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# POPULATION and EXPONENTIAL GROWTH INVESTIGATIONS: A MATHEMATICAL LENS

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## Abstract

*Overpopulation has potential to reduce the quality of our lives through finite resource depletion and environmental degradation. While population growth has potential to increase the standard of living through more robust economic growth within a strong and young labor market, the consequences for the incredible growth experienced during the 20th century has led to profound consequences for the planet. The purpose of this article is to present a problem used with teachers in an alternative certification teacher preparation program in a mathematics methods class in order to increase quantitative literacy about the consequences of runaway population growth. The teachers viewed exponential growth through the use of Albert Bartlett's Bacteria Problem.*

## Introduction

Overpopulation is a problem that reduces quality of life locally, as well as globally, as more resources are needed to support growing populations. With predictions of fresh water shortages, possible fossil fuel shortages, and agricultural shortfalls, it seems reasonable to invest in efforts to curtail the population growth through efforts such as family planning and contraception education. The greater the global population consuming limited resources, the fewer resources there are available for individuals. This is particularly true in the consumer driven markets in Western developed nations compared with the less resource intensive lifestyles in the developing world. Additionally, increased human birth rates exacerbate the greenhouse emissions of climate change, which may lead to severe consequences over the next several decades and later in the century. It is arguable that the planet is beyond its carrying capacity, which is the maximum population that can be sustained given current resources and circumstances.

In 1798 Thomas Malthus published *An Essay on the Principle of Population*, a book that addressed the consequences of increased population such as disease and starvation but incorrectly predicted such events in the 19$^{th}$ Century. In 1968 Paul Ehrlich's *The Population Bomb* reexamined the Malthusian argument and further incorrectly predicted impending mass starvation. While Malthus and Ehrlich were wrong in their predictions for near future consequences, their basic premise of the potential consequences of overpopulation are worthy of continued consideration. At the time of writing, the world population is 7.2 billion people (United States Census Bureau, 2015), and the United Nations predicts the world population could reach 9.6 billion by 2050 (United Nations, 2015). While the growth rate of world population growth is slowing compared to the high growth rates in the 20$^{th}$ century, and will continue to do so (United Nations Population Division, 2015), it is expected that there will be nearly about 2.4 billion more people in 2050 than in 2015. This represents a one-third increase in population between 2015 and 2050. Past wrong predictions do not preclude possible catastrophic consequences in the next several decades given finite non-renewable resources necessary for the lifestyles of billions of people and environmental toll this consumption places on the planet. It is quite possible that in the not too distance future technological innovations could reduce our dependencies on limited resources and help us find innovate ways to fuel our lives and the economy just as the Green Revolution affected the agricultural production to meet the demands of an exponentially growing world in the mid-20$^{th}$ century. The Green Revolution represents the use of technology to vastly increase productively in food production, and we may experience something similar with the use of genetically modified foods in the near future, despite some of possible safely risks such foods may pose to human health.

## Education and Population Growth

Education has the potential to reduce a person's fertility and consequently lead to a higher standard of living in a post-industrial economy. Throughout most of history having children was an economic benefit through having children labor on the family farm and the later appalling practice of the harsh conditions of child labor in the factories after the industrial revolution. However, today children are mostly economic liabilities in the modern Western economy and reduce the economic lifestyle of parents even if they enrich parents' lives in non-monetary ways.

Studies have shown a strong negative correlation between the level of female education and the fertility rate, particularly in the developing world (Osili & Long, 2008; Population Reference Bureu, 2007; Singer, 2009). In other words, the more educated a woman is, the fewer children she tends to have. Education in general, in addition to family planning and conception education, has the potential to reduce a society's fertility rate, which creates a better economic life for the potential parents while also putting much less strain on resources and the environment. However, just because something is good for the individual, and in this case good for natural resource availability/consumption and environmental preservation, it may have a negative effect on the future health of the nation's economy.

As a general rule we can assume that as the birthrate falls below replacement level, which is about 2.1 children per woman in much of the developed world and as high as 3.5 children per woman in the developing world (Espenshade, T. J., Guzman, J. C., & Westoff, C. F., 2003), the economy suffers through reduced labor availability. While an *LA Times* editorial emphasized, "Nations cannot indefinitely produce a larger and larger generations to support older ones" ("Defusing the Population Bomb," 2011, para 13), a lower population growth rate means fewer young workers are available to support more retired people through pension and social security programs. Predictions for the economies of Japan, China, Germany, Spain, Italy, Greece, Russia, among other countries, including many in Eastern Europe, range considerably based upon declining fertility in those countries and regions. However, the highest growth rates tend to cluster in Sub-Sahara Africa. It can be assumed that countries experiencing demographic decline will generally, with some exception perhaps, see their national economies move in the same direction. As a general rule the wealthier nations have been experiencing a lower birthrate while developing nations experience stronger demographic growth. In some cases, immigration has boosted a population while local birthrates continued to fall. The United States is somewhat unique in its robust growth rate compared to many other wealthy Organization for Economic Co-operation and Development (OECD) countries. The United States current fertility rate per woman is approximately 1.9, compared to Canada's 1.6, Germany's 1.4, Japan's 1.4, Spain's 1.3, Japan's 1.4, and South Korea's 1.2 (World Bank, 2015).

Here lies the paradox. Having fewer children, or none at all, positively affects the economic state of an individual, mostly. However, reduced national fertility rate negatively impacts the macro-economy. The paradox is similar to examples such as an individual not selling shares in a collapsing stock market for the good of the national economy, even if it is not good for the individual, or not removing money from the bank if there is a suspected bank run looming. Drivers are informed not to fill their gas tanks fully during times of gas shortages, a problem that would exacerbate the shortage. It would benefit the individual to keep a full gas tank, but it would not benefit, but rather harm, the greater society. Having more children is similar to these ideas if we are willing to hold the resources depletions and environmental consequences of having more children in place, aspects that clearly demonstrate advantages of lower birthrates. This makes population growth a complicated case because more growth can be bad for the resource availability and environmental health, but more growth could be good for a country's economic growth.

As developing countries continue to develop and approach levels of Western consumption, we see a shift from a less resource-intensive way of life to a more intensive consumption level as is notable in wealthier OECD countries such as found in Northern America, Western Europe, and the wealthy Australasia nations (Japan, South Korea, Singapore, Australia, New Zealand, etc.). As developing nations advance, citizens have ambitions to live more like citizens living in OECD countries. Given the variables of individual lifestyle benefits, resource consumption, and environmental degradation, it would behoove us to encourage people to have fewer children through family planning and contraception education.

**Population and Exponential Growth in the Mathematics Classroom**

As a mathematics educator, I am interested in developing engaging ways to analyze real-world mathematical problems. I discovered Albert Bartlett's Bacteria Problem to help students understand how exponential growth worked, particularly in comparison to population growth. Albert Bartlett was a professor of physics who spent most of his career at the University of Colorado at Boulder and was concerned about the negative effects of robust population growth. Bartlett argued that given the nature of exponential growth, "sustainable growth" was a contradiction (Bartlett, 1994). He developed the Bacteria Problem to help people understand the power of exponential growth in a population growth context.

Over the course of several semesters in a mathematics methods course for high school teachers I presented Bartlett's problem for the teachers to solve. The teachers came from a cohort of mathematics high school teachers currently enrolled in the New York City Teaching Fellows (NYCTF) alternative certification program. The NYCTF program is a program developed in 2000 in conjunction with The New Teacher Project and the New York City Department of Education to address the teacher shortages in New York City schools, particularly high-need schools (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007; NYCTF, 2015).

Bartlett outlined his Bacteria Problem in an article called "Forgotten Fundamentals of the Energy Crisis" (n.d.). The first part of Bartlett's (n.d.) problem follows.

> Bacteria grow by division so that 1 bacterium becomes 2, the 2 divide to give 4, the 4 divide to give 8, etc. Consider a hypothetical strain of bacteria for which this division time is 1 minute. The number of bacteria thus grows exponentially with a doubling time of 1 minute. One bacterium is put in a bottle at 11:00 a.m. and it is observed that the bottle is full of bacteria at 12:00 noon. Here is a simple

example of exponential growth in a finite environment. This is mathematically identical to the case of the exponentially growing consumption of our finite resources of fossil fuels. Keep this in mind as you ponder three questions about the bacteria: (1) When was the bottle half-full?

Let's stop there and examine Bartlett's first question. Many students and teachers presented with the problem suggest that at 11:30 the bottle would be half full. However, this represents a misunderstanding of how exponential growth works, particularly doubling time in this case. The correct answer, often to the surprise of students and teachers, is 11:59. Consider that the population doubles every minute to help understand how this happens. Working backwards allows us to see that one minute prior to noon the bottle was half full and subsequently double at noon to fill the entire bottle.

Bartlett (n.d.) continued with question two: "(2) If you were an average bacterium in the bottle, at what time would you first realize that you were running out of space?" Bartlett claimed there was no correct answer to this question. He did indicate that at 11:55 only 3% of the bottle is full, which leaves 97% of the bottle free for bacteria to continue growing. The problem with thinking there is no problem at 11:55 is that by 11:57 the bottle is 12.5% full, by 11:58 the bottle is 25% full, and as we now know by 11:59 the bottle is half full.

Bartlett's (n.d.) final question is as follows.

> Suppose that at 11:58 a.m. some farsighted bacteria realize that they are running out of space and consequently, with a great expenditure of effort and funds, they launch a search for new bottles. They look offshore on the outer continental shelf and in the Arctic, and at 11:59 a.m. they discover three new empty bottles. Great sighs of relief come from all the worried bacteria, because this magnificent discovery is three times the number of bottles that had hitherto been known. The discovery quadruples the total space resource known to the bacteria. Surely this will solve the problem so that the bacteria can be self-sufficient in space. The bacterial "Project Independence" must now have achieved its goal. (3) How long can the bacterial growth continue if the total space resources are quadrupled?

Unfortunately this does not give the bacteria considerably more time as might have been expected. The answer is it only gives the bacteria two additional minutes. At noon we know the bottle is full, by 12:01 the bacteria fill the second bottle, and by 12:02 they double again and fill all four bottles.

While well under half of the NYCTF teachers were able to answer the three questions correctly all of them indicated that before they thought about the questions they had not anticipated how quickly a population grows in the context of exponential growth. All NYCTF teachers indicated surprise by the results.

**Conclusion**

While it is possible to argue in favor of robust population growth in order to improve economic productively, robust pension and social security systems, and national Gross Domestic Product (GDP), it can also be argued that finite resource consumption, environmental degradation and climate change, and individual economic benefits should be seriously considered in educating young people in the benefits and consequences of curtailing population growth. Teachers given the Bacterial Problem were able to adapt their expectations about exponential growth and connect this process to a real-world problem such as population growth against finite natural resources, a lesson they could bring to the classroom. Better mathematical literacy focused on a basic concept such as exponential growth has potential to give young people the tools they need to make informed decisions for their own life choices. Frankly, it is their future and they will need to decide what that future should be for themselves and their world.

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