

ANALYSING UNIVERSITY STUDENTS' ABILITIES IN MAKING ASSUMPTIONS IN A BALLISTICS MODEL: A CASE STUDY

Sergiy Klymchuk

Auckland University of Technology, New Zealand

Zlatko Jovanoski

Abstract

This paper investigates abilities of two groups of university students in making assumptions in a ballistics model. The first group consisted of postgraduate students majoring in applied mathematics from a New Zealand university and the second group consisted of first-year science students majoring in applied mathematics from an Australian university. The students were asked to make reasonable assumptions in a ballistics model from mechanics. We started talking about stones thrown by catapults in ancient times and proceeded to discussing firing balls from cannons in medieval times and launching projectiles and missiles in recent history. Students' responses to the questionnaire on assumptions are presented and analysed in the paper.

Introduction

There are many diagrams produced by researchers and authors of textbooks that illustrate a mathematical modelling process. The diagrams produced by researchers tend to be rather complex like the one presented in table 1.

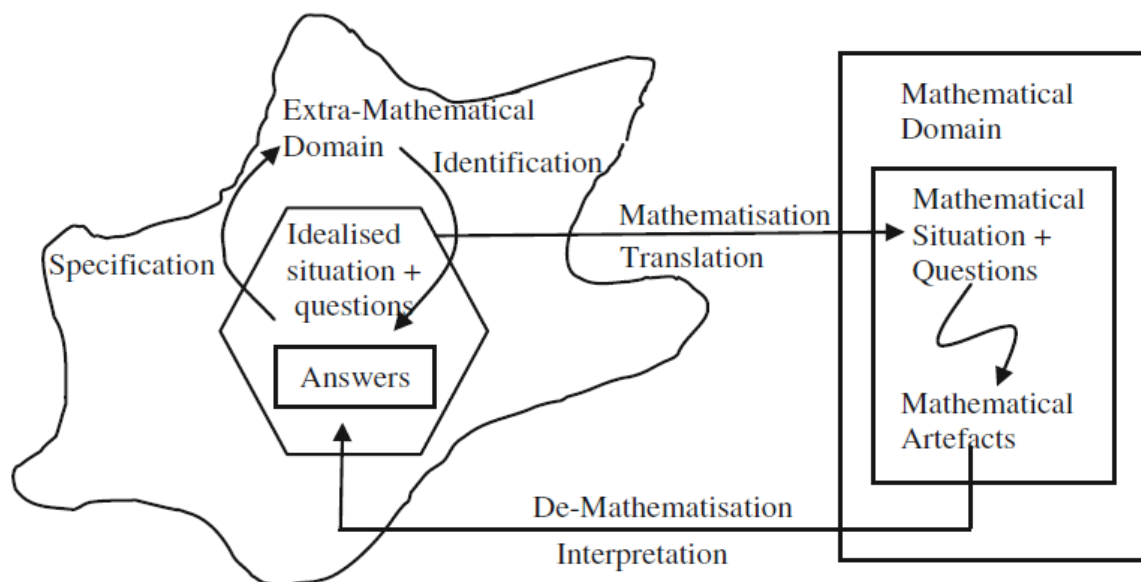


Table 1. Mathematical modelling process by Niss (2010).

Authors of mathematics textbooks tend to produce more practical diagrams that are easier to follow like the one presented in table 2.

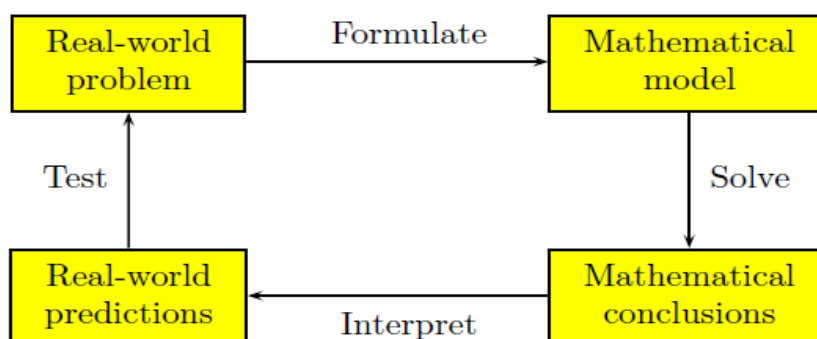


Table 2. Mathematical modelling process by Stewart (2006).

Making assumptions is a paramount and significant part of the formulation (construction/mathematisation) step of a mathematical modelling process. Some diagrams explicitly specify this step like the one presented in table 3.

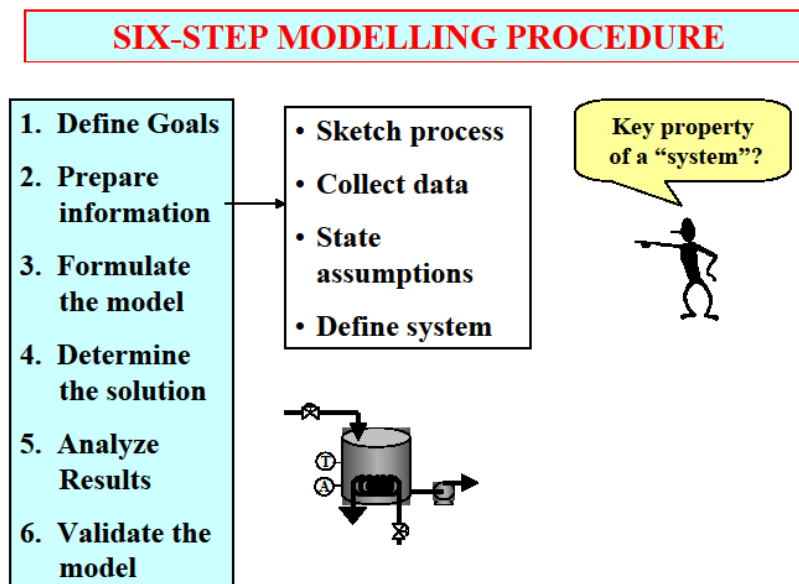


Table 3. Mathematical modelling process by Marlin (1995).

However some researchers express concern that “the role of assumptions in modelling activity has been over-simplified” (Galbraith & Stillman, 2001). Teaching experience shows that often students skip the assumptions stage and rush to “do the sums”. Seino (2005) regards assumptions as the foundation of the proposed model that define the balance between adequacy and complexity of the model: “the setting up of appropriate assumptions can be considered as the most important thing in performing mathematical modelling” (p.664). He proposed ‘the awareness of assumptions’ as a teaching principle to make students understand the importance of setting up assumptions and examine particular assumptions closely. It has a dual role: first, as a bridge to connect the *real world* to the *mathematical world*; second, as promotion of activities that reflect on the formulation step of the mathematical modelling process. Shugan (2007) made more general comments about the importance of assumptions in science: “Virtually all

scholarly research, benefiting from mathematical models or not, begins with both implicit and explicit assumptions...The assumptions are the foundation of proposed models, hypotheses, theories, forecasts, and so on. They dictate which variables to observe, not to observe, and the relationship between them”. (p. 450).

Sometimes assumptions are based on modeller’s intuition and common sense and not supported by calculations or experiments, especially in the education settings. Grigoras (2011) pointed out:

“The emerging hypotheses in students’ work are essentially of different nature than hypotheses in scientific research; like in physics, for example, where stated hypotheses are followed by experiments, and afterwards evaluated, therefore sustained or rejected. Here the students do not check many of these hypotheses, but simply state them and take them as granted. They are either led by intuition or the use of their background knowledge.” (p. 1020).

The purpose of the modelling exercise described below was to check university students’ intuition and common sense on making assumptions in a ballistics model and to illustrate the importance of making appropriate assumptions.

The Study

A Modelling Exercise

Two groups of university students were given a modelling exercise on assumptions in a ballistics model from mechanics. The first group consisted of 4 postgraduate students majoring in applied mathematics from a New Zealand university and the second group consisted of 61 first-year science students majoring in applied mathematics from an Australian university. The exercise was inspired by the book on applied mathematics (Tichonov & Kostomarov, 1984). We started talking about stones thrown by catapults in ancient times and proceeded to discussing firing balls from cannons in medieval times

and launching projectiles and missiles in recent history. In each of the four cases – a stone, ball, projectile and missile – the distance from the starting point to the landing point was given. In addition, the maximum height for a projectile and missile was also given. The students were challenged to think in each case about the appropriateness of the following four assumptions:

- *The Earth is flat;*
- *The Earth is an inertial system;*
- *Air resistance can be ignored;*
- *Acceleration due to gravity is constant.*

It was agreed that a relative error of less than 3% was not significant. Without doing any calculations the students were asked to indicate which of the above assumptions were reasonable and which were not in each of the four cases: a stone, ball, projectile and missile. They were asked to fill the following table putting “+” in the box if the assumption was reasonable and “-” if not.

OBJECT ASSUMPTION	<u>Stone from catapult</u> $l = 100 \text{ m}$	<u>Ball from cannon</u> $l = 1 \text{ km}$	<u>Projectile</u> $h = 20 \text{ km}$ $l = 200 \text{ km}$	<u>Missile</u> $h = 200 \text{ km}$ $l = 8000 \text{ km}$
Earth is flat				
Earth is an inertial system				
Ignore air resistance				
g is constant				

Table 4. Modelling exercise on making assumptions

After completing the modelling exercise the following correct solution was presented and discussed with the students.

OBJECT	<u>Stone from catapult</u> $l = 100 \text{ m}$	<u>Ball from cannon</u> $l = 1 \text{ km}$	<u>Projectile</u> $h = 20 \text{ km}$ $l = 200 \text{ km}$	<u>Missile</u> $h = 200 \text{ km}$ $l = 8000 \text{ km}$
ASSUMPTION				
Earth is flat	+	+	-	-
Earth is an inertial system	+	+	-	-
Ignore air resistance	+(2-3%)	-(15%)	-	+
g is constant	+	+	+	-

Table 5. Solution to the modelling exercise

The suggested correct solution was based on calculations from (Tichonov & Kostomarov, 1984) and consultations with experts. As an example the calculations of the relative errors of the distances in case of a stone and a ball if air resistance is included in the model are presented below:

An object is thrown with the initial velocity v_0 at the angle α to the horizontal.

$$x = t v_0 \cos \alpha; \quad y = t v_0 \sin \alpha - \frac{gt^2}{2}$$

$$y = x \tan \alpha - x^2 \frac{g}{2(v_0)^2 \cos^2 \alpha}$$

$$l = \frac{(v_0)^2}{g} \sin 2\alpha$$

a) Stone from catapult

$$l \approx 100m; \quad v \approx 30m/s$$

Including the air resistance into the model gives:

$$F = \frac{C\pi R^2 \rho v^2}{2}; C - \text{drag coefficient } (\approx 0.15),$$

$$R \approx 0.1m - \text{radius of the stone},$$

$$\rho = 1.3kg/m^3 - \text{density of air}$$

The force due to gravity:

$$P = mg = \frac{4\pi}{3} R^3 \rho_0 g; \quad \rho_0 = 2.3 \times 10^3 kg/m^3 - \text{density of stone}$$

The relative error:

$$\frac{\Delta l}{l} \approx \frac{F}{P} \approx 0.03$$

b) Ball from cannon

$$l \approx 1km, \quad R \approx 0.07m, \quad \rho_0 \approx 7 \times 10^3 kg/m^3, \quad v_0 \approx 100m/s$$

$$\frac{\Delta l}{l} \approx 0.15$$

The Questionnaire

After the discussion of the correct solution the following anonymous questionnaire was given to the students:

Question 1. How many correct assumptions out of 16 did you make?

Question 2. Is common sense and intuition enough to make correct assumptions? Why?

Question 3. Would special knowledge in physics help you to make correct assumptions?

If yes, in which way?

If no, why not?

Question 4. Which case (stone, ball, projectile or missile) was easiest to answer for you?

Why?

Question 5. Which case (stone, ball, projectile or missile) was hardest to answer for you?
Why?

Question 6. Which assumption out of 4 was the most difficult to estimate? Why?

Students Responses to the Questionnaire

The participation in the study was voluntary. The response rate in both groups was 100%. Students' comments in both groups were similar so we combined them. Below are summaries of the 65 students' responses and their typical comments.

Question 1. How many correct assumptions out of 16 did you make?

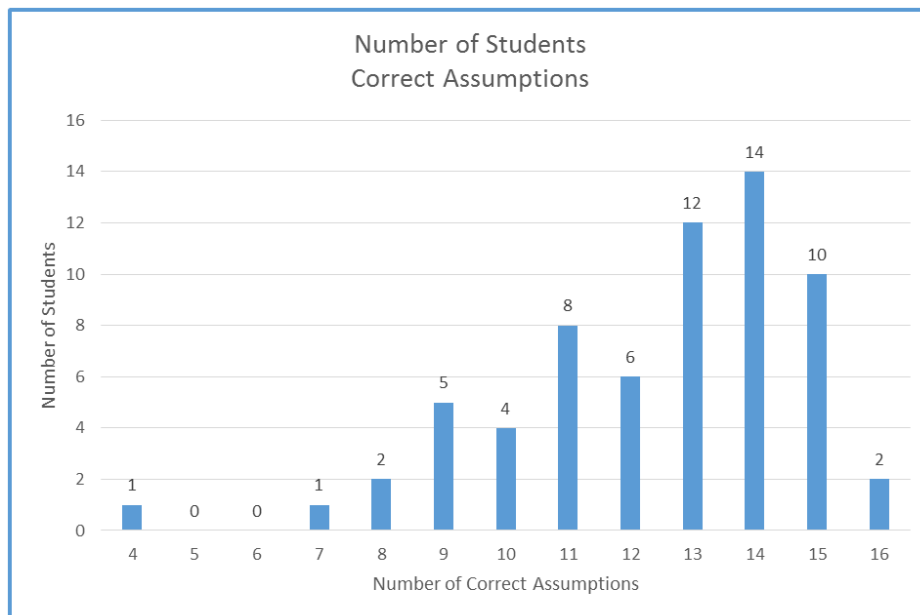


Figure 1. Number of students' correct assumptions

Question 2. Is common sense and intuition enough to make correct assumptions? Why?

Yes – 54%

No – 46%

“Yes, because when you don't quite know the correct answer that applies then common sense will usually provide the most accurate answer”

“Yes, these are simple, practical things we can relate to”

“To a point – but the rules bend in some scenarios and it is these previously learned tricks that are needed”

“No, you can never make assumptions unless it is completely universal knowledge”

Question 3. Would special knowledge in physics help you to make correct assumptions?

If yes, in which way?

If no, why not?

Yes – 86%

No – 14%

“Yes, I used a formula for gravitation in my head to help answer”

“No, physics tends to overcomplicate problems”

Question 4. Which case (stone, ball, projectile or missile) was easiest to answer for you?

Why?

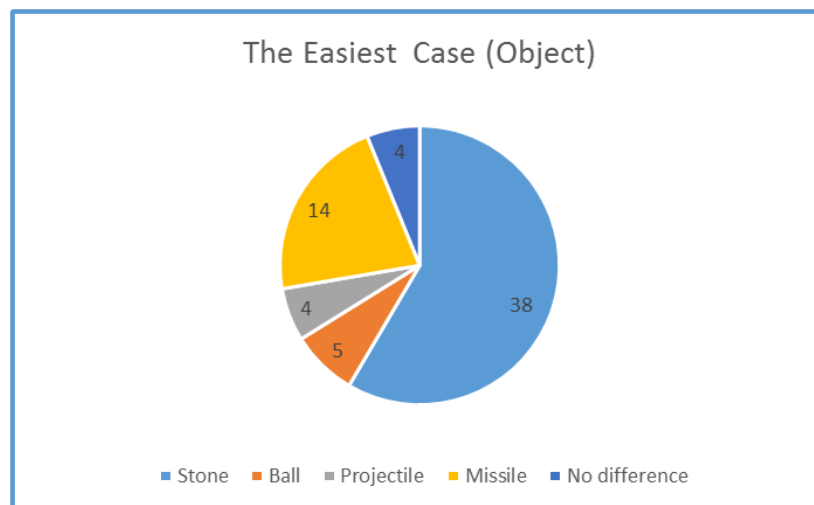


Figure 2. The easiest object out of the four cases.

“Stone, it made sense and was more relevant to my everyday experiences”

“Stone, we have all thrown one once in our life”

Question 5. Which case (stone, ball, projectile or missile) was hardest to answer for you?

Why?

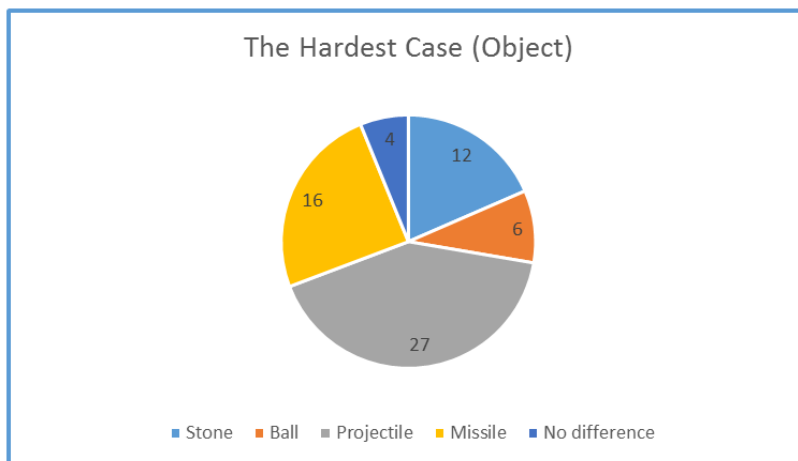


Figure 3. The hardest object out of the four cases.

“Missile, because it needs to consider about projectile force and its movement as it is fired by force that not happen in nature”

“Projectile - too many unknown factors”

“Projectile, it was in the middle range of the other objects”

Question 6. Which assumption out of 4 was the most difficult to estimate? Why?

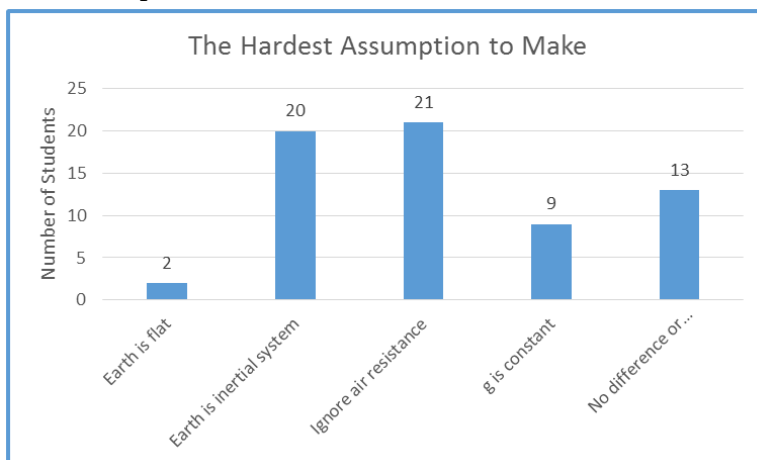


Figure 4. The hardest assumption to make.

“Didn’t know initially what ‘inertial’ meant in this context”

“Ignoring air resistance – it was hard to visualise such scenarios when they do not exist in real life”

Discussion and Conclusions

The distribution of students’ correct assumptions was clearly skewed to the left. Only 2 students out of 65 made all 16 correct assumptions. Another 24 students made 1 or 2 mistakes. So 39 students out of 65 or 60% made 3 or more mistakes. Students’ responses on the role of common sense and intuition in making correct assumptions were polarised – about 50-50. The vast majority of the students (86%) reported that special knowledge in physics helped them to make correct assumptions. Most of the students relied on the practical and familiar experiences in answering the questions about the easiest/hardest object. For most students (63%) the hardest assumptions to estimate across all four objects were “Earth is an inertial system” and “Ignore air resistance” for the reasons related to lack of knowledge (e.g. definition of the inertial system) and everyday experiences (difficult to visualise).

Discussions and observations in class and informal interviews with selected students revealed that the modelling exercise did increase ‘the awareness of assumptions’ (Seino, 2005) and their important role in the mathematical modelling process. In particular, it was consistent with Seino’s claim that it was possible (after the discussion of the correct solution) to develop an awareness of assumptions by making students recognise conditions of the problem as assumptions, by helping students realise how assumptions affect selection of formulas and functions, and whether there are other assumptions to consider. Students found that “what-if” questions were especially helpful in the modelling exercise on making assumptions.

References

Galbraith, P., Stillman, G. (2001). Assumptions and Context: Pursuing their Role in Modelling Activity. In Matos J.F., Blum W., Houston S.K., Carreira S.P. (Eds) *Modelling and Mathematics Education*, Horwood Publishing, 300-310.

Grigoras, R. (2011). Hypothesis and assumptions by modelling – A case study. In M. Pytlak et al (Eds) *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education*, Rzeszów, Poland, pp. 1020-1029.

Marlin, T.E. (1995). *Process Control*. McGraw-Hill. Slides for Chapter 3: Mathematical Modelling Principles. Retrieved from:
[http://www.pc-education.mcmaster.ca/Lecture Slides/Chap 03 Marlin 2002.pdf](http://www.pc-education.mcmaster.ca/Lecture%20Slides/Chap%2003%20Marlin%202002.pdf)

Niss, M. (2010). Modeling a crucial aspect of students' mathematical modelling. In R. Lesh et al (Eds) *Modelling Students' Mathematical Competences*, pp. 43-59. New York: Springer.

Seino, T. (2005). Understanding the role of assumptions in mathematical modelling: analysis of lessons with emphasis on the 'awareness of assumptions'. In P. Clarkson et al. (Eds) *Building Connections: Theory, Research and Practice, Proceedings of the 28th Annual Conference of the Mathematics Education Research Group of Australasia*, Vol. 2, pp. 664–671. Melbourne, Australia: MERGA.

Shugan, S.M. (2007). It's the findings, stupid, not the assumptions. *Marketing Science*, 26(4), 449-459.

Stewart, J. (2006). *Calculus: Concepts & Contexts*. Thomson Brooks/Cole.

Tichonov, A.N. & Kostomarov, D.P. (1984). *Introductory Lectures on Applied Mathematics*. Moscow: Nauka (in Russian).