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BREAKTHROUGH MOMENTS in PROBLEM SOLVING

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Abstract

Problem solving is critically important learning goal of mathematics as well as a necessary process for teaching mathematics. One of the most important moments of problem solving is when the student makes a cognitive breakthrough, also called an "aha!" moment, in the learning process. Teachers can serve an important role in scaffolding students to reach these moments of insight during the problem solving process. This article provides an example of the author's experience with "aha!" moments in his own mathematics education classroom with preservice and in-service teachers.

Introduction

Problem solving is critically important learning goal of mathematics as well as a necessary process for teaching mathematics (National Council of Teachers of Mathematics [NCTM], 2000; Posamentier & Krulik, 2008; Posamentier, Smith, & Stepelman, 2008). It is one of the NCTM's five process standards (NCTM, 2000), and has been considered by the National Council of Supervisors of Mathematics (NCSM) as the main reason for studying mathematics (NCSM, 1978). The NCTM (2000) considers problem solving to be a goal for learning mathematics in addition to being a means to do so. One of the most important moments of problem solving is when the student makes a cognitive breakthrough, also called an "aha!" moment, in the learning process. Teachers can serve an important role in scaffolding students to reach these moments of insight during the problem solving process. This article provides an example of the author's experience with "aha!" moments in his own mathematics education classroom with preservice and in-service teachers.

Conceptual Framework

There are three areas of work supporting the premise of this article: 1) Research in problem solving; 2) Vygotskyan social learning, scaffolding, and Zone of Proximal Development (ZPD); and 3) bisociative framework.

Problem solving is a process in which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation (Krulik & Rudnick, 1989). George Polya (1945) outlined the process for attempting to solve an unfamiliar problem: 1) Understanding the problem; 2) making a plan; 3) carrying out the plan; and 4) looking back. Problem solving can be considered the foundation of critical thinking and inquiry learning in a mathematics classroom, and it has been recommended that mathematics be taught from a problem solving perspective (Clark, 1997; Schoenfeld, 1985). The work of the NCTM, Clark, and Schoenfeld form a conceptual framework for the teaching and learning of problem solving.

Vygotsky is well-known for his theory that students learn best through social interactions and that teacher can provide the optimal amount of support, or scaffolding, for the learning process (Vygotsky, 1978). Learning through social interactions often is manifested in the classroom through the use of collaborative group learning in which students work together to solve a problem. Scaffolding "refers to the specific strategies or structures that help people move along in their development" and have been called "intellectual supports needed to reach new levels of understanding" (Nakkula & Toshalis, 2006, p. 10). Vygotsky called this the Zone of Proximal Development (ZPD), which "refers to the relative level of one's development in particular areas and is expressed as the difference between what a child can do without guidance and what he or she can do with assistance" (Nakkula & Toshalis, 2006, p. 10).

Bisociation is "a spontaneous flash of insight, which…connects the previously unconnected frames of reference and makes us experience reality at several planes at once" (Koestler, p. 45), which can be considered the "aha!" moment often experienced in a classroom. Bisociation is the antithesis of the Einstellung effect and is a foundation for creativity, inquiry, and discovery. The Einstellung effect is the habits people form in solving problems that keep them trying the same tested methods repeatedly rather than approach the problems from new and innovative perspectives. Bisociation is closely connected to the "aha!" moment students achieve during the problem solving process in a mathematics classroom (Czarnocha, Baker, Dias, & Prabhu, 2011). The foundation of this article and the process involved in problem solving is a the bisociative framework.

Teacher Education and Problem Solving

The author of this article teaches preservice and in-service teachers in a mathematics methods course at Pace University in New York. The teachers in his class are studying to become elementary school teachers and are required to take a class in mathematics pedagogy. The class is based on the NCTM *Standards* and Common Core, and is a reformed- and research-based class that addresses problem solving, conceptual mathematics understanding, and pedagogy. At the beginning of each class the author gives the teachers a "Do Now" problem that involves confronting an unfamiliar mathematics situation in which the teachers have the opportunity to work in collaborative groups to solve the problems. The class meets for 14 evenings through the fall semester each year, and the teachers solve 13 mathematics problems of this type at the beginning of every class except the first class.

The author first presents the problem to the class on an LCD projector and then allows the teachers time to read through the problem and begin to brainstorm the methods to solve the problem. Next, the teachers self-select to form their own groups in which they can discuss the problem and begin working together to solve the problem. After sufficient time has been given, the professor begins the scaffolding process by asking the right questions at the right time such as the following.

1) What information does the problem provide to you?
2) What information do you need?
3) What question are you trying to answer?
4) What methods do you have that might work in this situation?
5) Have you considered the problem from this particular angle?

While the teachers are working on the problem the professor is walking around the class to provide individual and group support when needed. It is during this process that teachers often raise their hands with excitement and call the professor over to explain their reasoning and solution. It is during this moment that teachers experience the "aha!" moment associated with the bisociative framework. In some cases, the teachers have a mistake in their process or reasoning, and have to return to the problem. In other cases, the correct solution had been found through correct reasoning. In cases in which the correct answers have been found, the professor follows up with a new set of questions, such as 1) What if the conditions were different?; 2) Can you generalize your solution?; and 3) Could you approach the problem differently with another method of solution?

Students are assessed based upon the professor's observation of the teachers' reasoning in the problem solving process. Additionally, selected teachers share their solutions with the class at the end of the session. Teachers also keep two reflective journals in the class. The first is a reflection on what they had learned with emphasis on the problem solving process. Secondly, teachers keep journals on what they observe in their own field observations in elementary school classrooms, which focuses on the application of problem solving in a classroom setting. Teachers express through own reasoning in the problem solving process along with the variables that led to the "aha!" moment for them. There are multiple goals for this process. First, teachers experience the problem solving process as simulation of the process their own students experience in the classroom. This helps teachers better scaffold the problem solving process for their own students. Second, teachers have the opportunity to reflect on their own problem solving and improve upon it. Third, teachers examine the variables that lead to the "aha!"

moment to refine their own thinking and improve their own problem solving. Fourth, teachers have the opportunity to reflect upon improving the conditions that lead to the "aha!" moment for their own students in their own classrooms.

Teachers have expressed an appreciation for the problem solving exercises even if they initially encountered frustration in the process. Teachers have indicated that beginning each class with a problem solving exercise helped them to incorporate problem solving within their own classrooms and better understand both their own and their students' reasoning and thinking.

Reflection and Conclusion

This process at the beginning of each class rests upon a pedagogical problem solving foundation using Vygotsky's social learning, scaffolding, and Zone of Proximal Development (ZPD) along with Koestler's bisociative framework. The combinations of these frameworks provide teachers and students with opportunities to "think about thinking" in their own problem solving processes. Being a reflective practitioner is important for quality teaching just as being a reflective problem solver is important for being a good problem solver. The reflective process in relation to teaching and problem solving helps shape better mathematics teachers for the classroom.

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