (taken from Plomp, 2013)

The self-evaluation stage involved the researcher independently reviewing and revising the draft of the learning environment model, which consisted of the financial literacy content guide, training module, Learning Management System (LMS), and the sequence of activities within the teacher professional development program. The product generated at this stage was referred to as the first prototype.

The second layer of formative evaluation consisted of one-to-one testing and the construct validation of the first prototype. The one-to-one phase aimed to assess the practicality of the initial draft of the learning environment model. Data were collected through observations, focusing on the practicality criteria of clarity, ease of use, and the appropriateness of the sequence of activities for supporting teachers' pedagogical mastery. Comments and suggestions gathered during the expert review and one-to-one evaluations were used to revise the first prototype.

Simultaneously, expert reviews were conducted to evaluate the content, construction, and language aspects of the components of the learning environment model. The experts involved included university lecturers specializing in financial literacy content, university lecturers in realistic

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mathematics education, widyaiswara (professional trainers) specializing in number content, learning development experts, and teacher learning community representatives. The results from both the one-to-one testing and the expert review served as the basis for revising the first prototype.

Subsequently, the second prototype was tested in a small group trial involving a limited number of participants. If the intervention at this stage could be implemented smoothly and produced the expected outcomes, the second prototype would be considered to have achieved an adequate level of practicality. The second prototype was then revised based on the findings from the small group trial, resulting in the third prototype. Finally, a field trial was conducted to evaluate the effectiveness of the final prototype of the developed learning environment model.

The evaluation criteria used to measure effectiveness are based on Guskey's (2000) five levels of professional development evaluation. The first level, *participants' reaction*, assesses teachers' satisfaction with the program through post-activity surveys. The second level, *participants' learning*, measures teachers' acquisition of knowledge using worksheets and their responses to content-related questions in mathematics and financial literacy. The third level, *organizational support and change*, evaluates the commitment of operational management through interviews to determine institutional backing for sustaining the model. The fourth level, *participants' use of new knowledge and skills*, examines how teachers implement financial literacy within mathematics instruction, assessed through classroom observations and supporting documentation such as photos and videos. The fifth level, *student learning outcomes*, determines whether students achieve the intended learning objectives by analyzing the results of assessments administered after instructional activities. Collectively, these five levels provide a comprehensive framework for evaluating both the process and impact of the implemented professional development model in this study.

As shown in Table 1, the stages of design research are systematically connected to the three main quality criteria for evaluating the developed intervention.

Phase	Criteria	Short description of activities
Preliminary	The primary emphasis was placed on content validity through the researcher's own evaluation	This stage involved conducting a literature review and developing an initial design of the intervention.
Prototyping	The focus at this stage was on construct validity and practicality, while the effectiveness of the prototype also began to be considered.	The prototype was developed for limited testing, validation, and refinement based on the results of formative evaluations. The initial prototype underwent formative evaluation through expert reviews and trials to assess its construct validity and the expected level of practicality.
Assessment	The focus was on assessing practicality and effectiveness.	Practicality was evaluated by determining whether the target users were able and willing to use the intervention. The assessment also examined the extent to which the intervention was effective.

Table 1. Research Phases and Intervention Criteria

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Participants

Participants were selected using a probability sampling technique, namely random sampling. As defined by Wellington (2015), random sampling is a technique in which each member of the population has a known and generally equal probability of being selected. In this study, the target population comprised 58 public and private junior high schools that had implemented the Merdeka Curriculum and exhibited similar characteristics.

From this population, participants were involved in three stages of formative evaluation. In the individual trial phase, two mathematics teachers from two different schools were engaged to review and provide feedback on first prototype of the learning environment model. This was followed by a small group trial, which involved three mathematics teachers from three different schools who tested second prototype, developed based on findings from the previous phase. Lastly, the final prototype was evaluated during a field trial involving fifteen mathematics teachers from fifteen different schools.

RESULT

This study resulted in the development of a learning environment model designed to improve mathematics teacher professionalism, hereafter referred to as the Mathematics in Financial-Flipped Classroom (Math Fina-Flip Classroom). The development of this learning environment was carried out through three phases: preliminary, prototyping, and assessment.

Preliminary: First Prototype

The learning environment design created in this study adopted one of the blended learning implementation models proposed by the Christensen Institute, namely a flipped classroom with the Before-During-After stages. In this case, the stages of improving teacher professionalism were carried out synchronously and asynchronously. The workshop methods were applied at the stages of the learning environment model being developed. Figure 2 below presents the concept of implementing the Math Fina-Flip Classroom with the stages before-during-after class in the in-class and out-of-class environments.

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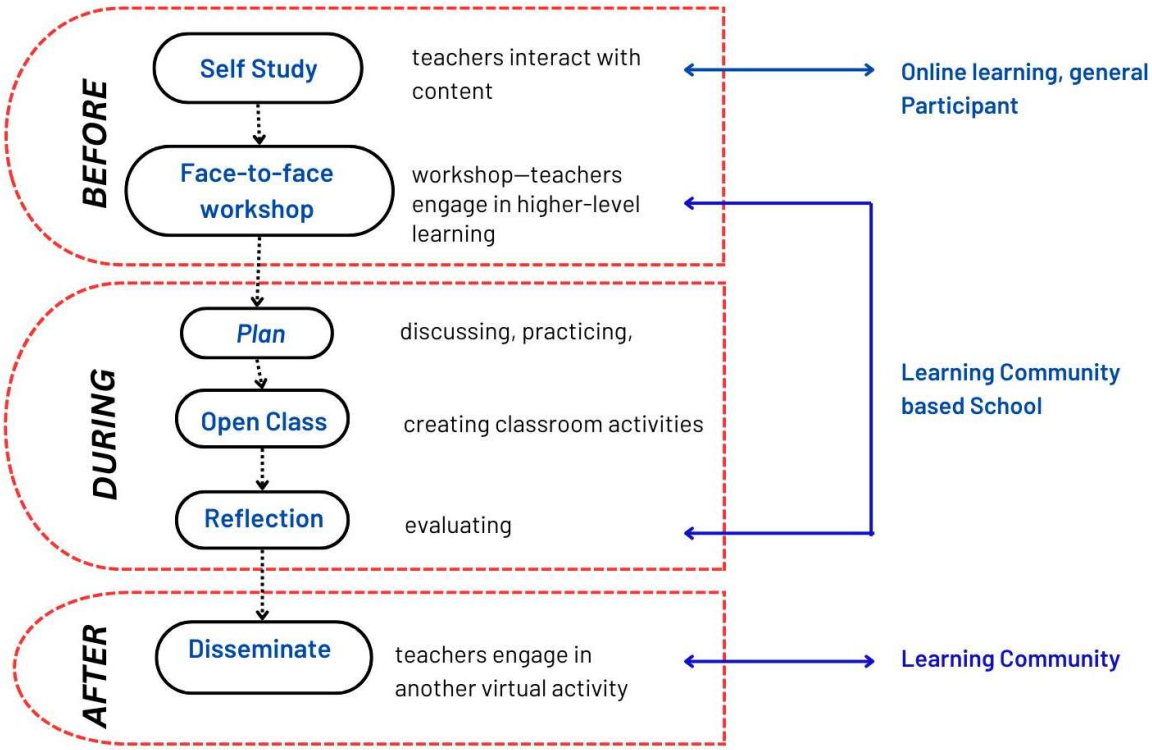


Figure 2: First Prototype Design Math-Fina Flipped Classroom Learning Environment

The duration of the implementation planned is 30 credits or equal to 25 hours. In addition, the ratio between total online and offline activities credit is 17 online credit (65.38%) and 9 offline credit (34.62%). This total credit that 30% to 70% blended learning interaction on the content is carried out online.

Prototyping: Second Prototype

The prototyping stage consists of five formative evaluation activities: self-evaluation, expert review, one-to-one evaluation, small-group evaluation, and a field test. Participant satisfaction and learning are addressed in the field test and subsequent assessment phase.

Self-Evaluation. At this stage, researchers examined the completion of six main components in the learning environment design. First, in terms of content, the development of a relevant financial literacy and mathematics learning module based on financial literacy was evaluated. Second, active learning was supported through the design of a teacher worksheet, which utilized a backward design approach to foster engagement. The third component, coherence, involved ensuring that the model aligned with the latest education policies, such as the Independent Curriculum. Regarding duration, the implementation of the flipped classroom model was set to last for 32 credits, equating

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to 26.7 hours. The fifth component, collective participation, focused on involving teaching communities in designing learning activities that reflected students' characteristics. Finally, the technology component was assessed by evaluating the LMS, which included features such as pretests, posttests, modules, and evaluation tools.

Evaluation was conducted on the completion of the design component that has been developed. The evaluation result gives significant input, such as the need for adding effective activities supporting the achievement of learning objectives in the module. In addition, features on the LMS need to be improved to make it more user-friendly and able to support optimal offline learning process. These improvements include the addition of guidelines of using LMS, integration of evaluation tools, such as interactive quiz and content enrichment in order to improve the participants' involvement.

Expert Review. Validation was carried out through directed group discussion with mathematics experts, financial literacy experts, and teacher professional development. Material drafts such as the content of financial literacy numbers and learning modules were evaluated based on legality, substance, language, and presentation aspects. The evaluation of the validity of the material shows that the material developed reaches the high to very high category. However, some input from experts is the basis for improvements to improve the quality of the material, including:

a) Addition of material depth: The materials on integers and rational numbers need to be deepened to be more comprehensive and in accordance with learning needs at the junior high school level.
b) Revision of material presentation: Material presentation should be more adjusted, in order to become more relevant to Junior High School Students' characteristics, both in terms of difficulty level and learning context.
c) Improvement of language and content integration: the language used in the material needs to be improved to be more communicative and in accordance with the standard language rules. In addition, the integration of content with the theme of Social Sciences (IPS) was carried out to create more holistic and contextual learning.
d) This improvement aims to ensure that the material is not theoretically valid but also practical and effective when applied in the learning process.

One-to-One. At this stage, two teachers from the participating school were involved to test the practicality of the prototype. The teacher implements a series of activities as designed in Figure 2 above. The professional development process followed several structured stages. First, through self-study, teachers independently learned the material provided via the LMS and completed the corresponding teacher worksheets. This was followed by a face-to-face workshop, where teachers reviewed the materials that they had studied and engaged in discussions and collaborative activities. In the designing contextual problems phase, teachers developed collaborative learning tools that integrated financial literacy concepts. In the practice in class stage, teachers implemented their learning designs in the classroom setting and recorded the sessions for later reflection. Finally, during the reflection and sharing knowledge phase, teachers evaluated their classroom practices

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within their learning communities and exchanged experiences across schools to enhance collective professional growth.

This stage produces several inputs aiming to improve the prototype 1 learning environment model. Teachers were generally satisfied with the activities in the learning model that was implemented. However, suggestions are given to increase the duration of online discussions so that teachers have more time to explore the material collaboratively. Analysis of the teacher worksheet showed that the average learning achievement reached 78%, indicating that most teachers were able to understand and implement well-designed activities. Furthermore, the posttest results showed an increase in teacher understanding of 19% compared to the pretest. This reflects the effectiveness of the learning model in improving teacher abilities, especially in mastering financial literacy and the application of mathematical concepts. Several errors were identified, especially in the analysis of financial literacy and the application of mathematical concepts. These errors provide important insights for improving the prototype, including adjusting learning activities and refining materials to support deeper understanding. The proposed improvements aim to ensure that the learning model is not only effective but also able to have a significant impact on teacher competence in integrating financial literacy into mathematics teaching.

Small Group. At this stage, an 11-day small group trial was conducted on the revised version of the second prototype, which was developed following the one-to-one evaluation stage. The small group trial sample consisted of three schools, including two public schools and one private school. The small group trial stage was carried out 11 days. Figure 3 presents an illustration of the Math Fina-Flipp Classroom learning environment model.

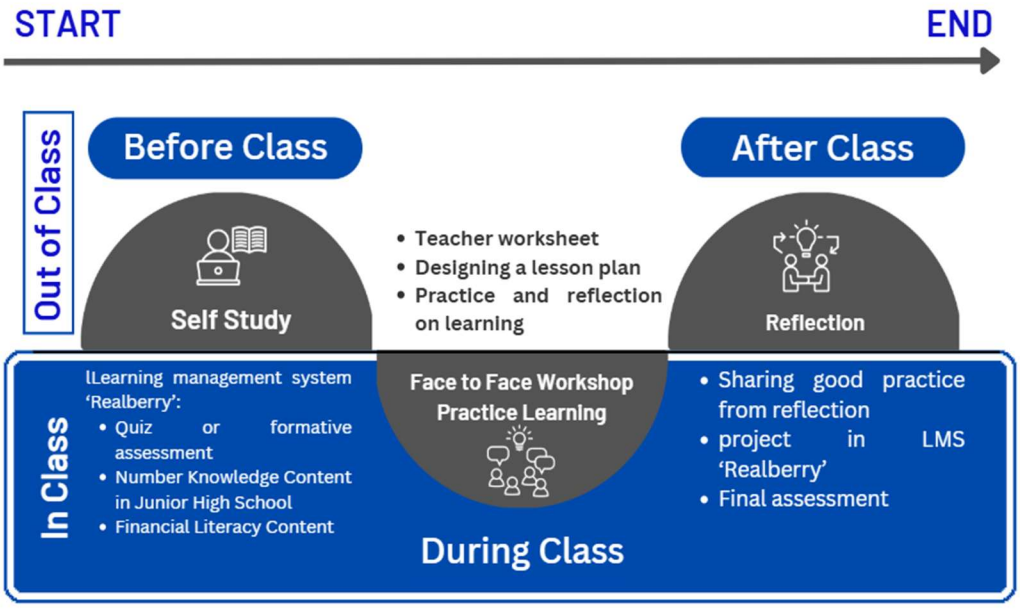


Figure 3. Second prototype learning environment model of math fina-flipp classroom

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Changes in the activity stages result in changes in the number of credits. In prototype 2, the comparison of the number of online and offline activity credits is 16 online credits (48.64%) and 19 offline credits (51.35%) as presented in Appendix 1.

Field Trial. The field trial stage constituted the final field testing of the third prototype of the Math Fina-Flip Classroom Learning Environment Model. This phase was implemented over a period from January 18 to February 2, 2024, involving fifteen teachers from seven different schools. The formative evaluation of third prototype identified a significant recommendation for further refinement, specifically concerning the insufficient duration allocated for offline activities. The offline components, conducted during in-class sessions, comprised several key phases: a face-to-face workshop, contextual problem design, classroom practice, and reflective discussions. Meanwhile, the activities during the online phase were integrated into the existing school-based professional learning communities, aligning with their regular weekly meeting schedules. In order to ensure the effectiveness and efficiency of each stage of the intervention, a minimum duration of approximately four weeks (one month) is recommended.

Findings from the field trial were analyzed through a combination of observations, interviews, and comprehension tests administered to teachers during the workshops. The results of this analysis are presented below.

Participation' satisfaction. Satisfaction among participants was gauged using a response questionnaire administered at the conclusion of the field test phase. The questionnaire results revealed that 95% of respondents answered "Yes" to 20 positive questions posed. This indicates a high level of alignment and satisfaction among teachers with the Math Fina-Flip Classroom Learning Environment Model and its five features for teacher professionalism development. However, there were some "Uncertain" responses regarding the duration aspects. The comments section provides further insights, summarized in column 1.

- *The duration of offline activities was too short, resulting in limited time for discussions.*
- *I faced difficulties uploading teaching videos due to large file sizes.*
- *Your feedback would be appreciated; perhaps extending the duration would allow more time for thinking, designing, and implementing activities in class. As teachers, we often struggle to find spare time for learning and completing tasks/worksheets during regular hours.*
- *After attending this training, I've realized that students' success isn't just about their visible results in answering worksheets but also about fostering a desire to learn, curiosity, and active participation in discussions. These changes show their enthusiasm and motivation to learn.*
- *Financial literacy is not limited to social arithmetic content; it can be applied across all subjects, especially in mathematics.*
- *This training program has broadened my knowledge in the field of education*.

Column 1: Participant Comments Summary

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The analysis of comments from column 1 leads to the conclusion that participants in the Math Fina-Flip Classroom have demonstrated an enhancement in teachers' competency to design and implement mathematics lessons integrated with financial literacy. Moreover, teachers have recognized that designing "Student Worksheets" effectively and tailored to students' characteristics can stimulate students' interest in learning, curiosity, and active engagement during discussions. Valuable feedback has been given regarding the short duration and scheduling conflicts with school activities.

Participation' learning. The first criterion for assessing learning participation is teachers' engagement during the training sessions. Their understanding of the material was evaluated through the completion of the teacher worksheet. The average scores of participants' professional development improvement using the Math Fina-Flip Classroom, as presented in Table 2 indicates that all participants were able to analyze financial literacy content within the sample teaching modules provided, select appropriate mathematics content aligned with financial literacy, set learning objectives, and determine suitable assessments to measure learning outcomes.

Teacher Worksheet Activities	Assessment Criteria	Mean	Standard Deviation
Conducting analysis of learning objectives and learning objectives direction	Accuracy in formulating objectives, analyzing the sequencing of learning objective flows, and analyzing student characteristics	77.86	4.88
Conducting analysis of financial literacy content	The accuracy and suitability of the selection of financial literacy content with mathematical content accompanied by comprehensive analysis.	67.14	5.67
Conducting analysis of learning objectives and financial literacy content on learning module	The explanation of the reasons is very clear and in accordance with the questions of each question.	73.57	14.35
Determining the learning objectives and objectives measurement indicators	Objectives and indicators are in accordance with KD/CP	91.43	2.44
Constructing assessment proof in accordance with learning objectives	1) The assessment evidence is in accordance with objectives 2) Activities are in accordance with assessment evidence and objectives	85.71	7.32
Constructing *learning plan*	1) Prior knowledge possessed by students 2) Mathematical models to be used 3) Containing students' conjectures of thought	81.43	8.02

Table 2. Recapitulation of assessment result on each teacher worksheet

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Looking at the standard deviation (SD) in Table 2. The highest SD value was observed in the activity of analyzing learning objectives and financial literacy content in the teaching modules. The high SD value suggests significant variation among teachers in this specific ability. This does not necessarily imply that teachers were unable to analyze the teaching modules effectively. However, it indicates that the explanations provided may be somewhat unclear and not entirely aligned with the questions for all objectives presented in the teaching modules.

The second criterion for evaluating learning participation is based on criteria outlined in Table 3, assessing teachers' professionalism after completing all stages of the Math Fina-Flip Classroom. Table 3 shows an 8% increase in the "correct answer" level from pretest to posttest, alongside an 85% decrease in "incorrect" answers and a 13% decrease in "wrong" answers. This indicates that participants still made mistakes in problem-solving or failed to provide explanations for their answers.

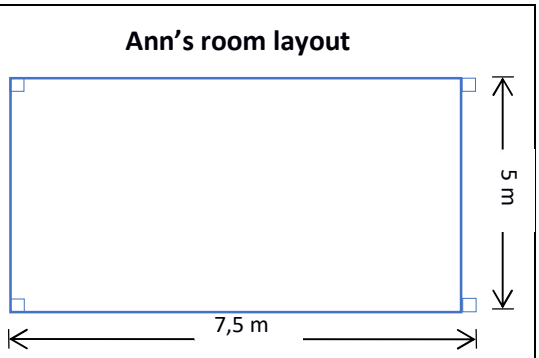
No	Question Item/Context	Correct Answer		Incomplete Answer		Incorrect Answer	
		PR	FA	PR	FA	PR	FA
1.	Rational or irrational	5	7	0	0	3	1
2.	Car selling price	3	3	0	0	5	5
3.	*Wallpaper border*	4	5	3	2	1	1
4.	Currency	8	8	0	0	0	0
5.	Definition of fraction, decimal, percentage	6	7	2	1	0	0
6.	Misconception of fraction, decimal, percentage	6	8	2	0	0	0
7.	Summation of fraction, decimal, percentage	6	6	2	2	0	0
8.	Sequencing fraction, decimal, percentage	8	8	0	0	0	0

Table 3. Result of Pretest and *final assessment*

Below is a discussion of the responses to the final assessment during the field test phase, focusing on the variety of "correct," "incorrect," and "wrong" answers provided by participants.

Ann is planning to install a continuous wallpaper border on the walls of her living room, as shown in the picture. She has three wallpaper options available for purchase:

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1. A roll priced at Rp25,000, measuring 5 meters in length (features: waterproof, comes with wall-paper adhesive). *2. A roll priced at Rp17,000, measuring 10 meters in length (features: waterproof).* *3. A roll priced at Rp10,000, measuring 2.3 meters in length (features: waterproof, comes with adhesive).* *Which wallpaper should Ann choose to maximize her budget efficiency?* 

Column 2. Question Context: Wallpaper Border (source: TEDS-M)

The accuracy of answers is assessed based on the ability to correctly understand the "wallpaper border" problem, utilize the perimeter concept to solve it, and make precise calculations for each of the three types of "wallpaper border" options, resulting in an effective decision on which wallpaper to purchase.

Aspect of MCK Show whether each of the following numbers is rational or irrational.

Column 3. Context of the issue: Rational or Irrational

Figure 5 depicts a participant's "incorrect answer" in solving the rational or irrational context problem during the post-test small group stage.

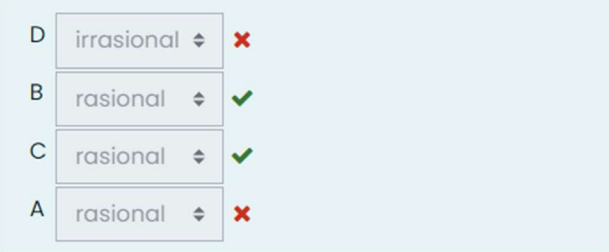


Figure 5: MYR's answers to the context of Rational or Irrational.

Following confirmation by the researcher, the error can be attributed to a misinterpretation regarding the definitions of rational and irrational numbers. One participant erroneously assumed that irrational numbers invariably bear a negative sign. Moreover, the confusion regarding the classification of pi (π) as a rational number stemmed from misconceptions about its representation, either as a fraction (22/7) or a decimal (3.14).

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Participants' Use of New Knowledge and Skills

The initial criterion for evaluating participant engagement in learning was examined through the lesson plans articulated within the hypothetical learning trajectory. The presented Table 4 below is a synopsis of the financial literacy and mathematics content formulated by the participants.

Teacher	Financial Literacy Content	Mathematics Content	Context
SBH	*Financial Awareness*	Rational Numbers	Price difference of chili
SNV	*Financial Awareness*	Rational Numbers	Price difference of chili
RKD	*Financial Awareness*	Using percentage concepts	Online shopping
TWA	*Budgeting Plan*	Using percentage concepts	Discounted prices
APR	*Financial Awareness*	Using percentage concepts	Discounted prices and online fruit trading
FAW	*Financial Awareness*	Using percentage concepts	Discounted prices and online fruit
IST	*Budgeting Plan*	Using percentage concepts	Up to 50% off: Becoming a smart consumer
MMR	*Budgeting Plan*	Rational Number arithmetic operations	Monthly shopping
NRA	*Financial Awareness*	Rational Numbers	Planning for Hajj early
RKW	*Budgeting Plan*	Ratio and comparison concepts	Profit or loss
SRU	*Budgeting Plan*	Ratio and comparison concepts	Profit or loss

Table 4. Summary of financial literacy content and mathematics content

The hypothetical learning trajectory and the design of a sample lesson plan are described as follows. At one public school in Yogyakarta, mathematics education revolves around ratios and comparisons, while financial literacy covers budgeting plans. The financial literacy content, exemplified by the scenario "market day: profit or loss?", aims to help students apply ratios and

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comparisons in financial contexts. The learning design includes two structured activities: determining product selling prices based on capital and calculating profits or losses for different beverages.

Additionally, teachers must possess knowledge of mathematics to create effective learning materials aligned with student needs. This specific design, termed knowledge of mathematics, particularly in integrating financial literacy into structured activities, showcasing teachers' abilities in lesson design that merges financial literacy with mathematics. The assessment of teachers' ability to apply acquired knowledge and skills is crucial. This level of evaluation was conducted using an observation sheet during the "practice in class" phase, this session lasted for three teaching hours (3 x 40 minutes), divided into two hours for activities based on the student worksheet and one hour for formative evaluation. Teachers began by outlining the learning objectives and presenting financial literacy scenarios that students must solve. For instance, column 4 showed RKN's introduction of ratios using the class's gender distribution aided students in completing subsequent activities.

Ratio and Proportion
Number of girls in class 7A = 18
Number of boys in class 7A = 16
Ratio of the number of boys to the number of girls = 16 : 18
Value of the ratio = 16/18

Column 4. Perception of the ratio of the number of men and women in the situation.

Figure 6 illustrates that at the 29th minute, the students had not yet realized that they could apply the concept of ratios or proportions to solve the given problem situation. To address this, RKN provided assistance by introducing a simple modeling approach for equivalent ratios using a ratio table. This was demonstrated using a scenario involving the purchase of markers.

RKN	*: "...Let's do an illustration. Suppose the price of one marker is ten thousand, what would be the price for two markers?"*
Student	*: "Twenty thousand."*
RKN	*: "And for three markers?"*
Student	*: "Thirty thousand."*
RKN	*: "How about four markers?"*
Student	*: "Forty thousand."*
RKN	*: "Good, now can you see a pattern here? One marker is ten thousand, two markers are twenty thousand, three markers are thirty thousand, and four markers are forty thousand. So, if you need to find the price for 100 markers, do you add ten repeatedly?"*
Student	*: "No, we multiply."*
RKN	*: "Exactly. So, what would be the price for 100 markers?"*
Student	*: "One million."*

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RKN : "Very good! It's the same concept as the example in student worksheet where two oranges require 50 grams of sugar. If you have 1 kg of sugar, how many oranges can you get? You can use these patterns in various situations."



Figure 6: Overview of the classroom situation during the 29th minute of the lesson

Another approach, which is more intuitive, is the unit-rate method. This method involves finding the unit value and then multiplying it by the known rate to determine an unknown value. RKN's explanation to assist students demonstrates knowledge of mathematics and pedagogical skills. This competency includes a teacher's ability to present mathematical concepts appropriately for students' cognitive levels, have a deep understanding of mathematical topics, and effectively connect different topics.

Organizational support and change. The Math Fina-Flip requires support from school management, teacher mathematics learning community, and School Supervisors specializing in Mathematics. The school's endorsement of the developed learning environment model is evident in the active involvement of the school's learning community during the stages of designing contextual problems, in-class practice, and reflection. The principal plays a role in evaluating the learning environment model to ensure its successful implementation. Additionally, the learning community

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and School Supervisors contribute by sharing knowledge to bolster effective teaching practices. These collaborative efforts contribute to the sustainability of the Math Fina-Flip Classroom.

Students' Learning Outcomes. Assessing the achievement of learning goals from the designed lesson plans involves administering tests to students. Throughout the learning process, teachers conduct formative assessments aligned with the learning objectives' criteria. Research findings indicate that students successfully meet the defined learning goals as outlined in the student worksheet and evaluation questions. This demonstrates the potential positive impact of the Math Fina-Flip Classroom, specifically tailored for teacher leaders, on students' learning outcomes.

DISCUSSION

The duration of the Math Fina-Flip Classroom learning environment model's implementation is planned for a total of 30 credits, equivalent to approximately 25 hours. This duration is aligned with the recommendations of Compen et al. (2019), who emphasize that teacher workshops are more effective when conducted for at least 14 hours. The model integrates the three phases of the flipped classroom approach—before, during, and after instruction—within a school-based professional learning community (see Figure 7).

These phases have been shown to effectively meet the ongoing needs for teacher professional development (Guskey, 2000; Timperley et al., 2008). Findings from this study further indicate that the structured implementation of these phases significantly contributed to enhancing teacher professionalism, particularly in designing student activities that integrate financial literacy content within mathematics instruction.

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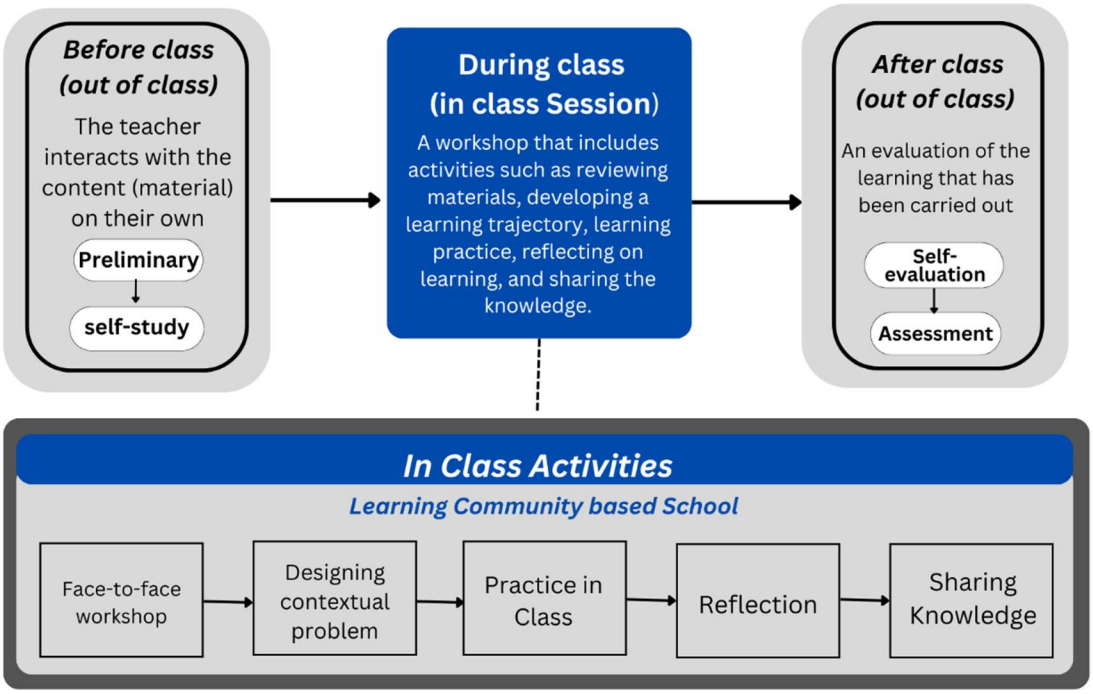


Figure 7: Math Fina-Flip Classroom Learning Environment Model

A detailed explanation of each phase, along with the underlying concepts and theoretical foundations, will be presented in the subsequent sections.

1. **Flipped Classroom Environment Out of Class: Before Class**

   The "out of class" environment was implemented through the Learning Management Systems (LMS) for a total of 13 credits or 585 minutes. In this phase, teachers engaged with content independently, allowing them to review material, take breaks, and then return to their studies (Burns, 2023). Firstly, there's the Self-study phase where teachers interacted with provided content autonomously. The aim is to establish foundational knowledge about numerical content (fractions, decimals, and percentages) and financial literacy content for secondary education. This includes learning objectives for each financial literacy content, sample activities integrating financial literacy into percentage-related math content, and examples of assessments for math content integrated with financial literacy.

   Based on observations at the end of the "out of class" phase, teachers gained basic understanding of financial literacy terms and how mathematics content integrates financial literacy elements after reviewing the provided material. This aligns with previous research emphasizing the significance of self-directed learning in teacher professional growth (Loughran, 2005), as it empowers teachers to tailor their learning to student needs and interests (Vanassche et al., 2021).

2. **Flipped Classroom Environment in Class: During Class**

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The "during class" phase of the in-class environment involves face-to-face workshop activities where teachers engaged in advanced learning with their peers. These activities were designed to empower teachers to actively design mathematics lessons infused with financial literacy, practice, evaluate, discuss, and refine their lesson plans. Through collaborative learning, sharing experiences, collaborating on curriculum development, and providing mutual support, teachers can grow and develop professionally. Engaging in a learning community to enhance teacher professionalism is an effective method for improving the quality of education (Paramita et al., 2020). Establishing a professionally run learning community can create an environment that fosters the exchange of ideas and best practices, ultimately benefiting the student learning experience.

The "during class" phase spanned 19 credits (with 1 credit equivalent to 45 minutes) and comprised five stages within the school-level learning community: face-to-face workshops, designing contextual problems, in-class practice, reflection, and knowledge sharing. During the face-to-face workshops, teachers reviewed the materials covered during self-study through discussions and collaborative lesson design activities facilitated by the learning community. This activity, grounded in methods suggested by Burns (2023) and Van Campenhout (2015), enables teachers to collaboratively apply the theories they have learned. Teachers developed draft lesson plans using a teacher worksheet designed according to backward design principles, which included prompting questions to guide them in setting learning objectives, identifying assessment evidence, and designing learning activities. Workshops have consistently proven to be an effective method for enhancing teachers' abilities to design instruction that incorporates financial literacy (Burns, 2023).

Following the workshops, the stage of designing contextual problems aimed to assist teachers in developing lesson plans that align with predetermined learning objectives and are tailored to students' characteristics. Within the school's learning community, teachers collaboratively designed mathematics lessons that integrated financial literacy, creating a hypothetical learning trajectory based on the teacher worksheet.

The practice in class stage involved teachers implementing their developed lesson plans with their learning community peers. This aligns with research findings indicating that learning community-based professionalism is most effective when it includes opportunities for practical application (Sims & Fletcher-Wood, 2021). During this stage, teachers executed their lesson plans while their peers observed and recorded the learning process for subsequent reflection.

The conversation transcript during classroom implementation revealed that students were able to determine an unknown value (the price of markers) by using a method called cross-multiplication. This method involves multiplying the quantity of markers by their price; however, cross-multiplication is considered relatively abstract and not very intuitive. According to Van de Walle et al. (2021), it is typically introduced only after students have successfully solved proportional problems using more accessible reasoning strategies and visual aids, such as tape diagrams, double number lines, or ratio tables. Another, more intuitive approach is the unit-rate

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method, which involves finding the unit value and then multiplying it by the known rate to determine an unknown quantity (Van de Walle et al., 2021).

Following the practice stage, the reflection stage was conducted through collaborative online activities within the school's learning community forum, where teachers collectively reviewed and discussed the learning activities they had implemented. Finally, the knowledge sharing stage was carried out offline, during which teachers evaluated and reflected on their instructional practices, shared challenges faced, and discussed feedback from students. This ongoing exchange promoted continuous improvement in teaching practices and strengthened the professional learning community.

3. **Flipped Classroom Environment Out of Class: After Class**

The flipped classroom learning environment wrapped up with activities on the LMS. In this "after class" setting, teachers tackled a final assessment comprising questions on mathematics content knowledge, financial literacy in mathematics content knowledge, understanding of student mathematical thinking, and familiarity with the curriculum. The final assessment phase spanned 1 credit, during which teachers worked through eight questions encompassing content knowledge, including mathematics content knowledge, understanding of student mathematical thinking, and familiarity with the curriculum.

This research underscores the importance of enhancing teachers' ability to design mathematics lessons infused with financial literacy, tailored to their students' characteristics. Aligned with prior findings, various factors, including professional development, supplemental curriculum materials, and teacher beliefs, have been identified as influential in shaping the quality of teaching practices (Sie & Agyei, 2024). Inadequate content knowledge and pedagogical content knowledge in financial literacy can lead teachers to employ financial contexts without meaningful connections, potentially causing students to struggle in applying mathematics to financial problem-solving. Therefore, effective and efficient learning environments need to be created by teachers. This research contributes a learning environment model, employing a framework for teacher professionalism development through blended learning and optimizing the school's learning community.

The Math Fina-Flip Classroom Learning Environment model aims to elevate the professionalism of mathematics teachers in financial literacy-infused number content. Unlike the existing teacher professionalism improvement programs in Indonesia, this model focuses on financial literacy-infused number content, facilitated through blended learning and involving the school's learning community. However, integrating financial literacy into mathematics education poses challenges. Nevertheless, interventions through the Math Fina-Flip Classroom Learning Environment model show promising effects, measured using Guskey's five levels. This aligns with Blue & Grootenboer's (2017) view that professional development programs emphasizing financial literacy

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can significantly enhance teachers' confidence and skills in integrating these topics into their curriculum.

CONCLUSIONS

This research emphasizes the need to improve teachers' ability to design math lessons that integrate financial literacy tailored to students' characteristics. Without strong content knowledge and pedagogical content knowledge in financial literacy, teachers may use financial contexts without meaningful connections, leading to student errors in applying math to financial problem-solving. Therefore, effective learning environments must be created. This study developed a learning environment model using a framework for teacher professionalism, designed with blended learning and optimizing school communities. The Math Fina-Flip Classroom enhances math teachers' professionalism in financial literacy-infused content. Unlike the existing programs, it focuses on financial literacy in math through blended learning involving school communities, though introducing financial literacy in math education is challenging.

Findings from discussions and evaluations with Math teachers in Yogyakarta City reveal several insights. First, enhancing professionalism with specific content positively impacts knowledge of mathematics and pedagogical skills, particularly in integrating financial literacy. Second, the model includes six professionalism features: content, program coherence, ownership, active learning, duration, participant engagement, and ICT integration. Third, the model follows a flipped classroom approach: "before, during, after" phases. Collaboration within learning communities supports lesson planning, observation, reflection, and sharing best practices. While the research follows procedures, weaknesses such as inadequate duration in the "during" phase were noted. This may impact program effectiveness, as participants may lack time to fully understand and apply materials. The model's continuous learning cycle could be improved to enhance teacher professionalism further.

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APPENDIX

Stage (Syntax)	Material	Activities	Objectives	Credit
Preliminary				
Preliminary	Program Design	Initial meeting to coordinate the activities	Delivering the program flow and schedule making	2 credits
Environment of Flipped *Classroom*: *Before Class (out of class)*				
Quiz	Materials that are in accordance with the training	Teacher work on questions consisting of *mathematics content knowledge*, *financial in mathematic content knowledge*, *knowledge of student mathematical thinking*, dan *knowledge of the curriculum*	Knowing the participants' initial knowledge about teachers' knowledge consisting of *mathematics content knowledge*, *financial in mathematic content knowledge*, *knowledge of student mathematical thinking*, dan *knowledge of the curriculum*	1 credit
Self-study	▪ Financial literacy guidelines ▪ Number book ▪ Learning module example	Teacher interacts with the content (material) that has been provided independently	▪ Teacher receives initial knowledge concerning the design of number content learning (decimal and percentage), financial literacy content, and learning module example of financial literacy-contained learning module for VII Class	4 credits
	▪ Teacher Activities Sheet -1 (LK-1) * ▪ Teacher Activities Sheet -2 (LK-2) * ▪ Teacher Activities Sheet -3 (LK-3) *	Teacher interacts with the Teacher Activities Sheet independently	▪ Teachers are able to determine learning objectives and essential questions by working on activity 1 ▪ Teachers are able to determine the achievement of number content learning competencies, financial literacy learning competency achievements, and are able to analyze the appropriate financial literacy content ▪ Teachers are able to formulate indicators of learning competency achievement.	8 credits

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Stage (Syntax)	Material	Activities	Objectives	Credit
Environment of Flipped *Classroom*: *During Class (in class)*				
Face-to-face workshop	Lesson plan draft	*Face-to-face* workshops that contain review activities of materials that have been studied by teachers in self-study through presentations of materials and discussions in order to develop learning designs. This activity is divided into two meetings, namely online and offline.	Teachers are able to make lesson plan draft based on predetermined learning objectives.	7 credits (3 online, 4 offline)
Designing contextual problem	Lesson plan making	Lesson plan making for number content containing financial literacy collaboratively in learning communities within schools.	Teachers are able to carry out learning activities based on learning objectives that have been determined based on the characteristics of students in each school.	4 credits
Practice in Class	Observation sheet	Learning practices from the lesson plan that has been developed. Teams in the learning community conduct classroom observations together.	Teachers implement learning practice in the classroom based on the designed plan and other teachers observe the class. The learning process was recorded as a joint reflection material.	3 credits
Reflection	Reflection sheet	Reflection based on learning practices was conducted in a learning community forum in schools.	Conducting learning reflection in a learning community	2 credits
Environment of Flipped *Classroom:* *After Class (out of class)*				
Sharing Knowledge	Follow-up plan	Online activities that aim to evaluate and discuss learning practices that have been carried out in the learning community	Finding and sharing best practice from each school	3 credits
Assessment *	Materials in accordance with the training	Participants work on final assessment questions that contain *mathematics content knowledge, financial in mathematics*	Knowing the participants' final knowledge about *mathematics content knowledge*, *financial in mathematic content knowledge*, *knowledge of student mathematical*	1 credit

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Stage (Syntax)	Material	Activities	Objectives	Credit
		content knowledge, knowledge of student mathematical thinking, and knowledge of the curriculum	*thinking*, dan *knowledge of the curriculum*	
Total Credit				**35 credits**

Appendix 1. Prototype 2 Description of activities and total credit of Math Fina-Flip Classroom Learning Environment Model

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Enhancing Communication and Connection Abilities through Differentiated Instruction and Writing to Learn in a Bilingual Context

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Abstract: This study aimed to investigate the effectiveness of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context to improve students' communication and connection abilities in mathematics. This study used mixed methods as a quasi-experimental design with a non-randomized control group. The experimental class applied DI using WtL in a bilingual context, while the control class used the expository approach. Quantitative data were analyzed using multivariate tests, while qualitative data were also analyzed. The results showed DI using WtL in a bilingual context was more effective than the expository approach in enhancing these abilities. Compared to the control class, the experimental class had a higher number of students who met each indicator of communication and connection abilities. The study indicated that the majority of students preferred and benefitted from the DI method employing WtL within a bilingual context. The observation indicated that the teacher employed three forms of differentiation: content, process, and product. The differentiation was tailored to students' learning readiness and interests, and students participated in various activities. Therefore, it can be concluded that applying DI using WtL in a bilingual context may serve as an alternative strategy to improve students' communication and connection abilities in mathematics.

Keywords: Differentiated instruction, writing to learn, bilingual, communication ability, connection ability, mathematics learning, data and diagrams.

INTRODUCTION

Mathematical literacy refers to the capacity of individuals to create, employ, and comprehend mathematics in many situations (Ojose, 2011; She et al., 2018). The context may involve both the

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application of mathematical reasoning and the utilization of facts, concepts, methods, and principles to make predictions about a phenomenon. Having a solid understanding of mathematics enables individuals to make informed and rational judgments and decisions (Haara et al., 2021). Furthermore, possessing mathematical literacy enables individuals to acknowledge the significance of mathematics in their lives and to effectively apply mathematical concepts in many scenarios they encounter (Rizki & Priatna, 2019; Sitopu, 2024). Thus, the acquisition of mathematical literacy is a crucial skill that can be cultivated at an early stage.

Although mathematical literacy is essential for students, the mathematical literacy of students in Indonesia still needs improvement. This is evidenced by the outcomes of the Programme for International Student Assessment (PISA). Indonesia has participated in PISA multiple times. PISA is an evaluation conducted by the OECD to measure competency in reading, mathematics, and science. PISA has been conducted since 2003, with the most recent assessment held in 2022. Based on data published by the OECD (https://www.oecd.org/pisa/publications/), the average mathematical literacy of Indonesian students has fluctuated. The data indicate that the highest score was achieved in 2006, followed by minor decreases and increases that were not statistically significant. In 2018, the score reached only 379. Based on the 2018 data, the OECD stated that, on average, 71% of students in Indonesia still needed to reach the minimum standard in mathematics. Indonesian students struggled when faced with problems or situations requiring problem-solving abilities. One of the reasons contributing to these challenges was that the interaction between teachers and students in learning did not stimulate analytical and higher-order thinking skills. The learning tended to be the same for all students, and they did not have equal opportunities to develop knowledge that fit their needs. In 2022, the math literacy score increased by five ranks compared to PISA 2018. However, the average of students' math literacy scores dropped by 13 points from 2018. From these points, it is imperative to sustain learning enhancements that offer students the chance to cultivate knowledge according to their own needs.

Mathematical literacy encompasses both problem-solving skills and the capacity to articulate mathematical concepts with clarity and precision. Consequently, mathematical communication is essential to mathematical literacy, as it enables students to articulate ideas, elucidate mental processes, and relate mathematical concepts to real-world contexts. Research findings indicate a significant correlation between mathematical literacy and mathematical communication ability (Paroqi et al., 2020; Triana et al., 2019). Communication ability can enhance students' understanding of mathematical concepts and procedural knowledge, as well as foster problem-solving and metacognitive abilities (Paroqi et al., 2020; Triana et al., 2019). Through verbal and written communication of their ideas, students can deepen their understanding of diverse concepts and discover more effective methods for resolving mathematical issues (Santos & Semana, 2015; Sanusi Siregar et al., 2018). This procedure can enhance mathematical literacy. Furthermore, mathematical literacy involves the understanding and application of mathematical concepts, as well as the capacity to relate these concepts both internally and to other disciplines and real-world mathematical situations (Asfar et al., 2022;

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Kenedi et al., 2019). Consequently, connection ability constitutes a vital component of mathematical literacy. Numerous studies indicate that individuals with strong mathematical connection ability exhibit superior numeracy and mathematical literacy, accurately resolve numeracy-related problems, and demonstrate a deeper understanding of mathematical concepts (Septian, 2022; Son, 2022). These findings indicate that connection ability is essential for students to enhance mathematical literacy. As essential abilities that contribute to the development and enhancement of mathematical literacy, communication and connection abilities must be cultivated within mathematics education. This instruction should provide opportunities for students to construct an understanding of mathematics through meaningful experiences (Lauria, 2010; Smale-Jacobse et al., 2019).

One approach that can foster these meaningful experiences is Differentiated Instruction (DI). DI represents an extension of best educational practices. (Chamberlin & Powers, 2010; Marks et al., 2021). Why is differentiation necessary? Two philosophical frameworks can be employed to explain this. Piaget categorized intellectual development into four stages. While these four stages are consistent for all students, the rate of development within each stage varies among individuals (Kamarulzaman et al., 2022). Therefore, teachers must design learning experiences that accommodate these differences. Additionally, each student's Zone of Proximal Development (ZPD) is unique, reflecting the diversity in the distance between their actual and potential levels of ability. (Bodrova & Leong, 2015; Yohanes, 2010). Teachers also need to consider this diversity when learning mathematics.

Differentiated Instruction (DI) must also be integrated with the Writing to Learn (WtL) approach. WtL is a process in which students write on paper to explain ideas, pose questions, determine solutions, develop and articulate arguments, as well as think critically, reflect, and describe subsequent steps (Finkenstaedt-Quinn et al., 2021; Nückles et al., 2020). In mathematics education, the Writing to Learn (WtL) approach encourages students to write effectively about a comprehensive understanding of the material or the steps involved in solving mathematical problems (Graham et al., 2020; Kenney et al., 2014). This indicates that numeracy and literacy cannot be separated when assessing students' mathematical achievements. Furthermore, Writing to Learn (WtL) can serve as a means of communication for students who experience math anxiety, particularly for those who feel unable to articulate mathematical concepts orally (Powell et al., 2021).

The integration of Differentiated Instruction (DI) and Writing to Learn (WtL) in education can address students' needs, enabling them to acquire maximum knowledge through writing activities. These activities are not limited to solving mathematical problems; they can also involve presenting solutions in the form of poems, short stories, rhymes, or articles. Through Writing to Learn, teachers can more accurately assess students' knowledge, identify misconceptions, and recognize areas of difficulty. The combination of DI and WtL is expected to enhance not only students' mathematical communication and connection abilities but also their ability to write with correct grammar. To further strengthen these skills and promote more robust mathematical literacy, the learning process is also delivered in a bilingual context (Garza Ayala, 2022; Sumarno et al., 2024). In this study, the

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term 'bilingual' refers to students engaging in writing activities in either Indonesian or English. This is facilitated by tasks that students can complete using both languages. In the context of Differentiated Instruction (DI), bilingual instruction can help students develop broader skills that are relevant to global needs. It can also present students with challenges in learning mathematics (Bal, 2023; Hidayati, 2020; Kamarulzaman et al., 2022; Marks et al., 2021; Owusu-ansah et al., 2023; Owusu-Ansah & Apawu, 2022). For these reasons, the study aims to investigate the effectiveness of DI utilizing WtL in a bilingual setting to improve students' communication and connection abilities in mathematics. This study addresses the following research questions:

a. Is there a significant difference in students' communication and connection abilities between students who underwent Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context and those in the expository approach?
b. How many students from each group have met the indicators of communication and connection abilities?
c. What are the students' perceptions of the Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context?
d. How was the Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context implemented?

LITERATURE REVIEW

Differentiated Instruction (DI)

A literature review indicates a growing interest in the application of Differentiated Instruction (DI) in educational settings. The review reveals an increasing use of DI, as evidenced by the numerous research articles published on the topic (Shareefa & Moosa, 2020). A separate review on the implementation of Differentiated Instruction (DI) in mathematics education found it to be beneficial for developing mathematical skills (Kurnila et al., 2025). DI is employed across all levels of education, from early childhood to tertiary education. The review also highlights several important principles that should be applied to mathematics instruction: identifying students' learning readiness, interests, and learning profiles; differentiating content, processes, and products; ensuring the quality of tasks; and conducting continuous assessment. DI is a strategy to enhance meaningful learning experiences. It involves providing materials or tasks with varying levels of complexity, employing diverse scaffolding techniques, and organizing different groupings (Lauria, 2010; Smale-Jacobse et al., 2019).

In Indonesia, DI is integral to the ***New Paradigm Learning***, which is a crucial element of the ***Merdeka curriculum*** introduced in 2021. The concept of new paradigm learning involves providing scaffolding through structured educational support to enhance students' individuality and potential. This distinctiveness and potential help educators identify and adapt instructional methods or interactions with students. The government has initiated the implementation of DI in the educational

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process to facilitate the development of this concept. Curriculum modification, teacher training in differentiated instruction design, the provision of supportive facilities, and the implementation of DI have been launched nationwide. These processes are currently ongoing. One effect has been the emergence of enhanced literacy in mathematics learning compared to prior years. According to the 2023 report by the Ministry of Education and Culture-Ristek, the application of certain DI principles in mathematics education significantly contributed to this enhancement. Consequently, the government is progressively endorsing the integration of differentiated instruction in educational institutions. The government expects educators to implement differentiated instruction alongside other pedagogical methods, models, or strategies to enhance student engagement in learning.

Writing to Learn (WtL)

Writing to Learn (WTL) is a pedagogical approach and method that employs writing exercises to enhance students' comprehension and cultivate their knowledge and critical thinking abilities. Unlike writing for assignments, which is evaluated for clarity and precision, WtL is informal, exploratory, and often unassessed. Its primary purpose is to enable students to utilize writing as a tool to investigate concepts, elucidate thoughts, and cultivate a deeper comprehension (Graham et al., 2020; Kenney et al., 2014). Research indicates that the WtL technique significantly enhances students' learning outcomes, cognitive abilities, and creative thinking, yielding substantial benefits. Additionally, the quality of students' writing enhances their cognitive capacity (Finkenstaedt-Quinn et al., 2021; Nückles et al., 2020). WtL is an educational methodology that incorporates writing as a fundamental instrument for learning and critical analysis, rather than as a formal writing assignment subject to evaluation, hence affording students the liberty and opportunity to cultivate their ideas and comprehension (Powell et al., 2021).

Bilingual Context

A bilingual context in education refers to the use of two languages. The languages employed are Indonesian and English, which facilitate the initiation of the learning process. The bilingual context aims to enhance linguistic intelligence and expand global perspectives (Garza Ayala, 2022; Sumarno et al., 2024). In this study, a bilingual environment signifies that writing serves as a bridge between two languages, enabling pupils to investigate concepts, express comprehension, and transmit knowledge across linguistic boundaries. This context has several key characteristics. Initially, writing becomes a cognitive process, with students using writing to investigate, structure, and enhance their comprehension in both languages. Writing enables students to clarify experiences and views in both Indonesian and English. Secondly, the bilingual context becomes a translanguaging process, allowing students to dynamically employ both languages to structure ideas, select vocabulary, and complete writing assignments (Garza Ayala, 2022; Sumarno et al., 2024). This procedure fosters linguistic innovation and interlingual comprehension. Furthermore, it enhances linguistic abilities. Bilingual writing exercises enhance vocabulary learning, sentence construction, and expressive ability in both languages. Teachers employ a transitional methodology to facilitate this process, initially instructing in Indonesian and then in English.

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Communication Ability

Communication ability enhances mathematical learning and strengthens students' metacognitive abilities. Students with this capability can deepen their comprehension of diverse topics and identify more effective methods or strategies for resolving mathematical problems (Paroqi et al., 2020; Triana et al., 2019). This capability encourages pupils to articulate their mathematical concepts both verbally and in writing. These concepts may manifest as symbols, images, graphs, diagrams, and text. In this study, researchers used several indicators of communication ability, including explaining the known and the unknown elements, conveying ideas by using mathematical symbols and terms in understanding the problem, conveying ideas using pictures, symbols, mathematical terms, or formulas in solving the problem, explaining relevant things to understand the problem, and presenting conclusions (Santos & Semana, 2015; Sanusi Siregar et al., 2018).

Connection Ability

Connection ability links various mathematical concepts, procedures, and representations both internally and externally. This ability enables students to understand the relationships between mathematical topics and their application in other contexts, such as other sciences or everyday life (Asfar et al., 2022; Kenedi et al., 2019). This ability helps students develop open thinking, broad insight, and the ability to solve problems in an integrated manner using their existing knowledge. In this study, the researchers used several indicators of connection skills, including formulating the problem, identifying mathematical facts, concepts, and principles, finding relationships between mathematical principles and between mathematics and other sciences or disciplines used to solve problems, using these relationships to derive new formulas for problem-solving, and writing conclusions (Septian, 2022; Son, 2022).

METHOD

Research Design

This study employed a quasi-experimental design, comparing two different treatments applied to the research subjects. A nonrandomized Control Group Design was utilized. The experimental group received treatment through Differentiated Instruction (DI) with Writing to Learn (WtL) in a bilingual context, while the control group received treatment using the expository approach. An important preliminary step taken by the researcher was to obtain permission from the school to conduct the study. The researcher also engaged teachers who were willing to participate and ensured that students had received permission from their parents. The selection of students also considers the effective implementation of the bilingual context. Consequently, the researcher involved students proficient in both spoken and written English. The English teacher, familiar with the students' proficiency, facilitated the selection process. Furthermore, the researcher maintained the confidentiality of the identities of the teachers and students involved in the study. The identities of the teachers and students were not disclosed without their explicit consent.

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The participants in this study were junior high school students aged 12 to 13 years. The study included 30 students in the experimental group and 30 students in the control group. These two groups were selected to possess similar characteristics in order to avoid bias. The groups included students with low, medium, and high learning readiness. These attributes are essential for the implementation of DI within the experimental group. Consequently, random assignment through a lottery was not feasible for the experimental and control groups. The researcher employed a proportionate matching procedure based on three kinds of learning readiness to ensure similar characteristics. Prior to matching, the researcher administered a diagnostic assessment to ascertain the learning readiness category of 90 students. The assessment results indicated that 26.7% of students exhibited low learning readiness, 53.3% showed moderate learning readiness, and 20% displayed high learning readiness. The researcher subsequently selected 30 students for the experimental group and 30 for the control group using a proportional matching process, ensuring that the percentage of students exhibiting high learning readiness was identical in both groups. Similarly, the percentage of students exhibiting other learning readiness categories was the same in both the experimental and control groups. The researcher applied the identical proportion as the diagnostic assessment outcomes in each category. Table 1 presents a summary of the percentage of students categorized by learning readiness.

Class	Percentage of students with low learning readiness	Percentage of students with medium learning readiness	Percentage of students with high learning readiness
Experimental	26.7%	53.3%	20%
Control	26.7%	53.3%	20%

Table 1. Percentage of students within each category of learning readiness.

The research instrument utilized in this study was a test. The researcher developed four story problems for the test, adapting the questions from a mathematics reference book for junior high school teachers published by the government. The format of the problems was aligned with the indicators of communication and connection abilities. The researcher adapted the communication skill indicators from NCTM (2000) and the PISA Framework (2015). Simultaneously, the connection ability indicators were adapted from Coxford (1995) and NCTM (2000). The researcher integrated the indicators of each ability based on these sources. Furthermore, the indicators used in the tests are presented in Table 2.

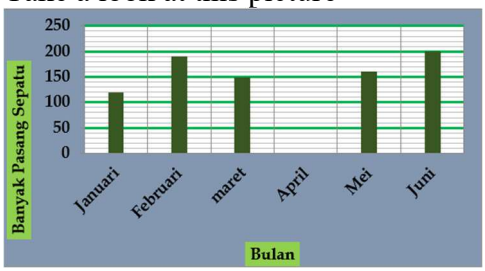
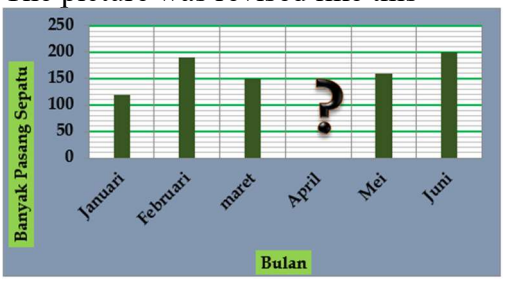
Number	Indicators of Communication Ability	Indicators of Connection Ability
1.	Explaining the known and the asked elements	Formulating the problem
2.	Conveying ideas by using mathematical symbols and terms in understanding the problem.	Identifying mathematical facts, concepts, and principles

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3.	Conveying ideas using pictures, symbols, mathematical terms, or formulas in solving the problem.	Finding relationships between mathematical principles and between mathematics and other sciences or disciplines, which are used to solve problems.
4.	Explaining relevant things to understand the problem	Using the relationships between mathematical principles and the relationships between mathematics and other sciences or disciplines in order to obtain a new formula for solving the problem
5.	Presenting conclusions	Writing conclusions

Table 2. Indicators of communication and connection abilities

Two validators assessed the test prior to its implementation in the study. The validators were university lecturers with expertise in primary and secondary school mathematics. Both validators also served as facilitators for teachers in the implementation of Differentiated Instruction (DI) in educational settings. They provided suggestions to enhance some of the illustrations in the story problems and recommended modifications to certain questions to better assess students' communication and connection abilities. The complete results of the validators' suggestions, along with the researcher's revisions to the questions intended for use in the study, are presented in Table 3, and the complete test is included in the appendix.

Validator	Validator's suggestions	Researcher's revisions
First validator	Take a look at this picture  Give the symbol in April so that students can easily understand the graph.	The picture was revised like this 
First validator	Take a look at the picture in the second question.	The picture was revised like this

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	 Remove the percentage of football or basketball so that students are more critical in answering this question. Choose one. The question needs to be changed; it is very simple. 'What is the number of students who like basketball and students who like football?' Also, add a question so students can infer from the data.	 The question was changed to 'What is the difference between the number of students who like basketball and the number of students who like football? What is the conclusion from the data?"
Second validator	The following questions are simple; they cannot measure students' connection and communication skills. e.g., "How much did you weigh in April?" and "In what month did you weigh the most?" Replace with other questions.	The questions were changed to a. In which month did the baby experience weight loss? b. How much weight was lost during that month? c. When was the most significant increase in the baby's weight? d. What is your conclusion regarding Baby Alya?
Second validator	a. The first instruction is 'Present the data in the table above in a bar chart. Eliminate the shape of the diagram so that students can determine which diagram is appropriate to present this data. b. Add a question allowing students to relate their mathematical knowledge to others.	a. The first instruction was changed to "Present the data in the table above in an appropriate diagram." b. Questions that have been added "If each shop must sell at least 80 pairs of men's shoes, which shop will suffer a loss? What is the percentage loss if one pair of shoes costs Rp.80,000?"

Table 3. The validators' suggestions and the researcher's revisions to the test to be used in the study

In addition to tests, observation sheets and questionnaires were utilized in this study to provide a complete and comprehensive understanding of the research questions. Both instruments underwent

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validation by the validators. The validators confirmed that the statements on the questionnaire and the descriptions of activities on the observation sheet were of acceptable quality; however, they suggested improvements to certain sentences to ensure compliance with the rules of the Indonesian language. Based on these recommendations, the researchers revised the observation sheets and questionnaires used in the study.

The data acquired in this study were evaluated quantitatively by Multivariate Analysis of Variance (MANOVA). The objective was to examine the simultaneous differences between the two dependent variables, communication and connection abilities, of the control and experimental groups. The researcher conducted a multivariate test following a series of learning activities and the administration of the post-test. The data utilized in this test comprised post-test results reflecting students' communication and connection abilities, collected from both the experimental and control classes. The purpose of this analysis was to determine whether there was a significant difference in the effectiveness of Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context compared to the effectiveness of the expository approach in enhancing students' communication and connection abilities. Furthermore, the multivariate tests were conducted on the significance values of the F statistic, specifically Pillai's Trace, Wilks' Lambda, Hotelling's Trace, and Roy's Largest Root. This analysis was performed using SPSS. If the F value exceeded 5.00 or the significance value was less than 0.05, it indicated a difference between the effectiveness of Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context and the effectiveness of the expository approach in enhancing students' communication and connection abilities. If a difference was identified, a t-test (independent samples test) would be conducted to ascertain which classes contributed to the overall difference. The criterion employed for this test was the Bonferroni criterion, with a significance level of 0.025, derived from dividing the degrees of freedom ($\alpha = 0.05$) by 2.

Before conducting the multivariate test, the researchers performed prerequisite analyses, comprising normality test and homogeneity test. The normality test was conducted using a univariate approach with the Kolmogorov-Smirnov test. If the significance value from this test was greater than 0.05, the data were considered to be normally distributed. Subsequently, the homogeneity test was conducted for both communication and connection abilities using the Levene Test. Additionally, the homogeneity of the two abilities could be assessed together by applying Box's M Test. If the significance values obtained from both homogeneity tests, conducted individually and collectively, were greater than 0.05, it indicated that the variance-covariance matrix for the dependent variable was homogeneous.

The obtained data were analyzed both quantitatively using multivariate tests and qualitatively. The researcher conducted a qualitative analysis by determining the percentage of students who met each indicator of communication and connection ability. The percentage of these two abilities were calculated for both experimental and control groups. Additionally, the results of the learning observations were utilized to describe the implementation of the learning process. The observations were carried out by two members of the research team with experience in implementing DI. This procedure was conducted twice: once during the learning phase and once on the recorded learning results.

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This technique provides accurate information about teacher activities and student attitudes. Furthermore, the researcher administered a questionnaire to assess students' satisfaction with the implementation of learning through Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context. Based on the 3-point Likert scale, nine questions were created to gauge students' opinions of the DI with WtL approach implemented in the experimental classroom. These three techniques were employed to provide a clearer understanding of the process of implementation of DI with WtL in a bilingual setting to enhance students' communication and connection abilities.

One Form of a Lesson Plan Utilizing Differentiated Instruction (DI) with Writing to Learn (WtL) in a Bilingual Context

Firstly, the teacher presented the material on data and diagrams in a traditional manner, utilizing various media, including PowerPoint, videos, and images. The use of these media aimed to accommodate students with different learning styles, specifically visual, auditory, and kinesthetic. The teacher occasionally provided explanations in English, and some of the material presented through these media was also in English. The various media used for explanations by the teacher are illustrated in Figure 1.



Figure 1. The teacher provides explanations using various media.

At the beginning of the lesson, the teacher allowed the students to record what they already know about data and diagrams in the 'Know' column and what they wish to learn in the 'Want to Know' column. Students were permitted to write in either Indonesian or English, thereby accommodating those who are more proficient in English.

Secondly, following the classical explanation, the teacher provided students with a worksheet as a form of content differentiation based on learning readiness, which is categorized into three levels: high, medium, and low. Subsequently, the teacher designed assignments tailored to each category. The difficulty level of the tasks was adjusted according to students' learning readiness while remaining sufficiently challenging to stimulate their communication and connection abilities. This approach

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reflects the implementation of Piaget's and Vygotsky's theories. The application of these theories was also evident in collaborative activities, where teachers provide assistance to students, particularly those with low to moderate learning readiness, as they complete their assignments. This support included intensive help, instructional materials articulated in clear and accessible language, and the engagement of students as peer tutors for their counterparts. One example of an assignment on the worksheet for each learning readiness category is as follows:

1. For students with low learning readiness
   Data on Mr Nadu's income over five years is presented in the following table.

Year	Income (in rupiah)
2016	16
2017	27
2018	20
2019	35
2020	45

   a. Present the data in the table above as a bar chart.
   b. How much was Mr Nadu income in 2017?
   c. Mr. Nadu's highest income was in the year ...
   d. How much was Mr. Nadu income in 2016 and 2020?
   e. What is the difference between Mr. Maicon's income in 2018 and 2019?
   f. Provide a conclusion based on the data.

2. For students with medium learning readiness
   The following is data on students' math test scores.

85	90	70	75	90	80	85	95	100	75
70	75	80	80	85	95	100	75	85	90
75	85	80	85	90	70	85	90	80	85
90	90	75	80	80	85	95	90	95	100

   a. How many students took the math test based on the data in the table above?
   b. Use a bar chart to Present the test score data in the table above.
   c. Can this data be presented in the other types of diagrams?
   d. How many students scored 95?
   e. How many students scored 75?
   f. If the Minimum Completeness Criteria is 78, how many students did not the criterion?
   g. If the Minimum Completeness Criteria is 78, how many students met the criterion?
   h. Provide a conclusion based on the data.

3. For students with high learning readiness

   Visitor data for Cepi Watu Borong tourist attractions in one year is presented in the following table.

Month	Visitors

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	Children	Adults
January	5%	15%
February	10%	20%
March	5%	10%
April	15%	5%
May	35%	25%
June	30%	25%

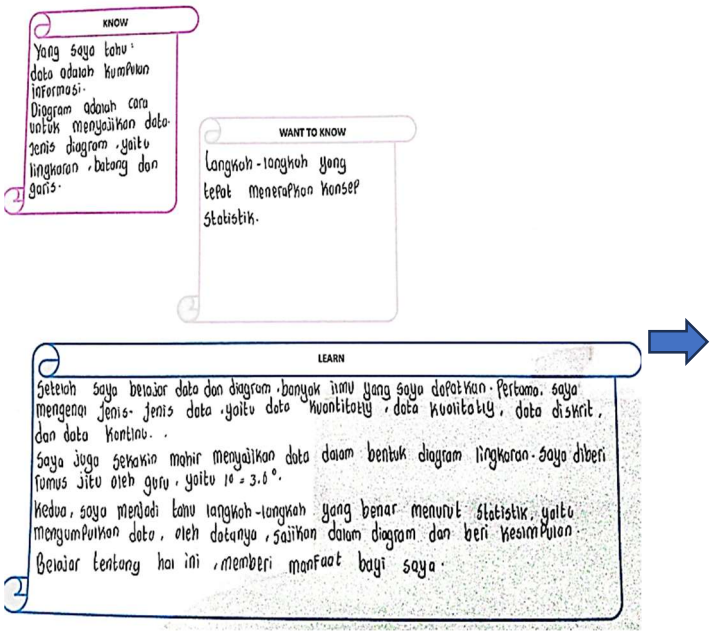
a. Present the data in the table above in a diagram.
b. If the total number of adult visitors in half a year is 2000 people, how many adult visitors in June?
c. If the total number of child visitors in half a year is 500, how many adult visitors are there in February?
d. If every visitor must pay an entrance ticket with the provisions of Children Rp.2000 and Adults Rp.5000, how much revenue is generated from the Cepi Watu Borong tourist spot in half a year?
e. Provide a conclusion based on the data.

The difference among the three worksheets lies in the types of questions tailored to students' learning readiness. Students with low learning readiness received questions involving single data points, which can be presented in a bar chart. This data had already been organized into a table for single data points. Some of the questions in this section are relatively easy but are designed to help students establish connections in order to draw conclusions from the presented data. Students with medium learning readiness received questions involving a larger set of data that has not yet been organized into a table. This data was still random and not systematically arranged, requiring students to first organize the data into a table before proceeding with the questions. The table may contain either single data points or frequency distribution data. The questions are of medium difficulty and require students to connect the data values to the Minimum Completeness Criteria. Consequently, the conclusions drawn involve not only the data values but also their relation to the established Minimum Completeness Criteria. Students with high learning readiness received questions involving compound data, which was presented in percentage form. In these questions, students were not provided with specific instructions regarding which diagram to use for presenting the data; instead, they were permitted to select the most appropriate diagram for their presentation. These questions necessitate the use of more complex algorithms and require students to make connections between the total number of child and adult visitors, as well as the revenue generated based on visitor data.

Thirdly, when the students were completing the tasks on the worksheet, they were allowed to utilize various media, such as books, videos, and the internet. Students had the freedom to use media in either Indonesian or English. They were also allowed to solve problems in Indonesian, English, or a combination of both languages. After completing the tasks, students presented their results. Furthermore, they were free to present their findings in any language. Following the presentation and discussion, students filled in the 'Learned' column, where they recorded what they had learned during

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the learning process. An example of how students filled in the three columns (in both Indonesian and English) is illustrated in Figure 2.



Translate from the learn column:
After we studied the data and diagrams, we obtained a lot of information.
Firstly, we recognized the types of data, namely quantitative data, qualitative data, discrete data, and continuous data.
Secondly, we became adept at presenting data as pie charts. We also know the formula
1^0=3.6%.
Thirdly, we learned the steps in statistics, namely collecting data, processing data, presenting data, and making conclusions.

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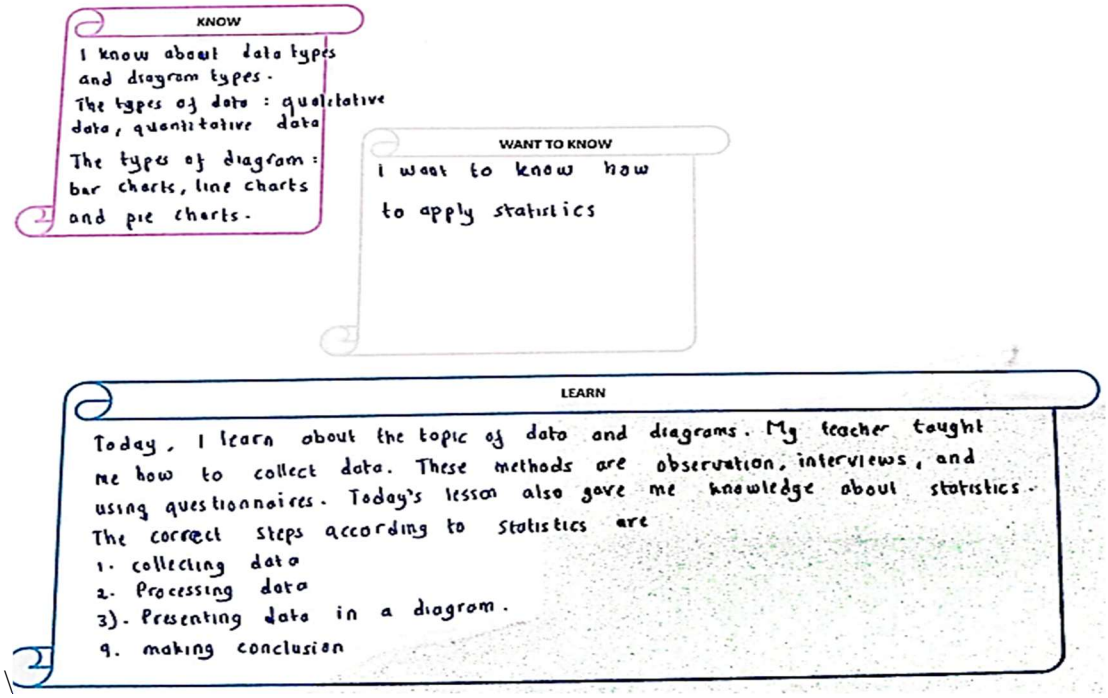


Figure 2: An example of how students fill in the three columns (in Indonesian and English)

Students were also free to write what they have learned in either Indonesian or English. This activity was then presented in a product created by the students, such as short stories, posters, poems, or other forms of artistic expression. The students and their products are illustrated in Figure 3.



Figure 3. The students and their products

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RESULTS

Results of Multivariate Test

The implementation of the learning activities was conducted five times for both the experimental and control classes. Subsequently, students in both classes received a post-test to measure their communication and connection abilities. The post-test results reflected the students' achievements in these two areas. Table 4 provides a concise overview of the test results.

Variable	Class	Mean	Standard Deviation	Highest score	Lowest score
Communication Ability	Experimental	83.11	3.82	93.5	69.67
	Control	75.89	3.12	89.5	63.33
Connection Ability	Experimental	79.43	3.10	89.3	72.3
	Control	73.37	3.16	80.5	69.3

Table 4. A concise overview of the test results

Table 4 displays the disparity in the mean scores of communication and connection abilities between the experimental and control classes. The experimental class exhibited a mean communication ability score of 83.11, while the control class demonstrated a mean score of 75.89. Additionally, the experimental class recorded a mean connection ability score of 79.43, whereas the control class achieved a mean connection ability score of 73.37.

The percentage of students in the experimental class who met the minimum criteria for both aspects of the communication and connection abilities assessment was 73.33%. In comparison, the percentage of students in the control class who met the minimum criteria for these two aspects was 60%. The results of the students' recapitulation based on the minimum criteria are presented in Table 5.

Class	Percentage of the number of students who reached the minimum criteria before treatment	Percentage of the number of students who reached the minimum criteria after treatment	The Improvement
Experimental	16.7%	73.33%	56.6%
Control	14.2%	60%	45.8%

Table 5. Percentage of students who met the minimum criteria before and after the treatment and the improvement

Table 5 indicates that the number of students who met the minimum criteria increased in both the experimental and control classes. However, the improvement was more pronounced in the experimental class compared to the control class.

Based on Tables 4 and 5, the experimental class demonstrated higher communication and connection abilities than the control class. Nevertheless, these results were insufficient to conclude that there

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was a significant difference between the effectiveness of Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context and the effectiveness of the expository approach in enhancing students' communication and connection abilities. Therefore, the next step involved conducting a multivariate test.

Prior to utilizing the multivariate test, it was essential to conduct normality and homogeneity tests. Once these two criteria were satisfied, the researchers could proceed with the multivariate tests to address the research questions. The normality test employed the Kolmogorov-Smirnov test. The test results obtained using SPSS indicated that the communication ability had a significance value (Sig.) of 0.092, while the connection ability had a significance value of 0.064. Since the p-values were greater than 0.05, the researcher concluded that the data on communication and connection abilities followed a normal distribution. The comprehensive results of the normality test conducted using SPSS are presented in Table 6.

Variable	Statistic	df	Sig.
Communication Ability	0.106	60	**0.092**
Connection Ability	0.111	60	**0.064**

Table 6. Results of normality test with Kolmogorov Smirnov using SPSS

The homogeneity test in this study employed Box's M test, which evaluated the similarity of the variance-covariance matrix. The computation results indicated that the significance value (Sig.) was 0.629. Since the significance value exceeded 0.05, the researchers concluded that the covariance variance matrix of the two populations was homogeneous. The results of the homogeneity test conducted using Box's M test in SPSS are presented in Table 7.

Box's M	F	df1	df2	Sig.
1.802	0.578	3	605520.000	**0.629**

Table 7. Homogeneity test results with Box's M test using SPSS

The Levene test was employed to conduct homogeneity testing on the two abilities. The calculation results indicated that the significance value (Sig.) for communication ability was 0.139, while the significance value (Sig.) for connection ability was 0.953. Since the significance values for both abilities were greater than 0.05, the researchers concluded that the variance matrices were the same. Consequently, the data in this study exhibited homogeneity. The results of the homogeneity test conducted using the Levene test in SPSS are presented in Table 8.

Variable	F	df1	df2	Sig.
Communication Ability	2.246	1	58	**0.139**
Connection Ability	0.004	1	58	**0.953**

Table 8. The results of the homogeneity test with Levene test using SPSS

After the data met the criteria for normality and homogeneity, the researchers tested the effectiveness of both Differentiated Instruction (DI) utilizing Writing to Learn (WtL) in a bilingual context and

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the expository approach on students' communication and connection abilities. This test employed a one-sample t-test. The calculation results indicated that the t-value for the communication ability of the experimental class was 4.785, while the t-value for the communication ability of the control class was 2.046. Additionally, the t-value for the connection ability of the experimental class was 7.824, and the t-value for the connection ability of the control class was 2.371. The t-values for all abilities were greater than 2.045. Thus, the DI using WtL in a bilingual context effectively improved both students' communication and connection abilities. Similarly, the expository approach also effectively enhanced both students' communication and connection abilities. Based on these results, the DI using WtL in a bilingual context and the expository approach were equally effective in improving these two abilities. The effectiveness test results from the one-sample t-test conducted using SPSS are presented in Table 9.

Variable	t	df	Sig. (2-tailed)	Mean Difference
The communication ability of the experimental class	**4.785**	29	0.000	11.11100
The communication ability of the control class	**2.046**	29	0.049	3.88833
The connection ability of the experimental class	**7.824**	29	0.000	4.43333
The connection ability of the control class	**2.371**	29	0.025	1.36667

Table 9. The effectiveness test results from the One-Sample t-test using SPSS

Furthermore, the testing employed a multivariate test. This test utilized the significance values of Pillai's Trace, Wilks' Lambda, Hotelling's Trace, and Roy's Largest Root, as well as the F value. The calculation results indicated that the significance value (Sig.) was 0.001, and the F value was 9.097. Since the significance value was less than 0.05 and the F value exceeded 5.00, there was a significant difference in the effectiveness of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context compared to the effectiveness of the expository approach in improving students' communication and connection abilities. The complete results of the multivariate test conducted using SPSS are presented in Table 10.

Test	Value	F	Hypothesis df	Error df	Sig.
Pillai's Trace	0.242	**9.097**	2	57	**0.001**
Wilks' Lambda	0.758	**9.097**	2	57	**0.001**
Hotelling's Trace	0.319	**9.097**	2	57	**0.001**
Roy's Largest Root	0.319	**9.097**	2	57	**0.001**

Table 10. Results of Multivariate test using SPSS

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Due to the differences in the effectiveness of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context compared to the expository approach, the researchers conducted a t-test to determine which specific groups contributed to the overall differences. The t-test was performed on each ability, beginning with communication ability. The calculation results indicated that the significance value (Sig. 2-tailed) was 0.019, and the t value was 2.407. Since the significance value was less than 0.025 and the t value exceeded 2.386, it was concluded that DI using WtL in a bilingual context was more effective than the expository approach in improving communication ability. Thus, the DI using WtL in a bilingual context was found to be superior in enhancing communication ability compared to the expository approach. The results of the t-test analysis on communication ability conducted using SPSS are presented in Table 11.

Variable	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Communication ability	2.246	0.139	**2.407**	58	**0.019**	2.16667	0.90009

Table 11. t-test results on communication ability using SPSS

Second, the test was conducted to measure students' connection ability. The calculation results indicated that the significance value (Sig. 2-tailed) was 0.001, and the t value was 3.794. Since the significance value was less than 0.025 and the t value exceeded 2.386, it was concluded that Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context was more effective than the expository approach in improving students' connection ability. Thus, the DI using WtL in a bilingual context was found to be superior in enhancing connection ability compared to the expository approach. The complete results of the t-test on connection ability, conducted with the assistance of SPSS, are presented in Table 12.

Variable	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Connection ability	0.004	0.953	**3.794**	58	**0.001**	3.06667	0.80824

Table 12. t-test results on connection ability using SPSS

The Evaluation Results Based on Each Indicator of Communication and Connection Abilities

According to the results of the multivariate analysis, Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context is more effective than the expository approach in improving communication and connection abilities. Descriptively, the data for these two abilities were evaluated according to their respective indicators. The researcher calculated the percentage of students who met each indicator for these two abilities. The percentage was determined by dividing the number of students who fulfilled specific indicators across all questions by the total number of students, then multiplying by 100%.

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Number	Indicators of Communication Ability	Experimental Class	Control Class
1.	Explaining the known and the asked elements	93.3%	83.3%
2.	Conveying ideas by using mathematical symbols and terms in the process of understanding the problem.	83.3%	73.3%
3.	Conveying ideas using pictures, symbols, mathematical terms, or formulas in the process of solving the problem.	76.7%	66.7%
4.	Explaining relevant things to understand the problem	76.7%	63.3%
5.	Presenting conclusions	66.7%	56.7%
	Average	79,34%	68,66%

Table 13. Percentage of students who fulfill each indicator of communication ability

Table 13 presents the percentage of students who met the communication ability indicators. The experimental class exhibited a higher percentage than the control class for each communication ability indicator. This result indicates that the implementation of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context is more effective in enhancing communication ability. The opportunity for students to write about what they know, what they want to know, and what they have learned actively engages them in the learning process. Students who may lack oral communication skills are motivated to express their ideas in writing. They can present a short paragraph using mathematical language in the form of symbols, pictures, graphs, diagrams, and words. The differentiation process is based on students' prior knowledge and their learning objectives, allowing the topics covered to be appropriately adjusted.

After learning a topic, students can articulate what they have learned through the concept of free writing. This approach provides a clear representation of the mathematical ideas that students possess, as well as their correct use of symbols, images, graphs, or diagrams. The subsequent step involves students presenting their work, which accommodates those with oral communication abilities. The use of both Indonesian and English further supports the learning process. Employing these two languages can enhance student engagement during instruction. Students with an interest in English are encouraged to express mathematical ideas using this language, which also motivates them to seek broader information from English-language media or teaching materials. This information can be shared with peers through group activities, allowing them to complement each other's knowledge.

From the five indicators of communication ability, the lowest percentage was observed in indicator 5. This trend was evident not only in the experimental class but also in the control class. Some students were able to identify the problem and articulate the relevant information; however, their

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conclusions required correction. Generally, students in the experimental class demonstrated the ability to solve problems with clear writing and presentations, and their argumentation was appropriate. In contrast, while some students in the control class presented good ideas for solving the problem, their opportunities for exploration were still limited, resulting in incomplete solutions to the problems.

Number	Indicators of Connection Ability	Experimental Class	Control Class
1.	Formulating the problem	93.3%	83.3%
2.	Identifying mathematical facts, concepts, and principles	80%	70%
3.	Finding relationships between mathematical principles and between mathematics and other sciences or disciplines, which are used to solve problems.	76.7%	70%
4.	Using the relationships between mathematical principles and the relationships between mathematics and other sciences or disciplines in order to obtain a new formula for solving the problem	73.3%	60%
5.	Writing conclusions	66.7%	56.7%
	Average	78%	68%

Table 14. Percentage of students who fulfill each indicator of connection ability

Table 14 presents the percentage of students who met the connection ability indicators. The experimental class exhibited a higher percentage than the control class for each connection ability indicator. Specifically, the percentage of students who met indicator 2 was 76.7% higher than that of the control class, while the percentage of students who met indicator 3 was 73.3% higher than that of the control class. This indicates that the implementation of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context is more effective in enhancing connection ability. One of the key characteristics of DI is the provision of quality tasks. In the experimental class, tasks were tailored to align with students' learning readiness. Although the tasks were adjusted to accommodate individual learning levels, they all required students to find connections between mathematical topics or between mathematics and other sciences, and to apply these connections in solving mathematical problems.

Writing to Learn (WtL) activities are integrated into these tasks through exercises that involve writing about what is known and what is desired to be known regarding the relationships between mathematical topics or between mathematics and other sciences. Furthermore, students engage in writing about what they have learned through the products they create. These products take the form of poems, news reports, or stories, which are tailored to align with each student's learning style and interests. Some students present their products in Indonesian, while others use English. Table 14

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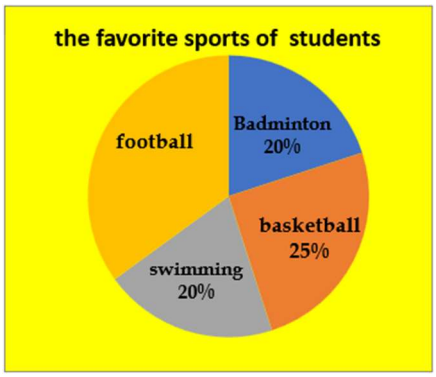
also indicates that only 60% of students in the control class fulfilled the fourth indicator. Some students were not yet able to connect mathematics with other sciences when solving problems related to the tasks.

Of the five indicators of connection ability, the lowest percentage was observed in indicator 5. This trend was evident not only in the experimental class but also in the control class. In the experimental class, students who did not meet this indicator tended to make connections effectively but drew incorrect conclusions. Conversely, in the control class, students who did not fulfill this indicator tended to make incorrect connections, which resulted in errors in their conclusions.

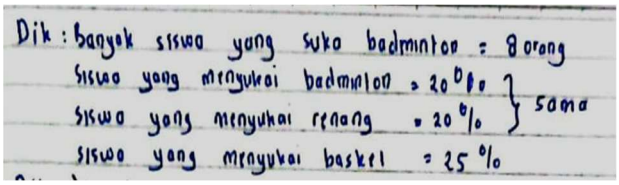
One of the students' answers to a question on the test.
Question:

Take a look at the pie chart.

Data on the favorite sports of seventh-grade students are shown in the pie chart beside the questions. Eight students like badminton.
What is the difference between the number of students who like basketball and those who like football?
Can students who like football make two groups of players in a football match?
What is the conclusion from the data?



Student's answer:

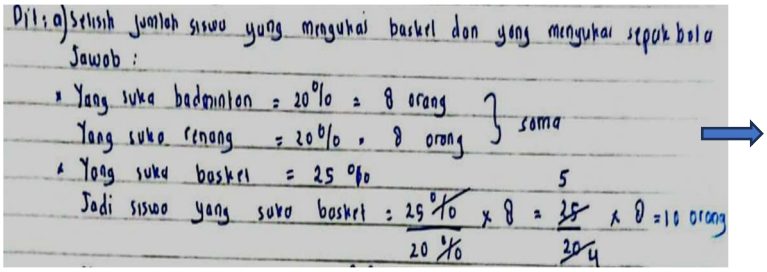


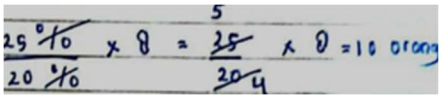
Known:
Number of students who like badminton = 8 people.
Percentage of students who like badminton = 20%, like swimming = 20%, and like basketball = 25 %

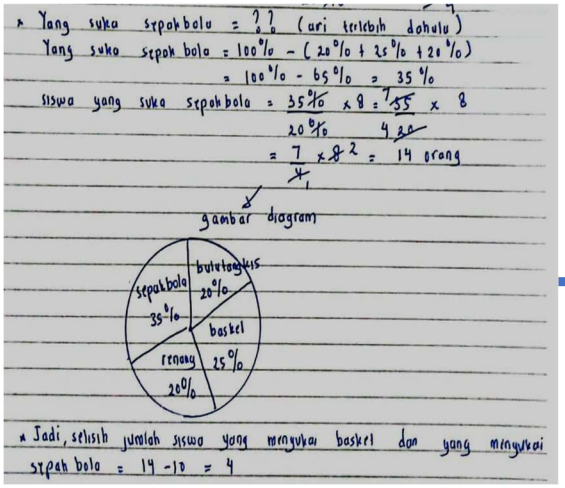
Asked: the difference between the number of students who like basketball and those who like football
Answer (The first Question)

- Like Badminton = 20% = 8 people, and like swimming = 20%. So, like swimming = 8 people.
- Like Basket ball = 25%, so the

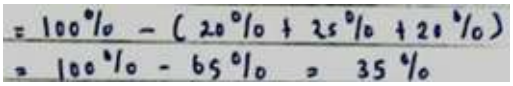
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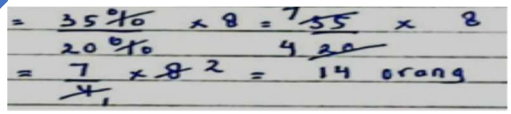




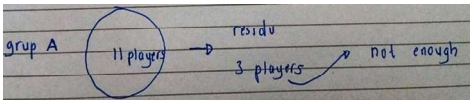
- Like football = ??
  Firstly, find the percentage of students who like football



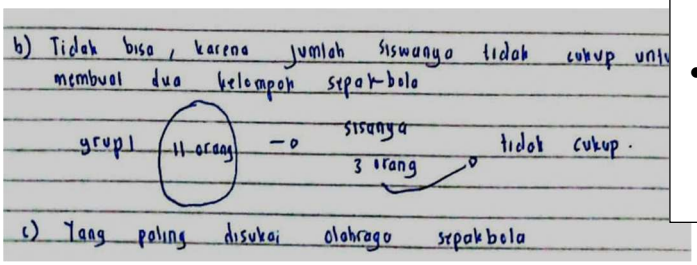
Second, Number of students who like football



Answer (The second question)
Can't, there are not enough students to make two groups of football players.



Answer (The Third Question)
Favourite sport is football



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Figure 4 The students' answers to a question on the test

Figure 4 illustrates the responses of students with medium learning readiness. Based on the indicators of communication ability, the answers reveal several important points. First, students are able to explain what is known and what is being asked in the problem, incorporating these two elements at the beginning of their responses. They describe relevant information according to the narrative presented in the problem and the accompanying pie chart. Second, students employed various symbols to interpret the problem when answering the first and second questions. In the first response, the student used curly braces to indicate that the percentage of students who like badminton is equal to the percentage of students who like swimming. This symbol was used to conclude that the number of students who like badminton is the same as those who like swimming. Additionally, students utilized symbols such as arrows, percentages, and slashes to express the simplification of fractions when calculating the number of students who like basketball and football. Furthermore, students redrew the pie chart, providing more comprehensive information than the initial diagram. While students attempted to use several symbols, some were not appropriate, such as the curly braces. In addition to symbols, students employed various mathematical terms to clarify the problem, specifically what is known and what is being asked. Third, students applied the correct formula to solve the problem. In the first response, the student utilized a formula to determine the number of students who like basketball and football, incorporating comparison, multiplication, division, and fraction simplification. In the second response, the student represented a group of football players with a circle and used an arrow to indicate that the remaining three individuals were insufficient to form another group of football players. Fourth, the student included relevant information to address the first question. The response initially presented information regarding the number of students who liked badminton, followed by information on the number of students who liked swimming. However, one of these pieces of information could determine the number of students who like basketball and football. The initial answer to this question indicates that students have not yet been able to sort through the relevant information necessary to solve the problem, as the solution only requires one piece of information: the number of students who like badminton. Fifth, students mentioned only one conclusion derived from the data displayed on the pie chart, although additional conclusions could be drawn from the data.

Based on the indicators of connection ability, Figure 4 highlights several important points. First, students are able to formulate the problem by determining what is known and what is being asked. With these two pieces of information, students can proceed correctly to the next step of the solution. Second, students include relevant facts, such as the number of students who like badminton and the percentage of students according to their favorite sports as depicted in the diagram. In addressing this problem, students also apply various arithmetic concepts, including the associative principle when determining the percentage of students who like football. Third, students utilized the correct formula to ascertain the number of students who like basketball and football. This formula involves operations of multiplication, division, and comparison or ratio. When applying this formula, students

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effectively connected multiplication, division, and the simplification of fractions. They also used slashes to express the simplification of fractions when calculating the number of students who like basketball and football. These processes demonstrate that students can identify and employ appropriate connections among several mathematical principles. Fourth, while the student provided the correct answer to the second question, they did not explicitly explain why the number of students was insufficient to form two groups of football players. The student merely described the information with a visual representation without clarifying that one football group comprises 11 players. Despite this lack of explicit explanation, the student demonstrated an understanding of the relationship between mathematics and sports science. Fifth, students mentioned only one conclusion derived from the data displayed on the pie chart, although additional conclusions could be drawn by connecting the first, second, and third responses.

Results of the Survey on Students' Opinions

Following the implementation of the learning activities in the experimental class, the researcher administered a questionnaire to gather students' opinions regarding the use of Differentiated Instruction (DI) with Writing to Learn (WtL) in a bilingual context. Through this questionnaire, the researcher obtained insights into students' feelings about the learning process and its effects on them. These opinions are summarized through the results of student feedback corresponding to the statements in the questionnaire. The answer choices in the questionnaire included "Agree," "Neutral," and "Disagree." The percentage for each answer choice was determined by dividing the number of students who selected a particular response for each statement by the total number of students, then multiplying by 100%. The results of the questionnaire are presented in Table 15.

Items	Agree	Neutral	Disagree
I feel happy to participate in learning about data and diagram topics.	83.3%	16.7%	0%
For me, learning about this topic is interesting.	83.3%	16.7%	0%
If there is similar learning in other subjects, I am eager to engage those subjects as well.	83.3%	16.7%	0%
Although my assignment was different from my friends', it helped me to understand the topic of data and diagrams better.	76.7%	13.3%	10%
Through writing activities, I can explore and increase my knowledge about data and diagrams.	83.3%	16.7%	0%
The freedom to use either Indonesian or English motivates me to learn more about data and diagrams.	73.3%	0%	26.7%
This learning model gives me the confidence to communicate my mathematical knowledge to others.	90%	0%	10%
This learning model helped me understand the relationship between data, diagrams and statistics.	76.7%	23.3%	0%

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With this learning model, I know the benefits of learning about data and diagrams in everyday life.	90%	10%	0%

Table 15. Students' opinions in experimental class

Table 15 indicates that the majority of students appreciated learning through Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual setting, with 83.3% expressing positive feedback. Additionally, 83.3% of students found the learning experience engaging. Furthermore, most students (83.3%) expressed eagerness to participate in future lessons utilizing a similar learning model. However, 16.7% of students did not express an opinion on these three aspects. Table 15 reveals that a significant majority of students (76.7%) agreed with the concept of receiving different assignments from their peers. Despite the variations in assignments, students remained enthusiastic about learning the material related to data and diagrams. Nonetheless, 13.3% of students did not provide an opinion on this matter, and a small percentage (10%) disagreed with the differentiation of tasks. Based on brief interviews with these students, the researcher identified them as having lower abilities. They expressed that when their tasks were different and easier than those of their peers, it diminished their confidence. They expressed a desire for more challenging tasks.

Table 15 also indicates that the majority of students enjoyed writing activities during learning, with 83.3% expressing positive feedback. Observations revealed that students wrote about what they knew, what they wanted to know, and what they had learned with enthusiasm and diligence. They were particularly motivated when given the freedom to present their ideas or mathematical solutions in the form of poems or short stories, which they found both unique and challenging. However, 16.7% of students did not express an opinion on this matter, viewing writing activities as a normal part of the learning process. Additionally, Table 15 shows that most students appreciated the bilingual learning package in both Indonesian and English, with 66.7% indicating their preference. Observations indicated that students were enthusiastic about this approach. They demonstrated confidence in expressing their opinions and writing in both Indonesian and English. When working in pairs or groups, students actively translated or shared information in both languages. However, 26.7% of students disagreed with the bilingual learning approach, stating that they were more accustomed to using Indonesian in their studies.

Table 15 reveals that the majority of students reported that learning through Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual setting enhanced their confidence in communicating their mathematical knowledge, both orally and in writing, with 90% expressing this sentiment. However, 10% of students disagreed with this assertion. Observations and interviews with the teacher who conducted the lesson indicated that the students who disagreed tended to experience high levels of anxiety. These students often remained silent during lessons and lacked the courage to express their opinions verbally. Additionally, Table 15 indicates that most students believed that learning with DI using WtL in a bilingual context helped them understand the relationship between data, diagrams, and statistics, with 76.7% affirming this belief. However, 23.3% of students did not assess this statement. Furthermore, Table 15 also shows that a significant majority of students (90%) stated that learning with DI using WtL in a bilingual setting taught them the practical benefits of

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data and diagrams in everyday life. This was evident during the learning process, as students were able to provide examples of the application of data and diagrams in various fields, such as education, economics, and health. Some students were able to explain in detail the steps involved in collecting and interpreting data presented in diagram form. Nevertheless, a small percentage of students (10%) did not provide an assessment of this statement.

In general, Table 15 indicates that the majority of students preferred and benefited from the Differentiated Instruction (DI) approach using Writing to Learn (WtL) in a bilingual context. This learning method enhanced students' enthusiasm and motivation to engage with the topics of data and diagrams.

The Observation Results of Learning Process in Experimental Class

In addition to employing the student opinion questionnaires, the researchers also conducted observations in the experimental classes focusing on two factors: teacher activities and student attitudes. These observations aimed to obtain an overview of the activities undertaken by teachers and the attitudes indicated by students during learning with Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context. Table 16 presents a summary of the teacher activities and student attitudes observed in the experimental class.

Factor	Experimental Class
Teacher activities	Begin by providing an apperception and motivating students about their surrounding environment by positioning themselves in front of students who differ in nature and background. Convey learning objectives by understanding students' interests and how to facilitate them, as well as the contextual scope of the material. Inspire students to be the best version of themselves. Provide sparking questions to start from their initial understanding and give all students the opportunity to respond (the teacher begins using WtL with the activity of writing what students know and want to know about data and diagrams. Students are given the freedom to write either inIndonesian or English). Provide varied reading and observational materials and allowing freedom to explore by utilizing various learning resources (e.g. bilingual reading materials and information from the internet) Encourage students to discuss in groups, collaborate, and position themselves as students' "friends." Reflect on the preliminary results from their exploration of the concepts they have discovered. Quickly respond to students' problems, and when needed, act as a mentor, coach, and partner for students.

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Factor	Experimental Class
	Invite students to visualize their findings in various forms of visual works, offering them choices (the teacher uses WtL activities by asking students to write what the students have learned through several products, such as poems, short stories, posters or other art products presented in either Indonesian or English) Allow students to express their ideas as their best work. Invite students to demonstrate their visual works. Conduct each sequence of activities in class with no abrupt transitions. Provide opportunities for students to ask questions and express their opinions about other students' visual work. Be an active listener (with empathy) when students share their work or opinions. Praise students in public and provide constructive feedback (*feedback on the ladder)*. Ensure fairness in determining student achievement targets using authentic assessment Plan and assign meaningful homework activities, avoiding tasks that lack purpose. Show enthusiasm and provide moral messages that will benefit students in the future.
Student attitudes	Students demonstrate their activeness in the classroom by participating in various activities, such as lively discussions, expressing opinions effectively in either Indonesian or English, and engaging in serious thinking activities, either individually or in groups. Students also show their creativity by producing some interesting products. Students are also more motivated in learning because they have the opportunity to develop their ability to use English alongside Indonesian.

Table 16. Observation results of teacher activities and student attitudes during learning in the experimental class.

Based on the results of statistical tests, the learning conducted in both the experimental and control classes was equally effective in improving connection and communication abilities. However, further analyses indicated that the learning in the experimental class was more effective than in the control class for these two abilities. The observation results presented in Table 16 illustrate this effectiveness, showing the number of positive activities in the experimental class—activities that not only enhanced students' academic abilities but also contributed to their character development. Teachers used three distinctions, namely content, process and product. Moreover, the learning approach accommodated students' needs according to their learning readiness and interests. This approach increased students' comfort during the learning process, motivating them to engage more actively in mathematical thinking activities. As a result, students in the experimental class demonstrated superior communication and connection abilities compared to those in the control class.

DISCUSSION

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Communication and connection abilities are two essential skills for students, as they significantly support mathematical literacy. Communication ability enhance students' understanding of concepts and the correct procedures for solving mathematical problems. Meanwhile, connection ability deepen students' comprehension of both internal and external relationships within mathematics. Therefore, it is imperative for teachers to implement meaningful learning experiences that encourage students' interest in seeking information, learning, and exploring the field of mathematics. Such an approach can effectively support the enhancement of both students' communication and connection abilities.

The study adopted the Differentiated Instruction (DI) approach to make learning more meaningful and enjoyable for students. The DI approach provides opportunities for students to explore according to their individual abilities. Each student acquires knowledge through a learning process tailored to their ability patterns, allowing them to leverage their strengths while addressing their weaknesses weaknesses (Lauria, 2010; Smale-Jacobse et al., 2019). This learning pattern strongly supports students' communication and connection processes. Students actively engage in mathematics learning by demonstrating curiosity, asking questions, making discoveries, critically analyzing topics, and utilizing both prior and additional knowledge to solve problems. This process can be implemented through various approaches or perspectives to facilitate the development of students' communication and interpersonal skills (Kamarulzaman et al., 2022; Kërënxhi & Gjoci, 2015). In this study, DI was combined with the Writing to Learn (WtL) approach, which emphasized students' practices of writing definitions and journals, understanding the material, and explaining the steps involved in solving mathematical problems. As previously mentioned, DI offers several advantages. Many prior studies studies (Hapsari, 2018; Herner-Patnode & Lee, 2021; Maula et al., 2024; Nurasiah et al., 2020; Prast et al., 2018) across various educational levels have demonstrated the effectiveness of DI. These studies found that the implementation of DI resulted in improvements in mathematical ability, engagement in learning, interest in learning, problem-solving skills, communication, and cooperation. Furthermore, the findings indicated that DI fostered an enjoyable classroom environment for both students and teachers in the context of learning mathematics (Marks et al., 2021; Shodiq et al., 2024). In this study, students' worksheets and certain tasks were presented in a bilingual format to facilitate the introduction of two languages: Indonesian and English. Students were also given the opportunity to engage in the WtL process in one of these languages. Thus, it can be concluded that utilizing DI in conjunction with WtL in a bilingual context not only enhances communication and connection abilities in mathematics but also contributes to proficiency in both languages.

This study indicated that both the Differentiated Instruction (DI) approach using Writing to Learn (WtL) in a bilingual context and the expository approach were effective in improving communication and connection abilities in mathematics. However, there were notable differences in the effectiveness of these two approaches regarding the enhancement of students' communication and connection abilities. In this study, the implementation of DI utilized three forms of differentiation: content, process, and product. These distinctions were based on three factors: learning readiness, interest, and learning profile. To facilitate effective communication in mathematics and to help students connect various mathematical concepts, the researchers integrated DI with WtL. WtL activities in mathematics learning proved beneficial for students, who received a worksheet containing three

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columns labeled "Know," "Want to Know," and "Learned." The activity began with the teacher distributing the worksheet and instructing students to complete the first two columns, specifically the "Know" and "Want to Know" columns. Through the responses provided by the students, the teacher could easily assess their initial knowledge and interests. During the learning process, students explored a new topic and wrote a short paragraph summarizing their understanding of the material. At the conclusion of the activity, students filled in the third column as evidence of their learning progress. This writing activity was conducted in a bilingual format, utilizing both Indonesian and English, with the teacher occasionally providing explanations in English. Furthermore, the expository approach to mathematics learning in this study was primarily characterized by the teacher's direct explanations. The teacher provided numerous explanations regarding concepts for solving mathematical problems, interspersed with questions and answers between the teacher and the students. However, the teacher's explanations were delivered exclusively in a monolingual format, specifically in Indonesian.

Furthermore, this study showed that the Differentiated Instruction (DI) approach using Writing to Learn (WtL) in a bilingual context was more effective than the expository approach in enhancing students' communication abilities. One form of DI implementation observed in this study involved the differentiation of processes tailored to various learning styles. The teacher employed several media to accommodate these learning styles, including PowerPoint presentations, images, and educational videos. This differentiation was conducted in a classical format, interspersed with explanations and question-and-answer sessions. Moreover, one of the WtL activities integrated into this differentiation occurred when the teacher sensed that the students were struggling to understand the lesson. The teacher paused the instruction and encouraged the students to articulate their difficulties. Students were able to write about what they knew in the "Know" column and what they wanted to learn in the "Want to Know" column. They could express their mathematical understanding using mathematical language, symbols, images, graphs, diagrams, and words. Based on this writing, the teacher could identify both easy and difficult material, as well as the sources of students' errors in understanding mathematical concepts. Additionally, the teacher could uncover any misconceptions held by the students. The subsequent process allowed students to gain distinctions based on the information derived from their writing. This differentiation encouraged students to enhance their comprehension of mathematical subjects according to their readiness and desire to learn. The teacher integrated this differentiation with tasks that involved solving mathematical problems, which stimulated a deeper understanding of the material covered in the lesson. The teacher then arranged the students in pairs and allowed each pair to "free write" about the mathematical concepts they had understood and how to apply them to solve mathematical problems. Furthermore, the students were asked to communicate their written work to their classmates. This learning style effectively stimulated students to improve their communication abilities in mathematics. These results are supported by several studies (Nurasiah et al., 2020; Sukri, A., Rizka, M. A., Purwanti, E., Ramdiah, S., & Lukitasari, 2022) that utilized DI in mathematics education. The findings of these studies indicated that implementing personalized instruction based on individual characteristics could enhance students' engagement in learning mathematics. Moreover, students exhibited curiosity, asked questions, explored novel concepts, engaged in critical thinking, sought solutions, and utilized prior knowledge

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to solve mathematical problems. This engagement, in turn, stimulated students to communicate mathematical concepts effectively during learning. Furthermore, previous studies (Graham et al., 2020; Miller, 2020) demonstrated that integrating WtL into learning further stimulated students' mathematical communication abilities in writing. According to these studies, students' communication skills became more organized in terms of general grammar and mathematical language. Although WtL primarily focused on improving written communication, its integration within DI also positively impacted students' oral communication abilities. Additionally, employing a bilingual approach in the learning process using DI with WtL not only improved students' mathematical communication in Indonesian but also in English.

The study also indicated that the Differentiated Instruction (DI) approach using Writing to Learn (WtL) in a bilingual context was more effective than the expository approach in enhancing students' connection abilities. One form of DI implementation in this study involved the use of quality tasks, which were presented differently based on students' learning readiness. The categories of students' learning readiness were classified as low, medium, and high. Although the tasks varied, they were designed to be more challenging and slightly above the students' current levels of readiness, thereby pushing them out of their comfort zones (Chamberlin & Powers, 2010; Hapsari et al., 2018; Hidayati, 2020; Mellroth et al., 2021; Prast et al., 2018; Rasheed & Wahid, 2018; Shodiq et al., 2024). In this study, one characteristic of the challenging tasks was to identify connections between mathematical topics or between mathematics and other scientific disciplines, and then to use these connections to solve mathematical problems. WtL activities were integrated into this task. Each student was encouraged to write about what they knew and what they wanted to learn regarding the relationships between mathematical topics or between mathematics and other disciplines. Furthermore, students worked on the tasks in pairs. Initially, they were asked to complete a task using the essential facts, concepts, and principles necessary for its completion. Subsequently, in pairs, the students created connections between mathematical topics or between mathematics and other disciplines based on their prior knowledge and inquiries. They then applied these connections to solve mathematical problems. The outcomes of these activities were presented in the form of short paragraphs, poems, or stories, which students created in Indonesian, English, or both languages. When completing their products, students utilized various media, such as books, videos, or social media, in both languages. The assessment of these products focused not only on the connections made between mathematical topics or between mathematics and other disciplines but also on the coherence of sentences, vocabulary usage, and proper spelling. This type of learning activity effectively enhanced students' connection abilities in mathematics. Additionally, these results are supported by several studies (Canque et al., 2021; Kamarulzaman et al., 2022; Maula et al., 2024), which indicate that DI can facilitate students in connecting ideas or concepts when solving mathematical problems. When learning is combined with WtL, students' connection abilities improve. Another study (Kenney et al., 2014) also demonstrated that WtL is an effective strategy for helping students establish relationships between mathematics and other subjects or disciplines. Although the process is conducted through writing, it can still stimulate both systematic and connective thinking processes. Moreover, WtL can assess students' understanding and provide valuable support for learning mathematical concepts.

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Additionally, this study showed that 79.34% of students in the experimental class met the communication ability indicators, compared to 68.66% in the control class. The experimental class exhibited a higher percentage across all indicators of communication ability. These findings suggest that the implementation of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context is more effective in enhancing students' communication abilities. Providing students with opportunities to write about what they know, what they want to know, and what they have learned actively engages them in the learning process. Students who may struggle with oral communication are encouraged to express their ideas through writing. Moreover, the integration of both Indonesian and English further supports and enriches the learning experience. Employing these two languages enhances student engagement during instruction. Students interested in English are encouraged to express mathematical ideas in this language, which also motivates them to seek broader information from English-language media or teaching materials. This information can be shared with peers through group activities, allowing them to complement each other's knowledge. Additionally, this study indicated that 78% of students in the experimental class met the connection ability indicators, compared to 68% in the control class. The experimental class exhibited a higher percentage than the control class for each connection ability indicator. This indicates that the implementation of Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context is more effective in enhancing connection ability. A key characteristic of DI is the provision of quality tasks. In the experimental class, tasks were tailored to align with students' learning readiness. Although adjusted to accommodate individual learning levels, all tasks required students to find connections between mathematical topics or between mathematics and other sciences, and to apply these connections in solving mathematical problems. Writing to Learn (WtL) activities were integrated into these tasks through exercises involving writing about what is known and what is desired to be known regarding these relationships.

The study also showed that the majority of students appreciated learning through Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual setting, with 83.3% expressing positive feedback. Observations indicated that the teacher implemented three forms of differentiation: content, process, and product, tailored to students' learning readiness and interests. The findings also suggest that most students enjoyed writing activities during learning. Observational data revealed that students wrote enthusiastically and diligently about what they knew, what they wanted to know, and what they had learned. They were particularly motivated when given the freedom to present their ideas or mathematical solutions in the form of poems or short stories, which they found both unique and challenging. Additionally, this study indicates that most students appreciated the bilingual learning package in both Indonesian and English, with 66.7% expressing preference for this approach. Observations revealed that students were enthusiastic, demonstrating confidence in expressing their opinions and writing in both languages. During pair or group activities, students actively translated and shared information in both Indonesian and English. The majority of students reported that learning through Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual setting enhanced their confidence in communicating mathematical knowledge both orally and in writing. Overall, these findings indicate that the majority of students preferred and benefited from the Differentiated Instruction (DI) approach incorporating Writing to Learn (WtL) in a

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bilingual context. This learning method enhanced students' enthusiasm and motivation to engage with the topics of data and diagrams.

Based on these research findings, the DI approach incorporating WtL in a bilingual context positively influences the mathematics learning process in the classroom. Therefore, teachers should consider applying this approach to other mathematical concepts or to enhance other mathematical skills. Furthermore, several important principles demonstrated by the application of this approach include the necessity for learning to accommodate student characteristics, the effectiveness of learning through social interaction, the encouragement for students to design their activities or products during the learning process, and the importance of exploring various student perspectives to deepen their understanding of mathematical concepts.

There are several notable contributions to bilingual learning, particularly the use of two languages in the educational process, which strengthens communication skills in both Indonesian and English. Students can also enrich their English vocabulary through this approach. Additionally, bilingual learning stimulates students' willingness to explore more information from various learning media in both languages, thereby enhancing their knowledge of mathematics. This type of learning encourages students to think more flexibly and to integrate mathematical concepts from both languages. These contributions suggest that students engaged in bilingual learning may benefit more than those involved in monolingual learning. Furthermore, if the activities designed for bilingual learning are applied to monolingual students, they may experience improved understanding of mathematical concepts. This study has several limitations. First, the conclusions are based on a limited set of research results derived from tests and observations. Future studies could benefit from integrating feedback from students or teachers to provide a more accurate representation of the research findings. Second, the sample size in this study was small, which restricts the generalizability of the results. The students involved in this study were aged 12 to 13 years. Third, the school classification selected for this study falls within the high category; thus, the results may differ for schools classified at medium or low levels. Fourth, other mathematical skills, such as critical thinking and problem-solving, are essential for supporting students' communication and connection abilities. Enhancing these mathematical skills will positively impact students' communication and connection abilities, which, in turn, will influence their mathematical literacy. Therefore, critical thinking and problem-solving skills should also be considered in future studies aimed at improving students' mathematical literacy.

Students also encounter challenges during the learning process. Specifically, they often struggle to identify writing topics and to connect mathematical concepts with everyday life in their writing. Additionally, teachers face difficulties when implementing Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context, particularly in modifying writing activities to meet students' needs. Therefore, it is essential to develop potential solutions to address these challenges in the future. First, the approach should be integrated with Problem-Based Learning (PBL) to enable students to analyze real-world problems and report their findings in writing. Second, the use of concept maps can facilitate students in formulating writing topics. Lastly, teachers should conduct brainstorming sessions prior to class to openly explore students' ideas or to compile a list of potential topics, thereby making it easier to design writing activities that align with students' needs. This

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solution represents a crucial component that must be considered when implementing this approach in the future.

CONCLUSION

Based on the research findings, four key conclusions can be drawn. First, both Differentiated Instruction (DI) using Writing to Learn (WtL) in a bilingual context and the expository approach effectively improved students' communication and connection abilities in mathematics. However, there were notable differences in the effectiveness of the two approaches. DI using WtL in a bilingual context was more effective than the expository approach in improving both students' communication ability. DI using WtL in a bilingual context was also found to be more effective than the expository approach in enhancing students' connection ability. Second, 79.34% of students in the experimental class met the communication ability indicators, compared to 68.66% in the control class. Additionally, 78% of students in the experimental class met the connection ability indicators, while 68% did so in the control class. Third, the majority of students preferred and benefited from the Differentiated Instruction (DI) approach using Writing to Learn (WtL) in a bilingual context. This learning method enhanced students' enthusiasm and motivation to engage with the topics of data and diagrams. Fourth, the observation revealed that the teacher employed three forms of differentiation: content, process, and product, all tailored to students' learning readiness and interests. To further enhance learning engagement, educators used the WtL method in a bilingual context. The observation also indicated that students participated in various activities during the learning process, guided by their requirements or interests. According to the study results, the application of DI using WtL in a bilingual context can serve as an alternative strategy for improving students' communication and connection abilities in mathematics. A critical component of this alternative strategy is the appropriate integration of DI, WtL, and bilingualism within the mathematics learning process to ensure coherence rather than overlap. Additionally, teachers must address potential challenges faced by both educators and students when employing this approach. For instance, integrating the approach with Problem-Based Learning (PBL), utilizing concept maps to assist students in identifying writing topics, and conducting brainstorming sessions prior to lessons to openly explore students' ideas can facilitate the design of writing activities that align with students' needs.

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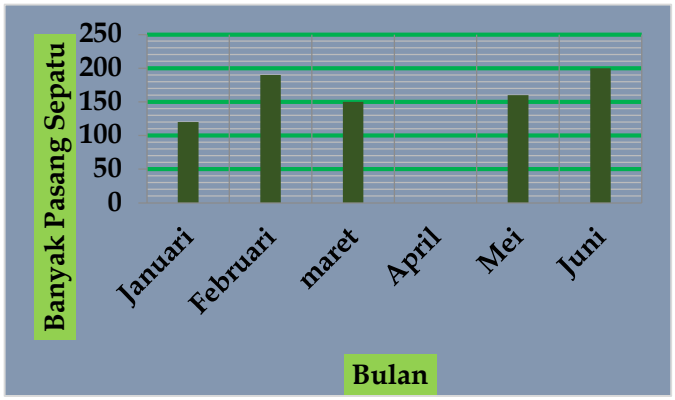
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APPENDIX
The Test Given to Students (in Indonesian and English)

In Indonesian

1. Berikut ini adalah diagram batang mengenai penjualan sepatu di Toko Murah.



Jika penjualan sepatu selama 6 bulan tersebut adalah 900 pasang sepatu, maka banyaknya penjualan sepatu pada bulan April adalah … Bagaimana kesimpulan dari data tersebut?

2. Perhatikan diagram lingkaran

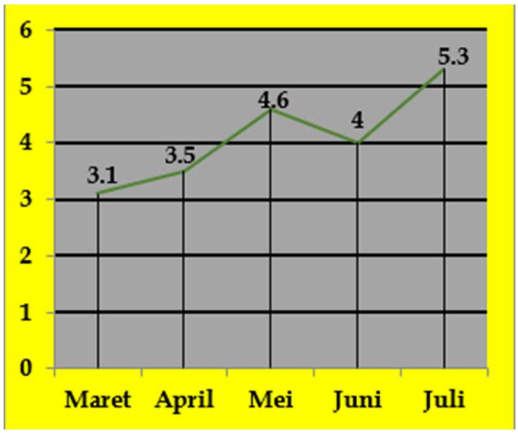
Data olahraga kegemaran siswa kelas 7 ditunjukkan dalam diagram lingkaran di samping. Banyak siswa yang menyukai bulu tangkis adalah 8 orang.
Berapakah selisih jumlah siswa yang menyukai basket dan jumlah siswa yang menyukai sepak bola?
Apakah siswa yang menyukai sepakbola dapat membuat dua grup pemain pada pertandingan sepakbola?
Bagaimana kesimpulan dari data tersebut?



3. Perhatikan diagram berat badan bayi Alya berikut ini.

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a. Pada bulan apa terjadi penurunan berat badan bayi?
b. Berapa besar penurunan berat badan yang terjadi di bulan tersebut?
c. Kapan terjadi peningkatan berat badan bayi yang terbesar?
d. Bagaimana kesimpulanmu tentang bayi Alya?



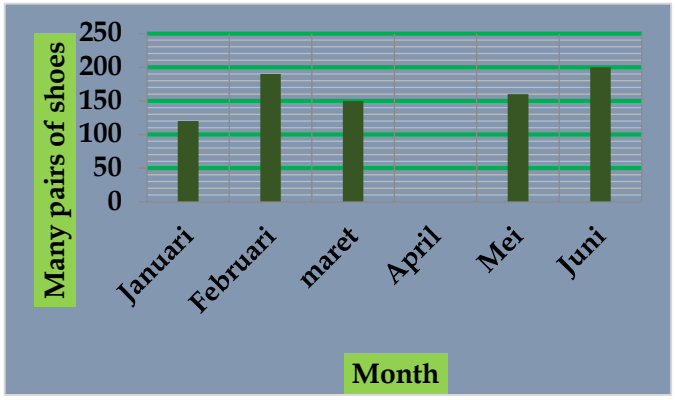
4. Data penjualan sepatu pria dan wanita dari empat toko yang berbeda dalam seminggu ada pada tabel.

Toko A		Toko B		Toko C		Toko D	
Pria	Wanita	Pria	Wanita	Pria	Wanita	Pria	Wanita
90 psg	70 psg	100 psg	80 psg	80 psg	50 psg	45 psg	90 psg

a. Sajikan data pada tabel di atas dalam diagram yang tepat.
b. Berapa selisih penjulan sepatu pria di toko A dan C?
c. Berapa selisih penjualan sepatu wanita di toko B dan C?
d. Jika tiap toko wajib menjual paling sedikit 80 pasang sepatu pria, manakah toko yang mengalami kerugian? Berapa persen kerugiannya, jika satu pasang sepatu harganya Rp.80.000.
e. Buatlah kesimpulan dari data ini.

In English

1. The following is a bar chart of shoe sales at Toko Murah.

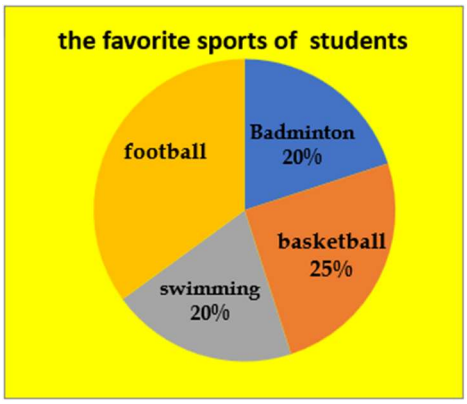


If the sales of shoes during the 6 months is 900 pairs, then the number of shoes sold in April is ...
What is the conclusion from the data?

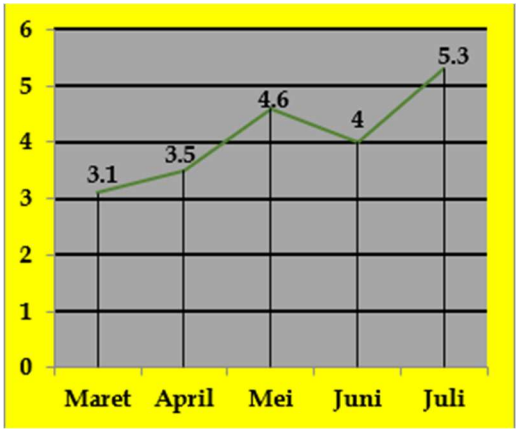
2. Take a look at the pie chart.

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Data on the favorite sports of seventh-grade students are shown in the pie chart beside the questions. Eight students like badminton.
What is the difference between the number of students who like basketball and those who like football?
Can students who like football make two groups of players in a football match?
What is the conclusion from the data?



3. Take a look at baby Alya's weight chart.
   a. In which month did the baby experience weight loss?
   b. How much weight was lost during that month?
   c. When was the most significant increase in the baby's weight?
   d. What is your conclusion regarding Baby Alya?



4. The sales data for men's and women's shoes from four shops in a week are in the table.

Shop A		Shop B		Shop C		Shop D	
Men	Women	Men	Women	Men	Women	Men	Women
90 psg	70 psg	100 psg	80 psg	80 psg	50 psg	45 psg	90 psg

   a. Present the data in the table above in an appropriate diagram.
   b. What is the difference between the sales of men's shoes in stores A and C?
   c. What is the difference in sales of women's shoes at stores B and C?
   d. If each shop must sell at least 80 pairs of men's shoes, which shop will suffer a loss? What is the percentage loss if one pair of shoes costs Rp.80,000?
   e. Draw conclusions from this data.

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Proficiency Level of Grade 9 Students on Quadratic Functions Through Process-Oriented Guided Inquiry Learning (POGIL)

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Abstract: This study investigated the impact of Process-Oriented Guided Inquiry Learning (POGIL) approach in the proficiency level of Grade 9 students along quadratic functions. It employed a quasi-experimental design with a qualitative approach. Two groups with 40 students each from Saint John Berchmans High School Incorporated were considered as the subjects of the study. Results revealed that the students' proficiency level on quadratic functions before and after exposure to POGIL approach improved. Whereas, the other group of students who were exposed under direct instruction remained at a low proficiency level in the same lesson. Students in both groups significantly improved their proficiency level in quadratic functions before and after the use of the both instructional approaches. However, results show that between two approaches used in this study, the POGIL approach of teaching quadratic functions to students is more effective than direct instruction. For the continual improvement of teaching quadratic functions to students using the POGIL approach, observers' feedback was gathered and these involved students' engagement, incorporation of higher-order thinking-skills and multimedia presentations, and time engagement in the given activities. These suggestions were carried out in the enhancement of the POGIL activities ready for utilization of Grade 9 mathematics teachers.

Keywords: direct instruction, higher-order thinking skills, inquiry-based learning, students' engagement

INTRODUCTION

Mathematics is a fundamental tool in our lives and in every scientific field since it fosters personal growth and development on a large scale (Sharma, 2021). Mathematics is also an effective means of establishing mental discipline and an essential subject since it equips students with a uniquely powerful set of tools to understand and change the world (Andamon & Tan, 2018; Sharma, 2021). These tools include logical reasoning, problem-solving skills, and the ability to think in abstract ways (Andamon & Tan, 2018). Mathematics Proficiency is not only essential for academic success but also for potential employment opportunities in various fields such as science, technology,

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engineering, and finance (Bone et al., 2021). However, students' mathematics performance has raised concerns in various countries, including the Philippines. Most students find mathematics difficult to deal with (Baring & Alegre, 2019). The results in the TIMSS 2019 and PISA 2018 showed that Filipino students performed poorly in Mathematics where the Philippines scored below the average of participating countries (Mullis et al., 2019;Department of Education, 2019). The poor performance of Filipino students in Mathematics is also evident in the National Achievement Test 2018 where the students from Cagayan Valley Region (Region II) gained a Mean Percentage Score in Mathematics which is far from the 75% passing rate. Moreover, problem solving and critical thinking are the 21st century skills with the lowest percentage (Department of Education, 2019).

One of the critical mathematical concepts that needs to be understood by the students is Quadratic Function since it plays a vital role in the development of mathematical concepts since they cross a range of mathematical content domains, including algebra and geometry. As such, Parent (2015) said that learning quadratic function is vital because the concepts will become foundational for advanced mathematics, particularly when working with higher polynomial functions. Moreover, according to Leinhardt et al. (1990), this topic holds significance due to two key reasons: (a) the growing acknowledgment of the function concept as a powerful organizing principle spanning from middle school mathematics to more advanced high school and college-level topics, and (b) the symbolic links it offers, which enhance students' ability to connect and understand the graphical and algebraic representations of mathematical ideas. Further, Hoon et al. (2018) added that students who understand quadratic functions, their properties, and applications will be able to construct and develop a good understanding of more complicated and various kinds of functions and concepts. In addition, acquiring knowledge in quadratic functions fosters the application of mathematical cognition and logical reasoning, which includes decision-making (Hoon et al., 2018). Similarly, Díaz et al. (2020) stated that quadratic functions have practical relevance in real-world mathematical thinking and reasoning. However, this lesson is regarded as one of the most conceptually challenging topics in the curriculum (Nielsen, 2015). Susandi (2021) stated that students' low critical thinking skills are associated with misconceptions, procedural errors, and calculation errors. Such errors are caused by a method of mathematics learning that prioritizes memorization over comprehension. There are several errors made by the students regarding this concept, these include: learners' inability to interpret graphs of functions, inability to represent functions graphically, and converting flexibly between different representations of functions (Dillingerová & Vankúš, 2020). Additionally, Arnal-Palacian et al. (2022) highlighted the challenges students face when working with quadratic functions. One common difficulty involves understanding the relationship between the coefficient of the linear term and the position of the parabola's axis of symmetry. Many students struggle to derive the coefficient of the quadratic term based solely on visual representations. The authors also noted that most student errors stem from a lack of conceptual understanding and procedural skills, particularly in situations that demand more than the straightforward application of formulas. Su et al. (2016) suggest that teachers need to replace the current math classes with meaningful mathematical experiences that teach students "how to think through math" rather than memorizing formulas. This encourages students to think critically and discover new relationships between mathematical concepts, which leads to innovation

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and the creation of ideas.

Nowadays, one of the innovative approaches in teaching mathematics that could enhance the students' proficiency in mathematics is Process Oriented Guided Inquiry Learning (POGIL). It is a method combining guided inquiry and cooperative learning in mathematics education (Muhammad & Purwanto, 2020). The three phases of learning in POGIL model are exploration, concept discovery, and application of concepts. Kartono & Shora (2020) stated that POGIL has seven stages of learning; elicit, engage, explore, explain, elaborate, evaluate, and extend. In the context of constructivist theory by Jean Piaget and discovery learning theory by Jerome Bruner, students will continuously build their knowledge when they are actively involved in the learning process (Pasion, 2019). Moreover, through inquiry-based approach to teaching, learning is more meaningful to learners when they have the chance to independently uncover the connections between various ideas (Dumigsi & Cabrella, 2019).

The implementation of POGIL demonstrates positive outcomes on students' conceptual understanding, analytical reasoning and problem-solving abilities which supports the twin goals of mathematics curriculum.

With this context, the researcher aimed to determine the effect of Process Oriented Guided Inquiry Learning (POGIL) on the Proficiency Level of Grade 9 students in Quadratic Function.

Specifically, this study sought to answer the following questions:

1. To determine the proficiency level of the Grade 9 students along quadratic function before and after being taught through:
    a. Process Oriented Guided Inquiry Learning (POGIL); and
    b. Direct Instruction.
2. To find the effect of the interventions in the proficiency level of Grade 9 students along quadratic function through:
    a. determining if significant difference exists between the pretest and posttest; and
    b. determining if significant difference exists in the level of proficiency of Grade 9 students along quadratic function when taught using the direct instruction and POGIL.
3. To enhance the student activities to integrate in the Process Oriented Guided Inquiry Learning (POGIL) approach.

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Figure 1 presents a research paradigm formulated based on the variables discussed and served as a guide in the conduct of the study.

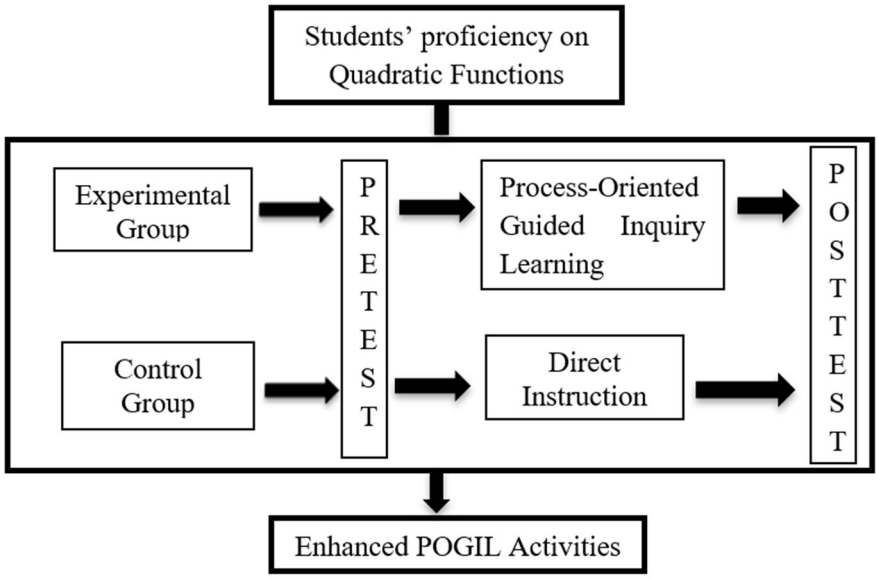


Figure 1. Research Paradigm

LITERATURE REVIEW

The POGIL teaching approach highlights the collaborative elements of constructivism used in the three phases of learning: exploration, concept discovery, and application of concepts (Kartono & Shora, 2020). In the exploration stage, students engage in activities such as observing, experiment designing, collecting, checking, analyzing data, investigating, and testing the hypothesis. In concept formation, students are asked to think critically and analyze the concept formation. In the application phase, students are involved in applying new knowledge in exercising, problem-solving, or even research situations. During this task, students collaborate in teams to deepen their learning and integrate it with the new concept they have (Muhammad & Purwanto, 2020). On the other hand, Kartono and Shora (2020) stated that POGIL has seven stages of learning: elicit, engage, explore, explain, elaborate, evaluate, and extend. Moreover, the members in POGIL approach have their specific roles, including leader, speaker, secretary, and strategy analyst/reflector. As for the teacher's role, Datu-dacula and Anda (2021) stated teachers assist students in their knowledge creation. Engaging students actively in the learning process will encourage them to generate information and develop critical thinking skills, which will fill gaps in their mental representations.

Numerous studies have explored the effectiveness of POGIL as a teaching strategy. POGIL is a successful teaching strategy that can help students build their mathematical HOTS, as demonstrated

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by Brion et al. (2022). Furthermore, Diniyyah et al. (2022) said that in the context of online learning, combining the POGIL model with a digital mind map might improve students' capacity for critical thought and problem-solving. According to research by Samosir (2022) and Aiman et al. (2020), students' critical thinking abilities are positively impacted by the POGIL methodology. Similar results were obtained by Kisworo and Gusman (2019), who found that the POGIL technique significantly improved students' critical thinking abilities and cognitive development. Furthermore, it has been demonstrated by Muhammad and Purwanto (2020) and Sanggara et al. (2018) that students taught utilizing the POGIL paradigm had a higher level of success in solving mathematical problems than students taught directly. This is in line with research by Andriani et al. (2019), which found that POGIL students outperform traditional learning methodologies in terms of logical reasoning and mathematical proficiency. Irwanto et al. (2018) also found that POGIL is a useful technique for raising students' performance, particularly in terms of their ability to solve problems and think critically. Additionally, POGIL enhances students' academic performance and cultivates a positive attitude toward learning, as demonstrated by Bug-os and Caro (2019). This bolsters the findings of Artuz and Roble (2021) that guided inquiry learning promotes students' active engagement in tasks, which improves learning outcomes. Additionally, this study showed that POGIL improved students' analytical, problem-solving, communication, and critical thinking abilities. Meanwhile, Kartono and Shora (2020) showed that POGIL can also develop the mathematical reasoning ability of the students. Additionally, Pradiyanasari et al. (2020) claimed that guided inquiry learning or POGIL can also develop students' understanding and mastery of concepts because it emphasizes a learning process that directs students to find answers to questions or problems on their own, allowing them to gain a deeper understanding. Sen et al., (2015) findings claimed that POGIL has a positive impact on students' mastery approach, self-efficacy for learning and performance, critical thinking, peer learning, metacognitive self-regulation, and effort regulation.

Method

Design

The study utilized both descriptive quantitative and qualitative approaches of researches and employed a quasi-experimental design. In this study, Process-Oriented Guided Inquiry Learning (POGIL) was used as an intervention for the experimental group while direct instruction was used for the control group. A descriptive comparative research design was also utilized to compare the proficiency level of the students in quadratic function in the Process-Oriented Guided Inquiry Learning (POGIL) and direct instruction. Moreover, qualitative method was used to identify the points of improvement on the students' activities in the 7 stages of POGIL approach.

Research Instrument

There were two main instruments that were used in this study, the pre-/post-tests and the observation note form. The pre-/post-tests were researcher-made tests with 50-item multiple choice type of test containing all the learning competencies in quadratic functions. These were parallel

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tests with equal difficulties of items covering the same competencies. The pre-/post-tests were validated by subject matter experts and tried out by the Grade 10 students of SJBHSI during S.Y. 2022-2023 to ensure reliability of the instrument. The reliability value of the test items was α=0.7831 which was reliable. The pretest was administered to identify the initial proficiency level of the students and a posttest to determine whether there was an effect in the students' proficiency level along quadratic functions after the execution of POGIL approach and direct instruction. Additionally, feedback from observers through observation note form was used as an instrument to identify the points of improvement in the students' activities in the 7 stages of learning in POGIL approach. The other materials that were used in the implementation of the interventions were the students' worksheets and the lesson plan.

Locale

The study was conducted within the Junior High School Department of Saint John Berchman's High School Incorporated (SJBHSI). It is a private institution located at Magsaysay, Cordon, Isabela. This 2023, selected Grade 7 and Grade 10 students participated in the Performance Appraisal in Math (PATH) Mathematics Challenge District Level held in the municipality and secured first place. However, in 2020, the Performance Assessment of Standards and Skills (PASS)results revealed that the students' proficiency in Mathematics is at the "Developing level".

Samples and Sampling Procedures

There were two main participants in this study namely: the subjects of the study and the observers. The subjects of the study were the Grade 9 students of SJBHSI enrolled in SY 2023-2024. Grade 9 students were chosen as the subjects of the study because they met the criteria of having a lesson on quadratic functions for the first quarter. Two comparable classes with the same level of performance in Mathematics 9, specifically the two low-performing classes were selected and became the experimental and control groups.

	Section Name	**Male**	**Female**	**Total**
Grade 9	St. Thomas of Aquinas	13	31	44
	St. Therese of Avila	23	21	44
	St. Jerome	23	22	45
	St. Augustine	27	18	45
	Sub total	86	92	178

Table 1. Population of Grade 9 students in SJBHSI for SY 2023-2024

From these four sections, St. Jerome class was selected as the experimental group and St. Augustine served as the control group. In Table 2, the proficiency level of the students for both

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classes was compared before their exposure to the interventions.

Group	Intervention	Mean	SD	T	df	Sig. (2-tailed)
Control Experimental	*Direct Instruction* *POGIL*	12.90 13.10	2.55 3.05	-0.32	78	0.751

*$p< 0.05$

Table 2. Comparison in the Initial Proficiency of the Two Groups in Quadratic Functions

It can be gleaned from the table that the students' proficiency level in quadratic functions of the control and experimental groups had no significant difference $t(78)$=-0.32, p=0.751. This implies that initially, the two groups had a similar level of proficiency in quadratic functions. The two classes that were selected have 45 students each; however, due to some limitations such as learners' unwillingness to join the study, lack of consent from their parents and absenteeism, only 40 students from each class were able to complete the interventions. The other students who were not able to take part in the interventions joined the other class of their mathematics teacher.

Meanwhile, three teachers were purposely selected and served as the observers during the execution of the POGIL teaching approach. Each teacher has more than 10 years of teaching experience and has the mandate to conduct instructional supervision. They used the observation note form during the POGIL implementation to identify the points of improvement in the students' activities on the 7 stages of learning in POGIL approach.

The constructed pre-/post-test on Quadratic functions used by the two groups was validated by subject matter experts and tried out by the Grade 10 students of SJBHSI during S.Y. 2022-2023 to ensure reliability of the instrument. The reliability value of the test items was α=0.7831 which was found to be reliable.

Data Treatment

To determine the students' proficiency on quadratic functions, Mean Percent Scores were used. To interpret the mean percent scores of the students, the five levels of proficiency adapted from the new standard of Department of Education (DepEd Order No.024, s.2022) was employed. To determine the effects of POGIL and direct instruction on students' proficiency in quadratic functions, t-tests for paired samples were utilized. Meanwhile, a one-way Analysis of Covariance (ANCOVA) was conducted to compare the effects of the interventions in the proficiency level of Grade 9 students in quadratic functions after controlling the pretest scores. Moreover, thematic analysis was used to analyze the feedback of the observers about the enhancements in the students'

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activities on the 7 stages of POGIL approach and was integrated into the proposed activities of Process-Oriented Guided Inquiry Learning.

Ethical Considerations

This study was submitted to Saint Mary's University Research Ethics Board (SMUREB) for ethics review. The researcher affirms that there was no conflict of interest in the study that influenced the validity of the findings. The researcher made sure that all the gathered data that personally identified the subjects of the study were treated with utmost confidentiality. No one was able to recognize them as the study subjects aside from the researcher. The data provided by the participants were transferred to excel in number coded format. After the study is completed and finally bound in a book, all the data collected were deleted for good. Since the participants are under 18 years old, parents' consent was secured by the researcher. A consent form was also given to research participants. Participants fully understood the given form as well as the research's purpose. The students were free to decide whether to participate or not and asked questions for any word or concept in the form that they did not understand. If they feel uncomfortable during the data gathering process, they were allowed to withdraw from the study at any moment without explanation. Any information they had provided was used in the study. They were allowed to join to the other sections of grade 9 if they did not want to participate on the intervention programs that were implemented by the researcher. Their non-participation or withdrawal did not affect their standing as a student. The study was funded by the Department of Science and Technology under the Capacity Building Program in Science and Mathematics Education.

Results and Discussion

A. Proficiency Level of Students along Quadratic Functions

Proficiency Level	**Before**		**After**	
	f	%	f	%
Highly proficient	0	0	0	0
Proficient	0	0	0	0
Nearly Proficient	0	0	30	75
Low Proficient	22	55	10	25
Not Proficient	18	45	0	0
Overall	**MPS= 26.2% Level: Low Proficient**		**MPS=54.4% Level: Nearly Proficient**	

Legend: MPS (Mean Percent Score); 0-24% (Not Proficient); 25-49% (Low Proficient); 50-74% (Nearly Proficient); 75-89% (Proficient); 90-100% (Highly Proficient)

Table 3. Frequency Count and Percent of Students' Level of Proficiency in Quadratic Functions in POGIL Approach

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It can be gleaned from the table that students of the POGIL group have low proficiency level in quadratic functions (MPS=26.2%) before they were taught in the POGIL approach of teaching. Majority (55% or 22) of the students have low proficiency level while 45% or 18 students are not proficient in quadratic functions.

After exposure to the POGIL approach in learning quadratic functions, the students' proficiency level increased (MPS=54.4%). Majority (75% or 30) of the students are nearly proficient while few (25% or 10) of the students are still in low proficiency level. This signifies that students' proficiency in quadratic functions improved after they were taught using the POGIL approach.

Related studies conducted by Irwanto et al. (2018), Artuz and Roble (2021), Brion et al. (2022), Samosir (2022), Aiman et al (2020), and Kisworo and Gusman (2029) found out that POGIL is an effective method in improving student performance. According to these authors, POGIL enhanced students' engagement, communication, critical thinking, and problem-solving skills.

To further see the proficiency of the students in the POGIL class, Table 4 shows the mean percent scores and proficiency level of the students for each competency of quadratic functions.

Learning Competencies	**Mean Percent Score**		**Proficiency Level**	
	Pretest	**Posttest**	**Pretest**	**Posttest**
1. Models real-life situations using quadratic functions.	45.0	89.4	Low Proficient	Proficient
2. Represents a quadratic function using: (a) table of values; (b) graph; and (c) equation.	29.8	56.9	Low Proficient	Nearly Proficient
3. Transforms the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$.	23.8	40.6	Not Proficient	Low Proficient
4. Graphs a quadratic function: (a) domain; (b) range; (c) intercepts; (d) axis of symmetry; (e) vertex; (f) direction of the opening of the parabola.	23.1	55.2	Not Proficient	Nearly Proficient
5. Analyzes the effects of changing the values of a, h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph.	20.0	57.5	Not Proficient	Nearly Proficient
6. Determines the equation of a quadratic function: (a) a	24.3	50.7	Not Proficient	Nearly Proficient

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table of values; (b) graph; (c) zeros.				
7. Solves problems involving quadratic functions.	21.4	38.6	Not Proficient	Low Proficient
Overall	**26.2**	**54.4**	**Low Proficient**	**Nearly Proficient**

Legend: 0-24% (Not Proficient); 25-49% (Low Proficient); 50-74% (Nearly Proficient); 75-89% (Proficient); 90-100% (Highly Proficient)

Table 4. Mean Percent Scores and Proficiency Level of Students in Each Learning Competency of Quadratic Functions in POGIL Approach

As shown in the table, the students' proficiency level before exposure to POGIL approach ranged from not proficient to low proficient. The mean percent scores of the students on five out of seven competencies is not proficient. These competencies are: transforming the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$; graphing a quadratic function: (a) domain; (b) range; (c) intercepts; (d) axis of symmetry; (e) vertex; (f) direction of the opening of the parabola; analyzing the effects of changing the values of a, h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph; determining the equation of a quadratic function: (a) a table of values; (b) graph; (c) zeros; and solving problems involving quadratic functions. This implies that prerequisite knowledge and skills have not been acquired or developed in the competencies of quadratic functions. Meanwhile, students were low proficient in modeling real-life situations using quadratic functions and representing a quadratic function using: (a) table of values; (b) graph; and (c) equation. This means that students possess the minimum knowledge, skills and core understandings, but need help throughout the performance of authentic tasks.

It can be gleaned from the table that the students' proficiency level for each competency improved when they were taught using the POGIL approach. Most of the students' mean percent scores were nearly proficient in the competencies of quadratic function. Students became proficient in modeling real-life situations using quadratic functions where they developed their fundamental knowledge and skills, and can transfer them independently through authentic performance tasks. Moreover, the mean percent scores of the students on four competencies increased to nearly proficient which implies that the students have developed fundamental knowledge, skills, and core understanding in representing and graphing quadratic functions, analyzing the effects of changing the values of the variables in graph of quadratic functions and determining the equation of quadratic function, but students still need little guidance from the teacher. The mean percent scores of the students also increased from not proficient to low proficient in the competencies; transforming the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$ and solving problems involving quadratic functions. This suggests that students only acquired minimum knowledge and skills, and still need help to develop their proficiency in these competencies. Thus, in implementing the POGIL approach in teaching quadratic functions, the teacher should provide enough time for the students to better grasp the lesson and further develop their proficiency level in the aforementioned competencies of quadratic functions.

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Proficiency Level	Before		After	
	F	%	F	%
Highly proficient	0	0	0	0
Proficient	0	0	0	0
Nearly Proficient	0	0	3	7.5
Low Proficient	22	55	34	85
Not Proficient	18	45	3	7.5
Overall	**MPS= 25.8% Level: Low Proficient**		**MPS=36.5% Level: Low Proficient**	

Legend: MPS (Mean Percent Score); 0-24% (Not Proficient); 25-49% (Low Proficient); 50-74% (Nearly Proficient); 75-89% (Proficient); 90-100% (Highly Proficient)

Table 5. Frequency Count and Percent of Students' Level of Proficiency in Quadratic Functions in Direct Instruction

It can be seen from the table that students of the non-POGIL group before they were taught using direct instruction have low proficiency in quadratic functions (*MPS*=25.8%). Majority (55% or 22) of the students were low proficient while 45% or 18 students were not proficient in quadratic function. It can be noticed that the number of students in not proficient and low proficient before exposure to direct instruction were the same with the number of students before exposure to POGIL.

After their exposure to direct instruction, the proficiency level of the students remained at a low proficiency level (*MPS*=36.5%). Majority (85% or 34) of the students were low proficiency while only few (7.5% or 3) of the students improved their proficiency in quadratic functions. This suggests that direct instruction did not enhance the students' proficiency in quadratic functions.

The result is similar with the study of Artuz and Roble (2021) which revealed that despite an increase in the mean scores of the students in non-POGIL , their scores remained below the expectation level, indicating they did not meet the set standards.

Moreover, Table 6 displays the mean percent scores and proficiency level of the students for each learning competency of quadratic functions in direct instruction.

Learning Competencies	Mean Percent Score		Proficiency Level	
	Pretest	Posttest	Pretest	Posttest
1. Models real-life situations using quadratic functions.	40.6	60.0	Low Proficient	Nearly Proficient
2. Represents a quadratic function using: (a) table of values; (b) graph; and (c)	29.0	39.8	Low Proficient	Low Proficient

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equation.				
3. Transforms the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x – h)^2 + k$.	22.5	24.4	Not Proficient	Not Proficient
4. Graphs a quadratic function: (a) domain; (b) range; (c) intercepts; (d) axis of symmetry; (e) vertex; (f) direction of the opening of the parabola.	22.9	39.4	Not Proficient	Low Proficient
5. Analyzes the effects of changing the values of a, h and k in the equation $y = a(x – h)^2 + k$ of a quadratic function on its graph.	24.4	25.6	Not Proficient	Low Proficient
6. Determines the equation of a quadratic function: (a) a table of values; (b) graph; (c) zeros.	21.8	36.1	Not Proficient	Low Proficient
7. Solves problems involving quadratic functions.	23.6	26.1	Not Proficient	Low Proficient
Overall	**25.8**	**36.5**	**Low Proficient**	**Low Proficient**

Legend: 0-24% (Not Proficient); 25-49% (Low Proficient); 50-74% (Nearly Proficient); 75-89% (Proficient); 90-100% (Highly Proficient)

Table 6. Mean Percent Scores and Proficiency Level of Students in Each Learning Competency of Quadratic Functions in Direct Instruction

Table 6 presents the results of the pretest and posttest given to Grade 9 students who were exposed to direct instruction. It can be noticed that the students' proficiency level for each competency before exposure to direct instruction was the same with the proficiency level of the students in POGIL class in their pretest. Wherein, students were not proficient in transforming the quadratic function; graphing a quadratic function; analyzing the effects of changing the values of a, h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph; determining the equation of a quadratic function; and solving problems involving quadratic functions. Additionally, the students were low proficient in modeling real-life situations using quadratic functions and representing a quadratic function using table of values, graph, and equation.

The students' mean percent scores in their posttest slightly increased for each competency; however, most of them were at low proficiency level. After exposure to direct instruction, the students were still low proficient in representing quadratic functions, graphing quadratic functions, analyzing the effects of changing the values of a, h and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph; determining the equation of a quadratic function; and solving problems involving quadratic functions. Some of the students remained not proficient in transforming the quadratic function defined by $y = ax^2 + bx + c$ into the form $y = a(x - h)^2 + k$. Furthermore, only one of the competencies reached the expected standard where students became

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nearly proficient in modeling real-life situations using quadratic functions. This implies that generally, the students have not yet developed their knowledge and essential skills in solving quadratic functions when they were taught using direct instruction.

B. The Effects of POGIL and Direct Instruction in the Proficiency Level of Students along Quadratic Functions

1. Significant Difference Between Pretest and Posttest of the Interventions

Intervention	Test	N	Mean	SD	t	df	Sig. (2-tailed)
POGIL	*Pretest*	40	13.10	3.05	-24.92	39	0.000
	Posttest	40	27.20	4.56			
Direct Instruction	*Pretest*	40	12.90	2.55	-11.98	39	0.000
	Posttest	40	18.25	4.57			

*$p< 0.05$

Table 7. Significant Difference between Mean Pretest and Posttest in POGIL Approach and Direct Instruction

As presented in the table, the proficiency level of the students after the POGIL intervention ($\overline{x}$=27.20, SD=4.56) was higher than their proficiency before($\overline{x}$=13.10, SD=3.05) they were taught using the POGIL approach in discussing the lesson on quadratic functions. Moreover, the result reveals that the post-test mean score of the students is significantly different from their pre-test mean score, $t(39)$=-24.92, p<0.0. This indicates that the POGIL approach used in teaching Quadratic Functions significantly elevated the students' proficiency level. Thus, the Process-Oriented Guided Inquiry Learning (POGIL) approach is effective in improving students' proficiency in Quadratic Functions. This result is supported by the findings of Artuz & Roble (2021) and Kisworo & Gusman (2019), which showed that POGIL teaching approach significantly improved the mathematics proficiency of the students, especially their critical thinking skills.

On the other hand, students mean scores in pre-test ($\overline{x}$=12.90, SD=2.55) slightly increased in their post-test ($\overline{x}$=18.25, SD=4.57) when they were exposed in Direct instruction. Additionally, there is a significant difference ($t(39)$=-11.98, p<0.0) between the pre-test and post-test scores of the students. However, the mean difference was low, which made their proficiency level remain in low proficiency but the gain is significant. Similarly, In the study of Bug-os & Caro (2019), it was found out that students mean scores in Non-POGIL class increased. The authors added that, regardless of strategy whether it is conventional way of instruction or POGIL approach, it will always provide some learning to the students.

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2. *Comparison in the Level of Proficiency of Grade 9 Students along Quadratic Functions Between Direct Instruction and POGIL*

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	2435.452[a]	2	1217.726	118.018	.000	.754
Intercept	202.729	1	202.729	19.648	.000	.203
PreTest	833.402	1	833.402	80.770	.000	.512
Treatment (Intervention)	1517.962	1	1517.962	147.116	.000*	.656
Error	794.498	77	10.318			
Total	44544.000	80				

a. R Squared = .754 (Adjusted R Squared = .748), *significant at 5% level

Table 8. Analysis of Covariance for Proficiency Level as a Function of the Intervention Using Pretest Scores as a Covariate

Results indicate that after controlling the pretest scores, there is a significant difference between the proficiency level of students in the two groups, one using the POGIL approach and the other direct instruction, $F(1, 77) = 147.12$, $p < 0.05$. The partial Eta Squared value of 0.66 indicates moderate effect size which means that the variance in the proficiency level of Grade 9 students along quadratic functions is explained by 66% of the interventions used, POGIL and direct instruction.

The post hoc test was carried out to determine significant difference in the proficiency level of Grade 9 students along quadratic functions in both interventions.

Interventions	Mean (Unadjusted)	Mean (Adjusted)	Std. Error	Mean Difference	Std. Error	Sig.[b]
Direct Instruction	18.25	18.366[a]	.508	8.718*	.719	.000
POGIL	27.20	27.084[a]	.508			

a. Covariates appearing in the model are evaluated at the following values: PreTest Scores = 13.0000.

b. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

*. The mean difference is significant at the .05 level.

Table 9. Pairwise Comparisons of the Adjusted and Unadjusted Means for Proficiency Level Using Pretest Scores as a Covariate

The estimated adjusted means after controlling the pretest scores in the proficiency level of Grade 9 students along quadratic functions surfaced almost the same with the unadjusted means. This simply tells that there is a significant difference in the proficiency levels of Grade 9 students in quadratic functions taught under POGIL and direct instruction as shown by the mean difference of

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8.718 with $p < 0.05$. For this adjusted means, after removing the effect of the pretest scores, Grade 9 students taught under POGIL approach ($M_{Adj} = 27.08$) have statistically higher proficiency level than the Grade 9 students taught under direct instruction ($M_{Adj} = 18.37$). This further implies that POGIL approach is a more effective in teaching quadratic functions to Grade 9 students than the direct instruction.

This result is consistent with the study of Bug-os and Caro (2019) which found a significant difference in the academic performance of students exposed in POGIL and non-POGIL class. Moreover, Sanggara et al. (2018) and Muhammad and Purwanto (2020) showed that the performance and mathematical problem- solving ability of students who are taught using the POGIL model is better than students who are taught using direct Learning. Also, Datu-Dacula and Anda (2021) found that POGIL approach is more beneficial than direct instruction in raising students' achievement. This further supports the findings of Dinniyah et al. (2022), Samosir (2022), Aiman (2022), and Andriani et al. (2019) that POGIL is an effective teaching method for enhancing students learning.

C. The Enhancement to be Integrated to the Students' Activities in POGIL

The observers gave their comments and suggestions in the given activities during the Process-Oriented Guided Inquiry Learning implementation through observation notes. Their feedback was further elaborated through dialogue with the researcher. Based on their feedback, four themes emerged: (1) Students' Engagement; (2) Students' Higher Order Thinking Skills; (3) Multimedia Presentations; and (4) Time Management.

1. *Students' Engagement*

By actively engaging in the learning process, students develop a deeper understanding of the subject matter and are more likely to retain the knowledge and skills they acquire. Based on the teacher-observers feedback, they indicated that students were actively engaged in the different activities during the POGIL instruction of the quadratic function.

Teacher	Comments/Suggestions
Teacher Gary	*"The review is well-participated by the students. This signifies that the previous lesson is well explained and understood by the students. The previous knowledge is properly used in presenting the new concept/lesson."* *"The students work collaboratively to attain the desired result."*
Teacher Zasha	*"The learners participated well in the recall."* *"Students were very active during recitation and the whole period. Students were able to identify the values of the variable correctly."* *"The results in the group activity can be presented on the board by the students and can become a point of discussion."*

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Teacher Regine	*"Collaboration of the members of each group is evident."* *"It must have been better if during the presentation of answers, the graphs of quadratic functions were shown on the board so that students could easily identify and compare their answers from the rest."*

Table 10. Observers' Comments and Suggestions on the Students' Engagement

During the lesson about graphs of quadratic functions, the teacher only asked the group reporters to read their answers or observations about the effects of changing the values of h, and k in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph. In this activity, the teacher should let the students explain their answers on the board so that students will be more engaged in the lesson. This promotes a dynamic and interactive learning environment, where everyone can contribute their thoughts and perspectives, leading to a deeper understanding of the topic. In the study of Li (2014), it was revealed that student group presentations increased students' retention and success rate about the lesson. The author further added that students were actively engaged in learning the materials instead of listening to the lectures. Moreover, Murillo and Tan (2019) showed that students in inquiry-based learning were more positively engaged than the students who were exposed in the traditional way of teaching.

2. *Students' Higher-Order Thinking Skills (HOTS)*

During the POGIL instruction, students were encouraged to think critically by asking them HOTS questions. Exploration activities were also in HOTS-based questions to develop students' critical thinking and problem-solving skills. The observers noted that incorporating HOTS questions helped the students understand better the lesson.

Teacher	Comments/Suggestions
Teacher Gary	*"The students are asked HOTS questions that enable them to think critically. The teacher processes students' understanding."*
Teacher Zasha	*"The lesson/activity is quite congested but delivered smoothly because the teacher guided the students through asking HOTS questions."*
Teacher Regine	*"Through HOTS questions, the students are able to identify the effect of changing the value of 'a' in the graph of the quadratic function."*

Table 11. Observers' Comments and Suggestions on the Students' Higher-Order Thinking Skills

It can be inferred from the observers' comments that HOTS-based questions foster critical thinking skills, helping students approach mathematical concepts with a deeper level of analysis, leading them to a better understanding of the subject matter. In the study of Brion et al. (2022), it was revealed that POGIL is an effective teaching strategy that successfully improved mathematical

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HOTS of the students. However, the findings of Zana et al. (2022)showed that students had difficulty in solving and answering HOTS-based questions due to low reading interest and diverse student abilities.

3. *Multimedia Presentations*

One of the instructional materials that could help students in grasping the lesson is through the use of multimedia presentations. According to the observers, the visual/video presentations were very useful in teaching the quadratic functions. They helped the students understand the concepts and answer correctly the questions that were asked.

Teacher	Comments/Suggestions
Teacher Gary	*"It must have been better if the students were able to identify the next lesson based on the video presented."*
Teacher Zasha	*"The use of visual presentation of the graph of quadratic function was very useful. It helped the student identify the concepts presented in the previous lesson. The students were able to answer the questions asked by the teacher correctly."* *"It must have been better if the descriptions of the concepts used were also given by the students together with the expected values of each concept."* *"Moreover, while asking for the values of the different properties of quadratic function, concepts must be reviewed and presented on the slide."* *"Students were engaged in doing their task however, some groups were not done in answering some of the questions. The problem and graph can be presented through video so that students can visualize the situation better."*
Teacher Regine	*"The video presentation enhances students' curiosity about the lesson" and that "the presentation of the lesson and its objectives was done smoothly."*

Table 12. Observers' Comments and Suggestions on the Students' Higher-Order Thinking Skills

In the lesson on quadratic function graphs, the identification and video-analysis stages were carried out smoothly through a visual presentation and a GeoGebra-based recorded video. However, in the exploration activity, some of the groups struggled to finish their tasks since they could not visualize properly the problem/situation. After the group activity, the teacher explained the problem with the use of video presentation, the students analyzed better how changing the values of h, and k affect the graph of quadratic functions.

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Murillo and Tan (2019) stated that visual presentation helps learners understand and retain information better by associating ideas, words and concepts with images. The authors' findings also revealed that visual learning tools increased the students' HOT skills. Additionally, the study of Debrenti (2015) showed that the use visual representations in teaching mathematics help students understand the problem and improve their mathematical reasoning. Furthermore, Lalian (2018) revealed that use of videos as a learning media in mathematics plays a role in improving students' motivation in learning, enhancing students' knowledge and understanding of the lesson and improving the students' achievements.

4. *Time Management*

One of the challenges that occurred in the implementation of POGIL is the insufficient time frame. The observers indicated that there were too many objectives and questions in the worksheet that led to unfinished activities of the students. This scenario happened many times in the 16-day instruction of the teacher using the POGIL approach. These activities that were not done served as their assignments and continued the next day.

Teacher	Comments/Suggestions
Teacher Gary	*"Some of the students' activities were not finished due to limited time."*
Teacher Zasha	*"The objectives are too many for a 1-hour class, one objective is enough to cater all the activities."*
Teacher Regine	*"Variations of assessment were provided. However, due to time constraints, some activities were not completely done."*

Table 13. Observers' Comments and Suggestions on the Students' Higher-Order Thinking Skills

Apparently, the number of items in the group activity and evaluation was one of reasons why students did not finish all the tasks within the limited time frame. Peer feedback was not also daily executed due to the numerous activities. Thus, it is better to eliminate the peer-feedback in the daily activities in POGIL and just do it before their summative exam. According to Leong and Chick (2011), time pressure is a real day-to-day classroom experience. Teachers have multiple goals to fulfil in their instructional work which include syllabus coverage, helping students learn the deeper meanings of the content, teaching mathematical reasoning, and encouraging students to participate in discourse about mathematics within limited time.

In using POGIL approach, the teacher should prioritize key concepts and consider the number of learning objectives that can be achieved within a 1-hour instruction. Additionally, the teacher should create activities with realistic time limits to maximize learning. Thus, the teacher should manage the time efficiently when using the POGIL approach, including setting goals and priorities and using time management methods.

From the qualitative responses of the observers, it can be deduced that the POGIL approach increased students' engagement and developed students' critical thinking and problem-solving

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skills through HOTS-based questions. Moreover, the use of multimedia presentations helped the students understand the mathematical concepts and problems properly. However, insufficient time frame became the challenge in the implementation of POGIL wherein some of the activities were not executed within the available time. Thus, POGIL activities used in the study were further improved by prioritizing key objectives, eliminating some of the items, making the given equations easier, allocating time for each activity, and incorporating multimedia presentations in the exploration activity. The table 10 shows the description of activities for 7 learning stages.

Learning Stages	Description of Activities
1. Elicit	This is a short activity that piques students' curiosity, eliciting their prior knowledge related to quadratic functions. This can be done by the students individually or by pair within 2 mins.
2. Engage	This activity is a problem or scenario that is associated to the lesson and can be accomplished within 3 mins. It can also be done individually or by-pair.
3. Explore	In this activity, students delve into the core content. They work on guided inquiry questions that prompt them to explore concepts, apply their knowledge, and formulate their own solutions. Aside from the worksheet to be given, the problem/scenario could also be presented through visual and video presentations. It can be done by groups with four members and can be accomplished within 15 mins.
4. Explain	Following exploration, students articulate their understanding of the concepts. This involves sharing of findings or output to the class. The teacher facilitates the discussion to clarify misconceptions and reinforce ideas. This can be done within 15 mins.
5. Elaborate	The teacher will have a short discussion about the topic and encourage students to participate on the discussion. This can be done within 15 mins.
6. Evaluate	This is an individual activity, following the procedures that were discussed by the teacher. This can be accomplished within 5 mins.
7. Extend	This is a problem-based activity that will let the students apply their learned concepts to a more complex scenario. This will serve as their assignment.

Table 14. Description of the Enhanced POGIL Activities for Each Leaning Stage

CONCLUSIONS

Based on the findings, the following conclusions are drawn:

1. Before instruction, both the POGIL and non-POGIL groups exhibited low proficiency in quadratic functions, with similar mean percentage scores—26.2% for the POGIL group and 25.8% for the non-POGIL group. In both groups, 55% of students were classified as low

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proficient, while 45% were not proficient. After being taught using the POGIL, the POGIL group showed significant improvement in proficiency, with the mean score increasing to 54.4%. A majority of students (75%) reached nearly proficient levels, while only 25% remained in the low proficiency category. Students showed noticeable gains across key competencies such as modeling real-life situations, graphing, analyzing parameter changes, and determining quadratic function equations. These results suggest that POGIL helped students build fundamental knowledge and apply it independently in authentic tasks, though some still needed minimal guidance. In contrast, the non-POGIL group taught through direct instruction showed only a modest increase in mean score (36.5%), and the majority (85%) remained low proficient, with only 7.5% showing improvement. Students continued to struggle with representing and transforming quadratic functions and did not meet the expected proficiency standards. These findings are supported by previous research from Irwanto et al. (2018), Artuz and Roble (2021), Brion et al. (2022), Samosir (2022), Aiman et al. (2020), and Kisworo and Gusman (2029), which consistently highlight POGIL's effectiveness in enhancing student performance. According to these studies, POGIL promotes deeper engagement, better communication, and improved critical thinking and problem-solving skills—leading to more meaningful learning outcomes compared to traditional teaching methods.

2. Students taught using the POGIL approach significantly improved their proficiency in quadratic functions. Their mean score increased from 13.10 (SD = 3.05) in the pretest to 27.20 (SD = 4.56) in the posttest, with a statistically significant difference, $t(39) = -24.92$, $p < 0.05$. This demonstrates that the POGIL method effectively enhanced students' understanding and performance. These findings are supported by Artuz & Roble (2021) and Kisworo & Gusman (2019), who also reported improved mathematics proficiency and critical thinking among students using POGIL. On the other hand, students who received direct instruction showed only a slight increase in scores—from a pretest mean of 12.90 (SD = 2.55) to a posttest mean of 18.25 (SD = 4.57)—although the gain was statistically significant, $t(39) = -11.98$, $p < 0.05$. However, their proficiency level remained low. This aligns with Bug-os & Caro (2019), who noted that while learning occurs under any teaching strategy, the depth of learning may vary. The adjusted mean score for the POGIL group (MAdj = 27.08) was significantly higher than that of the direct instruction group (MAdj = 18.37), confirming POGIL's greater effectiveness.These results are consistent with findings by Bug-os & Caro (2019), Sanggara et al. (2018), Muhammad & Purwanto (2020), Datu-Dacula & Anda (2021), Dinniyah et al. (2022), Samosir (2022), Aiman (2022), and Andriani et al. (2019), all of whom concluded that POGIL enhances students' academic performance, problem-solving skills, and overall learning outcomes more effectively than traditional methods.

3. The observer's feedback for the improvement of POGIL activities in teaching quadratic functions involved students' engagement, development of students' higher-order thinking skills, use of multimedia presentations, and time management in the given activities. Students were actively engaged during POGIL activities, leading to deeper understanding and better knowledge retention. This aligns with findings by Li (2014) and Murillo & Tan

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(2019), who emphasized the benefits of active and inquiry-based learning over traditional lectures. POGIL also encouraged critical thinking through HOTS-based questions, which helped improve students' problem-solving skills. This is supported by Brion et al. (2022), though Zana et al. (2022) noted challenges among students with low reading interest and varied abilities in tackling HOTS tasks. Multimedia tools, especially visual and video presentations, were effective in enhancing student understanding, motivation, and achievement. Studies by Murillo & Tan (2019), Debrenti (2015), and Lalian (2018) support the positive impact of visual learning on comprehension and reasoning in mathematics.However, time management was a notable challenge. Observers pointed out that extensive worksheets and objectives often led to unfinished tasks. This issue reflects broader classroom realities noted by Leong & Chick (2011), where teachers must balance deep learning goals with time constraints.

RECOMMENDATIONS

As manifested in the findings and conclusions of the study, the following are hereby proposed:

1. Mathematics teachers should employ the POGIL approach in teaching mathematics lessons that involve problem-based activities. On the other hand, direct instruction can be used on other topics such as the least-mastered competencies in mathematics.
2. POGIL approach may be used in teaching quadratic functions to improve students' proficiency.
3. The enhanced POGIL activities made by the researcher can be utilized by the mathematics teachers in teaching quadratic functions to Grade 9 students. This may be further developed to fully achieve its objective of assisting students in increasing their proficiency in quadratic functions.
4. Future researchers may also consider the impact of POGIL in other areas of mathematics and science such as trigonometry, geometry, calculus, biology, chemistry, and physics because these subjects offer valuable opportunities for cultivating the students' higher-order thinking skills.

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APPENDIX I

PRE-TEST

Name (Optional):_____Section:_____

MULTIPLE CHOICE. *Directions:* Read each item carefully and choose the correct answer from the options given and then write the letter of your answer on the line provided before the number.

_____1. The height of the object from the ground at time t is h, which is given by, $h(t) = -25t^2 + 15t + 28$. What function best describe the situation?

a. Linear function
b. Quadratic function
c. Radical function
d. Rational function

_____2. Throwing a ball, shooting a cannon, diving from a platform and hitting a golf ball are all examples of situations that can be modeled by _____.

a. Linear function
b. Quadratic function
c. Radical function
d. Rational function

_____3. Which of the following situation does NOT represent a quadratic function?

a. The rock is thrown off a 75 meter high cliff into some water. The height of the rock relative to the cliff after t seconds is given by $h(t) = -5t^2 + 20t$.
b. The area A of a room with width 5m more than the length.
c. The width of a frame in creating a border using all of the 10sq. feet of fabric for a quilt that is 4 feet by 5 feet.
d. The total number of credit hours C that remain for a student to earn a degree after semesters have been completed successfully is given by C = 130 -9s.

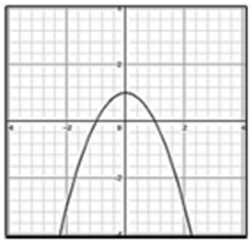
_____4. Which of the following situations represents a quadratic function?

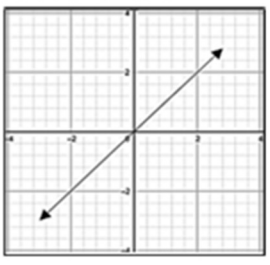
a. The train travels 45 miles every hour after departing the station.
b. The equation $h(t) = -4.9t^2 + 8t + 5$ represents Jeremiah's height (h) in meters above the water t seconds after he leaves the diving board.
c. The amount of money in a bank account increases by 1 percent each year.
d. The cost C of sending m text messages C = ₱1m.

_____5. Which of the following equations is a quadratic function?

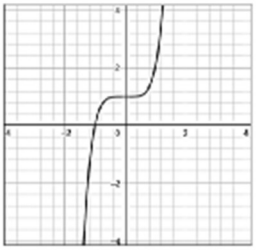
a. $y = 3 + 2x^2$ b. $3 + 2y^2 = x$ c. $y = 3x - 2^2$ d. $y = 2^x - 3$

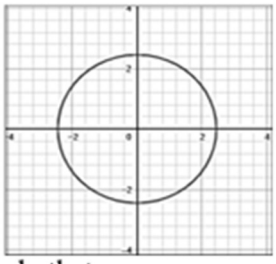
_____6. Which of the following graphs represents quadratic function?

a. 

c. 

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b. 

d. 

_____7. The graph of a quadratic function is a parabola that opens _____only.

a. upward and downward b. to the left c. to the right d. both b and c

_____8. In the quadratic function of the form $y = ax^2 + bx + c$, which of the following must NOT be equal to 0?

a. a b. b c. c d. x-term

_____9. What are the values of a, b, and c in the equation $y + 5 = 2x - 3x^2$, if written in the form $y = ax^2 + bx + c$?

a. $a = 5,\ b = -3,\ c = 2$

b. $a = -3,\ b = -5,\ c = 2$

c. $a = -3,\ b = 2,\ c = -5$

d. $a = -3,\ b = 2,\ c = 5$

_____10. If $y = (x + 2)^2 + 1$ is written in the form $f(x) = ax^2 + bx + c$, find $a + b + c$.

a. 10 b. 12 c. 15 d. 26

_____11. What is true about the equation $y + 2x^2 = 2x(x + 1) - 3$?

a. The equation represents quadratic function because the highest degree of the variable is 2.

b. If you will simplify and rearrange the function, it will be in the form of $y = mx + b$. Therefore, it is not quadratic function, but a linear function.

c. The highest degree of the variable is 3. Thus, it is not quadratic function.

d. If the function is expanded it will be $y = 2x^2 + 2x + 1$. Thus, the equation represents quadratic function.

_____12. Identify the table that represents the graph of quadratic function.

a.

x	-3	-2	-1	0	1
y	0	4	6	4	0

b.

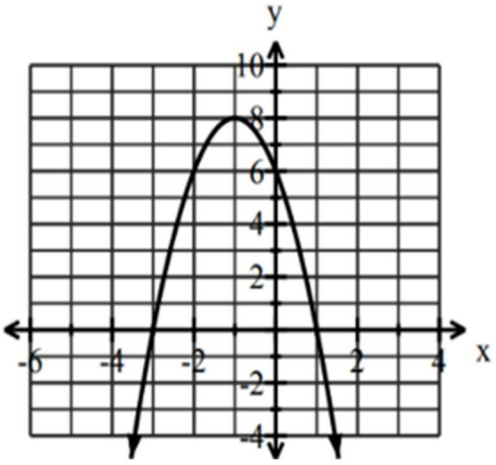
x	-3	-2	-1	0	1
y	0	6	8	6	0

c.

x	-4	-2	0	2	4
y	0	4	6	4	0

d.

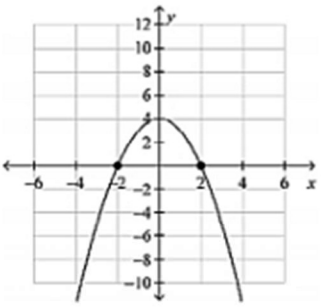
x	-4	-2	0	2	4
y	0	6	8	6	0



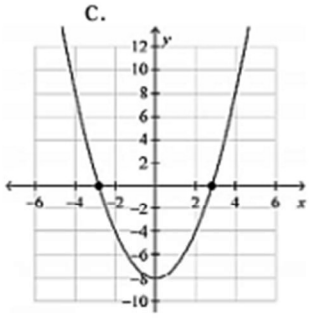
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_____13. Identify the graph that best represent the equation $y = x^2 - 4$.

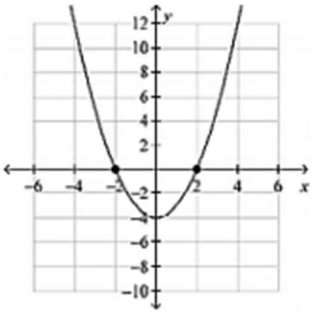
a.



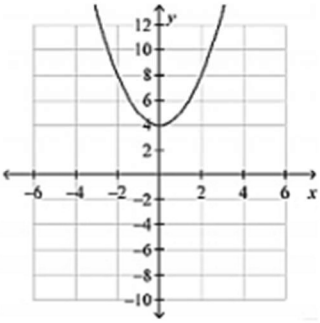
c.



b.



d.



_____14. Which of the following is true about this given table of values?

x	1	2	3	4	5
y	5	12	19	26	33

a. The y values are all positive. Hence, the table represents a quadratic function.
b. The y values are all positive. Hence, the table does not represent a quadratic function.
c. The first and second differences of the y values are equal. Hence, the table represents a quadratic function.
d. The first differences of the y values are already equal. Hence, the table does not represent a quadratic function.

_____15. Which of the following table of values does NOT represent a quadratic function?

a.

x	-2	-1	0	1	2
y	6	3	2	3	6

b.

x	-2	-1	0	1	2
y	5	2	1	2	5

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c.

x	-2	-1	0	1	2
y	4	1	0	1	4

d.

x	-2	-1	0	1	2
y	8	1	0	-1	-8

_____16. Given the equation $y = -2x^2 + 4x + 7$, what value of m and n make the table complete:

x	-3	-2	-1	0	1	2	3	4
y	m	-9	1	7	9	n	1	-9

a. -23 and 7 c. -23 and 1
b. -13 and 7 d. 1 and 7

_____17. What must be the value of c if $y = 3(x-1)^2 - 6$ is transformed into general form?

a. -3 b. 3 c. -9 d. 9

_____18. Express $f(x) = 5x^2 - 10x + 7$, in the form $f(x) = a(x-h)^2 + k$.

a. $f(x) = 5(x-1)^2 + 2$ c. $f(x) = 5(x-2)^2 + 1$
b. $f(x) = 5(x-1)^2 - 2$ d. $f(x) = 5(x-2)^2 - 1$

_____19. The quadratic function $y = x^2 + 2x - 5$, is expressed in vertex form as:

a. $y = (x+1)^2 - 6$ c. $y = (x+1)^2 + 4$
b. $y = (x+1)^2 - 4$ d. $y = (x+1)^2 + 6$

_____20. The quadratic function d $y = (x+3)^2 - 1$, is expressed in general form as:

a. $y = x^2 + x - 6$ c. $y = x^2 + 6x - 8$
b. $y = x^2 + 6x + 8$ d. $y = x^2 + 8x + 14$

_____21. It is the turning point of the parabola, which indicates the highest/lowest point of the graph?

a. Axis of symmetry b. Range c. Vertex d. Zeros

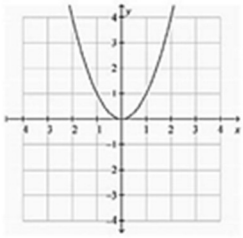
_____22. In the quadratic form $y = a(x-h)^2 + k$, what is the vertex?

a. (a, x) b. (h, k) c. (y, k) d. (y, a)

_____23. What is the value of (h, k) in the quadratic function $y = 2(x-3)^2 + 4$?

a. $(-3, -4)$ b. $(-3,4)$ c. $(3, -4)$ d. $(3,4)$

_____24. Identify the vertex of the graph. Tell whether it is a minimum or maximum.



a. (0,0); minimum c. (0,1); minimum
b. (1,0); maximum d. (2,0); maximum

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_____25. Which quadratic function opens upward and has a vertex of $(0,3)$?

a. $y = -(x-3)^2$ c. $y = (x-3)^2$

b. $y = x^2 + 3$ d. $y = -x^2 + 3$

_____26. Find the equation of the axis of symmetry and the coordinate of the vertex of the graph of $y = x^2 - 8x + 7$?

a. $x = 4; vertex(4,-9)$ c. $x = -4; vertex(-4,-9)$

b. $x = -9; vertex\ (-9,4)$ d. $x = 4; vertex(-4,-9)$

_____27. What is the y-intercept of the parabola defined by the equation $y = (x+2)^2$?

a. $(0,-4)$ b. $(0,-2)$ c. $(0,2)$ d. $(0,4)$

_____28. What is the x-intercepts of the quadratic function $y = 2x^2 - 4x - 6$?

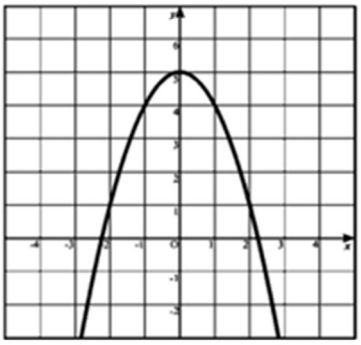
a. $(-1,0),(3,0)$ b. $(1,0),(-3,0)$ c. $(-1,0),(-3,0)$ d. $(-1,0),(3,0)$

_____29. Which quadratic function opens downward and has a vertex of $(0,7)$?

a. $y = -(x-7)^2$ c. $y = (x-7)^2$

b. $y = x^2 + 7$ d. $y = -x^2 + 7$

For items 30-31, refer to the graph at the right.



_____30. What is the range of the parabola?

a. $(-\infty,-5]$ c. $(-\infty,+\infty)$

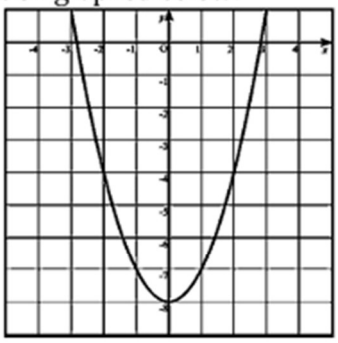
b. $(-\infty,5]$ d. $[5,+\infty)$

_____31. What is the domain of the parabola?

a. $(-\infty,-5]$ c. $(-\infty,+\infty)$

b. $(-\infty,5]$ d. $[5,+\infty)$

_____32. Which statement is correct for the quadratic function graphed below?



a. The graph of the function is $y = x^2 + 8$ with a maximum value of -8.

b. The graph of the function is $y = -x^2 + 8$ with a maximum value of -8.

c. The graph of the function is $y = x^2 - 8$ with a minimum value of -8.

d. The graph of the function is $y = -x^2 - 8$ with a minimum value of -8

_____33. In the quadratic function $y = a(x-h)^2 + k$, if $h > 0$ then the parabola moves:

a. h units downward c. h units to the right

b. h units upward d. h units to the left

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_____34. In the quadratic function $y = a(x-h)^2 + k$, if $k < 0$ then the parabola moves:

a. k units downward
b. k units upward
c. k units to the right
d. k units to the left

_____35. The graph of $y = x^2 - 3$ is obtained by sliding the graph of $y = x^2$:

a. 3 units downward
b. 3 units upward
c. 3 units to the right
d. 3 units to the left

_____36. Which of the following equations of parabola has a wider opening?

a. $y = \frac{1}{4}x^2$ b. $y = \frac{1}{2}x^2$ c. $y = 3x^2$ d. $y = 5x^2$

_____37. What are the zeros of the quadratic function $y = x^2 - 2x - 63$?

a. -9 and -7 b. -9 and 7 c. -7 and 9 d. 7 and 9

_____38. Which quadratic function has no zeros?

a. $y = x^2 - 4$
b. $y = -x^2 - 4$
c. $y = -x^2 + 4$
d. $y = x^2 + 4x$

_____39. Determine the quadratic function if the zeros are 5 and -3?

a. $y = x^2 - 2x - 15$
b. $y = x^2 - 8x + 15$
c. $y = -x^2 + 2x + 8$
d. $y = x^2 + 2x + 15$

_____40. Identify the zeros of the quadratic function $y = x^2 + 5x - 24$?

a. 8 and -3 b. -8 and 3 c. -3 and -8 d. 3 and 8

_____41. Which quadratic function contain the point (1,7)?

a. $y = -x^2 - 2x - 4$
b. $y = -x^2 + 2x - 4$
c. $y = -x^2 - 2x + 4$
d. $y = x^2 + 2x + 4$

_____42. What is the second difference of y-values of

x	-2	-1	0	1	2
y	-3	0	1	0	-3

a. -2 b. -1 c. 1 d. 2

_____43. Use the coordinates from the table to determine which equation models the table below:

X	-2	-1	0	1	2
Y	6	0	-4	-6	-6

a. $f(x) = x^2 - 3x - 4$
b. $f(x) = 3x^2 - 12x - 16$
c. $f(x) = x^2 + 3x - 4$
d. $f(x) = 3x^2 - 6x - 8$

_____44. A farmer has 120 m of fencing. He wants to put a fence around three sides of a rectangular plot of land, with the side of a barn forming the fourth side. What dimensions give this area?

a. 30 m by 60 m
b. 20 m by 30 m
c. 20 m by 40 m
d. 40 m by 60 m

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For question numbers 45-46: From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height S(t) of the ball above the building after t seconds is given by the function $S(t) = 80t - 16t^2$.

_____45. What maximum height will the object reach?

a. 50 ft b. 80 ft c. 100 ft d. 200 ft

_____46. How long will it take the object to reach the maximum height?

a. 1.5 secs b. 2.5 secs c. 3 secs d. 2 secs

For numbers 47-48: A projectile is launched from a point above the ground. The height at ground level is given by the equation $h = -3t^2 + 24t$, where h is the height in meters and t is the time in seconds.

_____47. What maximum height it can be reach?

a. 40 m b. 35 m c. 24 m d. 48 m

_____48. How long will it take to reach the maximum height?

a. 2 secs b. 4 secs c. 3 secs d. 5 secs

For numbers 49-50: A ball is launched upward at 14 m/s from a platform 30 m high. The height of the ball is given by the function $h(t) = -4.9t^2 + 14t + 30$.

_____49. Find the maximum height the ball reaches?

a. 50 m b. 20 m c. 80 m d. 40 m

_____50. How long will it take the ball to reach the maximum height?

a. 2.2 secs b. 1.81 secs c. 2 secs d. 1.43 secs

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APPENDIX II
POST-TEST

Name(Optional): _____Section: _____

MULTIPLE CHOICE. *Directions:* Read each item carefully and choose the correct answer from the options given and then write the letter of your answer on the line provided before the number.

_____1. There are many real-world situations that can be modeled by this function, such as throwing a ball, shooting a cannon, diving from a platform, and hitting a golf ball.

a. Linear function c. Radical function
b. Quadratic function d. Rational function

_____2. The height of the ball from the ground at time t is h, which is given by, $h = -16t^2 + 64t + 80$. What function best describe the situation?

a. Linear function c. Radical function
b. Quadratic function d. Rational function

_____3. Which of the following situation does NOT represents a quadratic function?

a. The employee is paid P570 for each day worked.
b. The maximum area of a rectangular cardboard that can be framed using 140 inches of boarder materials.
c. The rock is thrown off a 75 meter high cliff into some water. The height of the rock relative to the cliff after t seconds is given by $h(t) = -5t^2 + 20t$.
d. The area A of a room with width 5m more than the length.

_____4. Which of the following situations represents a quadratic function?

a. The train travels 50 miles every hour after departing the station.
b. The equation $h(t) = -6t^2 + 10t + 7$ represents John's height (h) in meters above the water t seconds after he leaves the diving board.
c. The amount of money in a bank account increases by 1.5 percent each year.
d. The cost C of sending m text messages C = ₱2m.

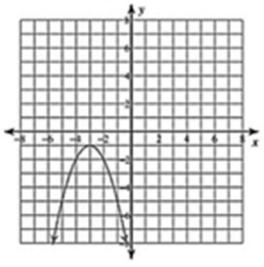
_____5. Which of the following equations is NOT a quadratic function?

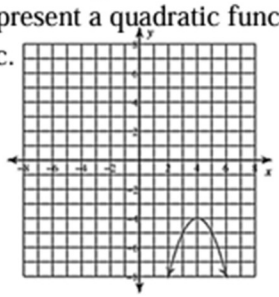
a. $y = 3 + 2x^2$ b. $1 + 5x^2 = y$ c. $y = 3x + 7$ d. $y = x^2 - 1$

_____6. The graph of a quadratic function is a parabola that opens _____ only.

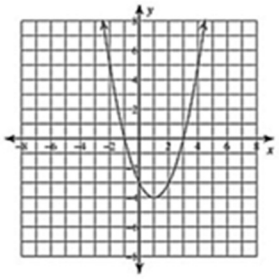
a. upward and downward b. to the left c. to the right d. both b and c

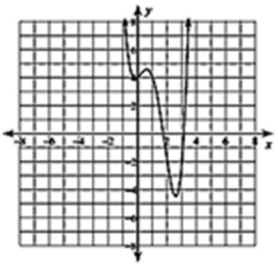
_____7. Which of the following graph does NOT represent a quadratic function?

a. 

c. 

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b. 

d. 

_____8. Quadratic function is any function of the form: $y = ax^2 + bx + c$, where a, b and c are real numbers and _____.

a. $a \neq 0$ b. $a < 0$ c. $a = 0$ d. $a \geq 0$

_____9. If $y - 6 = 1 + x^2 + 3x$ is written in the form $y = ax^2 + bx + c$, find a, b, and c.

a. $a = 1\,,\ b = 3\,,\ c = 7$
b. $a = 1\,,\ b = 3\,,\ c = -5$
c. $a = 1\,,\ b = 1\,,\ c = 3$
d. $a = 3,\ b = 1\,,\ c = 7$

_____10. If $y = (x + 1)^2 + 2$ is written in the form $y = ax^2 + bx + c$, find $a + b + c$.

a. -3 b. 5 c. 6 d. 10

_____11. What is true about the equation $y + x - 1 = 3x(x + 2)$?

a. The equation represents linear function because the highest degree of the variable is 1.
b. If you will simplify and rearrange the function, it will be in the form of $y = mx + b$. Therefore, it is not quadratic function, but a linear function.
c. The highest degree of the variable is 3. Thus, it is not quadratic function.
d. If the function is expanded it will be $y = 3x^2 + 5x + 1$. Thus, the equation represents quadratic function.

_____12. Which of the following is true about this given table of values?

x	-1	0	1	2	3
y	2	1	2	5	10

a. The y values are all positive. Hence, the table represents a quadratic function.
b. The y values are all positive. Hence, the table does not represent a quadratic function.
c. The first difference of y values are not equal but the second differences are equal. Thus, the table represents a quadratic function.
d. The first differences of the y values are already equal. Hence, the table does not represent a quadratic function.

_____13. Which of the following table of values represent a quadratic function?

a.

x	-2	-1	0	1	2
y	2	1	0	1	2

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b.

x	-2	-1	0	1	2
y	2	1	0	-1	-2

c.

x	-2	-1	0	1	2
y	4	1	0	1	4

d.

x	-2	-1	0	1	2
y	8	1	0	-1	-8

_____14. Identify the table that represents the graph of quadratic function.

a.

x	-2	-1	0	1	2
y	0	-3	-4	-3	0

b.

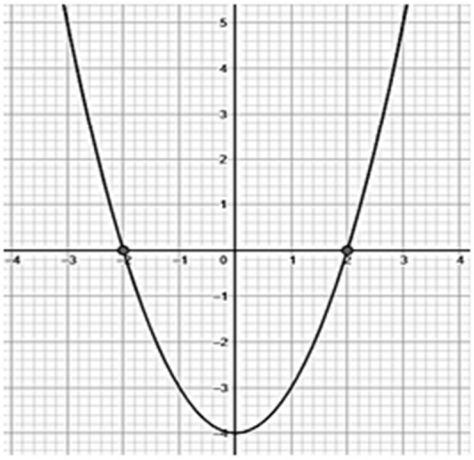
x	-2	-1	0	1	2
y	-3	0	-4	0	-3

c.

x	-2	-1	0	1	2
y	0	4	-3	4	0

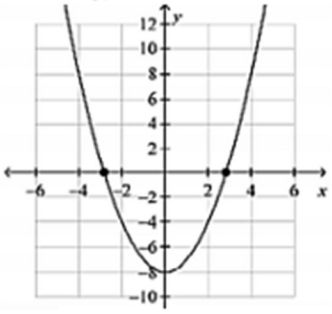
d.

x	-2	-1	0	1	2
y	0	-3	4	3	0

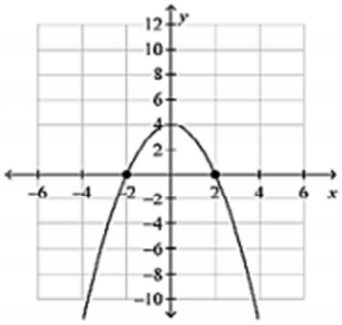


_____15. What is the graph of the equation $y = x^2 - 8$?

a.

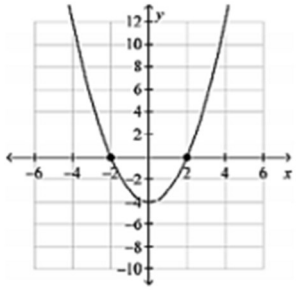


c.

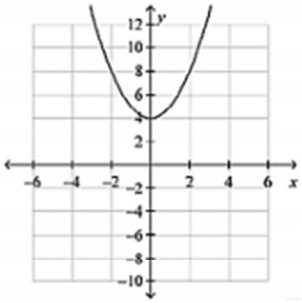


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b.



d.



_____16. Given the equation $y = x^2 + 3x + 11$, what value of s and t make the table complete:

x	-3	-2	-1	0	1	2	3	4
y	11	s	9	11	15	21	t	39

a. 9 and 25 c. -13 and 29
b. 9 and 29 d. -7 and 23

_____17. What must be the value of $\boldsymbol{b}$ if $y = 5(x-1)^2 + 10$ is transformed into general form?

a. -5 b. 5 c. -10 d. 10

_____18. Express $f(x) = 5x^2 - 20x + 21$, in the form $f(x) = a(x-h)^2 + k$.

a. $f(x) = 5(x-2)^2 - 1$ c. $f(x) = 5(x+2)^2 + 1$
b. $f(x) = 5(x-2)^2 + 1$ d. $f(x) = 5(x+2)^2 - 1$

_____19. Express the quadratic function $y = 3(x-2)^2 + 1$ into general form.

a. $y = 3x^2 - 12x + 13$ c. $y = 3x^2 - 8x + 12$
b. $y = x^2 + 13x - 7$ d. $y = x^2 - 11x - 13$

_____20. The quadratic function $y = x^2 - 4x + 3$, is expressed in vertex form as:

a. $y = (x-2)^2 - 1$ c. $y = (x+2)^2 - 1$
b. $y = (x-2)^2 + 1$ d. $y = (x+2)^2 + 1$

_____21. What is the vertex in the quadratic form $y = a(x-h)^2 + k$?

a. (h,k) b. (y,k) c. (a,x) d. (y,a)

_____22. It represents the minimum/maximum value of the graph of quadratic function.

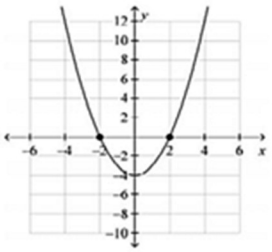
a. Axis of symmetry b. Range c. Vertex d. Zeros

_____23. Which must be the value of (h,k) in the quadratic function $y = 3(x-2)^2 - 5$?

a. $(-2,-5)$ b. $(-2,5)$ c. $(2,-5)$ d. $(2,5)$

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_____24. Identify the vertex of the graph. Tell whether it is a minimum or maximum.



a. $(0,-4)$; minimum c.$(-4,0)$; minimum
b. $(0,4)$; maximum d. $(4,0)$; maximum

_____25. What is the y-intercept of the parabola defined by the equation $y = (x+4)^2$?

a. $(-4,0)$ b. $(4,0)$ c. $(0,-16)$ d. $(0,16)$

_____26. What is the x-intercepts of the quadratic function $y = 3x^2 + 5x - 2$?

a. $\left(\frac{1}{3},0\right),(2,0)$ b. $\left(\frac{1}{3},0\right),(-2,0)$ c. $(3,0),(2,0)$ d. $(3,0),(-2,0)$

_____27. Which quadratic function opens downward and has a vertex of $(0,4)$?

a. $y = (x-4)^2$ c. $y = -(x+4)^2$
b. $y = -x^2 + 4$ d. $y = -x^2 - 4$

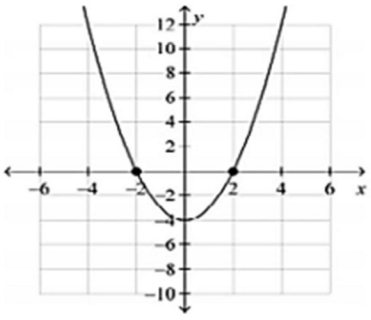
_____28. Find the equation of the axis of symmetry and the coordinate of the vertex of the graph of $y = x^2 + 10x - 3$?

a. $x = 5; vertex(5,-28)$ c. $x = -5; vertex(-5,-28)$
b. $x = -5; vertex\ (-5,28)$ d. $x = 5; vertex(5,28)$

_____29. Which quadratic function opens upward and has a vertex of $(3,4)$?

a. $y = -2(x-3)^2 + 4$ c. $y = -2(x+3)^2 + 4$
b. $y = 2(x-3)^2 + 4$ d.$y = 2(x+3)^2 - 4$

For items 30-31, refer to the graph at the right



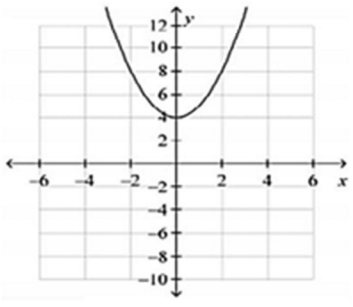
_____30. What is the domain of quadratic function?

a. $(-\infty,-4]$ c. $(-\infty,+\infty)$
b. $(-\infty,4]$ d. $[4,+\infty)$

_____31. What is the range of the quadratic function?

a. $(\infty,-4]$ c. $(-\infty,+\infty)$
b. $(-\infty,4]$ d. $[4,+\infty)$

_____32. Which statement is correct for the quadratic function graphed below?



a. The graph of the function is $y = -x^2 + 4$ with a maximum value of 4.
b. The graph of the function is $y = -x^2 - 4$ with a maximum value of -4.
c. The graph of the function is $y = x^2 + 4$ with a minimum value of 4.

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d. The graph of the function is $y = x^2 - 4$ with a minimum value of -4.

_____33. In the quadratic function $y = a(x-h)^2 + k$, if $h < 0$ then the parabola moves:

a. h units downward c. h units to the right
b. h units upward d. h units to the left

_____34. In the quadratic function $y = a(x-h)^2 + k$, if $k > 0$ then the parabola moves:

a. k units downward c. k units to the right
b. k units upward d. k units to the left

_____35. The graph of $y = x^2 - 6$ is obtained by sliding the graph of $y = x^2$:

a. 6 units downward c. 6 units to the right
b. 6 units upward d. 6 units to the left

_____36. Which of the following equations of parabola has a wider opening?

a. $y = \frac{1}{8}x^2$ b. $y = \frac{1}{4}x^2$ c. $y = x^2$ d. $y = 3x^2$

_____37. What is the second difference of y-values of

x	-2	-1	0	1	2
y	5	2	1	2	5

a. -2 b. -1 c. 1 d. 2

_____38. What are the zeros of the quadratic function $y = x^2 + 4x - 32$?

a. -8 and 4 b. 4 and 8 c. -4 and 8 d. -8 and -4

_____39. Determine the quadratic function if the zeros are -1 and 6?

a. $y = x^2 - 5x - 6$ c. $y = x^2 + 5x + 6$
b. $y = x^2 - 7x + 5$ d. $y = x^2 + 6x + 11$

_____40. Which quadratic function has no zeros?

a. $y = x^2 - 9$ c. $y = -x^2 - 9$
b. $y = -x^2 + 9$ d. $y = x^2 + 9x$

_____41. Identify the zeros of the quadratic function $y = x^2 + 16x + 63$?

a. -9 and -7 b. -9 and 7 c. -7 and 9 d. 7 and 9

_____42. Which quadratic function contain the point (2,5)?

a. $y = x^2 - 2x + 5$ c. $y = -x^2 - 4x + 7$
b. $y = -x^2 + 5x - 3$ d. $y = x^2 + x + 3$

_____43. Determine the equation that represents the table of values below:

X	-2	-1	0	1	2
Y	4	3	4	7	12

a. $f(x) = x^2 - 6x - 4$ c. $f(x) = x^2 + 4x - 2$
b. $f(x) = x^2 + 2x + 4$ d. $f(x) = 2x^2 - 2x - 6$

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_____44. A farmer has 160 m of fencing. He wants to put a fence around three sides of a rectangular plot of land, with the side of a barn forming the fourth side. What dimensions give this area?

a. 30 m by 60 m
b. 20 m by 30 m
c. 20 m by 40 m
d. 40 m by 80 m

For numbers 45-46: A ball is launched upward at 14 m/s from a platform 30 m high. The height of the ball is given by the function $h(t) = -4.9t^2 + 14t + 30$.

_____45. Find the maximum height the ball reaches?

a. 50 m b. 20 m c. 80 m d. 40 m

_____46. How long will it take the ball to reach the maximum height?

a. 2.2 secs b. 1.81 secs c. 2 secs d. 1.43 secs

For numbers 47-48: Elaine shoots an arrow upward at a speed of 32 feet per second from a bridge that is 28 feet high. The height of the arrow is given by the function $h(t) = -16t^2 + 32t + 28$, where t is the time in seconds.

_____47. How long does it take the arrow to reach its maximum height?

a. 1 sec b. 1.5 secs c. 2 secs d. 3 secs

_____48. What is the maximum height that the arrow reaches?

a. 24 ft b. 30 ft c. 44 ft d. 32 ft

For numbers 49-50: A ball is launched from a point above the ground. The height at ground level is given by the equation $S(t) = -3t^2 + 24t$, where S(t) is the height in meters and t is the time in seconds.

_____49. What maximum height it can be reach?

a. 40 m b. 35 m c. 24 m d. 48 m

_____50. How long will it take to reach the maximum height?

a. 2 secs b. 4 secs c. 3 secs d. 5 secs

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APPENDIX III

Observation Notes Form

Observer:_____**Date:**_____

Directions. Please give your general observations and feedback on the student's activities in the seven stages of Process-Oriented Guided Inquiry Learning.

General Observations:

Stages of Learning (Activities)	**Feedback**	**Suggestions for Improvement**
1. Elicit (Linking previous knowledge)		
2. Engage (Identify the need for learning)		
3. Explore (Exploration/Discovery)		

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4. Explain (Understanding and concept formation)		
5. Elaborate (Practice of applying knowledge)		
6. Evaluate (Assessment and Reflection)		
7. Extend (Apply knowledge to new concepts)		

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Cultural Relevance in Mathematics Education Using Indigenous Patterns for Teaching Algebra

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Abstract: The study identified the algebraic patterns present in the cultural artifacts in the mountain ranges of northeastern Luzon, Philippines, and explored their integration into teaching mathematics. Culturally responsive teaching is a practice applied both locally and internationally, designed to strengthen the link between students' cultural context and their academic learning. Qualitative research was applied through an ethnographic approach since it highlights the indigenous community's extensive collection of tangible cultural artifacts. To precisely identify cultural artifacts and their mathematical patterns, ethnographic research uses contextual interviews, passive observation, and archival analysis. It was found that bead patterns, baskets, and pendants all symbolize personal ornaments, livelihood activities, and status symbols, respectively. Additionally, gongs are employed in ceremonies and festivities, and baskets display regional expertise and practical artistry. The community's relationship with the land and sustainable living methods is reflected in architectural styles such as traditional homes. Further, by employing a mathematical investigation approach, the study found algebraic patterns such as sequences and series. The study also established a procedure for integrating the cultural artifacts' patterns into teaching mathematics, particularly in the teacher-education curriculum.

Keywords: Algebraic patterns, ethnomathematics, mathematics education, mathematical investigation

INTRODUCTION

The rich cultural heritage of the indigenous peoples in the mountain ranges of northeastern Luzon, Philippines, offers a unique and valuable perspective on the interplay between mathematics and culture. Indigenous groups such as the *Ifugao, Kalinga, Mountain Province,* and *Isnag* have developed sophisticated systems of knowledge that are reflected in their art, architecture, textiles, and other cultural artifacts (Capua et al., 2022; Hortelano & Lapinid, 2024; Abbacan-Tuguic, 2016). These artifacts often contain intricate patterns and designs that, upon closer examination,

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reveal deep mathematical principles, including algebraic patterns. Despite the evident presence of mathematics in these cultural forms, there is limited exploration into how these patterns can be integrated into contemporary educational practices, particularly in the curriculum for pre-service mathematics teachers (Hortelano & Roleda, 2024).

The study is motivated by the desire to bridge cultural knowledge with formal mathematical education, enhancing the learning experience for future educators (Howley-Rouse, 2023; Llaneta, 2017). Incorporating students' cultural practices into mathematics instruction fosters belonging and enhances understanding, as mathematics is a product of human activity influenced by cultural values (Danoebroto & Suyata, 2024).

Internationally, integrating cultural knowledge into mathematics education is recognized as a way to make learning more meaningful and relevant (Danoebroto & Suyata, 2024). Culturally responsive teaching uses familiar cultural resources to teach knowledge, skills, and attitudes, supporting students in understanding the world around them. Learning becomes more accessible when situated within a student's frame of reference, achieving greater success when taught through their cultural lens (Parker et al., 2017). For instance, research indicates that many families possess mathematical knowledge and skills related to cooking, weaving, and cultural activities, which can provide relevant contexts for teaching mathematics (Howley-Rouse, 2023). In Southeast Asia, AI (Artificial Intelligence) tools in mathematics education are being customized to align with students' cultural backgrounds, including language preferences and traditional learning practices (Payadnya et al, 2024). Ethnomathematics, which connects mathematics and daily life activities, recognizes mathematics as part of a broader knowledge system that helps people understand and interact with the world (Arya et al., 2024; Hamidah et al., 2024). This approach expands mathematical authorship and strengthens the relationship between learners and mathematics by drawing from diverse community resources.

In the Philippines, ethnomathematics is increasingly recognized as a valuable framework for culturally relevant mathematics education (Alangui, 2017). Studies continue to document the ethnomathematics of indigenous communities, such as the *Agta Tabangnon* (one of the communities in the Philippines), highlighting the mathematical concepts embedded in their cultural practices (Briones et al, 2023). Recent research emphasizes the potential of ethnomathematics to create culturally relevant and engaging learning activities, contributing to a more inclusive understanding of mathematics (Abay & Parola, 2024). For example, ethnomathematical concepts found in traditional games can be integrated into mathematics instruction to foster a deeper understanding and appreciation of mathematics among students (Abay & Parola, 2024; Villarin et al, 2024). This aligns with the broader goal of promoting equity and inclusion in mathematics education by connecting mathematical concepts to students' cultural backgrounds and lived experiences.

Culturally responsive mathematics teaching improves student performance, interest, problem-solving persistence, and confidence (Krings, 2019). For pre-service teachers, developing a culturally based approach to mathematical investigation is crucial for understanding how diverse students engage with mathematical concepts (Aga, 2024; Harris et al., 2024). Exposure to culturally responsive teaching methods helps them recognize and value cultural backgrounds while acquiring

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skills to integrate culturally relevant strategies into their practices. Artifacts from various cultures, which often feature intricate patterns and structures, provide valuable starting points for mathematical investigations (Thomas et al., 2024). These patterns, found in crafts, architecture, and indigenous art, offer real-world contexts to explore mathematical concepts like algebraic sequences and series (Lyublinskaya et al., 2024). Incorporating such patterns deepens students' understanding of abstract concepts and makes learning more inclusive and engaging for diverse groups. In culturally rich environments, this approach ensures that lessons are relevant and resonate with students from various backgrounds, promoting equitable learning (Thomas et al., 2024). Although pre-service teachers have few inadequacies in mathematical content knowledge (Dewantara et al., 2024), pre-service teachers who embrace this method are better prepared to create inclusive classrooms where all students feel represented and empowered in their mathematical learning (Aga, 2024; Thomas & Berry, 2019; Rigney et al., 2020).

This research aims to integrate algebraic patterns found in indigenous cultural artifacts into math teaching, offering a culturally relevant context for algebra. By incorporating local knowledge, it enhances the curriculum, highlighting mathematics as a dynamic, culturally connected discipline. With a focus on culturally responsive teaching, the study explores how these patterns can be included in training pre-service math teachers to promote a more inclusive and meaningful approach to education.

The study is guided by the following research questions:

1. What types of cultural artifacts are produced by the indigenous people in the mountain ranges of northeastern Luzon, Philippines?
2. What algebraic patterns can be observed in these cultural artifacts?
3. What material can be designed as a guide for teachers to integrate these culture-based algebraic patterns in teaching mathematical investigation?

Recent research in mathematics education highlights the importance of integrating local and cultural contexts to make math more meaningful for students. Studies have demonstrated the value of incorporating indigenous knowledge, particularly in areas like geometry and number systems. However, effectively integrating these insights into the formal curriculum remains challenging, especially in countries like the Philippines, where many indigenous cultures are underrepresented (Curtis et al., 2021).

REVIEW OF LITERATURE

Ethnomathematics and Indigenous Knowledge

Ethnomathematics provides a framework for understanding how cultural practices, including those of indigenous communities, reflect mathematical concepts (Mairing & Nini, 2024; Alangui, 2017). This approach recognizes that mathematics is not just an abstract discipline but is embedded in various cultural activities and artifacts (Hamidah et al., 2024; Batiibwe, 2024). In the context of indigenous communities in the Philippines, ethnomathematics can be a tool for culturally relevant

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mathematics education (Alangui, 2017; Briones et al., 2023). This involves exploring the mathematical ideas and practices within these communities and integrating them into the formal mathematics curriculum (Magni, 2017; Danoebroto & Suyata, 2024).

Mathematical Patterns in Indigenous Art and Textiles

Indigenous art and textiles often showcase complex geometric and algebraic patterns (Capua et al., 2022; Abbacan-Tuguic, 2016; Golding, 2018). For example, the traditional clothing of the *Ifugao* people in the Eastern Cordillera region features frieze group designs, demonstrating the weavers' ability to create geometric shapes without formal mathematical knowledge (Capua et al., 2022). Similarly, *Kalinga* tattoo designs introduce geometrical concepts such as points, lines, and polygons (Abbacan-Tuguic, 2016). The *Karara* designs of Maranao weaving incorporate algebra, number properties, operations, and geometry (Bitiibwe, 2024). Analyzing fractals in *Navajo* weaving reveals patterns in the Chief's Blanket, Two Grey Hills, and Ganado Red styles.

Integrating Indigenous Knowledge into Mathematics Education

Integrating indigenous knowledge into mathematics education can enhance student engagement, problem-solving skills, and cultural appreciation (Bitiibwe, 2024). By connecting mathematical concepts to familiar cultural contexts, educators can make mathematics more relevant and accessible to students (Hortelano & Roleda, 2024; Mairing & Nini, 2024; Alangui, 2017). This approach aligns with culturally responsive teaching, which uses cultural resources to support students' understanding of the world. For instance, ethnomathematical concepts found in traditional games, such as *Dakon* or *Congklak* (board games generally known as count-and-capture games), can be integrated into mathematics instruction to foster a deeper understanding and appreciation of mathematics among students (Bitiibwe, 2024).

A key challenge is connecting abstract mathematical concepts with the lived experiences of indigenous students. While indigenous students may be familiar with mathematical patterns in cultural artifacts, presenting them to a broader audience requires balancing cultural relevance with mathematical rigor (Sari et al., 2024). Additionally, pre-service math teachers need to develop the pedagogical skills to effectively integrate this approach in the classroom (Çilingir Altıner et al., 2023).

Furthermore, the methodology section in this study details data collection from indigenous communities in northeastern Luzon, analysis of cultural artifacts, and the framework for incorporating these findings into pre-service teacher training. The findings section identifies algebraic patterns in the artifacts and offers curriculum integration recommendations. Finally, the discussion addresses the study's implications for math education and teacher training, suggesting directions for future research and practices in culturally responsive teaching.

In sum, this study aims to uncover the algebraic patterns embedded in the cultural artifacts of Indigenous peoples in the region and to explore how these patterns can enrich the curriculum for pre-service mathematics teachers, contributing to a more inclusive, culturally responsive approach

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to teaching mathematics. The output of this research, although it was not yet implemented as part of the study, was a teacher's guide in integrating culture in teaching mathematical investigation.

METHODS

This section presents the research design, the sources of data, the different instruments and the procedure of collecting the data, and the methods in the analysis of the data.

Research Design

This study used a qualitative research approach. Specifically, an ethnographic design was applied to explore and describe the algebraic patterns in tangible artifacts. This design is well-suited to the study, as it focuses on the region's rich collection of physical cultural artifacts. Ethnographic research involves archival investigation, passive observation, and contextual interviews to accurately uncover cultural artifacts and their mathematical patterns.

Source of Data

The study took place in the mountain ranges of northeastern Luzon, Philippines, focusing on the provinces of Ifugao and Mountain Province. It included the municipalities of Alfonso Lista, Aguinaldo, Mayoyao, and Banaue in Ifugao, and Paracelis in Mountain Province. The study examined physical artifacts from these regions, such as pendants, musical instruments, architectural designs, baskets, and beadwork, to investigate their patterns.

The study conducted interviews with local individuals, including *crafting center owners* and *elders*, and *cultural experts*, to validate observations made during data collection. The elders and experts were senior community members and specialized in local traditions, while the crafting center owners created cultural items like native clothing, wine, tattoos, and traditional houses. The study included participants from dominant ethnolinguistic groups recognized by the National Commission on Indigenous Peoples (NCIP), with 25 participants interviewed. The study identified 25 participants through the NCIP, a government agency that protects and promotes the rights and welfare of Indigenous Cultural Communities and Indigenous Peoples, respecting their beliefs, customs, traditions, and ancestral domains while granting approval.

Instrumentation and Data Collection

The study's main goal was to find and examine physical artifacts by employing precise measurements and reliable measurement methods with instruments like protractors, calipers, and rulers. The researchers labeled sketches or diagrams, recorded these measurements, and examined dimensions, angles, and proportions. To find significant patterns that provide insights into cultural prac-

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tices, craftsmanship methods, and functional roles, they also took into account the larger archaeological context. This methodical approach provided a strong basis for interpreting the cultural significance of these artifacts, and it was reinforced by meticulous documentation and contextual analysis.

Additionally, photographs of the cultural artifacts and observation notes were taken to supplement the interview process. The researchers took a cautious approach to ethical issues, making sure to get people's permission before taking their pictures and honoring their desires for the use and distribution of their photos. The participants were not objectified in any manner, and they were treated with respect.

The study emphasized the importance of respecting cultural context and avoiding misrepresentation or exoticization of a community's identity. The researcher took responsibility for sharing photographs, considering their potential impact, and refraining from exploiting them. This ethical approach contributed to a more inclusive, respectful, and accurate portrayal of documented communities. A Certification Precondition from the National Commission on Indigenous Peoples (NCIP) was obtained for the study's conduct and dissemination.

Moreover, to triangulate the study's findings regarding the artifacts, participants were interviewed. A semi-structured questionnaire was employed to interview the owners of cultural artifact centers, elders, and culture experts. This approach allowed for a more personalized and flexible interview process, facilitating a deeper understanding of the subject matter.

The interview protocol was developed through a thorough process, starting with a thorough understanding of the research objectives and themes. Key areas were identified and participants' information was collected. Questions were designed to be clear, concise, and relevant to the research goals. A pilot interview was conducted, followed by debriefing sessions and revisions. Ethical considerations were taken into account, resulting in a comprehensive tool for gathering valuable data.

Prior to the interview, participants were provided with a letter detailing the nature and purpose of the study, and their consent to participate was obtained.

Data Analysis

A thematic approach was employed to systematically identify, analyze, and report recurring patterns (themes) within the dataset, which included sources such as interviews, surveys, and textual documents.

The mathematical investigation approach was used to analyze algebraic patterns in cultural artifacts from Northeastern Luzon. Also, the thematic analysis of the interview results involves examining recurring patterns and mathematical relationships, integrating archaeology, anthropology, and mathematics to understand their cultural significance. The process begins by compiling a va-

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riety of cultural items, systematically exploring algebraic patterns, and deriving conclusions. Attention is given to algebraic relationships, progressions, and formulas, ensuring no detail is overlooked. The concept of mathematical sequences and series is also considered in the analysis.

RESULTS AND DISCUSSION

The next section shows the summary of the descriptions of the different cultural artifacts found in the regions and the algebraic patterns explored. Also, at the end of this section is the integration of these findings into mathematical investigations for pre-service teachers.

The Cultural Artifacts

Pendants. The *Buung or Linglingo* (Lucky Charm Pendant), as shown in Figure 1, is a handmade artifact found at Barangay Bocos, Banaue, Ifugao. The *Buung* or *Linglingo* is a centuries-old symbol of fertility and luck since the Metal Age of the Philippines from 500-1000 AD. Individually made, each one has slight differences and individual markings. Figure 1(a) is bronze made from extra-small (XS) to extra-large (XL) in size with the measurement in diameter. Figure 1(b) is the *Linglingo* with the mold, and Figure 1(c) is the finished product of silver.



(a) (b) (c)

Figure 1: The Buung or Linglingo of Banaue, Ifugao

Musical Instruments. Figure 2 shows the *Gangha* or Gong artifact of Banaue, Ifugao. According to the respondents, the *Gangha* was made of pure bronze metal and silver nickel used for rituals and celebrations. They made a set of 3, a set of 4, and a set of 7. The set of seven signifies the '*do, re, mi, fa, so, la, ti, do'* sequence of making the *Gangha*. Moreover, it has been mentioned by the maker that the first *Gangha* obtained a thickness of 2.5 mm and the rest measured a thickness of 2mm.

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Figure 2: The *Gangha* or Gong of Banaue, Ifugao

Basketry. In Figure 3, it shows the *Pallungan* or a rectangular winnowing tool crafted from rattan or *anoh* (a type of climbing plant from the palm family). Its dimensions span 58 cm x 58 cm at the top and 47 cm x 47 cm at the bottom. This essential implement is employed by residents to exactly cleanse rice by removing impurities and husks. The top view (Figure 3(a)), side view (Figure 3(b)), and focused image (Figure 3(c)) provide insight into the intricate weaving technique employed in its construction, showcasing the skilled craftsmanship involved. The use of rattan or *anoh* underscores not only the resourcefulness of indigenous materials but also the preservation of traditional techniques passed down through generations. The *Pallungan* serves as a tangible link between past practices and contemporary household needs, embodying both practical functionality and cultural significance.



(a) (b) (c)

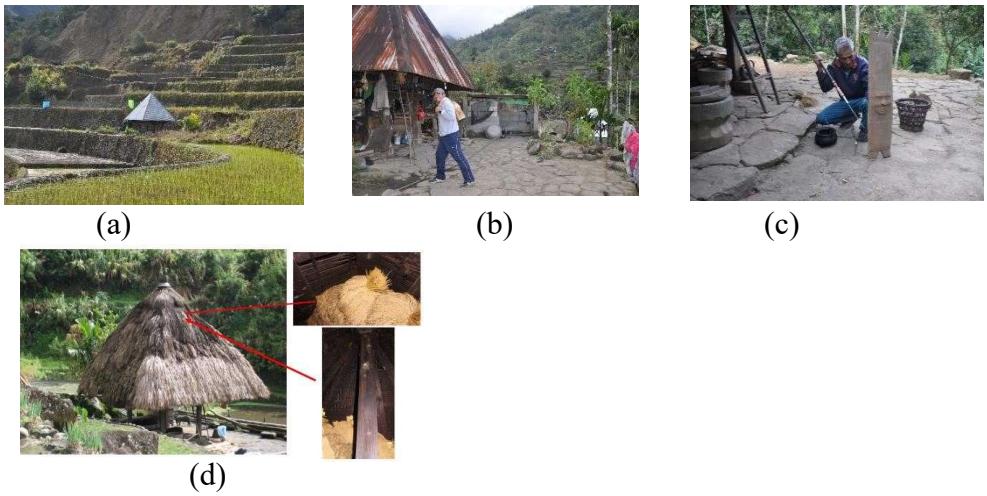
Figure 3: The *Pallungan* or a rectangular winnower of Barangay Maga

Additionally, there are other baskets with the same pattern found in both places Mayoyao and Aguinaldo such as the *Taga'e* (basket plate), *Hape'eng* (ritual object), *Labfa* (necklace), *Inomon Hape'eng* (traditional backpack), *Ho-op* (woven basket), *Inomon Pasikeng* (traditional backpack), and *Upping* (lunch box).

Architectural Designs. Figure 4 shows the modernized native house/ native house or *baleh* at Mayoyao, Ifugao. Figure 4(a) reflects the complete image of the modernized/ galvanized native

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house at a distance. Figure 4(b) and Figure 4(c) depict the surroundings of the native house. Figure 6(d) shows where in the attic of the native house. There are three levels of the *baleh.* The first level is the *daulon* or ground floor. Moreover, it has a multi-function. It is an open area within the level of the yard or *dolah*. The area for pounding rice is beside this *daulon* shaded by the roof. The *luhung* or mortar and *al-u* or pestle are used for pounding rice. The second level is the *bohongna* or the one-room quarter, which is accessible through a removable ladder. The family cooks, eats, and socializes inside this area. The third level is the *palah* or attic above the floor, purposely for *palay* (rice plant) storage. However, the researcher found the *patye* - a shelf extending from the wall boards to the underside of the roof. It serves to safely keep and organize the things and gadgets of the family, as well as provide structural support to the roof.



(a) (b) (c)

(d)

Figure 4. The Native House of Barangay Maga

The outer part of the house is embellished with rows of skulls of animals offered to the gods during their annual rituals, which are also a source of pride to the homeowner. The *baleh* type is considered the typical Ifugao native house. Residents mentioned the rice terraces and *baleh* caught the attention of pioneering Western anthropologists who came to the Philippines even before the Second World War.

Using Geometry and Trigonometry, indigenous peoples in the province determined the appropriate shapes of the structure of the house, such as a pyramid-shaped roof that lets strong typhoons pass by without impact on the structure, as well as the conservation of energy. The appropriate angle of inclination of the wall of about 30 degrees, and the roof that varies from 45 degrees to 60 degrees, were special angles of significance to the Egyptians when building their pyramids. They observed the perpendicularity of the king post to the center beam for the stability of the roof. The squareness of the *ambubulan* (central roof hub) near the apex of the pyramid roof has to be established and well-fastened to resist typhoons from twisting the roof frames since all the rafters are fastened to it. The strength of the four *bughol* (the main rafters) that formed the edges of the pyramid must be well fastened at the apex.

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Moreover, measurements of lengths, heights, and appropriate distances, for instance, of the rafters and the purlins, were estimated. The construction of the roof or the putting of the *runo* leaves (reed leaves) of the roof follows a mathematical pattern of sequence and series from the base until it reaches the apex of the pyramid. The *runo* or cogon is tied up in smaller bundles and fastened on the purlins (Figure 5).



Figure 5: The Roof of Native House in Mayoyao, Ifugao

Beadworks. The beadworks of the *Ga'dang* Community in Paracelis, Mountain Province contain double *ansulam or antalasik* (full diamond) design and *ansulam or antalasik* (half diamond). Each full and half diamond is formed by positioning the beads per row, making a minimum of 2 rows for the smallest half diamond and a minimum of 3 rows for the full-figured diamond.

The *Bakwat* (usually a belt) as shown in Figure 6 is a textile to enclose the skirt so that it will not be fall and designed with colorful beads. The colors of the beads are red, yellow, white, and black. Figure 6(a) shows the whole artifact of the *Bakwat*. Also, Figure 6(b) is the focus design of the *ansulam/antalasik* beads in its center and Figure 6(c) shows both ends of the design.



(a) *(b)* *(c)*

Figure 6: The Bakwat of the Ga'dang in Paracelis

Figure 7 shows the *Hambag* (bag) of the Ga'dang in Paracelis, Mountain Province. The Figure 7(a) is the pouch used where they can put facial make-up paraphernalia, whereas Figure 7(b) is the body bag used to put cellphones and other personal materials. Both of the bags embedded

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beads with geometric designs such as *ansulam/antalasik* (triangular), double *ansulam/antalasik* (diamond), and *lappaw*-circular beads formation.



(a) (b)

Figure 7: The Hambag of Ga'dang in Paracelis, Mt. Province

Furthermore, other similar patterns of beads from Alfonso Lista and Paracelis are *Abag* (loincloth/use by males to cover their private parts), *Kuton* (male vest and it is shorter than cheleko), *Barangal* (look like a handkerchief use for headdress or to wrap in the shoulder for males), *Tapit* (cape/kappa that wears by the males which signifies that he is a king or prince), *Aken (Innammata and lallad)* (skirt used by females), *Barawasi (Innammata and lallad)*(blouse use by females with long sleeve), *Sayay* (betel nut pouch use both men and women), *Singat* (forehead dress worn by ladies), *Tarabbung* (modernized hat for male and female for sun protection), *Bayoyung* (head band used by females), and *Cheleko* (modernized vest/ dress for males. It is longer than a kuton).

Indigenous artifacts showcase craftsmanship, creativity, and worldview, connected to environment, spiritual beliefs, and social organization (Cameron, 2024). Integrating these cultural elements into classrooms can make mathematics more relevant for indigenous students (Kadonsi, 2024; Sari et al., 2024). Teachers can highlight the connection between everyday cultural practices and mathematical concepts through patterns like symmetry, geometry, and counting systems (Agbata et al., 2024).

Indigenous students benefit from incorporating their cultural heritage into their education, as it validates their identity and makes learning more meaningful (Lunda et al., 2024). Further, by analyzing geometric patterns and mathematical concepts, they see the practical application of these concepts in their lives, fostering a sense of pride and active participation (Agbata et al., 2024; Xu & Ball, 2024). Additionally, incorporating local cultural artifacts into the curriculum enhances cultural awareness, inclusiveness, and appreciation for indigenous cultures, promoting understanding and appreciation among non-indigenous students (Kurniawan et al., 2023).

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Algebraic Patterns

In Table 1, the algebraic patterns are embedded in the different cultural artifacts, specifically on pendants, musical instruments, baskets, architectural roofing, and bead designs.

The Algebraic Patterns	Places and the Artifacts
$a_n=a_1+(n-1)d$ where $n\epsilon[1,+\infty)$	**Banaue** Buung/ Linglingo
$a_n = a_1 + (n - 1)d$ where $(1 \leq n \leq 7)$	**Banaue** Gangha (Gong)
$3 + 5 + 7 + \cdots + (2n - 1) = n^2+2$	**Mayoyao** Pallungan (winnower), Taga'e (Basket Plate), Hape'eng, Labfa, Inomon Hape'eng, Hag'cha, Ho-op, Uppig (Lunch box) **Aguinaldo** Pallungan (rectangular and circular winnower), Taga'e, Inamon/Inomon Pasikeng
$3 + 5 + 7 + \ldots + (2n + 1) = n^2 + 2n$	**Mayoyao, Ifugao** Native House Baleh
Half Diamond (ansulam/antalasik design) $1+2+3+4+5+\ldots+n=\frac{n(n+1)}{2}$ Full Diamond (double ansulam/antalasik design) $1 + 3 + 5 + \cdots + (2n - 1) = n^2$	**Alfonso Lista** Abag (Loincloth), Barangal, Tapit, Aken, Barawasi Bakwat, Sayay (betel nut pouch) **Paracelis** Abag (Loincloth), Kuton, Barangal, Tapit, Aken (Innammata and lallad), Barawasi(Innammata and lallad), Bakwat (Innammata and lallad), Sayay, Singat, Tarabbung, Bayoyung, Cheleko, Camisola, Hambag (pouch/body bag)

Table 1: The Algebraic Patterns that are found in the Artifacts of the Region

It is evident how deeply ingrained mathematical knowledge is in indigenous practices when these algebraic patterns are found in cultural artifacts. These trends are not random, rather, they represent the community's perceptions of time, social structures, and the natural world (Sari et al., 2024; Xu & Ball, 2024). Indigenous peoples have long used mathematical reasoning in their cultural practices, albeit in ways that are frequently disregarded in formal education systems, as evidenced by the use of such patterns in artifacts (Huencho & Chandia, 2023).

Development of a Culturally Based Approach to Mathematical Investigation

The next section provides the sample procedure of this study for using mathematical investigation to come up with a conjecture from the cultural artifacts. It was found that the patterns explored can be a basis for integrating culture when teaching in the classroom. According to Kurniawan et al. (2023), patterns found in cultural artifacts can be used as a basis to integrate culture into classroom teaching.

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On pendants, Table 2 shows the first five sizes and the diameters in inches of the *Linglingo*. The table presents the expanded form using the common difference from the 1st size a_1 to the 5th size a_5.

a_n *(where n is the size)*	Diameter in inches	Expanded form	a_n with respect to *n* and *d*
a_1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$} + left (0 right) left ({{1} over {
a_2	$\frac{5}{8}$	$\frac{3}{8}$} + {{1} over {4}} ^ {	$\frac{3}{8}$} + left (1 right) left ({{1} over {
a_3	$\frac{7}{8}$	$\frac{3}{8}$} + {{1} over {4}} ^ { $+\frac{1}{4}$	$\frac{3}{8}$} + left (2 right) left ({{1} over {
a_4	$1\frac{1}{8}$	$\frac{3}{8}$} + {{1} over {4}} ^ { $+\frac{1}{4}$} + {{1} ove	$\frac{3}{8}$} + left (3 right) left ({{1} over {
a_5	$1\frac{3}{8}$	$\frac{3}{8}$} + {{1} over {4}} ^ { $+\frac{1}{4}$} + {{1} ove	$\frac{3}{8}$} + left (4 right) left ({{1} over {
⋮	⋮	⋮	⋮
a_n			$a_1+(n-1)d$

Table 2: Sample exploration of the data gathered from the *buung* or *linglingo* pendants

Based on the exploration made, the following conjecture was found:

> *Let a_n be the nth size of the Linglingo, a_1 be the first size of the buung or linglingo, n be the rank of the linglingo (XS-XL…) or $n\epsilon[1,+\infty)$, and d is the common difference of the sizes. The nth size of the buung/linglingo equals the first linglingo plus one less than the order of the linglingo times the common difference. Its algebraic notation is $a_n=a_1+(n-1)d$.*

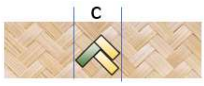
Thus, the sequence $a_n=a_1+(n-1)d$ where $n\epsilon[1,+\infty)$, is apparent in *Buung or Linglingo,* or a lucky charm pendant.

By exploring the other patterns in these artifacts, students deepen their understanding of math while gaining insight into the cultural significance behind the artifacts. This approach not only enhances engagement but also fosters cultural awareness, showing how math is used across different cultures and making learning more inclusive and relevant (Putra & Mahmudah, 2021; Arya et al., 2024; Danoebroto & Suyata, 2024).

Moreover, according to Pradhan (2020), cultural artifacts embed mathematical ideas and knowledge. Table 3 presents a comprehensive summary of artifacts, their associated investigative tasks, and the conjectures explored through them. These investigative tasks can be assigned by

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teachers to mathematics pre-service teachers as part of an activity focused on teaching mathematical investigation. This approach not only models the process of integrating culture into teaching but also immerses students in mathematical investigation within a context that resonates with their lived experiences (Mairing & Nini, 2024; Hunter & Miller, 2022). According to Magni (2017), these practices provide opportunities for pre-service teachers to develop mathematical concepts beyond the classroom (Çilingir Altıner et al., 2023). By studying artifacts, pre-service teachers explore mathematical ideas and appreciate how mathematics is developed and used across cultures (Kurniawan et al., 2023). With the teacher acting as a facilitator, students will be guided through the process of formulating conjectures (Wang, 2024).

Cultural Artifacts	Investigative Tasks	Conjecture
The Buung or Linglingo of Banaue, Ifugao	The *Buung* or *Linglingo,* of Banaue, is a circular pendant with increasing sizes from extra small (XS), small (S), medium (M), large (L), and extra-large (XL) and a diameter in inches 0.375, 0.625, 0.875, 1.125, and 1.375 respectively. Investigate the diameter as the size of the pendant increases! 	*Let* a_n *be the nth size of the Linglingo,* a_1 *be the first size of the buung or linglingo,* n *be the rank of the linglingo (XS-XL…) or* $n \in [1, +\infty)$*, and* d *is the common difference of the sizes. The nth size of the buung/linglingo equals to the first linglingo plus one less than the order of the linglingo times the common difference. It is algebraic notation is* $a_n=a_1+(n-1)d$.
Gangha or Gong artifact of Banaue, Ifugao	One of the musical instrument found in Banaue, Ifugao is the *Gangha* or commonly known as Gong. The first four sizes of the Gangha in inches are 13, 12.5, 12, and 11.5. These sizes are aligned to the first four pitches *do, re, mi*, and *fa.* As the size becomes smaller, the pitch also becomes higher. Investigate! 	*Let* a_n *be the nth diameter size of the gangha* a_1 *be the first size of the gangha,* n *be the order/rank of the gangha* $(s.t.\ 1 \leq n \leq 7)$*, and* d *is the common difference of the sizes. The nth size of the gangha equals to the first gangha size plus one less than the order of the gangha times the common difference. Its algebraic notation is* $a_n = a_1 + (n - 1)d$
Pallungan or a rectangular winnower	Among the baskets found in Mayoyao and Aguinaldo, a rectanglular winnwer called *Pallungan*, was crafted from split bamboo poser (*anoh*) and featured a rattan rim constructed in a one-over-two twill pattern.  Let  be x and let be y. Let  be $x \oplus y_n$, $n \in \{1, 2\}$, where there are two y's overlapping in x. The columns (c) in the design will be defined as the arrangement of $x \oplus y_n$ vertically. The symbol	*Let*  *be* x *and let* *be* y. *Let*  *be* $x \oplus y_n$, $n \in \{1, 2\}$, *where there are two y's overlapping in x. The columns (c) in the design will be defined as the arrangement of* $x \oplus y_n$ *vertically. If* $x \oplus y_n$ *is arranged by tilling it in columns where the number of x increases by 1 as the number of columns* (n) *increases from* $n > 1$*, then the total number of* x *and* y *is* $3+5+7+...+(2n+1)=n^2+2n$

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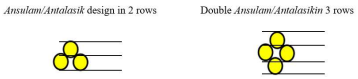
	⊕ is used to get the sum of x's and y's as it is tilled to form the basket. Arranged $x \oplus y_n$ by tilling it in columns where the number of x increases by 1 as the number of columns (n) increases from $n > 1$. Investigate the arrangement!	
Modernized native house/ native house *baleh* at Mayoyao Ifugao	The *baleh* type is considered as the typical Ifugao native house. A runo was used in its roof. The runo is tied up in smaller bundles and fastened on the purlins.  Let  be x where x is a small bundle of cogon or runo leaves. The roof of the native house *baleh* is constructed through small bundles of *runo* or cogon leaves (Note: we let 3 small bundles start from the apex down). Starting from the first and the highest row, down to the 2nd, 3rd ,4th , and 5th have a total number of bundled *runos* of 3, 8, 15, 24, and 35 respectively. Investigate the number of small bundles of *runo* leaves in each row.	*Let*  *be* x*-small bundle of cogon or runo leaves. Let*  *be the three small bundles of cogon or runo leaves where the* x*'s first arranged in a row* $x \oplus x_n$*. The row in the design will be defined as the arrangement of* $x \oplus x_n$ *horizontally. If* $x \oplus x_n$ *is arranged by connecting it in rows where the number of rows* (n) *increases from* $n > 1$*, then the total number of* x*- small bundle of cogon or runo leaves is* $3+5+7+...+(2n+1)=n^2+2n$
Bakwat and Hambag in Paracelis, Mountain Province	The beadworks of the *Ga'dang* Community in Paracelis Mountain Province contain double *ansulam* or *antalasik* design (full diamond) and *ansulam* or *antalasik* (half diamond). Each full and half diamond is formed by positioning the beads per row, making a minimum of 2 rows for the smallest half diamond and a minimum of 3 rows for the full-figured diamond (Note: We let 1 bead to form the smallest half and full diamond).  As the textile enlarges depending on the person wearing the material, the number of rows will also be increased as possible. Investigate the number of beads required by the ansulam or antalasik and double ansulam or antalasik constructed in the textile as the number of rows increases.	*a. Let n be the number of rows and h for the total number of beads for a half diamond (ansulam/antalasik design). If the beads are arranged in rows, where n increases from* $n \geq 1$*, then the total number of beads for half diamond (ansulam/antalasik design) (h) is* $1+2+3+4+5+ \ldots +n=\frac{n(n+1)}{2}$. *b. Let n be the number of rows, and f for the total number of beads for a full diamond (double ansulam/antalasik). If the beads are arranged in rows, where n increases from* $n \geq 1$*, then the total number of beads for a full diamond (double ansulam/antalasik)* (f) *is* $1+3+5+...+(2n+1)=n^2$

Table 3: The summary of artifacts with their corresponding investigative tasks and the conjectures investigated

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The Teacher's Guide for a Culturally-Based Mathematical Investigation

The findings of this study on the algebraic patterns found from the different artifacts were used in generating mathematical investigation tasks with their corresponding conjectures. Now, these concepts were concretized by converting them into a Teacher's Guide (see Appendix). The preliminary part gives information about the subject code, title, topic, faculty, and time frame. The preliminary part is followed by a 3-part format: Objectives, Procedure, and Evaluation. The Objectives tell what the students should learn after the lesson, and the Evaluation gives the students assessment activities. Further, the Procedure is divided into motivation, discussion, and generalization. Under the discussion part, the teacher will initiate a discussion about the nature of the artifacts, enabling students to engage meaningfully with the topic. Following this, the teacher will present and explain the investigative task, linking it to the prior discussion of the artifacts. Afterward, the students will be allowed to conduct the investigative activity, where they will organize and analyze the data to identify patterns. The patterns they uncover will guide them in formulating conjectures (Wang, 2024).

Next, because students often show difficulty in proving conjectures, the teacher will facilitate this process, ensuring that students understand the steps involved in mathematical reasoning (Hartono et al., 2024; Farkash et al., 2024). Finally, upon validating the conjecture, the teacher will encourage students to extend their findings, applying their insights to further explore the artifacts under investigation. This approach fosters a deeper understanding of both the mathematical concepts and the cultural context embedded in the study of the artifacts (Wang, 2024). In the generalization part, the students will be asked different questions about the artifacts discussed and the patterns discovered during the sessions.

CONCLUSIONS

The artifacts explored in the study are all important symbols of a community's identity, values, and traditions. Pendants, bead designs, and baskets serve as fortune bringers, personal adornments and status markers, and represent livelihood activities, respectively. Musical instruments like gongs are used in rituals and celebrations, while baskets showcase local knowledge and functional artistry. Architectural designs like traditional houses reflect the community's relationship with the land and sustainable living practices.

In different areas of the region, artifacts such as pendants and musical instruments were found to exhibit consistent and repeating patterns that align with mathematical sequences. For example, the geometric arrangements in beadwork in half and full diamonds often followed a predictable order, with colors or shapes repeating in a manner that mirrors the arithmetic series. Likewise, the intricate patterns in woven baskets often adhered to principles of symmetry and repeated units, which can be linked to concepts of the sum of odd numbers ($n \geq 3$). A similar pattern was explored in the roofing of the native houses.

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The results describe a teaching approach that integrates cultural artifacts with mathematical investigation, making math more engaging and meaningful by linking it to students' cultural backgrounds. Teachers lead students through discussions of the artifacts, introduce investigative tasks, and guide them in analyzing data to uncover patterns that form the basis for conjectures. Through supporting the process of proving conjectures and expanding the investigation, teachers help students strengthen their critical thinking and mathematical reasoning, while deepening their appreciation for both mathematics and the cultural relevance of the artifacts. This method creates a dynamic, context-rich learning environment that promotes both mathematical exploration and cultural understanding.

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APPENDIX

Sample Teacher's Guide in Integrating the Cultural Artifacts into Teaching Mathematical Investigation

1

Teacher's Guide
1st Semester, S.Y 2024, 2025

Subject: SEME 109. Problem-solving, mathematical investigation, and modeling
Topic: Mathematical Investigation (*Basketry of Mayoyao and Aguinaldo*)
Faculty: *TBA*
Time Frame: 2 sessions/ 1 hour per session

I. Objectives:

At the end of the lesson, the students should be able to:

1. describe the native baskets of Mayoyao and Aguinaldo;
2. identify distinct patterns found in the baskets of the region; and
3. apply the mathematical investigation activity to the algebraic pattern found in the baskets.

II. Procedure:

Session 1

A. Motivation

Review. Ask the following questions:

1. Provide a description of the Gangha from Banaue, Ifugao. What materials was it crafted from? How many pieces typically make up a set of Gangha? Share details about its sizes.
2. What pattern has been identified in the Gangha? Presentation of the topic.

Present the topic Mathematical Investigation of the Basketry of Mayoyao and Aguinaldo

B. Discussion

Localizing/ contextualizing

1. Distribute a copy of the material on the Basketry of Mayoyao and Aguinaldo (see Annex A) to each student and have them read and study it.
2. After the students read the material, ask the following questions (*try to catch their interest by relating the material to their experience in their community*):
   a. What types of baskets are commonly found in Mayoyao and Aguinaldo?
   b. How are each of these baskets used in daily life? What roles do they serve within the community?
   c. What materials are typically used to make these baskets? How are they crafted? Do you know anyone in your community who makes these baskets, and have you ever observed the process?
   d. Which of these baskets can you find in your own home? How do you use them, and who in your household typically uses them?

Data gathering

3. Show common images found in the baskets:



   a. Instruct the students to examine the images and identify any common patterns among the baskets (guide them through this activity). Allow time for them to analyze the images individually. Afterward, pair them up so they can discuss their observations and share initial thoughts. Next, have each student find a new partner

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to further discuss and refine their ideas. Finally, ask 3 to 5 students to share the patterns they've identified with the class, and encourage the others to reflect on whether they observed similar patterns.

b. Help them convert their ideas into mathematical expressions.

4. Evaluation (*for session 1*)
Essay: In 200 words, describe one type of basket found in the locality and discuss their roles in the community.

Session 2

Presentation of the Investigative task:

5. Present to the students the following task:

"Among the baskets found in Mayoyao and Aguinaldo, a rectangular winnower called Pallungan, was crafted from split bamboo poser (anoh) and featured a rattan rim constructed in a one-over-two twill pattern.



Let  *be* x *and let*  *be* y. *Let*  *be* $x \oplus y_n$, $n = \{1, 2\}$, *where there are two y's overlapping in x. The columns (c) in the design will be defined as the arrangement of* $x \oplus y_n$ *vertically. The symbol* $\oplus$ *is used to get the sum of x's and y's as it is tilled to form the basket. Arranged* $x \oplus y_n$ *by tilling it in columns where the number of x increases by 1 as the number of columns (n) increases from n > 1. Investigate the arrangement!"*

6. *Exploring systematically*
Lead the students to come up with this observation (facilitate this *activity*):

Number of columns (n)	Figure formed	$x \oplus y_n$
1		3
2		8
3		15
4		24
⋮	⋮	⋮

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Guide the students to complete the table where $n \geq 1$:

Number of columns (n)	Sum per column	$x \oplus y_n$,
1	$3 = 3$	$1^2 + 2(1) = 3$
2	$3 + 5 = 8$	$2^2 + 2(2) = 8$
3	$3 + 5 + 7 = 15$	$3^2 + 2(3) = 15$
4	$3 + 5 + 7 + 9 = 24$	$4^2 + 2(4) = 24$
⋮	⋮	⋮
n	$3 + 5 + 7 + \ldots + 2n + 1 = n^2 + 2n$	$n^2 + 2n$

Ask the students to find $x \oplus y_n$, when $n = \{5, 6, 7\}$, and try to verify them by drawing the figure formed by x's and y's.

7. *Writing the conjecture*
   Guide the students to come up with the conjecture:

   Let  *be x and let*  *be y. Let*  *be* $x \oplus y_n$, $n = \{1, 2\}$, *where there are two y's overlapping in x. The columns (c) in the design will be defined as the arrangement of* $x \oplus y_n$ *vertically. If* $x \oplus y_n$ *is arranged by tilling it in columns where the number of x increases by 1 as the number of columns* (n) *increases from* $n > 1$, *then the total number of x and y is*

   $$3 + 5 + 7 + \ldots + (2n + 1) = n^2 + 2n$$

8. *Proving the conjecture*:
   Ask the students how they would go about proving the conjecture. Guide them through the process of proving it using mathematical induction (*remind them of the steps involved*). Begin with the base case, demonstrating that the statement holds true for the initial value, typically $n = 1$. Next, lead them through the inductive step, where they assume the statement is true for $n = k$ (*the induction hypothesis*), and then show that it must also be true for $n=k+1$.

   Ask them to provide the conclusion.

C. Generalization
Ask the following (*provide metacognitive prompts to guide the students in answering these questions*):

1. Name a type of basket found in either Mayoyao or Aguinaldo and describe it in three sentences. What are its key characteristics, and what purpose does it serve in the community?
2. Explain the pattern observed in the baskets, focusing on the techniques and processes used in their creation. How does the design or structure of the baskets reveal this pattern?
3. What algebraic patterns can be identified in the design or construction of these baskets? Are there recurring numerical or algebraic relationships?
4. Is this pattern valid for all values of n? How was the pattern rigorously proven, and what steps were taken to verify its general applicability?
5. Describe the process of the investigation activity that led to the discovery and proof of the algebraic pattern present in the baskets. How did the you explore and confirm this pattern through observation and analysis?

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III. Evaluation

Do the following task in a group with 3 members.

Investigation Task:

Investigate the arithmetic sequence *4,9,14,19,...* by identifying the common difference and deriving the formula for the n^{th} term. Using this formula, calculate the 15^{th} term and the sum of the first *8* terms. Afterward, create a new arithmetic sequence with a negative common difference, and calculate the 6th term and the sum of the first *12* terms. Analyze how the negative common difference changes the behavior of the sequence and its sum.

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Integrating Realism in Mathematical Problem Solving: Insights from *Stand and Deliver*

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Abstract: Solving contextual math problems has been a central activity in mathematics education for decades, as they are believed to meet the requirements of being meaningful to students due to their contextual relevance and enable them to engage in deep learning characterized by the use of higher-order skills. Deep learning is activated when students are not provided with a clear computational procedure to follow, which is the case with some nonstandard word problems. Inspired by the movie Stand and Deliver, the paper focuses on using realistic considerations in solving math problems by presenting a modification of a word problem extracted from the movie and discussing didactical nuances of using realistic and open-ended approaches in mathematics. Moreover, the paper presents a study on pre-service primary teachers (N=54) focusing on using realistic considerations in solving a nonstandard word problem and their ratings of solutions to a similar word problem. The findings confirmed that most prospective teachers tend to disregard realistic considerations when solving math problems and prefer solutions that do not incorporate potential external real-life experiences.

Keywords: Mathematics education. Contextual problems. Realistic considerations.

INTRODUCTION

Problem-solving is a general teaching and learning method in mathematics education. It occurs when students are confronted by a mathematical problem for which they have no known solution (Chirinda, 2021; Schoenfeld, 2013). Specific problems in mathematics education are word problems designed to model real-life situations, enabling students to apply abstract mathematics in solving these problem situations. A word problem can be defined as a verbally described problem situation that the reader is somewhat familiar with, where one or more questions are posed. Answers to these questions can be found through applying mathematical operations to numerical data or logical deduction (Greer et al., 2002; Verschaffel et al., 2014). Several purposes of utilizing word problems within mathematics education have been stated in the literature (Verschaffel

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et al., 2000). Generally, solving word problems by students demonstrates that mathematics is not just about rote learning, but can also relate to real-life situations. Stressing the embodiment of mathematics in real-life situations may help students better understand the role and importance of mathematics in the world. Solving word problems also teaches students about mathematics as a language that can be used to translate a problem situation and subsequently manipulate data to solve the problem.

Word problems are considered one of the most challenging tasks in mathematics education (Tomková, 2012). Using challenging tasks in mathematics education accords with *Principles and Standards for School Mathematics* (NCTM, 2000, p. 52), particularly requiring teachers to provide students with "frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of effort." Solving complex problems is a reasonable requirement of the school curriculum, as solving complex problems is considered one of the essential skills for the 21st century (Gray, 2016). By solving complex problems, pupils may improve text comprehension, cognitive analysis, cognitive planning, computational procedures, and conceptual understanding (Uchida & Kawashima, 2008; Fuchs et al., 2015; Saridakis & Doukakis, 2020).

Contextual Problems in Mathematics Education

Effective teaching of mathematics requires using such content that students can relate to and find applicable in real-life situations (Posner et al., 1982). Therefore, the more realistic the situation described in a word problem is, the better it may serve for learning mathematics. However, incorporating a realistic approach to teaching mathematics isn't automatically straightforward (Sethole, 2004). The realistic approach is characterized by considering various factors in solving a contextual math problem; thus, the problem's solution is dependent on contextual factors taken into consideration during the problem-solving process.

The realistic approach contrasts with the traditional approach of solving contextual math problems, which is typical for mathematical modeling using only given data and denying any speculations regarding reality. However, to simulate the problem-solving process as it usually occurs in real life, students should be encouraged to consider many real-life aspects of the problem situation and to use various strategies in solving these problems. Contextual problems that offer more than one interpretation or whose solution can be altered based on considered aspects of the situation are called "problematic from a realistic point of view" (Verschaffel et al., 1997). Another term for these realistic problems is "ill-defined" problems, referring to solving problems in the real world where some information is missing (Costello, 2022). Realistic math problems refer to meaningful contextual situations and demand students to use their real-life knowledge together with applying mathematical principles and concepts. Additionally, considering multiple aspects

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requires students to use a deep level of thinking or higher-order thinking skills, which makes these problems more challenging than traditionally used closed problems that require an exact solving procedure from students (Vessonen et al., 2024). As a result, many positive effects of using realistic math problems in teaching mathematics have been identified, such as increased logical, critical, and creative thinking (Dhayanti et al., 2018; Ismunandar et al., 2020; Samura et al., 2022) and problem-solving skills (Nugraheni & Marsigit, 2021; Uyen et al., 2021).

To illustrate the difference between the traditional approach and realistic approach in solving word problems, let's consider the following math problem for primary school pupils: *A rope is used to fence a triangular area by drawing it around three distinct poles that are 7 ft, 10 ft, and 15 ft apart. What length of rope do we need to use?* (For more examples of realistic problems, see Verschaffel et al. (2000). Typically, pupils would solve the problem by creating a mathematical model of the situation, where the poles are modeled by the vertices of the triangle, and the lengths of the triangle's sides are added to find the triangle's perimeter of 32 feet. Although this solution seems correct, it would not be satisfactory in a real situation, as it assumes a rope that is not long enough to go around the poles while tying its ends. In contrast, a realistic solution to the question might be: "Because the lengths of the three sides add up to 32 feet, to tie the rope around the poles, we would need a rope longer than 32 feet." or "Depends on how many times we turn the rope around each pole and how thick each pole is." Such answers are referred to as realistic answers, realistic reactions, or solutions arrived at using realistic considerations.

Most of the studies on realistic math problems reported that students might tend to exclude realistic considerations when solving problems that are problematic from a real point of view (Verschaffel et al., 1994; Verschaffel et al., 1997; Inoue, 2005). The reason might be grounded in how math problems are usually solved and elaborated within a classroom (Greer, 1997). This is also due to traditional textbooks, which typically contain closed problems and do not encourage students to apply much of their real-life knowledge (Stigler et al., 1986; De Corte & Verschaffel, 1987; Schoenfeld, 1991; Tárraga-Mínguez et al., 2021; Kilienė, 2021). Fortunately, even if this tendency occurs, it can be mediated by implementing a proper teaching intervention (Verschaffel & De Corte, 1997; Renkl, 1999; Sepeng & Webb, 2012).

Along with the described realistic approach to solving word problems, realism in mathematical problem solving is present when solving word problems leads to multiple correct solutions regardless of the realistic considerations. This is explained by the fact that real-life problems can usually be solved in many ways. Math problems commonly having more than one correct answer are called "open-ended", although the term of open-endedness seems ambiguously defined by various authors (Bingölbali & Bingölbali, 2021). The opposites of these problems are closed math problems (also referred to as traditional problems) that are solved in only one way (Bahar & Maker, 2015). As closed math problems are those typically occurring in math textbooks, it is vital to have a strategy for transforming these problems into open-ended problems. One strategy

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is to provide students with less quantitative information in the problem. Let's illustrate this modification in the following example:

Original Closed Word Problem: *Cinderella was told by her stepmother to bake 11 pies. As she baked them, two of them got burned. How many edible pies can she serve to her stepmother?*

Modified Word Problem: *Cinderella was told by her stepmother to bake 11 pies. As she baked them, some of them got burned. How many edible pies can she serve to her stepmother?*

The original word problem asks students to solve it using the correct application of an arithmetic operation; in this case, subtracting 2 from 11. Solving such problems aims to teach students to interpret the situation and subtract one number from another mathematically. On the other hand, the modified version of the problem gives students some ambiguity. As they do not know the number of burned pies, they cannot answer the question using a straightforward arithmetic calculation. Exploring a solution to the modified problem necessitates a systematic approach, achievable through appropriate tools, such as tables. This encourages contemplation of more options and the relationship between two quantities, thereby contributing to the development of algebraic thinking (Russell et al., 2011).

The second strategy involves altering the question itself or employing additional questions to target various mathematical abilities of students. This implies that rather than simply asking students to solve a problem, a teacher might utilize different verbs that would change or enhance the richness of the task, such as asking students to explain, compare, prove, apply, construct, describe, and so on. Additionally, new questions may also focus on the actual aspects of the problem situation. Furthermore, metacognitive strategies, such as "What-If" questions, may be utilized to deepen inquiry (Payadnya et al., 2021).

Original closed word problem: *Cinderella was told by her stepmother to bake 11 pies. As she baked them, two of them got burned. How many edible pies can she serve to her stepmother?*

Modified word problem: *Cinderella was told by her stepmother to bake 11 pies. As she baked them, two of them got burned.*

a) *How many edible pies can she serve to her stepmother?*
b) *How many pies would she burn if she were told to make 22 pies? Explain your reasoning.*
c) *How long would it take her to bake 11 pies if she can bake up to two loads at once?*
d) *What do you think? Why does her stepmother need so many pies? How many people can she host if each guest should receive at least one-sixth of a pie?*

The third strategy is to provide students with the solution to the original problem and ask them to work towards the data that were initially stated but remain unknown for now.

Original Closed Word Problem: *Cinderella was told by her stepmother to bake 11 pies. As she*

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baked them, two of them got burned. How many edible pies can she serve to her stepmother?

Modified Word Problem: *Cinderella was told by her stepmother to bake some pies. As she baked them, some of them got burned. How many pies was she told to bake, and how many of them got burned if she served 9 edible pies to her stepmother?*

The modified problem does not provide students with all the numerical information. The intention is to enable them to work through the problem and discover multiple solutions or a general solution (the number of pies she was instructed to bake was at least 9, and the number that were burnt was 9 fewer). This method shares similarities with so-called "numberless word problems" or "problems without figures" (Gillan, 1909; Shaw, 1922; Williams & Gillian, 2018; Plank & Dyess, 2022).

Open-ended problems and a realistic approach to addressing these problems are the key ideas of this paper and should also serve as foundational elements in teaching school mathematics. The intersection of these approaches can be illustrated as two perpendicular axes (see Figure *1*). The horizontal axis illustrates the degree of intellectual freedom afforded to students when tackling a problem. A closed problem anticipates one specific solution and a clear solving strategy. An open-ended problem allows students to employ a more systematic approach, which promises greater knowledge acquisition. For instance, adding two integers using column addition is a closed problem. In contrast, finding multiple ways to determine the sum of two given integers is an open-ended problem. Similarly, a word problem that requires finding the sum based on quantitative information about two parts is a closed problem, while a word problem that does not provide all the necessary information is considered an open-ended problem. The vertical axis pertains to the context of the problems and how students approach them. The realistic approach demands contexts that are meaningful to students and solving processes that consider real-life aspects of the situation. In contrast, the non-realistic approach involves problems that make little sense and do not typically occur in real life.

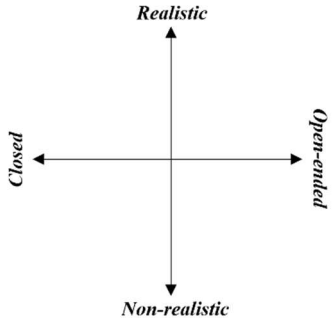


Figure 1. The overlap of realistic and open-ended pedagogy

Incorporating the realistic approach in solving math problems seems beneficial to students, as

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such problems resemble real problems and show the meaning of learning mathematics. Therefore, math instructors should adopt a teaching approach that supports taking real-life variables into account in solving problems, pose problems that cannot be solved straightforwardly, ask their students to devise alternative solutions, and incorporate using "What If" questions to extend the problem situations and activate higher-order thinking processes. Obviously, not all traditional word problems can be easily changed, such that having realistic considerations is an essential part of the solving process. From the previous studies on realistic reactions among students (a few of them described in Verschaffel et al., 2000), it can be assumed that word problems requiring realistic considerations are those that do not post numerical information in such a way that a simple calculation yields the correct answer or that somehow contradicts with a traditional way of thinking about solving math problems. For example, these problems require students' knowledge about out-of-school areas, contradict students' stereotypes about the presence of proportionality in all situations, demand students to check their results within the context, and challenge students to work on projects. Besides that, the realistic approach assumes that numerical information in designed problems reflects reality (Fatmanissa & Qomaria, 2021).

The present study aims to illustrate an example of transforming a seemingly closed problem into an open-ended one by considering realistic elements of the situation. The source of the problem is a scene from the film "Stand and Deliver," which serves as inspiration. To complement scientific research conducted in the realm of the realistic approach to solving mathematical problems (for example, Greer, 1993; Reusser & Stebler, 1997; Verschaffel et al., 2000; Bonotto, 2010; Dewolf et al., 2013; Van Dooren et al., 2019), an additional inquiry involving pre-service primary teachers was conducted, aimed at analyzing their preferred approach to solving a realistic mathematical problem and their preferences in evaluating students' solutions.

METHOD

The aim of this paper is to provide an example of how to modify a mathematical word problem into a more realistic one by considering additional factors that were not initially included in the original problem. The original contextual problem was extracted from the film "Stand and Deliver." This biographical film follows Jaime Escalante, a teacher who immigrated to the United States, was employed by a high school in Los Angeles, and chose to teach algebra and calculus to underperforming Latino students, preparing them for college studies. To motivate his students, Jaime's teaching approach consisted of using real-life examples and humor to build a relationship with them. In addition to modifying the problem by using a realistic approach, the study aims to investigate the use of realistic considerations among preservice primary teachers in solving a similar word problem that is problematic from a real point of view, as well as their attitudes towards students' solutions to a similar problem.

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Original Problem and Realistic Approach

Original problem: *Juan has 5-times as many girlfriends as Pedro. Carlos has one girlfriend less than Pedro. If their total number of girlfriends is 20, how many does each gigolo have?*

If translated into mathematical language, the word problem leads to solving a linear equation in a single variable. One notable aspect is the atypical context of the problem. It can be concluded that the context was designed to connect with students' experiences and provide entertainment in the classroom, as doing so generally increases students' motivation (Papanastasiou & Bottiger, 2004; Skinner & Pitzer, 2012). Given that most of the students in the movie were low-attaining, addressing such a realistic problem can enhance their learning (Barnes, 2005).

The idea to modify the math problem originated in the movie itself, as one of the characters notes, "You can't solve it unless you know how many girlfriends they have in common". That notion is not accurately true, as the problem can be solved using a traditional way of doing math as follows: Let's introduce variable x for a number of Pedro's girlfriends. Then, from the word problem, the following equation can be built: $5x+x+(x\text{-}1)=20$. Performing equivalent operations leads us to the conclusion that x equals 3, meaning that Pedro has three girlfriends, Juan has fifteen, and Carlos has two.

As the suggested modification concerns a socially controversial context, the problem may not be educationally appealing. Conversely, monogamy is a socially constructed concept that is not practiced in all societies around the globe. Therefore, discussing a scenario in which a girl has more than one partner may not only expand the math problem but also incorporate a social study topic.

The Realistic Solution to the Problem

Let us consider the case of intersecting sets. As there are three male individuals, their girlfriends can be divided into three sets. If two or three male partners share the same girlfriend, those sets must intersect. Thus, determining the number of common girlfriends requires examining the intersection of sets. The following Venn diagram illustrates three sets with potentially non-empty intersections, where J represents Juan, P represents Pedro, and C represents Carlos (see Figure *2*).

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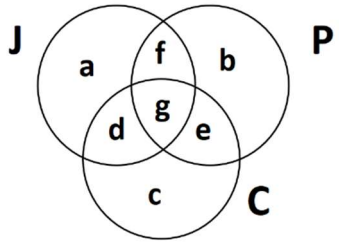


Figure 2. Venn diagram of Juan's, Pedro's, and Carlos's girlfriends

Assuming variables a, b, c, d, e, f, g, visualized in Figure 1 and the earlier described problem, we get the following four equations:

$$a+d+f+g=5x \quad (1)$$

$$b+e+f+g=x \quad (2)$$

$$c+d+e+g=x\text{-}1 \quad (3)$$

$$a+b+c+d+e+f+g=20 \quad (4)$$

To discover quantitative relationships between individual subsets, we will solve the first three listed equations for the variables a, b, and c (respectively) and substitute them into the last listed equation. Then we get:

$$a+b+c+d+e+f+g=20$$

$$(5x\text{-}d\text{-}f\text{-}g)+(x\text{-}e\text{-}f\text{-}g)+(x\text{-}1\text{-}d\text{-}e\text{-}g)+d+e+f+g=20$$

$$7x\text{-}d\text{-}e\text{-}f\text{-}2g=21$$

$$d+e+f+2g=7x\text{-}21$$

As 2g is a non-negative number, we obtain the following inequality:

$$7x\text{-}21=d+e+f+2g \geq d+e+f+g,$$

which implies further

$$7x \geq 21+d+e+f+g.$$

Because $d+e+f+g$ stands for the number of girlfriends (a non-negative number) who see more than one partner, x must be greater than or equal to 3. In the next step, we shall gradually elaborate on possible alternatives.

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- If x=3, then we get $21 \geq 21+d+e+f+g$, which implies that no girlfriends can be in common. That leaves us with solution (J,P,C)=(15,3,2).
- If x=4, then we get $24 \geq 21+d+e+f+g$, which means that there are utmost 3 girls who see more than one partner. To find out more, let's substitute x=4 in equations (1), (2), and (3). Then, we get the following system of linear equations:

$$a+d+f+g=20 \quad (5)$$

$$b+e+f+g=4 \quad (6)$$

$$c+d+e+g=3 \quad (7)$$

Because the number of all girlfriends is 20, from (5), we can deduce: b=c=e=0. Then, from (6) and (7), we get: $f+g$=4 and $d+g$=3, where the values d, f, g, and a can be found by simple experimentation (see Table 1).

	f	g	d	a
Case 1	4	0	3	13
Case 2	3	1	2	14
Case 3	2	2	1	15
Case 4	1	3	0	16
Case 5	0	4	-1	17

Table 1. Solution of the given problem when x=4

As d cannot be a negative number, there are four available solutions for x=4. However, this results in one solution based on the number of girlfriends each gigolo is supposed to have: (J,P,C)=(20,4,3).

- If $x \geq 5$, we deduce from (1) that Juan has more than or exactly 25 girlfriends, which contradicts the information that there are 20 girlfriends altogether.

Addressing problems that involve the intersection of sets can be challenging for students (Manurung et al., 2017). Although pupils formally study the intersection and union of sets during secondary education, they may become informally acquainted with these concepts even before starting elementary school (Šimčíková & Tomková, 2012). The difficulties associated with problems involving the intersection of sets can be mitigated when students engage with them through various activities and tasks. A real-world context that students are familiar with can also encourage them to be more receptive when tackling such problems.

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The Study of Using Realistic Considerations Among Pre-service Primary Teachers

To provide more than just a pedagogical approach to using word problems that require realistic considerations in mathematics education, this paper presents an inquiry into how pre-service primary school teachers engage with a realistic word problem. Furthermore, the study aims to evaluate how these teachers in training assess three potential solutions to a similar word problem (see Table *2*). The discussed "gigolo problem" focused on the possible intersection of three sets. To ensure consistency, the following word problem was solved by the participants:

> Carl has 5 friends, and Georges has 6 friends. Carl and Georges decide to have a party together. They invite all their friends. All their friends are present. How many friends are there at the party?

This problem and its modification have been previously utilised in numerous other studies on the realistic approach to mathematics education (Reusser & Stebler, 1997; Verschaffel et al., 1997; Yoshida et al., 1997; Verschaffel & De Corte, 1997; Tarim & Öktem, 2014; Aksoy et al., 2015; Kılıç & Şahinkaya, 2022).

The second task for the participants was to rate three potential solutions to a modified problem. The modification was done by changing the names of individuals from the problem and the number of their friends. Participants were asked to comment and rate these solutions on a scale of 1-7 (a greater number means a better rating).

Potential solution 1	Carl's number of friends and George's number of friends are added together to find out the total number of friends at the party.
Potential solution 2	As in "Potential solution 1", but including two more friends (Carl and George).
Potential solution 3	Carl and George are friends as well, and they may have some common friends.

Table 2. Potential solutions to the problem presented to participants

The study was conducted on 54 participants (53 females and 1 male, the average age being 34.3 years). The sample was skewed due to the consistent prevalence of women teachers in Slovakia. Therefore, the gender distribution of the sample reflects the gender distribution of primary teachers in Slovakia. About 75% of the participants had a professional working experience in education; however, none as a schoolteacher.

The word problem and the potential solutions were shared among participants in digital form through an online platform survey. Participants recorded their solutions and were offered the opportunity to provide a supporting comment. Following the collection of data, coding was conducted on an ad hoc basis.

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RESULTS

The responses of the pre-service teachers encompassed three main approaches to understanding the problem (see Table *3*). Some solutions featured a numerical error. For the purposes of the study, all responses that were substantiated and could be attributed to a specific solving strategy were included. Three responses were excluded as no solving strategy was discerned.

	Strategy description	All answers	Realistic answers
S1	Carl has 5 friends; George has 6 friends. Therefore, the situation involves two groups of people: Carl's group (6 friends) and George's group (7 friends). Then, the total number of friends is 13.	30	6
S2	Carl has 5 friends; George has 6 friends. If these two groups of friends are added together, the total number of friends is 11 (excluding Carl and George).	27	7
S3	Carl has 5 friends; George has 6 friends. Apparently, Carl and George are friends. Therefore, there is Carl, his 4 other friends, George, and his 5 other friends at the party. The total number of friends is 11.	4	0

Table 3. Pre-service teachers' solving strategies

Realistic answers were those that indicated the possibility of more than one correct solution or questioned the clarity of the problem's formulation. This approach was similar to that of Aksoy et al. (2015), who considered the realistic answer to be the one asserting that it is impossible to know the exact number of friends. Consequently, the total percentage of realistic reactions was 24%. Apparently, the majority of answers did not consider the possibility of Carl and George having common friends. Also, participants preferred to work arithmetically only with the given quantitative information without contemplating other possible situations.

The rating of three potential solution strategies revealed different preferences among participants (see Figure *3*).

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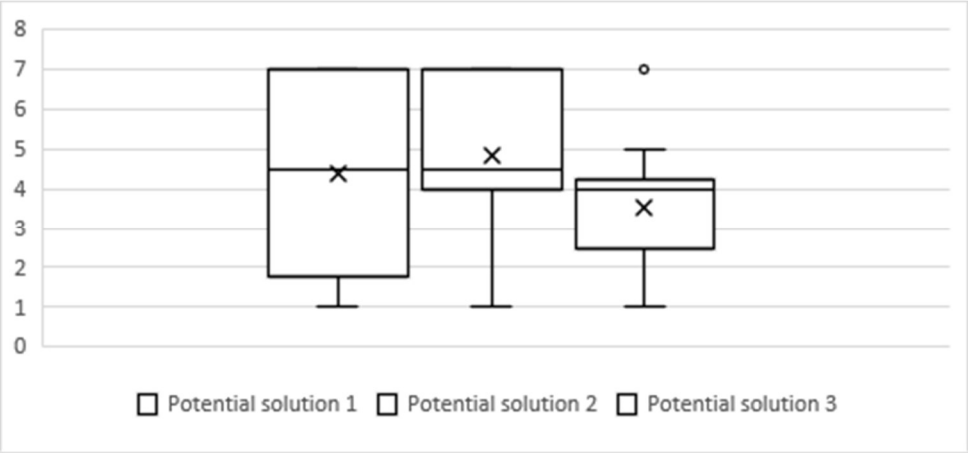


Figure 3. Boxplot illustrating participants‘ ratings of three different solutions.

The potential solution 1 (see Table *2*), which corresponds with the participants' strategy 2 (see Table *3*), earned an average score of 4.59 (SD=2.23, N=54, Mdn=5, Mode=7). The potential solution 2 (see Table *2*), which corresponds with the most frequently used participants' strategy 1 (see Table *3*), earned an average score of 4.98 (SD=2, N=54, Mdn=5, Mode=7). The potential solution 3 (see Table *2*), which involved a subjective but realistic approach, earned an average score of 3.78 (SD=2.02, N=54, Mdn=4, Mode=3).

For a more detailed statistical analysis, the data sets representing ratings of three potential solutions were tested for normality using the Shapiro-Wilk test. The results did not show normality among these data sets (PS1: W=.85383, p=.00001; PS2: W=.84628, p=.00001; PS3: W=.89884, p=.00026). Based on the normality test results, the Wilcoxon Matched-Pairs Signed-Rank Test was conducted to compare the ratings of three potential solutions (PS1 vs. PS2, PS1 vs. PS3, PS2 vs. PS3). The results are summarized in the following table (see Table *4*).

Variables	Test Statistic (T)	Z-Score	p-Value	Interpretation
PS2 PS3	146.0000	3.256	0.0011	Statistically significant difference; ratings between solutions are meaningfully different. The effect size seems strong.
PS1 PS3	197.0000	2.137	0.0326	Statistically significant difference; participants rated the solutions differently. The effect size seems moderately strong.
PS1 PS2	306.0000	0.935	0.3496	There is no statistically significant difference; ratings appear similar between solutions.

Table 4. The results of the Wilcoxon test of potential solutions

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The collected data and tests conducted revealed that pre-service teachers ranked the proposed potential solutions differently. Differences were discovered between potential strategy 1 and potential strategy 3, as well as between potential strategy 2 and potential strategy 3. This indicates that pre-service teachers did not value the strategy involving real-life aspects as much as the strategies that relied solely on the word problem statement. This is supported by the comments accompanying the rating of potential strategy 3. Further qualitative analysis of these comments demonstrated that nearly 40% of participants asserted that the word problem statement does not provide information about whether there are any mutual friends; consequently, they deemed the solution incorrect. This suggests a rigidity in how pre-service teachers view word problems, specifically regarding the information presented in the problem statement versus that which stems from external sources, such as real-life experiences.

DISCUSSION AND CONCLUSION

The study aimed to highlight the phenomenon of realism in solving word problems. According to the Principles and Standards for School Mathematics (NCTM, 2000, 14-15), the mathematics curriculum should focus on mathematics "that will prepare students for continued study and for solving problems in a variety of school, home, and work settings." This inevitably includes solving problems based on reality and students' experiences.

Previous studies have shown that mathematics education often neglects to consider real factors that may impact the solutions to word problems, which are set in specific situations (Verschaffel et al., 2000). The way mathematical abilities and perceptions of the relevance of mathematics to life are shaped is strongly influenced by teachers. This paper examined how pre-service teachers approach a word problem that allows for multiple solutions depending on the solver's approach, particularly how they interpret the problem in relation to reality. Furthermore, the study aimed to investigate how pre-service teachers assessed three different potential solutions to a similar problem. The word problem utilized in the study had been previously employed in other research concerning realistic considerations in solving word problems. Within the context of elementary and early secondary education, 10-20% of students were able to solve this problem using realistic considerations (Verschaffel et al., 1994; Yoshida et al., 1997; Koay & Foong, 1997; Reusser & Stebler, 1997; Renkl, 1997). Aksoy et al. (2015) conducted a study involving the same problem with prospective primary school teachers, revealing that approximately 30% provided realistic answers. Consequently, the results of the present study align with the earlier findings. The ratings of three potential solutions to a similar word problem indicated that pre-service teachers favor solutions that closely align with the problem statement. In other words, a solution that demonstrated an understanding of the situation based on real-life experience received a lower rating than those that adhered strictly to the information stated by the word problem. These results confirmed the earlier findings, revealing that teachers prefer the conventional approach to

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word problems, which also reflects their low appreciation for realistic reactions from students (Verschaffel et al., 1997). Simultaneously, teachers' appreciation for students' realistic reactions seems to be influenced by the training they receive (Xu, 2005; Chen et al., 2011). Therefore, educators should receive sufficient training to enhance their ability to recognise non-standard problems and to appreciate and promote realistic considerations in students (Ho & Hedberg, 2005; Bonotto, 2010). Specifically, teacher development programs should employ a greater number of non-standard problems, and problems leading to multiple solutions – divergent problems. A great emphasis should also be placed on solving methods, particularly working with various graphical representations of the problems. If the problem involves the intersection of sets, as described in the examples provided earlier, teachers could instruct their students on the Venn diagram or other strategies for tackling similar mathematical problems (Prídavková, 2021).

The study was inspired by the movie Stand and Deliver, in which the mathematics teacher does not take into account the realistic considerations of one of his students (not intended to undermine the quality and educational value of the movie – author's comment). Aside from the presented math problem, the movie Stand and Deliver features scenes where fractions are illustrated by cutting an apple, multiplication by 9 is demonstrated using fingers, negative numbers are linked to digging a hole, and other inspirational math scenes. In addition to the mathematical scenes, the movie offers an important commentary on math as an equalizer, a means to combat poverty and racism. Among cinematographic works, several more movies depicting mathematics or inspired by mathematicians have been made, such as A Beautiful Mind, Good Will Hunting, Hidden Figures, The Simpsons, and many others (Polster & Ross, 2012; Knill, 2025). The advantages of employing movies for mathematics education have been recognized for decades (Ficken, 1958), as movies can enhance mathematics education at various levels and serve both mathematical and didactical purposes (NCSCT, 1967). An example of a movie particularly aimed at educating the audience on mathematics is "Donald in Mathmagic Land," created by Walt Disney Productions, which introduces mathematical concepts to children. Another example is the educational series "The Forest Five," produced by Fool Moon, which features episodes designed to teach the youngest viewers various subjects across the curriculum, including mathematics. Movies featuring mathematics serve as dynamic representations that illustrate mathematical concepts or show actors engaging with math. Moreover, scenes from these movies can inspire educators to base their lessons on them. Students can enhance their mathematical understanding by observing how math problems are solved or by watching short movies on specific mathematical concepts (Fahlberg et al., 2007).

Another way of using movies in teaching mathematics is to design math lessons centered around movie scenes that are not explicitly mathematical. Russo & Russo (2020) developed lessons featuring tasks on proportional reasoning based on movie scenes. The findings indicated that students found these lessons enjoyable, leading to learning, and mathematically challenging. This approach to using movies embodies inquiry-based or project-based learning and enhances the

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connection between mathematics and real-world contexts. Such an approach may also prompt students to consider more realistic scenarios when tackling math problems. Therefore, greater emphasis should be placed on educating current and future teachers about the pedagogical opportunities that resources like movies can provide.

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Information and Communication Technology to Enhance Students' Conceptual Understanding and Reasoning Skills in Mathematics: A Case from Bangladesh

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Abstract: Nowadays, it is widely accepted that Information and Communication Technology (ICT) can promote positive changes in the teaching and learning process. This study explores how ICTs can be used in teaching and learning to enhance students' conceptual understanding and reasoning skills in mathematics. A mixed method case study (convergent parallel) approach was employed for this study, and the case was selected from a government school in Dhaka. The case constitutes an ICT-facilitated Grade 10 mathematics classroom, one mathematics teacher, and students of that classroom. Through a classroom-based intervention study, teachers' teaching practice with ICTs and students' conceptual understanding and adaptive reasoning in mathematics were investigated. The mathematical software GeoGebra was considered as the catalyst for the intervention, while other technologies were also used. It is found that students' mathematical concepts and reasoning ability increase if ICT is effectively applied in the teaching-learning process. Besides, different teaching-learning practices using ICTs were identified as effective pedagogical approaches to develop conceptual understanding and reasoning skills in mathematics. Finally, a few implications (theories and practices) for the findings have been articulated.

Keywords: ICT-facilitated classroom, conceptual understanding, reasoning skill, case study, intervention

INTRODUCTION

The world in the 21st century is changing fast due to the continuous advancement of technology. All such advancements are somehow connected to mathematics. Mathematics is no longer perceived as a subject for basic computations. Rather, mathematics knowledge is considered as one of the key catalysts of such advancement. According to Redecker and Johannessen (2013), students must be prepared with sufficient mathematical skills and competencies to cope with this 21st-

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century world. That indicates students must have to learn mathematics with proper understanding. Research shows that students fail to develop a meaningful understanding of mathematics if it is taught with an emphasis on drill and practice and rote learning of formulas (Hellum-Alexander, 2010). It is argued that learning mathematics becomes effective when the teaching and learning approach aims to develop students' conceptual understanding rather than focusing on the correct solution to a problem (Sultana et al., 2016).

In Bangladesh, students learn mathematics from primary to secondary levels as a compulsory subject. Thus, it is desired that they will grow up with good mathematical knowledge. In reality, students' performance in mathematics is inferior (Morshed, 2013). The study also showed that most Bangladeshi students of Grade 10 fail to represent a problem in multiple ways. Instead, they can identify information and perform routine procedures according to direct instructions only (Sultana et al., 2020). One of the primary reasons behind this could be the existing teaching practice. Most mathematics teachers have limited knowledge about modern teaching strategies and practice traditional approaches to teaching (Barua et al., 2020). In a recent study, Kabir and Jalali (2021) found that Bangladeshi teachers' focus on teaching is to disseminate mathematical knowledge to the students instead of making math meaningful, and they (teachers) fail to engage students in logical and systematic reasoning. As a result, students' ability to adaptively reason does not develop, and they fail to transform real-world problems into math problems.

Information and Communication Technology (ICT) refers to all technologies used for processing information and communication. Nowadays, there has been widespread interest in innovative projects leveraging ICT for delivering services (Khan, 2017; Zhang et al., 2025), for transforming organizations (Khan et al., 2015) and governance (Bukth & Khan, 2024), for promoting positive changes in the teaching and learning process, and whatnots. Like other countries, Bangladesh has also greatly emphasized incorporating ICTs in education. Bangladesh's government has taken several initiatives, such as developing multimedia classrooms, well-equipped ICT labs, a teachers' portal, digital content, students' e-portals, etc. (Islam & Ferdosh, 2019). Despite several initiatives, research shows no remarkable change between students' performance in general and multimedia classrooms (Imon, 2017). Besides, teachers still struggle to properly use digital materials (Ropum, 2022). That is why they are found not to regularly use digital devices in their classroom teaching-learning process (Khan et al., 2012). The use of ICT in Bangladesh is mainly limited to using scientific and ordinary calculators and PowerPoint slides in mathematics classrooms (Sultana et al., 2016). They found that most teachers use ICT tools to show pictures, video clips, and diagrams related to a topic to stimulate and engage students in the lesson rather than focusing on conceptual understanding and reasoning skills. Teachers are unaware of how to teach with ICT tools to develop

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a concept, and there are no specific guidelines on how to practice ICTs to build students' mathematical knowledge with proper understanding (Sultana & Khan, 2017). Besides, though the potential of ICTs (e.g., multimedia, math software, etc.) for mathematics teaching is well accepted for enhancing students' performance, their impact in developed countries may not be the same as in developing countries like Bangladesh due to contextual differences. Thus, the study aims to explore how ICTs can be applied in the mathematics teaching-learning process so that students' mathematical concepts, as well as reasoning abilities, are enhanced. The research questions the study seeks to address:

RQ1. Are there significant changes in students' conceptual understanding and adaptive reasoning due to the ICT-facilitated teaching-learning process?
RQ2. How does the ICT-facilitated teaching-learning process promote students' conceptual understanding and reasoning skills in mathematics?

In this study, the mathematics lesson topic was algebra (e.g., functions and linear equations). Given the broad scope of functions and linear equations, *basic concepts of functions and linear functions, slope and graphs of linear equations, and use of linear equations in real life* (all these topics are in line with the national curriculum for Grade 10) were considered.

Research Gap

Though the National Education Policy 2010 of Bangladesh emphasizes the use of ICT in science and mathematics, no specific approach has been suggested to be followed to develop students' mathematical concepts and reasoning skills. Thus, teachers in Bangladesh are unaware of how to develop students' mathematical concepts and reasoning skills with the help of ICTs (Sultana & Khan, 2017). In addition, limited studies have focused on the pedagogical approaches using ICTs to promote students' conceptual understanding and reasoning abilities, especially in the context of developing countries like Bangladesh. As such, by drawing on a case in Bangladesh, this study aims to add new knowledge to the existing literature on the mathematics teaching-learning process using ICT in a developing country context.

LITERATURE REVIEW

Students with proper conceptual understanding and adaptive reasoning in mathematics are vital in this fast-changing world. Conceptual Understanding (CU) is one of the mathematical abilities that enables students to organize their knowledge. Mathematical understanding means knowledge about unrelated concepts and the ability to explain their relationship (Kilpatrick et al., 2001; Kenedi et al., 2019). Students with CU can apply and adapt acquired mathematical ideas to explain new

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mathematical concepts (Kilpatrick et al., 2001). The authors claimed that students could use several mathematical representations and communicate their ideas if they have a conceptual understanding. Adaptive reasoning (AR) is thinking logically, estimating the answer with proper justification, and judging mathematical truth (Milgram, 2007). Consequently, students with adaptive reasoning skills have the ability to estimate, support their claims with arguments or proof, and draw inferences (Kilpatrick et al., 2001). In a study, Marasabessy (2021) claimed that students' logical thinking abilities will be formed by involving them in non-routine tasks. In this situation, students connect concepts and context by their logical thinking capability. Besides, reflective thinking skills (e.g., the ability to explain, the ability to justify) are also generated by logical thinking. Students' adaptive reasoning ability is vital as it helps them cope with real-life situations (Marasabessy, 2021).

Extant literature showed that ICT with appropriate pedagogical approaches greatly influences the development of students' higher-order thinking (Laborde, 2001). The use of ICT in the teaching-learning process not only develops students' mathematical procedural skills but also develops problem-solving, reasoning, and justifying skills if it is used strategically (Pierce & Stacey, 2010). While ICT has been seen to positively influence the quality of pedagogy and students' learning, its potential would be of no use if it is used only as a technological possibility rather than in the service of educational needs (Cuban, 2001). Thus, teachers must be skilled in determining when and how technology can enhance students' learning (International Society for Technology in Education [ISTE], 2008). When teachers use technology strategically in mathematics teaching, it helps to develop students' mathematical concepts, stimulate their interest, and increase their reasoning skills in mathematics (National Council of Teachers of Mathematics [NCTM], 2011; Das, 2019). Das (2019) stated that various types of ICT tools such as graphic calculators, specialized software (e.g., GeoGebra, SymPy, Maxima), spreadsheets, databases, etc., can be used for teaching mathematics, and with the effective use of these tools, the impact of ICT in mathematics education can be maximized (Becta, 2003). Again, Smith et al. (2018) claimed that to develop students' conceptual understanding of mathematics, teachers must create an environment for students to get involved in the discussions. The focus of the discussion should be reasoning rather than simply asking for the answer (Department of Basic Education [DBE], 2018). Since using mathematical language and explaining students' ideas behind their mathematical work are vital aspects of building students' conceptual understanding, teachers should provide opportunities for the learners to speak mathematically and explain their answers with proper justifications (DBE, 2018). However, teachers should explore students' misconceptions and use them as building blocks for deeper mathematical understanding (Anthony & Walshaw, 2009). During the question-and-answer session, teachers should create an environment where students have to respond by thinking critically and

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with proper justification (Anthony & Walshaw, 2009), and such initiatives ultimately promote students' adaptive reasoning ability. They also argued that when students struggle to deal with a problem, effective teachers should support them rather than provide complete solutions and encourage them to search for more information, try another method, discuss the problem with peers, and reflect on their own learning. In several studies (DBE, 2018; Wassie & Zergaw, 2018), researchers have demonstrated the potential of dynamic software and apps like GeoGebra and Desmos to enhance learners' conceptual understanding. These tools are well-recognized for their ability to reinforce mathematical concepts, improve visualization, and rectify misconceptions.

Conceptual Framework of the Study

The study considered the Technology Integration Panel (TIP) proposed by Li and Dawley (2019) as a theoretical base. This framework acknowledged the multifaceted nature of teaching-learning with technology while providing a clear outline to guide educators in their journey toward achieving their goals. Unlike conventional rubrics that oversimplify technology integration by equating it with either learning or teaching, TIP adopts a three-dimensional approach encompassing pedagogy, learning context, and access to technology (Li & Dawley, 2019). The feature "pedagogy" covers the learning objectives and the instructional design. According to Li and Dawley (2019), learning objectives are the instructional goals teachers emphasize in their teaching practices, such as whether

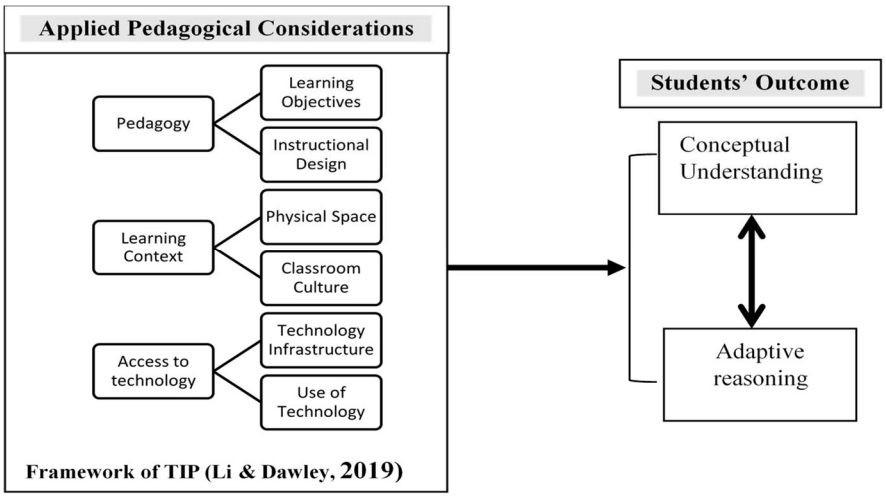


Figure 1: Conceptual Framework of the study

learning would be rote learning or meaningful learning/discovery learning. In contrast, instructional design could be student-centered or scaffolding. The "learning context" aspect includes physical space and classroom culture. In a technology-supported learning environment, classroom culture should be collaborative,

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and the classroom space should be designed to support a flexible and collaborative learning environment (Li & Dawley, 2019). The aspect of "access to technology" encompasses the availability of appropriate technical infrastructure and the use of technology. Based on these three-dimensional approaches, this study tried to explore the effective pedagogical considerations with ICTs to enhance students' mathematical concepts and reasoning skills, which are interconnected. The figure above (Figure 1) represents the conceptual framework of this study.

METHODOLOGY

This study employed a case study to understand the holistic and meaningful features of a complex real-life phenomenon (Yin, 2014). Again, to address the two research questions, a mixed method approach was adopted where the qualitative and quantitative data were collected simultaneously and analyzed separately to draw a more vivid comprehension of the phenomena (Creswell & Guetterman, 2019). Case study experts have recommended integrating qualitative and quantitative research in investigating the case (Yin, 2014). Thus, this study uses a *case study mixed method research approach* where the mixed method is specified as convergent parallel mixed methods (Guetterman & Mitchell, 2018). The case was chosen from a government school in Dhaka, in which an ICT-facilitated classroom, math teacher, and students of that classroom were the unit of analysis.

Research Design

An intervention was designed for the mathematics teacher in an ICT-facilitated classroom environment and the students of that class. The intervention process applied a cyclic approach of design, evaluation, and revision phases (McKenney et al., 2006). The aim of the design process was to understand teachers' mathematical practice while conducting classes with ICTs and improve the instructional sequence (if required) to enhance students' mathematical conceptual understanding and reasoning skills. The design process of the intervention is shown in Figure 2.

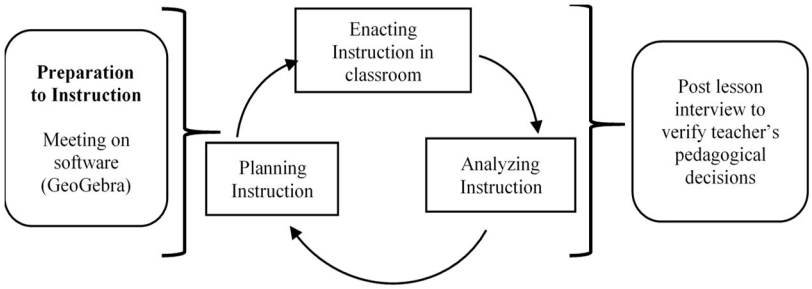


Figure 2: The design process of the intervention

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In this design process, the planning stage is the session for determining the instructional goals, selecting tasks, designing and developing ICT-supported materials and activity sheets, and structuring classroom action. In contrast, the enacting session is the actual instructional sequence in the classroom. During the planning sessions, the researcher clearly explained to the teacher the primary goal of the research study. The researcher and the teacher worked as a team to design the instructional materials for the enacting sessions. The teacher's implementation of instructions was analyzed at the last stage of this mini-cycle, and further improvement was suggested for the next class (if required). After completing each mini-cycle, post-lesson interviews were conducted to evaluate the instruction and investigate the teacher's mathematical practices. In this study, GeoGebra, along with other ICT tools, was used as a primary teaching-learning instrument since they have been found effective for the topics of study (e.g., function and linear equation) (Reezan, 2013).Prior to the cyclic approach of the intervention, the researcher arranged several meetings with the teacher to let him know about the usefulness, properties, and different functional options of GeoGebra software and provided some of the pre-prepared GeoGebra materials resource links so that he could get the idea to develop the GeoGebra supported instructional materials for the intervention sessions. The researcher also assisted the teacher in developing the materials when he asked for help.

Data Collection Method

Since the teacher could play a critical role in this research, the case selection depended on the mathematics teacher. This study considered one mathematics teacher and 30 Grade 10 students as sample. The participant teacher was selected purposively as the teacher had experience teaching with technology in the actual classroom setting and was willing to conduct the classes with the support of the mathematical software GeoGebra. Students were chosen by stratified random sampling to ensure all levels (based on academic performance) of students' representation. The total number of students was divided into three strata, and then 10 students were chosen randomly from each stratum. The whole process of data collection (pilot study and main study) was done from September 2022 to March 2023. A total of 6 classes were observed during the intervention. Each class was 50 minutes, and the post-lesson interview session was about 1 hour after each class.

Data were collected in 3 stages (before the intervention, during the intervention, and after the intervention). Before the intervention, a paper-pencil test was administered to determine the baseline understanding of students' mathematical concepts and reasoning skills. Classroom observations and semi-structured teacher interviews were conducted during the intervention. After the intervention, a paper-pencil test and FGD were done. The paper-pencil test and FGD supported the researcher in understanding whether students' CU and AR in mathematics developed or not. In contrast, classroom observation, semi-structured interviews, and FGD guided the researcher in finding out how students' CU and AR developed in an ICT-facilitated TL environment. Table 1 shows the

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research questions of this study, the data type, the source of data, the number of respondents, sampling techniques, and the instrument used for the respective questions in the study.

Research Questions	Data type	Source of data	Sampling techniques	Instrument
Are there significant changes in students' conceptual understanding and adaptive reasoning due to the ICT-facilitated teaching-learning process?	Quan. Qual.	30 Students of Grade 10 8 Students of Grade 10	Stratified random sampling	Paper-pencil test FGD (1)
How does the ICT-facilitated teaching-learning process promote students' conceptual understanding and reasoning skills in mathematics?	Qual.	6 Math classes of Grade 10 1 Math teacher of Grade 10 8 Students of Grade 10	Purposive sampling	Classroom observation Post lesson interview (6) FGD (1)

Table 1: Methodology matrix of the study

The interview questions were mainly open-ended and developed from existing literature on mathematical conceptual understanding, reasoning skills, and usage of ICT in mathematics TL. A "Tools validation workshop" was organized, where the participants were experts in the relevant field of this study. The experts' opinions were taken into account for further modifications and adjustments based on a pilot study. Besides, all interviews and observed classes were recorded (audio and video records, respectively), and field notes were taken wherever required during the observation.

A paper-pencil test was conducted before the intervention to measure students' baseline understanding of conceptual understanding (CU) and adaptive reasoning (AR). After the intervention, the test was again conducted to examine their level of proficiency in CU and AR enhancement. A total of five items (two for CU and three for AR) were administered in each paper-pencil test. All the test items were set based on the indicators (proposed by Kilpatrick et al., 2001; Milgram, 2007, Table 2) for the CU and AR skills in mathematics. The test scores range from 1 to 10 for CU and AR separately and were measured by comparing them with the standardized rubrics proposed by Lerís et al. (2017) (Table 2). As the marks of each component in the paper-pencil test were 10, to ensure an equal interval length for the proficiency levels, the range for the intervals was 2.5 (for details, please see Table 3). The test items before intervention and the test items after intervention were similar but not the same. The researcher used several data sources and data gathering methods

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for data triangulation to validate the data. The response validation technique was also used to validate the data.

Components	Indicator (Kilpatrick et al., 2001; Milgram, 2007)	Excellent	Good	Moderate	Poor
Conceptual Understanding (CU)	Ability to connect math concept to another one math concept/setting	Correct & complete	Correct & incomplete	Less complete	Incorrect/ not perform
	Ability to represent math in various ways	Correct & complete	Correct & incomplete	Less complete	Incorrect
Adaptive Reasoning (AR)	Ability to estimate the answer with proper justification	Correct & complete	Correct & incomplete	Less complete	Incorrect
	Ability to explain and draw conclusion	Correct & complete	Correct & incomplete	Managed to conclude	Incorrect

Table 2: Rubrics of the conceptual understanding and adaptive reasoning (Adapted from Lerís et al., 2017)

Proficiency level	Range of scores
Excellent	> 7.5 to10
Good	> 5.0 to 7.5
Moderate	> 2.5 to 5
Poor	0 to 2.5

Table 3: Proficiency level associated with the score range

Data Analysis

The quantitative data were analyzed using descriptive statistics (Tavakol & Dennick, 2011) to measure the average conceptual understanding and adaptive reasoning skills score. A paired sample t-test was also conducted to compare the pre- and post-test scores. Qualitative data were analyzed following the analytical approach used by Powell et al. (2003). The following seven sequential interacting phases were considered in analyzing the recorded (both audio and video) data (Figure 3).

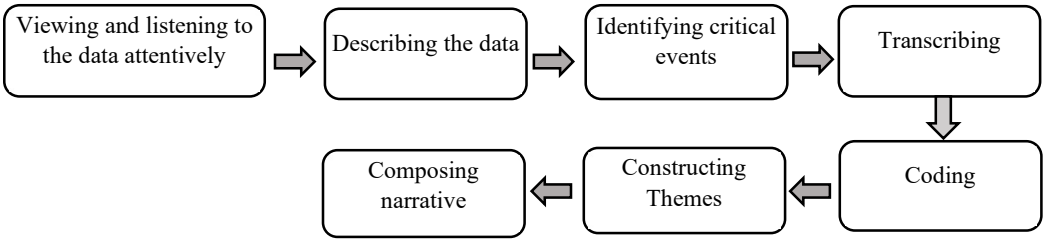


Figure 3: Sequential analytical approach for recorded data

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RESULTS AND DISCUSSION

Are there significant changes in students' conceptual understanding and adaptive reasoning due to the ICT-facilitated teaching-learning process?

Evidence from the paper-pencil test

From the analysis (Table 4) of the paper-pencil test, it is observed that before the intervention, the mean score of conceptual understanding in mathematics (i.e.*ability to connect math in a new situation* and *represent math in various ways*) was very low having a value of 2.07 (SD=1.26), whereas the mean score 3.63 (SD=2.48) for the adaptive reasoning (i.e., *ability to estimate the answer with proper justification* and *ability to conclu*de) is at a moderate level. However, after the intervention, students' CU and AR increased and reached 6.13 (SD=1.99) and 6.73 (SD=2.27), respectively.

Components	Test	Mean	Std. Deviation	Mean Difference	t-value	p-value
Conceptual Understanding	Pre-test	2.07	1.26	-4.06	-9.80	<.001
	Post-test	6.13	1.99			
Adaptive Reasoning	Pre-test	3.63	2.48	-3.10	-5.94	<.001
	Post-test	6.73	2.27			

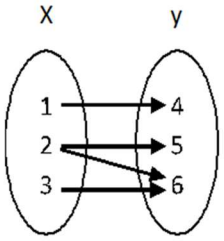
Table 4: Paired sample t-test for the conceptual understanding and adaptive reasoning

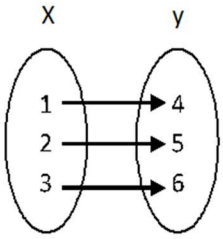
It is observed that there is a significant change ($p<0.05$) in the mean scores of the pre-test and post-test for both CU and AR (Table 4). This change means that students' ability of math conceptual understanding and reasoning skills have enhanced after attending the six experimental classes.

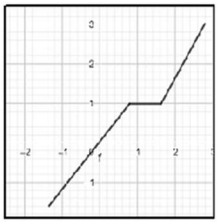
It was found that students' responses in two test items (Table 5, Q2 (b) and Q3) of the paper-pencil test improved significantly after the intervention. For test item Q2, students could identify the correct answer before the intervention but failed to explain the reason behind the answer. On the other hand, after the intervention, they could choose the correct answer with proper justification, which reflects their improvement in reasoning ability. Again, from the responses to test item Q3, it is observed that after the intervention, students could link mathematics to real-life situations and can apply the concept of mathematics in the new situation, which was not the case before the intervention.

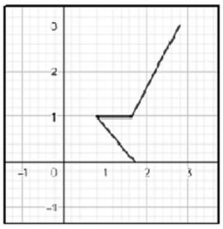
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Q2: a) Which of the following is not true for function? (Please √ the correct answer(s))

ii) 

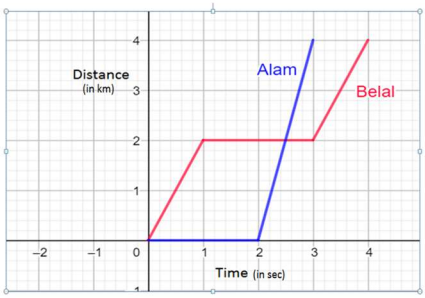
ii) 

iii) 

iv) 

b) Explain why they are not function.

Q3: The following graph shows the distance vs time relationship for journey of 2 people Alam and Belal.



Tell a story about the Belal's Journey including specific detail about time and distance.

Table 5: Two items of paper-pencil test

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Evidence from Focus Group Discussion (FGD)

Students' responses from FGD also reflect their development of CU and AR. During the FGD, the researcher asked them several questions about their experience in six experimental classes and provided them with a contextual problem (Figure 4) on "function". By analyzing their responses, it was found that students' conceptual understanding and reasoning skills developed very well after the intervention. They could explain the problem in different ways by connecting one concept to another, reflecting their CU. Besides, they could explain the reason behind their answer with proper justification and draw reasonable conclusions that reflect their ability of reasoning skills. For example, the FGD responses in Table 6 reflect students' CU and AR.

FGD Question	**FGD Responses (R=researcher, S=students)**
Mili and Shajib started from their home to the train station at the same time, and Ripon started 3 minutes later. The following graph shows the relation between the distance and time of their travels.	**R**: Can you talk about Mili's Journey? Here, the x-axis is in time(Minute), and the y-axis is in distance(meter). **S1:** Mili moves from a static position to 4 meters in 2 minutes with uniform velocity. Then there is no change at 4 meters for 2 minutes. That means she was not in motion for 2 minutes. After that, she went again with uniform velocity from 4 m to 7 m. **R:** Why uniform velocity, not acceleration? **S1:** Here, the line is straight. If the motion is accelerated, then the graph should be in curve form. **R:** Any other opinion? **S2:** I think it is acceleration as the curve is increasing.

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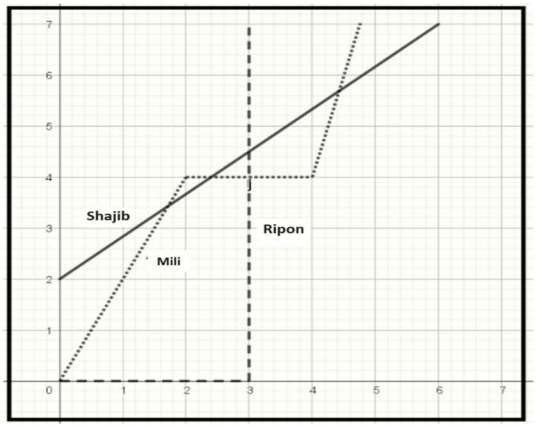


Figure 4: Distance vs time graph

S3: No, I think S1 is right. If it is acceleration, then velocity would change with time. But here, distance changes with time uniformly.
S2: Oh, yes. I did not notice that. It should be uniform velocity.

Table 6: A portion of FGD responses to Question

Thus, the paper-pencil test and FGD show that after experiencing classes in an ICT-facilitated environment, students' mathematical concepts and reasoning ability significantly increased. This finding conforms with the findings of Granberg and Olsson (2015), who reported that ICT tools assist students in engaging in creative reasoning to develop a solid conceptual understanding.

How does the ICT-facilitated teaching-learning process promote students' conceptual understanding and adaptive reasoning in mathematics?

By analyzing the data collected from classroom observation, teacher interviews, and FGD, the researcher identified several pedagogical approaches that foster students' CU and AR. She observed the pattern in the identified approaches and grouped similar patterns into 5 themes (Miles & Huberman, 1994). These themes collectively show the pedagogical considerations that the teacher uses when teaching mathematics in an ICT-facilitated environment.

This study found that a single approach cannot develop students' CU and AR. It is found that some of the approaches adopted by teachers directly enhance specific attributes while some have indirect influence. For instance, students develop their strong conceptual understanding while engaging in collaborative work. Thus, the pedagogical approach- "*Providing scope of- peer discussion, groupwork, whole class discussion"* has a direct influence on developing conceptual understanding, whereas the rearrangement of classroom setup to "*support collaborative learning space"* ultimately opens up scope for smooth collaboration and has an indirect effect on students' conceptual development. The pedagogical considerations for enhancing CU and AR are shown in Table 7.

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Dimensions	Applied Pedagogies for CU	Applied Pedagogies for AR
Learning Context	***Rearranging classroom amenities*** • Physical space support- collaborative learning • Physical space support- observation learning	***Rearranging classroom amenities*** • Physical space support- collaborative learning
Access to technology	***Ensuring Technology Accessibility and Its Appropriate Usage*** • Selection of tools- presentation, Geogebra, spreadsheet, Desmose • Involving learners directly in ICT • Using ICT (screen) to perform the task accurately	
Pedagogy	***Adopting different strategies to make learning meaningful*** • Linking and connecting math with real-life • Providing scope to apply acquired knowledge in a new situation • Exploring linking math with prior knowledge • Providing multiple ways of representation • Providing scope of- peer discussion, group work, whole class discussion • Providing the opportunity to speak/ use mathematical language • Offering challenging tasks and a variety of examples ***Offering multi-channel feedback*** • Feedback by the teacher, feedback by peer • Student's self-reflection ***Offering opportunities to identify error/ imprecision*** • Exploring misconception • Identify mistakes (by peer, by learner's own) • Providing counter example	***Adopting different strategies to make learning meaningful*** • Providing scope of- peer discussion, group work, whole class discussion • Providing a variety of examples Offering challenging tasks • Providing scope to explain students' thoughts with proper justification ***Offering multi-channel feedback*** • Feedback from the teacher • Feedback by peer • Student's Self-reflection

Table 7: Pedagogical considerations for developing Conceptual Understanding and Adaptive Reasoning

Pedagogical Considerations for Enhancing Conceptual Understanding

This study found that *classroom rearrangement* is an important pedagogical consideration to enhance students' conceptual development in mathematics. For conducting classes with ICT, the teacher used a portable whiteboard to link the screen. During the post-lesson interview session, he stated:

> *[…] Only one board was in the class. There was no extra facility to project the screen. So, I brought an extra portable board so that students could work on the board. Umm.. I also brought the board to explain some of my work. […] Sometimes, I use the board to*

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> *explain more about what was projected on the computer screen to give students a clear concept.*

It was also found that the teacher kept some extra chairs and a table in the class to engage students in group discussion. The study shows that if teachers intend to create an interactive and collaborative learning environment, all the features of *classroom rearrangement are vital* (e.g., board to link with the projected screen, round shape or face-to-face sitting arrangement, certain space among the groups, placement of board, projector screen and lighting arrangement, etc.). Such a supportive learning environment (observation and collaboration) creates scope for constructive discussions and helps develop students' CU. This finding is supported by Ozdemir and Pape (2012) as they stated that while students work collaboratively, they get the opportunity to explore the limits of their own understanding by dialogic talk (Wegerif, 2010).

The selection of appropriate ICT tools and their proper utilization for math teaching-learning is also found to be crucial for developing students' mathematical concepts. With the support of ICT, teachers can present and discuss mathematical topics in-depth, ultimately deepening and extending students' mathematical understanding. It was found that the teacher used the digital screen (i.e., displayed graph paper on the screen) to perform tasks. His intention behind this action was to allow students to draw the diagram by thinking from different perspectives so that they understand the concept more clearly. As he expressed:

> *Since the image of the graph paper was on the screen, there was no chance of error […] they can realize how the diagram was drawn in different ways, they understood what happens when the angle is acute, what happens when the angle is obtuse, they can compare these things. These things they quickly understood.*

The study also shows that while students learn mathematics by connecting it to real-life situations, learning becomes more authentic. To connect mathematics with real-life situations, teachers provide familiar contextual examples and use ICT so that learners can visualize the concept in real-life. While observing the class, it was found that the teacher used animated diagrams (Figure 5) to make the learning more visible, interesting, and concrete. Table 8 shows such a classroom context.

Such an approach helps students to deepen their mathematical concepts. According to the teacher:

> *When ICT tools are used for showing the pictures […], we can see the real shape […], and I think when students learn in this way, their understanding becomes concrete and long-lasting.*

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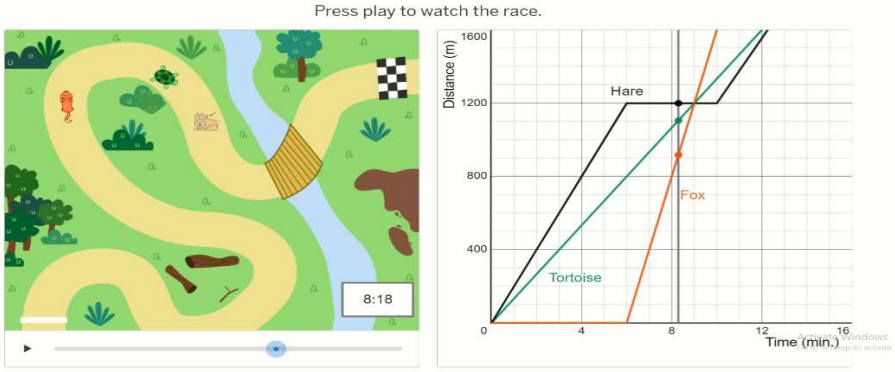


Figure 5: Use simulation to link math with real context

T: Here, you see three graphs (the teacher displays Figure 5 on the screen): one for the hare, one for the tortoise, and one for the fox. Now, I will play the button, and then you can see the real meaning of these graphs.
(teacher pressed the play button)
T: Where did the three animals meet?
S: at 1200 m
T: The graph of the hare is parallel to the x-axis for some time- what is the meaning of this?
S: The hare is at rest at that time, which means it has no motion.

Table 8: Classroom context for linking mathematical abstract concepts with real-life

Like the teacher, students during the FGD expressed their positive responses regarding the use of simulation as they found it helpful to visualize the meaning of mathematics in a real-life context. One of the students stated:

> *The example we have seen for uniform velocity or constant acceleration. In this case, we saw the animation, the graph changing with time, I think it was not possible in analog without a digital system. So that was very helpful to build up our concept.*

This finding is consistent with the findings of Wassie and Zergaw (2018), who reported that dynamic software like GeoGebra has the ability to enhance students' mathematical concepts. In addition, the study suggested that learners have to provide the scope to use mathematical language to make sense of the connection between mathematics and real-life. This viewpoint is well reported in the works (DBE, 2018) that ICT helps to visualize and connect math with the real world. Besides, *exploring students' prior knowledge, linking math with prior knowledge,* and *providing scopes to apply the acquired knowledge in new situations* are considered effective strategies to develop students' conceptual understanding. Similar findings were reported by Balka et al. (2010), who claimed that if teachers intend todevelop students' solid understanding of math concepts, they

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need to construct new knowledge with prior knowledge and utilize the new knowledge to solve problems in a new situation.

The study also found that students need to introduce or experience several examples from various dimensions. Working with problems sequencing from easier to harder helps learners interlink among the contents and ultimately enhances their mathematical concepts. This is consistent with the findings by Wolfram (2010), who found that harder problems, along with simpler ones, could develop students' conceptual understanding. In this study, *engaging learners in peer discussion, group work, and whole class discussion* are also found to be effective approaches for developing conceptual understanding, and the use of ICT in the TL process also saves time to do all those activities smoothly. Regarding this issue, the teacher stated, "*[...] if we can save that amount of time through digital technology, extra time can be given for group study*". It has been found that when students get involved in discussion and collaborative work, the concepts become clearer to them, and learners can develop their understanding of math. During the FGD, regarding this issue, one of the students argued:

> *Everyone does not think in the same way. I may know one rule to solve a problem, my friend may know another rule, and another may think of another way to solve the problem. So when we solve that problem together, we can enhance our knowledge by getting opinions from each other.*

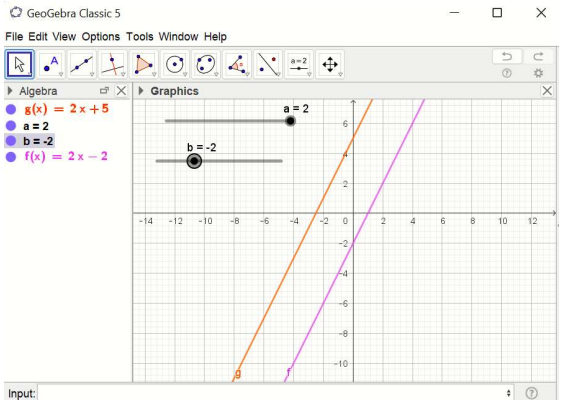


Figure 6: Exploring parallel line using slider

This finding is confirmed by researchers (DBE, 2018; Ozdemir & Pape, 2012), who showed that teachers should encourage and create the environment so that students can be involved in the discussion and progress with a solid understanding. This study also shows that *offering unfamiliar, non-routine, challenging tasks* is considered to be an effective strategy for developing a *conceptual understanding* of mathematics. During the classroom observation, it was found that the teacher offered students to draw a parallel line of $y=2x+5$, and students explored this issue by moving the slider (Figure 6) with proper understanding. In line with the work of Sultana et al. (2017), this study found that by dealing with unfamiliar, non-routine, and challenging tasks, students will be ready for realistic obstacles that exist in the real world.

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The findings of this study show that multi-channel feedback (e.g., feedback by teacher, Peer, and Students' self-reflection) can be applied in an ICT-facilitated teaching-learning environment to increase conceptual understanding. To ensure multi-channel feedback, the teacher guided students to the correct path rather than directly mentioning the right or wrong answer, and created the scope to open feedback by peers, which encourages them to think more. Regarding this issue, the teacher stated:

> *[…] When I give feedback to my students, I try to give precise feedback so they are not confused. I try to expose their wrong idea or mistakes through conversation. I think when students identify their mistakes in this way, their understanding becomes more concrete.*

Besides, students' self-reflection helps them to identify their own mistakes. In addition, exploration of students' misconceptions, encouragement to deliver counterexamples, and identifying mistakes are also found effective for developing conceptual understanding in this study. An ICT-facilitated teaching-learning environment opens up the scope to apply these strategies effectively. Many other studies support this finding (e.g., Granberg & Olsson, 2015; Ogwel, 2008), where it is argued that ICT offers challenging tasks, alleviates students' misconceptions, and provides the opportunity to show counterexamples.

Pedagogical Considerations for Enhancing Adaptive Reasoning

The study found that classroom rearrangement, suitable for collaboration, indirectly increases students' adaptive reasoning. During group work, peer work, and whole-class discussions, students engaged in arguments and explained their ideas to the group members or peers with proper logic, which enhanced their reasoning skills. This is well supported by Fang (2021), who reported that group discussions are a fruitful instructional design for enhancing students' reasoning ability. Though extant literature does not stress providing diverse dimensional problems for building adaptive reasoning, this study argues that various problems foster students' multi-dimensional thinking as students attempt to solve the problem using proper logic rather than rote learning. Again, challenging, non-routine, and a variety of problems create the scope for attempting multi-dimensional tasks from a logical point of view. This finding is consistent with the work of (Marasabessy, 2021), who stated that students' ability to think logically will be formed by dealing with non-routine tasks. In this situation, students apply logical thinking to connect concepts and contexts. Besides, learners need to get involved in creative reasoning and critical thinking to explain their thoughts with proper justifications. In every class, the teacher encouraged students to speak in mathematical language and asked them to talk about the reason behind their answers. That is, the teacher was not

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only concerned about right or wrong answers but also tried to understand the reason behind the students' thoughts. He often asked students to justify their answers. Regarding this issue, the teacher stated:

> *[...] When a student delivers a correct result, it does not mean he understands the problem properly. So, it is essential to know how he solves the problem and why he does it that way, and he needs to understand the logic behind his work.*

This viewpoint is well reported in the works (DBE, 2018), that teachers should encourage students to use mathematical language and explain their own ideas with logic. Besides,while teachers provide feedback, the instructions should be clear and lead students to think about the reason behind the mistakes and what needs to be done. Similar findings were reported by Shute (2008) that feedback is more effective when it not only provides information about the correctness of the answer but also elaborates on the qualities of student work or how to improve. Supporting the findings of Thonney and Montgomery (2019), this study also found that peer feedback makes students logical and constructive. The study found that in almost every class, teachers invited students to provide feedback to their peers with proper justifications, ultimately enhancing their reasoning skills. A sample of such a classroom situation is shown in Table 9.

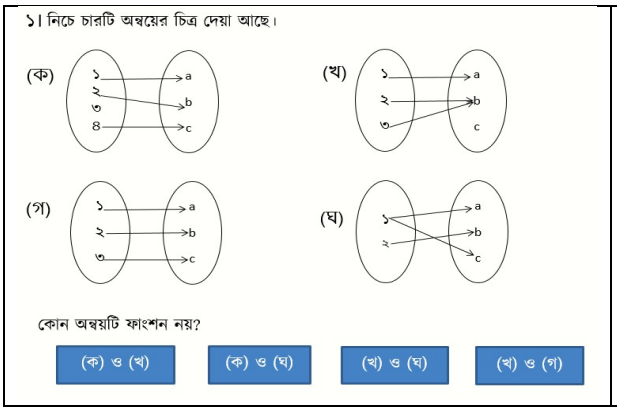
	**T:** S7, tell us if S11's answer is correct or not. **S7:** No sir. I think S11 is partially correct. The 4th diagram is also not a function. **T:** Explain to us why. **S7:** To be a function, every input must have exactly one output. But here, one input has two outputs, i.e., 'a' and 'c', which is not true for function. (Here, **T**=teacher, **S**=students)

Table 9: Sample of classroom scenario of peer feedback

The study's findings show that both the teacher and students showed positive views about peer feedback. During FGD, one of the students argued:

> *When our teacher told us to say what was right or wrong with our friend's work, we thought and tried to identify what mistakes he had made. I believe it insisted on us thinking more.*

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The study shows that students' self-reflection on their work also enhances their reasoning skill as this approach insists them to re-check their performed tasks and to think more with proper logic. It is consistent with the findings of Colley et al. (2012) where they suggested that reflection enhances students' knowledge system of thinking.

IMPLICATIONS AND LIMITATIONS

The findings of this study have a few implications for both theory and practice. By identifying several pedagogical approaches under five major themes that enhance students' conceptual understanding and reasoning skills in mathematics, this study adds new knowledge to the extant literature of the ICT-based mathematics teaching-learning process, especially in a developing country's context like Bangladesh. Besides, the study endorses that the policymakers should take the necessary initiatives to redesign the curriculum so that ICT-supported effective pedagogy for developing CU and AR could be addressed in the curriculum. Besides, the findings of this study would be helpful to teachers by providing an appropriate guideline for teaching ICT for students' conceptual development and reasoning skills in mathematics beyond the traditional approach. Moreover, the findings could be utilized by the teachers' trainer to develop the teachers' ICT-based professional skills, focusing on developing math concepts and reasoning skills.

While the findings of the study have several implications, the study has a few limitations that should be recognized before applying the findings to other situations. The case chosen for this study was from a Govt. school in Dhaka. Drawing on cases from different contexts (e.g., rural and urban contexts, Bangla medium and English medium schools, and high-tech and low-tech schools) would have strengthened the study's findings more. Thus, further study has been suggested for different settings. Besides, the sample size is limited to one mathematics teacher and 30 students of Grade 10. Though the researcher has collected data from different sources (e.g., classroom observations, post-lesson interviews,FGDs, and semi-structured interviews) to get a rigorous view and a clear essence of the scenario, more samples would have made the findings of the study more reliable and representative. Furthermore, the mathematics lesson topic for the study is limited to teaching *linear functions, slopes, and graphs of linear equations and real-life application*s. Thus, further study can be undertaken to see whether the findings of this study can be generalized for other mathematics content.

CONCLUSIONS

The study explored the effective pedagogical approaches to develop students' conceptual understanding and reasoning skills in mathematics. By applying a classroom-based (ICT-facilitated)

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intervention, the study intended to examine whether students' CU and reasoning skills in mathematics are enhanced. The study found that if teachers apply ICT effectively in their teaching, it will enhance students' mathematical concepts and reasoning abilities. The study highlighted some important pedagogical considerations such as rearranging classroom amenities to support collaborative and observation learning, ensuring technology accessibility as well as its proper use, adopting different strategies to make learning meaningful, offering multi-channel feedback and opportunities to identify imprecision for promoting students' conceptual understanding and adaptive reasoning skills. This in-depth exploration provides valuable insights into how teachers can adapt their teaching practices by integrating ICTs to develop students' conceptual understanding and adaptive reasoning in mathematics.

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The Effect of GeoGebra-Assisted Discovery Learning Considering Emotional Stability on the Achievement of Mathematical Reasoning

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Abstract: This study aims to describe and examine the effect of GeoGebra-assisted discovery learning on students' mathematical reasoning achievement in 3D geometry lessons. A Quasi-experimental research design with a post-test-only control group was used to conduct this study. This study involved 108 twelfth-grade students from a public senior high school in West Java, who were selected through purposive sampling. A valid and reliable mathematical reasoning test and emotional stability questionnaire, was used to collect the data. Effect size was computed using Hedges' equation, and two-way ANOVA was used to test group differences and interaction. Results showed that GeoGebra-assisted discovery learning significantly enhanced students' mathematical reasoning, exhibiting a strong positive effect (g = 1.247). Furthermore, significant differences in mathematical reasoning achievements were found among students exposed to GeoGebra-assisted discovery learning, discovery learning alone, and directed learning. However, no significant differences were found in the mathematical reasoning achievements among students with high, moderate, and low emotional stability. Additionally, there was no significant interaction between the learning model and emotional stability in affecting students' mathematical reasoning. These findings suggest that GeoGebra-assisted discovery learning is an effective intervention to enhance Indonesian students' mathematical reasoning.

Keywords: discovery learning, emotional stability, 3D geometry, GeoGebra, mathematical reasoning.

INTRODUCTION

Mathematical reasoning is central to mathematical education, encapsulating ideas, processes, and the essence of human thought. This process is recognized as a critical skill for students' development. It is emphasized by the Indonesian Ministry of Education and Culture and was identified by the National Council of Teachers of Mathematics (2000) as a key goal and one of the four pillars

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of mathematics education(Sukirwan et al., 2018). These endorsements highlight the crucial role of reasoning in enabling students to solve mathematical problems effectively. Mathematical reasoning is defined as the process of deriving solutions from presented problems. It not only enhances problem-solving skills but also makes complex mathematical concepts more accessible and understandable for students (Sunay & Erdogan, 2024). Additionally, it enhances students' ability to formulate logical arguments, thereby enriching their overall understanding of mathematics. By promoting mathematical reasoning, educational strategies aim to prepare students better to engage and overcome with challenging mathematical content, ensuring their proficiency and conceptual clarity.

The importance of reasoning starkly contrasts with the actual educational outcomes observed. According to a survey by OECD using the Programme for International Student Assessment (PISA), there are significant gaps in students' mathematical reasoning abilities. Since its inception in 2000, PISA has aimed to assess the knowledge, skills, well-being, and equity of 15-year-old students globally, with mathematical reasoning as one of the main evaluated competencies (Wijaya et al., 2024). In the 2022 PISA assessment, Indonesia's average mathematics score significantly decreased by 13 points to 366, down from 379 in 2018 (Wijaya et al., 2024). Additionally, numerous empirical studies have shown that Indonesian students' achievements in mathematical reasoning remain suboptimal (Lestari, 2019). Further, empirical evidence indicates that the mathematical reasoning of Indonesian high school students is also notably weak (Johar et al., 2017). This decline underscores an urgent need for targeted interventions to enhance mathematical reasoning skills among students in Indonesia.

Mathematical reasoning, a cornerstone of mathematical proficiency, extends beyond mere calculation abilities. It is a comprehensive cognitive skill that includes analysis, generalization, integration, and solving non-standard problems (Suparman et al., 2021). Regrettably, many students exhibit mathematical reasoning skills that are largely imitative, showing a dependency on memorization over genuine understanding. This approach restricts their capacity to apply learned concepts in new contexts (Marasabessy & Hasanah, 2021). To overcome these obstacles, there is a critical need for pedagogical strategies to improve students' analytical, reasoning skills, and boost their confidence in handling the complexities of mathematics. An integrative approach is crucial for fostering a deeper and more flexible understanding of mathematical concepts among students.

Research on three-dimensional (3D) geometry highlights the difficulties students face with abstract mathematical concepts, particularly their struggles with visualizing and describing 3D shapes effectively (İbili et al., 2020). The complexity of 3D geometry requires not only psychomotor skills but also critical thinking and reasoning abilities(Fujita et al., 2020). These demands often lead to difficulties in understanding and solving 3D geometry problems, emphasizing the need for improved educational strategies (Hallowell et al., 2015). Additionally, empirical studies have shown that students commonly make conceptual and procedural errors in 3D geometry (Moss & Lamberg,

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2019). These errors are primarily due to limited abstraction and visualization skills, which are essential for accurately describing spatial relationships and calculating distances between points and area in three-dimensional spaces. Such findings underscore the importance of instructional methods that effectively enhance these foundational skills.

Similarly, GeoGebra, has emerged as a leading technology in the new generation of mathematics learning tools. It has transformed the landscape of mathematics education, providing easy and free access to mathematical processes and tools for communities worldwide (Tamam & Dasari, 2021). According to (Celen, 2020), various studies have demonstrated the ability of GeoGebra to assist in fostering discovery and experimentation processes among students in classrooms. Its visualization features effectively assist students in formulating various mathematical conjectures, pointing towards its significant role in promoting an interactive and student-centered learning environment. Moreover, discovery learning is a mindset in which students incorporate concepts through observation, classification, hypothesizing, explaining, measuring, and concluding (Druckman & Ebner, 2018). Additionally, discovery learning is described as a model and strategy that emphasizes student activity, offering learning opportunities through exploration (Tran et al., 2014). It can be facilitated through various strategies in the classroom, highlighting its adaptability and effectiveness in fostering a deeper understanding and engagement among students. This model not only improves the learning experience but also prepares students for more complex problem-solving and knowledge application in real-world contexts (Simamora et al., 2018). Therefore, GeoGebra software supported the discovery, which can be applied in mathematics teaching to promote students' enhanced mathematical reasoning ability in geometry topics.

Numerous empirical studies have examined the use of GeoGebra software in discovery learning (Marion, Abdullah, & Rahman, 2023; Murni et al., 2017). The implementation of GeoGebra-assisted discovery learning significantly improves students' understanding of geometrical concepts (Marion, Abdullah, & Rahman, 2023). Additionally, discovery learning facilitated by GeoGebra software substantially enhances students' mathematical problem-solving and representation skills (Murni et al., 2017). Furthermore, GeoGebra-assisted discovery learning positively affects students' critical and creative mathematical thinking (Batubara, 2019). Other studies have also shown that integrating GeoGebra software with discovery learning significantly improves students' spatial visualization skills (Suparman et al., 2021).

Similarly, Thapa et al. (2022) demonstrated that the integration of GeoGebra in mathematics instruction fosters meaningful learning, promotes student motivation, and enhances active engagement, particularly in understanding geometric concepts such as circles. Moreover, Triet et al., (2024) showed that the use of a GeoGebra-supported 5E learning model significantly improves students' comprehension and retention of geometric concepts, such as the area of a trapezium, while also increasing engagement and positive attitudes towards mathematics. In addition, Nguyen et al. (2023) found that combining the flipped classroom model with GeoGebra software

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significantly enhances students' problem-solving abilities and learning attitudes, further supporting the role of GeoGebra in creating an effective, student-centered learning environment.

While there is a growing body of research on the use of technology in mathematics education, particularly with tools like GeoGebra, most studies primarily focus on its effectiveness in enhancing conceptual understanding, visualization, and problem-solving skills. Previous research has demonstrated that GeoGebra supports students in exploring mathematical concepts dynamically, yet limited studies specifically examine its role in developing mathematical reasoning, particularly in the domain of 3D geometry. Since mathematical reasoning is a critical competency in higher-order thinking, it is essential to investigate how GeoGebra-assisted discovery learning impacts this skill beyond general problem-solving.

Additionally, while discovery learning has been widely studied as an instructional approach that fosters student engagement and inquiry-based learning, few studies explore its interaction with students' emotional stability. Emotional stability is known to influence cognitive processes, learning motivation, and academic performance, but there is a lack of research on how it affects students' ability to engage in mathematical reasoning, especially in a technology-assisted learning environment.

The present study examines the effectiveness of discovery learning supported by GeoGebra on Indonesian high school students' mathematical reasoning achievement in 3D geometry, considering this aspect of students' emotional stability. The following research questions were projected to achieve the purpose of this study.

1. How is the profile of students' mathematical reasoning viewed by the learning model and emotional stability?
2. What is the size of the effect GeoGebra assisted discovery learning, and only discovery learning, has on students' mathematical reasoning acquisition?
3. Is there any significant difference in mathematical reasoning achievement between students who learn using only discovery learning, GeoGebra-assisted discovery learning, and directed learning?
4. Is there any significant difference in mathematical reasoning achievement between students who have high, moderate, and low emotional stability?
5. Is there any significant interaction effect between the learning model and emotional stability on the acquisition of students' mathematical reasoning?

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CONCEPTUAL FRAMEWORK

Mathematical Reasoning

Reasoning is defined as a process of thought used to make statements and reach conclusions in problem-solving that is not strictly based on formal logic and is not limited to evidence (Brown et al., 2010). Specifically, mathematical reasoning involves the ability to analyze, generalize, synthesize/integrate, provide accurate reasoning, and solve non-routine problems (Jeannotte & Kieran, 2017). (Hjelte et al., 2020) argue that mathematical reasoning is a fundamental objective of mathematics education. It represents a high level of mathematical thinking characterized by the capacity to think logically and systematically (Mueller et al., 2014). This implies that mastery of mathematics depends on mathematical reasoning, as it enables the generation of ideas and accurate interpretation of mathematical concepts.

Moreover, research suggests that emotional stability plays a crucial role in mathematical reasoning. Individuals with high emotional stability are more likely to exhibit logical and systematic thinking, as they can regulate negative emotions that may arise when faced with complex mathematical challenges (Hjelte et al., 2020). Conversely, individuals with low emotional stability are more prone to experiencing math anxiety, stress, and cognitive overload, which can hinder their ability to engage in effective reasoning (Pekrun & Linnenbrink-Garcia, 2012). Therefore, understanding the relationship between emotional stability and mathematical reasoning can provide insights into how affective factors influence students' cognitive abilities in mathematics.

Emotional Stability

Emotion is defined as the distinct feelings and thoughts that emerge in an individual in response to perceived changes in their body when confronted with specific situations (Pekrun & Linnenbrink-Garcia, 2012). These alterations arise from a complex assessment carried out by the individual's nervous system in response to internal or external stimuli. Emotions represent psychological states or feelings that vary across different circumstances. These emotional shifts can significantly influence an individual's behavior and activities, highlighting the importance of emotional stability in effectively managing these changes.

Emotional stability is defined as the condition in which an individual, upon exposure to emotional stimuli from external sources, does not succumb to emotional disturbances like depression or anxiety (Chaturvedi & Chander, 2010). Emotional stability can be observed through an individual's daily behavior, evident during social interactions, which include work performance, creativity, and tendencies toward worry. This trait involves the capability to manage and regulate one's emotions, incorporating the skills to recognize, understand, and control negative emotions such as fear, anger, and sadness, as well as to sustain positive emotions like joy and calmness (Barrick & Mount,

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2012). Individuals with high emotional stability are generally better equipped to cope with emotional fluctuations.

Emotional stability includes three factors such as emotional adequacy, emotional maturity, and emotional control (Li & Ahlstrom, 2016). Particularly, emotional adequacy refers to the appropriateness of emotional reactions to received stimuli, including the type, content, and direction emotions. Additionally, emotional maturity is characterized by the ability to respond emotionally in an appropriate manner in unpleasant situations and under certain conditions. Subsequently, emotional control refers to a crucial part of self-adjustment and mental health. Emotional maturity, control, and adequacy of functioning well and healthily indicate good emotional health and adjustment, ultimately supporting the achievement of stable emotions.

Emotional stability has significant implications for academic performance, particularly in subjects that require logical reasoning, such as mathematics. Students with high emotional stability are more likely to persist through challenging problems, remain calm under pressure, and effectively process information, which are essential qualities for mathematical reasoning (Li & Ahlstrom, 2016). In contrast, students with low emotional stability often experience higher levels of anxiety and frustration, which can impair their ability to think logically and systematically (Barrick & Mount, 2012). Research has also linked emotional stability to mathematics anxiety, where students with lower emotional stability tend to experience greater fear and apprehension when solving mathematical problems, negatively affecting their problem-solving abilities (Pekrun & Linnenbrink-Garcia, 2012).

Understanding the connection between emotional stability and mathematical reasoning is crucial in designing learning environments that cater to students' affective and cognitive needs. Investigating how emotional stability interacts with mathematical reasoning will provide insights into how students' affective states influence their ability to engage with complex mathematical problems and how educators can develop strategies to support students in overcoming emotional barriers to learning mathematics.

Discovery Learning

Discovery learning is a cognitive process where students acquire concepts through activities such as observation, classification, hypothesizing, explaining, measuring, and concluding (Druckman & Ebner, 2018). Tran et al., (2014) identified three main characteristics of discovery learning: (1) activities that promote the assimilation of new information into students' prior knowledge bases, (2) interest-based activities that are student-driven, allowing students to determine the sequence and frequency of their learning tasks, and (3) problem-solving activities that stimulate the creation, integration, and generalization of knowledge. Discovery learning is part of heuristic learning, an educational approach that utilizes methods designed to engage students in a process-oriented

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approach, encouraging them to lead, discover independently, and reflect throughout their learning activities.

In the context of discovery learning, emotional stability plays a critical role in how students respond to the open-ended nature of this learning approach. Since discovery learning requires active exploration, self-directed problem solving, and independent thinking, students must be able to manage frustration, persist through uncertainty, and regulate their emotions when facing difficulties (Druckman & Ebner, 2018). Students with high emotional stability are more likely to embrace challenges, stay engaged, and remain resilient when encountering difficult concepts. In contrast, students with low emotional stability may experience heightened stress, discouragement, or frustration when confronted with the ambiguity and cognitive demands of discovery-based tasks (Tran et al., 2014).

Thus, emotional stability serves as an essential factor in determining how well students adapt to discovery learning environments. By understanding his relationship, educators can develop strategies to support students who struggle with emotional regulation, ensuring that all learners can fully engage with discovery-based mathematics instruction and maximize their mathematical reasoning potential.

During the problem formulation phase, students identify various issues within the content. The data acquisition phase allows students to explore these problems by gathering information from online and offline sources, observing objects, conducting interviews, and performing experiments. In the data processing phase, students organize, classify, tabulate, and analyze the data. During the verification phase, educators help students validate their hypotheses based on their data analysis. Finally, in the generalization phase, students draw conclusions based on their verification results, incorporating feedback from educators and peers. These phases of discovery learning, especially when supported by GeoGebra software, are effectively used in geometry lessons.

GeoGebra was developed by Markus Hohenwarter in 2001. According to (Hohenwarter et al., 2008), GeoGebra represents a software program designed to integrate geometry, algebra, and calculus within a single dynamic environment. This program is available to download for free from www.geogebra.org. GeoGebra is a Java-based software that requires the Java Runtime Environment (JRE) to be installed on a computer as a prerequisite. It is a dynamic geometry system capable of constructing points, vectors, segments, lines, and conic sections, as well as functions, and altering these constructions subsequently in which equations and coordinates can be input directly (Celen, 2020). One of GeoGebra's unique features is that an object in the geometry window correlates to an equation in the algebra window and vice versa (Zengin et al., 2012). Hence, GeoGebra can handle variables for vectors, numbers, points, derivatives, and integrals of functions; it can even provide commands such as root or extremum.

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The GeoGebra program offers substantial benefits for both teachers and students (Hohenwarter et al., 2008). GeoGebra can be installed on personal computers and is available for usage at any time and location, in contrast to commercial software, which is usually only accessible at school. Teachers can utilize GeoGebra to construct dynamic online learning environments where students actively explore a variety of mathematical concepts. Additionally, the use of GeoGebra in mathematics learning offers several advantages: Firstly, it enables the quick and accurate production of geometric drawings, surpassing traditional tools like pencils, rulers, or compasses. Secondly, its animation and manipulation (dragging) capabilities offer students a clearer visual understanding of geometric concepts. Thirdly, it serves as a tool for feedback/evaluation, helping to verify the accuracy of the drawings produced. Fourthly, GeoGebra simplifies the process of investigating or demonstrating the properties of geometric objects, therefore improving the learning environment for both educators and students.

METHODOLOGY

Research Design and Approach

This study used a quantitative, quasi-experimental methodology to examine how GeoGebra-assisted discovery learning affected students' mathematical reasoning achievement. Numerous studies advocate for employing a quasi-experimental design within a quantitative framework as an effective approach to test and address hypotheses linked to theoretical assertions. Additionally, this study employed a post-test-only control group design. A post-test-only control group design was implemented to minimize potential threats to internal validity that may arise from pretest sensitization effects (Campbell & Stanley, 1963).

In this setup, two experimental classrooms and one control classroom were assigned as follows:

1. The GeoGebra-assisted discovery learning (DL+G) group engaged in interactive and visual exploration of 3D geometry concepts using GeoGebra software.
2. The standard discovery learning (DL) group participated in similar exploratory activities but without GeoGebra, relying solely on traditional learning tools.
3. The directed learning (DI-Control Group) group followed a teacher-led instructional approach, where concepts were introduced explicitly, and students were guided step-by-step through structured problem-solving activities.

The intervention was conducted over three sessions, with each classroom receiving ninety minutes of instruction per sessions to ensure comparability across groups. The instructional process within each condition followed a structured lesson plan, ensuring consistency in content while varying the mode of instruction.

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Use of GeoGebra for Analyzing Problems

To further support students in visualizing and analyzing geometric problems, GeoGebra was integrated into the lessons to enhance conceptual understanding. The specific usage included:

- *Provision of Figures:* The teacher initially provided problem scenarios, but students actively constructed and manipulated figures within GeoGebra.
- *Manipulatable Features:* Students were encouraged to drag points, adjust angles, and measure segment lengths to dynamically observe changes.
- *Key GeoGebra Features Used:* Measure Tool, Point Movement, and Dynamic Angle Adjustments

Differences in Learning Approaches

1. GeoGebra-assisted Discovery Learning (DL+G)

   Students actively manipulated objects, formulated conjectures, and tested hypotheses using GeoGebra (Hardi et al., 2021). The phases for implementing the discovery learning model include stimulation, problem formulation, data acquisition, data processing, verification, and generalization. In the stimulation phase, educators identify relevant content or pose questions to guide students in formulating problems. Exploring distance and angle in 3D geometry using the discovery learning model with GeoGebra as follows:

   (1) The distance between points or between a point and a line in 3D space.
   The following activities illustrates how students explore the concept of distance and angle in 3D space using GeoGebra. Specifically, they learn how to determine the distance between two points, or between a point and a line or plane, through step-by-step visual construction.
   a. Stimulation
   How do we measure the distance between two points in 3D space?

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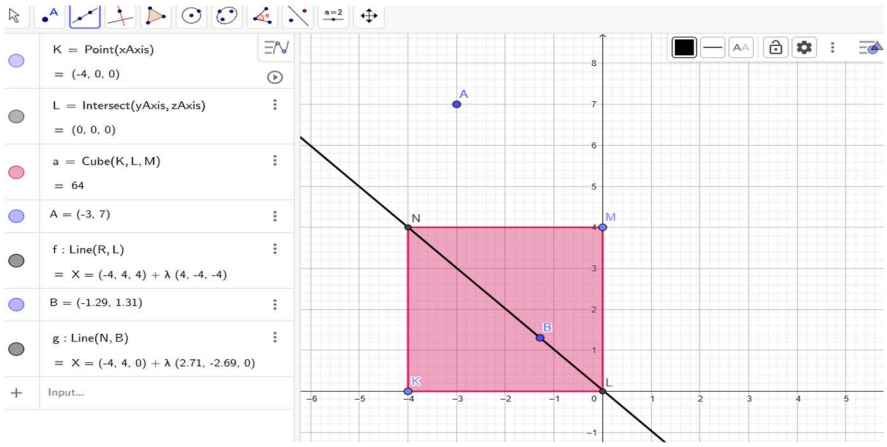


Figure 1. Students Begin Constructing 3D Representations by Defining Points, Lines, and Planes on a Two-Dimensional GeoGebra Interface

In figure 1, students begin by using GeoGebra to define basic geometric elements such as points and lines. Although the interface shown is two -dimensional, this step serves as a foundation for constructing three-dimensional representations. Students identify points (e.g., A, B, and C) and connect them with lines, preparing for spatial reasoning in the following stages.

b. Problem Statement, data collection, data processing and verification
After the instruction contained in the step “a” is done, then the result will look like in the following view

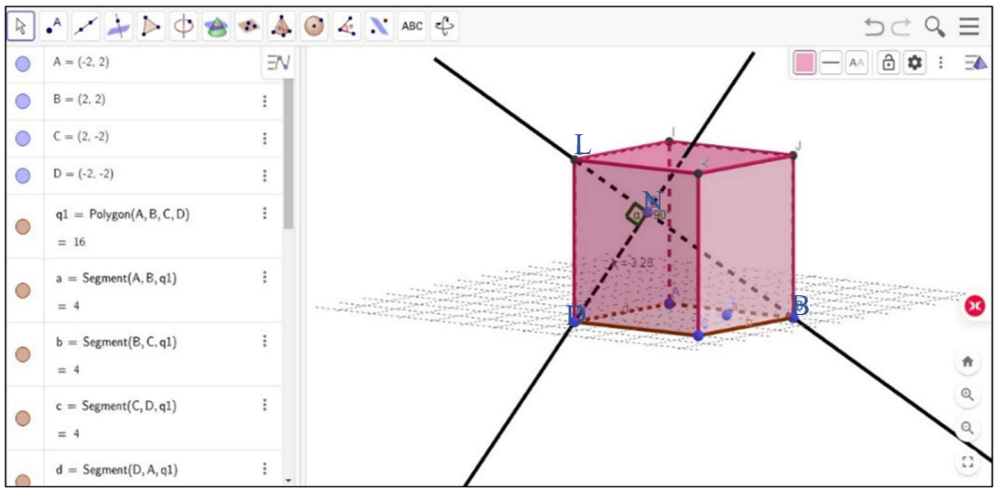


Figure 2. Outcome view of the distance between point and line

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Figure 2 displays the outcome of measuring the distance between a point and a line. In this stage, students will have completed the previous steps and are now able to visualize the relationships between geometric objects in 3D space. As shown in the figure, a cube is displayed with line LB intersecting with point D. The next task involves measuring the perpendicular distance between point D and line LB, as indicated by the highlighted perpendicular segment. This allows students to apply the concepts of distance and geometric relationships to a practical problem.

c. Generalization

At this stage, each group is asked determining the distance between point D and line LB with a side length of 4 cm by identifying the perpendicular line segment connecting point D to line LB. After this, the method involves using the ratio of the areas of two triangles to calculate the distance. This approach ensures accuracy in finding the shortest distance. The calculation follows these steps: Establish a perpendicular segment from D to LB. Relate the distances using geometric properties of triangles and apply proportionality of their areas. Derive the exact distance based on the given edge length of 4 cm.

Area(ΔDBL) Area(ΔBLD)

$$\frac{1}{2} \cdot DB \cdot LD = \frac{1}{2} \cdot BL \cdot DN$$

$$\frac{1}{2} \cdot 4\sqrt{2} \cdot 4 = \frac{1}{2} \cdot 4\sqrt{3} \cdot DN$$

$$DN = \frac{4\sqrt{6}}{3}$$

Here, DN represents the distance from point D to the line LB, as illustrated in Figure 2. This is the shortest distance and is determined by constructing a perpendicular segment from D to LB.

2. Discovery Learning Only (DL)

Students used paper-based tools for visualization and problem solving instead of GeoGebra, relying on conventional methods for exploration. This method follows structured phases of Discovery Learning as outlined by (Hardi et al., 2021), which include stimulation, problem formulation, data collection, data processing, verification, and generalization. It emphasizes self-guided learning, logical reasoning, and manual problem-solving without instant digital feedback. By following the six phases of discovery learning, students actively construct their knowledge while strengthening their ability to work with mathematical concepts systematically.

3. Directed Learning (DI)

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The teacher controlled the demonstration, and students observed the effects rather than exploring independently. The implementation of Directed Learning follows key phases of instruction, which include stimulation, explanation, demonstration, guided practice, independent practice, and assessment.

This implementation ensured that students not only visualized mathematical problems but also actively engaged in mathematical reasoning through interactive exploration.

To ensure the validity of findings, students were assigned to their respective classroom based on purposive sampling, with a focus on selecting those with lower early mathematics achievement to assess the impact of instructional interventions on students who typically struggle with mathematical reasoning. The effectiveness of each instructional method was measured through post-test scores, assessing students' ability to apply reasoning skills in solving complex mathematical problems.

This differentiation in instructional methods allows for a clear attribution of effects to the learning interventions, ensuring methodological rigor and facilitating replicability in future research.

Participants

This study involved 108 twelfth-grade students of a public senior high school in West Java, Indonesia. They were distributed in two experimental classrooms and one control classroom, with each classroom consisting of 36 students. Participants were selected using purposive sampling in collaboration with the school, which identified three intact Grade-12 classes that met the study's inclusion criteria (Etikan, 2016). To ensure equivalence between groups, we considered students' prior mathematics achievement based on their previous semester grades. This selection criterion ensured that both groups had similar ability before the intervention. Additionally, all students were from the same school and grade level.

Additionally, they were chosen as the participants in that the average early mathematics achievement of those classrooms was less than other classrooms in twelfth grade, ensuring that the intervention targeted students who might benefit the most from GeoGebra-assisted discovery learning. This selection was also intended to examine whether students with lower mathematical proficiency could improve their reasoning skills through the intervention while controlling for prior mathematical ability to reduce variability in the sample.

Instrument

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The instruments employed in this study were a mathematical reasoning test and an emotional stability questionnaire. The mathematical reasoning test consisted of four mathematical problems designed to assess core aspects of mathematical reasoning indicators, including: spatial reasoning, logical thinking, and generalization. (see Appendix 1). The maximum possible score for the test was 16 points (4 points per problem) (Saputri et al., 2017). The scoring rubric accounted for accuracy of the solution, depth of reasoning, and clarity of explanation.

This test was validated and proven reliable, as indicated by Pearson's correlation ($r \geq 0.75$) for each item and a Cronbach's Alpha of 0.91. The emotional stability questionnaire contained three indicators: emotional adequacy, emotional maturity, and emotional control. It was specifically used to measure students' emotional stability in the context of 3-D geometry learning (see Appendix 2). This questionnaire, based on the theory proposed by Hurlock (1978), which outlines the three components of emotional stability, was also validated and proven reliable, evidenced by Pearson's correlation ($r \geq 0.70$) for each item and a Cronbach's Alpha of 0.83. According to Özdemir et al. (2019), a Pearson's correlation r above 0.70 for each item indicates validity. Additionally, Bonett & Wright (2014) assert that a Cronbach's Alpha above 0.75 confirms the reliability of the instrument. These validations confirm that the mathematical reasoning test and emotional stability questionnaire are suitable tools for measuring students' mathematical reasoning and emotional stability, respectively.

Data Collection and Analysis

A mathematical reasoning test was administrated after the intervention in the GeoGebra-assisted discovery learning , discovery learning, and directed learning. Additionally, the questionnaire on emotional stability was also administered to students during the intervention process. Hedges'g equation was applied to measure the effect size of discovery learning promoted by GeoGebra on the acquisition of students' mathematical reasoning (Jaya & Suparman, 2021). Moreover, the value of Hedges' g was classified as $0.00 \leq g \leq 0.20$ (weak), $0.21 \leq g \leq 0.50$ (modest), $0.51 \leq g \leq 1.00$ (moderate), and $g > 1.00$ (strong) (Cohen et al., 2002). In addition, descriptive statistics were used to describe students' emotional stability in which classified students' emotional stability as $x<\overline{x}-s$ (low), $\overline{x}-s \leq x \leq \overline{x}+s$ (moderate), and $x \geq \overline{x}+s$ (high) (Azwar, 2022). Inferentially, a two ways ANOVA test was used to examine students' different mathematical reasoning viewed by learning model and emotional stability. According to Rutherford (2011), the data is assumed to be normally distributed, which justifies the application of a two-way ANOVA test.

RESULTS

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The Profile of Students' Mathematical Reasoning Viewed by Learning Model and Emotional Stability (RQ1)

Students' mathematical reasoning achievement from learning 3-D geometry for each classroom is shown in Table 1. It shows that the mathematical reasoning of students who learn using GeoGebra-assisted discovery learning was higher than the mathematical reasoning of students who learn using discovery learning and directed learning.

Classroom	Sample Size	Mean	Deviation Standard
Discovery Learning	*36*	*5.86*	*2.66*
GeoGebra-assisted Discovery Learning	*36*	*9.22*	*3.13*
Directed Learning	*36*	*5.53*	*2.71*

Table 1: The acquisition of students' mathematical reasoning for each learning model

Additionally, the acquisition of students' mathematical reasoning from learning 3-D geometry for each level of students' emotional stability is shown in Table 2. It indicates that students with high emotional stability have better mathematical reasoning scores compared to students with moderate and low emotional stability. This finding suggests that there are differences in students' mathematical reasoning scores across emotional stability levels, with higher scores observed among students with high emotional stability.

Level of Emotional Stability	Sample Size	Mean	Deviation Standard
High	*19*	*7.74*	*2.96*
Moderate	*74*	*6.59*	*3.29*
Low	*15*	*5.17*	*3.58*

Table 2: The acquisition of students' mathematical reasoning viewed by emotional stability

Based on examples of students' responses to the mathematical reasoning ability test categorized by emotional stability, the results are presented as follows:

1. High Emotional Stability

Students are able to accurately illustrate contextual problems using diagrams. They are also able to write down important information such as sides' length, as well as, the positions of the lamp and switch, that is a basis for determining distance. The calculation step uses the Pythagorean theorem that has been explained, showing the relationship between side length in the G'OG triangle and obtaining precise results.

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Answers from students with high emotional stability showed clarity, systematic structure, precision and consistency in solving problems sequentially, resulting in accurate answers. This indicates excellent focus during the analysis stage. The solution to the answer above is presented in the image below:

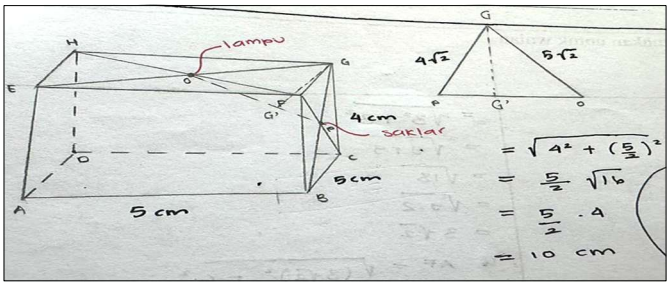


Figure 3. Sample Solution of a Student with High Emotional Stability in the Mathematical Reasoning Test

2. Moderate Emotional Stability

Students are able to illustrate contextual problems with diagrams and outline the calculation steps. However, there are minor errors, such as inputting incorrect values in the calculations, resulting in less accurate outcomes. Answers from students with moderate emotional stability show clarity, systematic organization, and detail, even though the results are incorrect. The problem-solving steps are sufficiently detailed, indicating good focus during the analysis stage, despite potential pressure while solving the problem. The solution to the answer above is presented in the image below:

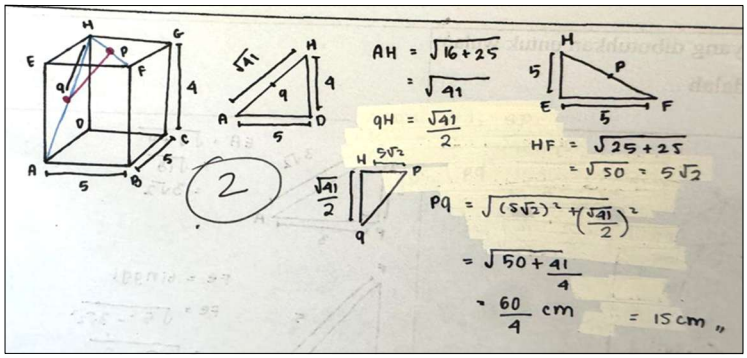


Figure 4. Sample Solution of a Student with Moderate Emotional Stability in the Mathematical Reasoning Test

3. Low Emotional Stability

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Students are able to illustrate contextual problems in the form of a rectangular prism and mark a point within the prism as the point whose distance to the space diagonal is calculated. Students apply the Pythagorean theorem but provide very minimal additional explanations. Students with low emotional stability still manage to complete the problem, even though the result is incorrect. This demonstrates an effort to maintain focus, prioritizing speed overelaborating the process. The solution to the answer above is presented in the image below:

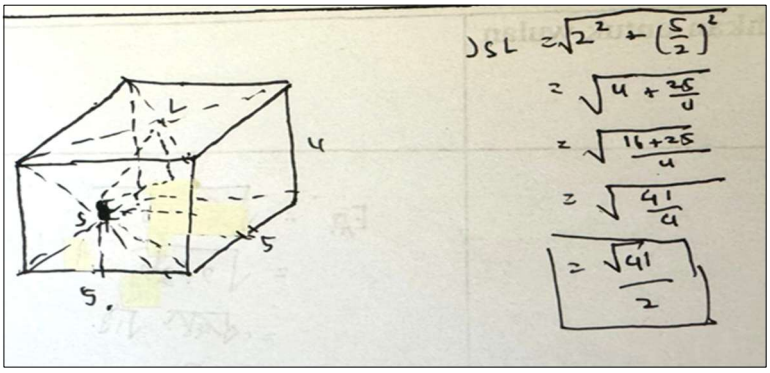


Figure 5. Sample Solution of a Student with Low Emotional Stability in the Mathematical Reasoning Test

The Effect Size of Experimental Intervention on the Acquisition of Students' Mathematical Reasoning (RQ2)

The effect size using Hedge's equation of experimental interventions, such as GeoGebra-assisted discovery learning and only discovery learning, on the acquisition of students' mathematical reasoning is shown in Table 4. It reveals that the implementation of GeoGebra-assisted discovery learning had a significantly strong positive effect on students' mathematical reasoning acquisition. In contrast, discovery learning alone showed a weak positive effect on students' mathematical reasoning. It indicates that GeoGebra-assisted discovery learning is more effective in optimizing students' mathematical reasoning compared to using discovery learning alone.

Experimental Intervention	Effect Size	Z-value	Sig.
GeoGebra-Discovery Learning	*1.247*	*4.88*	*0.00*
Discovery Learning	*0.122*	*0.52*	*0.60*

Table 4: The effect size of experimental intervention on the acquisition of mathematical reasoning

Students' Mathematical Reasoning Viewed by Learning Model and Emotional Stability (RQ3, RQ4, & RQ5)

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According to Rutherford (2011), it is assumed that the data are normally distributed, allowing for the application of a two-way ANOVA test. Consequently, a normality test using the Kolmogorov-Smirnov framework was performed (see Table 5). It shows that the significance values from the Kolmogorov-Smirnov normality test for each category exceeded 0.05, indicating that the data concerning students' mathematical reasoning, as viewed by both learning model and emotional stability, are normally distributed. Therefore, the precondition for normality has been met, justifying the use of a two-way ANOVA test.

Mathematical Reasoning	Category	Stat.	N	Sig.
Learning Model	GeoGebra-assisted discovery learning	0.938	36	0.201
	Discovery learning	0.959	36	0.194
	Directed learning	0.949	36	0.171
Emotional Stability	High	0.927	19	0.153
	Moderate	0.897	74	0.227
	Low	0.951	15	0.539

Table 5: The results of the normality test using the framework of Kolmogorov-Smirnov

The results of the two-way ANOVA are presented in Table 6. It indicates that the significance value of the *F* test for the learning model variable was less than 0.05, suggesting a significant difference in mathematical reasoning among students learning 3-D geometry through GeoGebra-assisted discovery learning, only discovery learning, and directed learning. Furthermore, the results of the post-hoc test using LSD, presented in Table 7, reveal significant differences in mathematical reasoning between students using GeoGebra-assisted discovery learning and those using only discovery learning, as well as between students using GeoGebra-assisted discovery learning and directed learning. However, Table 6 also indicates that students learning through only discovery learning and directed learning do not significantly differ in their mathematical reasoning.

Source	Mean Square	df	F-value	Sig.
Learning Model	*96.845*	*2*	*12.174*	*0.000*
Emotional Stability	*11.282*	*2*	*1.418*	*0.247*
Learning Model * Emotional Stability	*9.912*	*4*	*1.246*	*0.297*

Table 6: The results of two ways ANOVA test

Table 6 indicates that the significance value of the *F* test for the source of emotional stability was greater than 0.05, suggesting that there is no significant difference in mathematical reasoning between students with high, moderate, and low emotional stability. Additionally, Table 7 reveals that the significance value of the *F* test for the interaction between the learning model and emotional

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stability was also above 0.05, indicating that there is no significant interaction effect between the learning model and emotional stability on the acquisition of students' mathematical reasoning.

Learning Model	Mean Difference	Sig.
GeoGebra-assisted Discovery Learning – Only Discovery Learning	3.630	0.00
GeoGebra-assisted Discovery Learning – Directed Learning	3.443	0.00
Directed Learning – Only Discovery Learning	0.187	0.84

Table 7: The results of post-hoc test using LSD

DISCUSSION

The Effectiveness of GeoGebra-assisted Discovery Learning on the Achievement of Students' Mathematical Reasoning Considering Emotional Stability

This study examines the effect of GeoGebra-supported discovery learning on mathematical reasoning achievement among Indonesian high school students, with a particular focus on 3D geometry. Through dynamic visualization and interaction, GeoGebra has been recognized as a significant educational tool for fostering a deeper understanding of mathematical concepts (Celen, 2020; Zengin et al., 2012). We intend to explain the findings of the present study, shedding light on the implications of discovery learning supported by GeoGebra for improving students' mathematical reasoning skills.

A key finding of this study is that GeoGebra-assisted discovery learning significantly improves students' mathematical reasoning skills. Students in this group scored higher on average than those in traditional discovery learning and direct learning. Previous studies also support this finding (Marion, Abdullah, & Rahman, 2023). This outcome aligns with previous studies showing students' positive response to GeoGebra-assisted discovery learning, with significant improvements across learning cycles. It notably enhances students' mathematical reasoning in Cartesian coordinates. In his theoretical framework, Van Joolingen (1999) argues that discovery learning is a vital tool for promoting cognitive development within educational settings. The findings from this research support this theory, demonstrating that GeoGebra-assisted discovery learning significantly enhances mathematical reasoning with a strong effect size of 1.24. When GeoGebra is incorporated into discovery learning frameworks, it significantly boosts students' mathematical reasoning abilities. This substantial impact underscores the effectiveness of integrating GeoGebra into discovery learning environments.

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Furthermore, the study found no significant differences in mathematical reasoning abilities among students with varying levels of emotional stability. Although Cheng et al. (2022) suggest a positive correlation between emotional stability and academic achievement, the results indicate that GeoGebra-assisted discovery learning improves mathematical reasoning regardless of students' emotional stability levels. These findings highlight that while high emotional stability typically correlates with better academic outcomes, GeoGebra's influence on mathematical reasoning is independent of emotional stability.

Implications to Mathematics Learning

These findings have significant practical implications. The positive impact of GeoGebra-assisted discovery learning on mathematical reasoning suggests its effective use as an alternative instructional approach. The GeoGebra program enhances knowledge construction and reasoning skills by enabling students to visually explore three-dimensional concepts. Participants reported that GeoGebra simplifies the visualization of spatial relationships, transforming their perception of mathematics from challenging to accessible. This shift can motivate students to engage more deeply with mathematical concepts, leading to improved learning outcomes.

Limitations and Suggestions

The research, however, encountered limitations relating to GeoGebra-assisted learning's technological requirements. Since students were unable to bring their laptops to class, their dependency on laptops for implementing the learning process posed a challenge, inhibiting their full participation in learning activities. Due to this limitation, education institutions must ensure that students have adequate access to computer lab facilities or laptops in order to fully take advantage of GeoGebra-assisted discovery learning.

CONCLUSION

This study shows that learning through GeoGebra-based discovery learning significantly enhances the mathematical reasoning abilities of students compared to discovery learning without GeoGebra or direct instruction. GeoGebra offers a dynamic, visual, and interactive platform by which learners can investigate mathematical ideas in an easier and more meaningful way. This interactive approach not only strengthens students' conceptual understanding but also fosters critical thinking and reasoning skills. On the other hand, even though discovery learning without GeoGebra encourages active participation and exploration of concepts, it does not avail the same cognitive support like visualization facilities offered by GeoGebra. Meanwhile, direct instruction, which

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focuses on teacher-led instruction, reflects a lower impact on developing students' reasoning abilities.

Furthermore, this study focused on the effect of emotional stability on reasoning in mathematics. The students in the category of high emotional stability always outperformed others in the categories of medium and low emotional stability. This indicates that emotional stability and concentration is important in solving problems and deducing logically. However, the differences among groups of emotional stability are not statistically significant; hence, it indicates that there might be other factors which mediate the relationship between emotional stability and reasoning ability.

Interestingly, the findings did not show any significant interaction between emotional stability and the learning model on students' mathematical reasoning ability. This shows that the advantages of GeoGebra-based learning can be universally applied to students with varying levels of emotional stability. These findings indicate the importance of incorporating technology-enhanced learning tools, such as GeoGebra, to foster mathematical reasoning, while also taking into account individual emotional influences in student learning outcomes. Further studies may be designed to focus on the long-term impact of the proposed intervention and study how different types of students interact with technology in various educational settings.

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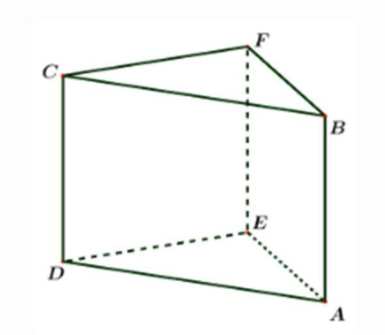
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APPENDIX

Appendix 1. Mathematical Reasoning Test

1. A child is assigned to observe a building from two locations. First, the child observes the building from a park with an elevation angle of 45°. Then, the child moves to a cake shop and observes the building with an elevation angle of 30°. The park is located south of the building, and the cake shop is east of the park. Given that the distance from the park to the cake shop is 100 meters, the height of the building is ....
2. Wynn's room is a block with length: width: height = 5:5:4, on the ceiling of the room, there is a lamp located the right at the center of the ceiling plane. On one wall of the room, there is a switch located the right in the center of the wall. Calculate the distance from the lamp to the switch and explain how to determine the distance!
3. Wulan will decorate her new cafe with decorative lights. The decorative lights will be installed in the center of the ceiling. To add the beauty, she plans to add LED lights installed from each corner of the ceiling to the center of the decorative lights. After conducting a price survey in the market-place it turns out that the LED lights sold in the market are in the form of rollers, where 1 roller is 3 m long with a price of 35,000 IDR/ roller. If the new cafe is 8 m x 8 m x 8 m, then it can be concluded that the amount of funds needed for her to buy LED lights is....
4. In prism ADE.BCF, DA = 6 cm and the distance of E to AD is 3 cm. What is the height of the prism? Decide whether the following statement (1) and (2) are sufficient to answer the question and give your reason!
    a. Volume of prism is 180 cm^3
    b. AF = 5 cm



Appendix 2. Emotional Stability Questionnaire

Every statement is scaled from 1 to 5 representing strongly disagree (1), disagree (2), adequate agree (3), agree (4), and strongly agree (5).

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Emotional Adequacy

1. I try to be someone who is liked by friends and teachers
2. I try to be a good listener to whoever I am talking to
3. I try to comfort my fellow friends who are experiencing sadness
4. I will care about friends who are experiencing a disaster
5. I always prioritize my interests over others
6. I will let my friends who are absent from school do the work given by the teacher
7. I feel that I do many things that are beneficial to others

Emotional Maturity

8. It doesn't take me long to adjust to something new
9. I am easily stressed if I get into trouble in my neighborhood
10. I get angry easily when my friends criticize me
11. During the learning process in class, I will study hard
12. I feel that the environment around me can accept me for who I am
13. I am aware of my strengths and weaknesses
14. I try to be ready and calm in facing tomorrow
15. I feel confident that I will succeed in what I am doing
16. I never make the same mistake a second time in carrying out a task
17. I often get tense and emotional when my work is criticized
18. I am easily offended when insulted by others
19. I feel that I am better than others
20. I feel anxious when I am among people I don't know
21. I often experience mood swings
22. I feel inferior when I hang out with friends
23. I feel worried about many things

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Emotional Control

24. I am easily discouraged by problems in the environment around me.
25. When I get angry, I can immediately think of the consequences or risks that arise when I get angry
26. I try to hold my anger even though someone always harms me.
27. Many things in my environment change my mood.
28. I feel excessive tension every time I complete an assignment or test given by my teacher
29. I rush to make decisions because of my emotions
30. I feel sad when I fail continuously
31. I am easily offended by anything that annoys me
32. If I get angry at a friend who offends me, I usually try to control my anger first
33. Even when I'm upset, I can still do my assignments well
34. I get caught up in problems that happen to me
35. I tend to show my anger in front of people I don't like
36. I panic easily if something happens to me
37. I am still kind to my friends even though they have done something wrong to me
38. Although I have a lot of assignments, I can still excel as a student
39. I try to relax and distract myself from problems that make me tense
40. I am easily distracted by the commotion around me

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Examining the Attitudes of Students Before and After a Course in Introductory Statistics

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Abstract: This study investigates the attitudes of undergraduate health science students towards statistics before and after completing an introductory statistics course. Given the critical role that attitudes play in students' academic performance, understanding how these attitudes evolve during the course is essential for improving teaching strategies and curriculum design. The study employed the Survey of Attitudes Toward Statistics (SATS-36), a well-validated instrument that measures six subscales of attitude: Affect, Cognitive Competence, Value, Difficulty, Interest, and Effort. Data were collected from 119 students at the University of Minnesota Rochester, with both pre-course and post-course assessments. The results indicated statistically significant changes in several attitude dimensions, suggesting that the course had a measurable impact on students' perceptions of statistics. These findings have important implications for educators in designing courses that not only improve statistical competence but also positively influence students' attitudes towards the subject.

Keywords: attitudes, statistics, education, assessment, pedagogy

INTRODUCTION

In recent decades, students' attitudes toward statistics have emerged as a central concern in statistics education research. Attitudes—defined as a predisposition to respond positively or negatively to statistics as a subject—have been shown to influence both engagement and academic success in statistics courses (Cashin & Elmore, 2005; Papanastasiou, 2000; Tapia & Marsh, 2001). For students in health science disciplines, statistical literacy is increasingly vital due to the rising emphasis on evidence-based practice, data-informed clinical decision-making, and the integration of research findings into patient care.

Despite its relevance, statistics often elicits anxiety and resistance among students. Therefore, it is essential to understand and address attitudinal barriers to learning.

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Several factors are known to influence students' attitudes toward statistics, including prior mathematical experience, perceived difficulty of statistical concepts, instructional methods, and the extent to which students can relate the material to their own field (Zientek et al., 2011; Gal, Ginsburg, & Schau, 1997). For health science students, who may not have a strong background or intrinsic interest in quantitative methods, statistics can be perceived as abstract and disconnected from their career goals—leading to negative attitudes that hinder learning and application. In fact, Dingel and Ayebo (2024) emphasize that students' early mathematical experiences, as reflected in their autobiographical narratives, often shape how they perceive and approach quantitative subjects later in their academic careers, including statistics.

Recent studies continue to underscore the significance of understanding and improving student attitudes toward statistics. For instance, Hannigan, Hegarty, and McGrath (2014) found that prior learning experiences significantly shaped graduate-entry medical students' attitudes. Similarly, Smith and Doe (2021) found that integrating real-world data analysis into undergraduate statistics courses significantly improved students' attitudes, enhancing conceptual understanding, engagement, and reducing anxiety. These findings align with the trend toward active learning environments that foster deeper comprehension and promote positive affective responses in statistics education. Ayebo and Dingel (2021) also explored how gender differences and mathematics attitudes among undergraduate health science students are linked to achievement, highlighting the importance of targeting attitudinal shifts to support learning outcomes in STEM fields.

To measure these complex attitudes, researchers have developed a variety of instruments, such as the Statistics Attitude Survey (SAS; Roberts & Bilderback, 1980), the Attitudes Toward Statistics (ATS; Wise, 1985), the Multifactorial Scale of Attitude Toward Statistics (MSAS; Auzmendi, 1991), and the Students' Attitudes Toward Statistics (STATS; Sutarso, 1992). Among these, the Survey of Attitudes Toward Statistics (SATS-36) developed by Schau et al. (2003) has become one of the most widely adopted tools. It measures students' attitudes across six dimensions: Affect, Cognitive Competence, Value, Difficulty, Interest, and Effort. Its design allows for the collection of both pre-course and post-course data, making it a useful tool for evaluating the effect of instruction on student perceptions.

Previous research using the SATS-36 has examined how various pedagogical approaches influence student attitudes. Active learning strategies (Carlson & Winquist, 2011), student-led projects (Carnell, 2008), and contextualized instruction (Liau, Kiat, & Nie, 2015) have all been found to foster more positive attitudes toward statistics. Recent work by Slootmaeckers, Kerremans, and Adriaensen (2014) emphasized the importance of engaging students emotionally and cognitively by enhancing the design of research methods courses through active learning and formative feedback. Their findings suggest that structuring learning experiences around clear expectations, practical relevance, and responsive support can improve student satisfaction and perceived learning outcomes—principles that are also applicable in introductory statistics education. Likewise, Çelik, Pektaş, and Karamustafaoğlu (2021) demonstrated that implementing flipped classroom models in higher education can enhance students' self-efficacy and foster more posi-

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tive attitudes toward learning. Together, these studies suggest that interactive, student-centered instructional approaches—such as flipped classrooms and formative feedback—can be especially effective in promoting engagement, perceived relevance, and positive attitudes in quantitative subjects like statistics.

In particular, studies have highlighted the benefit of aligning course content with students' career interests and demonstrating real-world relevance, which can significantly increase engagement and reduce anxiety (Ashaari et al., 2011; Coetzee & Van der Merwe, 2010; Dempster & McCorry, 2009). Integration of real-world datasets and application-based instruction has shown promise in bridging the gap between theoretical knowledge and practical use, particularly in health sciences where data-driven decisions are paramount (Makar & Rubin, 2018).

While much research has explored general student attitudes toward statistics, few studies have focused specifically on health science students in the early stages of their undergraduate education, particularly within U.S. academic settings. Moreover, while there is substantial evidence on the relationship between attitude and achievement, less is known about how introductory courses themselves shape attitudes over time, and which specific subcomponents of attitude are most affected by course participation.

This study aims to fill this gap by investigating changes in students' attitudes toward statistics before and after a one-semester introductory statistics course at a Midwestern university with a strong health sciences focus. Using the SATS-36, we assess six dimensions of attitude to determine which areas improve, remain stable, or decline following formal instruction. The findings offer valuable insights for educators seeking to improve statistics pedagogy, particularly for students in applied health science tracks, by highlighting the interplay between course design and student affect.

METHOD

Participants

The study involved 119 undergraduate health science students who were enrolled in a first-year statistics course at the University of Minnesota Rochester. The majority of the participants were female (76%) and primarily Caucasian (93%), with an average age of 18.25 years, ranging between 17 and 27 years. The University of Minnesota Rochester is recognized for its innovative educational approach, with a strong focus on teaching excellence and a commitment to the scholarship of teaching and learning.

This introductory statistics course is commonly taken by first- or second-year students in the fall semester. The curriculum covers essential statistical concepts, data representation techniques us-

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ing Microsoft Excel, the fundamentals of experimental design, probability theory, and statistical inference methods, including the Central Limit Theorem and the Law of Large Numbers. The course wraps up with a focus on quantitative statistical inference methods. The instructional team, which includes seasoned faculty members and undergraduate academic assistants, is attuned to the unique needs of the student population and actively works to assist students as they adapt to university life.

To foster a more positive learning environment, the course employed several innovative pedagogical strategies. These included collaborative group projects based on real-world healthcare data, the use of Excel to promote data literacy, and interactive simulations that allowed students to explore statistical concepts visually. Additionally, weekly formative assessments and reflective journals were used to encourage metacognition and reduce anxiety. These interventions aimed to boost students' confidence and demonstrate the relevance of statistics to health science fields.

Instrument

The Survey of Attitudes Toward Statistics (SATS-36), developed by Schau et al. (2003), was used to measure students' attitudes toward statistics. The SATS-36 consists of 36 items categorized into six subscales that capture different dimensions of student attitudes:

- ***Affect*** (6 items): Assesses students' emotional responses toward statistics, such as enjoyment, anxiety, and frustration (e.g., *"I will like statistics"* and *"I am scared by statistics"*).
- ***Cognitive Competence*** (6 items): Reflects students' self-perceptions of their intellectual ability and capacity to understand statistics (e.g., *"I can learn statistics"* and *"I will make a lot of math errors in statistics"*).
- **Value** (9 items): Measures the perceived usefulness and relevance of statistics in personal and professional life (e.g., *"Statistical skills will make me more employable"* and *"Statistics is irrelevant in my life"*).
- **Difficulty** (7 items): Evaluates how difficult students perceive statistics to be, including computational and conceptual complexity (e.g., *"Statistics involves massive computations"* and *"Statistics is a complicated subject"*).
- **Interest** (4 items): Captures the students' intrinsic interest in statistics and their desire to engage with statistical content (e.g., *"I am interested in learning statistics"*).
- **Effort** (4 items): Gauges the level of effort students intend to put into learning statistics (e.g., *"I plan to work hard in my statistics course"*).

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Items are scored on a 7-point Likert scale from 1 (strongly disagree) to 7 (strongly agree), with several items reverse-coded to ensure consistent interpretation. Higher scores generally indicate more favorable attitudes.

Statistical Analyses

The SATS-36 survey was administered to participants at two points: before and after one semester of instruction in introductory statistics. A total of 165 students initially participated, but due to attrition (e.g., course withdrawals and incomplete data), the final analysis included responses from 119 students.

To assess changes in students' attitudes, descriptive statistics (means and standard deviations) were calculated for each of the six SATS-36 subscales. Paired sample *t*-tests were conducted to examine statistically significant differences between pre- and post-course scores. Internal consistency reliability for each subscale was evaluated using Cronbach's alpha, with all subscales demonstrating acceptable to excellent reliability ($\alpha = 0.72$ to 0.91).

Criterion-related validity was assessed by correlating each attitude subscale with students' final course grades and a composite score derived from a statistical proficiency test. These correlations were analyzed both before and after the course to determine how relationships among affective domains and performance outcomes evolved over time.

Interpretations of score changes, especially for constructs such as Cognitive Competence, were grounded in both statistical significance and theoretical alignment with the SATS-36 instrument. For instance, an increase in this subscale suggests greater student confidence in their ability to understand and apply statistical concepts, even when item-level change was not uniformly significant. Taken together, these analyses offer a comprehensive view of how students' attitudes shifted across multiple dimensions of the learning experience.

RESULTS

Affect

The results indicated significant changes in students' attitudes across several domains. For the Affect subscale, participants' responses to statements such as "I will like statistics" and "I will enjoy taking statistics courses" showed a decrease over the semester, while statements indicating negative emotions towards statistics, such as "I will be under stress during statistics class" and "I am scared by statistics," showed an increase. These changes suggest that despite completing the course, some students may still harbor negative feelings towards statistics.

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Item	Pre-test		Post-test		Paired Sample	Comment on improvement
	M	SD	M	SD	*t*-test statistic	% increase or decrease
Affect 1	4.95	1.09	4.53	1.616	-2.83*	-8.48
Affect 2	4.53	0.51	4.63	1.705	0.58	2.21
Affect 3	4.60	1.59	4.14	1.857	-2.49*	-10.00
Affect 4	3.62	1.52	3.99	1.770	2.33*	10.22
Affect 5	4.57	1.05	4.24	1.580	-2.29*	-7.22
Affect 6	4.35	1.77	5.09	1.787	4.58**	17.01

Table 1: Mean, standard deviation and paired sample *t*-test scores on Affect
Key: M = mean; SD = standard deviation.; * $p < 0.05$; ** $p < 0.001$

These results underscore the complexity of students' emotional responses to statistics. While some may develop an appreciation or acceptance of the subject over time, others may find their anxieties and frustrations deepening, particularly if they struggle to grasp the material.

Cognitive Competence

In the Cognitive Competence subscale, which measures students' confidence in their ability to learn and understand statistics, most items showed a slight increase in mean scores from pre- to post-assessment, although only one item showed a statistically significant improvement. This suggests that while students may feel slightly more competent after completing the course, the increase in confidence is not uniform across all aspects of their statistical learning.

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Item	Pre-test		Post-test		Paired Sample	Comment on improvement
	M	SD	M	SD	*t*-test statistic	% increase or decrease
Cognitive Competence 1	4.74	1.31	4.88	1.74	0.89	2.95
Cognitive Competence 2	5.34	1.28	5.60	1.50	1.76**	4.87
Cognitive Competence 3	3.91	1.53	4.57	1.65	4.21	16.88
Cognitive Competence 4	6.25	0.98	6.11	1.26	-1.10	-2.24
Cognitive Competence 5	5.64	1.16	5.34	1.57	-1.85	-5.32
Cognitive Competence 6	4.26	1.41	4.44	1.67	1.29	4.23

Table 2: Mean, standard deviation and paired sample *t*-test scores on Cognitive Competence

Key: M = mean; SD = standard deviation.; * $p < 0.05$; ** $p < 0.001$

Despite these findings, the overall increase in cognitive competence is a positive outcome, indicating that the course may have been effective in building students' confidence in their statistical abilities, albeit to varying degrees.

Value

For the Value subscale, which assesses the perceived relevance and usefulness of statistics in students' lives, all items showed a decrease in mean scores from pre- to post-assessment. This finding is concerning, as it suggests that students may not fully appreciate the importance of statistics even after completing the course. The decrease in value-related attitudes may be linked to the challenges students face in applying statistical concepts to real-world scenarios and may indicate a need for the course to better connect statistical learning with practical, real-world applications that are directly relevant to students' future careers.

The Value subscale of the SATS-36 consists of nine items (Items 7, 9, 10, 13, 16, 17, 21, 25, and 33), which evaluate the extent to which students perceive statistics as relevant and useful in their

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personal and professional lives. The following table summarizes the pre- and post-course responses for each item.

Item	Pre-test		Post-test		Paired Sample	Comment on improvement
	M	SD	M	SD	*t*-test statistic	% increase or decrease
Value 1	6.05	1.007	5.71	1.43	-2.71*	-5.62
Value 2	4.93	1.318	4.71	1.49	-1.71	-4.46
Value 3	5.33	1.161	5.20	1.46	-1.12	-2.44
Value 4	5.21	1.367	5.12	1.40	-0.76	-1.73
Value 5	5.17	1.306	4.78	1.58	-2.71*	-7.54
Value 6	3.88	1.456	3.76	1.59	-0.85	-3.09
Value 7	4.86	1.362	4.81	1.311	-0.42	-1.03
Value 8	5.41	1.235	5.36	1.34	-0.41	-0.92
Value 9	5.53	1.281	5.14	1.40	-2.89*	-7.05

Table 3: Mean, standard deviation, and paired sample *t*-test scores for Value subscale (SATS-36 Items 7, 9, 10, 13, 16, 17, 21, 25, and 33)

Key: M = mean; SD = standard deviation.; * $p < 0.05$; ** $p < 0.001$

These results suggest that while students may recognize the value of statistics initially, the challenges they face during the course could lead to a reduction in their perception of its relevance. This highlights a potential area for curriculum improvement, where greater emphasis could be placed on demonstrating how statistical skills are essential in various health science professions.

Difficulty

The Difficulty subscale revealed that students perceived statistics as more challenging after completing the course, with several items showing significant increases in mean scores. This increase

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in perceived difficulty could be attributed to the course content becoming more complex as the semester progressed, or it could reflect a deeper understanding of the subject's inherent challenges.

The Difficulty subscale includes seven items (Items 6, 8, 22, 24, 30, 34, and 36), addressing students' perceptions of how difficult statistics is as a subject.

Item	Pre-test		Post-test		Paired Sample	Comment on improvement
	M	SD	M	SD	*t*-test statistic	% increase or decrease
Difficulty 1	4.19	1.11	4.90	1.33	5.53**	16.95
Difficulty 2	3.47	1.21	3.53	1.65	0.42	1.73
Difficulty 3	3.35	1.27	3.52	1.37	1.33	5.07
Difficulty 4	2.97	1.23	3.13	1.31	1.36	5.39
Difficulty 5	3.74	0.95	4.45	1.46	5.94**	18.98
Difficulty 6	3.60	1.06	3.58	1.27	-0.13	-0.56
Difficulty 7	3.72	1.23	3.85	1.34	1.05	3.49

Table 4: Mean, standard deviation, and paired sample *t*-test scores for Difficulty subscale (SATS-36 Items 6, 8, 22, 24, 30, 34, and 36)

Key: M = mean; SD = standard deviation.; * $p < 0.05$; ** $p < 0.001$

The significant increase in difficulty ratings suggests that as students progressed through the course, they found the material increasingly challenging. This may have contributed to the decline in their overall attitude towards the subject, as reflected in other subscales like Affect and Interest.

Interest

The Interest subscale, which measures students' enthusiasm for engaging with statistics, showed a significant decrease across all items. This decline in interest may indicate that the course con-

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tent or delivery methods did not sufficiently engage students or that the challenges they encountered dampened their initial enthusiasm for the subject.

The Interest subscale consists of four items (Items 12, 20, 23, and 29), measuring students' individual curiosity and enthusiasm for engaging with statistical material.

Item	Pre-test		Post-test		Paired Sample	Comment on improvement
	M	SD	M	SD	*t*-test statistic	% increase or decrease
Interest 1	4.99	1.33	4.20	1.67	-5.48**	-15.83
Interest 2	4.84	1.14	4.40	1.44	-3.78**	-9.09
Interest 3	5.29	1.09	4.57	1.39	-5.79**	-13.61
Interest 4	5.50	1.09	4.50	1.55	-7.54**	-18.18

Table 5: Mean, standard deviation, and paired sample *t*-test scores for Interest subscale (SATS-36 Items 12, 20, 23, and 29)

Key: M = mean; SD = standard deviation.; * $p < 0.05$; ** $p < 0.001$

These findings suggest a potential misalignment between the course structure and the interests of the students. To maintain and enhance interest, educators might consider integrating more interactive and applied learning experiences that directly connect with students' future professional goals.

Effort

Finally, the Effort subscale, which gauges the amount of effort students were willing to invest in learning statistics, also showed a decrease in mean scores from pre- to post-assessment. While the decrease was statistically significant for two of the four items, it suggests that students may have felt overwhelmed by the demands of the course, leading to a reduction in the effort they were willing to put forth.

The Effort subscale includes four items (Items 1, 2, 14, and 27), which reflect the amount of work students plan to expend in learning statistics.

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Item	Pre-test		Post-test		Paired Sample	Comment on improvement
	M	SD	M	SD	*t*-test statistic	% increase or decrease
Effort 1	6.82	0.69	6.71	0.96	-1.13	-1.61
Effort 2	6.83	0.66	6.34	0.99	-4.99**	-7.17
Effort 3	6.53	0.76	6.04	1.18	-4.78**	-7.50
Effort 4	6.81	0.63	6.71	0.88	-1.10	-1.47

Table 6: Mean, standard deviation, and paired sample *t*-test scores for Effort subscale (SATS-36 Items 1, 2, 14, and 27)

Key: M = mean; SD = standard deviation.; * $p < 0.05$; ** $p < 0.001$

The decline in effort may reflect students' growing frustration with the course material or a perception that their efforts were not yielding the desired outcomes, further contributing to negative attitudes towards the subject.

Correlational Analysis

The correlations among the subscales were examined both before and after the course. For the pre-assessment, all six subscales were generally significantly correlated with each other, except for Effort, which was only significantly correlated with Value and Interest. After the course, all subscales remained significantly correlated, with the Effort subscale continuing to show limited correlations with other domains. These correlations suggest that students' attitudes towards different aspects of statistics are interconnected, with changes in one domain potentially influencing attitudes in others.

	Affect	Cognitive Competence	Value	Difficulty	Interest	Effort	Final Grade
Affect	1						
CC	0.77**	1					
Value	0.43**	0.42**	1				

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Difficulty	0.47**	0.50**	0.14	1			
Interest	0.54**	0.46**	0.60**	0.22*	1		
Effort	0.15	0.15	0.20*	0.090	0.23*	1	
Final Grade	0.12	0.24**	0.09	0.097	0.07	0.05	1

Table 7: Correlations (Pre-test)

**Correlation significant at the 0.01 level (2-tailed); *Correlation significant at the 0.05 level (2-tailed).

	Affect	**Cognitive Competence**	**Value**	**Difficulty**	**Interest**	**Effort**	**Final Grade**
Affect	1						
CC	0.82**	1					
Value	0.50**	0.48**	1				
Difficulty	0.49**	0.47**	0.09	1			
Interest	0.54**	0.46**	0.71**	0.06	1		
Effort	0.020	-0.03	0.13	-0.03	0.22*	1	
Final Grade	0.36**	0.50**	0.27**	0.15	0.28**	0.05	1

Table 8: Correlations (Post-test)

**. Correlation is significant at the 0.01 level (2-tailed).

Final Grade

The correlations between the subscales and the final grade were also examined. For the pre-test, Cognitive Competence showed the strongest correlation with final grades, followed by Interest and Affect. However, in the post-test analysis, the correlation between Cognitive Competence and final grades strengthened, while the correlations for Affect and Interest also increased. This indicates that students who performed well in the course were likely to have higher cognitive competence and positive attitudes towards statistics by the end of the semester.

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Cronbach's Alpha

Cronbach's alpha values were computed to assess the internal consistency of the SATS-36 subscales for both the pre-test and post-test scores. The results, presented in Table 9, indicate that the survey instrument demonstrated good to excellent reliability across all subscales, with values ranging from 0.72 to 0.91. This high level of reliability suggests that the SATS-36 is a consistent and dependable measure of students' attitudes towards statistics.

Component	Time administered	
	Pre-test	Post-test
Affect	0.85	0.87
Cognitive Competence	0.81	0.87
Value	0.87	0.90
Difficulty	0.75	0.90
Interest	0.87	0.91
Effort	0.72	0.85

Table 9: Cronbach's alpha values for pre-test and post-test scores

DISCUSSION

The findings of this study provide important insights into how undergraduate health science students' attitudes towards statistics change over the course of an introductory statistics class. Despite completing the course, many students still held negative attitudes towards statistics, particularly in terms of affect and perceived difficulty. These results highlight the need for educators to not only focus on teaching statistical concepts but also on addressing students' attitudes and confidence in their ability to learn statistics.

One potential explanation for the observed decline in positive attitudes is the challenge that statistics presents to students who may not have a strong background in mathematics. The perceived

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difficulty of the subject could have led to increased anxiety and reduced interest, ultimately impacting their overall attitudes. Additionally, the decrease in the perceived value of statistics suggests that more effort may be needed to demonstrate the practical applications of statistical knowledge in the students' future careers, particularly in health sciences.

The results also suggest that students' cognitive competence and effort may be influenced by their emotional responses to the subject. For instance, students who feel anxious about statistics may also be less confident in their ability to understand the material and may be less motivated to put in the necessary effort to succeed. This interplay between affective and cognitive factors underscores the importance of addressing both emotional and intellectual aspects of learning in the classroom.

Implications for Teaching and Curriculum Design

The results of this study have several important implications for teaching and curriculum design in introductory statistics courses, particularly for health science students:

1. **Incorporating Real-World Applications**: To enhance the perceived value of statistics, instructors should incorporate more real-world applications that are directly relevant to students' future careers. For health science students, this could include examples from medical research, public health studies, and evidence-based practice. Demonstrating how statistical knowledge can be applied to real-world problems may help students see the relevance of what they are learning and increase their engagement with the subject.
2. **Addressing Student Anxiety**: Given the significant increase in perceived difficulty and anxiety related to statistics, it is important for instructors to address these issues head-on. This could involve integrating stress-reduction techniques into the course, such as mindfulness exercises or providing additional support through tutoring and office hours. Instructors might also consider starting the course with less intimidating, foundational content before gradually increasing the complexity of the material.
3. **Fostering a Growth Mindset**: To combat the decline in cognitive competence and effort, educators should focus on fostering a growth mindset in their students. This involves encouraging students to view challenges as opportunities for growth and learning, rather than as insurmountable obstacles. Strategies could include providing positive reinforcement, offering constructive feedback, and creating a classroom environment that celebrates effort and persistence.
4. **Reflecting on Pedagogical Impact**: The observed changes in students' attitudes—particularly the modest improvement in cognitive competence and the decline in perceived value and interest—may reflect the mixed impact of the course's instructional design. While the integration of real-world data, collaborative group projects, and formative assessments aimed to promote engagement and reduce anxiety, the results suggest that these interventions may not have fully addressed students' emotional responses or per-

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ceptions of relevance. It is possible that while students gained confidence in specific skills, they did not consistently perceive statistics as directly applicable to their future careers. This indicates a need for further alignment between course activities and students' professional goals, such as embedding more career-specific examples and facilitating reflective discussions about the role of statistics in health sciences. Strengthening this connection may help sustain students' interest and reinforce the value of statistical thinking beyond the classroom.

5. **Enhancing Student Engagement**: The significant decrease in students' interest in statistics over the semester suggests that more needs to be done to keep students engaged. Active learning strategies, such as group work, problem-based learning, and interactive simulations, can make the learning experience more dynamic and engaging. Additionally, providing opportunities for students to take ownership of their learning through projects or presentations can help maintain their interest.
6. **Continuous Assessment of Attitudes**: Finally, it is crucial for educators to continuously assess students' attitudes towards statistics throughout the course. This allows for timely interventions if negative attitudes begin to emerge. Regular check-ins, surveys, or reflective writing assignments can provide insights into how students are feeling about the material and where they might need additional support.

Limitations

While this study provides valuable insights, it is important to acknowledge its limitations. The use of convenience sampling and the lack of a control group limit the generalizability of the findings. Additionally, the study was conducted at a single institution with a relatively homogeneous sample, which may not reflect the experiences of students at other universities or in different fields of study. Future research could address these limitations by including a more diverse sample and comparing attitudes across different academic disciplines.

Future Research

To build on the findings of this study, future research could explore the following areas:

1. **Longitudinal Studies**: Conducting longitudinal studies that track changes in students' attitudes towards statistics over multiple semesters or across different courses would provide a deeper understanding of how these attitudes evolve over time. This could help identify the long-term impact of introductory statistics courses on students' perceptions and engagement with the subject.
2. **Comparative Studies**: Future research could compare the effectiveness of different teaching methods in shaping students' attitudes towards statistics. For example, studies could investigate whether active learning strategies, flipped classrooms, or online learning environments are more effective in improving students' attitudes compared to traditional lecture-based approaches.

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3. **Exploring the Role of Instructors**: The role of the instructor in influencing students' attitudes towards statistics is another important area for future research. Studies could examine how different teaching styles, instructor characteristics, and classroom environments impact students' attitudes and learning outcomes.
4. **Investigating Specific Subgroups**: Future research could also explore how attitudes towards statistics vary among different subgroups of students, such as those with different levels of mathematical background, those from different academic disciplines, or those with varying levels of interest in the subject. This could help identify specific groups of students who may benefit from targeted interventions.

CONCLUSION

This study highlights the complex and multifaceted nature of students' attitudes towards statistics. While some aspects of students' attitudes improved after completing an introductory statistics course, other areas, particularly affective responses and perceived difficulty, remained challenging. These findings underscore the need for educators to take a holistic approach to teaching statistics, one that addresses both the cognitive and emotional aspects of learning.

By integrating real-world applications, addressing student anxiety, fostering a growth mindset, and continuously assessing attitudes, educators can create a more supportive and engaging learning environment. This, in turn, can help students develop not only the skills they need to succeed in statistics but also a more positive and confident attitude towards the subject.

The insights gained from this study can inform the development of more effective statistics curricula, ultimately helping to better prepare health science students for the statistical demands of their future careers. As the importance of statistical literacy continues to grow, particularly in fields like health sciences where evidence-based practice is critical, it is essential that students leave their statistics courses with both the knowledge and the confidence to apply what they have learned.

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Appendix: SATS-36 Instrument

Affect – students' feelings concerning statistics

Affect 1: I will like statistics.

Affect 2: *I will feel insecure when I have to do statistics problems.

Affect 3: *I will get frustrated going over statistics tests in class.

Affect 4: *I will be under stress during statistics class.

Affect 5: I will enjoy taking statistics courses.

Affect 6: *I am scared by statistics.

Cognitive Competence – students' attitudes about their intellectual knowledge and skills when applied to statistics

CC 1: *I will have trouble understanding statistics because of how I think.

CC 2: *I will have no idea of what's going on in this statistics course.

CC 3: *I will make a lot of math errors in statistics.

CC 4: I can learn statistics.

CC 5: I will understand statistics equations.

CC 6: *I will find it difficult to understand statistical concepts.

Value – students' attitudes about the usefulness, relevance, and worth of statistics in personal and professional life

Value 1: *Statistics is worthless.

Value 2: Statistics should be a required part of my professional training.

Value 3: Statistical skills will make me more employable.

Value 4: *Statistics is not useful to the typical professional.

Value 5: *Statistical thinking is not applicable in my life outside my job.

Value 6: I use statistics in my everyday life.

Value 7: *Statistics conclusions are rarely presented in everyday life.

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Value 8: *I will have no application for statistics in my profession.

Value 9: *Statistics is irrelevant in my life.

Difficulty – students' attitudes about the difficulty of statistics as a subject

Difficulty 1: Statistics formulas are easy to understand.

Difficulty 2: *Statistics is a complicated subject.

Difficulty 3: Statistics is a subject quickly learned by most people.

Difficulty 4: *Learning statistics requires a great deal of discipline.

Difficulty 5: *Statistics involves massive computations.

Difficulty 6: *Statistics is highly technical.

Difficulty 7: *Most people have to learn a new way of thinking to do statistics.

Interest – students' level of individual interest in statistics

Interest 1: I am interested in being able to communicate statistical information to others.

Interest 2: I am interested in using statistics.

Interest 3: I am interested in understanding statistical information.

Interest 4: I am interested in learning statistics.

Effort - amount of work the student expends to learn statistics

Effort 1: I plan to complete all of my statistics assignments.

Effort 2: I plan to work hard in my statistics course.

Effort 3: I plan to study hard for every statistics test.

Effort 4: I plan to attend every statistics class session.

* *Asterisk indicates item is reverse-coded (i.e. 1 becomes 7, 2 becomes 6, etc.)*

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The Problem Corner



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The Purpose of ***The Problem Corner*** is to give Students and Instructors working independently or together a chance to step out of their "comfort zone" and solve challenging problems. Rather than in the solutions alone, we are interested in methods, strategies, and original ideas following the path toward figuring out the final solutions. We also encourage our Readers to propose new problems. To submit a solution, type it in Microsoft Word, using math type or equation editor, however PDF files are also acceptable. Email your solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country. Solutions to posted problem must contain detailed explanation of how the problem was solved. The best solution will be published in a future issue of MTRJ, and correct solutions will be given recognition. To propose a problem, type it in Microsoft Word, using math type or equation editor, email your proposed problem and its solution as an attachment to The Problem Corner Editor iretamoso@bmcc.cuny.edu stating your name, institutional affiliation, city, state, and country.

Greetings, problem solvers!

We are pleased to share that Problems 40 and 41 in *The Problem Corner* have received exceptional solutions, marked by both accuracy and originality. Highlighting these remarkable approaches aims to inspire others and foster a deeper appreciation for mathematical thinking worldwide.

New Problems

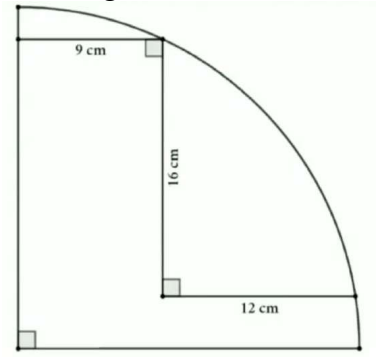
Problem 42
Proposed by Ivan Retamoso, BMCC, USA.

A 5-foot-tall fence stands parallel to a tall building, 2 feet away from it. Determine the length of the shortest ladder that can extend from the ground, over the fence, to touch the wall of the building.

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Problem 43
Proposed by Ivan Retamoso, BMCC, USA.

In the figure below find the radius of the circle.

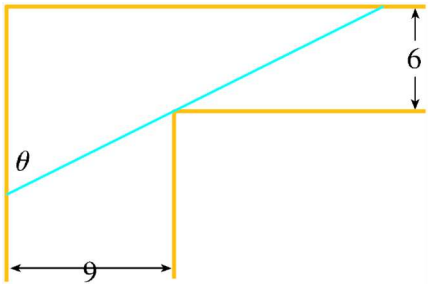


Solutions to **problems** from the previous issue.

"Steel pipe" problem.

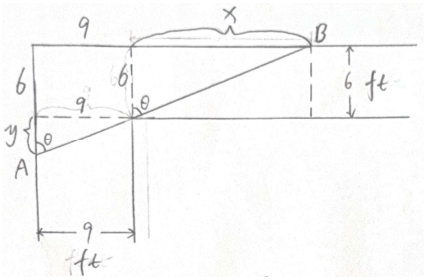
Problem 40
Proposed by Ivan Retamoso, BMCC, USA.

A steel pipe is being carried horizontally through a hallway that is 9 feet wide and turns at a right angle into a hallway that is 6 feet wide. What is the maximum length of pipe that can make the turn around the corner?



First solution to problem 40
By Stephen Chen, Borough of Manhattan CC, Fuzhou, China.
In this first solution, the solver provides a thorough explanation by first establishing a relationship between the length of the pipe and a clearly defined variable x. Through differentiation and a clever algebraic factorization, he identifies the critical point that leads to a well-justified solution.

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$tan\theta=\frac{x}{b}$, $tan\theta=\frac{9}{y}$, length of pipe $L=AB$

$\frac{x}{b}=\frac{9}{y} \Rightarrow y=\frac{54}{x}$

$L^2=(x+9)^2+(y+6)^2 \Rightarrow L=\sqrt{(x+9)^2+(\frac{54}{x}+6)^2}$

Taking the derivative of L, setting it equal to zero, and solving for x, we find

$$L'=\frac{2(x+9)+2(\frac{54}{x}+6)(-\frac{54}{x^2})}{2\sqrt{(x+9)^2+(\frac{54}{x}+6)^2}}=0$$

$$2x^4+18x^3-648x-5832=0$$
$$(2x+18)(x^3-324)=0$$
$$2x+18=0 \text{ or } x^3-324=0$$

$x=-9$ or $x^3=324$ then $x=6.868\ ft$

Since we have minimized L, the value of x found above will lead to the maximum length of pipe that can make the turn around the corner.

$$L=\sqrt{(x+9)^2+(\frac{54}{x}+6)^2}=\sqrt{(6.868+9)^2+(\frac{54}{6.868}+6)^2}=21.07\ ft$$

Second solution to problem 40

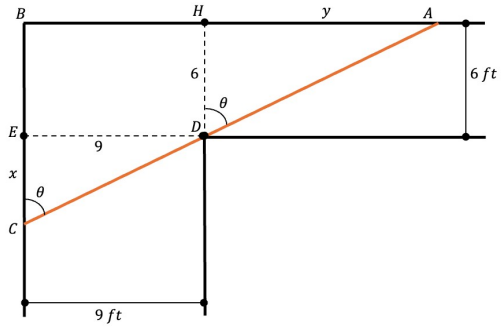
Dr. Abdullah Kurudirek, Al-Naji University, Baghdad.

In this second solution, the solver presents two approaches: one based on the given angle and the other on a properly defined distance x. Although some computational details are omitted, the work shown leads to the correct result for both distinct yet similar methods.

First, let's identify some points on the given diagram and form a right-angled triangle ABC. Accordingly, the maximum length of the pipe that can make the turn will be equal to the length of the hypotenuse of this triangle, which is $AC=L$. Then, to help us find this, let's draw some auxiliary lines to simplify the calculations, for example, lines like $DH=6\ ft$ and $DE=9\ ft$. Next, let's define $DA=L_1$ and $DC=L_2$, so that

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$L=L_1+L_2$. From this point, we can proceed in two different ways. I would like to briefly show the first method and then continue with the second one.



Method 1: $L=\frac{6}{cos\theta}+\frac{9}{sin\theta}$ by finding the minimum possible value of this expression with the appropriate value of θ, we determine the maximum pipe length L that can make the turn. So, we will proceed by setting $\frac{dL}{d\theta}=0$.

$$\frac{dL}{d\theta}=\frac{6sin\theta}{cos^2\theta}-\frac{9cos\theta}{sin^2\theta}=0$$

After performing the necessary algebraic operations, we express $tan\theta=\sqrt[3]{\frac{3}{2}}$. To simplify our calculations, let's set $tan\theta=\sqrt[3]{\frac{3}{2}}=a$. Then, if we continue our work by expressing $sin\theta=\frac{a}{\sqrt{1+a^2}}$ and $cos\theta=\frac{1}{\sqrt{1+a^2}}$ in terms of a, we proceed as follows.

$L=\frac{6}{cos\theta}+\frac{9}{sin\theta}=6\sqrt{1+a^2}+9\frac{\sqrt{1+a^2}}{a}$. Now substitute $a=\sqrt[3]{\frac{3}{2}}$ and get $L\approx 21.07\ ft$.

Method 2: In this method, we will proceed by using trigonometry and the concept of similarity in triangles, and by writing the following relationships.

From $tan\theta=\frac{y}{6}=\frac{9}{x}$, we can deduce that $xy=54$, and from there, we can write $x=\frac{54}{y}$.

Now, if we apply the Pythagorean Theorem in the right triangle ABC

$$AC=L=\sqrt{(x+6)^2+(y+9)^2}$$

After substituting $x=\frac{54}{y}$ here, we find that $L=\sqrt{(\frac{54}{y}+6)^2+(y+9)^2}$

then set $\frac{dL}{dy}=0$ as we did in the first method by finding the minimum possible value of this expression with the appropriate value of y which is $y\approx 6.87$ (*attention*, not y=-9), we determine the maximum pipe length L that can make the turn. *Finally, the maximum length of the pipe that can be carried horizontally around the corner is approximately as follows:* $L\approx 21.07\ ft$

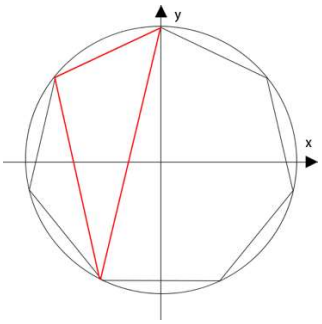
"Heptagon" problem.

Problem 41

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Proposed by Ivan Retamoso, BMCC, USA.

A regular heptagon is inscribed in a unit circle with a radius of 1 cm. Calculate the area of the red triangle.



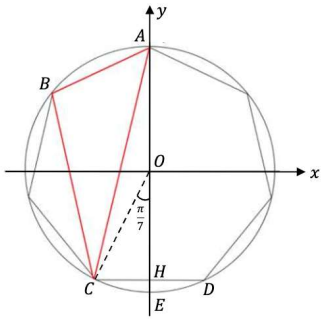
First Solution to problem 41
Dr. Abdullah Kurudirek, Al-Naji University, Baghdad.

Our solver has presented two detailed and well-structured solutions, each employing different trigonometric identities and formulas for expressing the area of a triangle in terms of the angle between its sides.

Method 1:
Let us first identify some points on the given figure and draw an auxiliary line. Using the concept of power of a point with respect to circles, we can write:

$$CH.HD=AH.HE$$

Here, let $HE=x$ and $AH=2-x$. Also, since a radius perpendicular to a chord bisects the chord, we know that $CH=HD$.



$$CH^2=(2-x)x$$

Now, in the right triangle OCH, we can use the sine theorem (law of sines) to write the following equality:

$$\frac{CH}{\sin\frac{\pi}{7}}=\frac{OC}{\sin\frac{\pi}{2}}$$

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From here, we obtain: $CH=sin\frac{\pi}{7}$. If we substitute this into equation (1), which was: $(sin\frac{\pi}{7})^2=2x-x^2$ and by using the discriminant formula, we obtain $x=1\pm cos\frac{\pi}{7}$. However, if we examine the given graph closely, it is evident that $HE<1$, so $x=1-cos\frac{\pi}{7}$.

Now, in the right-angled triangle AHC, we can use the Pythagorean Theorem to write the following:

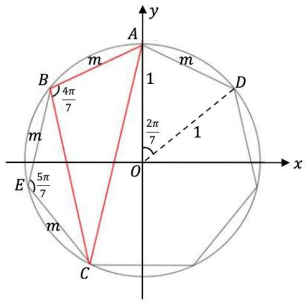
$$AH^2+CH^2=AC^2 \text{ then } (2-x)^2+(sin\frac{\pi}{7})^2=AC^2 \text{ where } x=1-cos\frac{\pi}{7}$$

Hence, $AC=\sqrt{(1+cos\frac{\pi}{7})^2+(sin\frac{\pi}{7})^2}$. Knowing that $<BAC=\frac{2\pi}{7}$ and $AB=2CH=2sin\frac{\pi}{7}$

Now, we can calculate the area of the desired red triangle using the sine area formula for triangles. Accordingly:

$$A(\Delta ABC)=\frac{AB.AC.sin\,(<BAC)}{2}=\frac{2sin\frac{\pi}{7}\sqrt{(1+cos\frac{\pi}{7})^2+(sin\frac{\pi}{7})^2}.sin\frac{2\pi}{7}}{2}\approx 0.661$$

Method 2: Again, let us first identify some points on the given figure and draw an auxiliary line. Next, let's denote one side of our regular heptagon as m; it was already given that the radius of the circle is 1 cm.



If we apply the law of cosines in triangle ΔADO

$m^2=1^2+1^2-2.1.1.cos\frac{2\pi}{7}$ then we obtain $m=\sqrt{2-2cos\frac{2\pi}{7}}$

Similarly, if we apply the law of cosines in triangle ΔBEC as well

$BC^2=m^2+m^2-2.m.m.cos\frac{5\pi}{7}$ and if we substitute the value of m here, we obtain

$BC=2sin\frac{2\pi}{7}$.

Now, we can calculate the area of the desired red triangle using the sine area formula for triangles. Accordingly:

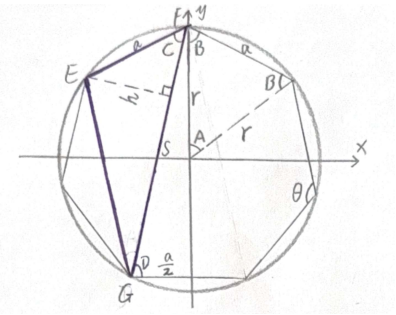
$$A(\Delta ABC)=\frac{AB.BC.sin\,(<ABC)}{2}=\frac{\sqrt{2-2cos\frac{2\pi}{7}}.2sin2\frac{\pi}{7}.sin\frac{4\pi}{7}}{2}\approx 0.661$$

Second Solution to problem 41

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This alternative solution employs a clever combination of geometric and trigonometric formulas to determine the required area of the triangle. The detailed computations are clearly presented, making it easier for readers to follow.



Each angle of the heptagon $\theta=\frac{180(7-2)}{7}=128.57°$, $r=1cm$

$LB=\frac{\theta}{2}=64.285°$, $LA=180°-2LB=180-2(64.285°)=51.43°$

$$\frac{sinA}{a}=\frac{sinB}{1}$$

Then $a=\frac{sin\,(51.43°)}{sin\,(64.285°)}$

$$LC=\frac{2\theta}{5}=51.43°$$

$h=asinC=\frac{sin^2(51.43°)}{sin\,(64.285°)}$

$$LD=\theta-\frac{2\theta}{5}=77.14°$$

$$S=\frac{\frac{a}{2}}{cosD}=\frac{\frac{sin\,(51.43°)}{2sin\,(64.285°)}}{cos\,(77.14°)}$$

$$A_{EFG}=\frac{1}{2}sh=\frac{1}{2}\frac{sin\,(51.43°)}{2sin\,(64.285°)cos\,(77.14°)}\frac{sin^2(51.43°)}{sin\,(64.285°)}=\frac{sin^3(51.43)}{4sin^2(64.285)cos\,(77.14°)}=0.66\ cm^2$$

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