

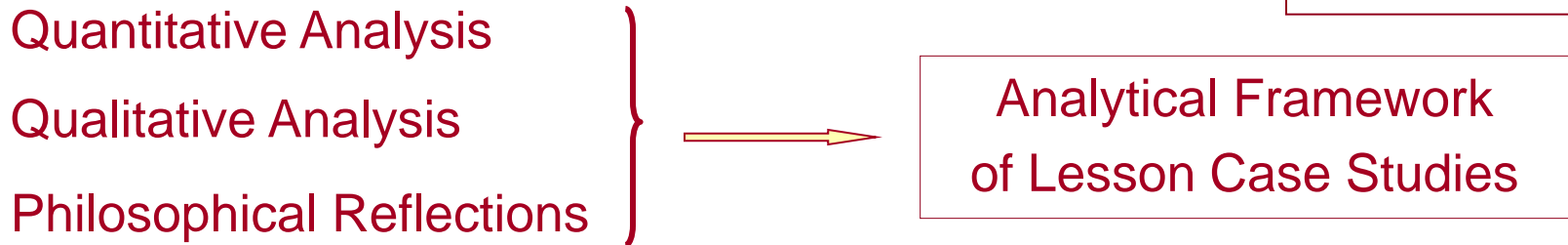
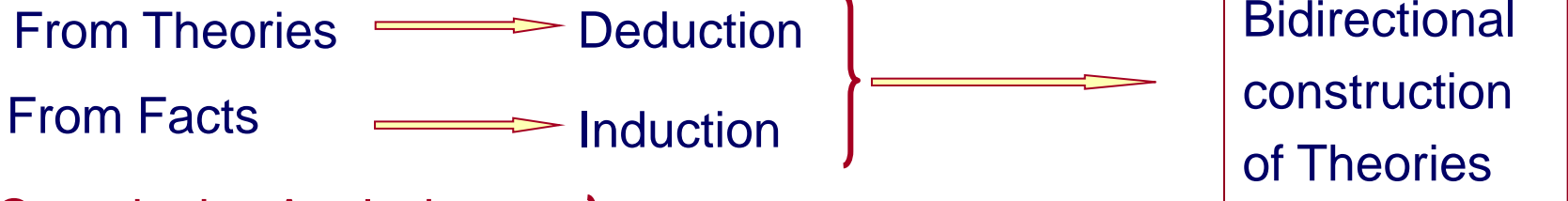
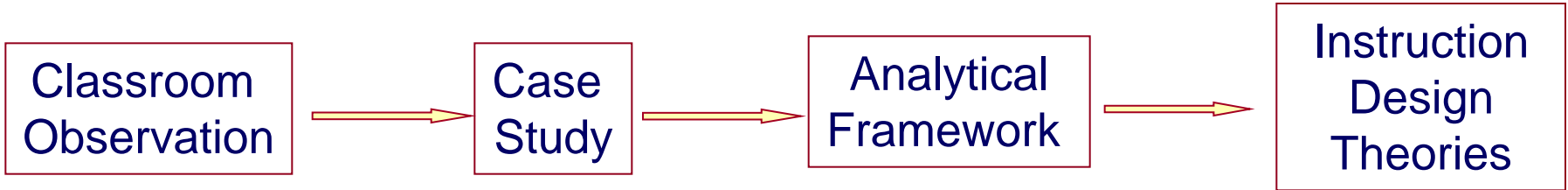
# **Constructing Own-Featured Mathematics Instruction Design Theories**

**Rongbao Tu**

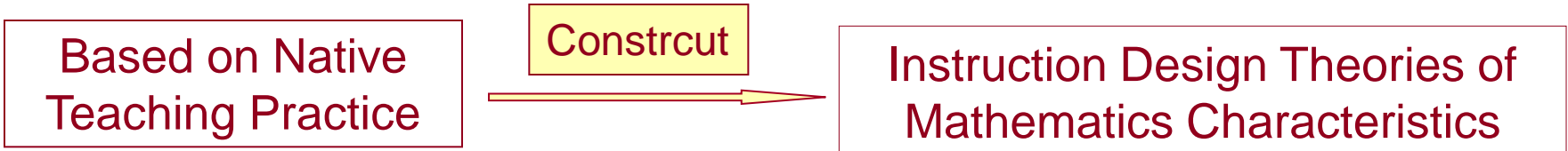
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# Origin of the Theories——Case Study of lessons



Be Short of Mathematics Teaching Theories of Own Features All This Time.



# Fundamental Idea

# What is the “*Scientific Development View in Education*”

The Overall Goal of Education is “Human Development”

Core Concept of Scientific Development View in Education

—— What kind of people we are going to educate !

The Education Objectives in China:

Cultivating socialism worker with all-round development of

morality, intelligence physique and aesthetic. ——Lays the Foundation

Worker——Creating value for society

and obtain benefit for own through work.

Socialism——Human’s society property and responsibility :

state and nation; human and history; now and future.

Condition——all-round development and Sustainable development

**The Scientific Development View in Education ① all-round Development**

**Condition—— (lifelong learning) Sustainable development**

# ***“Scientific Development View in Education”***

## **② Sustainable development**

- **Promote Students' Learning Enthusiasm**

Develop a positive learning disposition to explore the world  
with curiosity and interests.

The teacher should care, cultivate and encourage students' learning  
enthusiasm.

- **Teach Students How to Learn**

Master effective learning strategies and can obtain knowledge independently  
Master research method and learn from unknown to the core of problem,  
until the final construction and solving.

- **Develop Students' Cognitive Abilities——**

Ability to explore all kinds of things of the world.

scientific perspectives, creativity, imagination, insight, judgment,

foresee ability.

# “Scientific Development View in Education”

## ③ Values of Math Education

Core Question——How to contribute to the all-round  
Development and Sustainable Development

Value of math——Develop Human Ability of Thinking ——  
Make Human Became Smarter

- The ultimate goal of education is not to master knowledge  
but to develop cognitive ability.
- The knowledge may be forgotten, the left is education. ——Einstein
- What is the left education? The left is human’s cognitive ability.  
Bacon said: The knowledge is power.  
Einstein said: The imagination is more important than knowledge.
- Is knowledge important? Of course.  
Knowledge——common sense of life and foundation of professional development  
Knowledge——the only way to acquire cognitive ability. If one have no knowledge,  
the development of cognitive ability must be fall through.

But, the development of cognitive ability is more important than master knowledge.

Math has special powers which can achieve the development of cognitive ability  
——develop thinking and make human become more intelligent.

# The Double Corresponding Theories in Math Instruction

- The dynamic corresponding theory of teaching and learning  
—adapting instruction methods based on fulfilling students needs of learning.
- The dynamic corresponding theory of teaching and essence of math contents  
—overcome the tendencies of ‘far away mathematics’  
in teacher education and ‘activities far away mah’ in classroom.

- Corresponding teaching contents with mathematics knowledge.
- Corresponding knowledge structure of math with instruction
- Corresponding instructional situation with math essence
- Corresponding the thinking way of mathematical with instruction
- Corresponding educational research methods with math research methods
- Corresponding instructional expression with math expression
- Holding core math concepts throughout the instruction,  
and looking for the universal methods in math teaching.

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# **Mathematics Instruction Design Theories**

**——with thinking activities as core**

# 1. Teaching Students Learning to thinking

Instruction—Teaching students to ‘learn’ .

Teaching students ‘what to learn’?

Learning knowledge or thinking?

Learn how to think through gaining knowledge

◎ Proposing questions and topics

◎ Constructing new concepts and methods

◎ Learning how to find methods of problem-solving

◎ Learning universal method of math research

Learning  
to thinking

Teaching students ‘how to learn’?

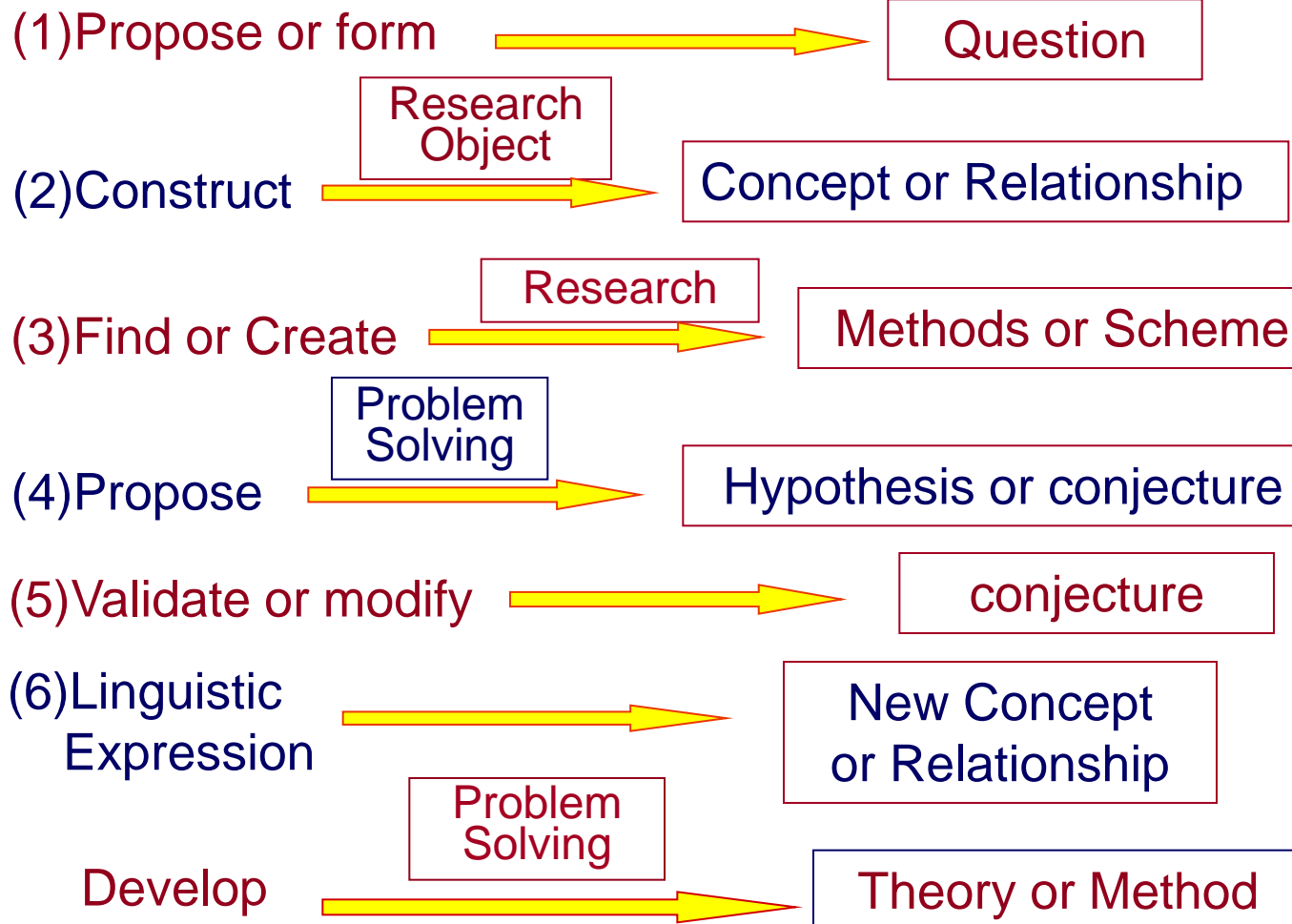
‘how to learn’—Using universal methods of  
mathematics research to learn mathematics.

learning to swim by swimming; learning by doing;

learning methods by using methods.

## 2. Using universal methods of math research to learn mathematics

### The universal methods of mathematics research



Forming Question;  
Constructing concept ;  
Finding Methods;  
Proposing Hypothesis;  
Validating Conjecture;  
Validating conjecture .

### 3. Promote instruction through problem structure

American Mathematician Halmos:

The question is the heart of mathematics.

- All the concepts, formulas, and methods were invented based on the needs of problem solving.
- When the existing concepts or methods cannot be used to solve new questions, we need to create new concepts or methods.

In the beginning of each lesson we need to propose a question, then solve it.

- Transfer the new teaching as a process of solve a new problem
- Transfer the learning activities as creative work
- Facilitate students to develop their consciousness of innovation creativity

**Teacher can propose questions, however,  
it is better to guide students oneself to propose questions**

Forming the “Problem→Solution→Problem→Solution”  
to promote the process of instruction

## 4. Creating situation, proposing problem

**Brubaker: The highest criteria for teaching art is proposing problem by students themselves.**

- But in mathematics learning activities,  
it is impossible to propose problems completely dependent students.  
It must depend on inspiration of teacher.
- **How to inspire students?** **Creating situation and proposing problem**  
—— This is the most effective method.
- Creating situation is the precondition and foundation;
- Proposing problem is the objectives and core.
- The basic criteria——
- Obviously, simplify and understandable, based on life, close to students, benefits to reveal mathematics essence.
- How to create situation?——
  - .Citing with old and new, .mathematics problem, .comment exercises,
  - .actual life, .economic manufacture, .Interesting news and historical narrative,
  - .supposition and simulation, .flaw and suspense, .construct through skill.

## 5. Inquiry from nonexistence to pass into existence

- What is exploring?
  - “from nonexistence to pass into existence”
  - If the things are clear at a glance, so they need not be explored.
- A process of development gradually from do not understand to understand.
- There are two ways of mathematics inquiry teaching
- Guided Inquiry——Inquiry be guided by clues
  - The strategies of teacher-guided investigation
  - The strategies of students investigate actively
- Discovery Inquiry——Inquiry by searching for clue
  - Students investigated independently

# Inquiry Teaching Strategies and Learning Ways

## Strategies of guided inquiry by teacher

1. Designing situation
2. Summary and evaluation
3. Textuality of key questions
4. Inspiration through cue words
5. Guiding via dynamic figures
6. Directing via body languages
7. Providing real life examples  
(one example surpass a dozen  
explanation)
8. Teaching based on students  
thinking (improve the occasion)

## Ways of students investigating actively

1. Proposing questions
2. Developing problem (example studies)
3. Guessing essential features  
(extracting common features,  
meaning studies)
4. Finding methods of problem solving
5. Validation and argument  
(rational thinking)
6. Summarization and induction
7. Reflection and extensions
8. Finding the theme  
(n-student repeated, over and over again)

## 6. Using heuristic cue words to guide students

- guided inquiry——Main way of inquiry math teaching

It is impossible for students to investigate math knowledge

without teacher's inspiration and guidance,

because math knowledge are abstract and formalized thinking materials.

- How to inspire students? —— hints and hidden hints.

Teacher should provide proper cue word for students to offer necessary hints or hidden hints.

Students get these hints through their own thinking activities.

- The way of offering hints —use heuristic cue words to ask

Three cue words: Meta-cognitive , Methodological, Cognitive

- The best teaching method is heuristic teaching.

The core of heuristic teaching is “question”——heuristic cue words

——find the answers by students themselves

The investigation should be all students' investigation.

—— How to for all students?

# 7. For all students, Inspiration from far and near.

Guiding students of different levels——

‘from far and near, asking depend on different levels’

- heuristic cue words——hints by ask, guides students finding clues.

The directivity of cue words should be not clearly. If it is clearly, students need not thinking.

The further we fall from the goal, the directivity is more hidden  
——It has more challenging of thinking.

The more it is attained, the directivity is more clearly  
——It has less challenging of thinking.

- Inspiration from far to near:

Hints far from  
the target

Transit

Hints close to  
the target

From far and near, asking depend on different levels,

every student has the chance to contribute.

Only asking not answering, if there is need to answer, earlier hidden and later clearly,  
avoiding the knower tell the know-nothing.

Thinking independently, students in different levels have attained different inspiration,  
everyone have a chance to progress.

Lastly, students can use the heuristic cue words to guide themselves.

So we can reach the goal that ‘teaching how to learn’, ‘teaching how to think’.

## 8. Carrying through 'reflective teaching'

- Reviewing, questioning, questioning closely, asking in retort  
——the common means of mathematics teaching.
- Reviewing the moment and past things;  
Reviewing the process and method——  
Is there any enlightenment? What result can I attain? Can I use it?  
Is there any laws? What results can I summarize? Any questions can I propose?
- Questioning:  
Why I can think so? Is it really so? Is it rigorous to solve the problem?
- Questioning closely:  
Where the ideas come to me? Why I think so? Are there different ideas?  
How can I think the next? Which benefits can I do it like this?
- Asking in retort:  
Do I have any questions? Can I improve it? Are there any flaws?  
If there are some flaws, what should be done next? Can I improve them?  
How to cope with them? If so(proposing counterexample), how to do?

## 9. Inductive in advance, Deduction follow-up

The basic ways of mathematics thinking activities——induction and deduction.

**Einstein:** The two pillars of western science development are induction and deduction.

**Zhenning Yang:** I have been learned deduction in China and learned induction in USA.

The traditional mathematics teaching in China

“valued deduction and despised induction in general”.

**Induction**——finding and proposing questions——providing invention and creation.

**Deduction**——proving conclusion——providing scientific and rigor.

**Seeking the right balance between induction and deduction——  
induction in advance and deduction follow-up.**

(1)special conditions

——common induction——guessing

(2)specific conditions

——abstract induction——guessing

(3)many to one

——empirical induction——guessing

(4)plausible reasoning

—— natural logic——guessing

Deduction

Rational Thinking

Proving and retorting

# 10. Understand the problem in solving problemteaching

How to think for a new question ——Where to start?

How to find a solution from nonexistence to pass into existence?

——Exploring from the existing to nonexistence.

- **“Understand the problem”**  
—— **The first step of problem solving learning.**
- Above all we must understand the problem in problem solving,  
but the step often be neglected by students.
- A good problem solver use a half time to understand the problem,  
and only use a half time to solve the problem.
- **The reason why students do not pay attention on understand the problem**
  - Have no sense to understand the problem
  - Have no habits to understand the problem
  - Have no methods to understand the problem

**Where to start? How to understand the problem?**

# ① Inspirational Cue Words about start to solve problems

- 1) What's the question? What's it want to solve or prove?  
—— Which category is the question? —— What need to be solved?
- 2) Which materials we have now? ——The conditions of the problem suppose.
- 3) Which tools we have now? ——The students have learned concepts, propositions, formulas and methods.
- 4) Do we lack any materials?
- 5) How to use the existing conditions and tools? From existing tools and materials.
- 6) Are there any conditions we missed? How to use them?

Actually, these are our eternal thoughts.

★ These thinking are not the browsing simply of words and sweeping over quickly inwardly, but the investigation thoroughly.

It needs to investigate significances, properties of every object and relations between objects.

★ Especially, we need to investigate how to transform the significances and relations?

★ These thinking are not proceeding isolate, but running through the above-mentioned all thinking processes.

★ They are basic methods, We can think step-by-step.

**How to investigate thoroughly?**

**How to transform?**

## ② Inspiration Cue Words about understand the problem

How to investigate thoroughly? Investigating the meaning of questions thoroughly.

How to transform? Transforming the forms of questions.

What is it? How to represent it? And other representation?

What properties it has? How to represent it? And other representation?

Are there any relations between them? How to represent it? And other representation?

Is it any relations about other questions? Can we use the relations?

What can we deduct from the conditions? And other results?

Are there any relations between the conclusions in the process of problem solving?

How to use them?

**It**——every sentence, term, concept, relationship, formula, symbol, the of the symbol, superscript and subscript of symbol, graph, dot, line and face of graph, and so on.

Teachers need effective strategies to teach students how to solve questions.

Inspirational cue words offered effective strategies to organize these thinking processes.

# **‘Teacher-main character of guide, students-subject of activities’**

**Rousseau:** The question lies not in telling the truth to him,  
but rather in what teach one how to discover the truth.

The teacher must really lead students to ‘re-discover’ and ‘re-create’ .

The teacher should use the way of discover and create to teach students

‘what to learn’ and ‘how to learn’.

As a teaching guide, teacher can teach students ‘what to learn’ and ‘how to learn’.

The character of teacher is ‘teaching guide’-----inspiration and guiding, not to offer answer.

**The teacher should play the leading role. ——— Teacher is the main character of guide.**

The teacher should put students in location of exploration and guide them.

The true exploration must depend on students themselves.

Students discover and explore the new things depend on themselves.

**The students should act as main body of exploration.**

- Students active mainly while teacher guiding subsidiary;
- Students’ thinking independently mainly while teamwork subsidiary;
- Students’ internal thinking mainly while external operation subsidiary;
- Students’ activity mainly while technical support subsidiary.

# Constructing Own-Featured Mathematics Instruction Design Theories

## 1. The theory of teaching Students How to Think

In the process of learning knowledge.

Students learning how to propose questions, constructing,

finding and universal method.

## 2. The theory of using universal methods of math research to learn math

Forming question;

Constructing concept;

Finding methods;

Proposing hypothesis;

Validating conjecture;

Linguistic Expression

## 3. The theory of promote instruction through problem structure

Each lesson should be problem-based.

Solving a new problem teaching-based.

Problem structuring.

## 4. The theory of creating situation, proposing problem

The basic criteria and methods of create situation.

## 5. The theory of inquiry from nonexistence to pass into existence

Guided Inquiry and Discovery Inquiry.

The strategies of teacher-guided and the ways of students' inquiry.

# Constructing Own-Featured Mathematics Instruction Design Theories

6. The theory of using heuristic cue words to guide students  
Cognitive cue words,  
Meta-cognitive cue words,  
Methodological cue words.
7. The theory of for all students, Inspiration from far and near  
The further fall from the goal transition the nearer.
8. The theory of carrying through 'reflective teaching'  
Reviewing, questioning, questioning closely, asking in retort
9. The theory of inductive in advance, deduction follow-up  
Seeking the right balance between induction and deduction
10. The theory of culling ideas as the core  
In solving problem teaching,  
students should learn how to solve the new problem.

**Thank you all**